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# JUDGMENTAL COMPETENCE OF INDIVIDUALS AND GROUPS IN A DICHOTOMOUS CHOICE SITUATION: IS A MAJORITY OF HEADS BETTER THAN ONE?

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We present a simple combinatorial model of group decision-making in a dichotomous choice situation. The independent variables in the model are group size and mean judgmental competence of group members. The dependent variables are the probabilities that the majority judgment of the group will be correct and that the best member of the group will be more likely to be correct than the group majority.

Two applications of this model are given: one a determination of isocompetence curves of groups of differing sizes and mean competence levels, the other an explication of the potential mechanisms accounting for Parkinson's only partly tongue-in-cheek observation that the point of ineffectiveness in a group seems to be reached when its total membership exceeds 20 or 21.

## I. INTRODUCTION

It is common sense "wisdom" that "people in groups are stupider than people taken by one," (Lathen, 1972:19) albeit it is also common sense "wisdom" that "two heads are better than one." Empirical evidence in the social psychological literature suggests that groups are better at certain kinds of problems (e.g. so-called Eureka-type problems) than are individuals, not necessarily because of qualities attributable to group deliberation but because of the statistical fact that the larger the group, the larger the probability of the group containing individuals capable of solving the problem in whole or part (Lorge & Solomon, 1955). Indeed, as one survey of the literature on group problem-solving notes:

> it is quite probable that group solution may have its advantages in stimulating one another for, and in inducing cooperation for, a common solution. Yet it must

> be recognized that group procedure may have disadvantages, too. A single member, or a coalition of members, may retard the group by holding out for its kind of solution--a consequence that may reduce the quality of the group product if the solutions so proposed are inadequate or unrealistic. (Lorge et al., 1958)

On the empirical level no clearcut resolution of the interaction between type of task, group size, and group deliberation in determining group output has been presented. On a hypothetical level, Steiner (1966) has specified a number of simple models (additive, conjunctive, and disjunctive) for the relationship between group size and group task performance (and some more complex models allowing for division of labor), but the models are of limited applicability to complex problem solving.

Clearly, the relationship between group size, group decision mechanisms, and the probability that the group will be "successful" in solving its problem or performing its task is an important question both for those interested in organization theory and for those interested in justifying (on empirical grounds) the superiority of democratic decision-making over oligarchic or dictatorial decision-making.[1] In this paper we shall present a simple model of group decision-making in a dichotomous choice situation and show what we believe to be some interesting implications for democratic theory.

## II. THE BASIC MODEL

Let us consider a group of N members[2] such that each member has some probability (p) of reaching a correct judgment in some dichotomous choice situation (e.g. with respect to the question of whether the number of balls in an urn exceeds some specified number). Let us initially assume that:

(a) this probability is the same for all group members;
(b) the group decides by simple majority vote;
(c) each member arrives at his decision independently of the views of the other members and maintains this decision regardless of the choices of other members.

Given these simplifying assumptions we may readily show that the probability of the group reaching a correct judgment increases with N as long as $p > 1/2$. The probability that exactly h members of the group will reach correct verdicts in a group of size N is calculated using the binomial theorem as

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[1]A very important case of group decision-making in a dichotomous choice situation (or sometimes a sequence of dichotomous choice situations, where there is a multiple indictment) is the criminal jury. Of course, in this context, the meaning of "correct" judgment is far from clear. For a discussion of this issue and some results related to those presented below, see Grofman (1976, 1978).

[2]For simplicity, unless stated to the contrary, N will be understood to be an odd number.

$$\binom{N}{h} p^h \; (1 - p)^{N-h} \qquad (1)$$

Similarly, the probability that exactly h members of the group will be wrong in their judgment is given by

$$\binom{N}{H} p^{N-h} \; (1 - p)^h \qquad (2)$$

Let m be a majority of the group, defined as $(N + 1)/2$.[3] The probability (for N odd) that a majority of the group will reach a correct judgment is

$$\sum_{h=m}^{N} \binom{N}{h} p^h \; (1 - p)^{N-h} \qquad (3)$$

*Result 1 (Condorcet's Theorem)*: For odd-size groups whose decision-making satisfied assumptions (a), (b), and (c), if $1 > p > 1/2$, then the larger the group the *more* likely it is that a majority of the group will reach the correct judgment; but if $0 < p < 1/2$, the larger the group the *less* likely it is that a majority of the group will reach a correct verdict; and if $p = 1/2$, the likelihood of a majority of the group reaching a correct judgment is independent of N and is equal to 1/2. Moreover, as $N \to \infty$, the probability that the group's judgment will be correct $\to 1$ if $p > 1/2$ and $\to 0$ if $p < 1/2$.

