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Comparing Bargaining Solutions in the Shadow of Conflict: How Norms against Threats Can Have Real Effects[1]

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In many economic environments agents make costly and irreversible investments (in "guns") that may enhance their respective threat payoffs but also shrink the utility possibilities set. In such settings, with variable threats and a variable utility possibilities set, it becomes possible to rank different bargaining solutions in terms of efficiency. We compare bargaining solutions within a class in which the influence of the threat point on the bargaining outcome varies across solutions. Under symmetry, we find that the solution in which the threat point is least influential—the equal sacrifice solution—Pareto-dominates the other solutions. Since the equal sacrifice solution puts the least weight on the threat point, norms against threats (that can be seen in many seemingly rhetorical pronouncements in adversarial relations) can mitigate some of the costs of conflict and therefore have efficiency-enhancing effects. *Journal of Economic Literature* Classification Numbers: C72, C78, D30, D70, D72, K42.

Key Words: contests; bargaining; division rules; variable utility possibilities set; variable threat payoffs.

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> (1) Aspiring sincerely to an international peace based on justice and order, the Japanese people forever renounce war as a sovereign right of the nation and threat or use of force as means of settling international disputes.
>
> (2) In order to accomplish the aim of the preceding paragraph, land, sea, and air forces, as well as other war potential, will never be maintained.
>
> (3) The right of aggression of the state will not be recognized.
>
> Article 9, Chapter II, *Japanese Constitution*

> It would be helpful now if Beijing were to respond in kind [to overtures from Taiwan]. Instead of threatening military action at every opportunity and criticizing American and Japanese support for Taiwan, it should recognize that peace and stability is to the benefit of all, and that the international community is as keen to prevent Taiwanese defiance as sabre-rattling from Beijing.
>
> Editorial, *South China Morning Post*, April 3, 1996

> Threats by force and its use, as well as sanctions, can only complicate existing problems and create new ones, the [joint] statement [by Yeltsin and Milosevic], carried by the state agency Tanjung, said.
>
> International News, *Agence France Presse*, June 18, 1998

> [Egypt and Saudi Arabia] urged the Israeli government to respond to the Arab and international peaceful mediations and refrain from the policy of force and threat and give the peoples of the region and the world the chance to realize their aspirations for the creation of a new Middle East where peace, stability and cooperation would prevail and the cycle of war and violence would disappear for good.
>
> *BBC Summary of World Broadcasts*, June 26, 1998

1. INTRODUCTION

If all countries were to abide by Article 9 of the Japanese constitution the world would be peaceful and more productive. Yet, although Japan does not abide by the letter of that article of its constitution (it introduced, for example, "self-defense" forces in 1956), there still appears to be some domestic resistance to Japan becoming a military power commensurate to its economic capabilities. Few Japanese, nevertheless, would support complete disarmament. Similarly, the news passages quoted above point to a reluctance to use force and make threats, but the agents involved in each quote are clearly pragmatic enough to understand that one must prepare for the eventuality that the use of force may be necessary. In addition, the quotes are very much representative of the attitude that can be found in the press every day, an attitude that is ambivalent toward arming and conflict: While there appears to be a norm against making explicit threats, everybody arms. Does this attitude simply reflect hypocrisy or, more charitably, a considerable capacity in human beings to rationalize and justify their own actions? Or does it perhaps also serve a useful purpose in controlling some undesirable outcomes of the self-seeking behavior of individuals and groups? While we remain agnostic about the former question, we suggest an affirmative answer to the latter for settings in which parties cannot write enforceable contracts on their level of arming. As long as the use of force, even though possibly very remote, can be the ultimate arbiter of disputes, it may well be advantageous to the interested parties to abide by norms, by rules of division, that are as insensitive as possible to threats. In other words, norms against threats may have real effects.

These real effects are related to the expenditure of resources that become unavailable for production and consumption in environments with incomplete contracting. Not just in international relations but in many other interactive settings, like litigation or political and business negotiations, agreements rarely conclude without first going through phases of (ex ante noncontractible) actions that attempt to improve each party's bargaining position. Many of these actions require the use of real resources: legal costs in the case of litigation, scarce staff resources, and other lobbying and influence costs in the case of political and business negotiations, or sizable defense expenditures by countries trying to settle even minor territorial disputes. These costs clearly can constitute a large fraction of the ex ante benefits to reaching an agreement. In some cases, as in those of total war, the costs can even dwarf any immediately apparent benefits. In some others, when the two parties perfectly trust one another, they can be avoided altogether. In most intermediate cases, however, some costs will be incurred, but they can vary widely depending on each party's expectations about the conventions or rules they are to follow to reach an agreement. Rules of negotiation—"norms"—that are insensitive to costly actions by the interested parties can be expected to yield lower overall costs, whereas rules that are more sensitive to such actions would yield higher costs. But the characteristics of rules and norms that induce lower costs appear to be unknown and, as far as we know, remain unexplored.[2]

