

Iannucci and Its Aftermath: The Application of the Banzhaf Index to Weighted Voting in the State of New York¹)

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Abstract: We review the use by New York State Courts of John Banzhaf III's game-theoretic inspired index of power as a measure of fair representation. We look at the extent to which game-theoretic arguments have been a) properly understood by the courts; b) integrated into constitutional and legal analysis; and c) properly applied. We pay particular attention to weighted voting in Nassau County, for which we provide a more detailed historical analysis.

1. Introduction

Recent decisions of the U.S. Supreme Court have stressed the requirement that apportionment and electoral systems at all levels of government approach the ideal of "one person, one vote." (See e.g., *Baker v. Carr*, *Reynolds v. Sims*, *Wesberry v. Sanders*.) In the 1960's unequally populated single-member legislative districts have been more or less eliminated due to court and legislative action. As a necessary consequence of adoption of a strict population standard, unit voting schemes which provided for one representative from each political subunit regardless of subunit population (such as those which had been used in 1960 at the county level in Michigan, Illinois, Wisconsin, New York and New Jersey and at the state level in eight state legislatures) have also been eliminated.

However, while single membership districting is the most common form of representation in the U.S., multimember districting and mixed single and multiple member

apportionments are to be found in various levels of government in the U.S.; and in one state (New York) weighted voting is the most common of the various systems in use for county government. In the late 1960's and 70's such systems have come under increasing challenge as violating 14th Amendment "equal protection" standards. In the past decade there have been well over a dozen lawsuits in New York alone challenging the constitutionality of local weighted voting and multimember district apportionment schemes on one man, one vote grounds.

In measuring deviations from the ideal of "one person, one vote — one vote, one value" in the case of weighted voting systems and systems which involve multimember districts, U.S. Courts have been urged by various plaintiffs to judge the fairness of voter/group/unit/legislator representation and weightings in terms of game-theoretic indices of power such as the Banzhaf index [*Banzhaf*, 1965, 1966]. The U.S. Supreme Court has rejected *Banzhaf's* reasoning as to the appropriate measure of voter power in the case of mixed single and multimember districts (*Whitcomb v. Chavis*); while New York State courts, on the other hand, have explicitly endorsed the use of the Banzhaf index as the appropriate measure of legislator power and as the criterion of fair representation in weighted voting schemes.

We propose to examine the nature and extent of the use by New York State courts of Banzhaf's game-theoretic inspired index of power as a measure of "fair" representation, making use of a number of criteria by which fair representation might be judged. We shall look at the extent to which game-theoretic arguments have been a) properly understood by the courts; b) integrated into constitutional and legal analysis; and c) appropriately applied.

2. The Banzhaf Index

In two articles which appeared in American law journals in the mid-1960's, *John Banzhaf III*, a lawyer and mathematician, proposed to evaluate representation systems in terms of the extent to which they allocated "power" fairly. *Banzhaf's* analysis makes use of game-theoretic notions in which power is equated with the ability to affect outcomes.

2.1 Equal Voter Power

Consider a group of citizens choosing between two opposing candidates. To calculate the power of the individual voter, we generate the set of all possible voting coalitions among the district's electorate. If there are N voters in the district, then there will be 2^N possible coalitions. Then we ask, for each of these possible coalitions, whether a change in an individual voter's choice from Candidate A to Candidate B (or from Candidate B to Candidate A) would alter the electoral outcome. If so, that voter's ballot is said to be *decisive*. A voter's power is defined as the number of times, in all possible coalitions, that his vote could be decisive, and can best be expressed as a percentage — i.e., the number of his decisive votes divided by the total number of all

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the decisive votes of all the voters (including himself). The higher the percentage of voter coalitions in which *his* vote is *decisive*, the higher a voter's power score.⁴) The Banzhaf index has considerable intuitive appeal; power is based on ability to affect outcome.

For single member district systems, each district having equal populations, all voters have identical power; the ability of the voter in one district to affect his district's electoral outcome is identical with the ability of another voter in a neighboring district to affect the outcome there. But what about the case of multiple member districts, with some districts of one size and others of another size? Here, since the voters who elect k representatives have k times as much importance as voters who can elect only one representative, we might expect that to equalize voter power we should assign the districts with k representatives k times as many voters as well, since with all votes of equal weight, intuitively, we would expect a voter's ability to decisively affect outcomes should be inversely proportional to district size. *Banzhaf* [1966] pointed out that this argument is mathematically incorrect.

