Political Research Quarterly
Volume 62 Number 4
December 2009 662-672

10.1177/1065912909338464
http://prq.sagepub.com
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An Introduction to Crisp Set QCA, with a Comparison to Binary Logistic Regression

Bernard Grofman
University of California, Irvine
Carsten Q. Schneider
Central European University, Budapest

The authors focus on the dichotomous *crisp set* form of qualitative comparative analysis (QCA). The authors review basic set theoretic QCA methodology, including truth tables, solution formulas, and coverage and consistency measures and discuss how QCA (a) displays relations between variables, (b) highlights descriptive or complex causal accounts for specific (groups of) cases, and (c) expresses the degree of fit. To help readers determine when QCA's configurational approach might be appropriate, the authors compare and contrast QCA to mainstream statistical methodologies such as binary logistic regressions done on the same data set.

Keywords: *comparative politics; political methodology; qualitative methods*

Introduction

Qualitative comparative analysis (QCA)[1] techniques are intended as methods for bridging the gap between qualitative (case study oriented) and quantitative (variable oriented) approaches in social scientific research. For simplicity of exposition here, we will limit ourselves to the dichotomous form of QCA, namely, to what is called *crisp set QCA*. The crisp set form of QCA allows for direct comparison to standard statistical techniques for handling variables treated as dichotomous and allows us to better compare and contrast the uses and theoretical objectives of QCA with those of more traditional methods so that readers may better judge for themselves when use of QCA is appropriate. To more clearly show the nature of differences between QCA and standard statistical approaches, we provide both a crisp set QCA and a binary logistic analyses of one particular data set, Charles Ragin's data on welfare states (see Ragin 2000, Table 10.6).

We begin with a discussion of four basic elements of QCA: (1) data tables, (2) truth tables, (3) solution formulas, and (4) parameters of fit. We then introduce three general aims for presentation of empirical analytic results that are not specific to QCA but to which QCA—due to its location at the intersection between case study and variable-oriented research—must pay particular attention. These aims consist of (a) displaying relations between variables, (b) indicating which descriptive or causal accounts apply to specific (groups of) cases, and (c) expressing the degree of fit of the proposed solution to the empirical data from which it was generated. For each of the standard elements of QCA we consider the degree to which it satisfies each of the three central goals of QCA data presentation identified previously.

Finally we compare QCA with logistic regression analyses of data on welfare states in sixteen countries.

Central Features of QCA

QCA is based on the twin ideas of *necessity* and *sufficiency*.[2] Its motivations include a concern for unraveling causally complex structures in terms of *equifinality*, *multifinality*, and *asymmetric causality* (see discussion in the following) that tend to be omitted or slighted in most discussions of mainstream

Bernard Grofman, Jack W. Peltason (Bren Foundation) Endowed Chair and Director, Center for the Study of Democracy, University of California, Irvine; bgrofman@uci.edu.

Carsten Q. Schneider, PhD Program Director and Assistant Professor, Department of Political Science, Central European University; schneiderc@ceu.hu.

statistical methods.[3] It is also explicitly configurational in approach (Rihoux and Ragin 2008). Moreover, unlike many statistical techniques, QCA does not require that at least some variables be measured at an interval or ratio level. In particular, for simple crisp set QCA, the data are in the form of (dichotomous) set membership scores in underlying concepts.

While QCA is sometimes thought to be strictly limited to small *n*, this is erroneous (see e.g., Ragin and Fiss 2008, and examples of large *n* analyses such as Ragin and Bradshaw 1991 and Miethe and Drass 1999). However, even when the number of cases is large enough that we might apply standard statistical methods appropriate for dichotomous or ordinal data, the goals of QCA are different from those of other statistical techniques, and the results it produces are also different. As Ragin (2008b, especially 176-89) emphasized, while standard statistical techniques are good at distilling the net effect of single variables, QCA, by virtue of giving premium to causal complexity, seeks to detect different conjunctions of conditions (configurations) that all lead to the same outcome.[4]

To better see the nature of such differences, we start with a data set large enough for some standard statistical analyses and for that data show the key differences between the QCA approach and methods such as logistic regression on dichotomous variables. Thus, rather than providing an abstract discussion of how QCA differs from other types of analysis, to see how QCA works we look first at an actual example of a QCA and then use the results from it to illustrate QCA's distinctive features.