The environment we consider involves bargaining. The role of norms is taken by division rules generated by alternative bargaining solutions and the role of threats by the disagreement payoffs in a bargaining problem (for surveys of axiomatic bargaining theory, see Roth [17], Moulin [11], and Thomson, [23]; for other overviews of bargaining, see Harsanyi [6], and Osborne and Rubinstein [15]). As we compare bargaining solutions in terms of efficiency, the reader will immediately notice that, in traditional bargaining theory, solutions cannot be Pareto ranked because typically both the threat point and the utility possibilities set are fixed—when one

[2] Recently, there have been important attempts to understand how norms emerge by the use of evolutionary models (see, for example, Samuelson [19], Skyrms [22], Young [24], and the references therein).

side does better with one bargaining solution, the other does worse. Our work differs in that both the utility possibilities set and the threat point are variable and endogenous to the actions of the two parties.

In particular, we examine an economic environment in which, first, each party makes costly and irreversible, up-front investments (in "guns") that enhance their respective threat payoffs but also shrink the utility possibilities set. In a second phase, after guns have been chosen and the induced threat payoffs and utility possibilities set determined, the two parties divide the benefits according to a predetermined bargaining solution. Different bargaining solutions in this second phase induce different division rules and different (equilibrium) investments in guns in the first phase, and therefore different payoffs to the two parties.[3] Our goal is to compare the effect of different bargaining solutions on these payoffs.

We compare bargaining solutions within a class that allows for different degrees of sensitivity to two points in the payoff space of a bargaining problem the *threat* point and the *ideal* point (please see Fig. 1). Although there is a continuum of solutions that could be considered, for simplicity and tractability we compare the two polar opposites within this class, the *split-the-surplus* and the *equal sacrifice* solutions, along with the well-known *Kalai–Smorodinsky* solution that is in the middle.[4] The split-the-surplus (or egalitarian) solution (Kalai [9], Roth [18]) divides the surplus over the threat point equally between the agents. The equal sacrifice solution (Aumann and Maschler [2], and O'Neill [14]) equalizes each agent's sacrifice from his or her maximum feasible payoff net of the threat payoff, whereas the Kalai–Smorodinsky solution (Kalai and Smorodinsky [10])

[3] As mentioned earlier, the bargaining problem is typically treated under the assumptions of a fixed threat point and a fixed utility possibilities set. In Nash's variable threats model (Nash [12]), the threat point is endogenous but the utility possibilities set is not. Here we examine an environment in which both elements of the bargaining problem (i.e., the threat point and the utility possibilities set) are variable. As in Nash's variable threats model, we follow a hybrid of the noncooperative and axiomatic approaches. Such an approach has been used by Grossman and Hart [5] in their approach to the theory of the firm, by Skaperdas [20] in studying the effects of risk attitudes on conflict and settlement, and by Skaperdas and Syropoulos [21] in an economic environment similar to that of this paper, but which has different concerns and does not compare bargaining solutions. Esteban and Ray [4] examine a model of conflict that employs a contest success function as we do in the case of our threat payoffs.

[4] The main reason we did not consider the Nash bargaining solution (Nash [12]) is because it is difficult to compare it to the other solutions analytically, owing to its very different (nonalgebraic) properties. However, in simulations we found that it resembles the Kalai–Smorodinsky solution more than the other two solutions. Since the two other solutions behave very differently, with the Kalai–Smorodinsky solution falling in the middle, this omission does not appear to affect the general import of our findings. However, we do plan to look more closely to the comparison of the two well-known solutions, the Nash and the Kalai–Smorodinsky solutions, in future work.

lies at the intersection of the Pareto frontier and the line connecting the ideal and threat points.[5]

Under symmetry, a clear ranking of the three solutions emerges: the equal sacrifice solution Pareto-dominates the Kalai–Smorodinsky solution which in turn dominates the split-the-surplus solution. Although a clear ranking is not in general possible under conditions of asymmetry, we find that when agents are sufficiently similar, or the stakes over the contested resource are high enough, the equal sacrifice solution still dominates; in contrast, when the agents are very different or the stakes are low we do not observe the emergence of a Pareto-dominant solution.