In a two-party candidate contest where all voters have equal weight, in order for a voter to be decisive in a district of size N , the rest of the voters (who are $N - 1$ in number) must split half for one candidate/party and half against. A straightforward combinatoric analysis reveals [*Banzhaf*, 1966; *Whitcomb v. Chavis* 403 U.S. at 145 n. 23; *Walther*, p. 11; *Lucas*, p. 52] that, if all combinations of vote outcomes are equally likely (i.e., each voter is equally likely to vote for either candidate/party), each member's decisive votes b are given by:

$$b = \frac{2(N-1)!}{((N-1)/2)((N-1)/2)} \quad (1)$$

We can examine the link between b and N by using Stirling's approximation [see *Feller*; *Banzhaf*, 1966; *Walther*, p. 12-13; *Lucas*, p. 53]

$$N! \approx e^{-N} N^N \sqrt{2\pi N} \quad (2)$$

to rewrite (1) as

$$b \approx \frac{2^N}{\sqrt{2\pi(N-1)}} \quad (3)$$

Thus, each member's Banzhaf Index, which we shall denote B_i , is simply

$$B_i \approx \frac{1}{\sqrt{2\pi(N-1)}} \quad (4)$$

This analysis can be applied to electoral systems involving both single and multimember districts. We see from expression (4) that B_i is approximately proportional to the

⁴) There are considerably more powerful mathematical tools to calculate the *Banzhaf* index than merely enumerating all 2^k possible coalition outcomes and identifying decisive voters in each. We shall not discuss such techniques here. [See *Walther*; *Brams/Affuso*.]

square root of N , district population. (This appears to have been first pointed out by *Penrose* [1946]; cf. also *Felding* [1973]). Thus, if we wish to assign all voters equal power to affect outcomes, we should assign each district a number of representatives proportional to the *square root* of district population, rather than directly proportional to district population. Doing so, however, violates the norm of allocating an equal number of citizens an equal number of representatives.

If we assign one representative for every 100 population in the square root of district size, then if there are 20,000 population spread equally over 2 smds, these voters (10,000 per district) would be entitled to have 2 representatives, 1 per district, since the square root of 10,000 is 100. Similarly, if there are 40,000 citizens spread equally over 4 smds (10,000 each) they would be entitled to 4 representatives. However, a single mmd of size 40,000 would be allocated only 2 representatives, since the square root of 40,000 is only 200. Thus, in this example, 20,000 voters would be entitled to as many representatives as 40,000 voters. If we follow the square root rule, the allocation of representatives depends on how voters are divided among the districts.

In a case decided in 1970, *Whitcomb v. Chavis* 403 U.S. 143, the Supreme Court dealt directly with *Banzhaf's* concept of voter power. The case involved Indiana's scheme of single and multiple member districts for its state legislature. The plaintiffs, citing *Banzhaf's* work, argued that voters in the multiple member districts were over-represented, claiming that citizens in the larger district had a power disproportionate to their population.

We can make this argument explicit as follows: if votes in a multi-member district (with population mN) elected m representatives, each voter in such a district would have a power of $\frac{m}{\sqrt{m}} / \sqrt{N}$, while those in smds with population of N would have a power of $\frac{1}{\sqrt{N}}$. Since $\frac{m}{\sqrt{m}} > 1$, for $m > 1$, this would be denying to all citizens an "equally effective voice in the election of members of his legislature." (377 U.S. at 565.)

The court in *Whitcomb* rejected the *Banzhaf* argument, both in the majority opinion and in Justice Harlan's dissenting opinion. Only Harlan's opinion, however, dealt forthrightly with the intellectual merits of the *Banzhaf* argument. Harlan lampooned the absurdity of *Banzhaf's* simplifying assumptions (clearly articulated by *Banzhaf* himself) e.g., the assumption that there exist no ingrained voting habits and that therefore each voter is equally likely to vote for either candidate before him. He pointed out with glee how minor variations in *Banzhaf's* assumptions can lead to major variations in results. Harlan's opinion of the *Banzhaf* index is best summed up in one sly footnote which quotes Mark Twain: "There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of reality." (403 U.S. at 169 n.5.)⁵

⁵) *Banzhaf* [1965] was previously cited briefly in *Kilgarlin v. Hill* 386 U.S. 120, 125; and at greater length in *WMCA Inc. v. Lomenzo*, 246 F. Supp 953 at 959.

Banzhaf's own views on the reasonableness in political terms of his index are worth mentioning. . . . Thus, in constructing a mathematical model, which must of necessity ignore many of the real problems of the system, one may hypothesize the representative to be no more than a vehicle for

2.2 Legislator Weight Proportional to Decisiveness

So long as each legislator has a single YEA or NAY vote on issues coming before the legislature, the question of legislative power does not have to be explicitly addressed. Thus, in the leading apportionment cases which have come before the U.S. Supreme Court, all of which have involved single or multiple-member districts with each elected representative eligible to cast a single vote, it seems to be simply *assumed* that the justification for examining the number of persons contained within each district is the fact that elected representatives by their vote wield decision-making power in the affairs of the polity; and that equality of apportionment thus indirectly results in equality of policy-making power among citizens.

But what about weighted voting schemes (also fractional voting schemes) where, say, a legislator from a district with 20,000 population casts two votes, while a legislator from a district with 10,000 population casts only one vote? Again, it was *John Banzhaf III* who pointed out the fallacy of such "common sense" apportionment schemes. Consider, for example, a three-member committee, with members A and B with two votes, and member C with only one vote. Despite the fact that *vote shares* (weights) are not equal, from the standpoint of *Banzhaf's* concept of decisive votes all committee members have equal *power* (1/3, 1/3, 1/3) when a majority (3 of 5 votes) is needed. This is shown in Table 1, which shows the 8 possible coalitions. When a two-thirds vote is necessary for passage, the power scores change. Now member C has no power at all (in the language of game theory, he is a *dummy*), while the other two members each hold 50 percent of the power. *Banzhaf's* argument is simply that when weighted voting schemes are designed, weights should be assigned in such a way that a legislator's *power* (as contrasted with the number of votes) should be made proportional to the number of citizens in his district.

