



An Integrated Perspective on the Three Potential Sources of Partisan Bias: Malapportionment, Turnout Differences, and the Geographic Distribution of Party Vote Shares

Bernard Grofman, William Koetzle and Thomas Brunell

Department of Politics, School of Social Sciences, University of California, Irvine, CA 92697, USA

Partisan bias refers to an asymmetry in the way party vote share is translated into seats, i.e., a situation where some parties are able to win a given share of seats with a lesser (share of the) vote than is true for other parties. Any districted system is potentially subject to partisan biases. We show that there are three potential sources of partisan bias: (1) differences in the nature of the vote shares of the winning candidates of different parties that give rise to differences in the proportion of each party's votes that come to be 'wasted'—differences which arise because of the nature of the geographic distribution of partisan support; (2) turnout rate differences across districts that are linked to the partisan vote shares in those districts, such that certain parties are more likely to have 'cheap seats' *vis-à-vis* turnout; and (3) malapportionment. In the context of two-party competition over single-member districts we provide a simple formulation to calculate the *independent* effect of each of these three factors. We illustrate our analysis with a calculation of the magnitude and direction of effects of the three determinants of partisan bias in elections to the US House and the US Senate in 1984, 1986 and 1988; then we consider how to extend the approach to a system with a mix of single- and multi-member districts or to a weighted voting system such as the US electoral college. We then apply the method to calculate the nature and sources of partisan bias in the 1984 and 1988 US presidential elections. © 1997 Elsevier Science Ltd. All rights reserved.

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In two-party political competition, there are two basic measures of the characteristics of a seats–votes curve showing the relationship between a party's vote share and its (expected) share of the seats: *partisan bias* and *swing ratio* (Tufte, 1973): The swing ratio, often denoted β , is a measure of the responsiveness of the electoral system to change in the vote. In two-party competition, the *swing ratio* is taken to be the expected size of the percent point increase in seat-share for each percentage point increase in a party's share of the aggregate vote above

50 per cent, i.e., swing is analogous to a tangent to the seats–votes curve (Tufté, 1973).¹ Partisan bias can be thought of as the (expected) advantage/disadvantage in seat-share above/below 50 per cent received by a given party that wins 50 per cent of the vote.² In two-party competition, *partisan bias* is customarily taken to be the difference between the seat-share a given party with exactly 50 per cent of the vote can expect to win and the seat-share that it should win if both parties were treated equally by the electoral rules, i.e., a seat share of 50 per cent (Tufté, 1973).

It is well known (Gudgin and Taylor, 1979; Johnston, 1981; Brady and Grofman, 1991b) that, in two-party competition, swing ratio is largely a function of the number of competitive districts. Similarly, it is well known that partisan bias is also, at least in part, a function of the asymmetry in the distribution of partisan voting strength across constituencies (Gudgin and Taylor, 1979; Johnston, 1981; Taylor *et al.*, 1986; Brady and Grofman, 1991b). In particular, if one party wins most of its seats by disproportionately large vote shares and loses most of the seats it loses by relatively narrow vote shares, while the reverse is true for the other party (or parties), then partisan bias exists against the first party. Such bias may have been caused by intentional gerrymandering or by an ‘accident’ of geography. Any districted system is potentially subject to partisan biases.³

The focus of this paper is on the determinants of partisan bias in two-party systems. The partisan bias that arises because of differences in the distribution of party voting strength across constituencies that creates differences between each party’s share of ‘wasted votes’ is only one of the three basic ways in which an electoral system may manifest partisan bias. The other two ways to create partisan bias are (a) through malapportionment, i.e., differences in population across districts (e.g., Baker, 1955; Rydon, 1968; May, 1974; Yamakawa, 1984; Jackman, 1994),⁴ and (b) through differences in turnout rates across districts (Campbell, 1996).⁵ However, neither malapportionment nor unequal turnout, *per se*, generate partisan bias; it is only when population or turnout differences across districts are linked to the distribution of party voting strength that we get partisan bias. While this fact is well known in the electoral systems literature (e.g., Jackman, 1994, Rydon, 1968), in discussions of partisan bias in the United States it is still customary to focus primarily (if not exclusively) on the distributional causes of partisan bias. While this is not that unreasonable in the case of the US House elections since the one-person, one-vote revolution, it does not make sense for other types of analysis, e.g., for analyzing partisan bias in the US Senate or in the US electoral college. Moreover, while population in US House districts is now almost perfectly equal within states, it is often forgotten that, across states, there can be dramatic differences in average House district size. In the 1990s apportionment, for example, the largest district in the United States had 1.7 times the population in the smallest (Grofman, 1992). Thus, despite the one-person, one-vote standard it is still quite reasonable to imagine that there might be a partisan bias in the US House due to malapportionment.

