Journal of Theoretical Politics 11(1): 57–74
0951-6928[1999/01]11:1; 57–74; 006736

THE POWER OF IDEOLOGICALLY CONCENTRATED MINORITIES

Samuel Merrill, Bernard Grofman, Thomas Brunell and William Koetzle

ABSTRACT

In two-party competition, it is well known that the party whose supporters/identifiers are more ideologically concentrated can pull the overall *median* in the direction of its party median, while the overall *mean* is essentially simply an average of the two party means weighted by the number of supporters/identifiers of each party. Yet the exact nature of the relationship between the overall median and the party-specific medians has never, we believe, been fully explicated. We consider three questions relevant to unidimensional two-party political competition: The first is 'What factors determine the location of the overall median relative to the medians in each political party?' The second is 'What are the factors that determine the location of the overall median relative to the overall mean?' The third, and potentially most interesting, is 'Under what circumstances, if any, will the median of the party with fewer supporters be closer to the overall median than that of the party with a preponderance of supporters?'

For party distributions which sufficiently overlap, we show analytically how (a) degree of party support, and (b) ideological cohesion of each party's supporters trade off with one another to determine the location of the overall median relative to the party medians. In general, if the smaller party is more concentrated ideologically and if the disparity in dispersion between the two parties exceeds the disparity in the number of party identifiers, then the overall median is closer to the median of the smaller party and, ceteris paribus, the smaller party can be expected to win.

KEY WORDS • ideological concentration • median • mixture distribution • spatial model

1. Introduction

In two-party competition in single-member districts, under what conditions should seats be safe or at least be favorable for one party over the other?

One answer, based on the Michigan party ID model (Campbell et al., 1960) is that districts where one party's supporters predominate should,

We are indebted to Dorothy Green for library assistance and to Nicholas Miller, Richard Potthoff, and two anonymous referees for helpful suggestions. Earlier versions of this paper were presented at the annual meetings of the Public Choice Society, San Francisco, 21–23 March, and of the American Political Science Association, Washington DC, 27–31 August, 1997. We also thank the University of Michigan ICPSR for data used in this article.

ceteris paribus, be safe for members of that party, since the Michigan model emphasizes the importance of long-standing and hard to reverse party loyalties. However, the ceteris in the previous sentence may not, in fact, be paribus.

Although there is a considerable body of literature showing a clear connection at the constituency level between party affiliation or party registration and partisan electoral success – at least outside the South (see, e.g., Cain, 1985; Glazer et al., 1987), Republican presidential popular vote victories occurred in the 1950s and 1960s – decades where Democratic identifiers/registrants outnumbered Republicans by at least three to two. Not only are there election-specific electoral tides to deal with, but the degree of party loyalty among partisan voters has been on the decline and the number of voters who identify themselves as independents has been on the rise (see, e.g., Wattenberg, 1991). Also, partisan differences in the likelihood of voting need to be taken into account. Moreover, party identification is, by and large, a better predictor of vote choice for Republican identifiers than for Democratic identifiers (see, e.g., Wattenberg, 1996).[1]

A second answer to the question of which districts should be more favorable to one party is based on the work of Anthony Downs (1957). That answer is that there should be no truly safe seats. In the classic Downsian approach to two-party competition over a single dimension, the ability of candidates to converge toward the views of the median voter should generally lead to competitive politics. However, taking into account institutional realities such as party primaries, the role of party activists, and the existence of simultaneous elections in multiple constituencies complicates this prediction.

A third answer to the question is based on work in the Downsian mode done since the publication of Downs' classic. Work, both theoretical (Coleman, 1971; Aranson and Ordeshook, 1972; Owen and Grofman, 1995) and empirical (Shapiro et al., 1990; Gerber and Morton, 1997), has shown that, although the Downsian centrist pressures are quite real, some degree of party divergence is to be expected. Clearly this divergence result has important implications for the conditions under which districts can be expected to be competitive, since now one party's candidate may be much further away from the district's median voter than the candidate of the other party.

We look at the determinants of competitive seats from a perspective that

1. Indeed, in the South, the 'split-level identification' thesis, which posits that southern voters have come to develop different partisan identifications at the presidential than at the sub-presidential level has been proposed to account for the anomaly of congressional results where Republican presidential candidates win in districts where, ostensibly, Democratic identifiers predominate.

combines ideas from the Michigan approach – in that identifiers of each party are distinguished and party nominees are located at the median positions of their identifiers – with insights from neo-Downsian modeling efforts that look at how institutional features of party competition can create party divergence and that focus on ideology as a determinant of voter choice.[2]

We posit certain stylized facts.

1. In any given constituency, the candidates of each party offer different ideological positions in a one-dimensional left/right spatial model, with the Democrat generally to the left of the Republican (Poole and Rosenthal, 1984; Grofman et al., 1990).
2. The ideological position of the candidates in any constituency is a function not merely of that candidate's party, but also of the ideological characteristics of the voters in that constituency, i.e., not all Democrats are alike, not all Republicans are alike (Grofman et al., 1996). In particular, each party's candidate can be expected to be located near to the median of that party's supporters in the constituency, although perhaps shifted somewhat in the direction of the overall median in the district (Shapiro et al., 1990).[3]
3. The candidate who is located closer to the overall median of the district will, ceteris paribus, be likely to win in the general election (Downs, 1957).

Drawing on these stylized facts, the conditions under which a district should be safe for one party can be examined. We will show:

First, and most obviously, the greater the ratio of supporters of one party to supporters of the other party, the safer, ceteris paribus, the district for the party whose supporters predominate.

