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Costly Enforcement of Property Rights and the Coase Theorem

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Abstract: We examine a setting in which property rights are initially ambiguously defined. Whether the parties go to court to remove the ambiguity or bargain and settle before or after trial, they incur enforcement costs. When the parties bargain, a version of the Coase theorem holds. However, despite the additional costs of going to court, other ex-post inefficiencies, and the absence of incomplete information, going to court may ex-ante Pareto dominate settling out of court. This is especially true in dynamic settings, where obtaining a court decision today saves on future enforcement costs. When the parties do not negotiate and go to court, a simple rule for the initial ambiguous assignment of property rights maximizes net surplus.

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1 Introduction

The meaning and implications of the Coase theorem are far from being settled more than four decades after its initial formulation in Coase [10]. For instance, Usher [36] has recently provided an elementary scrutiny of the possible meanings of the theorem and found them wanting, as suggested by the provocative title of his article: "The Coase Theorem is Tautological, Incoherent, or Wrong." At this late date, one would expect to have such issues settled, but it is surprising how little systematic follow-up has been to Coase's own call for examining positive transaction costs (see for example Coase [11], page 717).

Research on incomplete information as a source of transaction costs shows that the initial assignment of property rights matters for efficiency (see for example Myerson and Satterthwaite [31]; McKelvey and Page [30]. What is harder to determine, however, is whether a third party with limited information – a "bumbling bureaucrat" in Farrell's [18] terminology – can pick the efficient property rights structure.

This paper focuses on a relatively neglected aspect of the study of property rights: the fact that they are often costly to enforce and can be a significant source of transaction costs.[1] Apart from resorting to violence or the threat of it – a not uncommon form of enforcement in much of the world today – there are costs in securing title to assets in all economies. Even in developed economies it can be costly to enforce rights in real estate, intellectual property and nuisance disputes. In this paper we rule out income effects, incomplete information, bargaining costs and other asymmetries and focus on these costs. Specifically, we focus on the effect of litigation costs incurred in securing either a better settlement or a favorable court decision.[2]

Well-defined property rights create a starting point from which contracting can occur. If individuals are certain about what the initial legal regime is, then they will also know precisely which contracts are individually rational for them, and which are not. Thus, if property rights are perfectly clear (or are unclear but are costless to clarify), this would trivially be the end of the story. But if property rights are ill-defined, initial endowments, initial utilities and final utilities will be uncertain, and the parties may wish to take potentially costly actions to help determine the set of individually rational, mutually beneficial trades. We assume that parties cannot contract directly over these enforcement activities. After all, if individuals could mutually commit not to engage in these costly activities, the analysis would be trivial.

We examine a static setting in which parties can bargain over the distribution of an expected surplus both before and after going to court. Bargaining and settlement increase the size

[1] Notable exceptions include Stigler [35] and Becker and Stigler [3].

[2] The analysis of Anderlini and Felli [2] is complementary to ours. They focus on the role of up-front bargaining costs in sometimes foreclosing mutually beneficial exchange, even in the presence of complete information. We concentrate on the role of costs that are expended during the process of bargaining and which affect the bargaining outcome itself.

of the surplus and save on some enforcement costs. Pre-trial settlement is subgame perfect, and equilibrium enforcement efforts are independent of the initial imperfect assignment of property rights. However, parties can gain by committing not to bargain at all: if bargaining is ruled out, at least one party can be better off than if bargaining had been possible at the outset. Then, the pre-trial assignment of imperfect property rights matters for efficiency and a simple rule determines who should be assigned that right, even if it cannot be assigned perfectly.

Because going to court resolves all or part of the uncertainty over property rights, court decisions can reduce future enforcement costs. For a wide set of conditions in the multi-period version of our model, going to court in the first period is a subgame perfect equilibrium despite the absence of incomplete information or other complications typically associated with conflict.[3] Post-trial bargaining and settlement can still take place, and if it does a version of the Coase theorem holds. As in the static model, however, committing not to bargain can be efficient, and a bumbling bureaucrat could follow a simple rule for assigning the efficient property rights structure. Our analysis has interesting implications for the literature on the economics of trials, where to our knowledge it has yet to be rigorously demonstrated that going to court is a mutually-preferred outcome even when the players' perceptions are identical and information is complete.[4]

2 The Basic Setting

2.1 The Rancher, the Farmer, and Ambiguously Defined Property Rights

We consider two parties, a *farmer* (f) and a *rancher* (r). The rancher undertakes an activity (say, raising cattle) that produces output $x \geq 0$ which yields profits or private benefits $B(x) \geq 0$. B is strictly concave, and there is a unique $x_r > 0$ which maximizes the rancher's benefit, so that B is increasing for $x < x_r$ and decreasing for $x > x_r$. The rancher's production of x generates an external cost of $C(x) \geq 0$ to the farmer by, for example, having the cattle trample some of the farmer's crops. C is increasing and strictly convex. We further assume that $B(0) = C(0) = 0$, and that $C(x) < B(x)$ for at least one $x > 0$. The farmer's privately optimal level of x is clearly $x_f = 0$. Let x^* denote the socially optimal level of production, so that $x^* = \arg\max_x\{B(x) - C(x)\}$. The assumptions on $B(x)$ and $C(x)$ ensure that such a socially optimal level of x exists, and that it is unique. It is also straightforward to show that $0 < x^* < x_r$; that is, the profit-maximizing level of the rancher's activity is higher than the socially optimal level of the activity which, in turn, is higher than what the

[3] An analogous result for the occurrence of conflict and war has been shown in Garfinkel and Skaperdas [20]. In that setting, conflict can occur despite its costly nature because in dynamic setting there are compounding rewards to the winner and savings of future resources.

[4] Cooter and Rubinfeld [12] and Hay and Spier [24] are surveys of the literature.

farmer would prefer.