Data Tables

Let us begin with data in a form that is presented in the usual spreadsheet formulation, with variables as columns and cases as rows, but which is also arranged in a way to convey additional information. Our data is adapted from Ragin (2000, 286-300), who presents an analysis of the conditions for the existence of a generous welfare state (W) in advanced-industrial, democratic countries, in which he looks at four factors: strong left party (P), strong unions (U), corporatist industrial system (C), and sociocultural homogeneity (S).[5] We have reproduced a crisp set version of the relevant data in Table 1 as a data matrix. A score of one indicates that in a given case the condition or outcome is present and a score of zero that it is absent.[6] Cases are grouped in an order intended to make the table ultimately easier to read/interpret rather than alphabetically by case name. All cases that have identical combinations of the *outcome* and the *conditions/conditioning factors* are grouped together.[7]

Table 1
Data Matrix of "Generous Welfare State" and Four Conditions—Crisp Sets

Country	P	U	C	S	W
Austria	1	1	1	1	1
Denmark	1	1	1	1	1
Finland	1	1	1	1	1
Norway	1	1	1	1	1
Sweden	1	1	1	1	1
Australia	0	0	0	0	0
Canada	0	0	0	0	0
France	0	0	0	0	0
United States	0	0	0	0	0
Germany	0	0	1	0	1
Netherlands	0	0	1	0	1
Switzerland	0	0	1	0	0
Japan	0	0	0	1	0
New Zealand	0	1	0	0	0
Ireland	0	1	1	1	1
Belgium	1	1	1	0	1

Source: Adapted from Ragin (2000, Table 10.6).
Note: P = strong left party; U = strong unions; C = corporatist industrial system; S = sociocultural homogeneity; W = strong welfare state.

Truth Tables

In Table 2 we show the data from Table 1 in a form that makes clearer the relationship between cases, conditions, and outcomes. We will refer to the data as presented in Table 2 as a *truth table* (see the following).

Each row of a truth table such as Table 2 represents one of the 2^k logically possible combinations of the k (dichotomous) conditions, one column for each of the conditions.[8] The $(k + 1)$th column (final column) indicates the value of the outcome that those cases display that are characterized by the combination of conditions indicated in the respective row. For crisp set QCA, neglecting knife-edge cases where we are not sure of how to properly classify a case, any empirical case can be allocated to one (and only one) row of the truth table.

Sorting the information contained in Table 1 in a truth table reveals several pieces of information (see Table 2). First, out of the $2^4 = 16$ logically possible combinations, three are linked as sufficient conditions for the occurrence of a generous welfare state (W = 1,

Table 2
Truth Table of "Generous Welfare State" and Four Conditions

Row	Conditions				Outcome		
	P	U	C	S	W	*n*	Case Labels
1	1	1	1	1	1	5	AUT,DK,FIN,NOR,SWE
2	0	1	1	1	1	1	IRL
3	1	1	1	0	1	1	BEL
4	0	0	0	0	0	4	AUS,CAN,FR,USA
5	0	0	1	0	Contradictory	3	GER,NET,SWI
6	0	0	0	1	0	1	JAP
7	0	1	0	0	0	1	NZ
8	0	0	1	1	R	0	
9	0	1	0	1	R	0	
10	0	1	1	0	R	0	
11	1	0	0	0	R	0	
12	1	0	0	1	R	0	
13	1	0	1	0	R	0	
14	1	0	1	1	R	0	
15	1	1	0	0	R	0	
16	1	1	0	1	R	0	

Note: P = strong left party; U = strong unions; C = corporatist industrial system; S = sociocultural homogeneity; W = strong welfare state.

rows 1-3) and three are linked to its nonoccurrence (W = 0, rows 4, 6, and 7). Row 5 indicates a mixed outcome (a so-called contradictory row): two of the cases with the combination of conditions shown for that row (Germany and Netherlands) give rise to a generous welfare state, one does not (Switzerland). Thus, this set of conditions is (strictly speaking) neither sufficient for the presence of a generous welfare state nor sufficient for the absence of a generous welfare state.

Furthermore, despite having sixteen countries in the data set, the truth table reveals that limited diversity exists, that is, not all logically possible combinations between the conditions P, U, C, and S are empirically observed. The nine types of cases in rows 8 through 16 that are empirically absent from our truth-functional logic-defined universe of potential cases are indicated by an R ("logical remainder") in the W column. The phenomenon of limited diversity is omnipresent in all comparative approaches in the social sciences that are based on observational data (Ragin 1987, 104-18; Ragin and Rihoux 2004; Ragin and Sonnett 2004; Schneider and Wagemann 2006).

