THE MARKET FOR PROTECTION AND THE ORIGIN OF THE STATE

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ABSTRACT: We examine a stark setting in which security or protection can be provided by self-governing groups or by for-profit entrepreneurs (kings, kleptocrats, or mafia dons). Though self-governance is best for the population, it faces problems of long-term viability. Typically, in providing security the equilibrium market structure involves competing lords, a condition that leads to a tragedy of coercion: all the savings from the provision of collective protection are dissipated and welfare can be as low as, or even lower than, in the absence of the state.

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The collective good variously referred to as *security, order, protection of property rights*, or simply, *protection,* is a precondition for the provision of ordinary infrastructural public goods and generally for facilitating trade and economic development. Historically, it has also been the first type of good provided by states and is often considered the quintessential and defining attribute of the state.[1]

What sets protection apart, though, and its variations from other collective goods is the following characteristic: The inputs that are used for its production – soldiers and policemen, swords and guns – contain the seeds for the good's own destruction. Policemen and soldiers, by virtue of their positions, could extract even more than the robbers and bandits they are supposed to guard against. Similarly, rulers who provide protection against internal and external threats can use their power of extraction at an even grander scale. Army generals and colonels, ostensibly at the service of democratic governments, can, and regularly do, topple such governments. Clearly protection is not an ordinary good.

In this paper we argue that taking into account such peculiarities in the provision of protection leads to the understanding of two important tendencies, both in history and in the present. First, competition for the provision of protection often takes a very different form than the one we are accustomed to in economics: private providers of protection, instead of competing on the price of their service, typically compete with their means of violence over turf. Under such predatory competition, more competition leads to worse outcomes. Second, our approach helps understand the wide prevalence of autocracy, instead of self-governance, in the provision of protection and more generally in the organization of governance.

The type of competition usually examined within economics is one in which different jurisdictions attempt to attract mobile subjects through lower taxation, other privileges, and the provision of public goods. Whereas this type of competition is common nowadays and some economic historians (e.g., North and Thomas, 1973) have argued for its importance in the rise of the West, this is hardly the most widespread form that has existed in the past or the sole form of competition that is taking place today. From Ancient Mesopotamia to China, Egypt, Mesoamerica, or Medieval Europe, serfs were tied to the land and free peasants had few outside options, with rulers coming and going but without any change in their incentives for production. Emperors, kings, and princes were fighting for territory and the

[1] In the sense, following Weber's definition, that the provider of protection also has the monopoly in the legitimate use of force (Weber, 1978). Of course, in practice no actual state has a monopoly in the use of force. For example, the Russian Federal government exerts little control on some republics, mafias, or officials within its territory. Similarly, but less dramatically, US authorities exert little control in some American inner cities. Weber's usage of the term "legitimate" was likely meant to overcome this problem, although other questions emerge about the meaning of the term, especially for economists.

rents that come with it just as, in more recent times, mafiosi and warlords fight for turf and their accompanying protection rents. Under such conditions, the tribute or protection money paid depends on the relative ability of each side in the use of force. Promising a lower tribute on the part of a provider of protection is not credible unless it reflects that relative power of the two sides, the ruler and his agents on one side and the ordinary producer usually on the other.

In analyzing the behavior of for-profit providers of protection (or, states) we examine two market structures or regimes: the form of monopolistic competition that we have just described as well as monopoly. The most likely stable outcome that emerges endogenously is to have multiple for-profit states. Each state hires guards to protect its sequestered peasants from bandits, hires warriors to protect its borders from the other states, and receives income from tribute extracted from its peasant subjects. Moreover, under the regime with competing predatory states, under the conditions we examine total output can even be lower than without a state; all the savings accruing from the provision of internal protection are dissipated in fighting over the same rents created by those savings, whereas states can extract more than simple bandits. Thus, as far as the market for protection is concerned, and as another manifestation of the peculiar character of the provision of protection, competition is not a good thing.

Another set of market structures we examine involve self-governing groups of producers, with and without competition from for-profit states. The consensually organized, self-governing state could survive in the absence of predators, and although collective security would be underprovided and the state would be small in size, the welfare of peasants and bandits would be highest under such a market structure. In the presence of competing predators, however, we have found no long-run equilibrium in which a self-governing state would be viable. Because self-governing states face the free-rider problem, they have to be small. Being small in the presence of larger predators though, necessitates too much expenditure per person on external as well as internal security, leaving little room for production with a resultant welfare lower than even the subjects of a predator would enjoy. Thus, this finding helps understand the prevalence of autocracy.

The difficulty of establishing democracy and self-governance and the prevalence of autocracy is apparent from many recent experiences as well as more distant ones. From Indonesia to Africa, most post-colonial states have experienced coups and dictatorships to a much greater degree than democratic governance. Earlier, during the nineteenth century the first post-colonial states of Latin America have had similar fates. Our findings are also very relevant to the almost complete absence of self-governance during the time between the agricultural revolution and two centuries ago (see, e.g., Finer, 1997, or Mann, 1986).[2] Our approach is most relevant for this time period given that our model does not allow for the complex institutional web of modern mass representative democracy.[3]

Our approach is still helpful though in understanding what occurs in places in which the reach of the modern state is weak. That includes many "failed" states as well as the areas within modern states with power vacuums that allow warlords, gangs, and mafias to develop. As Gambetta (1993) argues the primary commodity sold by the Sicilian mafia is protection (for modeling dedicated to the activities of mafias and gangs see Grossman, 1995, Polo, 1995, Skaperdas and Syropoulos, 1995, and other contributions in Fiorentini and Peltzman, 1995, Konrad and Skaperdas,1997,1998).We tell a story with peasants and bandits which also applies to interactions among shopkeepers and robbers in Moscow, Los Angeles, or Lagos. In the latter case gangs and mafias come in to fill the gap vacated by the modern state, supplanting it and creating a near-monopoly of force in their area. We help understand why genuine community policing is difficult and why gangs arise in conditions with a power vacuum.

Compared to other work that has viewed the state as maximizing its revenue while providing a public good (Brennan and Buchanan, 1977,1978; Engineer, 1989; Findlay, 1990; Olson, 1991; Grossman and Noh, 1994; Marcouiller and Young, 1995; McGuire and Olson, 1996, Moselle and Polak, 2001), we take account of the aforementioned peculiar status of protection relative to other public goods. We also allow for the distribution of output, including taxation by the state, to depend explicitly on the relative ability of affected parties to use force. Thus taxation has a direct resource cost, whereas the cost of taxation in the existing literature is indirect, as deadweight loss or reduction in market activities. More importantly, in contrast to all this work which supposes a single Leviathan monopolistic state, we allow for different types – for-profit and self-governing – and the combinations

[2] Possible exceptions include city-states in early Mesopotamia, Ancient Greece, and late Medieval Italy. Of course, all of these are subject to many qualifications as the democratic franchise did not include slaves, women, and often most of the rest of the male population because of property qualifications.

[3] We should mention two analytically distinct but complementary reasons to the one we examine in this paper for the difficulties of self-governance's survival. First, coordination problems inherent in democratic decision-making might provide an advantage to the hierachical decision-making that usually prevails in the for-profit provision of protection. The formal incorporation of this reason in our model would not be difficult and would reinforce our results. In fact, representative democracy can be thought of as an attempt to get around the coordination problems of democracy. However, inherent in representative democracy is the second reason that limits self-governance, the so-called "iron law of oligarch" first identified by Michels (1962): the tendency of representative insitutions and organizations to be hijacked by their elected representatives and officers, primarily due to the informational asymmetry that develops between representatives and the represented. It would be difficult to incorporate this reason in our modeling, although it is clearly an important one that complements our own.

of market structures that become then possible. Usher (1989) is probably closest to this paper; but while we are interested primarily in contrasting the different types of states that can arise, Usher's main interest is in the alternation between despotism and anarchy. [4]

Because of the different market structures of different complexities that we examine, we start with the simplest one, anarchy, and gradually build to the more complex ones while trying to maintain comparisons with those analyzed earlier.

I. Peasants and Bandits in Anarchy

We begin with the simplest setting in which there is an absence of collective organizations. Individuals out of a population N sort themselves among peasant farmers and bandits where the latter make a living by preying on the peasants. A similar story could be told for an anarchic urban setting by having – instead of peasants and bandits – workers and robbers as the two possible occupations. Each peasant has one unit of a resource that he can distribute between work and self-protection – the higher is the level of self-protection, the lower is the amount of work and the lower is the output that can be produced. Denoting this self-protection activity by x, the peasant can keep a share $p(x)$ of output away from bandits, where $p(x)$ is increasing in x, $p(x) \in [0,1], p(0) = 0$ and $p(1) = 1$. Thus the payoff of a peasant is as follows:

$$U_p = p(x)(1 - x) \tag{1}$$

Each peasant chooses a level of self-protection x so as to maximize this payoff in (1). We suppose a unique such level, denoted by x^*. For the remainder of this paper we also denote the payoff associated with x^* by U_p^*.