Since the U.S. Supreme Court in *Whitcomb* rejected *Banzhaf's* "square root" argument regarding assigning representatives to multiple-member districts, one might have predicted that courts would also have rejected his closely related line of reasoning for weighted voting schemes as well. Such was not the case. Ruling on the constitutionality of a weighted voting scheme for Washington County and one for Saratoga County, the New York State Court of Appeals in *Iannucci v. Board of Supervisors* (1967) 28²

reflecting as best he can the votes of his constituents on certain issues. In such a model of the representative system, each representative would in effect poll his district on each issue and cast his vote according to the majority vote. For the limited purpose of establishing the outer boundaries of a fair representative system, it seems reasonable to assume this type of representative as an oversimplified model [*Banzhaf*, 1968, p. 817].

What little is known about how legislator's votes are influenced tends to cast doubt on any theory which would have a constituent's ability to affect his representative's vote depend solely on the population of the district. Such a theory would ignore party alliances, ethnic blocs, regional differences and interests, lobbying, influence peddling, and other realities of political life. Yet, so far, the Supreme Court has looked no further than population figures in deciding reapportionment cases. Moreover, the justification offered for multimember district systems also depends upon such a theory. If influence and representation cannot with some reasonable degree of accuracy be approximated by such a theory, then the justification fails and multimember district systems should be abandoned. On the other hand, if any such numerical theory can give even a reasonable approximation to political reality, it is submitted that the analysis contained herein is at least mathematically consistent and therefore more likely to be correct than the inverse ratio theory offered as justification for such systems [*Banzhaf*, 1968, p. 817].

A B C	Votes	Needed to pass Majority = 3	Decisive	Needed to pass 2/3rd = 4	Decisive
1 Y Y N	5	P	-	P	A, B
2 Y Y N	4	P	A, B	P	A, B
3 Y N Y	3	P	A, C	F	B
4 Y N N	2	F	B, C	F	B
5 N Y Y	3	P	B, C	F	A
6 N Y N	2	F	A, C	F	A
7 N N Y	1	F	A, B	F	-
8 N N N	0	F	-	F	-

Tab. 1

N.Y.S. 2d 502 proposed that the districts be assigned weight such that the *Banzhaf* power index for each district's representative would be approximately equal to that district's population share.

In the context of multimember districts whose representatives were assumed to vote as a bloc, this criterion (applied in *Iannucci* only to the weighted voting case) would assign a number of representatives to each district such that the *Banzhaf* power of that district's bloc would be approximately equal to that district's population share. New York is a state where there are counties with both mmds and smds. However, as far as we are aware, New York courts have never seen the connection between the line of reasoning in *Iannucci* (which has been accepted in subsequent weighted voting cases) and the constitutionality of mixed mmd-smd systems. Thus, a weighted voting system with weights 3, 1, 1 would be unconstitutional under *Iannucci*, yet a mixed mmd and smd system with one mmd with three representatives and two single-member districts would be held to be perfectly all right.⁶⁾

2.3 Equal Voter Decisiveness on Legislative Outcomes

Banzhaf [1966] has proposed that the most appropriate criterion of fair representation in a weighted voting system is neither equal voter power nor equal legislator power.

⁶⁾ In *Whitcomb* the court rejected the argument that representatives from multimember districts are *necessarily* more likely to vote as a bloc than representatives elected from the same area, elected from contiguous single-member districts (403 U.S. at 147-148), although it accepted the fact that "bloc voting tended to occur" in Marion county, and "defendant's own witnesses thought it was advantageous for Marion County's delegation to stick together." (403 U.S. at 147). Nonetheless the Court asserted that "nothing before us shows or suggests that any legislative skirmish ... would have come out differently had Marion County been subdistricted and its delegation elected from single-member districts" (403 U.S. at 148).

Moreover, the Court was not impressed with the notion that bloc voting led to influence for representatives from multimember districts more than proportionate to their numbers. *Brams/Affuso* [1976] have shown that, in power index terms, coalitions are not always more powerful than their members taken singly and the Court may have had some dim intuition of this when it claimed that "the theory that plural representation . . . unduly enhances a district's power and the influence of its voters remains to be demonstrated in practice and the day-to-day operation of the legislature" (403 U.S. at 147).

er but rather that the requirement that all voters have equal ability to affect the outcome of a vote in the legislature. *Walther* [1976] shows that to achieve this result requires that the ratio below should be the same for voters in each district.

$$\frac{\text{decisive outcomes for legislator(s) from district } j}{\text{square root of population of district } j} \quad (5)$$

The probability that voter i will be decisive in his district election of legislative representatives multiplied by the probability that the legislators (or legislative bloc) from that district will be decisive in the legislature is the product we wish to be the same for all voters if we wish to equalize voter ability to affect legislative outcomes. The second term is proportional to the combined number of decisive outcomes for legislators from that district and the first term is approximately inversely proportional to the square root of district population. Because of the approximate correspondence between population weight and *Banzhaf* weights in most weighted voter systems this criterion will, in general, be incompatible with the criterion of equalizing legislator power, but will be compatible with the criterion of equalizing voter power.