While distributional effects, malapportionment effects and turnout effects are not, in general, mutually exclusive, we can conceptually separate them in the following way by imagining three ideal types: In the first, all districts are equally populated⁶ and the same proportion of voters turn out in each (or, at least constituency population and turnout are uncorrelated with the distribution of party voting strength at the constituency level), but the distribution of voting strength across districts is such that one party’s victories are costlier than the others in terms of winning its seats by larger vote shares, on the average. In the second, all districts are equally populated (or, at least district population is uncorrelated with distribution of party voting strength at the constituency level) and the distribution of mean partisan voting strength across

districts does not generate any partisan bias, but one party's voters do tend to turn out at a lower level than do voters of the other party. In the third case, while the distribution of mean partisan voting strength across districts does not generate any partisan bias, and each party's voters tend to turn out at the same rate as do voters of the other party (or, at least, turnout is uncorrelated with distribution of party voting strength at the constituency level), now districts are not equally populated and the differences in population across districts is related to the partisan distribution of voting strength. We may think of these three examples as giving rise to pure forms of distributional, turnout and malapportionment-based partisan bias.

We may illustrate the first case, partisan bias in a legislature with equally populated districts and with identical turnout rates in each district, using a five-seat legislature. Imagine that there are two parties, Ds and Rs. Ds win two of the five seats, 100,000 to 50,000 each, and the Rs win three of the five seats, 80,000 to 70,000. Now, the Ds win their seats by a 2:1 ratio, while the Rs win theirs by only an 8:7 ratio. Clearly, the Rs are advantaged by this discrepancy in the average seat shares of the winning candidates of their party and those of the Ds. Indeed, in this example, the Ds get only 40 per cent of the seats even though they receive 54.7 per cent of the vote. Here, partisan bias is caused solely by the nature of the *distribution* of partisan voting strength across constituencies.

An illustration of the second case is based on turnout discrepancies across seven equally populated districts. We might imagine that the Ds win every seat they win by, say, 60,000 to 30,000; while the Rs win every seat they win by 80,000 to 40,000, i.e., turnout is higher in the areas where Rs do best, but the vote shares of all winners is the same, namely 2:1. If the Ds win four House seats while the Rs win three House seats, the Ds will have picked up their four seats with a total of 360,000 votes nationally, while the Rs will have picked up three seats with a total of 360,000 votes. Here, partisan bias in House outcomes is attributable to *differences in turnout rates* that act to favor the Ds.

An illustration of the third case is a five-constituency legislature with constituencies D and E exactly twice as populous as districts A, B, and C. Imagine that the Rs regularly win in A, B and C with 53.3 per cent of the vote (80,000 to 70,000) while the Ds regularly win in districts D and E with 53.3 per cent of the vote (160,000 to 140,000). Here the winner's average victory margin is uncorrelated with partisan vote share, and the turnout rate is the same in all districts. The Rs have 60 per cent of the seats in the legislature, even though their legislative candidates win only 520,000 votes, while those of the Ds win 530,000. Here, partisan bias is due simply to *malapportionment*.

The fact that there are three distinct sources for partisan bias that are not mutually exclusive gives rise to an important theoretical question in the study of electoral systems, namely "How can we develop an integrated theory of partisan bias that takes into account all three sources of such bias?" A number of authors have incorporated two of these three factors into a single model in a fashion that allows different effects to be separately estimated (see, especially, Gudgin and Taylor, 1979; Johnston, 1981; Taylor *et al.*, 1986; Jackman, 1994; Lee and Oppenheimer, 1997) but, as far as we are aware, no treatment exists that encompasses all three factors in this fashion. Our aim in this paper is to develop analytic tools to provide precise measurement of the *independent* impact of each of these three sources of partisan bias. Although developed independently, the approach we take is very similar to that in Jackman (1994).

Some notation is necessary to present our key results. We have deliberately chosen to separately represent raw votes (denoted by v 's) and vote shares (denoted by p 's). This makes our notation distinct from both that of Gelman and King (1994a) and Taagepera and Shugart

(1989). Also, although in this paper we present data analysis only for the case where there are two parties, we have expressed our results in a form that can be made applicable to the case where there are n parties competing. This makes for a more cumbersome notation but it also makes it easier to see how our results might generalize beyond the two-party case.

Let S be the size of the legislature, and N the number of separate constituencies.

We shall look initially only at legislatures all of whose members are elected from single-member districts, i.e., legislatures for which $S = N$.