The bandits roam the countryside looking for peasants to prey upon. Let N_p denote the number of peasants and let N_b represent the number of bandits. The bandit's payoff is as follows:

$$U_b = [1 - p(x)](1 - x)\frac{N_p}{N_b} \tag{2}$$

That is, bandits extract $1 - p(x)$ of output from each peasant who has not been previously robbed and the more peasants there are relative to bandits, the better it is for a bandit.[5]

[4] Another related area of research from economics is on the determinants of the size of states (Friedman, 1977, Wittman, 1991, Artzrouni and Komlos, 1996, Findlay, 1996). Our approach adds to this literature by deriving the determinants of size, as well as type, from an explicit optimizing model (Findlay also does this for a single state, an empire).

[5] We could obviously allow for a peasant to be robbed more than once by bandits, but such an assumption would unnecessarily complicate our model, especially in the later sec-

Given that a peasant's payoff is uniquely determined by the choice of x^* (and equals U_p^*), we are interested in an equilibrium state whereby the numbers of bandits and peasants adjust until a bandit's payoff equals that of a peasant. Formally, an *anarchic equilibrium* is a number of peasants N_p^*, a number of bandits N_b^*, and a bandit's payoff U_b^* such that $N_p^* + N_b^* = N$ and $U_b^* = U_p^*$.[6] In equilibrium the numbers of peasants and bandits are then given by:

$$N_p^* = p(x^*)N \text{ and } N_b^* = [1 - p(x^*)]N \tag{3}$$

The easier is to defend output from bandits, as captured by the properties of the function $p(\cdot)$ and the amount of self-protection induced, the more peasants there are relative to bandits. Total output, which we will use in welfare comparisons with collective forms of the organization of protection that we will examine later, equals:

$$N_p^*(1 - x^*) = p(x^*)(1 - x^*)N \tag{4}$$

Compared to the "Nirvana" condition without banditry, in which total output would equal N, the lower output under anarchy has two sources: (i) The fact that bandits do not contribute anything to production [the associated welfare loss equals $[1 - p(x^*)]N$] and (ii) those who become peasants have to divert a fraction of their resources toward self-protection [the associate welfare loss is $p(x^*)x^*N$].[7]

tions. We should mention that the function $p(x)$ could take a probabilistic interpretation, denoting the probability of the peasant prevailing in a conflictual encounter with a bandit. (Such a function can be justified axiomatically or in other ways as in Hirshleifer (1989), Skaperdas (1996), or Clark and Riis (1998).) Given the risk neutrality of the two types of agents in (1) and (2), a peasant and bandit would be indifferent between such conflict and dividing output with a share $p(x)$ going to the peasant and the remainder going to the bandit.

[6] To avoid unnecessary complications we allow for non-integer numbers of peasants and bandits.When these numbers are non-integer in equilibrium, we suppose that $Int[N_b^*]$ become bandits and $Int[N_p^*]$ become peasants (where $Int[z]$ denotes the largest integer that is smaller or equal to z). One agent, then, spends $v^* = N_p^* - Int[N_p^*]$ portion of his time as a peasant and the remainder as a bandit. The agent's portion of the payoff as peasant equals $p(\frac{x^*}{v^*})(v^* - x^*)$, with the optimal time spent on self-protection equalling x^*v^* which yields a payoff of $p(x^*)(v^* - x^*v^*) = v^*p(x^*)(1 - x^*) = v^*U_p^*$. With the payoff of the same agent from part-time banditry equalling $(1 - v^*)U_b^*$, the total equilibrium payoffthus equals that of any other agent, peasant or bandit. Grossman (1991, 1994) models every single agent, not just a marginal one, as allocating time between different occupations (farming, banditry, and soldiering). The justification of continuous variables in the different occupations that we employed here, also holds for the other occupations, the numbers of which we assume continuous in the rest of the paper (guards, warriors, praetorians).

[7] With $p(x) = x$, we have $x^* = 1/2$, there are as many bandits as peasants and total output is $1/4$ of potential output.

II. Collective Protection

In addition to each peasant taking self-protection measures against bandits privately, several peasants, a village, or a district could take protection measures collectively. Such measures can include simple warning systems about the presence of bandits in the area, the formation of a militia that becomes active when there is a threat, the building of rudimentary fortifications to protect crops or other property, or the employment of full-time guards and policemen. We abstract from the particular forms that collective protection takes and we simply suppose that collective protection can be provided more efficiently than self-protection. Letting $z \in [0, 1]$ denote the group's average per peasant expenditure on collective protection consisting of k peasants, the effective expenditure on collective protection (equivalent to expenditures on self-protection) received by each peasant is a function $f(z)$ with the following properties:

$$f(0) = 0; f(z) > z \text{ for all } z \in (0, 1); k \geq \bar{k} \text{ for some } \bar{k} > 1; f(\cdot) \text{ is concave, twice differentiable, except possibly at one point, and its inverse exists} \tag{5}$$

The share of own output retained by a peasant who has contributed x_i to collective protection is $p(x_i + f(z))$,[8] where $z = \sum_{j=1}^{k} \frac{z_j}{k}$ and z_j is the contribution of peasant j in the collective protection of the group. The key property in (5) is $f(z) > z$, for it implies that if each peasant in a group were to contribute z to collective protection, instead of contributing it to self-protection, he or she would receive a higher level effective protection overall. To have this type of protection truly collective, we require that the number of peasants in a group is at least as high as the minimum size $\bar{k}$.

To gain intuition about the effects of the collective protection technology and to facilitate comparisons with the non-cooperative choice we examine later, we briefly consider optimal choices of protection that maximize a welfare objective that takes the size (k) and composition of a group of peasants as given. The objective is to choose $x_i's$ and $z_i's$ $(i = 1, ..., k)$ so as to maximize the sum of the payoffs of the peasants belonging to the group:

$$\sum_{i=1}^{k} U_{pi} = \sum_{i=1}^{k} p(x_i + f(z))(1 - x_i - z_i) \text{ where } z = \sum_{j=1}^{k} \frac{z_j}{k} \tag{6}$$

Given that x_i and z_i have the same cost to a peasant but the average protection is higher with collective protection, we might expect that optimal protection should involve collective protection only. This is not the case,

[8] Similar functions that allow for both public and private protection or precautionary activities have been used in the law and crime literature (see, eg, Ben-Shar and Alon, 1995, or Hylton, 1996). Note that the domain of $p(\cdot)$, in order to allow for all possible values of $x_i + f(z)$, can be defined to be $[0, 2]$.

however, since given the concavity of $f(\cdot)$ the marginal return of collective protection could fall below the return to self-protection (which, given our specification, equals unity). Thus, the optimal choice involves choosing collective protection up to a certain point where $f'(z) \leq 1$. When $f'(z) > 1$, no self-protection is undertaken, whereas with $f'(z) = 1$ some self-protection could be undertaken. Whether or not some self-protection is optimal depends on the functional form.

Choosing the right levels of collective and private protection would require a benevolent agent who would also have the power to impose such choices. This would amount to effectively assuming away the problem we set out to examine. Thus, instead our task in the remainder it to explore different alternatives – different "industrial organizations" of protection – that could emerge from anarchy that utilize the more efficient collective protection technology.

III. Self-governance

One way of utilizing the higher efficiency of collective protection is for peasants to form a self-governing community and voluntarily contribute to collective protection, through a part-time peasants' militia, through the construction of fortifications, or other means.