The superiority of equal sacrifice solution originates in the lesser sensitivity of the division rule it generates to agents' threat payoffs and the greater sensitivity to their ideal payoffs—the threat payoffs play only an indirect role in the outcome through their impact on the agents' ideal payoffs. That is, guns under the equal sacrifice solution have the direct cost of diminishing one's (equilibrium) payoff by reducing his or her ideal payoff, while they have only an indirect benefit through the reduction of the opponent's ideal (and, thus, equilibrium) payoff. By contrast, the division rule under the split-the-surplus solution is directly sensitive to the threat point and only indirectly so to the location of the ideal point. The cost of guns of one agent under the split-the-surplus rule is essentially borne by both agents through the reduction of the total surplus, whereas the benefit of guns is enjoyed directly and solely by the agent who has invested in them through the greater claim he or she has on the total surplus. Thus, the overall tendency is for the equal sacrifice solution to induce less investment in guns than the split-the-surplus solution, as well as compared to the Kalai–Smorodinsky solution, which falls in between the two others in terms of its relative sensitivity to the threat and ideal points. Only when the two agents are sufficiently dissimilar and when the contested resource is unimportant is this tendency confounded by the other characteristics of the solutions we examine.

The remainder of the paper is as follows. Section 2 defines the economic environment within which we conduct our analysis. Section 3 compares the bargaining solution concepts mentioned above. Section 4 concludes.

[5] The split-the-surplus and equal sacrifice solutions lead to strikingly divergent outcomes in sufficiently asymmetric setups. The Kalai–Smorodinsky solution is typically closer to the Nash solution (and to several other solution concepts) than the split-the-surplus and the equal sacrifice solutions are. This is one reason for limiting our comparison to the three solutions mentioned above. Another reason is pragmatic: comparison of these solutions is analytically more tractable. Perhaps more importantly though this is an appealing selection of solution concepts because, by virtue of the fact that they belong to the same class of bargaining solutions, they help identify the relationship that seems to exist between efficiency and the reliance of division rules on threats.

2. THE ECONOMIC ENVIRONMENT

Two agents, labeled 1 and 2, have ownership claims to T_0 units of a productive resource, labeled "land" (T for "territory"). They can both use the contested resource, along with other factor inputs they possess, to produce a final consumption good. To establish ownership claims over the contested resource the agents have two options. They can either engage in open conflict, with an agent's probability of winning being determined by the quantities of guns both agents possess, or they can divide T_0 according to some focal division rule to which both agents adhere. Importantly, and as it becomes clear below, division of T_0 can only be conducted in the "shadow of conflict." Conflict and guns do not have to be interpreted literally here. For example, the model could be applied to other areas, including law and economics, with "guns" standing for expenditures on litigation and "conflict" representing going to court (see Hirshleifer and Osborne [8] for such a dedicated application).

Each agent $i(=1, 2)$ is endowed with T_i units of land and R_i units of human capital; in contrast to T_0, these endowments are secure and inalienable. The human capital R_i can be allocated into two different activities: "guns" (G) and productive "labor" (L). Labor is used with land to produce "butter." For simplicity we choose technology and units so that one unit of human capital can produce one gun or be converted into one unit of labor. Consequently, every agent i faces the following resource constraint

$$G_i + L_i = R_i, \qquad \forall i = 1, 2. \tag{1}$$

Each agent i's technology for butter is described by the function $F^i \equiv F(T_i, L_i)$ $(i = 1, 2)$ which satisfies Assumption 1 below. (In this assumption and elsewhere, subscripts that involve T, L, or G denote partial derivatives with respect to these variables.)

ASSUMPTION 1. $F(T_i, L_i)$ is twice continuously differentiable, increasing, and strictly concave in T_i and L_i, with F_L/F_T being nondecreasing in T_i and non-increasing in L_i $(i = 1, 2)$.

It is straightforward to verify that the last requirement in Assumption 1 implies that $F(\cdot)$ is a quasi-concave function. All homothetic functions that satisfy the other parts of Assumption 1 have this property but, of course, they are not the only ones.

Next, consider the possibility that the two agents resolve their dispute over T_0 through a winner-take-all contest. Let $p^1(\equiv p(G_1, G_2))$ and $p^2(\equiv 1 - p(G_1, G_2))$ denote the winning probability of agent $i = 1, 2$. The properties of this function that we maintain throughout the paper appear in Assumption 2.

ASSUMPTION 2. $p(G_1, G_2)$ is symmetric (i.e., $p(G_1, G_2) = 1 - p(G_1, G_2)$ for all (G_1, G_2)), twice continuously differentiable, increasing and concave in G_1, and decreasing and convex in G_2.[6]

Under winner-take-all conflict, each agent's payoff function is the expected consumption of the final good she or he produces. Given the constraint in (1) we can express the payoff functions as functions of (G_1, G_2).

$$U^i(G_1, G_2) = p^i(G_1, G_2)\, F(T_i + T_0, R_i - G_i) + (1 - p^i(G_1, G_2))\, F(T_i, R_i - G_i), \qquad \forall i = 1, 2. \tag{2}$$

