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Coalitions and Collective Action

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The General Irrelevance of the Zero Sum Assumption in the Legislative Context

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Abstract: Coalition formation models are proposed for two cases which exemplify the most common sorts of decisions facing legislators – one analogous to the traditional pork-barrel situation where there are a number of proposals for projects of benefit only to some legislators but whose cost will be shared by all, the other involving policy choice along some hypothesized unidimensional continuum. In both cases we show that the conflict is not zero sum and that minimal winning coalitions need not invariably be expected. We also examine the circumstances under which coalitions around the pork barrel are likely to give rise to cost-effective decisions as to which projects shall be funded.

1. Introduction

Coalition formation in small decision bodies such as legislatures or the U.S. Supreme Court has recently been the focus of a great deal of both theoretical and empirical attention [e.g., Francis; Ulmer; Hinckley, 1972; Koehler; Laakso; Rohde, 1972, 1973; Murray/Lutz; Li/Hinckley; Weingast, 1978, 1979].

Many of the above articles cite Riker's [1962] theorem that coalitions in an n -person zero sum game will be minimal winning, and examine the data to see if, indeed, minimal winning coalitions are observed. Since, by and large, minimal winning coalitions are not found [see Lutz/Williams for a review of the evidence from legislatures; see also interchange between Keating/Lee and Hardin, 1978; and Thompson; authors such as Hinckley [1972, 1973], Rohde [1972, 1973], Koehler [1972], and Glieber [1980] have tried to account for their absence in terms of alternative models of the group decision process, emphasizing risk aversion and over-time dynamics. In an important sense, Riker's theorem has been a bugaboo in the coalition literature. Riker's result has wrongly suggested to many authors the inevitability of

¹) The second section of this paper was inspired by and draws heavily on Barry Weingast, "Rational Choice Perspective on Congressional Norms," California Institute of Technology, Social Sciences Working Paper No. 142, Pasadena, California, October 1976; subsequently published in considerably revised form as Weingast [1979]. Translating my scribbles into final copy was accomplished by the staff of the Word Processing Center at the University of California, Irvine with their usual phenomenal speed and accuracy. This research was partially supported by NSF Grants No. SOC 77-24474 and SES 80-07915, Political Science Program. An earlier draft, which contained all of the major results, was completed in October 1977.

minimum winning coalitions in a wide range of contexts. These authors have neglected the very restrictive zero-sum conditions required for his result to hold.²⁾ Thus, a number of authors have taken minimum winning coalitions for granted as the “rational” outcome in the legislative or judicial context and have sought explanations (sometimes rather ad hoc ones) for the low proportion of coalitions of that size actually observed. In contrast, we believe that the zero-sum assumption for legislative choice is a misguided one, and neither of the two basic models of coalition formation we shall propose postulate a zero-sum situation. Our models also do not give rise to a uniform prediction of minimal winning coalitions.

We shall consider two cases. The first case is one analogous to the traditional pork-barrel situation, where there are a number of proposals for items of benefit only to some legislators but whose cost will be shared by all. The model we use was proposed by *Weingast* [1976], and our analysis builds on that of *Weingast* [1978, 1979]. The second case is that of policy choice among paired alternatives scalable along some unidimensional continuum. While the models we propose simplify the coalition processes somewhat so as to make them more tractable for analysis, e.g., by assuming symmetry of situation across legislators, and initially by omitting dynamic considerations (a defeat which we subsequently remedy); despite these simplifications we believe they represent the two basic types of decision situations faced by legislators.³⁾

²⁾ The mathematical validity of *Riker's* [1962] theorem has recently come under challenge [*Butterworth; Frohlich; Shepsle; Hardin*, 1976]. It seems clear to us that the theorem is wrong as stated, and that minimal winning coalitions can be expected to occur, even in the zero-sum issue, only under rather more restrictive conditions than those mentioned by *Riker* [see especially *Hardin*, 1976]. The correctness or lack thereof of *Riker's* theorem, however, is *not* relevant to our discussion of coalition formation models, since we (see below) shall not be dealing with the zero-sum case. In *Axelrod's* words, “(I)n studying a situation that is in reality non-zero sum, the application of a zero-sum theory may not be very successful [*Axelrod*, p. 180]. *Lutz/Williams* [1962, p. 13] assert in their review essay that “there appears to be a growing consensus that *Riker's* theory is not useful in the American legislative context, doubt that any theorem using a zero-sum assumption will be useful, and indications that policy distance (ideology) and issue content are important variables for explaining coalition behavior.” We shall return to these points in our discussion of non-zero sum models of coalition formation. [cf. *Weingast*, 1978, note 8.]