Second, and far less obviously, the smaller the ratio of the standard deviation of the distribution of one party's supporters on the ideology dimension relative to that of the other is, the more favorable are the prospects for the former party – provided that there is sufficient overlap among the two distributions. Independent influence on voting by party identification may moderate this effect but not change its nature. For those cases where the ideological distributions of each party's supporters are sufficiently overlapping, and for distributions that are symmetric (e.g., normal), we show that there is a remarkably simple and elegant numerical

2. See Enelow and Hinich (1990), Grofman (1993) and Hinich and Munger (1994), for general reviews of how institutional complexities may be taken into account in spatial models.

3. However, there may also be a pull rightward for Republicans and leftward for Democrats in districts whose constituency party medians are less extreme than their respective national party medians.

approximation that specifies to a considerable degree of accuracy exactly how (a) number of party supporters and (b) ideological cohesion of each party's supporters trade off with one another to determine the location of the overall median voter in the district. The location of this median voter will determine which party median is closer to the overall median, and thus which party is advantaged, ceteris paribus, in any electoral contest.

Third, for distributions that are approximately normal, we provide another simple analytic result which will allow us to compare the location of the actual median to the location of the median that would have arisen had the two parties been equal in variance. The difference between these two values gives us an estimate of the importance of ideological concentration in advantaging the views of the more concentrated party.

The basic outline of the model we present – based on the relative cohesion and relative size of two parties in one-dimensional competition – although independently derived, is not new. Miller (1996) introduces this model and evaluates the qualitative impact of the minority party on collective choice. Ceteris paribus, Miller concludes that '*minority impact on collective choice decreases as majority cohesion increases*' (emphasis in original). Miller, furthermore, emphasizes the critical importance of overlap of the party distributions for these inferences, observing that beyond a critical (but qualitative) threshold, '*minority cohesion has no effect on collective choice*'. The present paper can be regarded as a *quantitative* extension of Miller's insights. In it we determine the *quantitative* effects of party size and party cohesion on the location of the overall median voter, as well as formulas (both precise and rule of thumb) for the specific trade-offs between these two factors.[4]

2. Theoretical Results

Let P_1 be the proportion of Party 1's identifiers and P_2 the proportion of Party 2's identifiers (so that $P_1 + P_2 = 1$) in a one-dimensional spatial model in which voters have symmetric, single-peaked (Euclidean) preferences. Similarly, for $j = 1,2$, let S_j be the standard deviation of the ideological distribution of ideal points of party j's supporters. Let R_S be the ratio of the standard deviation of Party 2 to that of Party 1 (i.e., $R_S = S_2/S_1$),

4. There are, however, some minor differences between our approach and that of Miller. For example, we represent the party medians as the likely locations of the party candidates, whereas Miller speaks of the party medians as the collective choices of the respective parties. Miller also considers polarization (i.e., the distance between the party medians) as an additional variable. Ideological distinctness (i.e., lack of overlap) greatly diminishes the effect of disparity in cohesion, as Miller points out. However, once it is established that sufficient overlap exists, the location of the overall median depends only on the relative dispersion (and size) of the party distributions and not on the polarization.

and let R_P be the corresponding ratio of the numbers of their identifiers (i.e., $R_P = P_2/P_1$). Suppose that each party's supporters are normally[5] distributed, and, without loss of generality, centered at locations 0 and 1, respectively, and that M denotes the position of the overall median voter.

For groups whose ideological distributions overlap, the following result offers a way to calculate the tradeoffs between disparities in proportion of support and disparities in variance:

PROPOSITION 1 (analytic version): *If the ideological distributions of each party's supporters sufficiently overlap so that* $S_1 + S_2 \geq 1$, *and if each party's candidates are located at their respective party medians,*[6] *then, approximately,*

$$M \cong R_P/(R_S + R_P)$$

and ceteris paribus, $R_S < R_P$ *if and only if the median of Party 2 is closer to the overall median in the constituency than the median of Party 1. In this case, Party 2's candidate can be expected to win.*

A proof of this result is given in the Appendix. Although exact error bounds for this approximation are difficult to obtain analytically, they can be evaluated numerically. As long as neither proportion nor standard deviation is more than twice the other (i.e., $\frac{1}{2} \leq R_P \leq 2$ and $\frac{1}{2} \leq R_S \leq 2$) and $S_1 + S_2 \geq 1$, the error incurred in estimating M does not exceed 0.090 and is typically much smaller. If $S_1 + S_2 \geq 2$, the error under the conditions above only reaches 0.022. The error is greatest when the disparities in size and variance are at a maximum with the approximation overestimating the effect of a concentrated minority.

For small values of S_1 and S_2, the approximation no longer works well. Empirically realistic values of S_1 and S_2 for party ideological distributions of a mass public in a two-party polity should be in the range where the approximation does work quite well, as is suggested by the American data we analyze (see the Discussion in the next section).[7]

5. In the Appendix we show that the normal distribution may be replaced by any symmetric distribution whose density has a bounded derivative.

6. For a variety of reasons – such as the desire to win the general election – the locations of party nominees may be pulled inward. If this displacement from the party medians is equal for both parties, then nearness to the overall median is not reversed and the conclusions of the proposition are unaffected. Insofar as party identification affects candidate choice independently of spatial proximity, however, the advantage of a concentrated minority is reduced and may be reversed.

7. Values for S_1 and S_2 for a legislative body, such as the US Congress, may, however, be much smaller. Here any advantage of concentrated minorities may be muted and may in fact be reversed as the overall median is pulled close to the position of the majority party. However, if there is skewness in the distributions then variance effects may still obtain.