The result, of course, can be derived as a special case of the well known "law of large numbers." (For a historical discussion of this theorem and its derivation see Grofman, 1975).

An obvious corollary of this result is that, for a group satisfying the assumptions given above, the judgment of the majority is more likely to be correct than is the judgment of any single member when $p > 1/2$, less likely to be correct when $p < 1/2$, and as likely to be correct when $p = 1/2$.

Result 1 enables us to shed light on various seemingly contradictory proverbs, e.g. "Too many cooks spoil the broth," "A camel was a horse designed by a committee," "Two heads are better than one," and "Vox populi, vox dei." If the probabilities of correct judgment for each member of a group are each less than 1/2, then the majority group judgment is highly likely to be inferior to the judgment of the group's best member. The voice of the people is apt to be quite wrong, and the more people the more likely it is to be wrong. If, on the other hand, the group's probabilities of correct judgment are all even slightly better than 1/2 (and indeed, as we shall see, if the group's mean judgmental capability is greater than

[3]For N even, we might assume that $m = N/2$ and that the group flips a coin in the event of a tie or that a previously elected chairperson casts the tie-breaking vote.

1/2, even if some members have judgmental capabilities below 1/2) then the group verdict approaches infallibility as the group size approaches infinity. Numerical results for various values of p and N are shown in Table 1.

TABLE 1

The Probability That a Majority of Jurors Will Reach A Correct Verdict for Various Values of N and p [a]

| | | .2 | .4 | .5 | .6 | .8 |
|---|---|---|---|---|---|---|
| N | 1 | .2000 | .4000 | .5000 | .6000 | .8000 |
| | 3 | .1040 | .3520 | .5000 | .6480 | .8960 |
| | 5 | .0580 | .3174 | .5000 | .6826 | .9420 |
| | 7 | .0335 | .2858 | .5000 | .7102 | .9666 |
| | 9 | .0196 | .2666 | .5000 | .7334 | .9804 |
| | 11 | .0116 | .2466 | .5000 | .7534 | .9884 |
| | 13 | .0070 | .2288 | .5000 | .7712 | .9930 |
| | 15 | .0042 | .2132 | .5000 | .7868 | .9958 |
| | 17 | .0026 | .1990 | .5000 | .8010 | .9974 |
| | 19 | .0016 | .1860 | .5000 | .8140 | .9984 |

[a]N = group size, p = the probability that an individual member of the group will reach a correct judgment.

Table 1, which is merely based on the binomial expansion shows the probability that the majority will reach a correct decision as a function of group size and p, the probability that a given member of the group would arrive at the correct decision.

It is a natural question at this juncture to ask about the tradeoffs between group size and group judgmental competence. For example, how competent do members of a small "blue ribbon" panel have to be to have a higher probability of reaching a correct majority judgment (in some dichotomous choice situation) than the more numerous but less select members of some "ordinary" group? More formally, we may ask for what values of x and y do groups of size N + y and competence p - x have expected group (majority verdict) competence identical to that of a group of size N and competence p.

*Result 2*: Groups of size N + y and individual competence p - x are identical in expected correctness of group (majority) verdict to groups of size N and individual competence p if and only if

$$y = N\left[\frac{.25x(2p - 1 - x)}{p(1 - p)(p - x - .5)^2}\right] \qquad (4)$$

Proof: By the normal approximation to the binomial,[4] we wish to find x and y such that

$$\Phi\left(\frac{p - .5}{\sqrt{\frac{p(1 - p)}{N}}}\right) = \Phi\left(\frac{p - x - .5}{\sqrt{\frac{(p - x)(1 - p - x)}{N + y}}}\right) \qquad (5)$$

The desired result follows from some simple algebraic manipulation. In the above equation $\Phi(x)$ is, of course, the area under a normal distribution from $-\infty$ to x standard deviation units.

These results may be readily generalized (cf. Grofman, 1975:99-104). First, instead of treating the group's members as homogeneous in p, we may replace assumption (a) and treat the group as characterized by

(a') a mean value of p, binomially distributed, with variance equal to $p(1 - p)/N$.

Of course a normal distribution with mean p and variance $p(1 - p)/N$ approximates a binomial distribution of mean p, and this approximation is quite good even for relatively small N.