In the absence of third-party enforcement, laws, or any norms about who has the right to choose the level of activity x, private enforcement through the threat of violence would be the typical condition. However, Coase's own writings and much of the subsequent literature presupposes the existence of laws, courts, enforcement, and assignment of property rights, so in the remainder we assume the presence of these institutions, although a limiting case of our model could apply to the case of violence as well.[5]

We assume that the parties can clarify their legal positions (effectively, their initial endowments) by going to court and engaging in a *probabilistic contest*. The winner is awarded the right to choose x unilaterally, but the actual outcome is ex ante uncertain. The players can influence their winning probabilities by investing in "enforcement activities" – the costs of hiring of counsel and expert witnesses, payments to other legal and scientific researchers and private investigators, and other disbursements associated with the civil litigation process. The win probabilities depend directly on these enforcement activities and obey a *contest success function* (Skaperdas [34]). If e_f and e_r are the amounts that the farmer and the rancher invest, the rancher's winning probability is:

$$p(e_r, e_f) \equiv \begin{cases} \dfrac{\varphi f(e_r)}{\varphi f(e_r) + (1-\varphi) f(e_f)} & \text{if } e_r + e_f > 0 \\ \varphi & \text{otherwise} \end{cases} \tag{1}$$

where $\varphi \in (0,1)$, $f(0) = 0$, $f' > 0$ and $f'' \leq 0$.[6] The parameter φ is common knowledge and describes the pre-trial legal environment. Assuming $\varphi = 1$ or $\varphi = 0$ represents the situation in which binding legal precedent, legislation, or the facts of the case completely favor either the rancher or the farmer in gaining the right to choose x. Any other value of $\varphi \in (0,1)$ represents a situation where the legal or factual situation is not completely biased in favor of either party. We can also think of φ as a measure of the degree of ambiguity of property rights, or even the general "effectiveness" of the legal system, where effectiveness refers to the law's ability to generate and sustain well defined, widely applicable rules. [7]

These institutional features are difficult to change, even at the margin. Therefore, we further assume that the ambiguity of rights has the following simple structure: φ (and $1-\varphi$) can only take one of two values, φ' or $1-\varphi'$ (where $\varphi' > 1/2$). When $\varphi = \varphi'$, then the rancher possesses the ambiguous right to set x, and when $\varphi = 1-\varphi'$ the has the ambiguous right to

[5]Benson [ ] examines the development of private dispute-resolution markets in which arbitration services are supplied by private firms and parties agree to be bound by their decisions. Our analysis also applies to this environment.

[6]Clark and Riis [9] axiomatize the case $f(e) = e^m$, and show in an $n-$player setting that the assumption of φ entering multiplicatively in the CSF is logically equivalent to Luce's [29] Choice Axiom. Cornes and Hartley [13] use the same assumptions on f to prove existence and uniqueness in pure strategies. Farmer and Pecorino [16], Bernardo et. al. [5], and Hirshleifer and Osborne [26] all analyse legal contests using this functional form.

[7]Hirshleifer and Osborne [26] assume that φ represents a legal "fault factor" or the "advantage of having truth on one's side." Katz [28] and Farmer and Pecorino [16] call φ the "objective merits of the case."

set x. The *level* of φ' is beyond the control of government officials and the two parties. It can be changed only through major, long-term changes in governance. However, the particular initial *assignment* of the ambiguous right can be made in advance by administrative decision or regulation. Its ambiguity is due to the fact it can be challenged in court. Given this legal contest, we study a game with the following structure:

Stage 1: *Pre-trial discovery or litigation preparation* - Both parties invest some initial enforcement efforts e_r and e_f, which, if they were to end up in court, would influence their winning probabilities.

Stage 2: *Pre-trial negotiation* - The parties negotiate and possibly reach a pre-trial settlement, in which case the game ends.

Stage 3: *Trial* - If no settlement takes place, the case goes to court and the parties expend additional resources of βe_r and βe_f on the actual trial process.

Stage 4: *Post-trial negotiation* - After the court makes its decision, the parties again have an opportunity to engage in negotiation and reach a post-trial settlement.

2.2 Going to Court

Even though the parties may have an incentive to settle, many legal disputes actually *do* end up in court, so there may be good reasons for examining alternative scenarios to bargaining. Our first task then is to study the game consisting of stages 1 and 3 only, where the possibility of negotiation at any stage is completely ruled out. If both parties are risk neutral and the "American rule" (each party pays their own legal costs) is in place, [8] the expected payoffs are:

$$V_r^c \equiv p(e_r, e_f)B_r - (1+\beta)e_r \quad \text{and} \quad V_f^c \equiv -p(e_r, e_f)C_r - (1+\beta)e_f \tag{2}$$

where $B_r = B(x_r)$, $C_r = C(x_r)$, and $\beta > 0$. βe_r is the additional cost of actually going to court as opposed to just preparing to go to court.[9] Equating the two first order conditions for the equilibrium efforts e_r^c and e_f^c yields the equilibrium condition:

$$\frac{f'(e_r^c)}{f(e_f^c)} = c\frac{f'(e_f^c)}{f(e_f^c)} \tag{3}$$

where $c \equiv C_r/B_r > 0$ is the ratio of social costs to benefits at x_r. Equilibrium efforts are closely related to c and the ambiguity parameter φ.

[8] As opposed to the "English rule" in which the loser pays the winner's costs.

[9] There are different ways of modeling the costs of going to court: a fixed cost, a "melting" of part of the pie that is contested, and so on. None of out results depend on the particular way we model the costs of going to court.

Proposition 1 *If the two parties expect to go to court:*
(i) The party with more at stake devotes more resources to enforcement activities:

$$e_r^c \lesseqqgtr e_f^c \quad \text{as } c \gtreqqless 1$$

(ii) Given an unchangeable level of property rights ambiguity equal to $\varphi' > 1/2$, it is efficient to assign these rights to the party that has more at stake (to the rancher if $c < 1$, and to the farmer if $c > 1$); and
(iii) If $f(e) = e$ the enforcement costs and expected payoffs are:

$$(1+\beta)e_r^c = (1+\beta)\frac{\varphi(1-\varphi)cB_r}{(1+\beta)\left[\varphi+(1-\varphi)c\right]^2}\ ,\ (1+\beta)e_f^c = (1+\beta)\frac{\varphi(1-\varphi)cC_r}{(1+\beta)\left[\varphi+(1-\varphi)c\right]^2} \tag{4}$$

$$V_r^c(e_r^c, e_f^c) = \frac{\varphi^2 B_r}{(\varphi+(1-\varphi)c)^2} \ \text{and}\ V_f^c(e_r^c, e_f^c) = -\frac{\varphi(\varphi+2(1-\varphi)c)C_r}{(\varphi+(1-\varphi)c)^2} \tag{5}$$

Unless otherwise noted, all proofs are to be found in the Appendix. If the cost that the farmer is trying to avoid is greater (lower) than the benefit that the rancher will receive, then the farmer will exert greater (lower) effort. Proposition 2 below shows that this asymmetry does not occur when pre-trial bargaining is allowed, because the parties split all costs and benefits and bargaining is assumed to be symmetric. The task of efficiently assigning the initial ambiguous property right to the rancher ($\varphi = \varphi'$) or to the farmer ($\varphi = 1 - \varphi'$) depends only on the value of c and – as we now show – requires no knowledge of whether the parties will actually go to court or not.[10]

2.3 Pre-Trial Bargaining and Settlement: A Coasean Result

If the parties go to court one of them wins the right to choose x and post-trial bargaining could occur. Since the total surplus is maximized at x^* and has a value of $S^* = B(x^*) - C(x^*)$, the winning party could still be induced to choose x^* at the post-trial stage in exchange for a transfer from the loser. But of course the parties may wish to avoid costly trials altogether, so we now turn to the full game consisting of all four stages outlined at the end of section 2.1.