Thus, Party 2 can expect to win if $S_2/S_1 < P_2/P_1$ or, equivalently, if $P_1/S_1 < P_2/S_2$. This rule can also be expressed as follows: that candidate can be expected to win for whom the ratio P_j/S_j is larger. For example, suppose that Party 2 is supported by only 40 percent of the electorate, but that this group is more concentrated (with $S_1 = 1.0$ and $S_2 = 0.5$) so that the ratio of standard deviations ($R_S = S_2/S_1$) is 0.5 (see Figure 1, which presents the central portion of a mixed normal probability density). In this case, the ratio of proportions ($R_P = P_2/P_1$) is 0.4/0.6 = 0.667. Accordingly, variation is more disparate than proportion and Party 2 is closer to the overall median (by the approximation formula in Proposition 1, $M = 0.571$; the exact location is 0.553, as given in Table A1) and is expected to win.[8] Intuitively, the more widely spread party on the left loses to the opposing candidate more identifiers in its thick right tail than the more concentrated party on the right loses in its thin left tail.

In fact, under the hypotheses of Proposition 1, the height at the peak of each density is proportional to the ratio P_j/S_j and the following simple visual version of Proposition 1 can be employed.

PROPOSITION 1 (visual version): *Under the hypotheses of Proposition 1, the party with the higher peak density is closer to the overall median and is thus expected to win.*

See the Appendix for the proof. In the example in Figure 1, the smaller party on the right has the higher peak density and is expected to win.[9]

However, when the ideological distributions of the two parties do not overlap substantially because one or both have small standard deviations relative to the difference between their means, the advantage of the more concentrated party relative to the equal variance case drops to zero. In this case, however, as our common sense might suggest would always be true, the party with more supporters will almost always have its median closer to the overall median than the smaller party; and the larger the difference in proportions the greater, ceteris paribus, the difference in proximity to the overall median between the two parties.[10]

Figure 2 depicts the location of the overall median when the electorate is composed of a mixture of two groups with normal party-specific probability

8. The inferences from R_S and R_P are approximate, particularly as disparities increase. Numerical calculations (correct to three decimal places) show that for normal party-specific distributions and $S_1 = 1$, to compensate for, say, a two-to-one deficit in numbers, a standard deviation not just twice smaller but actually 2.38 times smaller is necessary. This does not substantially alter the message of Proposition 1.

9. For real data, peak density would be more sensitive to local fluctuations than the ratio P_j/S_j which integrates information over the entire spatial spectrum.

10. Compare the curve for $R_S = 0.5$ with that for $R_S = 1$ in Figure 2 later in this paper and see the discussion of that figure.

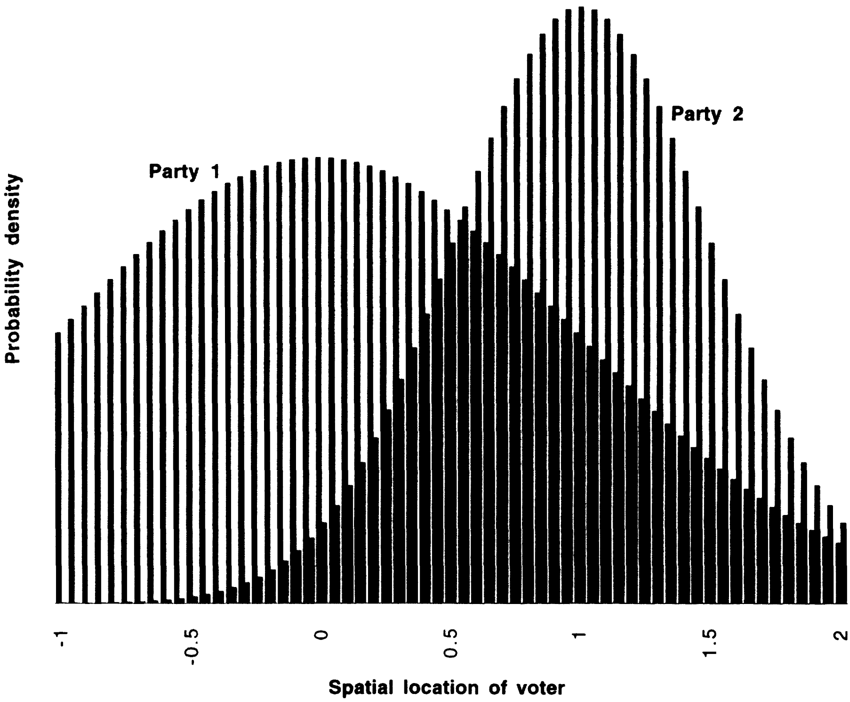

Figure 1. Typical voter distribution: Mixed normal probability density
$P_1 = 0.6$; $P_2 = 0.4$; $S_1 = 1$; $S_2 = 0.5$
In the example pictured, the overall median is at 0.553, substantially to the right of the overall mean at 0.400 (the value of P_2)

densities with respective means at 0 and 1 and which are moderately disparate in size ($P_1 = 0.6$, $P_2 = 0.4$). The figure plots the overall median versus S_1 for several values of the ratio of standard deviations, R_S.

Visual inspection of Figure 2 suggests that, as long as S_1 and S_2 are not small ($S_1 + S_2 \geq 1$), the (ideological) location of the median voter depends almost entirely on the ratios, $R_S = S_2/S_1$, and $R_P = P_2/P_1$, and not on their four constituent factors. Thus, the plots are nearly constant to the right of the dotted line representing $S_1 + S_2 = 1$. Exceptional cases occur only for extreme values of the two ratios, e.g., when one of the distributions is very small and narrow.

Note that the mean (as opposed to the median) of the overall distribution is always at the value P_2, i.e., the mean of the overall distribution simply reflects the relative sizes of the two groups. This follows because, by symmetry of the component distributions, their means are 0 and 1, respectively. The median voter, however, is generally skewed to the right of the overall mean, as long as the ratio R_S is less than 1 (i.e., if the right-hand group is more concentrated). The fact that the overall median and the overall mean are not the same is critical to our intutitive understanding.