Solution Formulas

At the heart of the analysis of data with QCA is the restatement of information that is contained in a truth table in terms of a parsimonious and encompassing truth-functional proposition or set of propositions. The most frequently used and, for all practical purposes, obligatory way of expressing the results of QCA is to write them down in the form of a solution formula, the end product of the process of logically summarizing/encapsulating the information stored in a truth table. In a solution formula the outcome and the causally relevant conditions are represented in letters that are linked with Boolean operators. The three basic Boolean operators are logical OR (+), logical AND (*), and logical NOT (where negation is customarily denoted in QCA by replacing an upper case letter with a lower case letter).

We can illustrate how these operators may be used with Ragin's (2000) data on the conditions linked to the existence of a strong welfare state shown in Tables 1 and 2. To show the logic of the three fundamental operators, OR, AND, and NOT, let us take a country with a crisp membership score in the set of homogeneous society (S) of zero and in strong union (U) of one.

The negation (logical NOT) is calculated by subtracting the original score from 1. Hence, the country's score in NOT-homogeneous is: s = 1 − S = 1 − 0 = 1. The membership of the country in the set of cases that are homogeneous society AND have strong unions (i.e., the intersection of sets) is determined by the minimum value of the two sets: $S*U = \min(S, U) = \min(0, 1) = 0$. The membership of the same country in the sets of cases that are homogeneous societies OR[9] have strong unions (i.e., the union of sets) is determined by the maximum value of the two sets: $S + U = \max(S, U) = \max(0, 1) = 1$. Similarly, if $S = 1$ and $U = 1$, then we could write $S + U = 1$, namely, $1 + 1 = 1$.

The three simple operations of AND, OR, and NOT suffice to express any feasible relationship between binary conditions and a binary outcome. In particular, we can specify a solution formula for the (strict) sufficiency of W simply by writing down in letters and Boolean operators each row of the truth table that displays W = 1 for each and every case in the row. For example, by inspection, we see that these are rows 1 through 3 in Table 2, but not row 5, which displays some cases with W = 1 but also one case with W = 0 (see original raw data in Table 1). This gives us three so-called primitive expressions that can be expressed in Boolean terms as follows:

$$\text{PUCS} + \text{pUCs} + \text{PUCs} \rightarrow \text{W}. \tag{1}$$

The presence of the → symbol indicates that the expression to its left implies the outcome to its right and for these data can be seen as a sufficient condition[10] for the outcome (Ragin and Rihoux 2004). More specifically, the presence of the logical OR indicates that there are three different sufficient conditions for the same outcome W, and the presence of the logical AND indicates that what causally matters are not individual conditions in isolation but specific combinations of them. Solution formulas of that type are common in QCA. In this article, for space reasons and simplicity of exposition, we will limit ourselves to a discussion of sufficient conditions.

Because we are looking at only four conditions, we were able to determine the sufficient (combinations of) conditions for W directly from the truth table by "eyeballing" the rows of the truth table. Had we a much larger set of conditions to search through, we might need to use a computer program to aid in quickly finding the sufficiency conditions.[11]

The QCA solution formula 1 for the data shown in Table 1 and Table 2 is not the most parsimonious form of logical expression to summarize all the information about sufficiency contained in the first seven rows of Table 2 (the rows for which there are corresponding cases). We may use a process of what in the QCA literature is called *logical minimization* (Ragin 1987, especially chaps. 6-7) to restate the information contained in the truth table (Table 2) in a much simpler way, yielding the following result, reflecting an intersection of conditions:[12]

$$\text{PUC} + \text{UCS} \rightarrow \text{W}. \tag{2}$$

In turn, this solution form may be rewritten (see discussion in Ragin 1987, 100-1; Caramani 2009; or any logic textbook) as

$$\text{UC (P + S)} \rightarrow \text{W}. \tag{2$'$}$$

The solution formulas 1 and 2 (or 2′) are logically equivalent. We see from this solution formula that there are two sufficient paths leading to a generous welfare state: a strong corporatist industrial system (C) AND strong unions (U) AND a strong left party (P) on one hand OR strong unions (U) AND a strong corporatist industrial system (C) AND socioeconomic homogeneity (S) on the other.[13]

Measures of Fit: Consistency and Coverage

The two key parameters for assessing the fit of QCA results to the underlying data are consistency and coverage (Ragin 2006; see also Goertz 2006). We will here present definitions only for crisp set QCA.

For sufficiency relations, the parameter of consistency expresses the proportion of the cases with the condition X where we also find the outcome Y, relative to all cases with X. For any given data set, the higher the consistency value of X, the closer is X to being a consistently sufficient condition for Y. If the consistency score is 100 percent, then X can be interpreted as (strictly speaking) sufficient for Y. As we see from Table 2, all six cases with PUC (rows 1 and 3) also display W, thus the consistency of PUC is 6/6 = 100% as a sufficient condition for W. Similarly, UCS has a consistency score of 100 percent as a sufficient condition for W. We represent these results in more familiar cross-tab format in Table 3.

