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COMPARING AND CONTRASTING THE USES OF TWO GRAPHICAL TOOLS FOR DISPLAYING PATTERNS OF MULTIPARTY COMPETITION

Nagayama Diagrams and Simplex Representations

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ABSTRACT

We compare two tools for displaying, in graphical form, information about vote outcomes in multiparty elections at the constituency level. One was recently proposed by Nagayama and introduced to the English-speaking world by Reed, who applied this method to Japanese and Italian election data. Reed labels the method Nagayama diagrams. Recently, Taagepera has shown how the domain of potential uses of Nagayama diagrams can be expanded significantly. A second graphical device has been used by a number of authors for various types of election analyses, but is not that well known in the comparative parties literature. This method, which uses barycentric coordinates (i.e. triangular) rather than the more familiar rectangular coordinates, has gone under a variety of names (e.g. trilinear plot, toroidal diagram and simplex representation), but we have chosen to use the last of these labels. We make use of both methods to visually present election data (by constituency) for the Italian national elections of 1994, 1996 and 2001. We show how different types of information may be readily gleaned from the two types of graph, and, perhaps most importantly, illustrate how we may improve the ready intuitive interpretability of each type of graph by specifying boundary constraints to define particular regions of the graph – a technique we call 'segmentation'.

KEY WORDS ▪ graphics ▪ political competition ▪ visual display

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A picture, it is said, is worth a thousand words. Similarly, graphical representation of data can often be more visually informative than presentation of the same data in tabular form. In this article we present two useful tools for representing a set of election returns for multiparty competition in individual constituencies. One is very new, the other has a venerable history and yet remains remarkably underutilized in the party and electoral systems literature.

The first method that we discuss, which we refer to as segmented Nagayama diagrams, is our own elaboration of the Nagayama diagram (Nagayama, 1997: cited in Reed, 2001) introduced to the English-speaking world by Reed (2001). These diagrams display the relative vote shares of the largest and second largest parties in a district. However, Taagepera (2003; Taagepera and Allik, 2003) has recently shown how the domain of potential uses of Nagayama diagrams can be significantly expanded when used to represent information about the feasible vote share values also of third parties, fourth parties, and so on.

A second graphical device, which uses barycentric (i.e. triangular) coordinates rather than the more familiar rectangular coordinates, has previously been used for various types of election analyses (see, e.g., Allen, 2002; Bartholomew and Bassett, 1971: 106–109; Cornford et al., 1995; Gaines, 2000; Ibbetson, 1965; Katz and King, 1997; Saari, 1994, 1995; Stray and Upton, 1989; Upton, 1976, 1994, 2001)[1] and in other domains, but is still not that well known in the comparative parties or electoral systems literature.[2] This method has gone under a variety of names (e.g. trilinear plot, toroidal diagram and simplex representation), and here we use the last of these terms. As with Nagayama diagrams, we propose the addition of a further element of visual display, i.e. a new form of presentation of the information in terms of defined segments of the triangle.

Nagayama diagrams and simplex representations can each be used to visually reveal data about the nature of party competition: in particular, the extent to which patterns of two-party competition have solidified at the district or national level. Because each type of figure can be used to represent the same data in a different fashion, each has slightly different advantages in terms of which features of the data are most intuitively grasped from visual inspection of the relevant diagrams. Nagayama diagrams help us understand the nature of competitiveness at the district level; simplex representations are more useful in determining the extent to which observed patterns are associated with particular parties/party blocs. We illustrate the use of each method with data taken from the single-member district results in the 475 single-member district constituencies in the Italian Chamber of Deputies in the elections of 1994, 1996 and 2001.

Neither of these methods is original, and we make no claims to any originality in our presentation. The primary purposes of this article are twofold. First, by comparing Nagayama diagrams and simplex representations for the same datasets, and by using each to search for insights into changes in patterns of party competition in Italy over the course of three elections, the contributions of each of the two methods can be more clearly revealed. Second, as previously noted, we extend each method in terms of what we call ‘segmentation’, a way of delimiting zones within each triangle to identify types of outcomes of particular interest, e.g., zones of one-party dominance. We believe that both methods should be more widely known and more widely used within the political parties and electoral systems literatures, and we hope that this article will contribute to that end.

The Basic Properties of Segmented Nagayama Diagrams and Simplex Representations

Segmented Nagayama Diagrams

For data at the constituency level in a single election contested by multiple parties, the x-axis in a Nagayama diagram is customarily used to show the vote share of the largest party, and the y-axis the vote share of the second largest party. Because the second largest party must receive fewer votes than the largest party, the feasible set of values in the diagram lies within a triangle bounded by the x-axis and segments of the lines $x - y = 0$ and $x + y = 1$. The former line segment represents the situation where the largest and second largest parties have equal vote shares; the latter line segment represents the situation where no third party receives any votes. By comparing the distribution of outcomes in the Nagayama diagram for different elections, it is possible simultaneously to view how electoral politics is shifting vis-à-vis the degree of competition between the top two vote-getting parties, on the one hand, and vis-à-vis the extent to which third, fourth, etc., parties are receiving any substantial share of the district level votes, on the other.

Although the Nagayama diagram is in only two dimensions, we can use it to determine the sum of the share of the votes for parties other than the first largest and second largest parties in a district simply by summing up the x and y values and subtracting from 1. We should also note, as pointed out by Taagepera (2003), that for multi-seat constituencies Nagayama diagrams can be used to visually represent seat shares at the constituency level.

Taagepera in his recent work (2003; Taagepera and Allik, 2003) has also shown how to generalize the uses of Nagayama diagrams in an interesting and elegant way by taking advantage of the fact that we can draw additional line segments within such triangles that indicate the maximum of values for the k^{th} largest parties, for any k, in addition to just using the *external* boundaries of the triangle to specify bounds on the share of the second largest party. We can illustrate Taagepera’s idea for third-party strength. The basic idea in Taagepera’s work is simple. A particular point in the triangle tells us that the vote (seat) strength is of the first largest party, which we may call v_1 (s_1), and of the second largest party, which we may call v_2 (s_2), and of the remaining parties, namely $1 - v_1 - v_2$ ($1 - s_1 - s_2$). But we also know that the third largest party can have no higher vote share than the second largest party, and thus no higher vote share than $(1 - v_1)/2$ (or $(1 - s_1)/2$, if we are looking at seat share). As we let v_1 (s_1) range from 1/3 (where the top three parties have an equal share of votes or seats) to 1, this formula defines a line segment that gives the maximum feasible vote (or seat) share of the third largest party. And, similarly, for the k^{th} largest party, where, for the formula $(1 - v_1)/(k - 1)$ (or $(1 - s_1)/(k - 1)$, if we are looking at seat share), we let v_1 (s_1) range from $1/k$ (where the top k parties have an equal share of votes or seats) to 1.

Here we introduce a different modification of Nagayama triangles, while still confining ourselves to reporting results for the two largest parties, by providing some additional labeling of segments of the Nagayama triangle that will facilitate visual comparisons of multiple Nagayama diagrams representing different elections[3] (or the same election in different geographical regions). From these segments we develop indices that are analogous to Gini coefficients.

We use the parameter z ($0 \le z \le 0.50$) as a free parameter to define these segments. For example, by varying z we can change our operationalization of what we mean by a competitive district.