Using equation 2 in the Appendix, we can show (after some algebra) that, given our approximation, the overall median exceeds the overall mean if and only if

$$S_2 < S_1,$$

i.e., if the distribution of Party 2's supporters is more concentrated than that for Party 1. In particular, the mean/median relationship depends only on the variance and not on the numbers of supporters.

We have seen that, ceteris paribus, the overall median is drawn toward

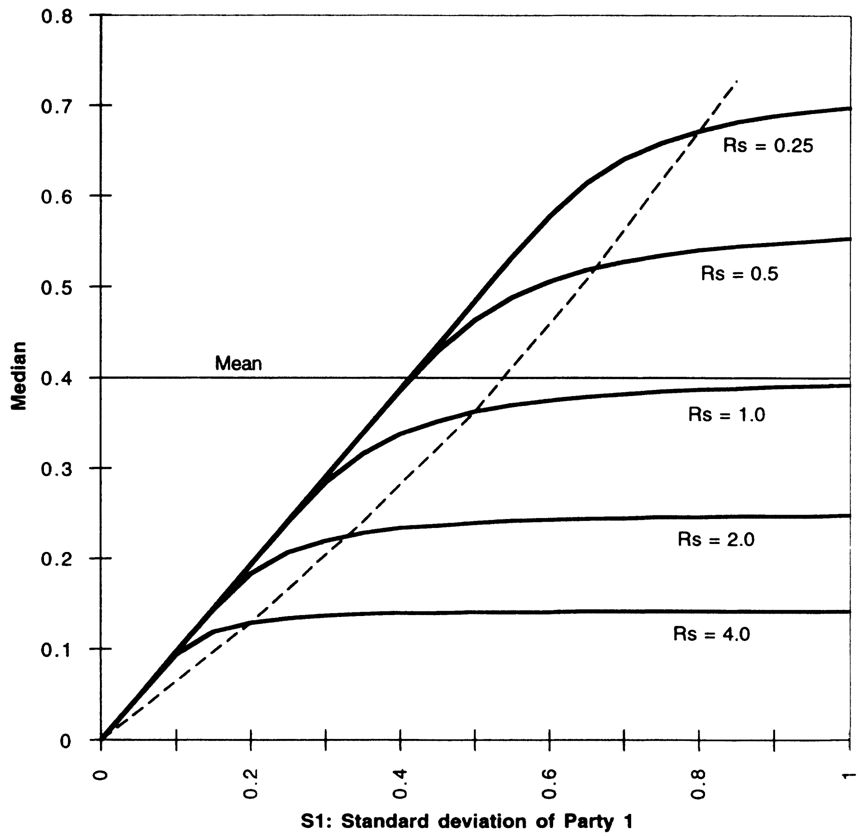

Figure 2. Median versus S_1 for a mixed normal probability density
$P_2 = 0.4$
The region to the right of the dotted line satisfies the condition: $S_1 + S_2 \geq 1$

the party whose supporters have small variance. Table A1 permits us to see this effect quantitatively for specific values of S_1, S_2, and P_2. Our intuition for the extent of this effect would be clearer, however, if we had a simple formula that estimates the amount of movement of the overall median due to disparity in variance. We will show that – under hypotheses similar to those of Proposition 1 – this displacement is approximately one-quarter the deviation of R_S from 1. Thus, for example, under the conditions of the proposition, if the disparity is moderate, say, $R_S = 0.8$, then the overall median is moved by about $(1 - 0.8)/4$, or 0.05 units from its equal variance position. We prove the following result in the Appendix.

PROPOSITION 2. *Fix R_S and R_P and assume that $S_1 + S_2 \geq 1$. Let M be the overall median and M^* the median for the same R_P but for $R_S = 1$, i.e., for equal variances. Then $M - M^*$ is approximated by*

$$\frac{R_P(1 - R_S)}{(R_S + R_P)(1 + R_P)}$$

In turn, if R_S and R_P are not far from 1, we have approximately[11]

$$M - M^* \cong \frac{1}{4}(1 - R_S)$$

3. Discussion

An intuition that motivated this paper was that, in American politics, the Democrats are more of a catch-all party than the Republicans. Thus, the standard deviation of the ideological distribution of Democratic identifiers would usually be higher than that for Republican identifiers. Using survey research data aggregated to the state level,[12] Grofman et al. (1999 forthcoming) have shown that Republican identifiers in the electorate within each state have lower standard deviations of their scores on a conservatism–liberalism scale than their Democratic counterparts in 44 of 50 states. If so, then, ceteris paribus, the Republicans should be advantaged relative to Democrats in state-wide electoral competition, i.e., the Republican position can be expected to be closer to the overall median (some-

11. Intuitively, the reason the formula for the difference in medians is not very sensitive to changes in R_P is that a change in R_P has a similar effect in both numerator and denominator. The difference is approximately linear over the range of interest. These expectations are borne out by exact calculations (not shown). Thus, the equalities in Proposition 2 are good approximations over the range for which it is likely to be applied when we use it for voter distributions that are approximately normal.

12. The American NES three-wave study (1988, 1990, 1992) of elections to the US Senate, which uses states as its sampling units. Each state sample included at least 100 respondents.

times even a lot closer) than we would expect from simply examining the relative numbers of each party's identifiers/supporters. Grofman et al. (1998 forthcoming) provide supporting evidence for this expectation but the facts relevant to the present study are summarized in the following paragraphs.

Following the normalization which places the Democratic and Republican medians at 0 and 1, respectively, values of S_1 range over the 50 states from 0.73 to 2.52 with a median of 1.22, whereas the values for S_2 range from 0.58 to 2.20 with a median of 1.11.[13] Values for the USA as a whole are 1.26 for S_1 and 1.09 for S_2, for a dispersion ratio of $R_S = 0.86$.