³⁾ There are at least two reasons that congressmen ... have an incentive to maximize the dollar benefits going to their constituencies, subject to the constraint that their decisions can be passed by majority vote in each house of Congress. First, many congressmen believe that “bringing home the bacon” will enhance their re-election possibilities. As Representative Richard Bolling (D-MA) has stated: ‘A great multi-million dollar flood control dam that affords protection to my congressional district plays a large part in my re-election over the years. My political reputation in my district is associated more with this project than with many stands I have taken on substantive issues including life, well being, and national security in our time.’ [*Bolling*, 109–110]. Similarly, James Murphy quotes a southern member of the House Public Works Committee as follows: ‘We are all national legislators in a sense and we have to be but the national issues don’t mean a damn thing back home – oh sure, they read about it in the newspapers but it doesn’t mean much to them. They’ve got to see something; it’s the bread and butter issue that counts – the dams, the post offices and the other public buildings, the highways’ [*Murphy*, 1968, 1970]. Even beyond helping with electoral possibilities, “many congressmen who define their role as one of constituency services would have an incentive to work for constituency related policy outputs for ... their district” [*Strom*, 713–714].

2. A Model of Legislature Coalition Formation Around the Pork-Barrel

Following *Weingast* [1978, 1979], let us assume that each member proposes one bill with cost c (shared by all legislators) and benefit (exclusive to his district) of b . (Initially, let us assume b and c to be the same for all legislators.) Let j = number of members whose bill passes. Let N = size of legislature. For simplicity, let N be odd. We may imagine that each bill represents a project of benefit only to the district in which it is located or where the benefits outside the district may safely be neglected. Let us assume that legislators identify their interests with those of their district.⁴) Thus, if j bills pass, a member whose bill passes receives the benefit of his project, b , and pays his share of the total costs, which is cj/N ; each of the $N - j$ members whose bills do not pass receives no benefit but must pay his share of the total cost, cj/N .

Clearly, no member should propose a bill for which $b < c/N$ since the expected benefits (to the district) do not exceed expected costs (to the district). *Weingast* [1978, 1979] posits that $b > c$. We find this unnecessarily restrictive as an assumption, and prefer to require only the assumption based on individually rationalistic calculations that $b > c/n$. [*Shepsle/Weingast*, offer a considerable further generalization to the case where b and c are not uniform for all legislators, but we shall neglect such further complications].

Consider the case, which we shall denote Case A, a *fixed coalition of size j*, where there is a coalition of size j , each member of which always supports the bills of all other members and votes against the bills of all members not in the coalition. We may ask what the optimum size of a fixed coalition would be, where optimum is defined in terms of maximizing *average* expected net benefits (EB) to members of the coalition.

We wish to find j , $0 \leq j \leq N$, for which EB, given by the expression

$$b - \frac{jc}{N} \quad (1)$$

is positive and maximal, but subject to the constraint that $j > N/2$; i.e., we require that a *winning* coalition be formed, since otherwise $EB = 0$.

⁴) While we believe it virtually certain that judicial opinion coalitions will include the median judge [cf. *Rohde*, 1972b] we are not as clear that the opinion coalition should be expected to be minimal. In the case of Supreme Court justices it is hard to decide what constitute rewards. Like *Hoyer/Mayer/Bernd* [1977, p. 392] we doubt that it makes sense to postulate that "only justices who sign the opinion should receive a payoff in the game." Furthermore, in the U.S. Supreme Court the potential disjunction between decision coalition and opinion coalition presents special problems in looking at coalition formation mechanisms [*Hoyer/Mayer/Bernd; Giles; Rohde*, 1977]. It is also not clear that the appropriate model for opinion coalitions in the U.S. Supreme Court, or other courts, is one of choosing between a point on some unidimensional issue continuum and a status quo point; or indeed that judicial choice should be modeled as sequential pairwise choice. In courts like the Supreme Court, at the opinion stage, it may be argued that "all policy options are simultaneously available for choice" [David Rohde, personal communication, June 29, 1977]. Thus, we regard the results we offer for our paired-comparison model in Section 3 as having only limited applicability to court decision making. Of course, we find no reason to suppose that either court decision coalitions or court opinion coalitions will be zero-sum conflict situations.