Let s_{ij} be the number of seats won by party i in the j th district. Let S_i be the number of seats won by party i nationally,⁷ i.e.,

$$S_i = \sum_j s_{ij}$$

Let v_{ij} be the number of votes won by party i in the j th district. Let V_i be the number of votes won (across all constituencies) by all candidates of party i , i.e.,

$$V_i = \sum_j v_{ij}$$

Let V be the total number of votes cast for legislative office, i.e.,

$$V = \sum_i V_i$$

Let p_{ij} be the proportion of votes won by party i in the j th district, i.e.,

$$p_{ij} = v_{ij} / \sum_i v_{ij}$$

If we have a two-party system, then i takes on values from the set $\{1, 2\}$.

Let P_i be the *average* proportion of the (two-party) vote (across districts) received by party i , i.e.,

$$P_i = \left(\sum_j p_{ij} \right) / S$$

Let R_i be party i 's share of the *total* national *raw* vote, i.e., party i 's share of the total votes won by that party's candidates across all the districts, i.e.,

$$R_i = V_i/V$$

Measuring the Distributional Element in Partisan Bias

The first source of partisan bias we wish to examine is that which springs from the nature of the distribution of partisan voting strength across constituencies. Such distributional differences may arise by the chance effects of geography or through intentional gerrymandering (e.g., Gudgin and Taylor, 1979; Johnston, 1981; Cain, 1985; Owen and Grofman, 1988).

All methods of calculating partisan bias have in common the need to specify each party's national share of the (two-party) vote as a baseline for calculating a seats–votes relationship from which bias is to be estimated. It is important to recognize that even though both P_i (party i 's vote share in each constituency averaged across all constituencies) and R_i (party i 's raw share of the total vote) can legitimately be regarded as party i 's national vote share, these two estimates of national party vote share are unlikely to be identical because they measure two different things. One, R_i , is based on *raw total votes*; the other, P_i , is based on *average vote shares at the district level*. Only if the district level turnout is totally uncorrelated with the distribution of party voting strength across constituencies (a special case of which would be that in which turnout levels are constant across all constituencies) will $R_i = P_i$. But we know that in the United States, for example, Democratic seats tend to have a lower turnout because Democratic identifiers are disproportionately lower turnout, lower income, and minority voters (e.g., Campbell, 1996; Grofman *et al.*, 1997).

Clearly, whether we use R_i or P_i as our national vote share value will directly affect our estimate of bias. Say, for example, we use P_i . If, instead, we had used R_i the effect would simply be to displace each x element on the seats–votes curve by an amount equal to $P_i - R_i$. But, in particular, this would mean that the seat share value when party i has a national vote share of 50 per cent would be displaced by an amount equal to $P_i - R_i$. But that is just another way of saying that *replacing P_i with R_i as our estimate of party i 's actual national vote share should (if our statistical estimation procedure were perfect) act to increase the estimated partisan bias by the amount $P_i - R_i$* . This simple link between choice of measure of national vote share and estimated partisan bias is an important observation that we will make crucial use of in developing our integrated approach to the determinants of partisan bias.

Measuring the Turnout Rate and Malapportionment Elements in Partisan Bias

Before we can show how to develop an integrated approach to partisan bias that separately measures distributional, turnout-related and malapportionment-related effects, some further mathematical analysis is very helpful in clarifying the underlying nature of partisan bias in seats–votes relationships. We begin by offering alternative definitions of both P_i and R_i , in which we show that *both* can be represented as a simple weighted function of the p_{ij} values, i.e., as a simple weighted sum of party i 's vote *shares* in each of the districts, of the general form

$$\sum_j (p_{ij} \times w^{(j)})$$

where the nature of the $w^{(j)}$ will, of course, be different for P_i and R_i , but will share the

characteristic that the weights are district specific. Later, we will show how an analogous representation as a weighted function of the p_{ij} values can be developed for a malapportionment-corrected measure of national party vote share. We will then use this malapportionment-corrected measure of national party vote share to derive an estimate of the nature of partisan bias due to malapportionment.

It is straightforward to represent P_i as such a weighted function. All we need do is take the weights to be

$$w^{(j)} = 1/S, \text{ for all } j$$

Here

$$\sum_j w^{(j)} = 1$$

This gives us

$$P_i = \sum_j (p_{ij} \times 1/S) = \left(\sum_j p_{ij} \right) / S$$

as desired.

Thus, we see that P_i may be defined as a weighted function of the p_{ij} values in which each constituency is weighted equally (i.e., with weight equal to $1/S$). Note also that, in calculating P_i as a weighted function of the p_{ij} values, the appropriate weights for each district may be taken to be the ratio of the number of seats in that district (here one) to total seats in the legislature.

Now we wish to show that R_i may also be defined as a weighted function of the p_{ij} , albeit with a different set of weights. To do so, some further notation is necessary.

