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SNTV, STV, and Single-Member-District Systems:  
Theoretical Comparisons and Contrasts\*

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SNTV elections have especially important similarities to three other types of election system: single-member-district (SMD) elections using plurality, limited voting (of which SNTV is a special case), and the single transferable vote (STV, also known as the Hare System). As shown in Table 1, each of these three election systems has exactly three of four basic characteristics in common with SNTV.

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Table 1 about here

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If we wish to better understand the mechanisms whereby SNTV produces its political effects, it can be useful to look at whether similar effects are found in polities which make use of electoral systems to which SNTV has a family resemblance. Lijphart, Pintor and Sone (1986) compare and contrast the proportionality of SNTV in Japan with that found under other forms of limited voting in Spain, and Lijphart (this volume) extends this comparison to STV in Malta. Bawn, Cox and Rosenbluth (this volume) compare the stability of party vote shares at the district level in Japan under SNTV with that in the U.S. under an SMD system. Here, rather than considering empirical evidence, I want to consider SNTV from a theoretical perspective, comparing it to the three systems shown in Table 1 and to three other systems to which it can also be seen as closely related, the next best well-known semi-proportional system,<sup>1</sup> cumulative voting; list PR under the D'Hondt rule; and a system of limited nomination. I will focus on three issues, proportionality of result, ease of achieving strategically

optimum behavior in terms of (party) decisions as to how many candidates to run, and incentives to cultivate a personal vote.

In the rest of the paper I will make use of the following notation.

Let

- $m$  = number of members being elected from a given district
- $n$  = number of parties contesting the election in some given district
- $L$  = number of seats in the legislature as a whole
- $v_i$  = vote share for party  $i$  in a given district
- $V_i$  = total national vote share for party  $i$

### Thresholds of Exclusion and Representation

We can make some useful comparisons among electoral systems in terms of two well-known indices:

$T_R$  = threshold of representation

and

$T_E$  = threshold of exclusion.

The threshold of representation (Rokkan, 1968; Rae, Hanby, and Loosemore, 1971; see also Rae, 1971; Grofman, 1975; Lijphart and Gibberd, 1977) is the minimum support necessary to earn a party its parliamentary seat, based on the most favorable case scenario in terms of how the other parties divide up their votes.<sup>2</sup> The threshold of exclusion (Rae, Hanby, and Loosemore, 1971; see also Rae, 1971; Grofman, 1975; Lijphart and Gibberd, 1977) on the other hand, is the maximum support that can be attained by a party while still failing to win even one seat.<sup>3</sup> The threshold of

representation provides a necessary condition for parliamentary representation, the threshold of exclusion provides a sufficient condition for it.<sup>4</sup>

We show in Table 2 values of  $T_R$  and  $T_E$  for the four electoral systems of Table 1 on the assumption that each party runs a full slate.

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Table 2 about here

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If there are more than two parties, it is easy to see from Table 2 that the threshold of representation is lower than the threshold of exclusion for all four election systems; we also see that, while there is some duplication of  $T_E$  values, each of the four systems has a unique value for  $T_R$ .<sup>5</sup> Of the four systems we consider, the threshold of exclusion is lowest for SNTV and STV, and highest for plurality. For  $m \geq 3$  and  $n \geq m$ , the threshold of representation is lowest for SNTV and highest under plurality.<sup>6</sup> Thus, SNTV makes it relatively easy for a minority voting bloc to gain some representation if its members are able to agree on who to vote for; if the bloc has support equal to a Droop quota then it cannot be denied representation, and if opponents do not allocate their support in an optimal manner, or if there are lots of voting blocs, then representation may be obtained even with much less than a Droop quota's worth of support.<sup>7</sup>

A further look at the entries in Table 2 makes it clear that SNTV and plurality may be taken as two ends of a continuum, with limited voting providing the middle ground.<sup>8</sup> If  $k = m$  then limited voting becomes plurality/plurality bloc voting. If  $k = 1$ ,  $m > 1$ , then we have SNTV. Since the values of  $T_E$  and  $T_R$  are the same for plurality bloc voting (i.e.,

plurality voting in multimember districts) as they are for plurality voting in single-member districts (Grofman, 1975), as we vary  $k$  between 1 and  $m$ , we obtain values of  $T_E$  and  $T_R$  that are intermediate for limited voting between those for plurality systems and those for SNTV. Roughly speaking, we may take  $(m - k) / m$  as an indicator of how likely a limited vote system is to be proportional in its effects; the closer that index is to one, the more proportional is the system.