Once guns are chosen, the probabilities of winning and every agent's output in each possible state of the world are determined. Then, *for any given choice of* (G_1, G_2), the strict concavity of $F(\cdot)$ in land implies that the following property holds

$$\begin{aligned} &p^i F(T_i + T_0, L_i) + (1 - p^i)\, F(T_i, L_i) \\ &\quad < F(p^i(T_i + T_0) + (1 - p^i)\, T_i,\, p^i L_i + (1 - p^i)\, L_i) \\ &\quad = F(T_i + p^i T_0, L_i), \qquad \forall i = 1, 2 \end{aligned} \tag{3}$$

where $L_i = R_i - G_i$. The relationship in (3) shows that each agent prefers to receive a share of the contested land equal to his or her probability of winning over entering a winner-take-all conflict with the same probability of winning. Our assumption of diminishing returns to land plays a role similar to risk aversion, with both sides preferring the division of the contested land over the uncertain outcome in winner take-all-contests. Dividing T_0 according to the winning probabilities of each agent, however, is typically only one of a continuum of possibilities for any given choice of guns. There are other ways to share the contested land that both sides would prefer over conflict.

The smallest share of T_0 that one could accept, which we denote by $\tilde{\alpha}^i$, is the one that provides the same expected payoff as under conflict and is implicitly defined by

$$\tilde{F}^i \equiv F(T_i + \tilde{\alpha}^i T_0, L_i) = U^i, \qquad \forall i = 1, 2 \tag{4}$$

[6] Assumption 2 is satisfied by the commonly used functional form $p(G_1, G_2) = h(G_1)/(h(G_1) + h(G_2))$ where $h \equiv h(G_i)$ is nonnegative, twice differentiable, increasing and concave in G_i $(i = 1, 2)$ with $h(0) \geqslant 0$ and $p(0, 0) = 1/2$.

with U^i satisfying (2). On the other hand, the largest share agent i can obtain is what would remain of T_0 if his opponent were to receive her minimum acceptable share of the contested land, or just $\tilde{\tilde{\alpha}}^i \equiv 1 - \tilde{\alpha}^j$, $(i \neq j = 1, 2)$. The utility associated with this maximal share, the "ideal" payoff, is

$$W^i(G_1, G_2) = \tilde{\tilde{F}}^i \equiv F(T_i + \tilde{\tilde{\alpha}}^i T_0, L_i), \qquad \forall i = 1, 2. \tag{5}$$

By the just-described definition of $\tilde{\tilde{\alpha}}^i$, agent i's ideal payoff depends indirectly on the conflict payoff, U^j, of her rival agent $j(\neq i)$. The important point is that, for any choice of guns, there will exist a range of possible divisions of the contested land leading to payoffs for each agent on the utility possibility frontier and Pareto-dominating their conflict payoffs. By the strict concavity of the production function in land, the utility possibilities set will be strictly convex.[7] We next explore the dependence of these payoffs on the bargaining solution concepts considered.

3. COMPARING BARGAINING SOLUTIONS

As we have just seen, given guns, the contestants will have an incentive to negotiate. But the quantity of guns each side may choose depends on the particular rule of division they expect to prevail. In turn, this rule will depend on the bargaining solution the agents employ. Here we study the effects of division rules generated by the three different bargaining solution concepts discussed in the introduction: the split-the-surplus (SS), the Kalai–Smorodinsky (KS), and the equal sacrifice (ES) solution concepts.

Each solution concept is defined for pre-determined guns and, consequently, for predetermined winning probabilities p and $1-p$, threat payoffs U^1 and U^2 in (2) and ideal payoffs W^1 and W^2 in (5). (For notational simplicity and wherever there is no risk of confusion we suppress the dependence of functions on (G_1, G_2).) For given guns, let $V^1(\alpha) = F(T_1 + \alpha T_0, R_1 - G_1)$ and $V^2(\alpha) = F(T_2 + (1-\alpha)\, T_0, R_2 - G_2)$ where $\alpha \in [\tilde{\alpha}^1, \tilde{\tilde{\alpha}}^1]$ represents a particular Pareto-efficient division of T_0. Each bargaining solution induces a division rule that depends on the threat payoffs, the ideal payoffs, and the Pareto frontier of the utility possibilities set, as described by $V^1(\alpha)$ and $V^2(\alpha)$. We now define the three division rules.

[7] However, both the location and the shape of the utility possibilities set will depend on the agents' secure factor endowments, the conflict and production technologies, and the resources they have committed to the production of guns.

DEFINITION SS. Let (G_1, G_2) be any combination of guns which uniquely define the threat payoffs U^1 and U^2 in (2) and the Pareto efficient pairs $(V^1(\alpha), V^2(\alpha))$ for $\alpha \in [\tilde{\alpha}^1, \tilde{\tilde{\alpha}}^1]$. The *split-the-surplus* division rule, α_{SS}, is defined by $V^1(\alpha_{SS}) - U^1 = V^2(\alpha_{SS}) - U^2$.

