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How many political parties are there, really? A new measure of the ideologically cognizable number of parties/party groupings

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Abstract

We offer a new measure of the ideologically cognizable number of political parties/party groupings that is intended to be complementary to the standard approach to counting the effective number of political parties – the Laakso–Taagepera index (1979). This approach allows the possibility of precise measurement of concepts such as polarized pluralism or fragmented bipolarism and is applicable to both unidimensional and multidimensional representations of party locations. Using recent CSES (Comparative Study of Electoral Systems) data on one-dimensional representations of party locations in four real-world examples (two of which are available in an online appendix), we find that Slovenia, treated initially as a five-party system, has its optimal reduction as a two-bloc/party system, as does Spain, which is treated initially as a four-party system. However, Canada, treated initially as a four-party system, has its optimal reduction as a three-bloc/party system if we look at a unidimensional representation of the party space, while it remains a four-bloc system if we draw on Johnston's two-dimensional characterization of Canadian political competition. Finally, the Czech Republic, initially a five-party system, is optimally reduced to a system with four party groupings.

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Introduction

How many political parties are there in the parliament of country Q? How many parties contested the election in which that parliament was elected? To anyone who has not thought much about these questions, they seem rather trivial problems – requiring for their answer only the kind of counting you learned to do in kindergarten. Yet, they are far from simple once we recognize that, as social scientists, what we want to do is to operationalize the number of parties at the electoral and parliamentary level in a way that will allow us to forge theoretical links between these variables and other key features of a country's politics. After reviewing the two most important efforts to address such questions: the Laakso–Taagepera index (LT index) of the effective number of parties (Laakso and Taagepera, 1979) and the Banzhaf power score modification of the LT index to take into account party decisiveness in a parliamentary weighted voting game (Dumont and Caulier, 2003; Grofman, 2006; Kline, 2009), which we will abbreviate the LTB index, we offer a new method of counting parties that integrates effective size considerations with party locations in policy space to develop what may be thought of as a measure of the ideologically cognizable number of parties/ party ideological groupings.

We believe such an approach is needed to allow precise quantification of Sartori's (1976) insight that party ideological location matters in counting how many parties/party groupings there are, 'really', a point that has recently been emphasized by Dalton (2008). For example, as we look at the evolution of the literature on cabinet-coalition-formation, we can see that evolution as demonstrating the need for theory-building that takes ideology into account. The early literature on cabinet-formation treated the number of members of the governing coalition as the critical variable, with major theories being minimal winning coalitions (Riker, 1962) and fewest actor coalitions (Leiserson, 1970). But it quickly became apparent from empirical work that models based only on number of parties, or on relative party strengths (e.g. Gamson, 1961), did not allow realistic predictions of cabinet-formation in post-World War II Western Europe (see, however, Schofield, 1976). The second generation of cabinet-formation models explicitly took the ideological location of parties into account, and took as a key input the distances between party location, as in de Swaan's (1970) work on least distance coalitions, or Axelrod's (1970) work on ideologically connected coalitions, or work on central actors (van Deemen, 1989; van Roozendaal, 1999). This work was joined by a plethora of models that moved from a unidimensional treatment of ideology to a multidimensional one, e.g. the McKelvey–Ordeshook–Winer competitive solution (1978; see also Winer, 1979), or Schofield's work on the heart (Schofield, 1995; Schofield and Sened, 2006); as well as cluster-theoretic models such as Grofman's (1982) dynamic model of proto-coalition-formation.[1] Just as success in predictive cabinet-coalition theory was minimal until the ideological characteristics of the parties (and, in some later models, of the cabinet ministers, themselves) were taken into account, we believe measures of party system properties that only 'count' changes in the 'number' or 'effective number' of parties over time are likely to be limited in their analytical power, since ideological structure is such an important aspect of party constellations.

Outside the United States there is evidence of the fragmentation of many party systems, but it is not clear whether that fragmentation is overstated by simply counting seat-winning parties or effective number of parties in that systems with multiple parties may involve competition among a number of ideological 'blocs' fewer in number than the number of seat-winning parties, or even the effective number of parties. Our approach is intended to give precision to the concept of polarized pluralism (Johnston, 2008) by characterizing systems with many parties in terms of a (perhaps) smaller number of (ideologically defined) 'party clusters'. In particular, we can assess the accuracy of the claim that given systems should be regarded as instances of fragmented bipolarism, i.e. for many purposes acting like two-party systems – a frequently made claim for post-1994 Italy (Bartolini et al., 2004), or much of post-World War II France.

In the next section we review the LT and LTB indices and show some illustrations of how these indices can conceal a great variation in the ideological structure of party constellations. We show that even constellations of parties with identical scores on the LT and LTB indices nonetheless appear to differ from one another in important ways, e.g. in terms of likely coalition dynamics, once we take into account where the parties are located in issue space. In the succeeding section we introduce our new measure and give a short overview of the underlying axiomatics, with a fuller exposition left to a methodological appendix to be found at Grofman's website, where a longer and more complete version of this article can be found.[2] In general, the (integer) number of ideologically grouped party clusters will be distinct from (and sometimes smaller and sometimes larger than) the effective number of parties.

In this article we offer both hypothetical and real-world examples of party competition. For our hypothetical examples, we consider both unidimensional and multidimensional party systems. However, the real-world data we draw on from the CSES (Comparative Study of Electoral Systems) project provide only one-dimensional representations of the four real-world examples that we make use of in this essay. We also recognize that CSES data, based on voter perceptions, are only one way of thinking about party space, and that other measures, e.g. expert witness judgments (Castles and Mair, 1984; Huber and Inglehart, 1995) or party manifestos data (Budge et al., 2001; Klingemann et al., 2006), or parliamentary voting patterns (Poole and Rosenthal, 2007), might also be illuminating. We use CSES data for three reasons: (1) their widespread use within the community of electoral and party system scholars, (2) the high level of standardization across countries in the metric used to locate parties ideologically, and (3) the fact that it has multiple waves that would, in further work, allow for longitudinal analyses. We chose the four countries we did simply for illustrative purposes, and because they were the same set of countries used for illustrative purposes in Dalton (2008), who also used CSES data for his analyses. As it turns out, these countries also work nicely to give us a considerable range of observed outcomes of the algorithm we make use of.