Result 1: The optimal coalition size for the fixed coalition case is

$$j = \begin{cases} \frac{N+1}{2} & \text{if } b > \frac{c}{2} \\ 0 & \text{otherwise} \end{cases}$$

Proof: Expression (1) is positive and non zero iff $b > jc/N$, i.e., iff $j < Nb/c$. If $b < c/2$, then Expression (1) will be negative for all j , $0 < j < N$, and thus the maximum value of Expression (1) occurs at $j = 0$. For $b > c/2$, Expression (1) is positive for all j and maximized for the minimal value of j , $0 \leq j \leq N$ which satisfies the constraint that $j > N/2$. Of course, this is simply $j = (N + 1)/2$.

In other words, if $b < c/2$, then no winning coalition should form (i.e., the pork-barrel proposal should be defeated); while if $b > c/2$, we expect, caeteris paribus, to find a minimal winning coalition. For the set of assumptions specified above, the case for expecting a minimal winning coalition can be made even stronger. *Weingast* [1979] has shown that the set of minimal winning coalitions constitutes the core of this fixed coalition legislative voting game (played cooperatively and without side payments). Thus, any non-minimal winning coalition is dominated by some minimal winning coalition and "once an MWC forms, no other condition can upset it" [*Weingast*, 1978, p. 5]. Given such a strong mathematical result, the question of why we do not observe minimal winning coalitions in pork-barrel situations takes on considerable interest.

Of course, if the fixed coalition forms on party lines in a two-party system, then we would anticipate that the winning coalition would be supra-optimal, i.e., greater than minimal winning, unless the parties were even in strength. *Murray/Lutz* [1974, 247–249] find that in a selected sample of roll-calls in 38 states those states with closely divided legislatures were twice as likely to approach minimalness in roll-calls as were those where one party had a sizable majority. Party division explained twice as much variance as any other factor examined [cf., *Koehler*]. As *Shepsle/Weingast* [1981, p. 109] point out, however, the importance of size of party majority will vary with the partisan cohesion of the legislature. "In a tightly organized partisan legislature..., a legislator knows (with certainty) that he will be a member of the victorious MWC, and his project authorized *only if* he is a member of the partisan of the majority party."

To account for the absence of bare majority coalitions in a more general fashion, let us again consider the situation where there is a winning coalition of size j , and bills of members in this winning coalition pass, and bills of members not in the winning coalition fail; *but now* [following *Weingast*, 1979] members are assuming to be choosing behind a veil of ignorance, behind which they do not know whether or not they will be in the winning coalition. Let us denote this as Case B, random assignment of legislators to the winning/losing side: choosing behind a veil of ignorance. Let expected value, EB, as before, be defined as the expected net benefit to a legislator's district, i.e., the value of the expected benefits brought to the district minus the district's share of the costs for all bills which are expected to be passed. If there are j members in the winning coalition, the probability of any given member belonging to it is simply j/N .

To maximize EB is to find j , $0 \leq j \leq N$, which maximizes net expected benefits minus net expected costs, i.e., to maximize

$$b\left(\frac{j}{N}\right) - j\left(\frac{c}{N}\right) = \frac{j(b-c)}{N}. \quad (2)$$

Result 2: Expression (2) is maximum when $b > c$ for $j = N$, and when $b < c$ for $j = 0$.

Proof: Expression (2) is monotonic in j . Since j is positive, maximum and minimum points of Expression (2) in the interval $[0, N]$ will occur at either $j = 0$ or $j = N$, depending as $(b - c)$ is positive or negative.