For pre-trial bargaining to occur, the two parties would have to agree on a choice of x (which we can assume to be x^*) and on a self-enforcing transfer that deters each of them from going to court. Given e_r and e_f, individuals can bargain over the surplus, but initial enforcement efforts e_r and e_f are *non-contractible.* If they were not, the parties could trivially agree to

[10] Glazer and Konrad [21] show that individual and aggregate tax-inclusive efforts in perfectly discriminating contests are unchanged if there is a proportional tax placed on them, suggesting that enforcement costs cannot be reduced using this policy tool. On the other hand, *non-linear* taxation schemes and cost-shifting rules *may* affect tax-inclusive equilibrium efforts (see, for example, Farmer and Pecorino [16]).

avoid them. In some countries parties can (under certain conditions) write contracts not to sue or not to go to trial. But this is by no means a universal legal practice, and even with such agreements there is the problem of cheating: parties can freely hire lawyers, explore litigation, and even litigate matters that are not directly related to the original agreements. Our non-contractibility assumption is similar to the incomplete contracts approach to the theory of the firm (see, for example, Grossman and Hart [22] or Hart [23]) in which the non-contractible quantities are relationship-specific investments.

Following standard practice we assume that bargaining outcomes depend on the surplus available for division and the disagreement (or threat) utilities that each party receives if bargaining breaks down. We also assume that the parties split the surplus, an assumption which is quite reasonable here: there are (intentionally) no income effects, so we have transferable utility, the Pareto frontier is a straight line, and the split-the-surplus solution coincides not only with the Nash bargaining solution but also with any other symmetric bargaining solution.[11] It is also the only bargaining outcome that would not provide one side with more exogenous bargaining power than the other.

We assume that neither party will engage in a subgame-imperfect manner. Since information is complete and going to court is costly, we can expect the parties to settle at the second stage and not go to court at all.[12] The two versions of the Coase theorem that are relevant for our analysis are:

The Efficiency Version: If transactions costs are sufficiently low, parties will agree on one of many possible efficient choices of x, but the presence of income effects means that this x will depend on the initial assignment of property rights.

The Invariance/Neutrality Version: Due to the absence of income effects there is a unique efficient level of x, and so if transactions costs are sufficiently low, any agreed-upon x cannot possibly vary with the initial assignment of property rights.[13]

Part (ii) of Proposition 1 shows that in the complete absence of bargaining, the assignment of ambiguous property rights matters for efficiency, and unsurprisingly neither version of the Coase theorem applies in that setting. But when the parties expect to engage in pretrial

[11] Binmore et al [6] explore noncooperative implementations of this solution. The appropriate noncooperative game in our case is one in which there is an exogenous risk of breakdown of the bargaining process. When utility is not transferable, different symmetric bargaining solutions can have qualitatively different outcomes that can even be Pareto ranked in some instances – see Anbarci et al [1].

[12] The structure of our model is similar to the general model of bargaining under uncertainty and "prior agreements" analyzed by Riddell [32]. In Riddell's setup the probabilities of various states of the world are exogenous, whereas in our model these probabilities can be influenced by both players.

[13] The efficiency version and the invariance/neutrality version are spelled out and analysed in detail by Hurwicz [27]. Hurwicz shows that the assumption of "parallel" preferences (or quasi-linear utility functions) is both sufficient *and* necessary for the invariance/neutrality version of the Coase Theorem to hold. For further discussions see Chipman [8], Varian [38] and Ulen [36].

bargaining, the particular assignment of such ambiguous property rights does not matter, and we obtain a version of the Coase theorem in a world of costly enforcement and ambiguously defined property rights.

Proposition 2 *If the parties expect to engage in pre-trial bargaining, then:*
(i) Their payoffs are:

$$V_r^b(e_r, e_f) \equiv \frac{S^*}{2} + p(e_r, e_f)\left(\frac{B_r + C_r}{2}\right) + \frac{\beta}{2}e_f - \left(1 + \frac{\beta}{2}\right)e_r \tag{6}$$

and

$$V_f^b(e_r, e_f) \equiv \frac{S^*}{2} - p(e_r, e_f)\left(\frac{B_r + C_r}{2}\right) + \frac{\beta}{2}e_r - \left(1 + \frac{\beta}{2}\right)e_f \tag{7}$$

and would be the same even if post-trial negotiation (stage 4) did not occur.
(ii) In equilibrium the parties choose identical enforcement efforts of e^b*, which is implicitly defined by:*

$$\frac{\varphi(1-\varphi)f'(e^b)}{f(e^b)}\frac{B_r + C_r}{2} = 1 + \frac{\beta}{2} \tag{8}$$

(iii) Given a level of ambiguity of property rights of $\varphi' > 1/2$*, an invariance version of the Coase theorem holds: both enforcement efforts and the net surplus available for division are independent of the initial ambiguous assignment of property rights*
(iv) The more ambiguous are property rights (the closer is φ' *to* $1/2$*), the higher are the equilibrium enforcement efforts and the lower is the net surplus; and*
(v) If $f(e) = e$*, the equilibrium efforts and payoffs are:*

$$e^b = \frac{\varphi(1-\varphi)(B_r + C_r)}{2+\beta} \tag{9}$$

$$V_r^b(e^b, e^b) = \frac{S^*}{2} + \varphi\frac{(\beta + 2\varphi)(B_r + C_r)}{2(2+\beta)} \quad \textit{and} \quad V_f^b(e^b, e^b) = \frac{S^*}{2} - \varphi\frac{(4 + \beta - 2\varphi)(B_r + C_r)}{2(2+\beta)} \tag{10}$$

The first term in each of the payoff functions (6) and (7) represents the share of the total surplus S^* received by each party. The remaining terms reflect the relative disagreement payoffs of the two parties and the bargaining power that emanates from them. The higher the winning probability of the rancher, the higher is the rancher's benefit B_r and the cost to the farmer C_r, the higher is the rancher's payoff and the lower is the farmer's payoff. The potential costs of going to court are shared equally, because bargaining takes place before they are incurred. Because bargaining is costless, the transferable utility assumption implies that in pre-trial bargaining the parties can take full account of what could occur in future stages, and so the payoffs do not depend on whether post-trial bargaining is possible.