Let us define the *ratio* of (two-party) turnout in the j th district to total turnout as $t^{(j)}$, i.e.,

$$t^{(j)} = \left(\sum_i v_{ij} \right) / V$$

and

$$\sum_j t^{(j)} = 1$$

Clearly, party i 's share of the two-party raw vote is just the sum over all districts, j , of the quantities that consist of party i 's share of the raw vote in each district multiplied by that district's share of the total raw vote. Thus, after some algebra, we obtain

$$R_i = Vi/V = \sum_j (p_{ij} \times t^{(j)})$$

This equation demonstrates that R_i may also be expressed as a weighted function of the p_{ij} . Here, the appropriate weights are the $t^{(j)}$ values, i.e., the appropriate district weights for calculating R_i as a weighted function of the p_{ij} can be defined as the *ratio of district raw turnout to total raw turnout*.

This way of thinking about both R_i and P_i shows that these measures can be expressed in a 'common language', where the difference between the two is a function of how we choose to weight. It is apparent that, in weighting constituencies equally, we neglect both turnout and malapportionment effects and have only distributional effects. While in weighting constituencies by turnout we incorporate turnout effects on partisan bias in addition to distributional effects.

While R_i captures both the distributional and turnout-related aspects of partisan bias, if national vote share is taken to be R_i in our calculation of the seats–votes curve (and features thereof such as swing and bias), we would not get separate measures of the impact of distributional and turnout-related factors on partisan bias—only a measure of combined impact. But we would like to be able to separate out the effects of these two factors. More generally, the question becomes: How can we specify the effects of all three factors—malapportionment, turnout rates, and partisan vote share distribution—on partisan bias in a way that allows us to separately estimate all three effects?

The approach to an integrated model of the three factors we develop below permits us to do so. In particular, when we let national party vote share be defined as P_i , rather than as R_i , the standard approach to bias pioneered by Tufte (1973) perfectly captures the concept of distributional bias in a fashion that excludes from consideration turnout and malapportionment effects. Thus, we can build our estimates of separate malapportionment effects and turnout effects *on top of* the analysis of distributional effects using the seats–votes curve that we have already created with P_i as our measure of national party vote share.⁸

Before we do so, we need to develop a malapportionment-corrected figure for national party vote shares. But it is easy to see how to do this. By analogy with the turnout-related weighting scheme, to establish a malapportionment-corrected figure, M_i , for national party vote shares, we simply weight the p_{ij} by $d^{(j)}$ = the ratio of raw population in the j th district to total raw national population, i.e., we set

$$M_i = \sum_j (p_{ij} \times d^{(j)})$$

Note that

$$\sum_j d^{(j)} = 1$$

We showed earlier that, when we change our measures of party i 's national vote share, we are, in effect, adding or subtracting partisan bias equal to the difference between the two measures. To create an integrated approach we begin by calculating partisan bias as in Tufte (1973) or Gelman and King (1994a) in a seats–votes equation in which national vote share is taken to be P_i . *We take this measure of partisan bias to be our pure measure of partisan bias due to distributional effects.*

Because this method does not take into account differential turnout rates across constituencies or malapportionment effects, we can then *use the difference between M_i and P_i (i.e., $M_i - P_i$) as our measure of that aspect of partisan bias that can be taken to be purely malapportionment-related in nature.*

However, to calculate the pure turnout-related effect on partisan bias we must be more careful, because some (or even all) of the differences in turnout rates across districts may be due to malapportionment and we do not want to count these effects on partisan bias twice. For example, if $t^{(i)} = d^{(i)}$, i.e., if turnout rate differences are simply a function of differences in the population base in each district rather than actual differences in turnout rates across district populations, then we really have no independent turnout-related effects. Thus, if $t^{(i)} = d^{(i)}$, we would want a measure of the pure turnout rate-related effects that was zero. *We will use the difference between R_i and M_i (i.e., $R_i - M_i$) as our measure of that aspect of partisan bias that can be taken to be purely turnout-related in nature* after we have corrected for both distributional bias and malapportionment bias.

Note that, now, all three effects are independent of one another, and the sum of the three effects may be thought of as the total partisan bias caused by all three factors.

Now that we have established how to calculate each of the three components of partisan bias, in the next section we illustrate those calculations with data from US House and US Senate elections in the 1980s. It is important, however, to recognize that these three estimates of partisan bias make sense only when taken together. For example, the turnout-related bias we estimate is after we have controlled for other sources of bias and is different from what we might estimate were we simply to look at, say, the correlation between turnout in the district and partisan success.⁹

Illustrative Applications of the Procedures to Estimate the Three Determinants of Partisan Bias

US House and Senate Elections 1984, 1986, 1988

Hitherto, for purposes of simplicity, we have largely treated the three sources of partisan bias separately, but there is no reason why more than one such factor might not be present in a particular situation, nor need they all operate in the same partisan direction. Thus, in looking at US House and Senate elections we would wish to take into account not just the effects of population-based malapportionment, but also the impact of the nature of the distribution of partisan support across states and of the partisan consequences of differences in turnout across states.