We would also emphasize that looking at a single dimension (which, for convenience, we treat as a left–right dimension) is not a limitation of our measurement approach; the same basic intuitions do apply in more than one, or even more than two, dimensions. However, results from unidimensional and multidimensional representations of party spaces need not yield the same optimal condensations, as we show when we compare our results for the CSES unidimensional representation of Canadian party space with our results for the two-dimensional representation of the same parties found in Johnston (2008), which is also based on voter perception data.

A brief overview of the Laakso–Taagepera and Laakso–Taagepera–Banzhaf approaches, along with some illustrative examples of how party constellations with identical LT (or LTB) scores can, nonetheless, differ greatly ideologically

Parties may be measured at either the electoral or the parliamentary level, i.e. as a function of their vote-share or seat-share (Taagepera, 1986; Taagepera and Shugart, 1989). In this essay we focus on seat-share.

To see why simple counting may not give us the only (or the best) answer to how many parliamentary parties there are, let us consider some examples.

First, compare two countries which each have four parties in the parliament. In the first country, the parties have parliamentary seat-shares of 25 percent each; in the second, the distribution of shares is 47 percent, 47 percent, 4 percent and 2 percent. Intuitively, it would seem quite clear that we might expect very different politics in the parliaments of the two different countries. Moreover, while it also seems quite clear that there really are four parties in the parliament of the first country, the 'real' number of parties in the second country seems less clear. Is it four, because four parties have representation? Or is it better characterized as closer to two, since there clearly are only two major parties? Or should we think of this as a three-party system, since any coalition with two of the first three parties in it is winning, but the fourth party is, in game-theoretic parlance, a dummy? Each of these intuitions gives rise to a different way to 'count' the number of parties.

In the simplest approach we simply count how many parties have at least one representative in parliament. The second approach, one taking into account party sizes, has been precisely formalized in the Laakso–Taagepera (LT) index. Let $p_i =$ the (seat) share of party i and $n =$ number of parties in parliament. Using the notation above, the effective number of parties (Laakso and Taagepera, 1979) is given by:

$$\mathrm{LT} = 1 \Bigg/ \sum_{i=1}^{n} p_i^2 \tag{1}$$

While there have been various alternatives proposed (e.g. Dunleavy and Boucek, 2003; Taagepera and Shugart, 1989 (Appendix C); Wildgen, 1971), and with what we now call the LT index only being one of a family of indices, of the form $1 \Big/ \sum_{i=1}^{n} p_i^k$, discussed in Laakso and Taagepera (1979), for various values of the parameter k, the

LT index has become the 'gold standard' for operationalizing the effective number of electoral or parliamentary parties. Over the past two decades, it has been used in virtually all comparative research on political parties and electoral system effects done by either political scientists or economists. When we calculate the LT index for the hypothetical data above, we get 4 parties for the first country and 2.25 effective parties for the second.

The intuition that suggests we ought to look at the parliamentary voting situation in which parties find themselves can be formalized in terms of game-theoretic models of pivotal power and decisiveness such as the Shapley–Shubik value (Shapley and Shubik, 1954) or the Banzhaf Index (Banzhaf, 1965). Here, we limit ourselves to the Banzhaf approach. Because we are looking at a majority rule voting game, we need only find Banzhaf scores for the cases in which a losing coalition is turned into a winning one (by symmetry, the power scores will be the same were we to count swings as the number of times a winning coalition is turned into a losing one). Given all possible coalitional configurations in this hypothetical example, there are a total of 12 minimal winning coalitions. To determine the normalized Banzhaf score for each of these parties, we count the number of times it is a swing player (i.e. the number of times it is able to turn a losing coalition into a winning one) and divide this number by the total number of coalitions which contain a swing (i.e. the minimal winning coalitions), in this case 12. Thus, the normalized Banzhaf score for these four parties is given by the vector (4/12, 4/12, 4/12, 0/12).

Dumont and Caulier (2003), Grofman (2006) and Kline (2009) have each independently proposed combining Banzhaf ideas with the Laakso–Taagepera approach to calculate what we may call the LTB index, by substituting power scores for seat proportions in the LT formula.

Let B_i = the normalized Banzhaf score of the ith party. Then,

$$\mathrm{LTB} = 1 \Big/ \sum_{i=1}^{n} B_i \tag{2}$$

There are two polar kinds of situation in which we might expect the proposed Banzhaf modification to the LT index to give an estimate of the 'effective' number of parties that is very different from the simple LT index. The first of these is when one party has a majority of the seats. Now the LTB index is always 1, while the LT index may be close to 2 if, for example, there are three parties with seat-shares 0.51, 0.48 and 0.01, respectively. The second is when there are three parties, two large and one small, such that any two of them form a winning coalition. Consider, for example, three parties with seat-shares 0.48, 0.48 and 0.04, respectively. The LT index is slightly above 2 (2.16), but the LTB index is 3.

Let us return to two examples from earlier in the article. When we calculate the LTB index for a party distribution of (0.47, 0.47, 0.04, 0.02), we get a value of 3, since three of the four parties have identical Banzhaf scores and the fourth a score of zero. Indeed, the distribution (0.47, 0.47, 0.04, 0.02) shows how we can get three different answers from the three different ways of operationalizing the number of parties in a political system:

simple counting (4), the LT measure (2.25) and the LTB measure (3). On the other hand, there are cases where all three methods give identical results. If, as in an earlier example, we have four parties with 25 percent seat-shares, then all four parties will have identical power scores using LTB, and thus LT, LTB and simple counting will also agree that this distribution is a four-party system.

Arguably, each of the three ways of counting parties has its merits. Sometimes we simply want to know how many parties have representation. Here, simple counting seems all that is required. Sometimes we may want to know how close the system is to one of *k* equally sized parties. Here the LT measure seems best. Sometimes we may want to use the number of parties to better understand coalition-formation processes. Here, the recently proposed LTB index would often seem to give us the most useful insights. But these three approaches do not exhaust all the useful ways of thinking about how many political parties there are in a given parliament. The fourth method we propose, which we refer to as a way of calculating the 'cognizable' number of parties from an ideological perspective, is intended to be complementary to the three approaches we have previously reviewed, rather than as a replacement for them. Before we introduce our own approach, we next provide some simple examples that illustrate why we might want to concern ourselves about party locations and not merely party sizes in deciding 'How many parties are there, really?'