For the value $z = 0.20$, we illustrate in Figure 1 what we refer to as a segmented Nagayama diagram. In it, we have blocked out portions of the Nagayama triangle that are of special interest. In particular, we show the *competitive districts* in the Nagayama triangle that lie on or between the line segments defined by the lines $x - y = 0$ and $x - y = z$; we also show the *districts with strong or complete two-party dominance* that lie on or between the line segments defined by the lines $x + y = 1 - z$ and $x + y = 1$. In addition, we show portions of the line $x - y = 1 - z$, and of the line

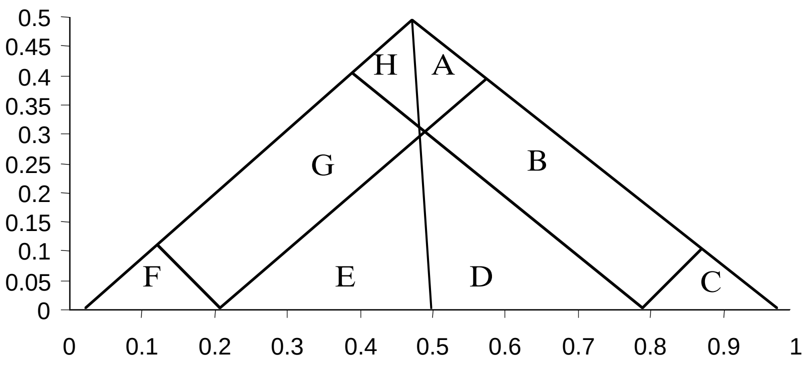

Figure 1. Segmented Nagayama diagram ($z = 0.20$)

$x + y = z$; and also, even more importantly, a portion of the line $x = 0.5$, which divides the triangle into two segments – in the triangular segment to the right the *largest party in the district receives 50 percent or more of the total vote*; in the triangular segment to the left, the *largest party in the district receives less than 50 percent of the vote*. The five line segments mentioned above, along with portions of the lines $x + y = z$, $x - y = 1 - z$, and the x-axis, divide the Nagayama triangle into eight mutually exclusive and exhaustive segments. Regions specified by different sets of segments can be interpreted in terms of the nature of the party competition reflected in the districts in the segments.

In the segmented Nagayama triangle shown in Figure 1, the area representing *districts with limited third-party strength*, i.e. those that lie on or between the line segments defined by the lines $x + y = 1 - z$ and $x + y = 1$, is given by the sum of the segments H + A + B + C.

The area representing *competitive districts*, i.e. those that lie on or between the line segments defined by the lines $x - y = 0$ and $x - y = z$, is given by the sum of the segments H + A + F + G.

These two areas overlap, sharing segments A and H, the combination of which we may think of as the zone characterized by *limited minor party strength and political competitiveness among the top two parties*.

Conversely, the area which is characterized by *neither strong or complete single- or two-party dominance nor political competitiveness among the top two parties* is given by the sum of segments D and E.

The area where the *largest party receives at least 50 percent of the vote* is given by the sum of segments A + B + C + D; the area where the *largest party receives no more than 50 percent of the vote* is given by the sum of segments E + F + G + H.

The area defined by segment F alone, i.e. the area that lies on or between the line segments defined by the lines $x - y = 0$ and $x + y = z$ and the x-axis, may be thought of as a zone of *extreme multiparty competition*.

The area defined by segment C alone, i.e. the area that lies on or between the line segments defined by the lines $x + y = 1$ and $x - y = 1 - z$ and the x-axis, may be thought of as a zone of extreme one-party dominance.

If we normalize the area under the Nagayama triangle to sum to 1, then, after some straightforward geometry (relying on the Pythagorean theorem and well known results about similar triangles), we may specify the areas in each of the eight segments (or any combinations thereof) as a simple function of z.[4]

Proposition 1: For segmented Nagayama diagrams, for a fixed z:

(a) $A = H = C = F = z^2$

(b) $D = E = 2z^2 - 2z + \frac{1}{2}$

(c) $B = G = -4z^2 + 2z$

Thus, for example, the area representing *districts with strong or complete two-party dominance*, given by the sum of segments H + A + B + C, equals $-z^2 + 2z$, as does the area representing *competitive districts* given by the sum of segments H + A + F + G. Similarly, the residual area shown in the triangle given by the sum of segments D + E (characterized by *neither strong or complete party dominance nor political competitiveness among the top two parties*) is given by $4z^2 - 4z + 1$.

By comparing the proportion of the actual districts found in any given region of the triangle with the areal share of that portion of the triangle we can calculate an index of over-/under-representation analogous in some ways to a Gini index. For example, we can create a simple function that runs between –1 and 1 that compares the proportion of districts lying in segments H + A + B + C (which we label O for observed) to the value $-z^2 + 2z$ (for a fixed z) that would be expected if points were uniformly distributed over the triangle (which we label U for uniform):

$$f_z(O, U) = (O - U)/ \max (U, 1 - U)$$

For a given value of z, this function can be taken as a *measure of the elimination of strong third parties at the district level*. Negative values of this index indicate systems where third parties remain stronger than would be expected under a uniform distribution; positive values of this index indicate systems where third parties are weaker than would be expected under a uniform distribution. Analogous indices can be generated for other segments, e.g. for the area representing *competitive districts* given by the sum of segments H + A + F + G.

There is, in our view, no natural or obvious value of z to pick. What value of z to use will depend upon the purposes of the researcher. In particular, there may be a trade-off between picking higher values of z for purposes of defining the absence of viable third parties and lower values for defining the degree of inter-party competition (e.g. a district where 80 percent of the vote goes to the top two parties seems clearly one of two-party dominance, but a district in which 20 percentage points separate the first and second parties may not seem that competitive).

We have initially chosen the value $z = 0.20$ to illustrate our results for two reasons. First, and most importantly, an 80 percent share for the top two parties seems a plausible breakpoint at which to operationalize the concept of ‘clear two-party dominance’; second, the value $z = 0.20$ has a nice mathematical property, as shown in the corollary to Proposition 1 below.

Corollary 1 to Proposition 1: For segmented Nagayama diagrams, if $z = 0.20$, then the region representing districts with strong or complete two-party dominance (H + A + B + C), the region representing competitive districts (H + A + F + G), and the residual region (D + E) characterized by neither strong or complete party dominance nor political competitiveness among the top two parties, are each identical in area.[5]

When these three areas are of equal size it is easier for the analyst to make direct visual comparisons of the degree of concentration of points in these three different regions of interest.

However, the three regions (H + A + B + C), (H + A + F + G) and (D + E) are not mutually exclusive in that the first region, representing districts with weak third, fourth, etc., parties, and the second region, representing districts competitive between the top two parties, share segments A and H. We can, however, readily define a set of three mutually exclusive and exhaustive regions by assigning A to one region and H to another, to yield {A + B + C}, {F + G + H} and {D + E} as our three regions. We can then find a value of z that will equalize these three areas.

Corollary 2 to Proposition 1: If $z = (1-1/\sqrt{3})/2$, i.e. if $z \approx 0.21$, then region A + B + C, region F + G + H and the residual region D + E are each identical in area.[6] This choice of z may be desirable for some purposes.

Also, and probably more importantly, because there is a trade-off between the value of z that may seem most desirable from the standpoint of operationalizing two-party dominance and that which may seem most desirable from the standpoint of operationalizing inter-party competitiveness, we often will not wish to restrict ourselves to a single value of z. Indeed, later in the article we add to some of our diagrams showing segments defined by $z = 0.20$ further segmentation specified by the value of $z = 0.10$. In our view, how many segments to show is a matter of art, not of science. Try to show too many segments and the graph becomes cluttered and hard to read.

Simplex Representations

Starting with Ibbetson (1965), authors who have made use of simplex representations showing outcomes/vote shares over a set of three alternatives generally portray the simplex as an equilateral triangle (see, e.g., Saari, 1994, 1995; Tabarrok, 2001), and we follow that convention.[7] In all the applications presented in this article, one vertex in the simplex (and the lines parallel to its facing edge) represents the principal national party or party bloc, another vertex (and the lines parallel to its facing edge) the second most important national party or party bloc, and the third vertex (and the lines parallel to its facing edge) the combined votes for all the remaining parties (see Figure 2).