Part (iii) of Proposition 2 states that if an administrator or regulator has ambiguously assigned the right to choose x to the rancher ($\varphi = \varphi'$) or the farmer ($\varphi = 1 - \varphi'$) before the game starts, then if the parties expect to bargain at the pre-trial stage this assignment makes no difference for the size of the net surplus. This Coasean result follows because the term $\varphi(1-\varphi)$ in (8) equals $\varphi'(1-\varphi')$, regardless of whether $\varphi = \varphi'$ or $\varphi = 1-\varphi'$. Since this is the only place that the value of φ enters in the determination of the common equilibrium effort e^b, this effort is independent of the administrator's initial choice. And since the surplus S^* is fixed, any variations in efficiency can only occur through variations in the *level* of efforts. Efforts do not vary with the *assignment* of rights, so the net surplus does not depend on the assignment of rights either – and this result does not depend on the particular value of φ. In other words, if the underlying conditions (absence of income effects) ensure that the invariance version of the Coase theorem holds when $\varphi = 0$, those same assumptions suffice to ensure that the invariance version will also hold even in a world of ambiguously defined property rights, when $\varphi > 0$.

2.4 Are There Mutual Gains from Committing Not to Bargain?

Whilst settlement does not involve the additional cost of going to court and parties agree that $x = x^*$, going to court could still be better for one or even both parties if the equilibrium enforcement costs are sufficiently low compared to those under settlement. To illustrate how this can happen, we consider the case where $f(e) = e$. Using (4) and (9), it is straightforward to show that settlement entails higher enforcement costs if and only if $\frac{2}{2+\beta} > \frac{c}{(\varphi+(1-\varphi)c)^2}$, which is satisfied when c is sufficiently small or sufficiently large.[14] Moreover, the payoffs in (10) could well be lower than the respective payoffs in (5).

Proposition 3 *At least one party may ex ante prefer going to court over pre-trial bargaining and settlement.*

If the parties go to court, greater payoff asymmetry creates disincentives to exert higher effort. If the effects of x_r on each party are sufficiently different (which can occur in the absence of settlement), both parties exert very low enforcement efforts, and the additional cost of going to court (embodied in β) can be overcome. If one or both parties were to ex ante prefer to go to court no matter what, we are back in the world of section 2.1, rather than section 2.2. The structure of the full game would have to be modified to allow this conflictual outcome as a subgame perfect equilibrium, because, regardless of the initial choice of enforcement efforts, if stage 2 is ever reached both parties would always prefer to settle. One party could, for example, commit in advance not to bargain by engaging in a "burn-the-bridges" act which eliminates any possibility of negotiation. We could also think of the same outcome obtaining if bargaining costs (not modelled in our analysis) are sufficiently high.

[14] The relevant inequality holds if $c > \frac{[2+\beta-4\varphi(1-\varphi)]+\{(2+\beta)[(2+\beta)-8\varphi(1-\varphi)]\}^{1/2}}{4\varphi^2}$ or if $c < \frac{[2+\beta-4\varphi(1-\varphi)]-\{(2+\beta)[(2+\beta)-8\varphi(1-\varphi)]\}^{1/2}}{4\varphi^2}$.

Coase [10], Demsetz [15] and others have suggested simple rules for assigning *unambiguous* property rights that are similar to ours. The static analysis in the preceding sections suggests that their results are robust: the net surplus is also maximized (and aggregate enforcement costs are minimised) when simple rules based on c are followed even when property rights cannot be assigned unambiguously. If an administrator, regulator or "bumbling bureaucrat" only has information about the cost-benefit ratio (c) and has no knowledge of whether the parties would actually want to go to court or not, the efficient action is to always assign the ambiguous property right to the party with the higher stake.

3 When the Future Casts its Shadow

The previous section examined a setting with a one-time interaction between the two parties or, trivially, as a multi-period repetition of the same exact conditions and outcomes in every period. However, once the time dimension is brought in, non-trivial dynamic considerations enter the picture. On the one hand, if one side has the ambiguous property right and agrees to settle early, could the property right become even more atrophied in the future (see Buchanan [7])? On the other hand, when courts make decisions they strengthen the property rights of the winner and presumably reduce or eliminate future enforcement costs. Such considerations might drive one or both parties to go to court in the first instance. To examine this possibility we consider a non-trivial dynamic extension of the model and for simplicity we allow for two periods.[15] The first period involves exactly the same characteristics and stages of the static model. If the parties have not gone to court in the first period, the second period also has the same characteristics and stage of the static model. If, however, the parties have gone to court in the first period, the court's decision also stands in the second period, and the winning party also gains the complete right to choose x in the second period. [16]

Both parties discount the second period by the factor $\delta \in (0, 1]$. We do not explicitly model the possibility that negotiation and settlement could erode one's property right, but it will become clear that our findings would, if anything, be strengthened by allowing for such a possibility. Before continuing we should re-emphasize that the parties cannot write binding contracts over enforcement costs. These costs can be eliminated in the second period but only if this can be induced as part of a subgame perfect equilibrium. This is only possible when a court decision has unambiguously assigned property rights in the first period (so

[15] The main ideas are easily generalizable to a finite horizon of arbitrary length and, with appropriate modifications, to an infinite horizon.

[16] The right in the second period need not be perfectly defined but just strengthened relative to the first period, without changing the nature of the results. For example, after the court's decision in period 1, the winner's still ambiguous right in the second period could equal $\varphi'' > \varphi' > 1/2$, where φ' is the favored party's period 1 right. That approach could be further generalized by allowing a greater number of periods, with each court decision refining the property right of the winner. The highest court's decision could be thought of as providing the perfectly defined property right. Thus, our approach is equivalent to the court's decision in the first period being final or not allowing any appeals.

$\varphi = 0$ or 1 at the start of period 2) and equilibrium second period efforts become zero.

3.1 Going to Court with Post-Trial Bargaining

We first show that going to court in the first period and then bargaining and settling in the second period is a subgame perfect equilibrium under a reasonable set of conditions. In the one-period model we have seen that whether post-trial bargaining can take place or not does not make a difference for pre-trial bargaining [Proposition 2, part (i)]. However, in the two-period model that we have just outlined, the resolution of uncertainty following a court decision has implications for the future that it did not have in the one-period model. Such a decision implies that one party has gained the unambiguous right to choose x both now and in the future and thus the two parties do not have to incur any enforcement costs in the second period. By contrast, if a pre-trial settlement were to be reached in stage 3 of period 1, enforcement costs will typically have to be incurred in the second period.