The comparisons between SNTV and STV are not quite as clear. These two systems have the same threshold of exclusion value, but we have forced a difference between the values of the thresholds of representation for the two systems by assuming that voters under STV each cast a long enough ballot so that the best any small voting bloc can hope for is that there will be one seat that won't be filled through vote transfers but will go instead to the candidate with the plurality of preferences among the remaining non exhausted ballots.

Under the very specific distributional assumptions we have made, the gap between  $T_E$  and  $T_R$  can be taken as a measure of the degree of "strategic play" in the system. When that gap is large then a party's success will depend upon how many opponents it has and on how they divide up the vote. Indeed, even when the specific distributional assumptions we use do not hold, we may still wish to interpret the gap between  $T_E$  and  $T_R$  shown in Table 2 as a very rough measure of the degree of "strategic play" in the system. For example, even if parties do not nominate full slates, as long as voters rank order their preferences for more than just a few candidates, it is generally hard for very small parties to "strike it lucky" under STV and win a seat with much less than a Droop quota of eventual

support. In contrast, in multi-party plurality contests in single-member districts it might not be that uncommon for the winner to have considerably less than half the vote.

In general, strategic calculations are not nearly as important under STV as under SNTV or the limited vote. Under STV, there are never reasons not to run a full slate.<sup>9</sup> In contrast, under SNTV, a party may overextend itself, by spreading its support "too thin" and must be sensitive to such strategic calculations. For example, under SNTV, with just under 60 percent of the vote as its expected share, in a three-member constituency, a party that runs three candidates may win only one seat in the worst case scenario of there being but a single opponent party that runs two candidates, and it will probably win only two seats even if it runs three candidates. In such a situation the party can "play it safe" by only running two candidates. With over 75 percent of the vote, however, running three candidates is a dominant strategy.

Under the limited vote, as noted earlier, the worst case scenario for any party is that in which there is but a single opposition party. If one party wishes to minimax; i.e., guarantee the selection of  $c$  representatives independent of how many candidates the other party puts up -- then it must look to the worst possible case (see Luce and Raiffa, 1957) and give its  $c$ th candidate more votes than the other party can possibly give its  $(m + 1 - c)$ th candidate. Hence, in two-party competition under the limited vote, in general, an "optimal" strategy for a party when it is able to divide its vote share evenly among its candidates, is to run exactly  $c$  candidates ( $k \leq c \leq m$ ) for the maximum  $c$  for which its expected vote percentage,  $v$ , is such that<sup>10</sup>

$$\begin{aligned}
 vk/c > (1-v)k/(m-c+1), & \quad \text{if } m-c \geq k \\
 vk/c > (1-v), & \quad \text{if } m-c < k.
 \end{aligned}$$

There are, however, a few special cases in which it cannot hurt to run one more than this number of candidates.<sup>11</sup>

We may restate the above inequalities by solving for  $v$ . The inequalities now become

$$\begin{aligned}
 v > c/(m+1), & \quad \text{if } m-c \geq k \\
 v > c/(c+k) & \quad \text{if } m-c < k.
 \end{aligned}$$

Rae, Hanby and Loosemore (1971) observe that, for each of the multimember voting schemes, thresholds of exclusion are inverse functions of district size ( $m$ ); thus, the thresholds decrease at a decreasing rate as  $m$  increases. This suggests that, since gains in increasing representation for small voting blocs diminish with increasing  $m$ , we can gain a good portion of the benefits of proportionality under SNTV (or STV) with "medium-sized" districts.<sup>12</sup> This argument is strengthened if there are independent reasons to keeping constituencies from being too large, such as a desire to enhance legislator-constituency ties, or a desire to prevent "narrowcasting" of a party's appeals.

While we usually focus on  $TE$  and  $TR$  as indicators of how large a party's voting strength must be in order to have a realistic chance to gain representation, we can also think of them as providing indicia of the incentives to develop a narrow- as opposed to a broad-based constituency. Even under the "worst" of circumstances, it is not necessary to try to gain more votes than represented by the threshold of exclusion to achieve

initial electoral success. Clearly, the smaller  $TE$ , the more "narrow-cast" can be a party's appeals.<sup>13</sup>

Of course, we must be careful not to misinterpret  $TE$ , in that the actual number of voters needed to win victory will also be a function of district size. Thus, in looking at the differences between, say, SNTV and SMD systems, it is sometimes said that, since under SNTV with, say,  $m = 3$ , a candidate only needs 25% of the vote to win ( $TE = 1/4$ ), while in a single-member district a candidate needs 50%+ of the vote to win ( $TE = 1/2+$ ), it is much easier (indeed, twice as easy) to win election under SNTV than under SMD. This is a quite misleading calculation.<sup>14</sup> A little thought will reveal that, if district magnitude is proportional to district population, then the three-member constituency has three times as many voters as the one-member constituency! Thus, under the assumptions above, if there are  $q$  voters per representative, it will take  $qm / (m + 1)$  voters in one's electoral support group to be sure of winning election under SNTV in  $m$ -member districts, and only  $q/2$  voters to be sure of winning election under SMD. Note that it actually takes more voters to be sure of winning under SNTV as under SMD. Indeed, in the limit, as  $m$  tends toward infinity, it takes twice as many voters to be sure of winning under SNTV as under SMD!