DEFINITION ES. Let (G_1, G_2) be any combination of guns which uniquely define the ideal payoffs W^1 and W^2 in (5) and the Pareto efficient pairs $(V^1(\alpha), V^2(\alpha))$ for $\alpha \in [\tilde{\alpha}^1, \tilde{\tilde{\alpha}}^1]$. The *equal sacrifice* division rule, α_{ES}, is defined by $W^1 - V^1(\alpha_{ES}) = W^2 - V^2(\alpha_{ES})$.

DEFINITION KS. Let (G_1, G_2) be any combination of guns which uniquely define the threat payoffs U^1 and U^2 in (2), the ideal payoffs W^1 and W^2 in (5), and the Pareto-efficient pairs $(V^1(\alpha), V^2(\alpha))$ for $\alpha \in [\tilde{\alpha}^1, \tilde{\tilde{\alpha}}^1]$. The *Kalai–Smorodinsky* division rule, α_{KS}, is defined by $(V^2(\alpha_{KS}) - U^2)/(V^1(\alpha_{KS}) - U^1) = (W^2 - V^2(\alpha_{KS}))/(W^1 - V^1(\alpha_{KS}))$.

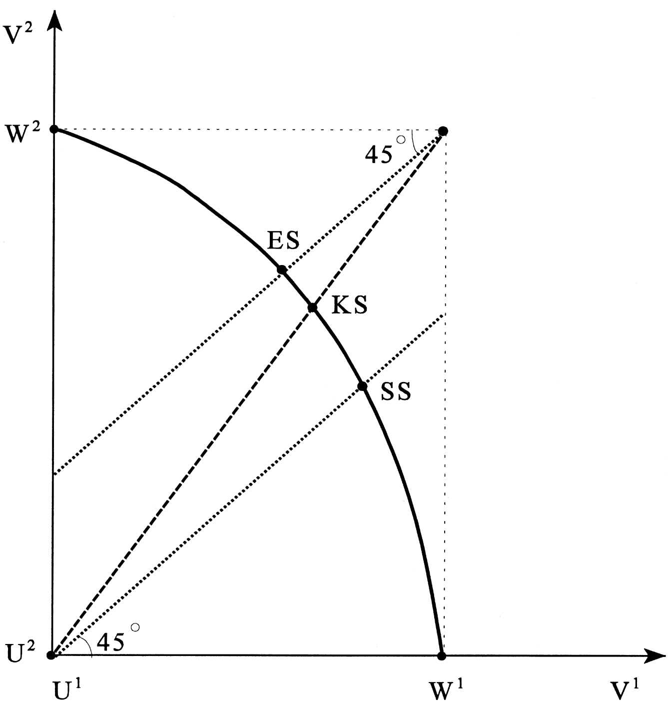

FIG. 1. U^i, threat payoff of agent i; W^i, ideal payoff of agent i.

Letting $J \in \{\text{SS, KS, ES}\}$ denote the solution concept and consequently the division rule considered, the two agents' payoff functions are

$$V_J^1(G_1, G_2) = F(T_1 + \alpha_J T_0, R_1 - G_1) \tag{6a}$$

$$V_J^2(G_1, G_2) = F(T_2 + (1 - \alpha_J)\, T_0, R_2 - G_2). \tag{6b}$$

Figure 1 illustrates how division of T_0 is determined under each of the three bargaining solutions. As can be inferred from (6a) and (6b), for given guns, payoffs do not differ across bargaining solutions and agents under conditions of complete symmetry. More generally, though, when guns are predetermined the bargaining solutions cannot be Pareto ranked. However, and as we now see, with guns being endogenously determined a Pareto-ranking becomes possible.

PROPOSITION 1. *Suppose that agents* 1 *and* 2 *have identical endowments of secure land and human capital* (*i.e.*, $T_1 = T_2$ *and* $R_1 = R_2$). *Moreover, assume the existence of a combination of symmetric, interior, and unique pure-strategy equilibria for each solution concept.*[8] *Letting* G_J *and* V_J *denote respectively the representative agent's* (*symmetric*) *equilibrium guns and payoffs under each bargaining solution* $J \in \{\text{SS, KS, ES}\}$, *we then have*

(i) $G_{\text{SS}} > G_{\text{KS}} > G_{\text{ES}}$

(ii) $V_{\text{SS}} < V_{\text{KS}} < V_{\text{ES}}$.