The minimum value of $S_1 + S_2$ over the states is 1.31 (for New Hampshire). Thus, the values of $S_1 + S_2$ exceed 1 for all states so that the non-overlapping criterion of Proposition 1 is met. In fact, $S_1 + S_2$ has a median of 2.30 and exceeds 2.0 for all but 12 states (and for the USA as a whole). Coupled with the fact that the values of R_S and R_P are all within the ranges specified in the discussion following Proposition 1 (values of R_S range over states from 0.63 to 1.05 with a median of 0.88; those for R_P range from 0.64 to 1.89 with a median of 1.16), the approximation in Proposition 1 should be accurate to about 0.02 for most states.[14]

Finally, we illustrate graphically for the state-by-state data the relationship between the overall median[15] and each of the three predictors: (a) the dispersion ratio, R_S, (b) the ratio of proportions of identifiers, R_P, and (c)

13. S_1 exceeds 2.0 for five states; S_2 exceeds 2.0 for one state. All of these high values of normalized standard deviation correspond to unusually low values of polarization. Unnormalized standard deviation ranges from 1.09 to 1.89 for Democrats and 0.98 to 1.50 for Republicans. State party distributions were found to be approximately normal. Tables of these parameters are given in Grofman et al. (1999 forthcoming). For consistency in evaluating the accuracy of the approximation in Proposition 1, we compute all statistics in the present paper based on the data set of respondents who place themselves on the seven-point liberal/conservative scale of the NES. This skews the number of identifiers toward the Republicans (converting a minority into a majority for the US as a whole) because a distinct majority of the respondents who do not place themselves on the scale identify as Democrats. In Grofman et al. (1999 forthcoming) we relax this restriction on respondents in computing P_1 and P_2 in order to provide more politically realistic results. Thus, for example, using the more realistic full data set to determine the ratio of identifiers for the entire US (for which R_P is 0.95), the overall median for the US predicted by Proposition 1 is .525, slightly closer to the median of the smaller, Republican party.

14. If the state party standard deviations in this survey are considered as a sample of possible standard deviations for the respective parties, a paired t-test can be applied and shows that the mean of $S_1 - S_2$ exceeds 0 ($p = .0001$), i.e., Democratic dispersion tends to exceed Republican dispersion.

15. In defining the party medians, we have omitted the 'pure independents', i.e., those respondents who identified themselves as independents and did not admit to leaning to either party. These respondents are also omitted in the calculation of the overall median. In most cases, the median of the pure independents is close to that of the two-party median.

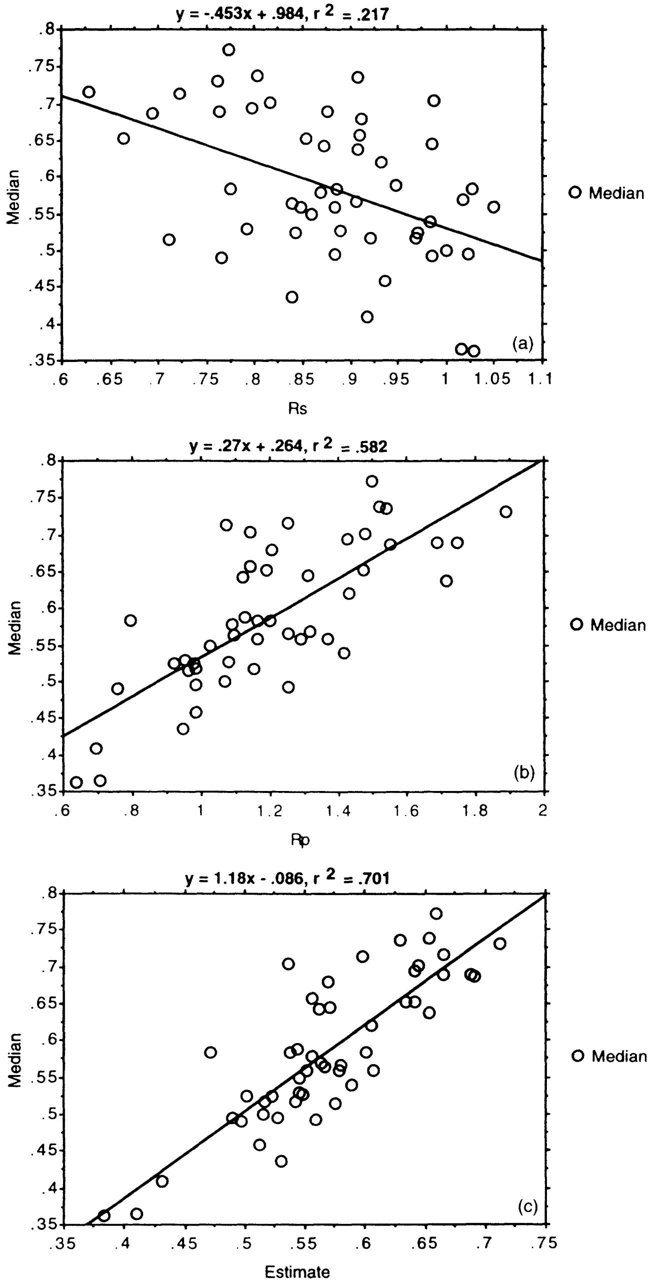

Figure 3. Plots of overall medians versus predictors, by state: (a) overall median versus dispersion ratio; (b) overall median versus ratio of numbers of identifiers; (c) overall median versus estimate of median from Proposition 1

our estimate for the overall median, $R_P/(R_S + R_P)$.[16] Note that the proportion of variance explained by the latter formula (0.701) is substantially greater than that of either ratio alone ($p = .0001$ in each case).