The calculation of coverage, the second parameter, only makes sense when it is applied to conditions that have turned out to be "consistent enough" to be regarded as sufficient for Y (Ragin 2006). For any condition X, which is sufficient for Y, coverage is the number (proportion) of cases with Y where we also find X, relative to all cases with Y. The higher the coverage score for X, the more cases displaying Y are covered (and thus explained—if accompanied by plausible theoretical arguments) by this sufficient condition.

Table 3
Sufficiency Conditions with W as the Outcome Variable

(a) Cross-Tab Showing PUC as a Sufficient Condition for Strong Welfare State	not PUC	PUC	
not W	7	0	7
W	3	6	9
	10	6	$N = 16$

(b) Cross-Tab Showing UCS as a Sufficient Condition for Strong Welfare State	not UCS	UCS	
not W	7	0	7
W	3	6	9
	10	6	$N = 16$

(c) Cross-Tab Showing {PUC or UCS} as a Sufficient Condition for Strong Welfare State	not (PUC or UCS)	PUC or UCS	
not W	7	0	7
W	2	7	9
	9	7	$N = 16$

Note: P = strong left party; U = strong unions; C = corporatist industrial system; S = sociocultural homogeneity; W = strong welfare state.

Table 2 shows a total of nine cases that display W. The solution formula PUC + UCS covers seven of them. Hence, the solution coverage, namely, the overall coverage of all sufficient conjunctions combined, is 7/9 = 78%. PUC alone covers six out of seven cases (rows 1 and 3). Its raw coverage, $\text{coverage}_{\text{PUC}}$, therefore, is 6/9 = 69%. UCS also covers six out of nine cases (rows 1 and 2). Its raw coverage, $\text{coverage}_{\text{UCS}}$, thus is also 6/9. The unique coverage of PUC, that is, all cases covered by PUC alone, is calculated by subtracting the raw coverage of UCS (6/9) from the solution coverage (7/9). Hence: unique $\text{coverage}_{\text{PUC}}$ = 1/9 = 11%. Similarly, the unique coverage of UCS is calculated by subtracting the raw coverage of PUC (6/9) from the solution coverage (7/9). Hence: unique $\text{coverage}_{\text{UCS}}$ = 1/9 = 11%.

Goals of QCA

Theoretical Issues

While most QCA applications investigate situations involving a moderate number of cases, and QCA researchers commonly see their tools as bridging the case study versus large *N* divide, as we pointed out earlier, it is not the number of cases that distinguishes QCA from more standard statistical techniques. Nor is it entirely the reliance on dichotomous variables, since there are standard quantitative techniques that can be used with dichotomies. Rather, it is the four differences in the following that we regard as most critical, which all have at their core the issue of causal complexity.

First, as already highlighted previously, in QCA a central concern is with the identification of conjunctions of factors that may be regarded as either *necessary* or *sufficient* for a given outcome. As QCA researchers, together with other qualitatively oriented scholars, point out, one strictly sufficient condition that covers only one or few cases (i.e., low unique coverage), and thus might give rise to coefficients that might not be regarded as statistically significant in say a binary logistic regression, can still be theoretically, empirically, and substantively highly informative. Such conditions might also serve as a good starting point for further (in-depth case study) research.

Second, QCA emphasizes equifinality, namely, the notion that different (combinations of) a small number of factors can be associated with the same outcome; or in causal terms, that we often expect to find different (sets of) causes giving rise to the same effect. In QCA, the emphasis is on explaining cases, and there is no expectation that the same (appropriately weighted) combination of factors will explain all cases.

Third, in QCA, the initial expectation is that *logical conjunctions of conditions*—not single variables in isolation or in additive combinations—are causally relevant for producing the outcome.

Fourth, QCA emphasizes asymmetric causality, namely, that the occurrence of a phenomenon and its nonoccurrence require separate analyses and explanations, and we need to distinguish between necessary and sufficient conditions.