While we have chosen to slice a Nagayama triangle into eight segments, perhaps the most natural way to partition the simplex representation is by drawing in each of the three perpendicular edge bisectors.[8] In this fashion, the simplex representation is ‘naturally’ divided into six sectors, corresponding to the six possible linear preference orderings over three alternatives {ABC, ACB, BAC, BCA, CAB, CBA}, as shown in Figure 3. That is how Saari (1994, 1995) makes use of the simplex representation.[9]

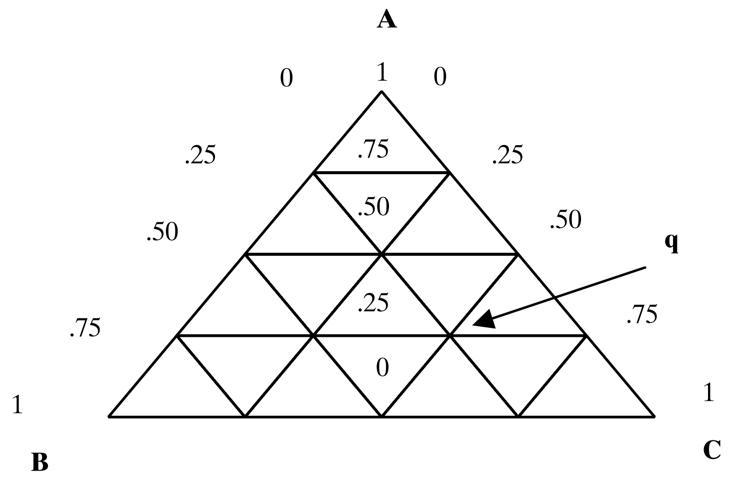

Figure 2. Simplex representation (partial grid for triangular coordinates)

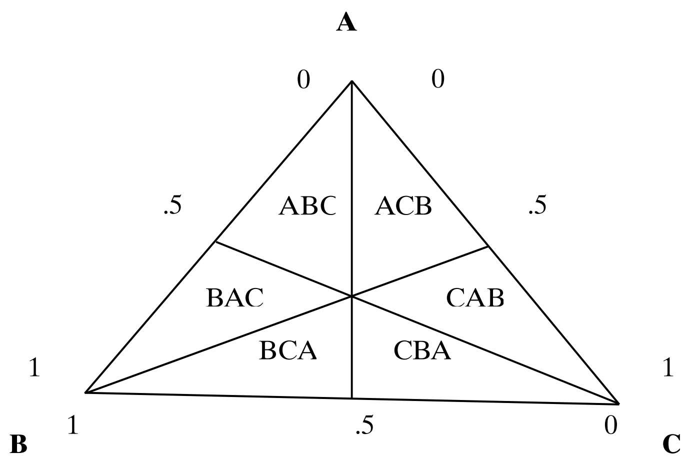

Figure 3. Simplex representation showing midpoint bisectors

In Figure 3, the closer you are to a given edge, the greater is the vote share of the party represented by that edge. Consider, for example, points on the line between vertex A and the midpoint of the BC edge (see Figure 3). Points on this line are those in which the division of votes between parties B and C is equal. These points differ only in the vote share of party A. Points to one side of this line reflect situations in which B has more votes than C; points to the other side of this line reflect situations in which C has more votes than B. In like manner, the other two perpendicular edge bisectors define the areas of the triangle in which vote share for A is greater than that for C or that for C greater than that for A, and in which vote share for B is greater than that for A or that for B greater than that for A, respectively.[10]

Even though it appears as if points in a simplex representation are in two-dimensional space, and thus duples, because the triangle is a projection from three dimensions onto two, each point in a simplex representation can also be thought of as a 'triple' showing the vote shares of each of the three candidates/parties/party groupings (as, of course, is true for the Nagayama diagram as well). It is important to understand how to interpret values within a simplex representation. But before we show how to do this in general, it is informative to consider the mapping of points in a simplex representation into vote shares for the three parties/party blocs: for the simplest cases, points on the vertices and edges of a simplex representation.

In triangular coordinates as we have defined them, a point located at the vertex associated with alternative A corresponds to the outcome (1, 0, 0), i.e. a vote share of 100 percent for party A and 0 percent for parties B and C; the vertex associated with alternative B corresponds to outcome (0, 1, 0); and the outcome (0, 0, 1) is at the vertex associated with alternative C. Similarly, any point on a given edge of the triangle specifies an outcome in which no votes are received by the alternative whose vertex is not on that edge. The points along each edge can be used to reflect the divisions of vote shares among the two parties/party blocs whose vertices define the edge, e.g. the midpoint of the AB edge reflects a vote division in a constituency in which the vote shares of party A and party B are equal and there are no votes for party/party bloc C.

Any point in the simplex representation can be uniquely defined in triangular coordinates as the intersection of three lines, each parallel to a given edge. But, as suggested earlier, because there are only two degrees of freedom involved in specifying the vote shares for three parties/party blocs, because these vote shares must sum to 1, once we have identified any two of these lines, then the third line is also determined.

In general, for points on a given line parallel to the opposing edge, the closer that line is to its facing vertex the greater is the vote share of the party located at that vertex.

In Figure 2 we have shown portions of a grid showing the family of lines parallel to each of the edges. We show lines in each family, for values of 0, 0.25, 0.50, 0.75 and 1. This diagram may be used to demonstrate how any point on or inside a simplex representation is uniquely defined by a set of three lines, one parallel to each of the edges. For any point, the intersection with the AB edge of the line parallel to the AC edge through that point gives us the value of B's vote share; similarly, the intersection with the AC edge of the line parallel to the AB edge through that point gives us the value of C's vote share in the constituency.[11] A's vote share in the district may be read off from the values associated with the horizontal line through the point – values shown above each of the illustrative horizontal lines parallel to the BC edge displayed in Figure 2.

Since this discussion is abstract it may be helpful to consider a straightforward example. Consider the point *q* in the simplex representation in Figure 2 that lies at the intersection of three lines. The point *q* is on the line parallel to AB that intersects the AC edge 1/2 of the way toward C. This tells us that the vote share for C must be 1/2. The point *q* is also on the line parallel to AC that intersects the BC edge 1/4 of the way toward B; and, finally, it is on the line parallel to BC that is 1/4 of the way toward A. Thus, *q* corresponds to the vector (1/4, 1/4, 1/2).

It can also be shown that the three subtriangles defined by the line from each of the vertices to any given point, *q*, inside the simplex, and the edges of the simplex, have areas proportional to the vote share of each of the three parties (Upton, 1976, 2001). This idea is illustrated in Figure 4, for the point, *q*, which we have previously identified as corresponding to the vote share vector of (1/4, 1/4, 1/2).

While the partition of a simplex representation into six regions representing the various linear orders (as shown in Figure 3) is the most common way that simplex representations are segmented into regions (see, e.g., Saari, 1994, 1995), other ways of segmenting the triangle may be more useful for some purposes of electoral analysis. Stray and Upton (1989), for example, in addition to discussing the sixfold partition, also show how to divide the simplex triangle into four equal triangular segments, each with particular interpretable properties vis-à-vis the degree of electoral competition; while Cornford et al. (1995) offer a different fourfold partition.

In general, in a simplex representation, points near a vertex represent single-party dominance, while points near an edge represent two-party

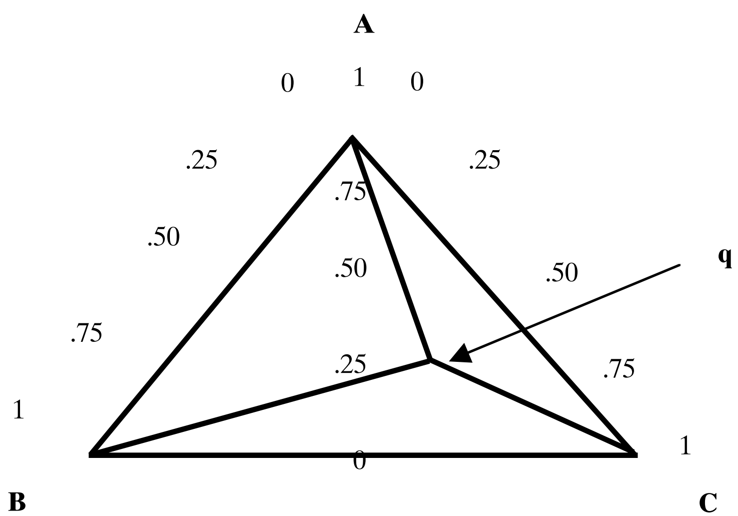

Figure 4. Simplex representation showing areas of subtriangles defined by vertices and *q*

competition, and points near the center (i.e. near the point where all three perpendicular bisectors intersect) reflect roughly even three-way competition. Thus, in a simplex representation, for example, the further away we are from the edge BC, the greater is the vote share of the A party/party bloc; indeed, all points on any given line parallel to the BC edge have the same vote share for party or party bloc A (recall that, as we are drawing it, the family of horizontal lines in an equilateral simplex representation represents the vote share for party bloc A).