Consider any (e_r^1, e_f^1) pair of enforcements efforts that have been incurred in stage 1 of period 1. To derive the threat payoffs at stage 2, we need to first examine what would occur in stage 4 once a court decision has been made. At that stage each part has paid $(1+\beta)\, e_i^1$ (for $i = r, f$). These costs are sunk and do not play any role in post-trial bargaining. There are two possible bargaining outcomes, depending on who has won in court. If the rancher has won, the rancher's threat payoff over the two periods would be $(1+\delta)B_r$ whereas the farmer's threat payoff would be $-(1+\delta)C_r$. Given that the surplus over the two periods is $(1+\delta)S^*$ and no enforcement costs are incurred in the second period, the split-the-surplus rule would imply the following payoffs if the rancher were to win:

$$W_{rr} = \frac{(1+\delta)(S^* + B_r + C_r)}{2} \quad \text{and } W_{fr} = \frac{(1+\delta)(S^* - B_r - C_r)}{2}$$

If the farmer has won in court, then the threat payoffs for either party would be 0 (since the optimal choice of x for the farmer is 0 and $B(0) = C(0)$). The ex-post bargaining payoffs in that case would be:

$$W_{rf} = \frac{(1+\delta)S^*}{2} \quad \text{and} \quad W_{ff} = \frac{(1+\delta)S^*}{2}$$

Let $p^1 \equiv \dfrac{\varphi f(e_r^1)}{[\varphi f(e_r^1) + (1-\varphi) f(e_f^1)]}$. Then the expected two-period payoffs before going to court in stage 3 of period 1 are:

$$W_r^e = p^1 W_{rr} + (1-p^1) W_{fr} - \beta e_r^1 \text{ and } W_f^e = p^1 W_{fr} + (1-p^1) W_{ff} - \beta e_f^1$$

and substitution from the expressions above yields:

$$W_r^e = \frac{(1+\delta)S^*}{2} + p^1 \frac{(1+\delta)(1+c)B_r}{2} - \beta e_r^1 \tag{11}$$

$$W_f^e = \frac{(1+\delta)S^*}{2} - p^1 \frac{(1+\delta)(1+c)B_r}{2} - \beta e_f^1 \tag{12}$$

These are the expected payoffs of going to court at the beginning of stage 3 in period 1. If the parties did not to go to court, they represent the threat payoffs at the beginning of stage 2 in period 1. Settlement at that stage will occur if and only if the surplus under settlement exceeds $W_r^e + W_f^e = (1+\delta)S^* - \beta(e_r^1 + e_f^1)$, the sum of the parties' expected payoffs of going to court. The surplus from settlement equals $(1+\delta)S^*$ minus any additional enforcement costs. Because no court costs would be incurred in period 1, there would be no additional enforcement costs in period 1. In the second period, however, the parties would face exactly the same conditions as those in the one-period model and therefore they would each incur the equilibrium cost of e^b. Thus the net payoff from settlement would be $(1+\delta)S^* - 2\delta e^b$. Comparing this to the surplus of going to court and then bargaining, we determine that the parties will go to court in stage 3 of period 1 if and only if:

$$2\delta e^b > \beta(e_r^1 + e_f^1) \tag{13}$$

The two parties will thus go to court if the enforcement efforts chosen in the first period are sufficiently small, if there are low marginal cost of going to court (i.e., low β), low discounting of the future (high δ), or high one-period equilibrium efforts e^b.

To determine whether equilibrium efforts will ever satisfy (13) first we need to define the appropriate payoff functions. For (e_r^1, e_f^1) combinations that satisfy (13), the parties will go to court and engage in post-trial bargaining in stage 4 of period 1. Otherwise, the parties will reach a pre-trial settlement and split the surplus $(1+\delta)S^* - 2\delta e^b$ with the payoffs in (11) and (12) as threat payoffs. So, the two-period payoff functions are:

$$W_r(e_r^1, e_f^1) = \begin{cases} \frac{(1+\delta)S^*}{2} + p^1\frac{(1+\delta)(1+c)B_r}{2} - (1+\beta)e_r^1 & \text{if } 2\delta e^b > \beta(e_r^1 + e_f^1) \\ \frac{(1+\delta)S^*}{2} - \delta e^b + p^1\frac{(1+\delta)(1+c)B_r}{2} + \frac{\beta}{2}e_f^1 - (1+\frac{\beta}{2})e_r^1 & \text{otherwise} \end{cases} \tag{14}$$

and

$$W_f(e_r^1, e_f^1) = \begin{cases} \frac{(1+\delta)S^*}{2} - p^1\frac{(1+\delta)(1+c)B_r}{2} - (1+\beta)e_f^1 & \text{if } 2\delta e^b > \beta(e_r^1 + e_f^1) \\ \frac{(1+\delta)S^*}{2} - \delta e^b - p^1\frac{(1+\delta)(1+c)B_r}{2} + \frac{\beta}{2}e_r^1 - (1+\frac{\beta}{2})e_f^1 & \text{otherwise} \end{cases} \tag{15}$$

For $f(e) = e$, it can be shown that going-to-court occurs if and only if $\frac{\beta}{2-\beta} < \delta$, or when the marginal cost of going to court is not too high and the second period is not discounted too heavily.[17] This result can be shown to be true more generally. Moreover, regardless of whether the two parties go to court,the initial assignment of property rights does not affect total enforcement efforts.

Proposition 4 *In the two-period model, where going to court in the first period determines who has the property rights in both periods:*

[17]From (9), we have $e^b = \frac{\varphi(1-\varphi)(1+c)B_r}{2+\beta}$. Assuming $2\delta e^b > \beta(e_r^1 + e_f^1)$ in (14) and (15), we find that $e_r^1 = e_f^1 = e^p = \frac{\varphi(1-\varphi)((1+\delta)1+c)B_r}{2(1+\beta)}$. It is straightforward to show that $2\delta e^b > \beta(e_r^1 + e_f^1) = 2\beta e^p$ if and only if $\frac{\beta(2+\beta)}{1+\beta} < \frac{2\delta}{1+\delta}$, which in turn is equivalent to $\frac{\beta}{2-\beta} < \delta$.

(i) There are combinations of costs of going to court (β) and discount factors (δ) for which going to court and post-trial settlement is the subgame perfect equilibrium; and
(ii) Irrespective of whether the two parties engage in pre or post-trial settlement in equilibrium, if the level of ambiguity of property rights is $\varphi' > 1/2$, the equilibrium efforts and the net surplus available for division between the two parties are independent of the initial assignment of these rights.

3.2 Going to Court Without Settlement

Even when the parties go to court, part *(ii)* of Proposition 4 shows that an invariance version of the Coase theorem holds. The fact that there is settlement after the trial is crucial for this result, because post-trial settlement allows the two parties to split the prize that they seeking, which in turn induces identical enforcement efforts in equilibrium.

There are, however, at least two potential problems with bargaining and settlement in a dynamic context. First, as mentioned earlier, any kind of bargaining – whether pre or post trial – might lead to erosion of a party's property right. If for example the farmer had acquired the right to choose x but acquiesced to choose x^* in exchange for some transfer from the rancher, the rancher could possibly use that choice of x as evidence against the rancher's right at some point in the future.