In this paper we have sought to provide new insights into the links between party medians and variances and the overall median of the distribution consisting of identifiers of two major parties. Our theoretical results have direct practical implications for two-party electoral competition. If the candidates of each party locate themselves at (or close to) their party's median, the conditions under which a district should be favorable (or even safe) for one party follow straightforwardly from our results. We may tabulate the proportion of the voters that are closer to, say, Party 2 as functions of P_2, R_S, and S_1, as is done in Table A2 in the Appendix. If the candidates of each party locate themselves at (or close to) their party's median, this proportion can be thought of as indicating which party is likely to win and also by how much.

APPENDIX: MATHEMATICAL DERIVATIONS

General Case: Expressing the Overall Median, M, as a Function of R_P, S_1, and S_2

If the party-specific distributions are uniform and overlapping, say, Party 1 on the interval $[-a, a]$ and Party 2 on the interval $[1 - b, 1 + b]$, and if M lies within the overlap, then the formula $M \cong R_P/(R_S + R_P)$ of Proposition 1 is exact. To see this, note that by the definition of a median,

$$\tfrac{1}{2} = \frac{P_1}{2a}(a + M) + \frac{P_2}{b}(m - 1 + b)$$

The formula follows by solving for M. A uniform distribution, however – with its sharp jumps on the left and on the right – is an unlikely model for the distribution of a party's identifiers.

We next derive a general relationship between R_P, S_1 and S_2, from which the proof of Proposition 1 will follow. Let the cumulative distribution functions of the two components be denoted by F_1 and F_2 and assume only that each is continuous, symmetric, and that they differ from each other with respect to location (0 or 1) and scale (S_1 or S_2) parameters but in no other way. For example, both could be normal or both symmetric triangular, or some other symmetric distribution.

16. The standard error of the estimate (root mean square) when the overall median is regressed on our estimate (as in Figure 3) is 0.055. Simulation analysis suggests that much of this error (about 0.04) is due to statistical variation inherent in the small state samples from which both medians and standard deviations are estimated.

LEMMA 1. *The overall median, M, satisfies*

$$R_P = \frac{\Psi\left(\frac{M}{S_1}\right) - \frac{1}{2}}{\Psi\left(\frac{1 - M}{S_2}\right) - \frac{1}{2}}, \tag{1}$$

where Ψ *denotes the common standardized form of* F_1 *and* F_2.

Proof. The overall median, M, must satisfy:

$$P_1F_1(M) + P_2F_2(M) = 0.5$$

so that

$$P_1\,[F_1(M) - \tfrac{1}{2}] + P_2\,[F_2(M) - \tfrac{1}{2}] = 0$$

Writing

$$F_i(M) = \Psi\left(\frac{M - \mu_i}{S_i}\right)$$

we have

$$P_1\,[\Psi\left(\frac{M}{S_1}\right) - \tfrac{1}{2}] = P_2\,[\Psi\left(\frac{1 - M}{S_2}\right) - \tfrac{1}{2}],$$

which simplifies to equation (1). □

If $P_1 = P_2 = 0.5$, this expression simplifies further so that

$$M = \frac{S_1}{S_l + S_2} = \frac{1}{1 + R_S}$$

where $R_S = S_2/S_1$ as before. Thus, if $R_S < 1$, then $M > 0.5$; if $R_S > 1$, then $M < 0.5$.

Expressing the Overall Median, M, as an Approximate Function of R_P and R_S

To gain insight about the relationships involved, a simple approximate formula is useful. We apply a Taylor series approximation to equation 1 to derive the following (approximate) formula for the overall median. Let ψ be the common standardized probability density for the party-specific distributions.

PROPOSITION 1 (ANALYTIC VERSION). *If the density* ψ *is symmetric and has bounded derivative,* $\psi(0) > 0$, *and* $S_1 + S_2 \geqslant 1$, *then, ceteris paribus, the following approximation holds:*

$$M \cong \frac{R_P}{R_S + R_P}. \tag{2}$$

Proof. Applying a first-order Taylor approximation to the standardized cumulative distribution Ψ, we obtain

$$\Psi(x) \cong \Psi(0) + x\Psi'(0) = \tfrac{1}{2} + x\Psi'(0)$$

Since $\Psi'(0) = \psi(0)$, equation 1 becomes

$$R_P \cong \frac{(M/S_1)\ \psi(0)}{([1 - M]/S_2)\ \psi(0)} = \frac{M/S_1}{[1 - M]/S_2}$$

so that, solving for M, we obtain equation 2.[17] □

COROLLARY 1. *Under the conditions of Proposition 1 and contingent on its approximation, the median of Party 2 will be closer to the overall median in the constituency than will the median of Party 1 if and only if* $R_S < R_P$.

Proof. By formula 2, $R_S < R_P$ if and only if $M > 0.5$.

Well-known distributions such as the normal and logistic satisfy the hypotheses of the Proposition. For normal component distributions, approximate values based on equation 2 for M – as a function of P_2, R_S, and S_1 – are presented in Table A1 along with numerically calculated values accurate to three places for comparison.

Table A2 views this same information for a mixed normal distribution from a slightly different perspective. In it – using numerically calculated values accurate to three places – we tabulate the proportion of the voters that are closer to Party 2 as functions of P_2, R_S, and S_1. When the candidates of each party locate themselves at the party median, this proportion can be thought of as indicating not only which party is likely to win but also by how much.

Proof of Proposition 1 (VISUAL VERSION). The component for the voters supporting party j of the mixed normal probability density is given by

$$f(x) = P_j \left\{ \frac{1}{S_j\sqrt{2\pi}} \exp\left[-(x - M_j)^2/2S_j^2\right] \right\}$$

where $M_1 = 0$ and $M_2 = 1$. Hence the height of each component of the density curve at its peak is

17. We thank an anonymous referee for simplifying and generalizing this proof. Previous work on mixtures of normal (or related) distributions has focused on sample estimates of the mean and other moments (see Cohen, 1967; Ord, 1972: 76–8; Titterington et al., 1985; Johnson et al., 1994); on the distribution of the sample median (see Patel and Read, 1996: 269–73); or on the numerical computation of the median of a mixture by an iterative process (see Al-Hussaini and Osman, 1997). We look instead at the relation between the median and the variances and weighting parameters of the components of the mixture.