Using a free parameter, z, similar to that we used for the Nagayama diagram, we present in Figure 5 a partition of the simplex representations into three areas, the lowest of which is the zone in which the sum of the votes for B and C is greater than or equal to $1 - z$. If we normalize the area of the triangle so that it sums to 1,[12] the area in this region is $2z/\sqrt{3}$.

While we have been making primary use of lines that are parallel to each of the edges of the simplex, it is also useful to recognize the properties of lines drawn through each of the vertices of a ternary simplex representation. We have already seen that lines that are perpendicular bisectors of an edge divide the simplex triangle into two parts: those where the party corresponding to one vertex on that edge has a greater vote share than the party corresponding to the other vertex on that edge, and those where it has a lower vote share. Note that, on the perpendicular bisector line itself, the two parties have an identical vote share. In general, as Ibbetson (1965) was one of the first to point out (and as has been emphasized by others: see, e.g., Stray and Upton, 1989), any line through a vertex has the property that the vote shares of the two parties with vertices on the facing edge are in some fixed ratio.[13] For example, if we consider the line $y = kx$, where k is a constant, then we would get $y/x = k$ for all points on that line. This 'constant ratio' property of lines through vertices for the two parties on the

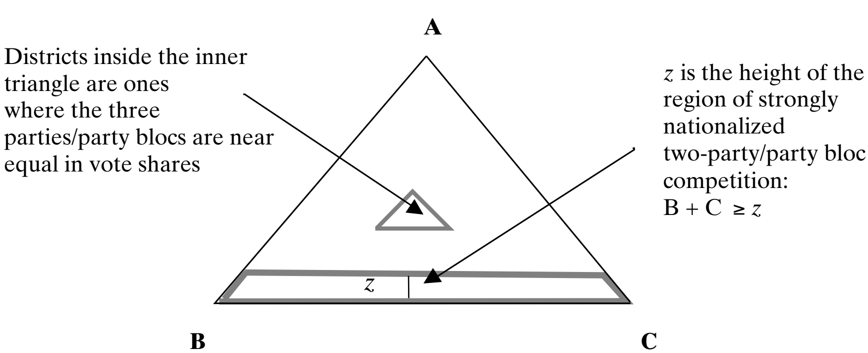

Figure 5. Equilateral simplex representation showing region where $B + C \geq z$ (the region of nationalized two-party competition) and region centred around the intersection of the perpendicular bisectors (region of near equal three-party competition)

facing edge can be useful in visualizing the nature of changes in party vote shares from one election to the next, and understanding the nature of inter-election swing, as is demonstrated in Ibbetson (1965).[14]

Comparisons of Nagayama Diagrams and Simplex Representations

The reader must be careful not to confuse the two types of graph. Even though they have many similarities (e.g. both are triangles, and each allows the vote share of two of the parties and the vote share for the remaining set of parties to be specified), there are many important differences between the two types of graph, not merely the fact that the Nagayama triangle is a right-angled triangle while we have chosen to draw the simplex representation as an equilateral triangle (as previously noted, we could have drawn the simplex representation as a right-angled triangle).

In particular, the two diagrams differ in important ways in what the axes mean. The Nagayama triangle is in standard rectangular coordinates. But in the Nagayama diagram the parties in each district have been 'pre-sorted' into largest and second largest, with the vote shares of the largest party on the x-axis and the vote shares of the second largest party on the y-axis. In contrast, the simplex representation is in triangular (barycentric) coordinates. In the simplex representation, vote shares for each party/party bloc are read by looking at the intersection with the adjacent edge of a line through each point parallel to the facing edge (or, for the horizontal edge, simply by reading values off the y-axis and then appropriately normalizing by rescaling values from 0 to 1).

Because of these differences, regions of the two types of triangle can have quite different meaning. For example, while in both the Nagayama triangle and the simplex representation, points that are near the *rightmost* edge of the triangle reflect a pattern of two-party/two-bloc dominance; in the simplex representation, for three-party competition, points that are near the *leftmost* edge also reflect a pattern of two-party/two-bloc dominance, but in the Nagayama triangle these points simply reflect competitive politics between the two largest parties. Similarly, in the simplex representation, for three-party competition, points that are near the bottom edge again reflect a pattern of two-party/two-bloc dominance, but they have no such meaning in the Nagayama triangle.

For the simplex representation, for three-party competition, roughly evenly balanced three-way party competition politics is found in points near the center of gravity of the triangle (the intersection point of the three perpendicular bisectors) in the interior of the triangle.[15] We show the locus of such points (location only approximate) in the inner triangle in Figure 4. In contrast, in the Nagayama diagram, roughly evenly balanced three-way party competition is found in points located near (1/3, 1/3) on the leftmost edge.

In short, we must be careful not to confuse the visual representations of data in the two graphs.

Also, because in a Nagayama diagram parties are, in effect, being 'generically' relabeled vis-à-vis their relative rank, information on the name of the party/party bloc that is, say, the largest vote-getter in a district, is lost; thus the Nagayama triangle does not allow us to read off the vote for any particular party/party bloc. In contrast, even if there are more than three parties/party blocs, the simplex representation always allows us to determine the vote shares for each of the two (named) parties/party blocs located at the vertices on the horizontal edge (as well as the vote share for the residual party grouping).

Extensions of Nagayama Diagrams and Simplex Representations to Illustrate Multiple Elections within the Same Diagram

In addition to showing how to extend Nagayama diagrams to illustrate results for more than three parties, Taagepera (2003) and Taagepera and Allik (2003) place (aggregated) information about multiple elections within a single Nagayama diagram to facilitate comparison across elections. Simplex representations have also been used to represent data on multiple elections within a single diagram. In particular, going back at least as far as Ibbetson (1965), they have been used to illustrate properties of inter-election swing between pairs of elections (see, e.g., Cornford et al., 1995). Perhaps the most interesting use for this end is that of Stray and Upton (1989), who illustrate inter-election changes in vote share at the constituency level (for the Labour, Conservative and Liberal parties in Great Britain, between pairs of adjacent elections from February 1974 to 1983) via a simplex representation, and then compare the observed patterns to what should be seen if a model of 'proportional swing' due to Reece (1985) were to hold (see also Moores, 1987). Using just the inter-ocular test (i.e. do the results jump out and hit you between the eyeballs?), they show that Reece's model can be completely rejected.[16]

Empirical Applications of Segmented Nagayama Diagrams and Simplex Representations to the Italian Elections of 1994, 1996 and 2001

Italian Election Data Visualized in Nagayama Diagrams

Nagayama (1967: cited in Reed, 2001) uses what Reed (2001) subsequently called Nagayama diagrams to study Japanese politics, with a focus on the question of whether Japanese politics had moved toward a two-party constellation of political conflict at the district level after elections under the single non-transferable vote were replaced after 1993 with a mixed system with both single-member district and list PR seats. For Italy, whose lower chamber shifted after 1993 from straight list PR to a mixed electoral system, Reed (2001) insightfully uses Nagayama diagrams for the single-member district component of the Italian mixed election system to study this same question in the elections of 1994 and 1996 (the first and second elections under the newly adopted mixed system), and convincingly demonstrates shifts toward two-party-dominant patterns of competition at the district level over that time period (i.e. ones in which the top two vote-getters in the district received the vast majority of the vote). Using Nagayama diagrams, here we add the data from the 2001 election in Italy to Reed's graphical analysis and produce the segmentation categories induced by $z = 0.20$ and $z = 0.10$.