Of course, the implications of this last calculation also need to be carefully thought through. As constituencies get more populous, groups that were not sufficiently geographically concentrated enough to make up the majority in any geographically compact single-member district, may have sufficient numbers in a multimember district to achieve representation. Thus, in order to determine whether candidates will be forced to "cast their nets more broadly"



If we assume that all seats are equally apportioned in per capita terms, for a fixed legislative size  $L$ , it is very important to appreciate the fact that  $E$  can be expected to be a monotonically increasing function of mean district magnitude for some candidate-centered systems (e.g., plurality bloc voting), since if we, say, cut the number of constituencies in half, thus doubling  $m$ ,  $E$  can also be expected to (roughly) double. However,  $E$  is a near constant function of  $m$  for some other candidate-centered systems (e.g., SNTV), since if we cut the number of constituencies in half, the population-weighted threshold of exclusion is  $1/2(m+1)$  as compared to  $1/(2m+1)$ , and the ratio of the two thresholds,  $(2m+2)/(2m+1)$ , stays reasonably close to one even though it increases slightly. Lastly,  $E$  can be an increasing function of  $m$  for some electoral systems (e.g., closed party list systems), since for closed party list systems increasing district size will increase  $E$ , albeit (for a given  $m$ ) the increase in size of  $E$  will generally be lower under closed party list systems than under plurality bloc voting because increasing district size will also permit some groupings whose size and/or lack of geographic concentration was not sufficient to permit them to win seats when  $m$  was low, to now do so.<sup>16</sup>

Looking at  $E$  suggests a new way to classify electoral systems. What is especially interesting about this classification scheme is that, for a fixed  $L$  and for a fixed  $m$ , plurality bloc voting is at one extreme (with a high  $E$  value) and other candidate-centered systems like SNTV and STV are at the other (with a low  $E$  value), while closed list PR is in the middle, with its exact location on the spectrum depending upon the distribution of voting strength across

in multimember district constituencies (e.g., SNTV) than in single-member districts, we need to be attentive to what kinds of interests might be represented and to the geographic distribution of those interests! Too simplistic a reliance on the threshold of exclusion can be quite misleading.

#### A New Index: Mean Electoral Constituency Size

Drawing on a well-known distinction in the American politics literature between electoral constituency and geographic constituency (Fenno, 1978), we will use the letter  $e$  to refer to the number of voters who voted for a given candidate or party, and  $E$  to the mean value of electoral constituency size in a legislature. In candidate-centered systems  $e$  is simply the vote received by the candidate; in closed party-list systems we take  $e$  to be the vote received by the party list in the district. In STV systems calculating  $e$  is more problematic. While a Droop quota elects a candidate, candidates who lack strong first-place support rely on second-place, third-place, etc. ballots transferred after other candidates have won/been dropped. This means that candidates' perception of the size of their constituency should be different under STV than under SNTV, because it was a matter of chance exactly which of the "excess" voters voting for winning candidates would be transferred to them. This leads me to believe that it might be better to think of the expected  $e$  value for STV as between one and two Droop quotas, but this is a matter which requires further analysis and thought. In the remainder of the paper, I will take the expected  $e$  for STV to be one Droop quota.<sup>15</sup>

voting blocs. The more even in voting strength are the groups the more closed list PR will look like SNTV and STV in terms of expected E value (i.e.,  $E = 1 / (m + 1)$ ); while if the distribution of voting strength is such that some groups are much larger than others, the E value for closed list PR will more closely resemble that for the plurality bloc voting case (i.e.,  $E = 1/2$ ).

### Semi-Proportional Systems: SNTV Compared to Cumulative Voting

Cumulative voting and the limited vote are often paired as being semi-proportional systems (see, e.g., Grofman, 1975, 1980a,b), and in the U.S. voting rights community they are invariably paired under the category of "alternative" voting schemes that might be proposed as remedies for observed dilution of minority voting under an at-large system in situations where a single-member district remedy does not seem desirable (Still, 1981, 1989 a,b; Zimmerman, 1986; Karlan, 1989; Guinier, 1994). SNTV and the cumulative vote have the same thresholds of exclusion. If we make the same distributional assumptions they will have the same threshold of representation as well. In many ways we may think of them as mirror systems.