For US House and Senate races in 1984, 1986 and 1988, Table 1 shows the three different measures of national vote share for the Democrats. It also shows the derived estimates for partisan bias of each of the three types. We use the Gelman and King (1994b) Judgelt program to calculate partisan bias based on mean partisan vote shares, with all districts/states equally weighted. We use that estimate as our value for partisan bias due to distributional effects.¹⁰

Table 1. Three ways of estimating democratic national vote share and three aspects of partisan bias in 1980s US House and Senate elections^a

Year	Chamber	P_i	M_i	R_i	Pure distribut. partisan bias	Pure malapport. partisan bias = $M_i - P_i$	Pure turnout partisan bias = $R_i - M_i$
1984	House	54.9	55.0	52.5	- 1.7**	0.1**	- 2.5**
1986	House	57.3	57.1	54.8	- 2.6**	- 0.3**	- 2.7**
1988	House	57.0	56.8	54.1	- 3.4**	- 0.3**	- 2.7**
1984	Senate	48.5	51.9	50.7	- 0.4 ns	3.4 ns	- 0.8 ns
1986	Senate	50.6	51.0	50.8	2.9 ns	0.4 ns	- 0.2 ns
1988	Senate	53.2	53.3	52.9	- 0.2 ns	0.1 ns	- 0.4 ns

^aPositive values of bias are pro-Republican.

**Significant at the 0.01 level or less.

We then use $M_i - P_i$ as our measure of that aspect of partisan bias that can be taken to be purely malapportionment-related in nature, and we use $R_i - M_i$ as our measure of that aspect of partisan bias that can be taken to be purely turnout-related in nature after we have controlled for malapportionment.

The statistical significance of the partisan bias calculated from P_i , M_i , and R_i are also reported in Table 1. However, the latter two of these are calculated differently from the first. The statistical significance of the partisan bias using the P_i value is provided by the Gelman and King Judgelt package. Since this bias is a mean value estimated from a simulation, there is an error variance associated with it. The significance level reported tells us the likelihood that the partisan bias attributed to distributional effects is nonzero.¹¹ In contrast, the statistical significances of the malapportionment bias and of the turnout bias are calculated using a difference of means test. For each district (or state) for each year we have an observed p_{ij} value, and observed values for $p_{ij} \times d^{(j)}$ and for $p_{ij} \times t^{(j)}$. If we neglect the issue of the up-to-datedness of the population figures for the different constituencies, all three of these values are actual values, not estimates. The significance reported for the $M_i - P_i$ column is the likelihood that the mean value of the p_{ij} is different from the mean value of the $p_{ij} \times d^{(j)}$ distribution. Similarly, the significance reported for the $R_i - M_i$ column is the likelihood that the mean value of the $p_{ij} \times t^{(j)}$ distribution is different from the mean value of the $p_{ij} \times d^{(j)}$ distribution.

We see from Table 1 that there is statistically significant partisan bias in the House that can be attributed to the geographic distribution of partisan vote shares, but that the findings on distributional bias for the Senate are not statistically significant.

We also see from Table 1 that for the House there is statistically significant partisan bias that can be attributed to malapportionment, although the actual magnitude of this bias is not especially large. However, for the Senate there is no statistically significant malapportionment bias. Indeed, with the exception of 1986, the partisan bias effects that might be attributed to Senate malapportionment are not that large. This may seem too implausible, given the dramatic malapportionment that exists in the US Senate, but, as noted earlier, we need to distinguish between malapportionment, *per se*, and malapportionment that generates partisan bias. In these

Senate elections there simply is no strong link between a state's population and how well either party does in that state.

Lastly, we see from Table 1 that there is a substantial and statistically significant partisan bias in the House due to turnout rate differences across constituencies. The Democrats are the beneficiary of this bias, i.e., Democrats win their seats, on average, in districts with lower levels of turnout than is the case for Republicans. This is the 'cheap seats' phenomenon that Campbell (1996) called attention to. However, for turnout-related bias, as with the other two potential causes of partisan bias, we find no statistically significant results for the Senate.

Of course, the fact that the n for the Senate is only 33 or 34 diminishes the likelihood of statistically significant effects. Nonetheless, even when we pool Senate data for the four years from 1984 to 1988 to raise our n to 100, we still get nonsignificant results for distributional bias. Moreover, even for this pooled data we still get statistical nonsignificance for partisan bias effects due to malapportionment or turnout as well.

If we look at the combined effects of all three sources of partisan bias over the 1984–1988 period we see that, by and large, in the House, they tended to reinforce one another to create a pro-Democratic bias. In the Senate, in contrast, they tended to work in a pro-Republican direction. Thus, we would expect that, in this period, the Senate would be more Republican in composition than the House—and it was.