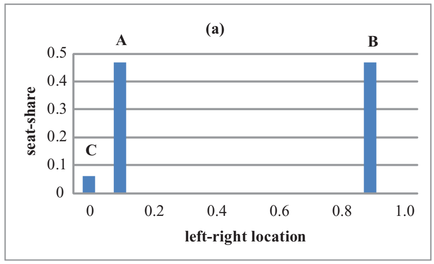
Let us consider a parliament where there are three parties, identified by their seat shares of 0.47, 0.47 and 0.06, which we identify as A, B and C, respectively. The LT index is 2.25; the LTB index is 3. But now let us imagine that these three parties are set at different points along the left–right dimension, as shown in Figures 1a–d, or in a two-dimensional space, as shown in Figures 1e–h.

Our claim is that, if one looks at the eight cases in Figure 1, even though they give rise to identical LT values and identical LTB values, one ought not to treat them as identical. Clearly, these different spatial arrangements have different potential implications for coalition-formation. For example, C and B seem very likely alliance partners in scenarios (b) and (e), since B is likely to be more interested in allying with small party C than with a party its own size in the simplified situation we have created in which any coalition of two parties is winning and B is equidistant from the other two parties. In contrast, in scenarios (a) and (f), C would appear a likely partner of A. In scenarios (c) and (g), C is in a position to be pivotal between A and B and would appear largely or entirely indifferent in policy terms as to which of the two with which to ally. This would seem to be the scenario in which C is most powerful in game-theoretic terms, but it might still seem to be basically a two-party contest in the sense that C's choice of alliance partner might not much affect the policy choices of its larger partner if we assume that policies of coalition governments represent the relative weights of the actors. Finally, in scenarios (d) and (h), the two larger parties A and B are so close together in ideological terms that they might well ally, leaving C irrelevant. However, if we assume that large parties, *ceteris paribus*, would prefer to ally with a small party to make a winning majority, rather than to ally with a large rival, we might still think the BC coalition the most likely. Slight variations of this scenario might sometimes give us AB coalitions and sometimes BC coalitions, depending upon exact party locations and sizes.

So, even though these eight cases have identical party size distributions, the likely alliance patterns can be very different due to the differences in the location of the parties

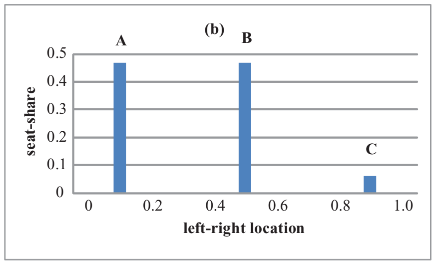
(a)

	power	location	seats
A	0.33	0.1	0.47
B	0.33	0.9	0.47
C	0.33	0	0.06



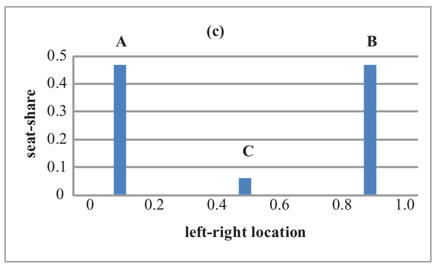
(b)

	power	location	seats
A	0.33	0.1	0.47
B	0.33	0.5	0.47
C	0.33	0.9	0.06



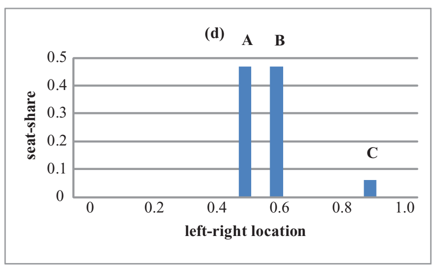
(c)

	power	location	seats
A	0.33	0.1	0.47
B	0.33	0.9	0.47
C	0.33	0.5	0.06



(d)

	power	location	seats
A	0.33	0.5	0.47
B	0.33	0.6	0.47
C	0.33	0.9	0.06



Two-Dimensional Examples

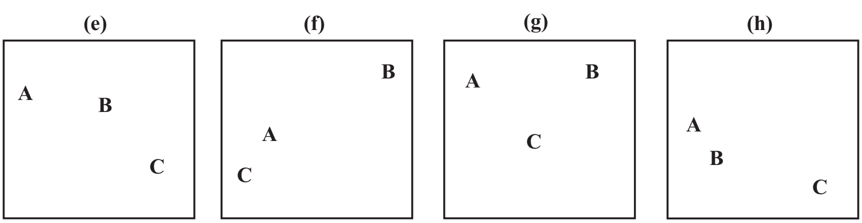


Figure 1. Hypothetical Party Locations in a Three Party System

in the policy space, with at least four different plausible scenarios even though we are only looking at three parties and two of these have identical weights. Moreover, if we look at the four unidimensional scenarios in polarization terms, using Dalton's (2008) measure of ideological polarization, the first and third show high polarization, the second shows mild polarization and the fourth low polarization.

Ideologically cognizable number of parties/party groupings: Theory

As Dalton (2008) argues, knowing the number of parties contesting for office, or the number of seat-winning parties, or the effective number of parties, is only a part of making sense of the structure of party competition and predicting its consequences – and the same is true even for the Banzhaf-adjusted effective number of parties. The method we briefly describe below, implemented using a clustering algorithm in Grofman (1982), is described in detail in a methodological appendix available on Grofman's website.

In this section we show how to provide 'reduced' representations/condensations of the existing party constellation that involve fewer parties than in the original constellation. At each stage, these reductions maintain intact one key parameter of the party system, namely mean party locations, and also have certain other desirable properties. Once we have developed a method to provide plausible smaller and smaller 'condensations' of the original party constellation, with our limiting case being a representation of the constellation in terms of only a single party, we offer a 'stopping rule' that will allow us to decide what is the fewest parties we can use that will allow us still to remain 'sufficiently' faithful to our original party constellation. The number of parties in that representation will then be taken to be our indicator of how many ideologically cognizable party groupings there are. The stopping rule we use is based on the size of the (party-size weighted) 'dislocations' in initial party locations we get as we 'condense' the original party constellation by representing it with fewer and fewer parties.