As of 1960, most New York counties used a unit voting system for their County Boards of Supervisors in which each town/city ward was given one representative. This scheme was struck down in *Graham v. Board of Supervisors of Erie County* (1967) 267 New York Supplement 2d 383. As of 1960, only Nassau county used weighted voting. In Nassau, weights were assigned directly proportional to population but with certain other peculiar features (see Section 4, below). In the 1960's, in response to the voiding of unit voting systems, a number of New York counties sought to preserve township-based representation while still complying with Court directives on "one man, one vote" by shifting to weighted voting schemes similar to that in use in Nassau County. Two cases involving such counties (Saratoga County and Washington County) were combined and decided by the New York Court of Appeals in an important decision, *Iannucci v. Board of Supervisors of the County of Washington* 282 N.Y.S. 2d 502. In that case, as we noted above, the court held that weighted voting was permissible only if the weights led to *Banzhaf* values proportional to population. We shall quote the Court's opinion at some length:

Although the small towns in a county would be separately represented on the board, each might actually be less able to affect the passage of legislation than if the county were divided into districts of equal population with equal representation on the board and several of the smaller towns were joined together in a single district. [See *Banzhaf*, 1965, p. 317] . . . *The significant standard for measuring a legislator's voting power, as Mr. Banzhaf points out, is not the number or fraction of votes which he may cast but rather his ability . . . by his vote, to affect the passage or defeat of a measure . . . (Ibid, p. 318).* And he goes on to demonstrate that a weighted voting plan, while apparently distributing this voting power in proportion to population, may actually operate to deprive the smaller towns of what little voting power they possess, to such an extent that some of them might be completely disenfranchised and rendered incapable of affecting any legislation. (Iannucci 282 N.Y.S. 2d at 507, emphasis ours.)

The principle of one man-one vote is violated, however, when the power of a representative to affect the passage of legislation by his vote, rather than by influencing his colleagues, does not roughly correspond to the proportion of the population in his constituency. Thus, for example, a particular weighted voting scheme would be invalid if 60 % of the population were represented by a single legislator who was entitled to cast 60 % of the votes. Although his vote would apparently

be weighted only in proportion to the population he represented, he would actually possess 100 % of the voting power whenever a simple majority was all that was necessary to enact legislation. Similarly a plan would be invalid if it was *mathematically impossible* for a particular legislator representing say 5 % of the population to ever cast a decisive vote. Ideally, in any weighted voting plan, it should be mathematically possible for every member of the legislative body to cast the decisive vote on legislation in the same ratio which the population of his constituency bears to the total population. Only then would a member representing 5 % of the population have, at least in theory, the same voting power (5 %) under a weighted voting plan as he would have in a legislative body which did not use weighted voting — e.g., as a member of a 20-member body with each member entitled to cast a single vote. This is what is meant by the one man-one vote principle as applied to weighted voting plans for municipal governments. A legislator's voting power, *measured by the mathematical possibility of his casting a decisive vote*, must approximate the power he would have in a legislature which did not employ weighted voting. (Iannucci 282 N.Y.S. 2d at 508; emphasis ours.)

The Court then went on to confess itself unable to determine whether the plans before it met the criterion proposed, and asserted that the Boards are not entitled to rely on the presumption that their legislative acts are constitutional. Rather,

. . . with respect to weighted voting . . . a considered judgment is impossible without computer analyses and, accordingly, if the boards choose to reapportion themselves by the use of weighted voting, *there is no alternative but to require them to come forward with such analyses and demonstrate the validity of their reapportionment plans.* (Iannucci, 282 N.Y.S. 2d at 510; emphasis ours.)

With these words the Court ushered in the age of computerized weighted voting in New York county government and helped a New York mathematician and consultant, Lee Papayanopoulos, to supplement his income in the next decade by providing New York counties with weighted voting schemes acceptable under the Iannucci guidelines. In the next few years, New York counties were more likely to adopt weighted voting and mixed multimember and single-district systems than they were to shift to single-member districting. (See Table 2) In Most of these systems, a handful of towns controlled a majority of votes [see Tables 3 and 4 in *Grofman/Scarow*].

Single Member Districts (20)

Albany	Franklin	Onondaga
Broome	Herkimer	Orange
Cayuga	Genessee	Otsego
Chemung	Lewis	St. Lawrence
Clinton	Monroe	Suffolk
Dutchess	Niagara	Westchester (P) ¹)
Erie	Oneida	

Multi-Member of Uniform District Size (1)

Allegany

Multimember of Unequal District Size (12)

Cattaraugus	Rockland	Tioga
Chatauqua	Schenectady	Tom King
Greene	Schuyler	Ulster
Rensselaer	Steuben	Yates

Simple Weighted Voting (3)

Orleans ²)	Oswego	Putnam
Computerized Weighted Voting (21)		
Chenango (P)	Jefferson	Schoharie
Columbia	Livingston	Seneca (P)
Cortland ³) (P)	Madison	Sullivan (P)
Delaware	Montgomery	Warren
Essex	Nassau (P)	Washington
Fulton	Ontario	Wayne
Hamilton	Saratoga	Wyoming

¹) (P) refers to counties for which we know Papayanopoulos to have prepared a weighted voting scheme.