Second, instead of limiting the group to simple majority as the de jure decision rule, we may replace assumption (b) with

(b') the de jure decision rule for the group is K/N, $K \geq (N + 1)/2$.

Third, instead of treating the group's members' decisions as totally independent of one another we may replace assumption (c) with an assumption of partial independence.

(c') the de facto group decision rule is simple majority, such that whenever a majority of members is in accord their view becomes the prevailing one.

Note that we are now no longer requiring that the de jure decision rule for the group be simple majority rule, but only that the de facto one be so in that the majority persuades (or browbeats) the minority to achieve the necessary K/N consensus. Experimental studies of jury decision-making under a unanimity rule have found support for decision processes very similar to a de facto K/N's rule, where K/N is either a simple or a 2/3 majority. (For a review, see Grofman, 1976, 1977.) Since many decision processes will operate under de jure majority rule, and others under de facto majority rule, we do not find the assumption of an effective decision rule of majority an empirically unreasonable one for many small group processes.

These modifications to our initial assumptions yield

*Result 1'*: In odd-size groups whose decision-making

[4] We shall neglect the continuity correction to the normal approximation to the binomial in the discussion that follows, since, for $N \geq 9$, this correction is not significant.

satisfied assumptions (a'), (b'), and (c') if $1 > p > 1/2$, the larger the group the *more* likely it is that the group will reach a correct judgment; if $0 < p < 1/2$, the larger the group the *less* likely the group will reach a correct judgment; and if $p = 1/2$, the likelihood of a correct judgment is independent of N and is equal to 1/2.

*Result 2'*: For groups whose decision-making satisfied assumptions (a'), (b'), and (c'), groups of size $N + y$ and mean competence $p - x$ are identical in expected correctness of group verdict to groups of size N and mean competence p if and only if equation (5) holds.

We show in Figure 1 isocompetence curves for groups of various sizes for p = .55, .6, .7, .8, and .9. This figure shows the tradeoff between group size and mean group competence necessary to maintain a constant level of expected correctness of group (majority) judgment.

These isocompetence curves shed interesting light on the relative attractiveness (judgmental competence) of democracy and dictatorship (or oligarchy) as a function of the mean competence of the dictators (oligarchs) versus the mean competence of the larger (and presumably less competent, on the average) democratic mass. If the mean competence of the democratic electorate is > 1/2, the majority rule (for N large enough) may indeed be regarded as "divinely" inspired, and to be preferred to the judgments of any dictator or any bank of oligarchs who are not themselves infallible. We see from Figure 1, for example, that a dictator of .9 competence is as likely to reach a correct judgment in a dichotomous choice situation as 41 individuals of average competence .6 reaching a judgment by majority verdict. Similarly, a group with 59 or more members of average competence only .55 will have a greater probability of reaching a correct (majority) verdict than will a single individual of competence level .9 or a group of 9 members with average competence of .7. *Thus increasing the size of the group, even though it reduces the mean competence of the group, may actually increase the probability of the group reaching a correct verdict.* Note, however, that the superiority of the group to the individual in this last example is in no way a product of group deliberation but is purely an "artifact" of the law of large numbers and a de jure or de facto majority decision rule.

## III. THE JUDGMENTAL COMPETENCE OF A GROUP VS. THAT OF ITS OWN BEST MEMBER

Another interesting question which has been treated very unsatisfactorily in the literature on group problem-solving is the following: "Is a group's judgment better than the judgment of its best member?" The early psychological literature on group behavior manifests considerable confusion in disentangling the effects of group size, per se, vs. those due to the impact of group discussion/conformity processes. This confusion has not been fully resolved. Lorge et al. summarize the rather contradictory literature by asserting that