Second, evidence suggests that very little bargaining (if any) actually takes place after court decisions are made. For example, Farnsworth [17] "examines twenty nuisance cases and finds no bargaining after judgment in any of them; nor did the parties' lawyers believe that bargaining would have occurred if judgment had been given to the loser. The lawyers said that the possibility of such bargaining was foreclosed not by the sorts of transaction costs that usually are the subject of economic models, but by animosity between the parties and their distaste for bargaining over the rights at issue." Animosity and the use of emotions for strategic purposes has been noted by some economists (Schelling [33; Hirshleifer [25] Chapter 10) as a commitment device. Is it possible, (as it was in the static model) that going to court and not bargaining could yield higher ex ante payoffs than those that allow for bargaining?

To answer this question, we first define the two-period payoff functions when both parties expect to go to court in period 1 and not engage in post-trial bargaining:

$$W_r^c(e_r^1, e_f^1) \equiv p(e_r^1, e_f^1)(1+\delta)B_r - (1+\beta)e_r^1 \tag{16}$$

and

$$W_f^c(e_r^1, e_f^1) \equiv -p(e_r^1, e_f^1)(1+\delta)C_r - (1+\beta)e_f^1 \tag{17}$$

Note that these expected payoff functions differ from those of the one-period model in (2) only in that the first term of each of them is multiplied by $(1+\delta)$. Since the parties will go to court in the first period, the court's decision will determine who has the property right in both periods, and no bargaining will ever take place; what matters is the total "prize" over the two periods which is the sum of the first period prize and the discounted sum of the second period prize. All enforcement effort is undertaken in the first period.

Given this similarity, it is trivial to show the same properties of equilibrium for the two-period model as those described in Proposition 1. Furthermore, although the welfare comparisons are not exactly the same in the two-period model as they were in the one-period model, a two-period version of Proposition 3 holds here as well: going to court can be better for at least one party than allowing any bargaining. Summarizing:

Proposition 5 *In the two-period model, suppose the two parties expect to go to court and their payoff functions are described in (16) and (17). Then:*
(i) The party with more at stake devotes more resources to enforcement activities:

$$e_r^{1c} \lesseqqgtr e_f^{1c} \quad \text{as } c \gtreqqless 1$$

(ii) Given a level of ambiguity of property rights of $\varphi' > 1/2$, it is efficient to assign these rights to the party that has more at stake (to the rancher if $c < 1$; to the farmer if $c > 1$); and
(iii) At least one party may ex ante prefer going to court over bargaining and settlement.

4 Concluding Remarks

This paper has intentionally put all asymmetries of information or power, concavities or income effects aside so that the conditions conform as closely as possible to the basic formulation of the Coase theorem with zero transaction costs. In this way we have been able to focus on the effect of enforcement costs. Somewhat surprisingly, going to court can be an equilibrium or ex ante Pareto superior when the costs of enforcement are taken into account. This happens because enforcement costs and the probability of success at trial are endogenous, and depend on the environment that parties expect to find themselves in. Not only does the Coase theorem not hold in this case, but a very simple rule – based on the cost and benefits of the activity that produces the externality – can be used to assign an efficient property rights structure. This optimality of a targeted assignment of property rights occurs because the absence of a negotiated settlement introduces an asymmetry in the payoffs of the two parties, which translates into different enforcement efforts. The introduction of other asymmetries in the model would similarly induce different enforcement efforts. Two types of asymmetries that could be readily introduced are differential bargaining power or a liquidity constraint for one party that limits its ability to incur enforcement costs. Despite the different enforcement efforts that would be induced, it is unclear whether simple rules for the initial assignment of property rights can be found as we have found here.

Appendix

Proof of Proposition 1, Part (i): To determine how the equilibrium efforts are related to the cost benefit ratio c, define $g(e) \equiv f'(e)/f(e)$. Then, from equation (3) we have:

$$g(e_r^c) = cg(e_f^c) \tag{18}$$

So $g(e_r^c) \gtreqqless g(e_f^c)$ as $c \gtreqqless 1$. But g is a decreasing function:

$$g'(e) = \frac{f''(e)f(e) - [f'(e)]^2}{[f(e)]^2} < 0 \tag{19}$$

where the last inequality follows from the assumptions on f. This then gives us the result.

Part (ii): Suppose $B(x_r) < C(x_r)$. We compare $e_f^c + e_r^c$ when $\varphi = 1 - \varphi'$ to the same sum when $\varphi = \varphi'$. Define the notation:

$$\underline{e_f} \equiv e_f^c\big|_{\varphi=1-\varphi'} \quad , \quad \underline{e_r} \equiv e_r^c\big|_{\varphi=1-\varphi'} \quad , \quad \overline{e_f} \equiv e_f^c\big|_{\varphi=\varphi'} \quad \text{and} \quad \overline{e_r} \equiv e_r^c\big|_{\varphi=\varphi'}$$

In any Nash equilibrium, $g(e_r) = cg(e_f)$ so $g(\overline{e_r}) = cg(\overline{e_f})$ and $g(\underline{e_r}) = cg(\underline{e_f})$. If $\underline{e_r} < \overline{e_r}$, then $cg(\underline{e_f}) = g(\underline{e_r}) > g(\overline{e_r}) = cg(\overline{e_f})$ and so $g(\underline{e_f}) > g(\overline{e_f})$, which means that $\underline{e_f} < \overline{e_f}$. If, on the other hand, $\underline{e_r} > \overline{e_r}$, we must, by the same reasoning, also have $\underline{e_f} > \overline{e_f}$. Thus, to prove the result, we just need to show that $c > 1$ implies that $\underline{e_r} < \overline{e_r}$.

Suppose to the contrary that $c > 1$ and $\underline{e_r} \geq \overline{e_r}$. Then $\underline{e_f} \geq \overline{e_f}$, and the first order conditions for the rancher imply that

$$1 = \begin{cases} \dfrac{(1-\varphi')\,\varphi' f'(\underline{e_r}) f(\underline{e_f})}{[(1-\varphi')\,f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2} B(x_r) & \text{when } \varphi = 1 - \varphi' \\ \dfrac{(1-\varphi')\,\varphi' f'(\overline{e_r}) f(\overline{e_f})}{[\varphi' f(\overline{e_r}) + (1-\varphi')\,f(\overline{e_f})]^2} B(x_r) & \text{when } \varphi = \varphi' \end{cases}$$

which imply:

$$\frac{f'(\underline{e_r}) f(\underline{e_f})}{[(1-\varphi')\,f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2} = \frac{f'(\overline{e_r}) f(\overline{e_f})}{[\varphi' f(\overline{e_r}) + (1-\varphi')\,f(\overline{e_f})]^2}$$