Consider a group of k $(\geq \bar{k})$ peasants, with the k initially given, who voluntarily choose between production, contributions to collective protection, and self-protection. That is, each peasant i belonging to the group chooses x_i and z_i (and, therefore, production which equals $1 - x_i - z_i$) so as to maximize his payoff as given by

$$U_{pi} = p(x_i + f(z))(1 - x_i - z_i) \text{ where } z = \sum_{j=1}^{k} \frac{z_j}{k} \tag{7}$$

These choices are made simultaneously by all peasants in the group so that they form a Nash equilibrium. To analyze such equilibria, first consider peasant i's incentives to choose x_i and z_i as indicated in the following partial derivatives:

$$\frac{\partial U_{pi}}{\partial x_i} = p'(x_i + f(z))(1 - x_i - z_i) - p(x_i + f(z)) \tag{8}$$

$$\frac{\partial U_{pi}}{\partial z_i} = p'(x_i + f(z))(1 - x_i - z_i)\frac{f'(z)}{k} - p(x_i + f(z)). \tag{9}$$

The first term of each equation represents the marginal private benefit of each protection activity, whereas the second term represents its marginal cost. Note how the marginal private benefit of contributing to collective protection in (9) is just $1/k$ of the value of its marginal social benefit. By comparing (8) to (9), it can be seen that a peasant's marginal benefit of increase in x_i exceeds his of her marginal benefit of an increase in z_i if and only if $f'(z) > k$. A more efficient collective protection and a smaller group size increase the incentives for individual contributions to collective protection. In this setting three different types of equilibrium can occur:

(a) $z_i = 0$, $x_i = x^*$ for all i (quasi-anarchy with only private protection).

(b) $z_i = \hat{z}$ for some $\hat{z} > 0$, $x_i = 0^*$ for all i (only collective protection used).

(c) $z_i = \hat{z}$ for some $\hat{z} > 0$, $x_i = \hat{x}$ for some $\hat{x} > 0$ (both types of protection used).

Using standard techniques, the following properties can be shown to hold (for proofs please see a Supplementary Appendix that is available on request):

Property (*i*): Equilibrium collective protection is non-increasing in group size k and strictly decreasing in k for type (c) and for type (b) provided $p(f(\hat{z})) < 1$.

Property (*ii*): Equilibrium self-protection is constant with respect to group size for types (a) and (b) and strictly decreasing for type (c).

Property (*iii*): The level of protection (i.e., the share retained by each peasant) is at least as high as under anarchy; it is strictly higher for type (c) and for type (b) if $p(f(\hat{z})) < 1$.

Property (*iv*): Individual welfare is non-increasing in group size. It is always constant for type (a) and it is always strictly decreasing for type (c) and for type (b) if $p(f(\hat{z})) < 1$.

Property (*v*) of the mixed type (c) equilibrium implies that, if we were to allow for an endogenous determination of group size, the size that would most likely emerge is the minimal one for which collective protection is feasible (i.e., for $\bar{k}$). We now introduce the possibility of the endogenous determination of groups.

A *self-governing equilibrium* is a number of peasants $\hat{N}_p$, a number of peasant groups $\hat{n}_p$, and a number of bandits $\hat{N}_b$ such that:

(I) Each peasant belongs to a group, chooses self-protection and collective protection strategically, and has no incentive to join another group or become a bandit;

(II) Each bandit does not have an incentive to join a peasant group;

(III) $\hat{N}_p + \hat{N}_b = N$.

Part (I) of the definition implies that in a self-governing equilibrium peasants have equal payoffs across groups and, therefore, all groups must have the same size. Moreover, group size must equal $\bar{k}$, since otherwise there would be an incentive for some peasants to form a smaller group as, by property (*iv*), have higher payoffs. Part (II) implies that the payoff of bandits must equal that of peasants, provided security is less than complete and there is a positive number of bandits in such an equilibrium. The following Proposition summarizes the main attributes of self-governing equilibria.

Proposition 1 *Consider a type (b) equilibrium with $p(f(\hat{z})) < 1$ or a type (c) equilibrium. Such an equilibrium has the following properties: (i) Each group is of minimum size $\bar{k}$; (ii) the number of peasants ($\hat{N}_p$) is higher than the number of peasants under anarchy (N_p^*) and the number of bandits ($\hat{N}_b$) is lower than the number of bandits under anarchy (N_p^*); (iii) the welfare of peasants belonging to a group and the welfare of bandits is higher than that under anarchy.*

Since the minimal scale for collective protection against bandits, $\bar{k}$, can be considered small the self-governing groups that will form will be of small size. We should note that self-governance involves considerable coordination and decision costs which would also favor small size and, in combination with the free-rider problem, could render self-governance more problematic than it appears thus far.

IV. Protection for Profit

Instead of having peasants voluntarily provide a portion of their time for collective protection, an entrepreneur – Leviathan, the chief, local lord, or Mafia don – could hire full-time guards to protect peasants against bandits in return for tribute. His objective would be to maximize the difference between his receipts from tribute minus his costs. Receipts from tribute are likely to be higher the better is the level of protection and the larger is the number of peasants. Thus, it appears that as far as collective protection is concerned an entrepreneur could have incentives to provide it for profit. The catch is of course what the peasants could get out of this, for the machinery protection against bandits can double as that of extortion against peasants.

IV.A Monopoly protection by Leviathan

We begin with the simpler form of market structure, whereby protection is provided monopolistically by Leviathan. Monopoly is also virtually the only form of market structure that has been studied in other work, starting with Brennan and Buchanan (1977), on the profit-maximizing state and therefore we can make appropriate comparisons more easily.

Leviathan can utilize the same collective protection technology introduced in section 3. He hires guards to protect peasants against bandits but also, at least indirectly, to extract tribute from the same peasants. Letting N_g denote the number of guards, $f(\frac{N_g}{N_p})$ represent the units of collective protection received by each peasant.[9] The extraction of tribute is also facilitated by an elite corps, the praetorians, who also monitor the guards

[9] For some of the results in a previous version of this paper we allowed for $f(\frac{\alpha N_g}{N_p})$ where $\alpha \in (0, 1]$ represents the proportion of guards that can be used for genuine protection, with the rest of the guards being used towards the extraction of tribute from peasants. Because our findings do not change qualitatively with this added complication, we chose not to include it in this version of the paper.

in the duties, contribute to administration, and they generally serve as a *portmanteau* variable for factors we cannot completely specify here.[10]

The payoffs of the occupations of guard and praetorian are determined by how much the peasants manage to keep. For given numbers of guards and peasants, and self-protection level x by a peasant, the maximum share of output that could theoretically be retained by the peasant is $p(x+ f(\frac{N_g}{N_p}))$. However, as Leviathan has all the coercive machinery of guards and praetorians at his disposal, peasants can retain only whatever they can keep from being snatched away from them. One possibility is that peasants can keep away from Leviathan what they keep away from bandits, $p(x)$. More generally, however, we can suppose a *resistance* function $\rho(x)$ that indicates the share of output a peasant can keep away from Leviathan and his agents for any given level of self-protection x, where $\rho(x) \in [0,1], \rho(0) = 0$, and $\rho(1) = 1$. Bandits take away $[1 - p(x+ f(\frac{N_g}{N_p}))](1 - x)$ from each peasant, Leviathan takes $[p(x+ f(\frac{N_g}{N_p})) - \rho(x)](1 - x)$, and each peasant retains $\rho(x)(1 - x)$ of output. Each peasant chooses x to maximize $\rho(x)(1 - x)$ and we suppose a unique such choice $\hat{x}$. Therefore, the payoff of peasants – as well as that of bandits, guards, and praetorians – is $\rho(\hat{x})(1 - \hat{x})$.

When $\rho(x) < p(x)$ for all x, we can say that peasants *resist* Leviathan *less* than they can resist bandits and, equivalently, Leviathan can *extract* from peasants *more* easily than bandits can. As Leviathan is more organized than individual bandits are we should perhaps expect this to be the more likely condition. Under such conditions, it can be shown that $\rho(\hat{x})(1 - \hat{x}) < p(x^*)(1-x^*)$.[11] That is, when Leviathan is better at extraction than bandits, the welfare of peasants under Leviathan would be lower than that under anarchy. Although we will first examine this case to understand some of its effects, for analytical convenience we will revert to the simpler case in which $\rho(x) = p(x)$.

Since the numbers of peasants and bandits depend on the level of protection, which in part depends on the number of guards, Leviathan needs to take account of the effect his choice of N_g has on the number of peasants, N_p. If $p(\hat{x} + f(\frac{N_g}{N_p})) < 1$ and therefore security is less than perfect, there will be a positive number of bandits $N_b = N - N_g - N_p - N_{pr}$. With the payoff of a bandit equalized to that of a peasant in this case, the number of peasants that would emerge can be implicitly derived. When security is perfect and there are no bandits, the number of peasants simply equals $N - N_g - N_{pr}$. Overall, for each choice of N_g there will be an induced number

[10] Wintrobe (1998) specifies a detailed model of a monopolist-for-profit state in which, in addition to repression, resource expenditures to increase the loyalty of subjects is examined.