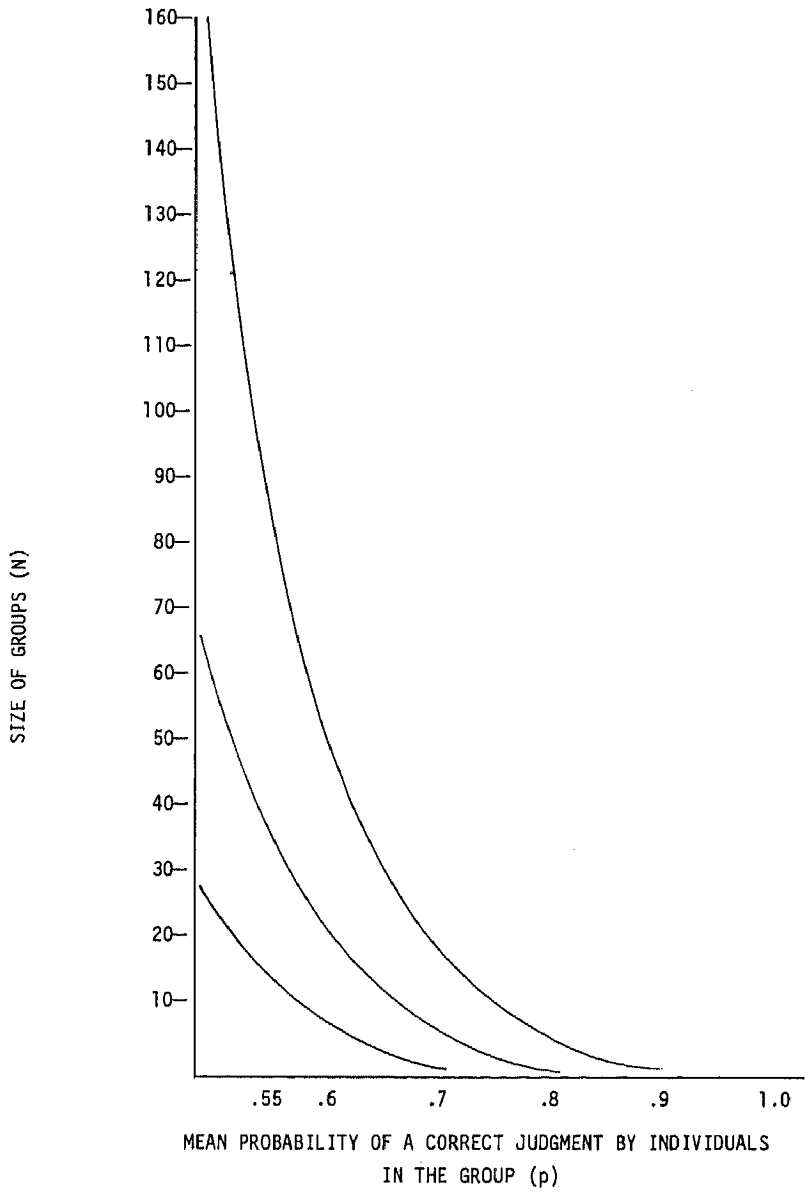


Figure 1 Isocompetence curves for various values of mean group competence (p).

"at best, group judgment equals the best individual judgment but usually is somewhat inferior to the best individual" (1958:344).[5] Similarly, a recent textbook on group dynamics

[5]Cf. Lorge et al. (1958:344-348). Although they are aware that "Group superiority depends upon the quality of judgments and the range of judg-

summarizes by asserting that the question "'Does the quality of group performance exceed that of the most proficient member of the group?' must be answered negatively, although under some circumstances the group performance might be better than that of any individual in the group" (Shaw, 1971:63).

We shall present some more exact results which, however, reveal a confused situation in which generalizations are hard to come by. Clearly, we must distinguish the case where group members interact with what Lorge et al. (1958) call "statisticized" groups, i.e. groups composed by combining responses of noninteracting individuals. However, as long as the effective decision rule is majority our model applies to both.[6]

In a dichotomous choice situation, the expected probability of a correct judgment (henceforth denoted $P_C$) in a group of N members whose decision making satisfied assumptions (a'), (b'), and (c') is of course approximately

$$\Phi\left(\frac{p - .5}{\sqrt{\frac{p(1-p)}{N}}}\right) = P_C \qquad (6)$$

This probability (Expression 6) may be thought of as the judgmental competence level of that group qua group. The probability (henceforth denoted $P_L$) that a randomly chosen member of the group is below that competence level is given by

$$\Phi\left[\frac{\Phi\left(\frac{p - .5}{\sqrt{\frac{p(1-p)}{N}}}\right) - p}{\sqrt{\frac{p(1-p)}{N}}}\right] = P_L \qquad (7)$$

*Result 3*: In a group whose decision-making satisfies (a'), (b'), and (c') the probability (henceforth denoted $P_B$) that *none* of the group's N members exceed $P_C$, the competence level of the group, is given by

$$1 - \Phi\left[\frac{\Phi\left(\frac{p - .5}{\sqrt{\frac{p(1-p)}{N}}}\right) - p}{\sqrt{\frac{p(1-p)}{N}}}\right]^N = P_B \qquad (8)$$

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of individual members of the group" (p. 348), the exact nature of this relationship was not discussed.