The function $p_r(e_r, e_f) = \dfrac{\varphi(1-\varphi) f'(e_r) f(e_f)}{[\varphi f(e_r) + (1-\varphi) f(e_f)]^2}$ is decreasing in each of its arguments separately when $c > 1$ and $\varphi < 1/2$. To see this, note that:

$$\operatorname{sgn} \frac{\partial p_r}{\partial e_r} = \operatorname{sgn} \left\{ f''(e_r)[\varphi f(e_r) + (1-\varphi)\,f(e_f)] - 2\varphi\,[f'(e_r)]^2 \right\} < 0$$

where the inequality follows from the assumption that $f'' \leq 0$. Also, when $c > 1$ and when $\varphi < 1/2$, we have:

$$\begin{aligned}
\operatorname{sgn} \frac{\partial p_r}{\partial e_f} &= \operatorname{sgn} \{[\varphi f(e_r) + (1-\varphi) f(e_f)] - 2(1-\varphi) f(e_f)\} \\
&= \operatorname{sgn}[\varphi f(e_r) - (1-\varphi) f(e_f)] < \operatorname{sgn}[\varphi f(e_f) - (1-\varphi) f(e_f)] \\
&= \operatorname{sgn}[f(e_f)(2\varphi - 1)] < 0
\end{aligned}$$

where the second last inequality follows from the fact that $e_r < e_f$ when $c > 1$ and the final inequality follows from the fact that $2\varphi - 1 < 0$ when $\varphi < 1/2$. Therefore, assuming that $c > 1$ and $\overline{e_r} \leq \underline{e_r}$ we have:

$$\begin{aligned}
\frac{f'(\underline{e_r}) f(\underline{e_f})}{[(1-\varphi') f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2} &\leq \frac{f'(\overline{e_r}) f(\underline{e_f})}{[(1-\varphi') f(\overline{e_r}) + \varphi' f(\underline{e_f})]^2} \leq \frac{f'(\overline{e_r}) f(\overline{e_f})}{[(1-\varphi') f(\overline{e_r}) + \varphi' f(\overline{e_f})]^2} \\
&< \frac{f'(\overline{e_r}) f(\overline{e_f})}{[\varphi' f(\overline{e_r}) + (1-\varphi') f(\overline{e_f})]^2} = \frac{f'(\underline{e_r}) f(\underline{e_f})}{[(1-\varphi') f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2}
\end{aligned}$$

The first inequality follows because $p_r(e_r, .)$ is decreasing in e_r and from the assumption that $\overline{e_r} \leq \underline{e_r}$. The second inequality follows because $p_r(., e_f)$ is a decreasing function of e_f when $\varphi = 1 - \varphi' < 1/2$ and the fact that $\overline{e_r} \leq \underline{e_r}$ also implies that $\overline{e_f} \leq \underline{e_f}$. The last inequality follows because $c > 1$ implies that $\overline{e_r} < \overline{e_f}$, so we must also have $(1-\varphi') f(\overline{e_r}) + \varphi' f(\overline{e_f}) > \varphi' f(\overline{e_r}) + (1-\varphi') f(\overline{e_f})$. The last equality follows from the equality of the first order conditions when $\varphi = 1 - \varphi'$ and $\varphi = \varphi'$, and gives a contradiction. Thus, it must be the case that $\underline{e_r} < \overline{e_r}$, from which it also follows that $\underline{e_f} < \overline{e_f}$, and so $\underline{e_f} + \underline{e_r} < \overline{e_f} + \overline{e_r}$, as required. The second part of the result, that $c < 1$ implies that $\overline{e_f} + \overline{e_r} < \underline{e_f} + \underline{e_r}$, can be proved in a similar fashion.]

Proof of Proposition 2, Part (i)

Consider any given (e_r, e_f) and the associated win probability of the rancher $p \equiv \frac{\varphi f(e_r)}{[\varphi f(e_r) + (1-\varphi) f(e_f)]}$. Our objective is to find the appropriate payoff functions taking into account that bargaining and settlement will take place. The disagreement or threat payoffs at the pre-trial bargaining stage (stage 2) are those that would be induced from going to court. In turn, these payoff would depend on what can be expected to occur at the stage of post trial bargaining. We therefore proceed by backward induction, beginning with the last stage.

Stage 4: The court has made a decision at this stage, so there are two possible bargaining outcomes depending on who has won. If the rancher has won, the threat payoffs would be B_r for the rancher and $-C_r$ for the farmer. Given that the surplus is S^*, the split-the-surplus rule would then give us:

$$V_{rr} = \frac{S^* + B_r + C_r}{2} \quad \text{and} \quad V_{fr} = \frac{S^* - B_r - C_r}{2}$$

e_r and e_f do not appear in these expressions because enforcement expenditures have already been incurred and are sunk costs at the post-trial bargaining stage. If, on the other hand, the farmer has won the right to choose x, the disagreement payoffs for the rancher and the farmer would both be 0, giving us the following post-trial bargaining payoffs:

$$V_{rf} = \frac{S^*}{2} \quad \text{and} \quad V_{ff} = \frac{S^*}{2}$$

Stage 3: The expected payoffs just before going to court are the probability weighted sums of the stage 4 payoffs, less the costs of going to court:

$$\begin{aligned} V_r &= pV_{rr} + (1-p)V_{fr} - \beta e_r = \frac{S^*}{2} + p\frac{B_r + C_r}{2} - \beta e_r \\ V_f &= pV_{fr} + (1-p)V_{ff} - \beta e_f = \frac{S^*}{2} - p\frac{B_r + C_r}{2} - \beta e_f \end{aligned}$$

Note that the costs of going to court for each party are included here since the have yet to be incurred.

Stage 2: The split-the-surplus rule gives us the following payoffs:

$$\begin{aligned} V_r^{ab} &= \frac{S^* + V_r - V_f}{2} = \frac{S^*}{2} + p\frac{B_r + C_r}{2} + \frac{\beta e_f}{2} - \frac{\beta e_r}{2} \\ V_f^{ab} &= \frac{S^* + V_f - V_r}{2} = \frac{S^*}{2} - p\frac{B_r + C_r}{2} + \frac{\beta e_r}{2} - \frac{\beta e_f}{2} \end{aligned}$$

Rolling back to Stage 1, we obtain the payoff functions in Proposition 2 by subtracting the expenditures of each party at the first stage of the game (e_r for the rancher and e_f for the farmer). The proof of parts (ii), (iv) and (v) follow by simple algebra. The proof of part (iii) is explained and proved in the main text.