²) Orleans' system of weighted voting is presently under court challenge.

³) Cortland uses weighted voting with single-member districts of approximately equal size. See *Slater v. Board of Supervisors of Cortland County* 330 NYS 2d 947.

Tab. 2: County Governing Bodies in New York State as of June 1977

In Iannucci New York's highest court has relied on the *Banzhaf* index as the measure of fair representation for legislators in weighted voting systems. In Whitcomb three years later, the U.S. Supreme Court considered and then rejected the relevance of voter power calculations based on a very similar line of reasoning (see discussion in Section 2 above). We might expect New York courts to have subsequently repudiated the Iannucci doctrine. They didn't. Instead, nearly two dozen New York counties shifted from unit voting to weighted voting; and with only a dwindling handful of exceptions (now only 3 in number), the apportionments rested on computerized schemes intended to satisfy the Iannucci doctrine. (Indeed in some cases, these apportionments were devised by New York district courts themselves in response to challenges to existing apportionments.) How can we account for this seeming divergence between New York and U.S. Supreme Court rulings?

First, in all the New York county apportionment cases involving weighted voting decided after Iannucci, it is Iannucci which is looked to for guidance; given the nature of our federal system, Supreme Court cases are treated as gloss. In Iannucci only the legislator power argument of *Banzhaf* [1965] is discussed; the voter power criteria of *Banzhaf* [1966] are not mentioned, despite the fact that *Banzhaf* himself entered an amicus curiae brief in the Iannucci case in which he sets forth the basic arguments of both *Banzhaf* [1965] and *Banzhaf* [1966].

Second, we should note that the conflict is more apparent than real. The argument before the Supreme Court in Whitcomb involved the square-root law as a means to maximizing voter equality. On the other hand, the argument before the New York Court of Appeals in Iannucci involved the use of the *Banzhaf* index as a measure of legislator strength. As pointed out previously, for weighted voting schemes (or mixed schemes with bloc voting) the two lines of argument lead to conflicting apportionment criteria. Thus, it would seem possible to accept one line of argument without accepting

the other. The divergence between these two lines of reasoning was pointed out in an insightful article by Johnson [1969], and had even earlier been recognized by some New York courts.⁷)

Third, there is an important difference between the line of reasoning used by the New York Court of Appeals in Iannucci and that used by the U.S. Supreme Court in Whitcomb which explains, in part, the divergent conclusions reached by the courts as to the usefulness of the *Banzhaf* index as a major component of a measure of "fair representation." In Whitcomb, as we previously pointed out, the Supreme Court rejected the use of the *Banzhaf* index on the grounds that it did not take into account "any political or other factors . . ." (403 U.S. at 406). However, in Iannucci, the New York Court of Appeals asserted that the sole criterion is the mathematical voting power which each legislator possesses in theory — i.e., the indices of representation — and not the actual voting power he possesses in fact — i.e., the indicia of influence." What was a sin for the U.S. Supreme Court was a virtue for the New York Court of Appeals. The one condemned *Banzhaf's* approach because of its lack of realism, the other applauded it because its abstractness and timelessness did not require constant revisions of apportionment decisions in the light of new election returns or changing political alignments.

3. Weighted Voting Case Study: Nassau County

Conflicting court rulings re the constitutionality of various Nassau County Legislature appointment schemes reveal how difficult determining the nature of "fair" representation in a weighted voting scheme can be, even if the Iannucci guidelines are followed.

⁷) The Supreme Court of the State of New York in *Westchester County (The Town of Greenburgh and the Town of Yorktown vs. the Board of Supervisors of Westchester County*, August 23, 1967), anticipating the Supreme Court in Whitcomb, held that the reasoning leading to the conclusions in *Banzhaf* [1966] "ignores all the realities of representative government and the conclusion reached can be no more reliable than the premise from which it starts."

In a slightly earlier Westchester case (*Town of Greenburgh v. Board of Supervisors of Westchester County* 277 N.Y.S. 2d 855), the Westchester County Supreme Court reviewed and rejected *Banzhaf's* square root argument.

It is obvious and conceded that under any plan which employs electoral districts of substantially unequal populations, effective population-based legislative representation cannot be obtained by distributing legislative seats, or votes in proportion to population square roots. Whatever this may accomplish in giving a legislator voting power commensurate with that of the citizen, in his district, it will not give him a legislative influence, proportional to the population of the district which he represents. For instance, in the situation . . . with Districts A, B, and C having population of 400 each and District D a population of 2500, whether votes or legislative seats are to be distributed 2, 2, 2 and 5, the results will be the same. In either case the three small districts with a total population of 1200 will be able to outvote District D with a population of 2500 and control, by minority rule, the deliberations of a legislative body consisting of the four representatives (277 N.Y.S. 2d at 897-898).

In a post-Iannucci case which contrasts the arguments of *Banzhaf* [1965] and *Banzhaf* [1966], the Supreme Court of Seneca County in *Glessing and Glessing v. Board of Supervisors of Seneca County*, October 26, 1967, rejects the test of equalizing effective votes of citizens in electing their legislators, and asserts that "the test laid down by the Court of Appeals in Iannucci is the legislator's decisive voting power and not the effective voting power of a citizen in electing the legislator" (emphasis in original).