[6]We shall, however, avoid the issue of whether "bad" solutions drive out "good" and shall assume that the effective decision rule in a group remains invariant with group size. A useful discussion of both points is found in Padawer-Singer and Barton (1975).

That is,

$$P_B = 1 - (P_L)^N \qquad (9)$$

This result follows readily from Expressions (6) and (7) and the binomial theorem.

Table 2 shows $P_C$, $P_L$, and $P_B$ for various values of p and N.

In Table 3 we show the maximum $P_B$ value as a function of p and also show for what N this maximum value occurs. For p in the range $.51 < p < .85$, as p increases, $P_B$ reaches its maximum at lower values of N, and the magnitude of this maximum value decreases. For $p \geq .85$, as p increases $P_B$ reaches its maximum at higher values of N and the magnitude of this maximum value increases.

In Table 4 we show for each of various values of p, the value of N at which $P_B$ falls below 1/2, i.e., the group size such that, for a given mean group competence p, the group's judgment is less likely to be correct than is the judgment of its best member for all groups at or above that size. Note that for all p, the probability that a group's judgment will be more likely to be correct than the judgment of its best member falls off as the group size increases past a certain point. This point may be thought of as the optimum size of a democratic decision making body of that mean competence level.[7] Beyond that size the group would be better off to entrust its judgments to the group's most competent member (assuming, of course, that such an individual can be correctly identified).[8] For p in the range $.51 < p < .78$, as p increases, the optimum size group for democratic decision-making decreases: for $p = .55$ it is 35, for $p = .65$ it is 13, and for $p = .77$ it reaches its minimum value of 9. On the other hand, for $p > .78$, as p increases, the optimum size group for democratic decision-making increases: for $p = .79$ it is 11, for $p = .85$, it is 19, etc. Thus, for very low levels of mean group competence ($p < .55$) democracy is to be preferred to rule by the best even in groups of relatively large (> 35 members) size. For medium levels of group competence ($.55 < p < .77$) democracy is a good idea only in very small groups. Finally, for groups of high mean competence level (say $p > .85$), as with groups of a very low mean competence level), democracy is preferred to rule of the best even in relatively large-sized groups.[9]

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[7] This size is that of the optimum democratic decision-making body since for any smaller sized body it follows from Result 1 that the probability of a correct judgment would necessarily be less; and for any larger sized body, democracy would not produce as good results as dictatorship by the group's best member.

[8] For some thoughts on how this identification might be achieved see Grofman (1975: Note 17).

[9] Of course, if group competence is not roughly normally distributed. and with variance as specified above, the above results do not apply. In the country of the blind, the one-eyed man ought to be king.

TABLE 2

The Relationship Between Group Size and Mean Competence Level and the Superiority of a Group (Majority) Judgment Over That of the Group's Average and Best Members[a]

| N | $P_C$ | | | | | $P_L$ | | | | | $P_B$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p = | .55 | .60 | .70 | .80 | .90 | .55 | .60 | .70 | .80 | .90 | .55 | .60 | .70 | .80 | .90 |
| 5 | .59 | .68 | .84 | .94 | .99 | .57 | .64 | .75 | .80 | .77 | .94 | .90 | .77 | .66 | .73 |
| 9 | .62 | .73 | .90 | .98 | .99+ | .66 | .79 | .91 | .92 | .84 | .98 | .88 | .57 | .53 | .79 |
| 13 | .64 | .77 | .94 | .99 | .99+ | .75 | .89 | .97 | .96 | .89 | .98 | .77 | .31 | .40 | .79 |
| 21 | .68 | .83 | .97 | .99+ | .99+ | .88 | .98 | .99 | .99 | .94 | .92 | .31 | .06 | .21 | .75 |
| 29 | .71 | .86 | .99 | .99+ | .99+ | .95 | .99+ | .99+ | .99 | .96 | .74 | .05 | .01 | .10 | .66 |
| 35 | .72 | .89 | .99 | .99+ | .99+ | .98 | .99+ | .99+ | .99 | .98 | .49 | .01 | .00+ | .05 | .58 |
| 41 | .74 | .90 | .99 | .99+ | .99+ | .99+ | .99+ | .99+ | .99+ | .98 | .26 | .00 | .00+ | .03 | .49 |

[a]N = group size, p = mean individual competence, $P_C$ = probability that group (majority) judgment is correct, $P_L$ = probability that average member of the group is less likely to be correct than the group's (majority) judgment, $P_B$ = probability that the best member of the group is less likely to be correct than the group's (majority) judgment.