We may interpret this result as saying that, if they must act behind a veil of ignorance as postulated above, members who wish to maximize their EB should wish the legislative norm governing support of pork-barrel legislation (i.e., the norm determining size of the winning coalition) to be "vote for those bills with $b > c$; vote against all others." Weingast's "universalism theorem" [1978, p. 8] expresses a virtually identical result. As Weingast [1979] discusses at some length, such reasoning might lead to consensus in the group around reciprocal backscatching norms—of the sort which have been observed in many legislature [Mayhew].

Result 3: Neither the fixed coalition nor the random assignment legislative game is zero sum; thus Riker's [1962] claim that coalitions in n -person zero sum games will be minimal winning is inapplicable. The value of the game in both these cases equals $j(b - c)$, which varies with j .

Proof: The total benefits to those in the winning coalition are $(b - jc/N)j$; the total losses to those in the losing coalition are $-jc/N(N - j)$. The sum of those is simply $j(b - c)$. This is not a constant, but varies with the size of the winning coalition. (This point is recognized by Weingast [1978, p. 8] but never explicitly stated.)

Result 4: If legislators act as if they were behind the veil of ignorance postulated in Case B, or under consensus norms which could be derived from such a model, then government spending on the pork barrel will be efficient,⁵ since projects with cost benefit ratio less than one will not be funded and those with cost benefit ratio greater than one will be funded.

Proof: See Expressions (1) and (2).

It is interesting to observe that *the rule which optimizes EB in the veil of ignorance case is also the rule which maximizes the value of the game, i.e., which maximizes the*

⁵) We neglect the opportunity costs of alternative uses of the funds. Davis/Meyer [1972] define optimal budget size in terms of approval of those bills for which the expected sum of benefits minus costs is positive.

expected net benefit to the legislators as a whole. Hence, acting (as if) behind such a veil of ignorance can be argued, at least in some limited sense, to serve the public interest. However, Weingast [1978, 9–11] points out that legislators, once such a universalistic norm is adopted, have a strong incentive to backslide from efficiency by proposed projects which benefit their district $b > c/N$, but are not necessarily collectively desirable ($b > c$).

We have already noted the point, made by Murray/Lute [1974], among others, that party cohesion puts strains on minimalness of winning coalitions, except when legislatures are evenly divided. We believe there is an even stronger reason why universalistic norms rather than fixed simple majority coalitions can be expected in pork-barrel-like situations. Pork-barrel legislation comes in several different types [Ferejohn], which makes virtually impossible the creation of a unique dominant pork-barrel coalition which is also minimal winning. For example, not all states have an equal interest in dam construction. Similarly, the pork-barrel coalition around naval shipyards is necessarily distinct from that around other types of military bases. Highway construction (often thought of as a form of pork barrel) has different relevance to large vs. small, or predominantly urban vs. predominantly rural states. If there were distinct minimal winning coalitions around distinct pork barrels, we could have a situation looking something like this:

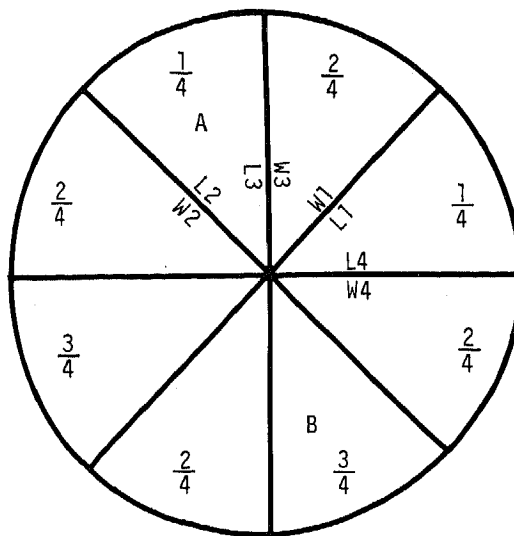


Fig. 1: Legislative Cleavages with Four Distinct and Independent Pork-Barrel Dimensions. Numbers indicate proportion of minimal winning coalitions in which each segment is to be found; e.g., Segment A is a winner only on Issue 1, while Segment B is a winner on Issues 2, 3, and 4.