In Nassau County, from 1900 (the year in which the county was formed, being carved out of Queens County) until 1917, the County Board of Supervisors was composed of the three town supervisors — one from the town of Hempstead, one from the town of North Hempstead, and one from the town of Oyster Bay. Each cast a single vote. Beginning in 1917, however, a system of *weighted voting* was introduced awarding votes to a supervisor in proportion to the population of his town. The following year a second elected position of supervisor-at-large was created for the town of Hempstead, specifically for the purpose of doubling the town's representation, since the town by this time had over half the population of the county. The net result of these two changes was a voting scheme whereby the town of Hempstead's two supervisors were able to cast a total of eight votes, whereas the supervisors from the other two towns were able to cast only two votes each. Even though Nassau's two cities were incorporated about this time, with each city being awarded one vote, the town of Hempstead was still able to dominate the proceedings.

Thus, from 1922 until 1936 the distribution of votes was as follows:

Town of Hempstead #1	4
Town of Hempstead #2	4
Town of North Hempstead	2
Town of Oyster Bay	2
City of Glen Cove	1
City of Long Beach	1
	<u>14</u>

A new charter was adopted in 1936. This charter continued the system of weighted voting, awarding one vote for every 10,000 population, but contained a major constraint on weighted voting apportionment: *no town could cast a majority of the weighted votes.*

Applying the one vote per 10,000 voters formula to the 1936 population the distribution of legislative votes was:

Hempstead #1	9
Hempstead #2	9
North Hempstead	6
Oyster Bay	3
Glen Cove	1
Long Beach	1
	<u>29</u>

In this case, a majority is 15 ($(29 + 1)/2$) votes and Hempstead's combined voting power was 18 votes. Clearly the one-vote-per-ten-thousand-citizens provision of the charter is incompatible with the charter provision which prohibits any town from wielding majority voting power. What was to be done? In December 1937, the Nassau County attorney proposed an ingenious method to "reconcile" the conflict between the two clearly incompatible charter provisions. He proposed that the votes of any town having over a majority be reduced to just under a majority. Thus, Hempstead's

total vote would be reduced from 18 to 14 (7 + 7). This would produce the following distribution:

Hempstead #1	7
Hempstead #2	7
North Hempstead	6
Oyster Bay	3
Glen Cove	1
Long Beach	1
	<u>25</u>

However, the majority would be left at 15 (not 13)! Similarly, when a two-thirds vote was required, the county attorney proposed that a two-thirds vote (a constitutional two-thirds) be calculated at 20 votes (not 17 votes), and that Hempstead would still only be allowed to cast 14 votes. This "interpretation" of the 1936 charter was accepted by the Board and went into effect in January 1938 although corrected census figures which excluded aliens changed the actual vote allocations slightly. The county attorney's recommended procedure was used to determine votes in the three subsequent reapportionments: 1942, 1962, and 1972.⁸) As of 1968, Hempstead, which had 57% of Nassau's population, was given only the effective equivalent of 49.6% of the votes on the Board of Supervisors in order to comply with the charter prohibition against any town having more than 50% of the votes. (Of course, given the majority required to pass legislation, this scheme gave Hempstead's representatives the power to block legislation — since no bill could be passed without the support of at least one Hempstead representative.) In 1968 the Nassau County Supreme Court held that this apportionment scheme was unconstitutional in that it deprived "citizens of Hempstead of their right to substantial equality of representation." The New York Court of Appeals upheld this decision *Franklin v. Mandeville* (1970) 308 N.Y.S. 2d 375, but allowed the existing scheme to remain in effect until the 1970 census, at which time the Board was directed to draw up a reapportionment scheme consistent with the principle of one man, one vote.

In ruling the scheme unconstitutional, neither the Supreme Court nor the Court of Appeals addressed the question of whether the scheme satisfied the test that it allocated to each legislator "voting power, measured by the mathematical possibility of his casting a decisive vote, approximately equal to the power which he would have in a

⁸) These vote reduction and special majority requirement procedures are not well known; most authors who have discussed weighted voting in the Nassau County Board of Supervisors were unaware of them. For example, *Thomas* [1960], in a book on Nassau County government mistakenly asserts that the 1942 apportionment violates the Charter provision that no town be given voting majority in that "Hempstead has 18 out of the 30 votes . . . clearly more than fifty percent of the total." The most cited article on weighted voting in the legal literature [*Banzhaf*, 1965] also makes this mistake, claiming that both the 1942 and the 1962 apportionments resulted in three of the six Nassau County Board members having zero voting power and the remaining three having equal power (as measured by the *Banzhaf* Index). Other authors [*Brams*] repeat *Banzhaf's* mistake, some laboring under the misapprehension that these special procedures did not go into effect until 1971 *Lucas* [1974, p. 421] or 1972 *Andelman* [1972].

legislative body which did not employ weighted voting." The finding in *Franklin v. Mandeville* was based on more straightforward grounds. It simply held that a scheme which forever denied majority representation to residents of a town which had 57 % of the county population was, on the face of it, invalid.