While we use the Grofman (1982) clustering algorithm, our goal is different. The aim of that article was to predict coalition-formation using ideological proximity. The algorithm was applied until a winning coalition was reached. Here, using our threshold stopping rule, the process of amalgamation may very well end before we get to a winning coalition, and may also continue beyond that point.

We begin with some given distribution of party location (x_i) and seat-shares (p_i). Our ultimate goal is to create 'optimal' reduced configurations that best reflect that initial constellation while also reflecting the underlying structure of party competition. In such a way we can re-configure a multiparty system that exhibits, say, fragmented bipolarism, as a two-party system that is as faithful as possible to the key parameters of the original multiparty constellation. To achieve this goal, we look for a way of sequentially creating 'reduced' party constellations (i.e. configurations with fewer parties than in the original) that satisfy three secondary aims:[3] (1) to reduce the number of parties by exactly one at each stage of the sequential merger process; (2) to preserve the mean ideological location of the initial party constellation at each stage of the sequential merger process, since this is a key parameter of the initial distribution; and

(3) to minimize the dislocation from their original locations of the new 'pseudo-parties' that are created as merged party units at each stage of the process.

To address the first of these aims we require that the merger process at each stage involve exactly two parties (or party groupings). That way the merger will reduce the existing number of parties/party groupings by one.

To achieve the second aim we provide a useful result about the kinds of merger process that are means preserving, i.e. ones in which pairwise mergers are located at the centre of gravity of the two parties (see Proposition 1), and then offer an algorithm that has this property. We seek to preserve the mean location rather than the median location because we are looking at quantitative measures of party location and we expect that actual distance, and not just relative placement in ordinal terms, will matter for bloc structure. Of course, we recognize that, in one dimension, location of the median voter or party may have important consequences for other aspects of political competition, such as policy outputs (see, e.g., McDonald and Budge, 2005).

In the process of achieving our third aim we create a measure that allows us to directly evaluate how good a job of reflecting an existing configuration each alternative reduced party configuration provides, by looking at the extent to which it creates a (normalized party-seat-share-weighted) displacement of parties/party groupings from their initial locations to their new location as part of a merged (pseudo) party. This measure of dislocation has the property that, when two parties merge, the smaller party is 'pulled' further from its initial location than is the larger party (see Propositions 2 and 3 in the online appendix). Party-weighted dislocation is important because it is a far more 'visible' change to a given party constellation if a large party changes its location by a given amount as a result of a merger than if a small party changes its location by that same amount.

Another key property of the dislocation measure is that it is based on minimizing 'mutual' dislocation among the set of possible pairwise mergers.

Definition: A pairwise merger between Party *i* and Party *j* is said to be a mutually least-dislocating merger if, for all k, $k \neq i$, $k \neq j$,

$$\text{abs}\,(x_i - (p_i x_i + p_j x_j)/(p_i + p_j)) \leq \text{abs}\,(x_i - (p_i x_i + p_k x_k)/(p_i + p_k))$$

and

$$\text{abs}\,(x_j - (p_i x_i + p_j x_j)/(p_i + p_j)) \leq \text{abs}\,(x_j - (p_k x_k + p_j x_j)/(p_k + p_j)).$$

In other words, a mutually least-dislocating merger is one where there is no other (single) merger partner that Party *i* could join with that would involve a smaller displacement of Party *i* from its original position than it gets when it merges with Party *j*, and there is no other (single) merger partner that Party *j* could join with that would involve a smaller displacement of Party *j* from its original position than it gets in merging with Party *i*. At any given stage of the merger process there may be more than one mutually least-dislocating merger, but (barring knife-edge ties) we can always pick the merger that has the lowest total displacement to get a unique outcome (see Proposition 4 in the online appendix). The Grofman (1982) model can be viewed as treating the party compression process as a kind of sequential dyadic marriage market.

There are a number of technical details required for identifying such a sequence of optimal mutually least-dislocating mergers, but, for simplicity of exposition and space reasons, we have relegated these to the online methodological appendix that contains the statement and proofs of propositions 1–11. The further key result in the appendix that we would emphasize is that an algorithm devised several decades ago for a different (but related) purpose, the Grofman (1982) algorithm for sequential proto-coalition formation, creates pairwise mergers that are both means-preserving and mutually least-dislocating, and thus provides a direct way to operationalize the sequential process of condensing ideologically like-minded parties into clusters with the three key desired properties identified earlier.

For simplicity of exposition, and because all but one of our empirical analyses are based entirely on unidmensional party locations, we present below only the version of that algorithm that applies in one dimension, but the algebra readily generalizes to multi-dimensional party locations.[4] Begin with some set of parties with seats shares, p_i, and locations in unidimensional space at x_{i}. In that model, whenever any two parties, i and j, merge we treat the merged party grouping as located at the centre of gravity of the two, i.e. at a point x^* given by $(p_ix_i + p_jx_j)/(p_i + p_j)$. If this merger occurs, we regard the new party grouping to be of size $p^* = (p_i + p_j)$.

In the standard version of this model, a merger between i and j occurs if and only if the j for which abs $(x_i — (p_ix_i + p_jx_j)/(p_i + p_j))$ is minimized equals the i for which abs$(x_j — (p_ix_i + p_jx_j)/(p_i + p_j))$ is minimized. This allows for multiple mergers to take place during a single stage of the proto-coalition process. In our empirical applications we modify the algorithm slightly to identify only a single merger at each stage, that which mimimizes the total dislocation, so that each step of the algorithm's application reduces the number of parties/party groupings by exactly one. As we move through the process of combining parties we will have two indices of dislocation; one is a raw measure, the other a normalized measure that runs from zero to one (see methodological appendix for details).