[11] Given that x^* maximizes $p(x)(1 - x)$, we have $p(x^*)(1 - x^*) \geq p(\hat{x})(1 - \hat{x})$. Since $p(x^*) > \rho(x^*)$ by the fact that Leviathan can extract more easily than bandits can, we have $p(\hat{x})(1 - \hat{x}) > \rho(\hat{x})(1 - \hat{x})$, thus yielding $p(x^*)(1 - x^*) > \rho(\hat{x})(1 - \hat{x})$.

of peasants which we denote by the function $\nu(N_g)$.[12] Leviathan's objective is to maximize his net receipts by the choice of N_g, provided these receipts are positive, while taking into account the effect on the number of peasants as described by $\nu(N_g)$: [13]

$$V_L = \nu(N_g)\left[p(\hat{x} + f(N_g/\nu(N_g))) - \rho(\hat{x})\right](1-\hat{x}) - (N_g + N_{pr})\rho(\hat{x})(1-\hat{x}) \tag{10}$$

The first term in (10) represents Leviathan's gross revenues (number of peasants times tribute rate times output per peasant). The second term represents the cost of hiring guards and praetorians.

We first show by example what can occur when Leviathan can extract tribute from peasants more easily than bandits can. Suppose $p(x) = x$, $\rho(x) = x^2$ and $f(z) = z^{\frac{1}{2}}$. Then under anarchy $x^* = \frac{1}{2}$, the payoff of peasants is $\frac{1}{4}$, half of the population are bandits and half peasants, and total output is $\frac{N}{4}$. Under Leviathan, $\hat{x} = \frac{2}{3}$, the payoff of peasants and those of the other occupations is just $\frac{4}{27}$, but security is perfect and the number of peasants is $0.9(N - N_{pr})$, higher than that under anarchy for most values of N_{pr} that yield a positive payoff for Leviathan. However, because peasants who are under Leviathan's heavy boot do not produce as much, total output is $\frac{3}{10}(N - N_{pr})$ which for $N_{pr} > \frac{1}{6}N$ (but not too high, so Leviathan´s payoff is positive) is lower than total output under anarchy. Thus, contrary to some of the arguments in McGuire and Olson (1996), Leviathan not only may not improve output compared to anarchy but also may actually leave a scorched earth of lower total output, as well as lower welfare for everyone except Leviathan (or, possibly some of his entourage which could be easily incorporated into the model). The key to this finding is a high extractive capacity of Leviathan combined with an inability to commit against using this capacity.

Having made this point, for convenience we will focus on the remainder on the simpler case in which Leviathan and bandits have exactly the same extractive capacity ($\rho(x) = p(x)$ for all x). The following Proposition summarizes our findings with (part (i)) and without (part (ii)) the higher extractive capacity by Leviathan.[14]

Proposition 2: (i) *If Leviathan can extract from peasants more easily than bandits can (i.e., $\rho(x) < p(x)$ for all x), then total output under*

[12] The details of the derivation of this function and its properties below are to be found in the Supplementary Appendix.

[13] A justification for using continuous variables for the different occupations both here and later in the paper, please see footnote 6.

[14] It can be shown that when Leviathan *cannot* extract as easily as bandits can (i.e., $\rho(x) > p(x)$) is the only case in which peasants would be better off under Leviathan than under anarchy. Grossman (1998) also finds conditions that lead to a similar finding (it occurs when bandits can take a lot from peasants). For our case, we cannot think of circumstances that would lead bandits to be better at extraction than Leviathan.

Leviathan´s rule can be lower than total output under anarchy.

(ii) *Suppose Leviathan and bandits can extract equally well from peasants (i.e., $\rho(x) = p(x)$ for all x). Further, suppose $p(\cdot)$ is concave and $f(\cdot)$ satisfies* (5). *If the fixed number of praetorians, N_{pr}, is sufficiently low, there is a choice of guards that maximizes Leviathan's payoff at a positive level. Such a choice has the following properties:* (a) *Total output under Leviathan is higher than total output under anarchy;* (b) *Total output under Leviathan may be higher or lower than total output under self-governance; the lower N_{pr} is and the higher $\bar{k}$ is , the higher the ratio of the two outputs is and, therefore, the more likely that output is higher under Leviathan.* (For the proof of part (ii), please see the proof of Proposition 3 in the 1997 working paper version.)

When Leviathan is not better than bandits in extracting tribute from peasants, total output is higher then under anarchy (iia) but does not have to be higher than that under self-governance, despite the latter's free-rider problem. The cost of taxation as manifested in the high self-protection levels of the peasants, along with a high fixed cost and small minimum size for the collective protection technology, can make output under self-governance higher.

Large profits typically attract competitors. With Leviathan appropriating all the extra income, we can expect competitors who would vie for a portion of profits.

IV.B Competing Lords

Instead of having a single Leviathan and small-time challengers contesting his rule, we will now examine the case in which all individuals *ex ante* are potential little Leviathans or lords; they can choose this occupation just as they would chose to be peasant, bandit, praetorian, or guard. A lord's job is similar to that of Leviathan in the hiring of praetorians and guards and in receiving tribute from peasants. We continue to maintain the same assumptions about the technology of collective protection and about the sharing of the surplus between lord and peasant. For simplicity, we continue to suppose that $\rho(x) = p(x)$ so that a peasant contributes x^* to his private protection and his payoff equals $p(x^*)(1 - x^*)$.

The lord, though, has a major headache that Leviathan did not have. Other lords are now after tribute received from peasants, and he needs to defend that tribute against them. He can do that by hiring warriors to keep the other lords outside his territory (and keep the sequestered peasants in) and possibly gain additional territory at their expense. But then the other lords will respond in kind. Thus the new element in the lords competing against one another is that they will have to hire warriors as well.

In this setting peasants have limited options. They are tied to their land and at the mercy of the lords who compete over how to divide them up.[15] This is a rather different type of competition than the one typically assumed by economists, whereby different jurisdictions attempt to attract mobile subjects through lower taxation or other privileges. Whereas this type of competition takes place in much of the world today and some economic historians (e.g., North and Thomas, 1973) have argued for its importance in the rise of the West, this is hardly the most widespread form of competition that has existed in the past or the only form of competition that is taking place today. From Mesopotamia to China, Egypt, Mesoamerica, or feudal Europe, serfs were tied to the land and free peasants typically had no outside options, with rulers coming and going but without any change in their incentives for production. Even in the past two centuries, with the rise of the rights of man, the most liberal of states have sequestered their citizens with barbed-wire borders and passport controls. While we do not deny the importance of tax-and-privilege competition of mobile subjects, we find the complete lack of study of this other significant form of competition based on the use of force as providing ample reasons for a first look.

Let n_{wl} denote the number of warriors hired by lord l. For a given number of lords N_l and peasants N_p, the number of peasants that lord l can sequester, and receive tribute from, is given by

$$q(n_{wl}, n_{w-l})N_p \tag{11}$$

where $n_{w-l} = (n_{wl}, ..., n_{wl-1}, n_{wl+1}, ..., n_{wN_L})$ is the vector of warriors hired by the other lords. Also, $q(\cdot)$ satisfies the following properties:

$q(\cdot) \in [0, 1]$ is a symmetric, twice differentiable function which is increasing in its first argument and decreasing in the remainder $N_L - 1$ arguments with $\sum_{j=1}^{N_l} q(n_{wj}, n_{w-j}) = 1$

Letting n_{pr} be the fixed number of praetorians and n_{gl} the number of guards hired by lord l, the payoff of the lord can now be written:

$$\begin{aligned} V_l \;=\; & q(n_{wl}, n_{w-l})N_p \\ & [p(x^* + f(n_{gl}/(q(n_{wl}, n_{w-l})N_p))) - p(x^*)]\,(1 - x^*) - \\ & (n_{wl} + n_{gl} + n_{pr})p(x^*)(1 - x^*) \end{aligned} \tag{12}$$

The main difference of (12) from Leviathan's payoff in (10) is the determination of the number of peasants: Whereas in (10) the chosen number

[15] In alternative interpretations, the peasants could conceivably decide to go it alone, but they would then receive the same payoff as under a lord. They could also go "into the woods" where they would receive a payoff that may be higher or lower that the prevailing payoff under a lord; effectively we are assuming that such payoff is not higher than that under a lord. The peasants might also try to join a self-governing state, a possibility that we will allow in the next section (where we can only find conditions that lead to an even lower payoff than under a lord or under anarchy).

of guards induces the number of peasants through $\nu(\cdot)$, here the number of peasants is determined by the number of warriors the particular lord has relative to other lords.