In 1972, the Nassau County Board of Supervisors in accord with the Court's earlier directive, proposed a reapportionment scheme based on 1970 census data. The Board had employed a computer analyst, the aforementioned Lee Papayanopoulos, who reviewed over 2,000 different combinations of votes and voting. The final plan proposed involved weighted voting and, indeed, was substantively identical to that previously rejected by the Court. This plan was declared unconstitutional by the Nassau County Supreme Court (72 Misc. 2d 104, 338 N.Y.S. 2d 561). However, the New York Court of Appeals reversed this ruling. (*Franklin v. Krause* 1973 344 N.Y.S. 2d 885.)

The Appeals Court rejected the view offered by the Nassau County Supreme Court that weighted voting was, per se, unacceptable as a matter of law, and also rejected the claim that the new plan had the same flaw as the apportionment scheme previously rejected as unconstitutional. In *Franklin v. Krause* the Court cleverly finesses the question of what (if anything) is *different* about the 1972 plan to improve it over the 1962 scheme (rejected as unconstitutional in *Franklin v. Mandeville* 308 N.Y.S. 2d 375), other than the fact that it was drawn up by a computer analyst (*Franklin v. Krause* at 887). Instead, the court addresses the 1972 plan *de novo* on its merits. The principal test used by the Court was the difference between population share and power share (as measured by the *Banzhaf* index, for majority votes only) for each of Nassau's three towns and two cities, *under the assumption that the two Hempstead supervisors voted independently of one another*. The maximum deviation was +3.8, and the total deviation was only 13.9. The deviation range was 7.3 (-3.5 to +3.8).

The court held (*Franklin v. Krause* at 888) that these deviations were within the Iannucci guidelines.

There are a number of problems with the Appeals Court's decision in *Franklin v. Krause*.

- a) By the criterion used in that decision, the plan rejected as unconstitutional in *Franklin v. Mandeville* is constitutional. The sum of the deviations in the 1962 plan was only 14.6 (as compared to 13.9 for the 1972 plan), and the maximum deviation difference was again only +3.8 while the range was 7.5 (-3.7 to +3.8). Furthermore, the two plans are *identical* in the power they assign representatives under majority voting. See Table 9, Column 4 in *Grofman/Scarrows* [1978].⁹)
- b) By the criterion used in *Franklin v. Mandeville* the plan accepted in the *Franklin v. Krause* decision should have been rejected. Under the 1962 plan, Hempstead's two representatives have 57.1 % of the population and 49.6 % of the vote share. Under the 1972 plan, Hempstead's two representatives have 56.2 % of the population and 50.0 % of the vote share. Under both plans, a township with over 50 % of the population is denied the possibility of ever obtaining a vote share of over 50 %. In *Frank-*

⁹) The data on power indices for the 1962 plan were made available to the court in the plaintiff's and cross-respondent's briefs. The court chose to disregard them and to decide *Franklin v. Mandeville* on other grounds. The court was reminded of these data in the cross-respondent's brief in *Franklin v. Krause*. Again, they disregarded them.

lin v. Mandeville the Court worried whether Hempstead was unconstitutionally *underrepresented* and concluded that it was. In *Franklin v. Krause*, with the same power distribution and virtually unchanged population fractions, the court worried whether Hempstead was unconstitutionally *overrepresented*, and concluded that it was not. Both the question and the means of answering it shifted.

In fairness to the Court of Appeals we should note that it defended its seeming inconsistency by pointing out (344 N.Y.S. 2d at 891) that, in the light of very recent Supreme Court cases extending the range of permissible deviations from strict population standards, the validity of standards in *local* apportionment decisions applied to its former decision had been very significantly altered.

- c) The Court cannot make up its mind as to whether to treat Hempstead's two representatives as independent or as a voting bloc. For the purpose of calculating the power index, they are treated as voting independently of one another. Yet, for purposes of warding off the danger of a constituency unit having 100 % voting power, they are treated as a voting bloc, as illustrated in the following passage:

It was also noted in Iannucci that a weighted voting plan would be invalid if over 50 % of the population were represented by a legislator entitled to cast over 50 % of the votes for them; in reality, he would possess 100 % voting power, at least as to measures requiring a majority vote for passage. The instant plan would violate that injunction, of course, were it not for its provision that for passage of a measure requiring a majority, 71, and not 66, votes are required; and for measures requiring a two-thirds vote, 92, and not 87, votes are required. Thus, while the Town of Hempstead Supervisors together possess 70 votes, more than a majority of the total 130, they cannot have 55 % voting power which would ordinarily be 100 % voting power in a 'pure majority' situation. This admittedly artificial voting requirement, in reality, gives the town of Hempstead a greater disenfranchisement than would otherwise be the case in certain voting combinations. (*Franklin v. Krause*, 888, emphasis ours.)¹⁰)

If the court were to treat Hempstead's two representatives as a voting bloc, as they do in the above passage, then the *Banzhaf* power indices would be different -- based on a five-member rather than a six-member board. Under the assumption that Hempstead's representatives vote together, in a majority vote under both the 1962 and 1972 plans, Hempstead has 89 % of the voting power, and the remaining four towns divide up in the remaining power equally, 3 % per town. With these assumptions, the sum of the differences between population and power are huge. In 1972 the sum is 68.2, and the maximum deviation is +32.7. In 1962 it was 66.9 with a maximum deviation of +31.3. [See *Grofman/Scarrows*, Table 9 for details.] Of course it can be argued that to treat the Hempstead supervisors as the voting bloc they in fact *are* is to violate the stipulation offered in Iannucci as to the irrelevance of actual voting patterns.