TABLE 3

Maximum Probability That a Group's Best Member is Less Likely To Be Correct Than the Group's (Majority) Judgment as a Function of p and N [a]

| p | Maximum value of $P_B$ | Value of N at which $P_B$ attains its maximum |
|---|---|---|
| .55 | .98 | 11 |
| .57 | .95 | 9 |
| .59 | .92 | 7 |
| .61 | .89 | 7 |
| .63 | .86 | 5 |
| .65 | .84 | 5 |
| .67 | .81 | 5 |
| .69 | .78 | 5 |
| .71 | .76 | 3 |
| .73 | .75 | 3 |
| .75 | .73 | 3 |
| .77 | .72 | 3 |
| .79 | .70 | 3 |
| .81 | .69 | 3 |
| .83 | .68 | 3 |
| .85 | .67 | 5 |
| .87 | .70 | 7 |
| .89 | .76 | 11 |
| .91 | .83 | 13 |

[a] N, p, and $P_B$ are explained in the footnote to Table 2.

These results are far from intuitively obvious. Moreover, they allow us to develop a rationale for Parkinson's famous, and not really tongue-in-cheek assertion, based on the history of the British Cabinet, that "the point of ineffectiveness in a cabinet is reached when the total membership exceeds 20 or 21. The Council of the Crown, the King's Council, the Privy Council had each passed the 20 mark when their decline began" (Parkinson, 1957:41). A few pages later, Parkinson goes on to assert that "Somewhere between the number

TABLE 4

Optimum Size of a Democratic Decision-Making Body As A Function of Its Mean Competence Level, p [a]

| p | Value of N at which $P_B$ is first less than 1/2 |
|---|---|
| .55 | 35 |
| .57 | 27 |
| .59 | 21 |
| .61 | 17 |
| .63 | 15 |
| .65 | 13 |
| .67 | 13 |
| .69 | 11 |
| .71 | 11 |
| .73 | 11 |
| .75 | 11 |
| .77 | 9 |
| .79 | 11 |
| .81 | 11 |
| .83 | 15 |
| .85 | 19 |
| .87 | 25 |
| .89 | 35 |
| .91 | 49 |

[a]N, p, and $P_B$ are explained in the footnote to Table 2.

of 3...and approximately 21 there lies the golden number" (Parkinson, 1957:44). We see from Table 4 that in groups of mean competence p, $.59 \leq p \leq .85$, rule of the majority is preferable to a dictatorship by the most able only the the group size is 21 or fewer. This range should, we would expect, cover the majority of cabinets. (Presumably, cabinets which were wrong more often than they were right wouldn't last long.) Thus we would propose that over-sized cabinets are replaced by inner cabals when the more competent members perceive such decision-making mechanisms (i.e. decision-making by a smaller group with higher mean competence level) would be likely to be more efficient. If the cabinet's decisions are more likely to be correct than the decisions of any one of its members, the feasibility of a superior decision mechanism is less likely to be seen than when some individual (or individuals) in the group realize(s) that he (they) would be better off going it alone. Thus, the entries of Table 4 can be thought of as "tipping points." For groups of mean competence $.65 \leq p \leq .81$ this "tipping" will occur with a greater than 50% probability even for groups as small as 11. Since groups of high mean competence will tend to reach more accurate judgments than the best individual within them, replacing the old cabinet (or Council of the Crown, King's Council, or whatever) with a new and more select body will indeed be likely to improve the situation, i.e. increase the likelihood of correct (majority) judgments. Moreover, adding members to this new group will, up to a point, improve its decision-making even if the mean competence level of the additions is such as to lower somewhat the mean competence level of the group as a whole. Thus, we would expect the new group to grow. However, at some point we would expect the process to begin again, either because the new additions are *so* much less competent than their fellows that the probability of a correct decision drops, or because the limits of democracy for a group of that mean competence level have been reached. This cycle of growth and parturition is, Parkinson notes, the saga of the British Cabinet and its various precursors (Parkinson, 1957: 36-37). Of course, we could not expect a model based on dichotomous choice decision-making to be really descriptive of group growth and disintegration, but we do believe that a rationale for these processes based on the issue of group effectiveness is a step in the correct direction.

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