In the succeeding empirical section of the article we report the results of our search for mutually least-dislocating mergers using the algorithm. The reader may wish to think of what we are doing as developing a normalized measure that is similar to the squared correlation coefficient in multivariate regression. Here, like the squared correlation coefficient, we have a measure that runs from 0 to 1 indicating degree of fit. However, rather than more variables improving fit, fit improves as we allow for more parties. Here, rather than a best-fitting regression line, for each r, there is a best-fitting party constellation involving exactly r parties/party groupings, i.e. one that can be thought of as 'closest' in seat-share-weighted terms to the original party locations. Rather than the baseline comparison being to the total variance we get when we simply predict all values are at their means, we take as our baseline the maximum dislocation when all parties are located at the same point (the party-seat-share-weighted mean) for the distribution of party locations and shares that maximizes that dislocation. Finally, just as we can fit any dataset perfectly if we have as many variables as there are cases, we can, tautologously, fit the original constellation perfectly if we allow n parties, giving us a goodness-of-fit measure of 1. So the question then becomes: 'How much worse do we do with fewer than n parties?'

Comparing the loss in fit (the change in the normalized index of dislocation) as we reduce the number of party groupings by allowing parties to merge can be thought of as directly analogous to determining in a multivariate regression whether it makes sense to add additional variables to our regression by looking, inter alia, at what happens to the (adjusted) R^2 value of the regression when additional variables are brought in.

The greater the change in dislocation, the less good is the 'fit' of the (further) reduced party configuration to the initial party constellation. Thus, an important issue in our modelling is specifying a stopping rule to determine when the dislocations involved in continuing the process by combining an additional two parties into a merged pseudo-party is 'too great'. We use as our rough rule of thumb that we continue reducing the number of parties as long as the raw index of dislocation is below 0.025, i.e. as long as the normalized index of dislocation is below 0.05.

We recognize that this 0.05 value is arbitrary, but so are conventional levels of statistical significance such as 0.05 or 0.01, or the frequently used restriction to eigenvalues greater than 1 in factor analyses, or the requirement of a 90 percent predictive accuracy in Guttman scaling. However, based on both our hypothetical and our real-world examples, we observe that this level of dislocation has three nice properties. First, it is low enough that the distortion introduced by reducing the party configuration at any given stage does not seem that great in substantive terms. Second, it is high enough that we almost always find ourselves able to generate a simplified representation of the constellation of parties with fewer party groupings than we began with. Third, the threshold we have set is not so high that it always reduces to a two-party configuration. Also, we can make use of the algorithm for any specified threshold, and, as we show in an appendix provided online, the empirical results we present are relatively robust to the exact threshold chosen.

Ideologically cognizable number of parties/party groupings: Applications

Three party examples: Hypothetical unidimensional data

In the general case, we first specify the location of each potential party merger. We then proceed as follows: we show how far away each party to that merger is shifted from the party's original location; we find which party each party wishes to join in terms of being in the merger that results in the least shift for that party; we look for pairwise matches; we look for the optimal pairwise match; and we repeat the process for the new stage 2 (partly) merged party constellations, etc. To help the reader get a sense of how the algorithm works, it is instructive to calculate the index of dislocation for one of the unidimensional examples we used in Figure 1.

Table 1 shows how we make calculations for the hypothetical party configuration previously shown in Figure 1a. First we show where each merged party grouping would locate, then we show the party-weighted displacement of the parties (or party groupings) in that merger from their new location. This gives us a matrix of dislocations. To find the mutually least displacing dislocation we look to find the situation where a cell is both the minimum displacement in its row and the minimum displacement in its column. As discussed in the methodological appendix, the Grofman (1982) algorithm guarantees that there will be at least one cell for which this is true. (If there is more than one instance we pick the cell with the smallest entry.) We can see from Table 1 that the mutually least displacing dislocation – that between parties A and C – has a normalized dislocation score of 0.02 (see the emboldened cell in the second 3×3 matrix). This newly merged party is located at 0.09 and has a seat-share of 0.53. The ideological mean in Figure 1a is

Table 1. Dislocation for the Hypothetical Party Configuration in Figure 1(a)

Stage 1: Locations and seat shares	Party	Location	Seat Share	
	A	0.1	0.47	
	B	0.9	0.47	
	C	0	0.06	
	Party Wtd. Mean	0.47		
Stage 1: Merged Locations	Party	A	B	C
	A	X	0.50	0.09
	B	0.50	X	0.80
	C	0.09	0.80	X
Stage 1: Normalized dislocation	Party	A	B	C
	A	X	0.75	0.02
	B	0.75	X	0.19
	C	0.02	0.19	X
Stage 2: Locations and seat shares	Party	Location	Seat Share	
	A+C	0.09	0.53	
	B	0.9	0.47	
	Party Wtd. Mean	0.47		
Stage 2: Merged Locations	Party	A+C	B	
	A+C	X	0.47	
	B	0.47	X	
Stage 2: Normalized dislocation	Party	A+C	B	
	A+C	X	0.81	
	B	0.81	X	

at 0.47. If all parties were to merge into a single party this would be where they located. The normalized dislocation for that case is 0.81. Under our 0.05 cutoff for normalized dislocation, only the first of these two mergers would take place, and thus only a two-party condensation would result.

For the special case of three parties, however, it is possible to simplify the calculations since, neglecting ties, there can be only one mutually least-dislocating merger in the first stage, and the median party is necessarily a partner party in that merger. Thus, in the three-party case, the optimal merger is the one that involves the minimum displacement for the median party (see Proposition 11 in the online appendix). In the three-party example in Figure 1a the median party is, by definition, sandwiched between the leftmost and rightmost parties, so the merger process we have outlined above must lead initially to a merger with whichever of them is closer to the median party in weighted distance terms. This results in the least dislocating merger being that between parties A and C, with a normalized dislocation value of 0.02, located at their weighted mean of 0.09 and with seat-share of 0.53, as obtained above.