Proof. *Part* (i). Since agent 1's share, α_J, of the contested land T_0 is a function of the bargaining solution concept $J \in \{\text{SS, KS, ES}\}$ and the quantities of guns (G_1, G_2) produced, we can differentiate his/her payoff function in (6a) with respect to G_1 to obtain

$$\frac{\partial V_J^1}{\partial G_1} = (T_0 F_T^1) \frac{\partial \alpha_J}{\partial G_1} - F_L^1, \qquad J \in \{\text{SS, KS, ES}\}. \tag{7}$$

[8] Existence of interior equilibria can be assured by imposing mild Inada-type conditions on the conflict and production technologies. Uniqueness of equilibrium under the split-the-surplus solution can also be established, but not generally so for the other two solution concepts. However, if the production function is CES and the conflict technology is described by $h(G_i) = G_i^m$, $m \in (0, 1]$, all symmetric equilibria are unique. Still, even if there are multiple equilibria under KS and ES, the findings in Proposition 1 carry through qualitatively. First, it can be shown that the solution that relies more directly on the threat payoffs, i.e., the split-the-surplus solution, is Pareto-dominated by the other solutions. Second, the most efficient division under ES Pareto-dominates all other solutions.

The first term in the RHS of (7) captures agent 1's marginal benefit of producing an additional gun: it is the increase in output of the final good he or she would enjoy by controlling $T_0(\partial\alpha_J/\partial G_1)$ extra units of land. The second term in the RHS of (7) captures agent 1's opportunity cost of producing guns which is the reduction in output s/he would incur by producing an additional gun. To rank the equilibrium quantities of guns under the bargaining solutions considered, we must examine how the division rules implied by these concepts condition each agent's *net benefit* of producing an extra gun. Owing to symmetry, though, in equilibrium each agent will receive one-half of the contested land T_0 (i.e., $\alpha_J = 1/2$) and their marginal products of land and labor will not differ across $J \in \{\text{SS, KS, ES}\}$. Further, it is sufficient to examine only one agent's (say, agent 1's) incentive to build guns.

From (7) it can be seen that agent 1's incentive to build guns will differ across solution concepts only if the $\partial\alpha_J/\partial G_1$ term varies with $J \in \{\text{SS, KS, ES}\}$; hence to prove part (i) it is sufficient to show that

$$\frac{\partial\alpha_{\text{ES}}}{\partial G_1} < \frac{\partial\alpha_{\text{KS}}}{\partial G_1} < \frac{\partial\alpha_{\text{SS}}}{\partial G_1}. \tag{8}$$

To find $\partial\alpha_J/\partial G_1$ for each $J \in \{\text{SS, KS, ES}\}$ we proceed as follows. First, we define

$$\Phi = \Phi(\alpha, G_1, G_2) \equiv \frac{V^2(\alpha) - U^2}{V^1(\alpha) - U^1} \quad \text{and}$$
$$\Omega = \Omega(\alpha, G_1, G_2) \equiv \frac{W^2 - V^2(\alpha)}{W^1 - V^1(\alpha)}. \tag{9}$$

From our definitions of the three bargaining solutions it can be seen that the division rules they give rise to solve the equations

$$\Phi(\alpha_{\text{SS}}, G_1, G_2) = 1 \qquad (\textit{split-the-surplus}) \tag{10a}$$

$$\Omega(\alpha_{\text{ES}}, G_1, G_2) = 1 \qquad (\textit{equal sacrifice}) \tag{10b}$$

$$\Phi(\alpha_{\text{KS}}, G_1, G_2) = \Omega(\alpha_{\text{KS}}, G_1, G_2) \qquad (\textit{Kalai–Smorodinsky}). \tag{10c}$$

These equations define the α_Js as functions of (G_1, G_2). We may thus find $\partial\alpha_J/\partial G_1$ by differentiating each one of these equations appropriately and then collecting terms to obtain the corresponding expressions

$$\frac{\partial \alpha_{\mathrm{ES}}}{\partial G_1} = \frac{W^1_{G_1} + F^1_L - W^2_{G_1}}{T_0(F^1_T + F^2_T)} \tag{11a}$$

$$\frac{\partial \alpha_{\mathrm{SS}}}{\partial G_1} = \frac{U^1_{G_1} + F^1_L - U^2_{G_1}}{T_0(F^1_T + F^2_T)} \tag{11b}$$

$$\frac{\partial \alpha_{\mathrm{KS}}}{\partial G_1} = \gamma \left[\frac{\Phi(W^1_{G_1} + F^1_L) - W^2_{G_1}}{T_0(\Phi F^1_T + F^2_T)} \right] + (1-\gamma) \left[\frac{\Phi(U^1_{G_1} + F^1_L) - U^2_{G_1}}{T_0(\Phi F^1_T + F^2_T)} \right], \tag{11c}$$

where in the derivation of (11c) we used the facts that $\Omega = \Phi$ under KS and $\gamma \equiv (V^1_{\mathrm{KS}} - U^1)/(W^1 - U^1) \in (0, 1)$.