SNTV achieves minority representation by restricting the voter options that had existed under plurality bloc voting by moving from  $m$  votes per voter to one vote per voter; cumulative voting achieves its effect by expanding voter options by permitting voters to cumulate their votes on one (or a few) candidate(s). The strategic issue under SNTV is how many candidates to run; the strategic issue under cumulative voting is how to distribute votes among candidates. In both systems, strategic

miscalculations can yield a party a much lower share of seats than that which is proportional to its voting strength; similarly, in both systems, strategic miscalculations on the part of opposing parties can yield a party a much higher share of seats than that which is proportional to its voting strength.<sup>17</sup>

It is straightforward to conduct an analysis similar to that we did for the limited vote of optimal candidacy strategies under cumulative voting; i.e., how many candidates should each party put up in any given constituency if it wishes to maximize the expected number of its candidates selected.<sup>18</sup> Using reasoning analogous to that for the SNTV case, a party with vote share  $v$  should prudentially nominate  $c$  candidates, where  $c$  is the largest value which satisfies the inequality below.

$$mv/c > m(1-v)/(m+1-c)$$

Solving for  $v$  we obtain

$$v > c/(m+1).$$

This is the same inequality as the first inequality that we obtained earlier for SNTV.<sup>19</sup> Note also that the value of  $v$  needed for it to be prudential to nominate exactly  $c$  candidates is  $c$  times the threshold of exclusion.

### SNTV Compared to List PR Under the D'Hondt Rule

All of the systems listed in Table 1 and Table 2 are systems in which voters give their vote(s) to particular candidates; the same is true for cumulative voting. In contrast, of course, list PR systems require that voters choose a party. Nonetheless, just as SNTV is a "kissing cousin" of cumulative voting, so too, it has important resemblances to list PR under D'Hondt. In particular, the threshold of exclusion is the same under the two systems.<sup>20</sup> However, the threshold of representation is not the same under the two systems because votes in excess of what is needed to win office that go to a particular single candidate are "wasted" under SNTV, while votes under list PR go to a party and not a candidate, and thus cannot be wasted in the same way.<sup>21</sup> Thus, strategic calculations under the two systems are not really equivalent (cf. Cox, 1991).

### Limited Voting Compared to Limited Nomination

Another "mirror image" of limited voting is a system of limited nominations. In a few U.S. states, to reduce the electoral uncertainty caused by the combination of winner-take-all plurality competition in multimember districts and volatile electoral tides, a scheme of limited nomination has been adopted for some local (partisan) elections under which parties are limited in how many candidates they can nominate -- to fewer candidates than there are seats -- but with plurality bloc voting (Grofman, 1982). Limited nomination assures that the one of the two major parties with the least support will not be denied representation completely.<sup>22</sup> An important advantage of limited nomination over limited voting is that the former greatly simplifies strategic calculations. Under limited

nomination, parties will run as many candidates as they are legally permitted since there are no real electoral disincentives to do so.

## II Concluding Heretical Proposition

I believe that too much emphasis has been placed on PR systems versus majoritarian/plurality systems as the principal cleavage line of electoral system choice. In the standard approach STV and list PR are taken as the two pure forms of PR, with semi-proportional systems treated as in the middle on the PR vs. plurality divide but tending toward the PR side as judged by their degree of proportionality of result. I know this is how I (e.g., Grofman, 1975) and many other scholars have classified electoral systems. I do not wish to suggest that the PR versus plurality continuum is not significant, but I now would wish to argue that the distinction among electoral systems between systems in which voters cast their votes for individual candidates (regardless of whether or not those candidates have an attached party label) and those in which voters' only choice is to vote for a party is at least as important as that between PR versus plurality.<sup>23</sup> Here, into the intermediate category fall "mixed" systems such as in Germany, and "open" list systems such as that in Finland (Tornudd, 1968) and Brazil (Ames, 1987, 1995; Mainwaring, 1991) where voters may cast both a party and a personal vote.<sup>24</sup> In this classification scheme, STV, though proportional, has more in common with SNTV or even SMD or multimember plurality bloc voting, than with list PR (even D'Hondt list PR, whose TE value it shares).<sup>25</sup>

After I finished a draft of this paper, I read a similar, but much more elaborated, electoral system classification scheme in a paper by Carey and Shugart (1995) that I believe is destined to

become a classic. They propose a continuum of electoral systems in terms of the incentives that each provides to "cultivate a personal vote." They rank systems in terms of four variables: (a) lack of leadership control over access to ballot/ballot position,<sup>26</sup> (b) degree to which candidates can be elected independent of the vote shares of co-partisans; and (c) single intra-party vote possessed by the voters as opposed to multiple intra-party votes or a single party-level vote, and (d) district magnitude,  $m$ . They treat these variables as dichotomous and weigh the first three factors equally to arrive at a composite index.