The results of the 1994, 1996 and 2001 elections to the Italian Chamber of Deputies are shown in Figure 6. It is apparent that the constituencies are growing more 'concentrated' vis-à-vis the various regions of the figure,[17] and that an even higher proportion of outcomes are located in districts characterized by two-party competition in 2001 than in 1996 (and much higher than in 1994), just as Reed (2001) showed for Italian Chamber of Deputies elections in 1996 as compared to 1994.

The Nagayama diagram is helpful in indicating how dramatic the changes in Italy are in two directions: toward a combination of two-party or two-party-bloc-dominant competition, on the one hand, and toward a more competitive environment between the candidates from each of the two largest parties or party blocs in the district, on the other.[18] Outcomes involving both two-party or two-bloc dominance and competitive politics are located in segments A and H, and it is apparent that the share of the constituencies located in these two regions has grown dramatically. Still, even with segment bounds shown, it is not that easy to interpret overall shifts in competitiveness in the Nagayama diagram, since a shift toward the rightmost edge of the triangle reflects change away from multipartyism toward two-party or two-party-bloc competition, while a shift toward the leftmost edge of the triangle reflects a change toward more competitiveness. Assigning these different meanings to rightward and leftward shifts (rather than having them symmetrical) may be cognitively troubling – at least until one gets used to Nagayama diagrams. Moreover, as noted earlier, the Nagayama diagrams do not indicate whether it is the same two parties (or candidates of the same party blocs) who are the top two vote-getters in each of the districts.

Italian Election Data Visualized in Simplex Representations

Recall that, in a simplex representation, each vertex represents a specific named alternative, while in a Nagayama triangle alternatives are defined in relative terms (e.g. the candidate with the largest vote).[19] In Figure 7 we provide equilateral simplex representations showing the results from the

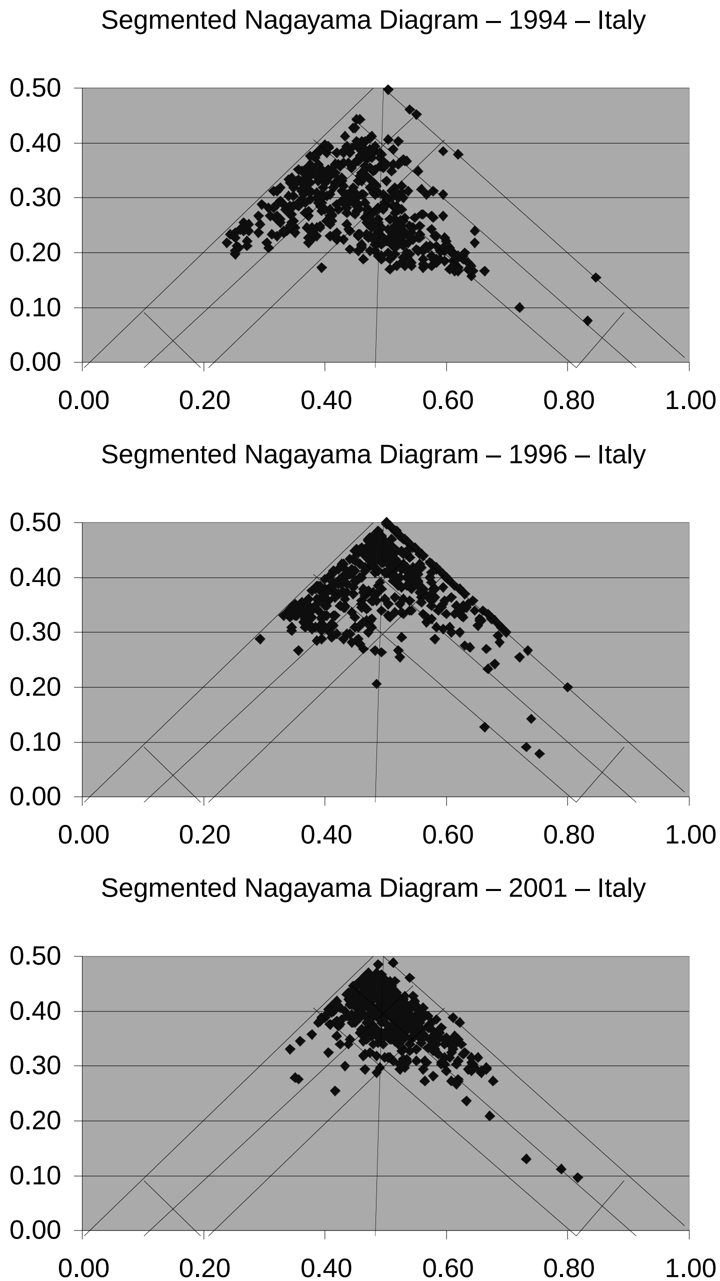

Figure 6. Segmented Nagayama triangle showing district level results of 1994, 1996 and 2001 Italian elections to the Chamber of Deputies in the single-member component of the mixed system

1994, 1996 and 2001 elections to the Italian Chamber of Deputies, drawing on the same dataset we used to create the Nagayama diagrams of Figure 6. In looking at the Italian election data, we have chosen to use vertices B and C to represent the two major national party blocs: the center–left coalition (presently Ulivo) and the center–right coalition (presently Casa delle Libertà).

In Figure 7, to make it easier to visualize outcomes for the two major parties/party blocs, rather than showing the equilateral simplex representations centered at the point where the three perpendicular bisectors intersect, and with the edges spaced symmetrically around this origin, we have chosen to distinguish the vertices B and C by placing them on the x-axis, at (0, 0) and (1, 0), respectively.[20] While points near any of the three edges would show two-party competition at the district level, when outcomes are concentrated near the edge defined by the B and C vertexes (i.e. near the x-axis), this indicates a *nationalization* of competition effect.

Outcomes in which the B and C vote sum to 0.80 or above are shown by the area under the dark lines in Figure 7. If we want to know whether B and C are in close electoral competition in the district, we can simply look at the region we get when we project upward the vertical lines perpendicular to the x-axis intersecting that axis at say, 0.45 and 0.55. Points which lie in that region and are also close to the BC edge exhibit both two-party/two-party bloc dominance and competitive politics between the major party blocs.

Visual comparisons of the simplex representations for each of the three election years reported on in Figure 7 reveal clearly that politics in the Italian Chamber of Deputies has been steadily moving both toward the dominance of the two major national blocs (center–right and center–left) at the district level, but also a reasonable degree of competition between them in a substantial proportion of the 475 single-member districts in the Chamber of Deputies. Indeed, visually what can be seen clearly over the course of the three elections is an emptying out of the center triangle (the elimination of districts displaying genuine three-way competition) and a shifting of sands to the bottom of the hourglass, i.e. toward districts where the only real competition, if any, is between the two major blocs.