Initially, suppose the number of lords is given at $\bar{N}_l > 1$. Then, a short-run lordship regime consists of numbers of peasants N'_p, bandits N'_b, and for each lord l guards n'_{gl} and warriors n'_{wl} such that:

(I) Each lord $l = 1, 2, ..., \bar{N}_l$ takes N'_p as given and chooses n'_{gl} and n'_{wl} simultaneously with other lords so that these choices form a Nash equilibrium;

(II) $N'_b = \sum_{j=1}^{\bar{N}_l} n'_{bj}$ where for all j $n'_{bj} = q(n'_{wj}, n'_{w-j})N'_p(1 - p^j)/p(x^*)$

and $p^j \equiv p(x^* + f(n'_{gj}/(q(n'_{wj}, n'_{w-j})N'_p)))$

(III) $N = \sum_{j=1}^{\bar{N}_l} n'_{gj} + \sum_{j=1}^{\bar{N}_l} n'_{wj} + \bar{N}_l n_{pr} + N'_p + \bar{N}_l + N'_b$

Part (I) is straightforward: The lords compete for "market share" through hiring of warriors and the protection they provide to peasants, although each lord individually does not take account of his own effect on the number of peasants. Part (II) states that the number of bandits equals the sum of the bandits in each lord´s territory, and the number of bandits in each lord´s territory is such that the utility of bandits and peasants is equalized. Clearly, the number of bandits in territory j is inversely related to the total protection level p^j and when there is perfect security ($p^j = 1$) there are no bandits in territory j. Finally, part (III) is a "market clearing" condition, so that the Nash equilibrium choices of warriors and guards, the induced numbers of peasants and bandits, and the fixed numbers of praetorians and lords add up to the total population N.

The problem of existence of such a regime, although analogous to the problem of existence of competitive equilibrium in neoclassical economics, is nontrivial. The proposition that follows provides information on existence, uniqueness, and characterization of the short-run lordship regime.

Proposition 3: *Suppose $q(\cdot)$ is concave in its first argument, $p(\cdot)$ is concave, and $f(\cdot)$ satisfies* (5).

(i) *Then, each lord´s payoff, V_l, is concave in n_{gl} and n_{wl} and for any given N_p a Nash equilibrium in n_{gl} and n_{wl} exists.*

(ii) *If the Nash equilibrium n_{wl}'s and n_{gl}'s are continuous functions of N_p on the interval $\left[0, N - \bar{N}_1(1 + n_{pr})\right]$, then a short-run lordship regime exists.*

(iii) *Under any short-run lordship regime, each lord provides the same level of protection.*

(iv) *If the following condition is satisfied*

$$q(n_{wl}, n_{w-l}) = h(n_{wl})/[\sum_{j=1}^{\bar{N}_l} h(n_{wj})] \quad (13)$$

where $h(\cdot)$ is a positive, increasing, and concave function a short-run lordship regime is unique in the number of lords and symmetric, whereby every lord chooses the same number of guards and warriors. In such regimes, (a) the number of peasants is strictly decreasing in the number of lords and (b) each lord's payoff is strictly decreasing in the number of lords. (the proof is in the Appendix.)

The sufficient condition for existence in part (ii) is analogous to the continuity of demand functions in the theory of competitive equilibrium. The properties of the short-run lordship regime in parts (iv), (a) and (b) are intuitively plausible. When an additional lord enters the fray, each lord would increase his number of warriors for a given number of peasants. Since the number of peasants is endogenous, however, their number should decrease in equilibrium with the total number of warriors increasing. A smaller number of peasants shared among a larger number of lords is eventually shown to also yield a smaller payoff for lords.

Additional properties would require employing specific functional forms. For example, consider the following special case of (13):[16]

$$q(n_{w1}, n_{w-1}) = {n_{w1}}^m / (\sum_{j=1}^{N_l} {n_{wj}}^m) \quad \textit{where } 1 \geq m > 0$$

The parameter m is a measure of how easy it is for a lord to increase his dominion when he increases the number of warriors he hires by a small amount, the effectiveness of conflict. Then, under examples for other functions we employed earlier,[17] as the technology of conflict becomes more effective (m increases), the total number of peasants becomes smaller. It appears that this occurs because lords compete more intensely when conflict becomes more effective by hiring additional warriors, without however changing their share of peasants. The effect of this additional demand for manpower is to decrease the population pool from which the peasants are drawn. The end result of an increase in m is a smaller number of peasants, a larger number of warriors per lord, and a smaller number of guards per lord. Such an increase in m also reduces each lord's profits.

[16] Hirshleifer (1989) has examined the properties of this functional form; Skaperdas (1996) has axiomatized it as well as the more general form in (13).

[17] That is, under $p(x) = x$ and the technology of collective protection is $f(z) = z^\beta$ ($\beta \in (0,1)$) the total number of peasants equals

$$N_p' = \frac{2^{\frac{1}{\beta}} \bar{N}_l [N - \bar{N}_l (1 + n_{pr})]}{\bar{N}_l (1 + 2^{\frac{1}{\beta}}) + (\bar{N}_l - 1) 2^{\frac{1}{\beta}} m (1 - \beta)}$$

In the long-run lords should be allowed to exit and potential lords should be allowed to enter and establish their own state. Since lords come from the population, N, we suppose the long-run number of lords is determined by the reservation payoff in this economy, which is the peasant's payoff $U_p^*(= p(x^*)(1-x^*))$. There will be no incentive for lords to exit and potential lords to enter as long as the existing lords earn a payoff that is at least as high as that of peasants, and if an extra lord were to enter he would receive a lower payoff than that of peasants. Let $V_l^{N_l}$ denote an (equilibrium) payoff of lord $l = 1, 2, ..., N_l$ under a short-run lordship regime with N_l lords. We then define a long-run lordship regime to be a short-run lordship regime (that satisfies $(I) - (III)$) and a number of lords $N_l^{'}$ such that

(IV) $V_l^{N_l^{'}} \geq U_p^*$ for all $l = 1, 2, ..., N_l^{'}$ and
$V_l^{N_l^{'}+1} < U_p^*$ for at least one $l = 1, 2, ..., N_l^{'}, N_l^{'} + 1$.

Proposition 4: *Suppose $q(\cdot)$ is concave in its first argument and satisfies (13), $p(\cdot)$ is concave, and $f(\cdot)$ satisfies* (5). *Furthermore, suppose that if only one lord were to exist, he would receive a higher payoff than a peasant. Then,* (i) *a unique (in the number of lords) and symmetric long-run lordship regime exists;* (ii) *the number of peasants and the output of such a long-run lordship regime approximates from above, respectively, the number of peasants and the output under anarchy; in particular:*

$$N_p^{'} = N_p^* + N_l^{'}(V_l^{N_l^{'}} - U_p^*) \tag{14}$$

Part (ii) of the Proposition states that output and the number of peasants is almost the same as those under anarchy. Free entry of lords essentially eliminates all the extra production that can be achieved by the use of the collective protection technology. What was previously taken by bandits under anarchy is now taken by praetorians, warriors, guards, lords, and, possibly, by bandits as well, without essentially affecting the total output that is produced. (If of course lords can extract more efficiently than bandits can, output could be even lower than anarchy.) Literal anarchy is replaced by a more organized, higher-level anarchy of predatory states.

V. Why Self-Governance is Difficult

We now show how self-governing states can not in general coexist in the presence of predatory states that are run by lords, even when there is no free-rider problem in providing for defense against other states. We first define an appropriate notion of short-run equilibrium that allows for both lords and self-governing states to coexist. We then show that, under the examples we have used in various parts of the paper, the equilibrium payoff of peasants belonging to a self-governing state would always fall short of the payoff a peasant could receive in anarchy or under a lord (when $p(x) = \rho(x)$). Thus, it would not be profitable for such a state to form and a long-run equilibrium with self-governing states would not exist.

Suppose there are $\bar{N}_l > 1$ lords and a number of $S \geq 1$ of self-governing states with k peasants each. The lords behave as in the previous section and their payoff functions are as in (12) (except for the slight modification of $q(\cdot)$ below, which has to take account of the warriors of self-governing states). Peasants in self-governing states, in addition to contributing to private and collective protection need to contribute to fighting for their independence by spending some of their time as warriors. Let w_s denote the total resources spent on fighting (external enemies) by self-governing state $s \in \{1, 2, ..., S\}$. We suppose that each citizen-peasant contributes an equal portion w_s/k to fighting; contributions to private and collective protection are as before voluntary. (Clearly, if contributions to fighting external enemies were voluntary, the viability of a self-governing state would be even more problematic.) Thus the payoff of a peasant-citizen is:

$$U_{pi} = p(x_i + f(z))(1 - x_i - z_i - w_s/k) \qquad [z = \sum_{j=1}^{k} z_j/k] \tag{1”}$$

To maintain their independence, the citizens of state $s \in \{1, 2, ..., S\}$ have to expend effort on war, w_s, so that

$$q(w_s, w_{-s}, \bar{n}_w)N_p = k \tag{15}$$

where w_{-s} is the vector of war efforts by all the other self-governing states, $\bar{n}_w$ is the vector of warriors of all the lordships, and $q(\cdot)$ is the contest success function defined in (11) and appropriately modified to include the war effort of the self-governing states. We are now ready to define an appropriate notion of equilibrium for these states, which is an extension of the short-run lordship regime defined in the previous section.