Contrary to the claim in Lancaster (1986), Carey and Shugart (1995) reach the conclusion that higher district magnitude actually increases incentives for clientalism in what they call "personal vote" seeking systems, even though it decrease such incentives in party-list systems or other systems with a great deal of centralized party control over the nomination process.<sup>27</sup> I generally share their views about the contingent effects of  $m$  on localism. However, I believe we can make this idea more precise by expressing the incentives for "personal voting" in terms of  $E$  (mean electoral constituency size) rather than  $m$  (district magnitude), since, as demonstrated earlier, the relationship between  $E$  and  $m$  will depend upon the type of election system. Moreover, I believe we can get a more fine-tuned analysis by estimating personal-vote incentives as a function of (average)  $e$ , because  $e$  is a quantitative rather than qualitative variable (albeit strength of party control over the nomination process might still need to be treated as some type of polychotomy).



When we take this approach, we get yet a third way to define a continuum of electoral system types. I am coming to the view that this way of classifying electoral systems provides us at least as many insights into the real political consequences of electoral laws as the standard PR versus plurality classification (for very similar views see Reed, 1994). For example, E lends itself quite nicely to a discussion of the policy consequences of at-large systems as opposed to SMD systems, a debate which has been important in the American local politics literature (see, e.g., Karnig and Welch, 1982).

I believe that, with the publications of Taagepera and Shugart (1989), Lijphart et al. (1992) and the recent sophisticated work by Gary King and Andrew Gelman on statistical models of estimating seats-votes relationships (e.g., King, 1990; Gelman and King, 1990, 1994), electoral systems research has largely resolved the central issues, both theoretical and empirical, about seats-votes proportionality relationship across different types of electoral systems. Although there remains a great deal to be said about those relationships in the ever growing varieties of electoral system type and about various technical measurement issues (Grofman, 1983; Gallagher, 1992), it nonetheless seems to me desirable to change the focus to other issues. Among the more important of these I would put the questions of how electoral systems impact incentives for localism/particularism/corruption and for candidate centered as opposed to party-centered politics.<sup>28</sup>

Note that the three ways we have identified for classifying the four main groups of electoral systems (plurality/plurality bloc voting, SNTV and cumulative voting, STV, and list PR) lead to three

very different ways of placing these systems along a continuum -- as shown in Table 3.

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Table 3 about here

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In one of these (the standard PR versus plurality continuum), plurality is at one end and list PR and STV are at the other, with SNTV and cumulative voting in the middle; in one (candidate-centered politics versus party-centered politics), SMD plurality, STV, SNTV, and cumulative voting are all together, with closed party list PR at the other end, and open list PR as an intermediate category; in the third (small electoral constituencies systems versus large electoral constituencies), one end of the continuum is plurality bloc voting, but now systems such as SNTV and STV anchor the other end, and closed list PR, remarkably, is an intermediate category.<sup>29</sup>

Still a fourth continuum might be developed were we to try to classify electoral systems according to the difficulty voters/parties have in developing optimal strategies (see, e.g., Cox, 1987a and discussion above).<sup>30</sup>

A **fifth** continuum along which electoral systems might usefully be differentiated is in terms of incentives toward conciliation. It is often taken for granted that the proportionality of an electoral system is a measure of its openness to the representation of extreme points of view, but that is too simplistic. Systems like STV and list PR may, for a given  $m$ , be roughly identical in their proportionality but may have quite different consequences for extremist politics, e.g.,

in terms of their degree of encouraging intra-party as opposed to inter-party competition and in terms of E, expected mean electoral constituency size.<sup>31</sup>

Recognizing that the degree of similarity between any two electoral systems will depend upon the research question at issue is a point of whose importance I hope the readers of this paper have been persuaded. Of course, in viewing electoral rules as an embedded institution we must be sensitive to the possibility that other factors (e.g., nature of the party system) might intervene.<sup>32</sup>

## ENDNOTES

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<sup>1</sup>There is a dispute in the electoral studies literature as to the appropriateness of the use of the term "semi-proportional" to characterize systems such as SNTV or cumulative voting. As Lijphart (this volume) notes, when we control for district magnitude, at least in Japan and Taiwan, SNTV operates essentially like a PR system in terms of seats-votes proportionality. Also, as noted below, the threshold of exclusion is the same for SNTV as for cumulative vote as for D'Hondt list PR. Nonetheless, I will continue to use the term "semi-proportional" in referring to SNTV or cumulative voting, since their proportionality in seats-votes (or lack thereof) is contingent on the degree of optimality of strategic choices made by parties as to how many candidates to run in a way that is not true for list PR systems or STV.