¹⁰) Immediately after this passage comes a sentence of remarkable ambiguity: "This is precisely the point which caused our rejection of the former plan, which, although based on different scales and values, contained the same sort of bar preventing the town of Hempstead supervisors from having 100 % voting power." (*Franklin v. Krause*, 888, emphasis ours.) What does the "this" refer to? The court *rejected* the 1962 apportionment scheme. Furthermore, in 1962 the Court was worrying about whether Hempstead was *underrepresented*, not about whether they were *overrepresented*.

Nor will practical experience in the use of such plans furnish relevant data since the sole criterion is the mathematical voting power which each legislator possesses in theory — i.e., the indicia of representation — and not the actual voting power he possesses in fact — i.e., the indicia of influence. (20 N.Y. 2d at 252.)

We agree that observed patterns of voting coalitions, which effect legislators' power indices (since they lead to situations in which not all coalitions are equally likely) may be disregarded. If we were to look at observed coalition patterns, any legislature dominated by a single party with a very high index of party cohesion might be taken to be in violation of the Iannucci doctrine that no members be shut out from the possibility of ever being decisive, since minority members' votes would be irrelevant if the majority party always voted as a bloc. We believe, however, that the Hempstead case does not fall under the Iannucci rubric in that both supervisors represent the *same* constituency and are elected at the same election. Thus, to lump their votes together as *the* Hempstead vote seems the more reasonable procedure, *regardless* of how they actually vote. In fact, virtually no case is known in which they have ever voted differently. Furthermore, since the "not more than 50%" clause of the 1936 Nassau County Charter was obviously intended to protect the smaller towns from domination, and since Hempstead had always had two representatives, even prior to this charter, the drafters of the charter clearly operated under the assumption that Hempstead's supervisor and deputy supervisor would indeed vote together as a single unit. The arithmetic gyrations called for in the 1938 county attorney's opinion are also based on that same premise. We do not see why the court did not take judicial notice of these facts, although we must admit that none of the briefs offered in either *Franklin v. Mandeville* or *Franklin v. Krause* call this issue to the court's attention. If the court had proceeded on the assumption that Hempstead's voting power should be based on the combined vote strength of its two representatives, it is inconceivable that the 1972 apportionment plan could have been found constitutional under the Iannucci guidelines — the discrepancies between population share and power share are simply too vast.

The question of how to treat multiple representatives of a single constituency is a troublesome one, and in none of the New York cases has there been any consideration of empirical evidence on the bloc voting tendencies of such representatives. [See further discussion of this point in *Grofman/Scarrow*, 59–60.]

- d) Even if we treat Hempstead's representatives as separate from one another, the Court used an inappropriate measure to compare township population and township power. The New York Court of Appeals looked at the difference between population share and power share (measured in percents). We believe the appropriate measure should have been that used by the New York Supreme Court in 342 N.Y.S. 2d 189, to wit:

$$\frac{\text{population share} - \text{power share}}{\text{population share}} \quad (6)$$

For example, the U.S. Supreme Court in *Mahan v. Howell* 1973 410 U.S. 315 has looked at deviations from ideal representation using the formula

$$\frac{\text{mean district population share} - \text{population share}}{\text{mean district population share}} \quad (7)$$

and examined the range of differences. The calculation is based on the deviation from the ideal (= mean district population share — population share) measured in terms of (i.e., divided by) the ideal (= mean district population share). Ideally, each district would have identical population; that is, each district would have a population equal to the mean district population. The analogue of expression (7) for an assembly using weighted voting is given in expression (6). Ideally, each town would have a power share equal to its population share. The calculation used by the Court in *Franklin v. Krause* looks only at the numerator of this expression. Thus, for example, Glen Cove is found to deviate only 3.3% (= .056 — .023) from its ideal representation (*Franklin v. Krause*, 889); yet, in actuality Glen Cove deviates 143% = (.056 — .023/.023) from its ideal representation; that is, Glen Cove has more than twice the representation (measured in power share) that it is entitled to (measured in population share). This, we believe, indicates that the Iannucci guidelines, *when these are properly construed*, are violated — even when the Hempstead supervisors are treated separately [cf. *Johnson; Imrie*]. In Monroe County a small township with one vote was overrepresented in power terms (as measured by expression (6)) by more than 200%, and this was held by the local Supreme Court to violate the principle of one man, one vote (342 N.Y.S. 2d 189). Had the Court of Appeals reasoned similarly in *Franklin*, the Nassau scheme would have been voted unconstitutional, since it involved a discrepancy of over 200% for Long Beach. [For details see *Grofman/Scarrow*, Table 10.]¹¹⁾

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¹¹⁾ For discussion of power discrepancies when 2/3 vote is required see *Grofman/Scarrow* [1978, Table 11 and note 37].