Tabular and Histogram Presentation of Information from the Segmented Nagayama Diagrams for the Italian Elections of 1994, 1996 and 2001

Up till now we have emphasized graphical presentation of data, but the way in which information is represented in Nagayama and simplex diagrams can also be portrayed in more traditional tabular form and, as suggested earlier, can be compiled into indices of various aspects of party competition (e.g. the degree to which district competition is characterized by the dominance

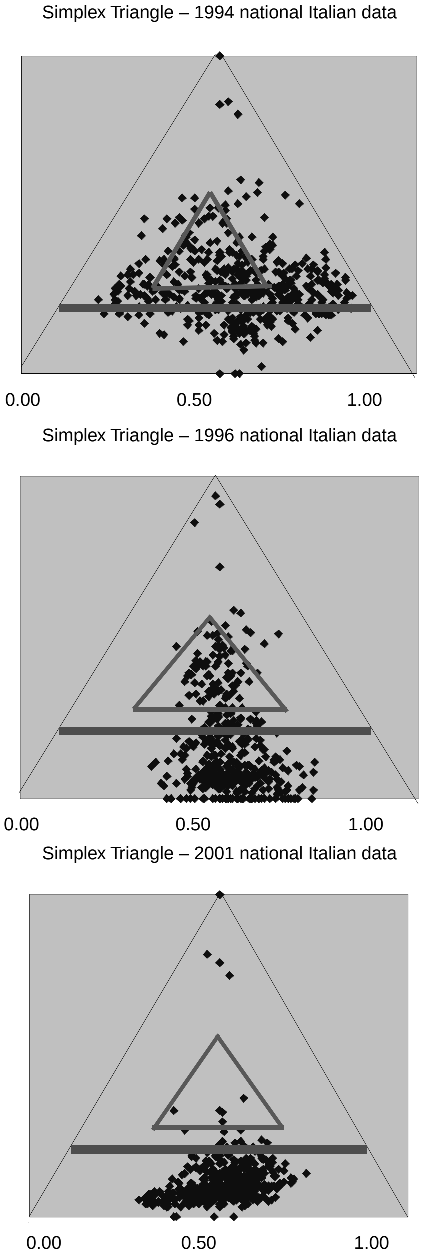

Figure 7. Equilateral simplex representations showing district level results of 1994, 1996 and 2001 Italian elections to the Chamber of Deputies in the single-member component of the Italian mixed system. Dark line (location approximate) shows region where B + C ≥ 0.80. Dark triangle (location approximate) shows areas of balanced three-party competition

of no more than two parties). We focus on tabular display of the information on Italian elections shown in the Nagayama triangles earlier.

We show in Table 1 the proportion of cases (*N* = 475) from the single-member component of the Italian national elections of 1994, 1996 and 2001 that fall into each of the segments A through H of a segmented Nagayama diagram. The defining characteristics of each of these segments were specified in our earlier discussion of the theoretical properties of segmented Nagayama triangles.

In Figure 8 we reproduce the same data shown in Table 1, but now with the proportion of cases in each of the eight segments delineated within a segmented Nagayama triangle. We suggest the reader compares Figure 8 with Figure 6 to see how scatterplots within a Nagayama triangle can be more revealing than numerical presentations within that same triangle, even if we cannot get a precise count of how many points fall within each segment from the scattergram representation. Of course, ideally, we want both the scattergrams of Figure 6 and the numerical tabulations in Table 1 (or Figure 8) to make sense of our election data.

Based on the data in Table 1, in Table 2 we show in the first part the proportions of cases in each of the three years that fall within two important categories: (1) districts that are at least competitive (defined by segments A + F + G + H, i.e. districts where the largest party/party-bloc vote share is less than 20 percentage points higher than the vote share of the second largest party/party bloc); and (2) districts with no substantial third party/party-bloc strength (defined by segments A + B + C + H, i.e. districts where the sum of votes for the largest party/party bloc plus votes for the second largest party/party bloc is at or in excess of 80 percent of the vote). We see that there was a dramatic movement over the three elections toward the elimination of districts where a third party/party bloc had substantial voting strength, and also a substantial increase from the first to the second election in the proportion of competitive seats, but with that proportion left essentially unchanged in the 2001 election.

The second portion of Table 2 provides the theoretical proportion of the

Table 1. Segmented Nagayama diagram segment proportions in Italian elections 1994, 1996 and 2001 (single-member district component, *n* = 475)

	1994	*1996*	*2001*
A	0.04	0.28	0.34
B	0.10	0.15	0.18
C	0.00	0.00	0.00
D	0.19	0.01	0.00
E	0.11	0.01	0.00
F	0.00	0.00	0.00
G	0.45	0.25	0.06
H	0.11	0.30	0.43

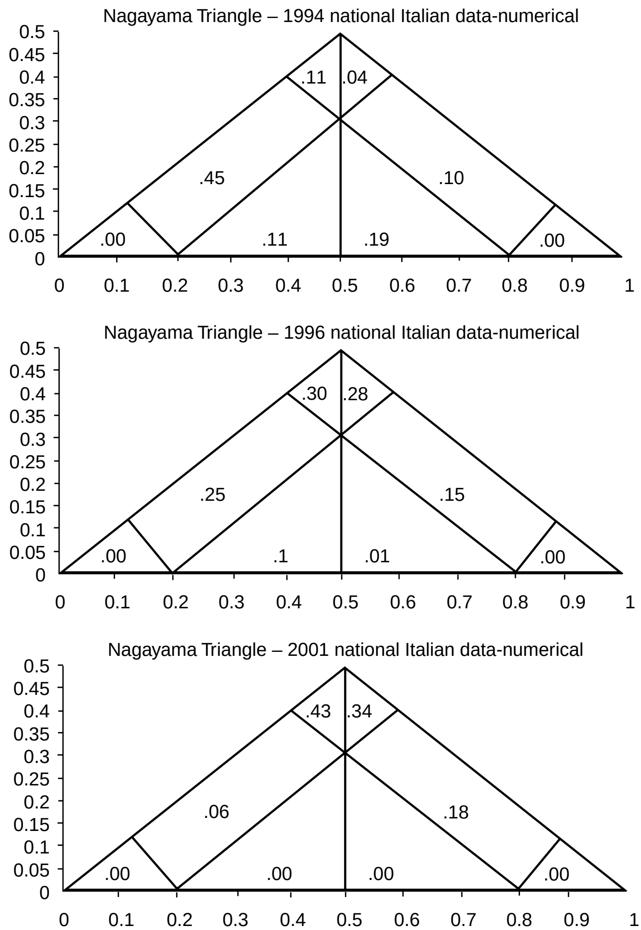

Figure 8. Segmented Nagayama triangle showing district level results of 1994, 1996 and 2001 Italian elections to the Chamber of Deputies in the single-member component of the mixed system in numerical form

electorate that would be *expected* in each of the two segment sets identified above if there were a uniform distribution of outcomes over the entire triangle (see earlier discussion); while the third portion of the table provides the values of the function $f_{z\,=\,0.20}$(O, U) for each of the two segments, for each of the three elections. Here we observe that, in 1994, because our index takes on a negative value, we have more districts with substantial third party/party-bloc strength than would be expected by chance (i.e. a uniform distribution) alone, but by 1996 the opposite is true, and by 2001 our index, which is bounded from above by 1, takes on a value of 0.9. Here, as is visually apparent from earlier figures, we are very far away indeed from a

Table 2. Tabular display of information about party competition in the single-member district component of Italian elections, 1994, 1996, 2001

	1994	*1996*	*2001*
Districts with no substantial third party strength = A+B+C+H	0.25	0.73	0.94
Competitive districts = A+F+G+H	0.60	0.84	0.82
Theoretical value of A+B+C+H for z = 0.20 and a uniform distribution	0.44		
Theoretical value of A+F+G+H for z = 0.20 and a uniform distribution	0.44		
$f_{z = 0.20}$(O, U) for A+B+C+H (no substantial third party strength)	–0.35	0.53	0.90
$f_{z = 0.20}$(O, U) for A+F+C+H (competitive districts)	0.28	0.71	0.68

uniform distribution – almost as far away as it is possible to get. When we use the index based on the function $f_{z = 0.20}$(O, U) to examine changes in political competition we see dramatic changes from 1994 to 1996 and stasis thereafter. Naturally, the findings derived from this index parallel those we get from directly counting cases in each of the segment sets. What we gain from using the index is an ability to make comparisons with what would be expected were the distribution of cases to have been uniformly distributed over the Nagayama triangle.

We could provide similar tabular presentations for the simplex representation presentations of this same Italian data, but we will not bother. Here we believe that observation of the changes in the proportion of points within the center of the simplex over the course of the three elections (the emptying out of the center, and thus of three-party/party bloc competition) and observation of the changes in the proportion of points below the 80 percent line toward the bottom of the simplex triangle are so stark in Figure 7 that there is no need to replicate these results in tabular form. Of course, if the results were not so dramatic, and if we needed to conduct statistical tests (other than the most powerful statistical test of all, the inter-ocular test), then providing the data in tabular/numerical form would certainly have been desirable.