<sup>2</sup>"(T)his analysis . . . . is optimistic (Panglossian), for it presumes that established parties are obliging enough not to form alliances against an emergent party and even go so far as to divide their votes to its best advantage" (Rae, Hanby, and Loosemore, 1971, pp. 479-480).

<sup>3</sup>Rae, Hanby and Loosemore (1971: 480) observe that calculation of the threshold of exclusion is simplified by the fact that a small party's opponents "have no better strategy than either to (a) let one of their number stand alone against the party in each district, or (b) form a wholesale electoral alliance to oppose it in each district."

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Thus, we may calculate  $T_E$  by supposing that a party with vote share  $v_i$  faces a single adversary with a vote proportion of  $1-v_i$ .

<sup>4</sup>In the analysis that follows I neglect exclusion rules such as those that deny representation to parties with less than a minimum percentage of the national vote. Such rules can be very important for denying representation to small parties, usually to the benefit of the largest and second largest parties in the system, but are best considered separately from electoral system type, per se.

<sup>5</sup>This is true also for the seven systems compared in Grofman (1975, Table 1, p. 313).

<sup>6</sup>Of course, we must be careful in interpreting this conclusion because we have made different distributional assumptions for these two systems in calculating  $T_R$ .

<sup>7</sup>See also the discussion of the maximum/minimum seats/votes curve in Grofman (1975: 318-319), based on ideas in Dahl (1956).

<sup>8</sup>Charles Dodgson (a.k.a. Lewis Carroll, author of Alice in Wonderland) was apparently the first to investigate the properties of the limited vote. He wrote almost a century ago (Dodgson, 1884), but his work remained unknown or misunderstood until the economist Duncan Black, who was both an authority on voting methods and an authority on Carroll, restated Carroll's arguments and calculations in

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a clearer form (Black, 1967). The limited vote was made use of for parliamentary elections in Great Britain from 1967 to 1884 for some constituencies: voters had two votes in the twelve three-member constituencies (Berrington, 1975).

<sup>9</sup>Technically, this is not quite right because it is possible to construct scenarios under STV voting in which the order in which ballots are exhausted will matter (Doron and Kronick, 1977), but I regard such scenarios as so contrived and impossible to predict in practice that they may be safely disregarded.

<sup>10</sup>The formula below corrects an error in Grofman (1975).

<sup>11</sup>In a 3-seat 2-vote district, if one party has less than 50 percent, it should run two candidates; if it has more than 60 percent, it should run 3. With 50 + to 60 - percent of the vote a party must win 2 seats if it contests 2 seats. If it contests 3 seats, it will win only 1 seat if the other party contests 2 seats. With 40 + to 50 - percent of the vote, a party should contest 2 seats but expect to win at most 1 of them. In such a case, if the other party errs and runs more than its optimum number of candidates, running the extra candidate will pick up an extra seat, and it can't ever hurt. If there is no c value for which the above inequalities are satisfied, the party's situation is hopeless and it may just as well run a full slate.

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<sup>12</sup>Dodgson (1884) advocated the single nontransferable vote. He also was interested in the question of optimum district size. He notes that the "change from single member to two-member district changes the percentage of unrepresented electors from 49 to 32 . . . ; whereas the change from five-member to six-member districts only changes the percentage from 16 to 14 . . . The conclusion is that the important point is to have as few single member and even as few two-member districts as possible; but that, when we have got as far as to districts returning four or five members each it is hardly worthwhile to go further" (Dodgson, 1884, 25-6, cited in Black, 1967: 16, emphasis in original).

<sup>13</sup>Game-theoretic arguments about incentives for narrow-casting are found in Cox (1990) and Myerson (1993a,b; see also Carey and Shugart, 1995).

<sup>14</sup>It is remarkable how even sophisticated scholars can make this mistake.

<sup>15</sup>Lowenberg and Patterson (1979: 192) make the point that the "(l)inkage between legislators and their constituents depends upon how members of the legislature characterize their constituency . . . ; on their ability to maintain contacts with their constants through various means of communication; and on their ability to act in a manner responsive to constituents. Each of these factors is to some extent determined by properties of the political system in which a

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legislature acts." While E certainly does not fully capture even the first of these aspects of representative-constituency linkage, it is nonetheless a useful beginning. There is a considerable recent comparative politics literature on the nature of representative-constituency linkages in different countries, much of it published in Legislative Studies Quarterly. For example, Judge and Ilonski (1995: Table 4, 169) show that, in the 386-member 1992 Hungarian Parliament, where 176 members were elected from single-member districts, 152 were elected through regional lists, and 58 were elected off of national party lists, the degree of constituency orientation is far lower among those elected through party lists than among those elected in a particular constituency.