Table A1. Location of Overall Median for Mixed Normal Distribution: (a) Locations Exact to Three Decimal Points, Based on Numerical Calculations and (b) Errors Incurred by Using the Approximation Formula (Proposition 1) for Overall Median[a]

		$R_S = S_2/S_1$				
S_1	P_2	.25	.5	1	2	4
(a) Locations exact to three decimal points based on numerical calcuations						
1.00	.7	.913	.837	.714	.549	.373
1.00	.6	.866	.760	.608	.433	.274
1.00	.5	.800	.667	.500	.333	.200
1.00	.4	.698	.553	.392	.248	.142
1.00	.3	.529	.418	.286	.173	.096
.75	.7	.919	.847	.725	.557	.376
.75	.6	.872	.769	.615	.436	.275
.75	.5	.800	.667	.500	.333	.200
.75	.4	.659	.534	.385	.246	.142
.75	.3	.423	.379	.275	.171	.096
.50	.7	.934	.871	.757	.582	.387
.50	.6	.891	.793	.637	.447	.279
.50	.5	.800	.667	.500	.333	.200
.50	.4	.484	.463	.363	.240	.141
.50	.3	.283	.282	.243	.163	.094
(b) Errors incurred by using the approximation formula (Proposition 1) for overall median[a]						
1.00	.7	.010	.013	.014	.011	.005
1.00	.6	.009	.010	.008	.004	.001
1.00	.5	.000	.000	.000	.000	.000
1.00	.4	−.029	−.018	−.008	−.002	−.001
1.00	.3	−.103	−.044	−.014	−.003	−.001
.75	.7		.023	.025	.019	.008
.75	.6		.019	.015	.007	.002
.75	.5		.000	.000	.000	.000
.75	.4		−.037	−.015	−.004	−.001
.75	.3		−.083	−.025	−.005	−.001
.50	.7			.057	.044	.019
.50	.6			.037	.018	.006
.50	.5			.000	.000	.000
.50	.4			−.037	−.010	−.002
.50	.3			−.057	−.013	−.003

[a] Cell entries in Table A1(b) are the difference between numerically correct values and approximate values. Blank cells correspond to conditions that do not satisfy the hypotheses of Proposition 1.

Table A2. Proportion of Voters Closer to Party 2 for Mixed Normal Distribution

		$R_S = S_2/S_1$				
S_1	P_2	.25	.5	1	2	4
1.00	.3	.509	.468	.423	.396	.381
1.00	.4	.576	.522	.462	.425	.405
1.00	.5	.643	.575	.500	.454	.429
1.00	.6	.710	.628	.538	.483	.453
1.00	.7	.777	.682	.577	.512	.477
.75	.3	.457	.409	.371	.349	.338
.75	.4	.525	.461	.410	.381	.366
.75	.5	.593	.513	.449	.413	.395
.75	.6	.661	.565	.489	.446	.423
.75	.7	.729	.617	.528	.478	.452
.50	.3	.363	.318	.291	.276	.269
.50	.4	.432	.372	.335	.315	.305
.50	.5	.500	.425	.379	.354	.342
.50	.6	.568	.478	.423	.393	.378
.50	.7	.637	.532	.467	.432	.415

$$f(x) = \frac{P_j}{S_j\sqrt{2\pi}}$$

which is proportional to the ratio $\frac{P_j}{S_j}$.

Proof of Proposition 2. By equation (2),

$$M - M^* \cong \frac{R_P}{R_S + R_P} - \frac{R_P}{1 + R_p} = \frac{R_P\,(1 - R_S)}{(R_S + R_P)(1 + R_P)}$$

Now, write $R_P = 1 - \epsilon$ and $R_S = 1 - \delta$ where both ϵ and δ are relatively small (as would be expected in applications). Then

$$M - M^* \cong \frac{R_P\,(1 - R_S)}{(R_S + R_P)(1 + R_P)} = \frac{(1 - \epsilon)(1 - R_S)}{(2 - \epsilon - \delta)(2 - \epsilon)}$$

$$\cong \frac{(1 - \epsilon)(1 - R_S)}{4(1 - \epsilon - \delta/2)} \cong \frac{1}{4}\,(1 + \delta/2)(1 - R_S)$$

where the approximations are obtained by discarding second-order terms. Finally, noting that the last expression is equal to $\delta/4 + \delta^2/8$ and dropping the second-order term, we have a second approximation, $M - M^* = \frac{1}{4}\,(1 - R_S)$.

REFERENCES

Al-Hussaini, Essam K. and Magued I. Osman (1997) 'On the Median of a Finite Mixture,' *Journal of Statistical Computation and Simulation* 58: 121–44.

Aranson, Peter and Peter C. Ordeshook (1972) 'Spatial Strategy for Sequential Elections', in R. Niemi and H. Weisberg (eds) *Probability Models of Collective Decision Making*. Columbus, OH: Merrill.

Cain, Bruce (1985) 'Assessing the Partisan Effects of Redistricting', *American Political Science Review* 79: 320–33.

Campbell, D. E., P. E. Converse, W. E. Miller, and D. E. Stokes (1960) *The American Voter*. New York: Wiley.

Cohen, A. C. (1967) 'Estimates in Mixtures of Two Normal Distributions', *Technometrics* 9: 15–28.

Coleman, James S. (1971) 'Internal Processes Governing Party Positions in Elections', *Public Choice* 11: 35–60.

Downs, Anthony (1957) *An Economic Theory of Democracy*. New York: Harper and Row.