There is one other form of graphical presentation of party competitiveness data, the old-fashioned histogram, the results from which we offer for purposes of comparison. A histogram of the distribution of the sum of the vote shares of the two largest parties in the 475 single-member districts in the Italian elections of 1994, 1996 and 2001 is shown in Figure 9.

Clearly, viewing the data in this way is informative, at least compared to the compressed tabular presentation in Table 2 for two-party dominance,

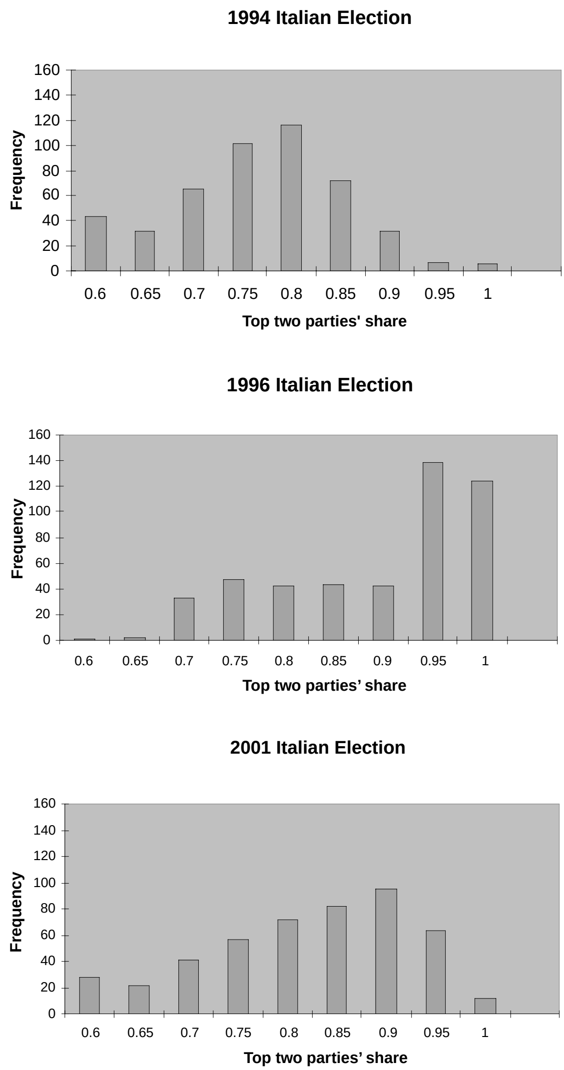

Figure 9. Histograms showing distribution of district vote to the top two vote-getting parties/party blocs in the district: 1994, 1996 and 2001 Italian elections to the Chamber of Deputies in the single-member component of the Italian mixed system

where we only enumerated districts where the sum of the two-party vote share was over 80 percent. In particular, we see that, despite the fact that 1996 and 2001 were similar in the number of districts with more than an 80 percent vote share going to the two largest parties/party blocs, there were more districts in 1996 than in 2001 with two-party/party bloc vote shares above 90 percent.

Of course, this same information can also be read off the Nagayama triangles for the two years. Indeed, while histograms of particular election properties of interest are clearly useful, no single histogram is likely to convey as much relevant information as can be gleaned from a Nagayama triangle visualization by a reader familiar with how to interpret the data represented therein (especially if the number of cases located in each of the eight – or more – critical segments is also identified, as we have done in Table 1 and Figure 8).

Discussion

Because this article is viewed as primarily a methodological one, we leave to other work a fuller discussion of the substantive implications of the 2001 Italian election results.[21] We hope to have illustrated; (1) how segmented Nagayama diagrams and simplex representations may be created and visually interpreted, (2) how they each reflect different aspects of multiparty competition, and (3) how useful each type of diagram can be in compactly presenting information in a visual field that can allow quick interpretation of key facts – aided, or so we would like to believe, by an appropriate segmentation of the triangles so that rough and ready comparisons of the density in each can readily be made.[22]

While this may, in part, be a subjective judgment, we found the picture told by comparing the points displayed in the three equilateral simplex representations in Figure 7 marginally easier to intuit and attach meaning to than the corresponding points in the three Nagayama diagrams of Figure 6. Also, simplex representations give us additional data on the nationalization of Italian politics. In particular, when we look at Figure 7 we see the Italian political system 'dropping down' over the period 1994–2001 into a pattern of outcomes near the x-axis, i.e. patterns where the two candidates of the two national party blocs in sum hold the preponderant share of the vote in the vast bulk of the single-member districts. Of course, we could modify the Nagayama diagram to show data only on the two major national parties/party blocs. But, using the x-axis as our orienting line, as we have done with the simplex representations we have presented, makes it easier, we believe, to grasp, holistically and quickly, the patterns of electoral competition shown in the data.

On the other hand, the Nagayama diagram presents information about relative sizes of first and second parties (as opposed to named parties), that is distinct from what we can easily read from a simplex representation. Moreover, the recent work of Taagepera (2003; Taagepera and Allik, 2003) has shown that more can be done with Nagayama triangles than just portraying first-party and second-party vote shares. Thus, we believe that both diagrams have important uses. Furthermore, as we argued above, it may be easier to grasp what is essentially cross-tabular information when that information is presented in the appropriate segments of either a Nagayama triangle or a simplex representation than when the information is merely displayed in some type of cross-tabular format – especially when what is being sought are comparisons across multiple elections. So we end this article by slightly rephrasing the statement that we began with: to wit, with respect to the Italian elections of 1994, 1996 and 2001, and comparisons among them: 'three pictures can be worth three thousand words'.

Notes

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1 For other uses of such diagrams, see Schmid and Schmid (1978: 150–2), Wainer (1995), Upton (2001) and Allen (2002). Rein Taagepera (personal communication, April 2003) has observed that this graphic technique has been used in the physical sciences for a variety of purposes since well before World War II. In particular, one of the anonymous referees of this article noted its use in the study of soil composition.
2 For psephology buffs, we might note that, in his commentary on the remarks of Ibbetson (1965), David Butler (1965) remarks that 'a diagram such as he suggests has been worked out by Graham Pyatt, who was involved with me on B.B.C. election night programme with the Elliott computer. Only technical difficulties are preventing me from including his diagram in the book which we are preparing at Nuffield College on the 1964 election. This kind of diagrammatic representation of election results seems to open up very exciting new possibilities in the diagrammatic presentation of election results, especially if we are moving more and more into a three-party situation'.
3 As Taagepera (2003) demonstrates, if we look only at aggregate outcomes (e.g. outcomes at the national level), then we can represent multiple elections within the same Nagayama triangle by using one point for each election. Of course, we could report data at the constituency level for multiple elections within a single Nagayama diagram by using different symbols for the cases from the different elections. However, it is likely that this would yield a cluttered and less comprehensible diagram.
4 Because the mathematics involved is essentially trivial, we do not bother to show details of the calculations.
5 The proof is left as an exercise for the reader.
6 We do not bother to show the proof, which only requires using the standard formula for solving a quadratic equation.
7 Of course, with appropriate renormalization (to assure that all values sum to 1) the third axis may also be used to represent the votes for the largest party or party bloc in the district other than the two principal national parties/party blocs, or to show the votes for a particular third party. Also, as we discuss below, information may be conveyed about more than one election within a single triangle.

8 Note that for an equilateral triangle the line between a vertex and the midpoint of the opposite edge is perpendicular to that edge.

9 In this fashion we are using the simplex representation to represent the linear order *permutohedron* for three alternatives. This representation may be extended to binary orders more general than linear orders, or to more than three alternatives, but we shall not consider such extensions here (see, e.g., Falmagne, 1996: 76; Regenwetter et al., forthcoming).