<sup>16</sup>For example, consider three voting blocs, with bloc A having  $4/7$  of the vote, bloc B having  $2/7^+$  and bloc C having  $1/7^-$  of the population. Let  $L = 8$ . If we have two four-seat districts then, in each, under closed list D'Hondt PR, if each bloc's voting strength is proportionally the same in each district as it is overall, then bloc A will win three seats (each with an e value of  $4/14$  of the national vote) and bloc B will win one seat (with an e value for that seat of  $2/14$  of the national vote); while bloc C will win no seats. Thus, E will be  $1/4$  ( $= (3 \times 4 + 1 \times 2) / (4 \times 14)$ ). If we have only one eight-seat district, then bloc A will win five seats (each with an e value of  $4/7$  of the national vote), bloc B will win two seats (each with an e value of  $2/7$  of the national vote), and bloc C will win 1 seat (with an e



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value of  $1/7$  of the national vote). Now  $E$  will be  $25/56$  ( $=(5 \times 4 + 2 \times 2 + 1 \times 1) / (8 \times 7)$ ). The  $E$  ratio in the two cases is 1.78.

<sup>17</sup>In principle, it is possible for a party to choose an expected utility maximizing strategy by subjectively assigning probabilities to the number of candidates it expects its opponents to run and the vote share it believes its opponents command, but we shall not pursue this matter further here (see Luce and Raiffa, 1957; and Brams, 1975, p. 112).

<sup>18</sup>If we look at the case where  $n = 2$ , it is easy to see that we have a zero sum two-person game (see Luce and Raiffa, 1957). The first person to realize this and to apply game theoretic notions to cumulative voting apparently was Glasser (1959). For other game-theoretic work on cumulative voting see Sawyer and MacRae (1962), Brams (1975), Glazer, Glazer and Grofman (1984).

<sup>19</sup>If we do not simplify by positing two-party competition, the analysis becomes more complex. The minimax strategy still makes sense as a prudential one and is optimal if one's opponents are acting in concert. However, if one's opponents are divided and do not run only  $m + 1 - c$  candidates, the minimax strategy may not achieve the maximum representation possible given the opposition's "irrationality." See Glasser (1959) for more detailed analysis.

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<sup>20</sup>If we wished a list PR system that even more closely resembled SNTV, we could add on some form of preference voting (such as in Finland) that would permit voters to reorder the party list.

<sup>21</sup>Votes can, however, be "wasted" under list PR in the sense that two parties could do better by combining their vote shares than they did running separate slates (Loosemore and Hanby, 1971), i.e., "remainders" that alone would not be sufficient to gain a seat might, when combined, do so.

<sup>22</sup>While this proviso might be sidestepped by "kluging" up "phony" slates, the advantage of party name recognition is sufficiently strong in the U.S., that this apparently does not happen.

<sup>23</sup>Relatedly, Andrew Reynolds (see, e.g., Reynolds and Grofman, 1992) has argued that, if list PR in large-m constituencies is adopted, its tendencies toward strong party control over legislative careers should be "balanced" by "opening" the list ordering to direct voter influence.

<sup>24</sup>More generally see Katz (1986); see also Cox and Shugart (1995) for their discussion of how the Colombian list system approximates SNTV incentives.

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<sup>25</sup>This classification is somewhat different than that given in Bogdanor (1985c, 11), although the underlying ideas are closely related.

<sup>26</sup>In particular, as Bogdanor (1985a) notes, when we combine SMD elections with party primaries, as is common in the U.S., we end up with a system which is quite different in terms of voter control over the candidate selection process than is SMD standing alone.

<sup>27</sup>Ramseyer and Rosenbluth (1993: 20-21) summarize an important historical argument in Cox (1987b) on changing incentives for the personal vote in great Britain in the 19th century: "Cox (1987b) outlined the rise in Britain of inter-party electoral contests on the basis of party platforms and the concurrent decline in the use of public policy for pork between the 1830s and the 1880s. As the British Parliament enlarged districts and gradually eliminated multi-member districts over the course of those decades, political parties found they had greater success appealing to the median voter with policy programs than in trying to buy off blocs of voters with particularistic favors. Larger districts made particularism a more costly strategy for individual politicians to woo for support. At the same time, the adoption of single-member districts made particularism less necessary, because parties needed to field only a single candidate in each district. This eliminated the politicians' need to build a personal following as a way of competing with candidates of their own party" (See also Myerson, 1993a; Cox, 1990). We believe

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the theoretical argument we give (and that in Carey and Shugart, 1995) shows why strengthening parties and lowered district magnitude reduced the quest for pork in 19th century Great Britain. Of course, it is also the case that the size of the eligible electorate was dramatically changing in Great Britain over this same period.