Examples with four or more parties: Real-world data

We now turn to two of the four examples discussed in Dalton (2008), namely Canada 2004 and Slovenia 1996, in order to show how our approach works with real-world data as well as how different the results of our approach can be from either LT or LTB

Table 2. Canadian Paty System Data, 2004 Election

Stage 1: Locations and seat shares	Party	Location	Seat Share		
	NDP	0.34	0.06		
	Bloc -Queb.	0.37	0.18		
	Liberal	0.51	0.44		
	Conservative	0.63	0.32		
	Party Wtd. Mean	0.51			
Stage 1: Merged Locations	Party	NDP	Bloc -Queb.	Liberal	Conservative
	NDP	X	0.36	0.49	0.58
	Bloc -Queb.	0.36	X	0.47	0.54
	Liberal	0.49	0.47	X	0.56
	Conservative	0.58	0.54	0.56	X
Stage 1: Normalized dislocation	Party	NDP	Bloc -Queb.	Liberal	Conservative
	NDP	X	0.005	0.036	0.059
	Bloc -Queb.	0.005	X	0.072	0.120
	Liberal	0.036	0.072	X	0.089
	Conservative	0.059	0.120	0.089	X
Stage 2: Locations and seat shares	Party	Location	Seat Share		
	NDP+B-Q	0.36	0.24		
	Liberal	0.51	0.44		
	Conservative	0.63	0.32		
	Party Wtd. Mean	0.51			
Stage 2: Merged Locations	Party	NDP+B-Q	Liberal	Conservative	
	NDP+B-Q	X	0.46	0.52	
	Liberal	0.46	X	0.56	
	Conservative	0.52	0.56	X	
Stage 2: Normalized dislocation	Party	NDP+B-Q	Liberal	Conservative	
	NDP+B-Q	X	0.092	0.147	
	Liberal	0.092	X	0.089	
	Conservative	0.147	0.089	X	
Two Party: Locations and seat shares	Party	Location	Seat Share		
	NDP+B-Q	0.36	0.24		
	Lib. + Cons.	0.56	0.76		
	Party Wtd. Mean	0.51			

calculations. Owing to space constraints, the cases of Spain 2004 and the Czech Republic 2002, also illustrated in Dalton (2008), are analysed separately and are available online at Grofman's website.[5]

Canada 2004

The basic information about the Canadian election in 2004 (party names, seat-shares and locations) is given in Table 2. Party location data are taken from CSES. Data have been normalized to treat this as a four-party contest involving the parties for whom we have CSES data. Data are normally shown to two significant figures unless we have to report a third significant digit to break a tie. We have converted CSES left–right location to a (0, 1) scale by dividing the original 10-point scale values by 10. The party-weighted mean is 0.51 and this is the location at which we would expect to find a multiparty merger which resulted in a single party. The merger process we are using preserves this value intact at each and every stage. In addition, for each possible round of mergers, Table 2 contains the locations of the merged parties for all possible pairwise mergers and the associated matrix of dislocation containing the normalized dislocation index values for each of these possible pairwise mergers. The emboldened entries highlight the mutually least-dislocating merger partners for each possible round of mergers. At each stage, the mutually least dislocating merger takes place if the associated normalized index of dislocation is less than 0.05. For expository purposes we have included the relevant data for all possible rounds of mutually dislocating mergers up until the party system is condensed to two groups, even if these mergers surpass our threshold of 0.05. In the first round there is only one mutually least-dislocating merger: one between the NDP and its nearest neighbour, the Bloc Quebecois (BQ), since the column minimum for the NDP is in the BQ row, while the column minimum for the BQ is in the NDP row. This results in a merged (NDP+BQ) party grouping located at 0.36, with seat-share 0.24. The raw party-weighted dislocation for this three-party/party grouping is 0.0025. The index of dislocation is simply double this, namely 0.005. Note that, in this configuration, there is as yet no majority party. We continue to use as our rough rule of thumb that we reduce the number of parties until the normalized index of dislocation is greater than 0.05. The shift from four to three easily satisfies that criterion.

In this reduced three-party/party grouping configuration, the Liberals are the median party, so their preferences are determinative as to what merger takes place at the next round. We can see from Table 2 above that the Liberals would have to move 0.089 normalized party-weighted units if they merged with the Conservatives. The location of the new agglomerated two-party grouping would be at 0.56, with seat-share 0.76. If, instead, the Liberals merged with the NDP + BQ party grouping, at the next stage of the process the new grouping would be located at 0.46, with a weight of 0.68. The normalized displacement of this merger would be 0.092. Since the two different merger possibilities are very close to a tie (to two-digit accuracy) in terms of dislocation of the Liberal Party, we might imagine both the merger between the Liberals and the NDP+BQ combination and the Liberal + Conservative merger as nearly equally feasible. However, since the index of dislocation for these mergers is above our threshold value, we would stick with a three-party configuration. Thus, our algorithm, in this case, yields a number of parties, three, which is quite close to the LT index for this configuration, 3.01, as well as the LTB for this configuration, 3.27.

This representation gives us a large and centrally located party, the Liberal Party with two substantially sized parties/party groupings to either side of it ideologically {NDP +

BQ, Conservatives}, and at similar distances from the more central party. The Canadian example thus demonstrates that we can find an optimal party reduction that has more than two parties. If, however, we use the two-dimensional ideological representation of the Canadian party system in Johnston (2008) we again get the liberal party and parties to either side of it, but now the BQ takes up a location of its own in the two-dimensional space, giving us a four-party representation when we apply our algorithm (details omitted for space reasons). However, as Johnston (2008) emphasizes, outside of Quebec, what we find is essentially three-sided competition. In the actual politics of this example, the Liberals were able to form a minority government after the NDP fell one seat short of providing the liberals with a minimal winning coalition.

Slovenia 1996

The basic information about the Slovenian election in 1996 is given in Table 3. Here we have six parties, but we omit the Democratic Party of Retired Persons with a 5.5 percent share of the seats and other smaller parties because we do not have CSES data on their ideological locations. The five parties we have information for control 87.6 percent of the parliamentary seats. We have normalized the seat-shares of the five parties for which we have CSES data so that their normalized seat-shares sum to 1. We have again converted left–right location to a (0, 1) scale by dividing the original CSES 10-point scale values by 10. The mean (party-weighted) is 0.51.

The matrix of dislocations for each of the column parties is given in Table 3 for each possible pairwise merger as we reduce from 5 to 4, 4 to 3 and then from 3 to 2 parties. In the first round there is only one mutually least-dislocating merger: one between the SDP and the party to its immediate right, the CD. The result of this merger is a merged party located at 0.64 with a combined seat-share of 0.33. The normalized index of dislocation for this merger, at 0.012, is well below our threshold value of 0.05, and thus we continue to apply our algorithm. At $r = 4$, our new party grouping is {USLD, LDP, SPP, SDP + CD}.