10 In general, any perpendicular to an edge of a simplex representation divides the triangle into two regions, each of which has a different sign on an inequality involving the alternatives on that edge. Consider, say, the edge BC. Let the perpendicular to the point on that edge be located k^{th} of the distance between B and C; that perpendicular divides the triangle into two regions: in only one of which the difference between the vote share of B and the vote share of C is greater than $2k - 1$; in the other, the difference between the vote share of B and the vote share of C is less than $2k - 1$.

11 Of course, once we have identified two of the lines, the vote share value corresponding to the intersection point of the third line with the relevant edge can be calculated simply by subtracting the sum of the two already obtained values from 1. This calculation tells us which line parallel to the remaining edge the point must lie on.

12 In developing data representations in rectangular coordinates for the empirical applications below, we set the coordinates of B and C as (0, 0) and (1, 0), respectively. Hence, the vertex A of the simplex representation is located at the coordinates $(1/2, \sqrt{3}/2)$. For these vertices, the center of gravity of the simplex representation is at $(1/2, \sqrt{3}/6)$. However, before we present the data, we renormalize so that values for A's vote share may be read directly off the y-axis. To do so we transformed the scale on the y-axis by multiplying the y-axis value by $2/\sqrt{3}$.

13 Intuitively, that is because lines through a vertex have the property that the constant term must be zero.

14 See also Stray and Upton (1989).

15 If we set the coordinates of B and C as (0, 0) and (1, 0), respectively, thus placing vertex A at $(1/2, \sqrt{3}/2)$, this intersection point is $(1/2, \sqrt{3}/6)$.

16 They also show that the 'uniform swing' model is clearly better than the proportional shares model, but that it, too, fails to account for all the observed patterns in the data; and it varies in fit across different pairs of (adjacent) election years.

17 Patterns of electoral competition do, however, vary significantly among the three main geographic areas of Italy (the North, the South and the Center). Since the purpose of this article is to introduce particular graphical tools, teasing out geographical variations in the data patterns must be left to other work. It should be apparent that we can create separate Nagayama diagrams or simplex representations for different regions of a country, and thus visually illustrate the extent to which 'national' patterns of party competition in fact exhibit significant regional variation (for an example of this type of analysis, see Bartolini et al., forthcoming).

18 Actually, in the case of Italy, in many instances since 1993 it is party blocs that agree on which party in the bloc will get to choose the candidate who will run in any given district. The two major blocs are both multiparty coalitions (D'Alimonte, 2000; Bartolini and D'Alimonte, forthcoming).

19 Recall also that (to make the trilateral symmetry involved in using triangular coordinates more visually apparent) we are presenting simplex representations as equilateral triangles rather than right-angled triangles. However, it is possible to present the same information given in the simplex representations in the form of simplex representations that are constructed as right-angled triangles.

20 Recall that, in simplex representations, the most relevant lines are not those specified by the usual Cartesian coordinates, but rather the set of the three families of lines parallel to each of the edges. (Of course, for purposes of simplicity of visualization, one of these three families of lines has been drawn parallel to the x-axis by locating the associated edge on the x-axis.) As noted earlier, each point in the triangle can be uniquely defined as the intersection of one line from each family, and knowing which lines from each family pass through the point tells us, with appropriate rescaling, the vote shares for the alternative at the vertex facing the given line. (However, we must perform some simple analytic transformations of the raw vote share data to plot these data as points in a two-dimensional space.)

21 See especially Bartolini et al. (forthcoming).

22 For example, in both types of figure, it is relatively easy to identify outcomes in which the two major party blocs are competitive. In the simplex representation, these are near the 0.50 location on the x-axis. In Nagayama diagrams, these are near the leftmost edge.

References

Allen, Terry (2002) ‘Using and Interpreting the Trilinear Plot’, *Chance Magazine* 15: 29–33.

Bartholomew, D. J. and E. E. Bassett (1971) *Let’s Look at the Figures: The Quantitative Approach to Human Affairs*. Penguin Books: Harmondsworth, Middlesex, England.

Bartolini, Stefano, Alessandro Chiaramonte and Roberto D’Alimonte (forthcoming) ‘The Italian Party System between Parties and Coalitions’, *West European Politics*.

Bartolini, Stefano and Roberto D’Alimonte (eds) (forthcoming) *The Italian Elections of 2001* (title tentative).

Butler, David E. (1965) ‘Comment on Hugh B. Berrington, “The General Election of 1964” ’, *Journal of the Royal Statistical Society, Series A (General)* 128(1): 55.

Cornford, J. R., D. F. Dorling and B. S. Tether (1995) ‘Historical Precedent and British Electoral Prospects’, *Electoral Studies* 14: 123–42.

D’Alimonte, Roberto (2000) ‘Mixed Electoral Rules, Partisan Realignment, and the Party System in Italy’, in Mathew Soberg Shugart and Martin P. Wattenberg (eds) *Mixed-Member Electoral Systems: The Best of Both Worlds?* pp. 323–50. Oxford: Oxford University Press.

Falmagne, J.-C. (1996) ‘A Stochastic Theory for the Emergence and the Evolution of Preference Relations’, *Mathematical Social Sciences* 31: 63–84.

Gaines, Brian J. (2000) ‘From Duverger to Cox and Beyond’, *Japanese Journal of Political Science* 1: 151–6.

Ibbetson, D. (1965) ‘Comment on Hugh B. Berrington, “The General Election of 1964” ’, *Journal of the Royal Statistical Society, Series A (General)*, 128: 54–5.

Katz, Jonathan N. and Gary King (1997) 'A Statistical Model for Multiparty Electoral Data', *Social Sciences Working Paper 1005*. Division of the Humanities and Social Sciences, California Institute of Technology, Pasadena, California (May).

Moores, B. (1987) 'Some Consequences of a Three-Party Split in a UK General Election', *Journal of the Operations Research Society* 38: 569–76.

Nagayama, Masao (1997) 'Shousenkyoku no kako to genzai' ('The Present and Future of Single-Member Districts'). Presented at the Annual Conference of the Japan Political Science Association, September 4–6 (cited in Reed, 2001).

Reece, G. (1985) *Voter Representation: A Study of the British Electoral System and Its Consequences*. London: Conservative Action for Electoral Reform.

Reed, Steven (2001) 'Duverger's Law is Working in Italy', *Comparative Political Studies* 34: 312–27.

Regenwetter, Michel, Anthony Marley and Bernard Grofman (forthcoming) 'The General Concept of Majority Rule', *Mathematical Social Sciences*.

Saari, Donald (1994) *Geometry of Voting*. Berlin: Springer-Verlag.

Saari, Donald (1995) *Basic Geometry of Voting*. Berlin: Springer-Verlag.

Schmid, C. F. and S. F. Schmid (1978) *Handbook of Graphic Presentation*, 2nd edn. New York: John Wiley.

Stray, Stephanie and Graham Upton (1989) 'Triangles and Triads', *Journal of the Operations Research Society* 40: 83–92.

Taagepera, Rein (2004) 'Extension of the Nagayama Triangle for Visualization of Party Strengths', *Party Politics* 10(3): 301–06.

Taagepera, Rein and Mirjam Allik (2003) 'Duverger Psychological Effect and the Size of Parties', unpublished manuscript, University of Tartu, Estonia, March 26.

Tabarrok, Alexander (2001) 'President Perot, or Fundamentals of Voting Theory Illustrated with the 1992 Election', *Public Choice* 106: 275–97.

Upton, Graham J. G. (1976) 'Diagrammatic Representation of Three Party Contests', *Political Studies* 24: 448–54.

Upton, Graham J. G. (1994) 'Picturing the 1992 British General Election', *Journal of the Royal Statistical Society*, Series A. 157: 231–52.

Upton, Graham, J. G. (2001) 'A Toroidal Scatter Diagram for Ternary Variables', *The American Statistician* 55: 240–3.

Wainer, Howard (1995) 'Trilinear Plots', *Chance Magazine* 8: 48–54.

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