<sup>28</sup>Recent work has heralded the rise of candidate-centered politics in the U.S. (Fiorina, 1987; Wattenberg, 1992), and a growing constituency orientation among politicians in the U.K. (Cain, Ferejohn and Fiorina, 1987).

<sup>29</sup>Of course, these are theoretically derived expectations as to placement. In particular, it would be important to look at how different electoral systems actually differ in their value of E.

<sup>30</sup>Attempting exact measures of how strategy-prone different systems is beyond the scope of this paper. To do it right would require us, in my view, to look not just at the number of parties/number of candidates nominated but at their ideological proximity—a complication which previous attempts to measure the strategy-proneness of electoral systems have not incorporated.

<sup>31</sup>Yet another continuum that has been suggested might be called "opaqueness." Edwin Winckler (personal communication, June 1995) has argued that "Japanese and Nationalist elites chose SNTV because

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it is an electoral system that is singularly open to manipulation from behind the scenes, thereby reducing their risks from democracy."

<sup>32</sup>For example, Shaun Bowler (personal communication, April 30, 1995) points out that the degree of localism among representative elected under STV appears very different in Ireland from what it is in Australia (cf. Rydon, 1985; Farrell, 1985; Bogdanor, 1985b). Similarly, the different degree of party control over the nomination process in the U.S. and the U.K. leads to very different campaign styles and orientations toward pork-barrel politics in the two countries (Cain, Ferejohn and Fiorina, 1987; Bogdanor, 1985b).

Table 1  
 Four Key Aspects of Electoral Systems  
 and the Place of SNTV in the Family of Electoral  
 Systems

	SNTV	Limited Voting ( $k > 1$ )	SMD Plurality	STV
single- vs. multi- member	multimember	multimember	single- member	multimember
single vote vs. multiple vote	single vote	multiple vote	single vote	single vote
transferable vs. nontransfer- able	non- transferable	non- transferable	non- transferable	transferable
party- focused vs. candidate- focused	candidate- focused	candidate- focused	candidate- focused	candidate- focused

Table 2  
Indices of Representation and Exclusion  
and the Place of SNTV in the Family of Electoral  
Systems\*

	SNTV	Limited Voting ( $k > 1$ )	SMD Plurality	STV
TE	$1 / (m + 1)$	$k / (m + k)$	$1/2$	$1 / (m + 1)$
TR	$1 / m (nm - m + 1)$	$(mk - m + 1) / m (nm - m + 1)$	$1/n$	$2 / (m + 1)$ (n)
TE - TR	$(m^2n - m^2 - 1) / m (m + 1) (nm - m + 1)$	$km^2n - 2km^2 + 2km - m - k^2m + k) / m (m + 1) (nm - m + 1)$	$(n-2) / 2n$	$(mn + n - m - 2) / (n (m + 1))$

\* Adapted and expanded from Grofman (1975: Table 1, p. 313). As in that table we assume that each party runs a full slate of candidates for multi-seat offices. However, the assumptions used to calculate TR in this table for SNTV and the limited vote and for STV are different from those in my previous work. Under SNTV, TR could be as low as one vote when all but one of the votes are evenly distributed among  $m-1$  candidates. To provide a more realistic assessment of TR under SNTV, the first  $m-1$  seats are assumed to each be filled by a Hare quota ( $1/m$ ) fraction of the voters, with the remaining  $mn - m + 1$  of the candidates each receiving an equal share of the remaining vote. For the limited vote, the first  $m-1$  seats are assumed to use up  $(m - 1)/km$  fraction of the vote, with the remaining  $mn - m + 1$  of the candidates each receiving an equal share of the remaining  $(km - m + 1)/m$  fraction of the vote. For STV, to calculate TR, we assume that  $m-1$  seats are each filled by Droop

quota ( $1/(m + 1)$ ) fraction of ballots and the last seat is decided by the remaining votes, which are assumed to be evenly divided among  $n$  blocs of voters. For plurality, to calculate  $T_R$  we assume that the  $n$  parties are equal in support.



Table 3  
 Three Continua of Classification  
 and the Place of SNTV in the Family of Electoral  
 Systems\*

CONTINUUM	most	intermediate	least
PR versus	list PR	SNTV	plurality bloc
plurality	STV	cumulative	voting
(proportion-		voting	
ality)		mixed systems	
candidate-	SMD plurality	open list PR	closed list PR
centered	STV	mixed systems	
politics versus	SNTV		
party-centered	cumulative		
politics	voting		
(candidate			
focus)			
large electoral	plurality bloc	closed list PR	STV
constituencies	voting	mixed systems	SNTV
systems versus			cumulative
small electoral			voting
constituencies			SMD plurality
(particularism,			
E)			