Solution formulas 1 and 2, which express the type of configurational causal complexity that QCA has comparative advantages in detecting, illustrate the four features of QCA highlighted previously. First, the solution formulas can be read in terms of sufficiency: each of the two conjunctions gives a combination of conditions that implies the outcome welfare state and can thus be interpreted as sufficient for the outcome in the given data. Second, the existence of strong welfare states exhibits equifinality because there are different "paths" (PUC and/or UCS) leading to that outcome. Third, conjunctural causation is at work because single conditions alone do not produce the outcome alone or additively, rather it is logical combinations of factors. Fourth, the emphasis is on asymmetric causality. Had we analyzed the sufficient conditions for not-W, the solution formula would not be identical to that for W (see the following).[14]

Presentational Issues in QCA

Regardless of the data analysis technique employed, the presentation of analytic results in comparative social research can have three different aims: (a) displaying relations between variables in a readily comprehensible fashion, (b) highlighting specific (groups of) cases in terms of alternative descriptive or causal accounts, and (c) expressing the fit of the result obtained to the data at hand. By and large, scholars engaging in case-based qualitative comparison tend to focus more on the second of these aims, understanding/explaining (how/why) what is going on in specific cases. In contrast, for quantitatively oriented scholars, the focus is on variables and on how much of the variation they are able to explain, the first and third of the three general goals. QCA, being at the intersection of qualitative and quantitative research, aims at incorporating all three research aims but highlights the second of these.

Most presentational methods deal better with some of these goals than with others, and this is true for the four QCA tools previously identified: solution formulas, truth tables, and measures of coverage and consistency.

Solution formulas display conjunctive (OR) and disjunctive (AND) equifinal relationships in a reader-friendly way. By making use of Boolean operators, solution formulas are a powerful tool to succinctly express fairly complex relationships among conditions and an outcome. Solution formulas as such, however, do not identify for the reader which cases fit which combination of conditions, nor do they express the degree to which the solution fits the general patterns in the data. Simply reporting the solution formula (especially in the compressed form shown in solution formula 2), the reader is left in the dark which cases follow which of the different paths toward the outcome, which of the paths might be empirically more important (in the sense of more cases displaying that path), and how well the solution component fits to the data at hand. Solution formulas thus seem best adapted only for the first of our three presentational and methodological goals.

Truth tables help to sort the information obtained on the cases in a logically structured way. They thus help to (a) bring to the fore analytic similarities and differences between cases and (b) reveal contradictory rows, namely, cases with identical combinations of conditions that show, nonetheless, differences in the outcome (identified in Table 2 as rows with "contradictory" outcomes) and (c) the degree of empirical "spread" in the data, namely, which logically possible combinations of conditions are and are not empirically observed. All these pieces of information, when examined appropriately, can help the researcher to think (again) about the universe of cases, the set of conditions, and the conceptualization of linkages between conditions and the outcome. For purposes of theory building, tables like Table 2 can play an important heuristic role.

The *coverage* and *consistency* parameters directly address the third of our three key methodological aims: to provide information on the overall and path-specific goodness of fit of solution formulas. But the aim of accounting for specific (groups of) cases is neglected. A simple way of overcoming this shortcoming, and one that is used by most QCA analysts (see e.g., Schneider 2008, chap. 6) is to report consistency and coverage values for each solution formula and list the labels of cases that are covered by the different conditions directly underneath the solution formula, as is done automatically in some software.[15] Applied to the data in Table 1, this can give us the kind of display shown in Table 4.

Comparisons between QCA and Binary Logistic Regression

To see directly how regression differs from QCA we can run a binary logistic regression on the data in Table 1.[16] If we run all four variables (P, U, C, and S)

Table 4
Summary of Consistency and Coverage Solution for Outcome W

	PUC +	UCS	→ W
Consistency	100%	100%	
Raw coverage	67% (6/9)	67% (6/9)	
Cases covered	AUT,DK,FIN,NOR,SWE,BEL	AUT,DK,FIN,NOR,SWE,IRL	
Unique coverage	11% (1/9)	11% (1/9)	
Cases uniquely covered	BEL	IRL	
Solution consistency			100%
Solution coverage			78% (7/9)

Note: P = strong left party; U = strong unions; C = corporatist industrial system; S = sociocultural homogeneity; W = strong welfare state.

Table 5
Binary Logistic Prediction in Cross-Tab Format with W as the Dependent Variable

(a) Model with P, U, C, and S Entered	Predicted as not W	Predicted as W	
not W	6	1	7
W	0	9	9
	6	10	$N = 16$

(b) Model with Just C Entered	Predicted as not W	Predicted as W	
not W	6	1	7
W	0	9	9
	6	10	$N = 16$

(c) Model with Just P Entered	Predicted as not W	Predicted as W	
not W	7	0	7
W	3	6	9
	10	9	$N = 16$

Note: P = strong left party; U = strong unions; C = corporatist industrial system; S = sociocultural homogeneity; W = strong welfare state.

in a binary logistic regression with W as the dependent variable, the overall regression fit (a Cox and Snell R^2 of .68; a Nagelkerke R^2 of .91) is very good, even though none of the variables are statistically significant. As shown in Table 5(a), we can convert the results of this binary logistic regression into the kinds of cross-tabs shown in Table 3. Table 5(a) might appear to have generated a better fit than even the QCA results reported in Table 3(c), since there is only one off-diagonal prediction in Table 5(a) but two in Table 3(c) and three each in Table 3(a) and (b).