US Presidential Elections 1984 and 1988

While we presented our analysis in the previous section solely for the case of single-member districts, it is straightforward to generalize it to districted systems with a mix of single- and multi-member districts or, analogously, to weighted voting systems like the US electoral college. We replace the weight $1/S$ in our earlier formula with $s^{(j)}/S$, where $s^{(j)}$ is simply the number of seats elected from the j th constituency. We apply this extension to calculate the three aspects of partisan bias in the US electoral college in 1988. Table 2 shows data for the presidential election of 1988 paralleling that in Table 1 for House and Senate elections.

We see from Table 2 that, in the electoral college, unlike what we found for the House, none of the three effects have any statistically discernible impact on partisan bias. This, too, is a surprising finding considering how much has been written about supposed (pro-Republican) bias in the electoral college of that period. Elsewhere (Grofman *et al.*, forthcoming) we show why partisan bias in the electoral college has generally been overestimated.

Table 2. Three ways of estimating democratic national vote share and three aspects of partisan bias for the US electoral college 1984 and 1988^a

Year	Equally weighted states estimate of Democrat. vote share	P_i (electoral college)	M_i	R_i	Pure distrib. partisan bias	Pure malapport. partisan bias = $M_i - P_i$	Pure turnout partisan bias = $R_i - M_i$
1984	39.7	40.5	40.6	40.8	- 0.8 ns	- 0.1 ns	0.2 ns
1988	46.0	46.0	46.0	46.0	- 1.7 ns	0.0 ns	0.0 ns

^aPositive values of bias are pro-Republican.

**Significant at the 0.01 level or less.

Discussion

When we think of partisan bias as having the three explanatory factors of partisan distribution of vote share, population malapportionment, and party-specific differences in turnout rates that translate into constituency-specific differences in turnout rates, we are in a position to resolve a long-standing dispute in the literature on elections about whether P_i or R_i should be used to measure national vote share. Some authors (e.g., Gudgin and Taylor, 1979; Campbell, 1996) argue for the latter, while most authors who have made use of seats–votes measures of bias (e.g., Grofman, 1983; Cain, 1985; Campagna, 1991; Brady and Grofman, 1991a; Gelman and King, 1994a) use the former.

The way to resolve the dispute is to recognize that, as we demonstrated earlier, when bias is calculated simultaneously with swing ratio in a formulation in which each party's vote share nationally is calculated as the *average* of its partisan vote *share* in each constituency (which, in effect, weights all constituencies equally), bias so calculated becomes a *pure* measure of bias of the first type, i.e., of distributional bias. In contrast, when bias is calculated simultaneously with swing ratio in a formulation in which each party's national vote share tally is taken to be its share of the *total* vote cast for its party's candidates for that office (which, in effect, weights each constituency by the constituency's proportion of the total national turnout), bias as so calculated is a *combined* measure of bias of the first and second and third types. Thus, controversy in the electoral systems literature as to which of these two methods is the 'correct' method for calculating partisan bias is misguided. Both can be said to be 'correct'; they simply measure different things.

Nonetheless, as we previously argued, use of P_i is preferred, since it is an uncontaminated measure of distributional effects. Of course, we must also recognize that use of P_i does not capture turnout rate-related or malapportionment-related effects, and thus, if we use P_i as our measure of national vote share, we need to separately account for these effects. Showing how this can best be done has, of course, been the central point of this paper.

We have demonstrated that it is possible to separately estimate turnout, malapportionment and distributional effects on partisan bias and that, for US elections, these do not necessarily all go in the same direction or operate with the same magnitude in different electoral contexts. We did see, however, that in the House, the sum of these three sources of partisan bias tended to reinforce a Democratic advantage in that body. The results shown in Table 1 are consistent with an important empirical phenomenon in the 1980s, namely the fact that, in this period, the Democrats did better for the House than for the Senate. We saw that distributional bias for the House is pro-Democratic and the only large distributional bias estimate for the Senate is in a pro-Republican direction. Similarly, we found both strong and statistically significant partisan bias in favor of the Democrats in the House in terms of bias that could be attributed to turnout differences. In the House, only with respect to malapportionment-related bias were there no biasing effects that were both statistically significant and strongly in favor of the Democrats.