A short-run integrated equilibrium consists of numbers of peasants $N_p^{'}$, bandits $N_b^{'}$; for each lord l guards $n_{gl}^{'}$ and warriors $n_{wl}^{'}$; for each self-governing state s a war effort $w_s^{'}$; and for each citizen-peasant in self-governing states choices of private and collective protection such that:

(Ia) Each lord $l = 1, 2, ..., \bar{N}_l$ takes $N_p^{'}$and the $w_s^{'}s$ as given, and chooses $n_{gl}^{'}$ and $n_{wl}^{'}$ simultaneously with other lords so that these choices form a Nash equilibrium.;

(Ib) Each self-governing state $s = 1, 2, ..., S$ chooses $w_s^{'}$ so that (15) is satisfied;

(Ic) Each citizen-peasant takes $w_s^{'}$ as given and chooses private and collective protection levels so that they form a Nash equilibrium;

(II) $N_b^{'} = \sum_{j=1}^{\bar{N}_l} n_{bj}^{'} + \sum_{s=1}^{S} n_{bs}$ where for all j $n_{bj}^{'} = q_j N_p^{'}(1-p^j)/p(x^*)$, for all s $n_{bs} = kp^s/p(x^*)$; and $p^j$and p^s are the shares of output kept away from bandits in lordship j and self-governing state s;

(III) $N = \sum_{j=1}^{\bar{N}_l} n'_{gj} + \sum_{j=1}^{\bar{N}_l} n'_{wj} + \bar{N}_l n_{pr} + N'_p + \bar{N}_l + N'_b + Sk$

We will now derive the integrated equilibrium under the following functional forms: $p(x) = x$, $f(z) = z^{1/2}$, and the modification of (13) where the share of peasants of lordship j is $q_j = n_{wl}/(\sum_{j=1}^{\bar{N}_l} n_{wj} + \sum_{s=1}^{S} w_s)$.

It can be shown that lords choose to provide perfect security and all choose the same number of guards $n'_{gl} = (N'_p - Sk)/4\bar{N}_l$. Lords also choose the same equilibrium number of warrior $n'_{wl} = (N'_p - Sk)(\bar{N}_l - 1)/\bar{N}_l^2$. All the self-governing states choose war effort $w' = k(\bar{N}_l - 1)/\bar{N}_l$. In turn, all citizen-peasants choose contributions to collective production of $z' = 1/4k^2$ and private protection of $x' = 1/2\bar{N}_l - (2k-1)/8k^2$. The equilibrium payoff of citizen-peasants can be found by substituting w'/k, x', and z in (1"), and equals

$$U'_p = 1/4\bar{N}_l^2 - (2k-1)^2/64k^4. \tag{16}$$

We are interested in comparing this equilibrium citizen-peasant payoff to that of a peasant under a lord, which (since $p(x) = \rho(x)$) also equals the peasant's payoff under anarchy, $p(x^*)(1 - x^*)$. Under the example we are examining this payoff is 1/4, which we need to compare to U'_p in (16). Straightforward algebra shows that $U'_p < 1/4$ holds for all $N_l > 1$ and for all k. Thus a citizen-peasant's payoff under a short-run integrated equilibrium is always lower than the payoff of a peasant under a lord or under anarchy. Consequently, there would be no incentive to form a self-governing state under such circumstances and thus self-governance could not be viable in the long-run.We summarize the finding of this section in the form of a Proposition.

Proposition 5: *Consider the short-run integrated equilibrium under the following functional forms: $p(x) = \rho(x) = x$, $f(z) = z^{1/2}$, and the share of peasants of lordship j is $q_j = n_{wl}/(\sum_{j=1}^{\bar{N}_l} n_{wj} + \sum_{s=1}^{S} w_s)$. Then, the equilibrium payoff of every peasant in a self-governing state is lower than under a lord or under anarchy.*

The burden of defense against other states, imposes such a cost on the individual citizen-peasants so that there are not many resources left for internal protection against bandits and for production.

We should emphasize that we do not completely rule out the possibility of self-governing states being able to survive under some set of functional forms that would allow this to occur. We consider then our counterexample to the coexistence of self-governance and lordships and our inability to find any examples in which this can occur as strong theoretical evidence for the difficulty of self-governance surviving in the presence of predators. Of course, discovering conditions in other models that would yield the viability of self-governance is an important topic for future research.

VI. Concluding Comments

We have examined the provision of protection within a simple and stark context. The framework we have employed has allowed us to make inferences both about the internal organization of the states that could emerge and about their market structure. While self-governance yields higher welfare for predator and prey alike, the small size of self-governing states along with the coercive machinery that can be employed by predatory states make the long-run viability of self-governance problematic. Hence hierarchy and predatory behavior towards subjects is the more stable form of internal organization; and competition among such states for the rents thus created is the dominant market structure. But, contrary to ordinary economic markets, the more competition there is in the market for protection, the worse it is - competing lords and their entourages extract what would have been taken in their absence by simple bandits.

A possibly helpful analogy is to think of the state as an onion, albeit with layers that have different character and color. Layers of autocratic and coercive habits lie below others with more democratic conventions, constitutions, legal codes, ideologies, or norms that govern interactions in most of today's states. Once in a while, something occurs and pierces the modern layers leading to the previous ones that lay dormant. Outbreaks of violence, coercion, and horrors can ensue. Our purpose in this paper has been to improve understanding of what lies in these deeper layers, in the subcortex of the State's brain. In much of economics these outer layers of the state have been taken for granted, a practice that in the somewhat tranquil post-World War II period may have been harmless and typically useful for understanding economic behavior in industrialized countries. But the inner layers of the state have always been making their ugly presence felt in much of the developing world and more recently have systematically confronted transition economies. Ignoring the fundamental problem in providing security and protection, and treating systematic deviations from ideal notions of the state as aberrations would not appear to be a fruitful attitude. Looking into the inner layers of the state is a comparatively easy task, because of their starkness and relative simplicity. Understanding how the outer layers of the modern state, including representative democracy, have appeared among seas of coercive governance appears to be a more difficult task.

APPENDIX

We will employ Claim 1 in the proof of *part* (i) of Proposition 3.

Claim 1: $A(z) \equiv p\left(x^{*}+f\left(z\right)\right)-p\left(x^{*}\right)-p'(x^{*}+f\left(z\right))\, f'\left(z\right) z>0$ for all $z\in[0,1]$ when $p\left(\cdot\right)$ is concave and $f\left(\cdot\right)$ is strictly concave.

Proof: Since $p\left(0\right)=0$ and $p\left(\cdot\right)$ is concave we have $\left[p\left(x^{*}+f\left(z\right)\right)-p\left(x^{*}\right)\right]/f\left(z\right)\geq p'\left(x^{*}+f\left(z\right)\right)$. Therefore, substitution yields:

$$A(z)\geq p'\left(x^{*}+f\left(z\right)\right)\left[f\left(z\right)-f'\left(z\right)z\right]$$

Since $f\left(0\right)=0$ and $f\left(\cdot\right)$ is strictly concave we also have $f\left(z\right)>f'\left(z\right)z$. Hence the term inside the brackets in the right-hand-side of the inequality is positive which, together with the positivity of $p'\left(x^{*}+f\left(z\right)\right)$, implies $A\left(z\right)>0$. ■

Proof of Proposition 3: *Part (i)*: For compactness, let $q^{l}=q\left(n_{wl},n_{w-l}\right), p=p\left(x^{*}+f\left(n_{gl}/\left(q\left(n_{wl},n_{w-l}\right)N_{p}\right)\right)\right)$, and $f=f\left(n_{gl}/\left(q\left(n_{wl},n_{w-l}\right)N_{p}\right)\right)$. Then, V_{l} in (12) is as follows:

$$V_{l}=q^{l}N_{p}\left[p-p\left(x^{*}\right)\right]\left(1-x^{*}\right)-\left(n_{wl}+n_{gl}+n_{pr}\right)p\left(x^{*}\right)\left(1-x^{*}\right) \qquad ((12))$$