But, from the perspective of QCA it is Table 3(c), or even Table 3(a) and (b), that give the superior results because QCA (as we are using it here for illustrative purposes) is only looking for the best fit in terms of sufficiency, *not* the best overall fit. In a logistic regression involving dichotomies, the program seeks to maximize fit, which in this case means maximizing the cases that are placed on the main diagonal. This is the equivalent of weighting deviations from sufficiency and deviations from necessity as equally important, since sufficiency requires one off-diagonal cell to be zero, the upper right hand cell (see Table 3), while necessity requires the other off-diagonal cell (the bottom left cell) to be equal to zero (see online appendix at http://prq.sagepub.com/supplemental). In logistic regression the program is indifferent to how it maximizes the number of cases on the main diagonal in terms of whether it is the upper right hand cell or the lower left hand cell that is brought closer to zero.

Looking more closely at how Table 5(a) differs from Table 3(a-c) we see that its predictive error comes in the upper right cell and that all of the cross-tabs in Table 3 have a zero in that cell. Yet, for QCA purposes this single incorrect prediction is critical since it is a predictive error with respect to sufficiency.

There are a few other points about QCA comparisons with binary logistic results (see also Grendstad 2007) we would like to make.

First, for the data set in Table 1, we can do just as well in overall predictive power with one variable as we can with all four, as shown in Table 5(b), where we do a bivariate logistic regression with C as the sole independent variable, namely, Table 5(b) is identical to Table 5(a).

Second, even though the predictive fit is the same, the pseudo R^2 values for the single variable equation (a Cox and Snell R^2 of .62; a Nagelkerke R^2 of .83) are (slightly) lower than those for the regression involving all four variables. That is because the closer the accurately predicted values are to one (or zero), the higher will be the associated pseudo R^2 values.

Third, for a logistic regression, it does not matter how the dichotomous dependent variable is coded. Flipping the values from zero to one and one to zero will yield the same coefficients (just with the direction of the signs inverted) and predictive power of the model. In QCA, this is not the case because causal relationships are assumed to be asymmetric. For this reason, the occurrence of the outcome and its no-occurrence require different explanations and thus separate analyses. Analyzing the sufficiency conditions for the nonoccurrence of W based on the data in Table 1 yields the following result:

$$\text{puc} + \text{pcs} \rightarrow \text{w}.$$

This solution formula for not-W is neither the arithmetic inverse nor the logical negation of the solution formula for W, PUC + UCS.[17] From the solution formula for the occurrence of the outcome one cannot derive the solution formula for the non-occurrence of the same outcome without running a separate analysis.

Using Interaction Terms in Binary Logistic Regressions to Mimic QCA Solution Formulas

Since regression models with interaction effects among "basic" variables can be constructed, it might appear that we could use such interaction-based regression models to mimic the kinds of insights into sufficient (or necessary) composite conditions we get from QCA. The first key point to recognize is that for dichotomous variables, there is a direct parallelism between the AND property of QCA propositions and multiplication of terms for purposes of specifying interaction in regression models. That is because for dichotomies, AND(i, j, k, . . .) is equivalent to MIN(i, j, k, . . .) and returns a value of zero if any of the values in it are zero; while the expression I*j*k similarly returns a value of zero if any of the dichotomies in the expression have values of zero.[18] The second point to recognize is that for dichotomies, OR(i, j, k, . . .) is equivalent to MAX(i, j, k, . . .) and returns a value of one if any of the values in it are ones. Thus, to translate solution formula 2 to their regression equivalents, we simply define new variables from the old PUC, UCS, and PUC + UCS, as P*U*C, U*C*S, and MAX(P*U*C, U*C*S), respectively.

We will consider a binary logistic regression with all three variables, namely, P*U*C, U*C*S, and MAX(P*U*C, U*C*S), but also one with just the first two and one with just the last (composite) variable. The prediction results from these logistic regressions are shown in Table 6(a-c). For all three of these regressions the Cox-Snell R^2 value is .54 and the Nagelkerke R^2 value is .72. Moreover, the prediction cross-tabs we generate are identical for all cases and are identical to what we got from the QCA solution formula analysis shown in Table 3(c).