As we look ahead to the next round of potential mergers, as r is reduced from four to three parties, the optimal mutually least-dislocating merger is that between ULSD and LDP, as indicated in Table 3. This merger has a normalized index of dislocation of 0.017. Thus, at $r = 3$, our party groupings are {USLD+LDP, SPP, SDP+CD}. As r goes to 2, the unique mutually least-dislocating merger is between SPP and SDP+CD, which has an associated normalized index of dislocation of 0.025, i.e. still quite small and still well below our threshold stopping point value.

Because the further reduction to a single bloc produces an unacceptably large dislocation (according to the cutoff parameter we have chosen), we opt for the two-party configuration of {ULSD+LDP, SPP+SDP+CD}. We feel that this is justified because these seem to form two natural party groupings which have a fairly large inter-group distance (0.38–0.62 = 0.24), but little within-group variation. The distance between the original positions of the two parties in the left-party grouping {ULSD+LDP} is only 0.05, and the distance between the two extremal parties' original positions in the right party grouping is only 0.07. In this case, our algorithm, which identifies a two-bloc partition, differs

Table 3. Slovenian Paty System Data, 1996 Election

Stage 1: Locations and seat shares	Party	Location	Seat Share			
	ULSD	0.34	0.11			
	LDP	0.39	0.32			
	SPP	0.59	0.24			
	SDP	0.62	0.20			
	CD	0.66	0.13			
	Party Wtd. Mean	0.51				
Stage 1: Merged Locations	Party	ULSD	LDP	SPP	SDP	CD
	ULSD	X	0.38	0.51	0.52	0.51
	LDP	0.38	X	0.48	0.48	0.47
	SPP	0.51	0.48	X	0.60	0.61
	SDP	0.52	0.48	0.60	X	0.64
	CD	0.51	0.47	0.61	0.64	X
Stage 1: Normalized dislocation	Party	ULSD	LDP	SPP	SDP	CD
	ULSD	X	0.017	0.077	0.082	0.077
	LDP	0.017	X	0.109	0.113	0.098
	SPP	0.077	0.109	X	0.013	0.023
	SDP	0.082	0.113	0.013	X	0.012
	CD	0.077	0.098	0.023	0.012	X
Stage 2: Locations and seat shares	Party	Location	Seat Share			
	ULSD	0.34	0.11			
	LDP	0.39	0.32			
	SPP	0.59	0.24			
	SDP+CD	0.64	0.33			
	Party Wtd. Mean	0.51				
Stage 2: Merged Locations	Party	ULSD	LDP	SPP	SDP+CD	
	ULSD	X	0.38	0.51	0.56	
	LDP	0.38	X	0.48	0.52	
	SPP	0.51	0.48	X	0.62	
	SDP+CD	0.56	0.52	0.62	X	
Stage 2: Normalized dislocation	Party	ULSD	LDP	SPP	SDP+CD	
	ULSD	X	0.017	0.077	0.100	
	LDP	0.017	X	0.109	0.158	
	SPP	0.077	0.109	X	0.025	
	SDP+CD	0.100	0.158	0.025	X	
Stage 3: Locations and seat shares	Party	Location	Seat Share			
	ULSD+LDP	0.38	0.43			
	SPP	0.59	0.24			
	SDP+CD	0.64	0.33			
	Party Wtd. Mean	0.51				
Stage 3: Merged Locations	Party	ULSD+LDP	SPP	SDP+CD		
	ULSD+LDP	X	0.45	0.49		
	SPP	0.45	X	0.62		
	SDP+CD	0.49	0.62	X		

most dramatically from the other approaches. The LT index in this case is 4.39 and the LTB 3.82.

The actual coalition formed after the 1996 election was one between the LDP and the SPP, a merger which is almost certain to contain the median voter. Here, ideological proximity of parties representing 12 percent of the seats that were omitted from our analysis because of lack of data may have been important in shaping coalition considerations to create a majority government.

Discussion

Just as the LT (or LTB) indexes allow us to take into account with precise quantitative measurement the intuition that party size matters in counting how many parties there are, 'really', so our new approach allows us to take ideological location into account directly. In this article, we have demonstrated how ideas about initial party locations and sizes and about the nature of party coalition processes can be used to make judgments about how best to make an estimate of the number of party blocs along the lines first suggested by Sartori (1976), but in a much more precise way. In particular, by looking at party locations and proximities we can make sensible decisions about which party constellations exhibit ideologically fragmented bipolarism, and which require three-bloc or more than three-bloc representations to accurately capture key aspects of the existing party constellation. We have provided an algorithmic method for combining parties into new groupings (located at the weighted ideological mean of the parties that are being joined) in such a way that the ideological structure of party competition is best preserved. By showing how to create an optimal dislocation minimizing reduction of the (unidimensional) space of party competition we have provided a way by which to specify an 'ideological cognizable number of parties' when we take into account both party size and each party's ideological proximity to other parties.

In addition to considering applications of the idea of an ideologically cognizable number of parties to purely hypothetical examples designed to illustrate how the algorithm works, we have applied our methodology to four real-world party systems (Canada, Spain, Slovenia and the Czech Republic) using data on a recent election in each for the set of major parties reported by CSES. Canada is initially treated as a four-party system; its optimal unidimensional reduction is as a three-party system. Spain is treated as a four-party system, but its optimal reduction is as a two-bloc/party system, although a reduction to a three-party system already creates a majority bloc. Slovenia is treated initially as a five-party system, but its optimal reduction is all the way down to a two-bloc/party system, while the Czech Republic – the most fragmented system – was reducible only from a five-party to four-party system. We saw that our approach often gave similar results to the LT and LTB approach in terms of apparently 'equivalent' numbers of parties/party blocs, but we need to recognize that the effective number of parties is not conceptually the same as the number of ideological party blocs. Moreover, in the case of Slovenia, we get very different numerical results. Here, our method captures the fact that some parties in Slovenia are ideologically proximate and thus can be combined with little distortion of the initial distribution. In contrast, Laakso–Taagepera and LTB are attentive only to the fact that three of the Slovenian parties are similar in size, and the remaining two are large enough to make a non-trivial contribution to the calculation of the effective number of parties.