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Notes

1. Since the publication in 1973 of Tufte's seminal article, numerous authors have approached the analysis of seats–votes relationships in two-party systems by looking at the twin concepts of partisan bias and swing ratio (e.g., Niemi and Deegan, 1978; Grofman, 1983; Brady and Grofman, 1991a; Cain, 1985; King and Browning, 1987; Campagna and Grofman, 1990; Campagna, 1991; Niemi and Jackman, 1991; King and Gelman, 1991; Garand and Parent, 1991; Gelman and King, 1994a). There are several different methods for simultaneously calculating swing ratio and bias, but two are most important. The first is the log-odds method developed by Tufte (1973) and used by many subsequent authors (e.g., Campagna, 1991; Brady and Grofman, (1991a, b)). The second is the averaging technique developed by King and Gelman (1991) and instantiated in the computer program *JudgeIt* used by these authors (Gelman and King, 1994a, b) and by a number of others (e.g., Garand and Parent, 1991).
2. Customarily, in two-party competition, both swing ratio and the distributional aspect of partisan bias are estimated at a (hypothetical) vote share of 50 (Tufte, 1973), or for a range of vote shares relatively near to 50 per cent and symmetrically distributed around that point. In this paper, following Gelman and King (1994a, b), we estimate values over the 0.45 to 0.55 vote share range. Swing ratio and bias can also be specified at any point on the seats–votes curve or averaged across any range of points (Grofman, 1983), but we shall neglect such complications here. In a two-party contest, the bias for party A is simply the negative of the bias for party B.
3. We shall consider only two-party contests in this paper, although the concepts of swing ratio and bias can both be generalized to multi-party competition. Grofman (1975), Taagepera and Shugart (1989) and Lijphart (1994) discuss the seats–votes relationship across other types of electoral systems.
4. Clearly, the concept of malapportionment needs to be defined with respect to some basis. In the United States, unlike most other democracies, apportionment is on the basis of total population (persons) rather than on the basis of citizen population or potentially eligible electorate (e.g. citizen voting age population) or registered voters or past turnout. Obviously, the choice as to the basis for apportionment can have important implications for what we conclude about the presence or absence of malapportionment (e.g., Grofman, 1992; Scarrow, 1992). In the remainder of this paper, except where otherwise indicated, the reader may take the word 'population' as a generic term, referring to whatever may be the basis of apportioning seats in the country under investigation. Since the actual data we analyze are from the United States, this usage should not be a cause of confusion.
5. By turnout rate we mean the ratio of votes cast to the apportionment base in the district. Obviously, the actual number of voters will not be the same as the apportionment base. Implications of that fact for the equity of representation have been discussed by a number of authors (for a review of the US debate see Brace *et al.*, 1988; Grofman, 1992).
6. Recall that we use 'population' as a generic term to refer to the basis of seat apportionment.
7. For simplicity, here we shall act as if the legislature we are analyzing is a national parliament. Exactly the same analyses go through for state or regional legislatures as well.
8. Campbell (1996) has identified a phenomenon that he refers to as 'cheap seats', in which one party wins its seats with fewer raw votes per victory, on average, than does the other party. He argues that the party that has the cheap seats is advantaged in terms of partisan bias. But the cheap seat phenomenon may arise in one or more of three ways we have previously identified. As with calculating bias via an equation in which national vote share is defined as R_i , the method proposed by Campbell to calculate the partisan bias caused by cheap seat effects (a method that calculates a function of the difference in each party's average total wasted votes) actually measures the combined impact of all three of these factors (distributional differences, apportionment differences, and turnout rate differences) in such a fashion that the independent impact of the factors cannot be disentangled.
9. Also, even if we eliminated malapportionment and turnout-related bias, as long as we still permitted distributional bias to come into play there are many districting plans which will yield the same raw vote totals but which will differ greatly in their partisan consequences. Moreover, it may in practice be impossible to redraw district boundaries so as to ensure both equal turnout and equal population.
10. For the House our estimates are different from those given in King and Gelman (1991) because we do each election separately and only use the actual election outcomes as input rather than attempt to estimate a predictive multiple regression equation based on election data from a longer time period.
11. For example, the House distributional bias figure of -1.7 reported in Table 1 has an associated

standard error of 0.44. Since this value is almost five times its standard error, the estimate is significant at well above the conventional 0.01 level.