Let us now impose the requisite symmetry on the structure of the model, including the production of guns, so that $G_1 = G_2$. Under these conditions, $\Phi = 1$ and $\alpha_J = \frac{1}{2}$, $\forall J \in \{\mathrm{SS}, \mathrm{KS}, \mathrm{ES}\}$. Utilization of this point in (11c) reveals that $\partial \alpha_{\mathrm{KS}}/\partial G_1$ can be written as the weighted sum of $\partial \alpha_{\mathrm{ES}}/\partial G_1$ and $\partial \alpha_{\mathrm{SS}}/\partial G_1$, i.e.,

$$\frac{\partial \alpha_{\mathrm{KS}}}{\partial G_1} = \gamma \frac{\partial \alpha_{\mathrm{ES}}}{\partial G_1} + (1-\gamma) \frac{\partial \alpha_{\mathrm{SS}}}{\partial G_1}. \tag{12}$$

To complete the proof we need to show that $\partial \alpha_{\mathrm{ES}}/\partial G_1 < \partial \alpha_{\mathrm{SS}}/\partial G_1$. Utilizing the definition of W^i in (6), and differentiating expressions with respect to G_1 gives

$$W^1_{G_1} + F^1_L - W^2_{G_1} = -\tilde{\tilde{F}}^1_T \left[T_0 \frac{\partial \tilde{\alpha}^2}{\partial G_1} \right] - \tilde{\tilde{F}}^1_L + F^1_L + \tilde{\tilde{F}}^2_T \left[T_0 \frac{\partial \tilde{\alpha}^1}{\partial G_1} \right]. \tag{13}$$

Differentiation of the expressions in (4) (which define the $\tilde{\alpha}^i$s implicitly) gives

$$\frac{\partial \tilde{\alpha}^2}{\partial G_1} = \frac{U^2_{G_1}}{T_0 \tilde{F}^2_T} \quad \text{and} \quad \frac{\partial \tilde{\alpha}^1}{\partial G_1} = \frac{U^1_{G_1} + \tilde{F}^1_L}{T_0 \tilde{F}^1_T}.$$

Substituting these expressions in (13) and rearranging terms gives

$$W^1_{G_1} + F^1_L - W^2_{G_1} = -\frac{\tilde{\tilde{F}}^1_T}{\tilde{F}^2_T} U^2_{G_1} - \tilde{\tilde{F}}^1_L + F^1_L + \frac{\tilde{\tilde{F}}^2_T}{\tilde{F}^1_T} [U^1_{G_1} + \tilde{F}^1_L]. \tag{14}$$

Due to symmetry, $\tilde{\tilde{F}}{}^2_T = \tilde{\tilde{F}}{}^1_T$ and $\tilde{F}^1_T = \tilde{F}^2_T$, so after some rearrangement and simplification, we can rewrite (14) as

$$W^1_{G_1} + F^1_L - W^2_{G_1} = \frac{\tilde{\tilde{F}}{}^1_T}{\tilde{F}^1_T}[U^1_{G_1} - U^2_{G_1}] + F^1_L - \tilde{\tilde{F}}{}^1_T\left[\frac{\tilde{\tilde{F}}{}^1_L}{\tilde{\tilde{F}}{}^1_T} - \frac{\tilde{F}^1_L}{\tilde{F}^1_T}\right]. \tag{15}$$

Now consider the RHS of (15). First, the ratio of marginal products in front of the first square brackets satisfies $\tilde{\tilde{F}}{}^1_T/\tilde{F}^1_T < 1$ because $F^1_{TT} < 0$. Moreover, the expression inside the last pair of square brackets is nonnegative because, by Assumption 1, F^1_L/F^1_T is nondecreasing in land and $\tilde{\alpha}^1 < \frac{1}{2} < \tilde{\tilde{\alpha}}{}^1$. It follows that

$$W^1_{G_1} + F^1_L - W^2_{G_1} < U^1_{G_1} + F^1_L - U^2_{G_1}. \tag{16}$$

Utilizing (16) in (11a) and (11b) readily confirms the validity of (8). Further, and in light of our earlier discussion on payoffs, we have

$$\frac{\partial V^1_{\mathrm{ES}}}{\partial G_1} < \frac{\partial V^1_{\mathrm{KS}}}{\partial G_1} < \frac{\partial V^1_{\mathrm{SS}}}{\partial G_1}. \tag{17}$$

Now assume the existence of a unique equilibrium in the interior of the strategy space under each solution concept which requires that $\partial V^1_J/\partial G_1 = 0\ \forall J \in \{\mathrm{SS, KS, ES}\}$. It follows from (17) that the pure-strategy equilibria cannot be related in a way other than $G_{\mathrm{SS}} > G_{\mathrm{KS}} > G_{\mathrm{ES}}$ (where agent specific indexes are dropped for simplicity) as described in part (i) of the proposition.

Part (ii). This part follows immediately from the fact that, under conditions of symmetry, each agent acquires $\frac{1}{2}T_0$ but each agent expends more human capital (thus leaving less labor for the production of the butter) under SS, than under KS, than under ES.