To show the concavity of V_{l} in n_{wl} and n_{gl}, we will show that the Hessian of V_{l} (w.r.t. those two variables) is negative definite. Letting q_{1}^{l} and q_{11}^{l} denote the first and second partial derivatives of q^{l} with respect to its first argument (n_{wl}), successive differentiation of V_{l} yields:

$$\partial V_{l}/\partial n_{wl}=\left(1-x^{*}\right)\left\{\left(q_{1}^{l}/q^{l}\right)\left[q^{l}N_{p}\left(p-p\left(x^{*}\right)\right)-p'f'n_{gl}\right]-p\left(x^{*}\right)\right\}$$

$$\partial V_{l}/\partial n_{gl}=\left(1-x^{*}\right)\left(p'f'-p\left(x^{*}\right)\right)$$

$$\partial^{2}V_{l}/\partial n_{wl}^{2}=\left(1-x^{*}\right)\left\{\begin{array}{c}\left(q_{11}^{l}q^{l}N_{p}/q^{l}\right)\left[\left(p-p\left(x^{*}\right)\right)-p'f'\left(n_{gl}/q^{l}N_{p}\right)\right]+\\ +\left(q_{1}^{l}\right)^{2}N_{p}\left[p''\left(f'\right)^{2}+p'f''\right]\left[n_{gl}/\left(q^{1}N_{p}\right)\right]^{2}\end{array}\right\}$$

$$=\left(1-x^{*}\right)\left\{\left(q_{11}^{l}q^{l}N_{p}/q^{l}\right)A+\left(q_{1}^{l}\right)^{2}N_{p}\left[n_{gl}/\left(q^{l}N_{p}\right)\right]^{2}B\right\} \qquad \text{(A1)}$$

where

$$A\equiv\left(p-p\left(x^{*}\right)\right)-p'f'\left(n_{gl}/q^{l}N_{p}\right)$$

and

$$C\equiv p''\left(f'\right)^{2}+p'f'',$$

$$\partial^{2}V_{l}/\partial n_{wl}^{2}=\left(1-x^{*}\right)C/\left(q^{l}N_{p}\right)$$

and

$$\partial^{2}V_{l}/\left(\partial n_{wl}\partial n_{gl}\right)=-Cq_{1}^{l}N_{p}$$

Note that A is the same as $A(z)$, defined in Claim 1, with $z = n_{gl}/\left(q^l N_p\right)$. By Claim 1, then, A is positive. Since $p(\cdot)$ is concave and $f(\cdot)$ is strictly concave, C, as defined above, is negative. Finally, since q is concave in its first argument, q_{11}^l is non-positive. Altogether, those properties readily imply the negativity of both $\partial^2 V_l/\partial n_{wl}^2$ and $\partial^2 V_l/\partial n_{gl}^2$. Consequently, the determinants of the first principal minors of the Hessian of V_l are negative, as is necessary for the concavity of V_l.

The determinant of the Hessian itself is $\overline{H} = \left[\partial^2 V_l/\partial n_{wl}^2\right]\left[\partial^2 V_l/\partial n_{gl}^2\right] - \left[\partial^2 V_l/\left(\partial n_{wl}\partial n_{gl}\right)\right]^2$ which, given the calculations above, can be shown to equal $(1-x^*)\, q_{11}^l AB/q^l$. Given that $q_{11}^l < 0$. $A > 0$, and $C < 0$, that determinant is positive. It follows that the Hessian of V_l is negative definite and, therefore, V_l is concave in n_{wl} and n_{gl}. Then, for the given number of lords, $\overline{N}_l$, and a number of peasants N_p, a Nash equilibrium exists.

Part (ii): Let $g_l(N_p)$ and $w_l(N_p)$ denote the continuous functions mentioned in the "if" part of (ii)'s statement. Then, note that the induced number of bandits for any given N_p, and assuming the lords play Nash equilibrium strategies that induce $g_l(N_p)$ guards and $w_l(N_p)$ warriors for lord l, is a function $B(N_p) = \sum_{l=1}^{\overline{N}_l} b_l(N_p)$ where $b_l(N_p)$ is the induced number of peasants in lord l's territory. Because $b_l(N_p)$ is a continuous function of the numbers of guards and warriors (compare with *part II* of definition of short-run lordship regime), $B(N_p)$ is a continuous function as well.

Thus far, we have shown that, for a given N_p, the induced guards $g_l(N_p)$, warriors $w_l(N_p)$ for $l = 1, ..., \overline{N}_l$, and the induced number of bandits, $B(N_p)$, satisfy *parts I* and *II* of the definition of the short-run lordship regime. To show the existence of that regime, then, amounts to showing the existence of an $N_p^{'}$ that induces numbers of guards, warriors, and bandits that satisfy the following version of *part III* of the regime's definition:

$$\left[N - \overline{N}_l\left(1 + n_{pr}\right)\right] = \sum_{l=1}^{\overline{N}_l} g_l\left(N_p^{'}\right) + \sum_{l=1}^{\overline{N}_l} w_l\left(N_p^{'}\right) + N_p^{'} + B\left(N_p^{'}\right)$$

or, that the function $H(N_p) \equiv \sum_{l=1}^{\overline{N}_l} g_l(N_p) + \sum_{l=1}^{\overline{N}_l} w_l(N_p) + N_p + B(N_p)$ has a point in its domain, $N_p^{'}$, such that $H\left(N_p^{'}\right) = \left[N - \overline{N}_l\left(1 + n_{pr}\right)\right]$. Now, note that for $N_p = 0$ it is optimal for every lord to choose no guards or warriors and thus have $g_l(0) = w_l(0) = 0$. Similarly, since without any peasants around being a bandit provides zero payoff, we must have $B(N_p) = 0$. Hence, we have $H(0) = 0$. Next, note that, since the numbers of guards, warriors, or bandits cannot be negative $H\left(N - \overline{N}_l\left(1 + n_{pr}\right)\right) \geq N - \overline{N}_l\left(1 + n_{pr}\right)$. These two properties along with the continuity of $H(\cdot)$ imply the existence of the $N_p^{'}$ we were looking for, with $n_{wl}^{'} = w_l\left(N_p^{'}\right), n_{gl}^{'} = g_l\left(N_p^{'}\right)$, and $N_b^{\prime} = B\left(N_p^{'}\right)$.

Part (iii): Consider a short-run lordship regime. The same level of protection would be provided by each lord if $p\left[x^* + f\left(n_{gl}^{\prime}/\left(q^l N_p^{\prime}\right)\right)\right]$ were to be identical for all $l = 1, ..., \overline{N}_l$ or, given the costliness of guards and warriors, if $z^l \equiv n_{gl}^{\prime}/\left(q^l N_p^{\prime}\right)$ were also to be identical across lords. First note that $\partial V_l/\partial n_{gl}$ evaluated at $n_{gl} = 0$ equals $(1 - x^*)\left(p'\left(x^*\right) f'(0) - p(x^*)\right) = (1 - x^*)\left(p'\left(x^*\right) f'(0) - p(x^*)(1 - x^*)\right) = (1 - x^*)\, p'\left(x^*\right)\left(f'(0) - (1 - x^*)\right)$ which is positive since the concavity of $f'(\cdot)$ along with $f(0) = 0$ imply $f'(0) \geq 1 > 1 - x^*$. In turn, this property implies that $n_{gl}^{'}$ is always positive for all l. Therefore at the lordship regime values, we must have either

$$\partial V_l/\partial n_g = (1 - x^*)\left[p' x^* + f(z^l)) f'(z^l) - p(x^*)\right] = 0,$$

or $p\left(x^* + f(z^l)\right) = 1$ (where in the latter case $\partial V_l/\partial n_g$ evaluated at z^l would be positive). The solution in terms of z^l, because $f(\cdot)$ is strictly concave and $p(\cdot)$ concave, is in either case unique and identical across the different lords. Therefore, each lord provides the same level of protection.