The above diagram [inspired by Schattschneider] shows the number of minimal winning coalitions each (hypothetical) segment of the legislature falls in if there are four axes of cleavage, i.e., four separate and independent types of pork-barrel issues.

Some legislators are in more than half of the winning coalitions, others are in less, even though *each* coalition is itself minimal winning. Clearly, if the coalitions are independent, then the average legislator obtains an expected payoff of only $(b - c)(N + 1)/2N \approx (b - c)/2$ per pork-barrel dimension, as compared to the $|b - (N + 1)/2N|/c \approx b - c/2$ obtained as a member of a single unique minimal winning coalition. *This situation is closely analogous to choosing behind a veil of ignorance.* As in Result 2, unless a legislator can guarantee being in every minimal winning coalition, which clearly not all legislators can do if these minimal winning coalitions are independent, then the only way in which *every* legislator can maximize his/her expected payoff is to insure that every coalition is winning (i.e., that every coalition is unanimously supported), which yields an expected payoff of $b - c$, which is clearly greater than $(b - c)/2$ if $b > c$.

This argument for reciprocity and a norm of unanimity is buttressed by *Hinckley's* [1973] emphasis on the over-time stability of legislative coalitions and by considerations of cognitive balance. It is difficult for a legislator to be in a permanent coalition with legislator y on one set of pork-barrel issues and in a permanent coalition against legislator y on another set of pork-barrel issues – since these issues are by and large not matters of ideological difference but rather simply questions of which constituencies shall receive benefits.⁶ [For other sorts of arguments as to why unanimity/reciprocity might be expected see *Eulau*].

Weingast [1978, 1979] restricts himself to the case where no side payments are permitted. We may consider what happens when sidepayments are permitted.

Result 5: In the fixed coalition case where a minimum winning coalition forms, even where side payments are possible, it will be impossible for excluded members to buy their way into that minimum winning coalition unless $b > c/2$. If $b > c/2$, the coalition will grow to a size k , where

$$k + 1 > \frac{Nb}{c} > k. \quad (3)$$

⁶ Cf. *Mayhew* [1974, p. 114] "(T)he best way for members to handle the particular is to establish inclusive universalistic standards. Thus, the House Interior Committee churns out an enormous number of bills, and of the ones that pass the House about 95 per cent go through without formal roll calls at all. In other areas the same effect can be achieved by use of 'omnibus' bills. Hence, on particular benefits there is no reason to expect to find minimal winning coalitions or close roll calls." The House Interior Committee, as described by Richard Fenno [*Fenno*, 58 cited in *Mayhew*, p. 89] "takes as its major decision rule a determination to process and pass *all* requests and to do it in such a way as to maximize the chances of passage in the House. Succinctly, then, Interior's major strategic premise is: *to secure House passage of all constituency-supported, member-sponsored bills.*" Similarly, House Public Works, according to James T. Murphy has a "norm of mutual advantage"; in the words of one of its members: "We have a rule on the committee, it's not a rule of the committee, it's not written down or anything, but it's just the way we do things. Any time any member of the committee wants something, or wants to get a bill out, we get it out for him ... Makes no difference – Republican or Democrat. We are all Americans when it comes to that" [*Murphy*, 1968, p. 23 cited in *Mayhew*, p. 90].

Proof: If we add a k th member to a coalition of size $k-1$, the k th member must compensate each of the $k-1$ others for the loss to them of c/N which arises from his joining the coalition and pay himself at least c/N to make it worthwhile to join the coalition. This is possible iff

$$b - \frac{kc}{N} > 0 \quad (4)$$

i.e. iff $b/c > k/N$. But if $k > N/2$, this is impossible unless $b > c/2$.

Corollary to Result 5: In a fixed coalition situation, where side payments are possible; if $b > c/2$ the final coalition should be larger than minimal winning – in particular, if $b \geq c$, then the final coalition will be expected to be of size N .

Of course, such supraminimal coalitions are not stable, since some members receive more benefits than others and would prefer a different coalition structure in which their constituents' payoffs were larger. Nonetheless, such short-run supraminimal coalition structures may be expected to form if sidepayments are possible.