Enelow, James and Melvin J. Hinich (eds) (1990) *Advances in the Spatial Theory of Voting*. New York and London: Cambridge University Press.

Gerber, Elizabeth R. and Rebecca B. Morton (1997) 'Primary Election Systems and Policy Divergence', Unpublished manuscript, Department of Political Science, University of California, San Diego.

Glazer, Amihai, Bernard Grofman and Marc Robbins (1987) 'Partisan and Incumbency Effects of 1970s Congressional Redistricting', *American Journal of Political Science* 30: 680–701.

Grofman, Bernard (ed.) (1993) *Information, Participation and Choice: An 'Economic Theory of Democracy' in Perspective*. Ann Arbor, MI: University of Michigan Press.

Grofman, Bernard, Robert Griffin and Amihai Glazer (1990) 'Identical Geography, Different Party: A Natural Experiment on the Magnitude of Party Differences in the U.S. Senate, 1960–84', in R. J. Johnston, F. M. Shelley and P. J. Taylor (eds) *Developments in Electoral Geography*, pp. 207–17. London: Routledge.

Grofman, Bernard, Michael McDonald, William Koetzle and Thomas Brunell (1996) 'Split-ticket Voting and Divided Government', Paper presented at the Conference on Strategy and Politics, Center for the Study of Collective Choice, University of Maryland, College Park, MD, 14 April.

Grofman, Bernard, Samuel Merrill, William Koetzle and Thomas Brunell (1999 forthcoming) 'The Potential Electoral Disadvantages of a Catch-All Party: With an Application to Ideological Variance among Republican and Democratic Identifiers in the 50 States', *Party Politics*.

Hinich, Melvin J. and Michael C. Munger (1994) *Ideology and the Theory of Political Choice*. Ann Arbor, MI: University of Michigan Press.

Johnson, Norman, Samuel Kotz, and N. Balakrishnan (1994) *Continuous Univariate Distributions, Vol. I*, 2nd edn. New York: John Wiley.

Miller, Nicholas (1996) 'Majority Rule and Minority Interests', in Ian Shapiro and Russell Hardin (eds) *Political Order Nomos XXXVIII*, pp. 207–50. New York: New York University Press.

Owen, Guillermo and Bernard Grofman (1995) 'Expressive Voting and Equilibrium in Two-stage Electoral Competition Involving Both Primaries and a General Election'. Paper prepared for delivery at the Annual Meeting of the Public Choice Society, Long Beach, California, 24–26 March 1995. (A revised version presented at the Conference on Strategy and Politics, Center for the Study of Collective Choice, University of Maryland, College Park, MD, 14 April 1996.)

Ord, J. K. (1972) *Families of Frequency Distributions*. New York: Hafner.

Patel, J. K. and C. B. Read (1996) *Handbook of the Normal Distribution*, Second Edition. New York: Marcel Dekker.

Poole, Keith T. and Howard Rosenthal (1984) 'The Polarization of American Politics', *Journal of Politics* 46: 1061–79.

Shapiro, Catherine R., David W. Brady, Richard A. Brody, John A. Ferejohn (1990) 'Linking Constituency Opinion and Senate Voting Scores: A Hybrid Explanation', *Legislative Studies Quarterly* 15: 599–623.

Titterington, D. M., A. F. M. Smith and U. E. Makov (1985) *Statistical Analysis of Finite Mixture Distributions*. New York: John Wiley.

Wattenberg, Martin P. (1991) *The Rise of Candidate-Centered Politics*. Cambridge, MA: Harvard University Press.

Wattenberg, Martin P. (1996) *Decline of American Political Parties 1952–92*. Cambridge, MA: Harvard University Press.

SAMUEL MERRILL, III, is Professor of Mathematics and Computer Science at Wilkes University, Pennsylvania. He received a PhD in Mathematics from Yale University. His current research involves mathematical and statistical modeling, particularly in political science. He is the author of *Making Multicandidate Elections More Democratic* (1988: Princeton University Press) and has published in a number of journals including the *American Political Science Review*, the *American Journal of Political Science*, *Public Choice*, and the *Journal of the American Statistical Association*. ADDRESS: Department of Mathematics and Computer Science, Wilkes University, Wilkes-Barre, PA 18766, USA. [email: smerrill@wilkes.edu]

BERNARD GROFMAN is Professor of Political Science and Social Psychology at the University of California, Irvine. He is a specialist in the theory of representation. His major fields of interest are in American politics, comparative election systems, and social choice theory. He is co-author, with Lisa Handley and Richard N. Niemi, of *Minority Representation and the Quest for Voting Equality*, Cambridge, 1992; and he has also edited a number of books including *Information, Participation & Choice: An Economic Theory of Democracy in Perspective*, Michigan, 1995; *Quiet Revolution in the South: The Impact of the Voting Rights Act, 1965–1990* (co-edited with Chandler Davidson), Princeton, 1994. ADDRESS: School of Social Sciences, University of California, Irvine 92697, USA.

THOMAS L. BRUNELL received his PhD from the Department of Politics & Society, University of California, Irvine. He is a 1998–99 American Political Science Association Congressional Fellow. His areas of interest include congressional elections, electoral systems, political parties, and European integration. His research has appeared in the *American Political Science Review*, *Journal of Politics*, *Legislative Studies Quarterly*, and *Electoral Studies*. ADDRESS: School of Social Sciences, University of California, Irvine 92697, USA.

WILLIAM KOETZLE is a 1997–98 APSA Congressional Fellow. He received his PhD from the University of California, Irvine in 1997. His research has appeared in the *Journal of Politics*, *Legislative Studies Quarterly*, *Party Politics*, *The Harvard International Journal of Press/Politics*, and *PS*. ADDRESS: School of Social Sciences, University of California, Irvine 92697, USA.

Paper submitted 23 July 1997; accepted for publication 28 March 1998.