Part *(iv)*: We first show symmetry and then uniqueness. Consider any short-run lordship regime and let " ' " over a variable denote the value of the variable under the regime. From part *(iii)* we know that $z^l \equiv n_{gl}^{\prime}/\left(q^l N_p^{\prime}\right)$ and $p\left(x^* + f(z^l)\right)$ take the same values for all lords. Therefore, $A \equiv p\left(x^* + f(z^l)\right) - p(x^*) - p'\left(x^* + f(z^l)\right) f'(z^l) z^l$ is identical across the different lords. Then, we can write:

$$\partial V_l/\partial n_{wl} = (1 - x^*)\left[q_1^l N_p A - p(x^*)\right]$$

Note that these derivatives can be different across lords (and, for the same lord, across different points) only by the value of $q_1^l = \partial q\left(n_{wl,} n_{w-l}\right)/\partial n_{wl}$. By the contest success function in (13), it can be shown that

$$q_1^l = h'\left(n_{wl}\right)\left[\sum_{i \neq l} h\left(n_{wi}\right)\right] / \left[\sum_{all\ i} h\left(n_{wi}\right)\right]^2 \tag{A2}$$

Consider any two lords j and k and suppose, contrary to what we want to show, that $n_{wj}^{\prime} > n_{wk}^{\prime}\ (\geq 0)$. Then, by the concavity of the payoff functions, the relationship between these two lords' partial derivatives, each evaluated at the lord's regime point, must be at follows:

$$\partial V_k/\partial n_{wk} \leq \partial V_j/\partial n_{wj} = 0$$

In turn, from the above this relationship implies $q_l^k \leq q_l^j$ or, given (A2),

$$h'\left(n'_{wk}\right)\left[\sum_{i\neq k} h\left(n'_{wi}\right)\right] / \left[\sum_{all\ i} h\left(n'_{wi}\right)\right]^2 \leq h'\left(n'_{wj}\right)\left[\sum_{i\neq k} h\left(n'_{wi}\right)\right] / \left[\sum_{all\ i} h\left(n'_{wi}\right)\right]^2$$

Since the denominators of the two expressions are identical, we also have

$$h'\left(n'_{wk}\right)\left[\sum_{i\neq k} h\left(n'_{wi}\right)\right] \leq h'\left(n'_{wj}\right)\left[\sum_{i\neq k} h\left(n'_{wi}\right)\right] \tag{A3}$$

Since, by supposition, $n'_{wj} > n'_{wk}$ we have $\sum\limits_{i\neq k} h\left(n'_{wi}\right) > \sum\limits_{i\neq k} h\left(n'_{wi}\right)$ and, by the concavity of $h(\cdot)$, $h'\left(n'_{wk}\right) \geq h'\left(n'_{wj}\right)$. These two inequalities, taken together, contradict (A3). Therefore, our original supposition $n'_{wj} > n'_{wk}$ is false. By a similar argument we can show that $n'_{wj} < n'_{wk}$ cannot be true either. Hence, we must have $n'_{wj} = n'_{wk}$ for any two lords j and k. This property, in turn, implies that $q^j N_p = q^k N_p$ and, given that $z^j = z^k$, we also have $n'_{gj} = n'_{gk}$. This establishes that any lordship regime is symmetric.

To show uniqueness, let n'_w and n^2_w denote the choices of warriors associated with two different regimes and w.l.o.g. suppose $n^2_w > n'_w (\geq 0)$. Then, the following relationships would hold between the pairs of derivatives:

$$\begin{gathered} \partial V'_l / \partial n_{wl} \leq \partial V^2_l / \partial n_{wl} = 0 \\ \Longrightarrow q_1^{l'} \leq q_1^{l2} \\ \Longrightarrow h'(n'_w)/h(n'_w) \leq h'(n^2_w)/h(n^2_w) \end{gathered}$$

But the concavity of $h(\cdot)$ along with $n^2_w > n'_w$ contradict this last inequality. Therefore our initial supposition of two different short-run lordship regimes must be false; there is only one symmetric regime.

Part *(iv), (a)*: We have just shown that a unique and symmetric short-run lordship regime exists for any given number of lords. The number of peasants in such a short-run lordship regime is:

$$N_p = N - N_b - N_l(1 + n_{pr} + n_w + n_g), \tag{A4}$$

where all variables are assumed to be at the regime values. From the proof of part (*iii*), it can be shown that, regardless of N_l,

$$n_g = \gamma(N_p/N_l) \text{ for some } \gamma > 0. \tag{A5}$$

That property also implies that the same level of protection is provided across different regimes, that $\overline{p} \equiv p(x^* + f(z))$ does not vary across regimes (and depends only on the technologies of private and collective protection).

In turn, that property along with condition (II) implies that the number of bandits is related to the number of peasants as follows:

$$N_b = \left[(1 - \overline{p})/p(x^*)\right]/N_p. \tag{A6}$$

Using (A5) and (A6), we can eliminate n_g and N_b from (A4), which after re-arranging can be written as:

$$EN_p + N_l(1 + n_{pr} + n_w) = N \text{ where } \mathrm{E} \equiv \left[1 - \overline{p} + p(x^*)(1 + \gamma)\right]/p(x^*) \quad \text{(A4')}$$

If n_w were 0, an increase in the number of lords, N_l, would clearly lead to a reduction in the number of peasants, N_p. Thus, for the rest of this proof we assume an interior (Nash equilibrium) choice of guards ($n_w > 0$). Then, the first-order condition of the symmetric equilibrium under (13) implies:

$$h'(n_w)/h(n_w) - N_l^2 d/\left[N_p(N_l - 1)\right] = 0 \text{ where } d \equiv 1 - \overline{p} + p(x^*)(1 + \gamma) \tag{A7}$$

N_p and n_w are simultaneously determined through (A4') and (A6) and a change in the number of lords also changes the values of these variables. Although we define lordship regimes for integer values of N_l, (A4') and (A6) are defined for real values of N_l. Moreover these functions are differentiable in N_l, as well as N_p and n_w, and the conditions for an implicit function theorem are satisfied. The marginal effect of N_l on N_p can then be shown to be:

$$\partial N_p/\partial N_l = (1/D)\left[-(1 + n_{pr} + n_w)HN_p(N_l - 1) + dN_l^2(N_l - 2)/(N_l - 1)\right] \tag{A8}$$

where

$$D = (d/p(x^*))HN_p(N_l - 1) - N_l(N_l - 1)dh'(n_w)/h(n_w)$$

which is negative since

$$H = h''(n_w)/h(n_w) - \left[h'(n_w)/h(n_w)\right]^2 \leq 0$$

(it is the second derivative of the first argument of $q((\cdot)$ which, by assumption, is concave). Since H is non-positive the term of $\partial N_p/\partial N_l$ inside the brackets is positive and, since D is negative, the effect of an increase in the number of lords on the number of peasants must be negative.

Part *(iv), (b)*: Next we seek to show that each lord's payoff is strictly decreasing in the number of lords. Note that in the symmetric regime the payoff of each lord is as follows:

$$\begin{aligned} V_l &= \left[Total\,output - (N_p + N_b + N_w + N_g + N_{pr})p(x^*)(1 - x^*)\right]/N_l \\ &= \left[N_p(1 - x^*) - (N - N_l)p(x^*)(1 - x^*)\right]/N_l \\ &= (1 - x^*)(N_p + p(x^*)N_l - p(x^*)N)/N_l \\ &= p(x^*)(1 - x^*) + (N_p - N_p^*)/N_l \end{aligned} \tag{A9}$$

Since $p(x^*)(1-x^*)$ and N_p^* are constant and we have just shown that N_p depends negatively on N_l, V_l must also be strictly decreasing on N_l.■

Proof of Proposition 4: *Part (i)*: By the assumptions stated in the Proposition, which are the same as those of Proposition 3, part *(iv)*, a short-run lordship regime exists for any number of lords, which is unique and symmetric - in particular all lords receive the same payoff. Moreover, by part *(iv) (b)* of Proposition 3, the lords' payoff is strictly decreasing in the number of lords. For sufficiently small n_{pr} and with $V^l(N_l = 1) \geq U_p^*$, there is a number of lords that yields a lord's payoff higher than that of a peasant (which equals U_p^*). In addition, we can always find a large enough number of lords (say, N) that yields a payoff to a lord that is lower than U_p^*. Then, since the lords' payoff is strictly decreasing in the number of lords, there must exist a unique number of lords, N_p', that satisfies condition (IV). Therefore, a unique long-run lordship regime exists.

Part (ii): Total output is proportional to the number of peasants (it equals $N_p(1-x^*)$), so we only need consider the number of peasants. From (A9), we have $V_l^{N_l'} - U_p^* = (N_p' - N_p^*)/N_l'$. Solving for N_p' in terms of the other variables yields equation (14) in the main text. Since the payoff of lords is strictly decreasing in the number of lords and, by the definition of a long-run lordship regime, $V_l^{N_l'} - U_p^*$ should be typically rather small, the number of peasants approximates from above the number of peasants under anarchy.■

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