So far we have acted as if benefits and costs were objective factors, identically perceived by all legislators. It is reasonable to believe, however, that for each legislator, the benefits of pork-barrel items allocated to his home district are more directly visible to his constituents than the costs associated with these projects. Costs are absorbed into the general tax load [see e.g., *Mayhew*], and may extend over a long period of time, while benefits may be expected to have immediate visibility (and hence payoff to the legislator) once the project is announced, even before construction is actually complete. Thus, since legislators can be expected to orient themselves toward the perceptions of constituents insofar as those are relevant to winning reelection, we would expect that costs and perceived costs would not be the same, e.g., that individual legislators would discount c by some factor which need not be the same for all legislators. [This point is expanded upon in *Shepsle/Weingast*.] Let us for simplicity, however, assume a uniform discounting factor, r , $0 < r < 1$. In other words, if the true costs (increased tax burden) of a project is c , the legislator will act as if the cost of that project were only rc . In the extreme case, a legislator concerned only with short-term maximization might behave as if r were effectively 0.

Applying this discount factor we readily obtain results analogous to those above. E.g., in the veil of ignorance case, members who wish to maximize EB should vote only for those bills with $b > rc$. Under the above assumptions, there will be a divergence between socially optimal choice (= choose only those projects for which $b > c$) and the actual choice mechanism (= choose those projects for which $b > rc$); i.e., projects which are not efficient will nonetheless be funded. In general, legislators will propose any project for which $b > rc/N$; hence, inefficient projects (i.e., those with $c/N > b > rc/N$) may be proposed. (Of course, when legislators do what is in their interest and in the common interest of all legislators, the public interest need not thereby be served, since the costs which are being minimized are constituent-perceived costs rather than the "true" social costs.)

Similarly, in a fixed coalition situation we would expect coalition sizes as follows for the case where there is a single type of pork-barrel issue in which (perceived) costs and benefits are symmetric across legislators.

If no sidepayments:

$$\text{if } b > \frac{rc}{2}, \text{ then } j = \frac{N+1}{2}$$

$$\text{if } b \leq \frac{rc}{2}, \text{ then } j = 0.$$

If sidepayments permitted:

$$\text{if } b > rc, \text{ then } j = N$$

$$\text{if } rc > b > \frac{rc}{2}, \text{ then } j = \frac{Nb}{rc}$$

$$\text{if } b < \frac{rc}{2}, \text{ then } j = 0.$$

There is again a divergence between policies which are socially efficient and those which we can expect to be chosen. With or without sidepayments, inefficient projects (i.e., those with $c > b > rc/2$) will be chosen; but also socially efficient projects will be chosen. However, the size of the coalitions in the two cases will be different when $b > rc$. Introducing a discount factor r increases the likelihood of social inefficiency. It also may dramatically increase the likelihood of a unanimity norm. The closer r to zero, the greater the inefficiency and the more certain it is that the chosen group decision rule will be that of unanimity. *Weingast* [1978, 12–13] has pointed out that some legislators may be concerned only with those constituents in their own district who provide their support and thus may discount costs borne by constituents not in their supporting coalition. Such an analysis leads to results analogous to those reported above, where we now interpret r to be the proportion of constituents in the legislator's support coalition. Similarly, we may drop the assumption of b and c identical for all legislators without substantially changing the tenor of our conclusions [cf. *Weingast*, 1978, 9–10]. *Weingast* [1978, 14–15] has also made the important point that, in the model considered above, greater social efficiency will customarily result from a universalism norm than from one involving minimum winning coalitions.

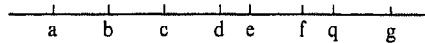
3. Choice Among Alternatives on an Unidimensional Continuum

Consider a decision-making body of size N , N for simplicity odd, choosing a policy position along some unidimensional continuum such as liberal-conservative dimension. [For evidence that legislative and/or judicial decisions may reflect such policy continua see *Rohole/Spaeth: MacRae*.] Assume that each actor has a most

preferred point somewhere on the continuum and evaluates the relative desirability of each alternative in terms of its distance from that most preferred point. For simplicity, let us assume the utility function to be the Euclidean distance metric, although this assumption will not prove crucial to our subsequent analysis. [For alternative assumptions see the discussion of spatial modeling in *Riker/Ordeshok*.]⁷⁾ Finally, let us also assume that the status quo can be located as a point along that continuum.⁸⁾ We shall label this unidimensional policy choice situation as Case C, a legislative body engaging in pairwise choice among alternatives which may be aligned on a unidimensional (policy) continuum. For simplicity, we shall give proofs based on a seven-member decision body. More general proofs may readily be constructed.