These facts might lead us to think that we could substitute binary logistic analyses for QCA, but that would be erroneous. Even an interaction-based regression model is not oriented to finding a 100 percent fit for as many cases as possible, rather than a very good fit for all the cases, on average. While once we have completed QCA we can use what we have learned to mimic its results with more traditional methods such as binary logistic regression, without QCA we would not have detected the complex interaction effect that allowed us to correctly predict seven of nine cases (rather than six of nine) in terms of sufficiency.

Also, if we do decide to use binary logistic methods to mimic QCA, we would add three strong notes of caution. First, while such methods do in principle allow for statistical inference, because of how we are generating the composite variables through a QCA search program, any statistical significance estimates we get from binary logistic regression are essentially meaningless, although we can meaningfully interpret the overall goodness-of-fit measure. Second, even if we use interaction terms, binary logistic regression remains insensitive to the differences between necessity and sufficiency. Finally, it is virtually unheard of to make use of even three-way interactions in most regression modeling, since there are so many different ways such an interaction could take place, the interpretation of the coefficients quickly gets out of

Table 6
Binary Logistic Prediction in Cross-Tab Format for Composite Variables with W as the Dependent Variable

(a) Model With {PUC}, {UCS}, and {PUC or UCS}	Predicted as not W	Predicted as W	
not W	7	0	7
W	2	7	9
	9	7	$N = 16$

(b) Model With {PUC} and {UCS}	Predicted as not W	Predicted as W	
not W	7	0	7
W	2	7	9
	9	7	$N = 16$

(c) Model With Just {PUC or UCS}	Predicted as not W	Predicted as W	
not W	7	0	7
W	2	7	9
	9	7	$N = 16$

Note: P = strong left party; U = strong unions; C = corporatist industrial system; S = sociocultural homogeneity; W = strong welfare state.

hand, and the violation of statistical assumptions inevitable (e.g., Kam and Franzese 2007).

Attempts to mimic QCA within the statistical framework are under way (see e.g., Braumoeller 2003, 2004). Those suggestions are promising but still come short of the complexity unravelled by QCA, while imposing data requirements that are hardly, if ever, met in (macro-)comparative social sciences.[19]

Conclusion

The primary target audience of this article is quantitatively trained political scientists who may have heard about QCA without really knowing much about it and who would benefit from a basic introduction to it that demonstrated its similarities and differences to more traditional statistical tools that arise because of QCA's emphases on conjunctural causation, equifinality, and asymmetric causality. We hope that our having spelled out the meanings of the major presentational forms used in QCA work will help interested quantitatively trained readers to make better use of QCA-based research done by others.[20] We hope, too, that the insights given here into how to make use of QCA and how it differs from methods like binary logistic regression will, in the future, lead more of those scholars to give QCA a try—something made much easier because by now it is possible to perform QCA in Stata or R, a software environment with which many are familiar. In our view, whenever there are good reasons to think that the phenomenon under study is best understood as the result of causally complex structures involving hypotheses about necessity or sufficiency, then the application of QCA should be considered.

Notes

1. We use the acronym QCA to refer to different variants of "comparative configurational methods" (Rihoux and Ragin 2008) based on set theory and Boolean algebra, namely, crisp set QCA (csQCA), fuzzy set QCA (fsQCA), and multivalue QCA (mvQCA). csQCA makes use of binary values with one indicating membership in a given set and zero indicating nonmembership. fsQCA allows for any value in the interval [0,1] indicating partial (non)membership in sets (Ragin 2000, 2008b). mvQCA (Cronqvist and Berg-Schlosser 2008) allows for multinomial conditions where the outcome needs to be a crisp set.

2. In appendices (online at http://prq.sagepub.com/supplemental) we review the logic of necessity and sufficiency and assemble a useful set of propositions linking necessity and sufficiency to conditions in cross-tabs, and we consider notational and terminological issues that may lead to misunderstanding of QCA results for readers used to standard statistical models.

3. Detailed introductions into the logic of QCA can be found in Ragin (1987, 2000), Schneider and Wagemann (2007), Rihoux and Ragin (2008), and Caramani (2009).

4. QCA thus is geared toward detecting so-called INUS conditions (Mackie 1974, 62; Goertz 2003, 68; Mahoney 2008), causally relevant factors that are almost always overlooked when standard statistical techniques are applied. INUS stands for "insufficient but necessary part of a condition which is itself unnecessary but sufficient for the result" (Mackie 1974, 62).

5. Ragin (2000) provided a data set for eighteen countries with their respective fuzzy membership scores in these four conditions and the outcome. For reasons that are irrelevant for present purposes we will confine ourselves to only sixteen of his eighteen cases. There are of course other variables that have been linked to strong welfare states than the four shown in the table,

such as the form of the electoral system (McDonald 2006), but in this article our aim is not to make substantive contributions to the literature on welfare states but only to illustrate QCA methodology, so we focus on these four dichotomized conditions.