Part (i) indicates that guns are highest under the split-the-surplus solution and lowest under the equal sacrifice solution, with those under the Kalai–Smorodinsky solution falling in between. Because with symmetry each agent receives 1/2 of the contested land T_0 under each bargaining solution, it must be the case that equilibrium payoffs are ranked in reverse order (part (ii) of the proposition). To understand why the ranking is as it is, we can compare the marginal costs and marginal benefits of investing in guns under the three solutions. By symmetry, each agent's marginal cost of producing guns (i.e., the butter she or he foregoes by allocating less human capital in its production, captured by the marginal product of labor) does not differ across bargaining solutions. Therefore, the representative agent's incentive to build guns will differ across solution concepts only if his or her marginal benefit of doing so differs, but this marginal benefit is proportional to the change in the agent's share of the contested land T_0 caused by a change in the production of guns (e.g., $\partial\alpha_J/\partial G_1|_{G_1=G_2}$ for agent 1 with $J \in \{\text{SS, KS, ES}\}$). Part (i) of Proposition 1 shows that the intensity of this effect is determined by the extent to which the three solution concepts rely on the threat point payoffs. Under the SS solution the agent's share is more sensitive to his production of guns because of its direct reliance on threat payoffs. In contrast, this share is less sensitive under the ES solution because changes in guns affect the agent's actual payoff through the implied changes in the ideal payoffs which, as shown earlier, depend on threat point payoffs only indirectly. Under the KS solution, an agent's incentive to build guns is a weighted sum of the incentives under the other two solutions and thus lies in between. In short, then, incentives to produce guns differ across these regimes as shown owing to their differential reliance on threat payoffs.

Having shown the Pareto-dominance of *equal sacrifice* over the other two bargaining solutions under conditions of symmetry, the question arises as to whether this dominance is preserved when asymmetries in technology or in the agents' secure resource endowments are considered. This is not a trivial problem. One difficulty stems from the fact that asymmetries themselves play a direct role in the determination of the share each agent receives and thus his or her preferences over the bargaining solutions considered. The second difficulty is that differences across agents affect their incentives to build guns asymmetrically under the solution concepts considered. For these reasons, the comparison of these concepts is analytically difficult, if not impossible, in the presence of asymmetries.

In Anbarci *et al.* [1] we investigated numerically the effects of inter-agent asymmetries in secure factor endowments and technologies in the context of widely used functional forms for the conflict and production technologies. Apart from demonstrating the existence of interior equilibria under each solution concept, our analysis there unveiled the following three points. First, with sufficiently large asymmetries a Pareto-dominant solution may not exist; second, these asymmetries have to be substantial for the Pareto-optimal ranking in Proposition 1 to fail; third, the SS and KS bargaining solutions were never Pareto-dominant.

4. CONCLUDING REMARKS

In actual negotiations between countries, organizations, or individuals, considerable resources are often expended on bettering the bargaining position of each party. Arming, lobbying, legal costs, and various other influence activities can increase one party's bargaining position but, as these are costly, they also deduct from what is obtainable for all from the final settlement. It is therefore in the collective interest of all parties to limit those costs. Conventions, rules, or norms of division that somehow provide suitable restraints would then be preferable by all parties. We have shown that the bargaining solution that gives rise to a division rule that is least sensitive to the threat point, the equal sacrifice solution, is Pareto-preferable within the class of bargaining solutions we have examined, provided the players are similar enough (when they are not, the factors that make bargaining solutions non-comparable within fixed bargaining problems take over). Placing little importance to the threat point when the time to settle differences comes appears, then, to be efficiency-enhancing. The seemingly self-serving pronouncements of diplomats, politicians, or strike negotiators about the inappropriateness of making threats can be viewed as attempts to maintain the visibility of norms against threats.

Our findings obviously have implications for bargaining theory itself (as well as for the applied literatures that employ bargaining solutions). Up to now, bargaining solutions have been compared only in terms of how reasonable their respective axioms are—there was no possibility of Pareto-ranking them. We have shown though that when bargaining solutions are embedded within an economic environment of threats that have resource consequences, all bargaining solutions are not created equal in front of the Pareto criterion. It is worth investigating then to what extent our findings carry through in other economic environments in which the threat point and the utility possibilities set are endogenous.[9]

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[9] One applied area of research with such characteristics is the incomplete contracts theory of the firm (for an overview, see Hart [7]). As argued recently by Chiu [3] and Rajan and Zingales [16] how bargaining is conducted can be critical for which ownership structure is optimal. Our findings suggest that the rules of bargaining employed can have important consequences for efficiency even across ownership structures in the presence of nontransferable utility, an issue that is not examined in this literature as utility is considered transferable.

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