Proof of Proposition 3: We want to find parameter values for which $V_r^c(e_r^c, e_f^c)$ in (5) is higher than $V_r^b(e^b, e^b)$ in (10) or $V_f^c(e_r^c, e_f^c)$ in (5) is higher than $V_f^b(e^b, e^b)$ in (10). First, $V_r^c(e_r^c, e_f^c) > V_r^b(e^b, e^b)$ is equivalent to:

$$\varphi B_r \left(\frac{\varphi}{(\varphi + (1-\varphi)c)^2} - \frac{(\beta + 2\varphi)(1+c)}{2(2+\beta)} \right) > \frac{S^*}{2}$$

The left-hand-side is continuous in c and its limit as $c \to 0$ exists and equals the value of the left-hand-side at $c = 0$. That limit can be shown to equal $B_r(2-\varphi)/2 > B_r/2$. Note that $B_r = B(x_r) \geq B(x^*)$ since x_r maximizes $B(x)$. Therefore, $B_r(2-\varphi)/2 > B(x^*)/2 > (B(x^*) - C(x^*))/2 = S^*/2$ and the limit of the left–hand-side of the equation above as $c \to 0$ is strictly greater than its right-hand-side. Hence, for c sufficiently close to 0, we must have $V_r^c(e_r^c, e_f^c) > V_r^b(e^b, e^b)$. Next, $V_f^c(e_r^c, e_f^c) > V_f^b(e^b, e^b)$ can be shown to be equivalent to:

$$\varphi B_r \left(\frac{(4 + \beta - 2\varphi)(1+c)}{2(2+\beta)} - \frac{(\varphi + 2(1-\varphi)c)c}{(\varphi + (1-\varphi)c)^2} \right) > \frac{S^*}{2}$$

Again, we will follow the same reasoning and consider the limit of the left-hand-side of the inequality as $c \to 0$ which equals $\varphi B_r \frac{4+\beta-2\varphi}{2(2+b)}$. For φ sufficiently large, this limit can be shown to be greater than $S^*/2$, and so for c close enough to 0, we must have $V_f^c(e_r^c, e_f^c) > V_f^b(e^b, e^b)$. [Note that the conditions for the farmer's payoff being higher under going-to-court are more stringent than the equivalent conditions for the rancher. However, the opposite can be shown to hold when c is sufficiently small. In that case the conditions for the farmer are much less stringent, whereas for large c it is impossible for the rancher's payoff under going-to-court to be higher than that under bargaining.]

Proof of Proposition 4, Part (i): Suppose initially that (13) is satisfied ($2\delta e^b > \beta(e_r^1, e_f^1)$) and derive the implied Nash equilibrium using the payoff functions in (14) and (15). Such an equilibrium is symmetric with $e_r^1 = e_f^1 = e^p$, which is implicitly defined by:

$$\frac{\varphi(1-\varphi)f'(e^p)}{f(e^p)}\frac{(1+\delta)(1+c)B_r}{2} - (1+\beta) = 0$$

Condition (13) then reduces to $\delta e^b > \beta e^p$, where e^b is implicitly defined in (8) (note that $(1+c)B_r = B_r + C_r$). (13) is automatically satisfied for combinations of $\beta = 0$ and any $\delta > 0$. Both e^b and e^p are differentiable, and therefore continuous,functions of β. Thus, (13) must be satisfied for other combinations of β and δ, with β close enough to zero.

Part (ii): From the implicit definition of e^p above, it is clear that e^p does not depend on whether $\varphi = \varphi'$ or $\varphi = 1-\varphi'$. Therefore, when the two parties bargain ex ante, equilibrium efforts and net surplus are independent of the initial assignment of rights. When the two parties bargain ex ante, it is straightforward to show the same result.

Proof of Proposition 5: The proofs of parts (i) and (ii) of the Proposition are virtually identical to the proofs of parts (i) and $(ii)$of Proposition 1. (The only difference is that the payoff functions in the two period model are $(1+\delta)$ multiples of the one period payoff functions, but the comparative statics can easily be shown to be identical.) We therefore concentrate on proving part (iii), a major part of which is identical to the proof of Proposition 3. As in that proof, we consider the case of $f(e) = e$. Then, the equilibrium payoffs under (16) and (17) are:

$$W_r^c = \frac{\varphi^2(1+\delta)B_r}{(\varphi+(1-\varphi)c)^2} \text{ and } W_f^c = -\frac{\varphi(\varphi+2(1-\varphi)c)(1+\delta)cB_r}{(\varphi+(1-\varphi)c)^2}$$

Note that these payoffs are just $V_i^c(e_i^c, e_i^c)$ multiplied by $(1+\delta)$. Compare these payoffs to those that correspond to the equilibrium under (either ex ante or ex post) settlement with the payoff functions in (14) and (15). When the two sides settle ex ante, the comparison is identical to that in the proof of Proposition 4, except that all payoffs are to be multiplied by $(1+\delta)$ without affecting the comparisons. When the two sides settle ex post under (14) and (15), with $2\delta e^b > \beta(e_r^1 + e_f^1)$ in equilibrium, the equilibrium payoffs become:

$$\begin{aligned} W_r^{ep} &= \frac{(1+\delta)S^*}{2} + \frac{\varphi^2(1+\delta)(1+c)B_r}{2} \\ W_f^{ep} &= \frac{(1+\delta)S^*}{2} - \frac{\varphi(2-\varphi)(1+\delta)(1+c)B_r}{2} \end{aligned}$$

Note first that $W_r^c > W_r^{ep}$ if and only if:

$$\varphi^2 B_r \left(\frac{1}{(\varphi + (1-\varphi)c)^2} - \frac{1+c}{2} \right) > \frac{S^*}{2}$$

The left-hand-side of this inequality is continuous in c and its limit as $c \to 0$ exists and equals the value of the left-hand-side at $c = 0$. That limit can be shown to equal $B_r(2 - \varphi^2)/2 > B_r/2$. Note that $B_r = B(x_r) \geq B(x^*)$ since x_r maximizes $B(x)$. Therefore, $B_r(2 - \varphi)/2 > B(x^*)/2 > (B(x^*) - C(x^*))/2 = S^*/2$ and the limit of the left–hand-side of the equation above as $c \to 0$ is strictly greater than its right-hand-side. Hence, for c sufficiently close to 0, we must have $W_r^c > W_r^{ep}$. Next, we have $W_f^c > W_f^{ep}$ if and only if:

$$\varphi B_r \left(\frac{(2-\varphi)(1+c)}{2} - \frac{(\varphi + 2(1-\varphi)c)c}{(\varphi + (1-\varphi)c)^2} \right) > \frac{S^*}{2}$$

Again, as above, the left-hand-side of this inequality is continuous in c and its limit as $c \to 0$ exists and equals the value of the left-hand-side at $c = 0$. The limit at $c = 0$ equals $\varphi(2 - \varphi)B_r/2$, which for sufficiently large φ is greater than $S^*/2$ and the above inequality holds. By continuity, then, $W_f^c > W_f^{ep}$ for c small enough and large enough φ. Thus, as required in part (*iii*) of the Proposition, we have found conditions under which going to court and never negotiating is preferable by at least one party.

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