6. To code the outcome W = 1 we required a fuzzy membership value of greater than .66 in Ragin's original coding, and for all four conditions, an original fuzzy membership score >.5 yields a crisp score of 1. The dichotomized data show in Table 1 only a particular crisp set transformation of the original fuzzy set data found in Ragin (2000), Table 10.6, and is intended only for illustrative purposes (see Ragin 2008a for ways of creating a crisp truth table representation of fuzzy set data).

7. Although most QCA researchers eschew the terms *dependent variable* and *independent variables*, arguing that this language presupposes an independence across conditions that may not be present (Rihoux and Ragin 2008; Schneider and Wagemann forthcoming), to make the presentation more readable by readers trained in standard statistical methods, in the discussion that follows we use *dependent variable* as synonymous with *outcome* and *independent variables* as synonymous with *conditions*.

8. Strictly speaking, however, the columns indicating the number of cases ("*n*") and the case labels ("Case Labels") in Table 2 are not integral parts of a truth table, but we believe it very helpful to also record this information in such tables to simplify the reader's task of interpretation. Information stored in these columns is also used by several software packages when presenting QCA results.

9. This is a nonexclusionary OR, that is, one and the same element (case) is allowed to have memberships in both or in just one of the two sets (conditions) being combined.

10. The sign for necessary condition is the arrow in the opposite direction (←).

11. There are various programs to do this for QCA, such as the specialized software fsQCA (Ragin, Drass, and Davey 2006) and Tosmana (Cronqvist 2006), but mainstream statistical programs Stata (Longest and Vaisey 2008) and R (Dusa 2007) now also allow QCA-related calculations. We will not discuss the algorithms used since that discussion takes us beyond the scope of the present article. Here we have deliberately chosen a data set whose crisp set form can be analyzed "by hand."

12. Note that when we represent the data in Table 1 in the form < PUC + UCS → W> we are making no simplifying assumptions about what is happening in the *logical remainders* (i.e., hypothetical cases with no data) in rows 8 through 16. For different treatments of these logical remainders, see Ragin and Sonnett (2004), Schneider and Wagemann (2006), or Rihoux and De Meur (2009, 59-65).

13. Note that it might be tempting to interpret both U and C alone as necessary condition for W because both conditions are part of all sufficient conditions for W. However, only under specific and rather rare empirical situations does the analysis of sufficient conditions also correctly identify the presence of necessary conditions. Therefore, standards of good QCA practice dictate that the analysis of necessary conditions should be performed separately from that for sufficient conditions (Ragin 2008a; Schneider and Wagemann 2007, forthcoming).

14. In addition to these four differences, QCA scholars tend to emphasize one other point, *multifinality*, namely, that the same factor can play a different role in different contexts. Imagine, for example, purely hypothetically, that the solution formula we had arrived at was PC + pU → W. In the first subformula, strong left parties in conjunction with corporatism are associated with strong welfare states; in the second subformula, weak left parties in conjunction with strong labor unions are associated with a strong welfare state. If this were what the QCA data analysis had shown, it would tell us that sometimes the presence of strong left parties facilitates a welfare state and sometimes their absence does, depending upon what other conditions are present. We will not deal with multifinality in this article.

15. The latter option is offered by the Tosmana and R software, whereas the former is offered by the fsQCA 2.0 and Stata software.

16. Katz, vom Hau, and Mahoney (2005) compared ordinary regression to fuzzy set QCA, as did Seawright (2005b). See also Ragin (2005) and Seawright (2005a).

17. The complement, or negation, of PUC + UCS would be ps + u + c. While the former term describes a subset of cases described by the latter term, substantively they differ quite a lot and the latter solution term includes assumptions on the outcome value of logically possible combinations of conditions for which, however, no empirical evidence is at hand.

18. Note, however, that for interval and metric scale variables, multiplication is not equivalent to the logical AND. This is why there is a (subtle) different between *interactions* and *intersections*.

19. We note, however, that it is in principle possible to develop inferential tests to determine, say, whether the degree of consistency (or coverage) of some particular combination of variables is more than would be expected from chance relationships uncovered through data mining (see e.g., Ragin 2000, 109-15, for statistical tests of sufficiency; Dion 2003, 102-9, for necessity; or Eliason and Stryker forthcoming; see also Schield and Burnham 2002).

20. We also hope that our discussion in the online appendix at http://prq.sagepub.com/supplemental of potential notational and terminological sources of confusion will be useful to readers.

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