The basic idea of our approach is that we kept reducing the number of parties by one (combining two parties or two-party groupings) until we got to a configuration that is 'too far away' from the original ideological configuration of *n* parties to be satisfactory in representing the ideological features of that initial constellation. A rough rule of thumb we have applied in our discussion of the real-world cases is that the raw dislocation for the new configuration is no greater than 0.025 and thus the normalized dislocation index is not above 0.05. The reader may, perhaps, be concerned about the robustness of that cutoff rule. In an appendix available online we show the results of the mutually least-dislocating merger in each possible stage of reduction for our four cases as we reduce the party groupings to two for each case. What we see from these calculations is that any cutoff in the range from 0.025 to 0.075 would not have changed our conclusions about the cardinality of the party blocs in each of the optimal configurations. Various substantive interpretations of this parameter may be offered, but one natural way to think of it as the degree to which coalitions with other parties have political costs in terms of pulling a party away from its own platform (or, more precisely for the data we reported, away from what voters believe to be its platform).

While Dalton's work was an inspiration for this article, it is useful to contrast the two approaches. Dalton (2008) proposes using ideological polarization as a measure of the ideological structure of party competition. We have integrated party size considerations and ideological considerations of the sort reflected in his polarization measure into one single index. We see our two approaches as complementary. Although Dalton's polarization measure is also based on ideological location, essentially looking at the (normalized) standard deviation of the ideological distribution, our approach differs from his in that we distinguish cases which, in polarization terms, would be identical. In 'counting' the number of ideological 'blocs' we are sensitive to more than variance. Consider for example two different scenarios. In one we have five equally sized parties located at 0.20, 0.40, 0.60, 0.80 and 1. This gives us a standard deviation of 0.32, and if we take the scale as 0 to 1 rather than the 10-point scales used by Dalton (2008) we get a normalized Dalton polarization score of 0.57. In this scenario we have five parties under our measure, and even if we move some party locations very slightly, we would still get a four-party or, at its most reduced level, a three-party scenario. In contrast, consider a scenario with two equally sized parties, located at 0.20 and 0.77. This is clearly a two-party system, and yet it has the same standard deviation (and thus the same polarization score) as our previous example.[6]

In addition to Dalton's study, other recent studies have demonstrated that the ideological dispersion of political parties in a system is an important variable. Alvarez and Nagler (2004) create a measure of party system 'compactness' – essentially party polarization normalized by the dispersion of citizen preferences – which they employ to examine its effects on issue voting. Ezrow (2007) demonstrates a positive correlation between the dispersion of citizen preferences and the dispersion of party positions. In this study, we have provided a framework that can potentially be utilized for understanding the effects of ideological dispersion on assessing the number of underlying party groupings in a system. Given the politically substantive importance of such dispersion (or lack of it), as demonstrated by these studies, we hope that this article inspires further substantive investigations of the effects of party size and party-system dispersion, including more attention to the (coalitional) importance of (small) parties that may not neatly fit the model of unidimensional competition.

With appropriate data, the methodology we have used can be applied to many more countries, just as is true for the Dalton (2008) polarization measure, and it need not be restricted to one-dimensional representations of party space. It is increasingly common in studies of party systems or electoral competition to report a time series for Laakso–Taagepera values at the vote and/or seat level. Our methodology can also be applied to making sense of changes in party constellations over time in a way that is usefully complementary to the more standard approaches of simply counting changes in seat-winning parties or in the effective number of parties. We hope that this article will inspire the development of similar time series on changes in the ideological bloc structure of party competition.

Notes

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1. Also important are game-theory inspired models derived from the Aumann–Maschler bargaining set (1964) and related ideas (see, e.g., Schofield, 1976 and Schofield and Laver, 1987).
2. Available at: http://www.socsci.uci.edu/~bgrofman/.
3. In many ways the approach we take is analogous to techniques for data reduction, such as factor analysis (Harman, 1960) or multidimensional scaling (Poole and Rosenthal, 2007; Romney et al., 1972), which look for ways to accurately capture relationships found in a given dataset using fewer 'dimensions' than in the original data array. It also bears analogy to the notion of a covering relationship in graph theory (Harary et al., 1965), and to techniques in the physical and biological sciences for partitioning spatial or attribute arrays into a fixed number of units (see, e.g., Richter-Gebert et al., 2003; Mitchell, 2003: references for which we are indebted to Roland G. Freyer Jr., Department of Economics, Harvard University); but the method we use is closest in spirit to cluster-theoretic approaches to data reduction found in sociology and in the biological and physical sciences (Romesburg, 1990).
4. The general formulation can be found in Grofman (1982), Straffin and Grofman (1984) and Grofman et al. (1996).
5. Available at: http://www.socsci.uci.edu/~bgrofman/.
6. For further aspects of comparison, see Methological Appendix B to this article, included in the longer version available on line at: http://www.socsci.uci.edu/~bgrofman/.

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Author Biographies

Bernard Grofman received his BS in Mathematics at the University of Chicago in 1966 and his PhD in Political Science at the University of Chicago in 1972. He has been on the faculty of the University of California at Irvine since 1976, and Professor of Political Science since 1980. His research deals with behavioural social choice, including mathematical models of group decision-making, legislative representation, electoral rules and redistricting. Currently, he is working on comparative politics and political economy, with an emphasis on viewing the United States in comparative perspective. He is co-author of four books, all published by Cambridge University Press, and co-editor of twenty-one other books; he has published over 200 research articles and book chapters, including work in the *American Political Science Review*, the *American Journal of Political Science*, the *Journal of Politics*, the *British Journal of Political Science*, *Electoral Studies*, *Legislative Studies Quarterly*, *Social Choice and Welfare* and *Public Choice*. Professor Grofman is a past President of the Public Choice Society. In 2001 he became a Fellow of the American Academy of Arts and in 2008 the Jack W. Peltason (Bren Foundation) Endowed Chair and Director of the UCI Center for the Study of Democracy.

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