References

- Baker, G. E. (1955) *Rural Versus Urban Political Power: The Nature and Consequences of Unbalanced Representation*. Greenwood Press, Westport, CT.
- Brace, K., Grofman, B., Handley, L. and Niemi, R. (1988) Minority voting equality: the 65 percent rule in theory and practice. *Law and Policy* **10**, 43–62.
- Brady, D. W. and Grofman, B. (1991a) Sectional differences in partisan bias and electoral responsiveness in U.S. House elections. *British Journal of Political Science* **21**, 247–256.
- Brady, D. W. and Grofman, B. (1991b) Modeling the determinants of swing ratio and bias in U.S. House elections, 1850–1980. *Political Geography Quarterly* **10**, 54–262.
- Cain, B. (1985) Assessing the partisan effects of redistricting. *American Political Science Review* **79**, 320–333.
- Campagna, J. C. (1991) Bias and responsiveness in the seat–vote relationship. *Legislative Studies Quarterly* **16**, 81–90.
- Campagna, J. C. and Grofman, B. (1990) Party control and partisan bias in 1980s congressional redistricting. *Journal of Politics* **52**, 1242–1257.
- Campbell, J. E. (1996) *Cheap Seats: Democratic Party Advantage in U.S. House Elections*. Ohio State University Press, Columbus, OH.
- Garand, J. C. and Parent, T. W. (1991) Representation, swing and bias in U.S. presidential elections, 1972–1988. *American Journal of Political Science* **35**, 1011–1031.
- Gelman, A. and King, T. (1994a) A unified method of evaluating electoral systems and redistricting plans. *American Journal of Political Science* **38**, 514–554.
- Gelman, A. and King, G. (1994b) *Judgelt* (computer program available free via anonymous FTP from latte.harvard.edu.).
- Grofman, B. (1975) A review of macro-election systems. In *German Political Yearbook (Sozialwissenschaftliches Jahrbuch für Politik)*, ed. R. Wildenmann, Vol. 4, pp. 303–352. Verlag, Munich.
- Grofman, B. (1983) Measures of bias and proportionality in seats–votes relationships. *Political Methodology* **9**, 295–327.
- Grofman, B. (1992) An expert witness perspective on continuing and emerging voting rights controversies: from one person, one vote to political gerrymandering. *Stetson University Law Review* **21**(3), 783–818. (A revised and expanded version appears under the title What happens after one person–one vote: implications of the U.S. experience for Canada. In *Drawing Boundaries*, ed. J. Courtney and D. Smith, pp. 156–178. Fifth House, Saskatoon, Saskatchewan, 1992.)
- Grofman, B., Collet, C. and Griffin, R. (1997) Analyzing the turnout–competition link with aggregate cross-sectional data. *Public Choice* (in press).
- Grofman, B., Brunell, T. and Campagna, J. (forthcoming) Distinguishing between the effects of swing ratio and bias on outcomes in the U.S. electoral college, 1900–1992. *Electoral Studies*.
- Gudgin, G. and Taylor, P. J. (1979) *Seats, Votes and the Spatial Organization of Elections*. Pion, London.
- Jackman, S. (1994) Measuring electoral bias: Australia, 1949–93. *British Journal of Political Science* **24**, 319–357.
- Johnston, R. J. (1981) *Political, Electoral and Spatial Systems*. Oxford University Press, London.
- King, G. and Browning, R. X. (1987) Democratic representation and partisan bias in congressional elections. *American Political Science Review* **81**, 1251–1273.
- King G. and Gelman A. (1991) Systemic consequences of incumbency advantage in United States House elections. *American Journal of Political Science* February, 110–138.
- Lee, F. F. and Oppenheimer, B. I. (1997) Senate apportionment: competitiveness and partisan advantage. *Legislative Studies Quarterly* **22**, 3–24.
- Lijphart, A. (1994) *Electoral Systems and Party Systems: A Study of Twenty-Seven Democracies, 1945–1990*. Oxford University Press, New York.
- May, J. D. (1974) Democracy and rural overrepresentation. *Australian Quarterly* **46**, 52–56.
- Niemi, R. G. and Deegan, J. (1978) A theory of political districting. *American Political Science Review* **72**, 1304–1323.

- Niemi, R. G. and Jackman, S. (1991) Bias and responsiveness in state legislative districting. *Legislative Studies Quarterly* **16**, 183–202.
- Owen, G. and Grofman, B. N. (1988) Optimal partisan gerrymandering. *Political Geography Quarterly* **7**, 5–22.
- Rydon, J. (1968) Malapportionment: Australian style. *Politics* **3**(2), 133–147.
- Scarrow, H. (1992) 'One man, one vote': tracing its roots and consequences. In *Drawing Boundaries*, ed. J. Courtney and D. Smith, pp. 179–191. Fifth House Publishers, Saskatoon, Saskatchewan.
- Taagepera, R. and Shugart, M. (1989) *Seats and Votes: The Effects of Determinants on Electoral Systems*. Yale University Press, New Haven, CT.
- Taylor, P. J., Gudgin, G. and Johnston, R. J. (1986) The geography of representation: a review of recent findings. In *Electoral Laws and Their Political Consequences*, ed. B. Grofman and A. Lijphart, pp. 183–192. Agathon Press, New York.
- Tufte, E. R. (1973) The relationship between seats and votes in two-party systems. *American Political Science Review* **67**, 540–547.
- Yamakawa, K. (1984) A Lorenz curve analysis of the allocation of the seats in the House of the Representatives by the general elections. *Kansai University Review of Law and Politics* **5**, 1–26.