Results 6: Policy choice for a decision-making body making pairwise choices from alternatives ordered along a unidimensional continuum does not constitute a zero sum game.

Proof: Consider the example below of a seven-member legislative body.



We have identified the most preferred points of our seven legislators (a through g) and we have labeled the status quo as q . Consider the choice between q and the ideal point of the voter f . Let $\bar{f}q$ represent the distance between f and q . Six voters gain by this move, $\bar{f}q$ each, and one loses $\bar{f}q$. The net gain is $5\bar{f}q$. A choice between q and e results in a net gain of $5\bar{f}q + 3\bar{e}f$. A choice between q and d (the ideal point of the median voter) yields the maximal net gain possible of $5\bar{f}q + 3\bar{e}f + \bar{d}e$.

The sum of the loser's losses and winner's gains is not a constant. It is true that there is considerable conflict of interest in this legislative game between those to the left of q and those to the right of it, but conflict of interest in this situation is more complex [e.g., both d and b prefer e to q , while g does not; both d and g prefer e to c while b does not; cf. *Axelrod*].

Result 7: Consider a choice between two alternatives, s and t (where s or t may be the status quo point, q). Policy choice for a legislative body under the above assumptions, for s and t on opposite sides of the median voter's ideal point, results in minimal winning coalitions only when there are exactly as many members with ideal points between s and the ideal point of the median legislator as there are between t and the ideal point of the median legislator. When s and t are both on the same side of the median voter's ideal point, unless they are both between that ideal point and that of the ideal point of the next nearest legislator, then the winning coalition is never minimal winning. In a choice between the ideal point of the median legislator and an alternate representing the ideal point of some other legislator, only when that alternative

⁷⁾ Although we do not discuss this point further, we believe that the results we present below could be generalized to the multidimensional case.

⁸⁾ See footnote 3.

represents the ideal point of the legislator immediately to the right or left of the median legislator will the winning coalition be minimal winning.

Proof: Straightforward. Details left as exercise for the reader.

We may interpret Result 7 to imply that minimal winning coalitions will be the exception rather than the rule if alternatives representing the ideal points of each legislator are randomly paired against one another. Indeed, the likelihood of an *exactly* bare minimal winning coalition goes to zero as the size of the legislature grows.

It might be argued that the assumption of random pairing in Result 7 is grossly unrealistic in that we would expect that the status quo would be the outcome of a spatial voting game which resulted in an outcome reflecting the ideal point of the median voter [*Downs*; see also *Black*], and that only when the median preference had shifted would any change to the status quo be proposed. It might, furthermore, be expected that any such changes would, at least in the short run, probably reflect only limited movement from the previously established outcome and thus give rise to choices which nearly evenly divided the legislature and thus resulted in minimal winning or near minimal legislative coalitions. While we would agree that the assumption of random pairing is unrealistic, so, too, is the assumption that the status quo (and all proposed alternatives to it) lies close to the ideal point of the median voter.

When a policy area comes up year after year for a vote, presumably the status quo will be reasonably close to the ideal point of the median legislator and members will have very accurate perceptions as to where the median preference in the body lies. In such a case we would expect that one or more alternatives very close to ideal point of the median legislator will be proposed. Thus, we would expect that, under standard amendment procedure [See *Farquharson*; *Grofman*, 1969], the final vote in a well-ploughed issue area will pair two alternatives (one of which is the status quo) which are both close to the ideal preference of the median legislator.⁹⁾ Hence, this final vote should give rise to a coalition which is close to minimal winning. In a well-ploughed issue area, even earlier votes might do so if most of the alternatives being posed were near to the ideal point of the median voter.¹⁰⁾ Of course, the location of the median preference may shift over time due either to turnover in the legislature or shifts in policy judgments, which may be generated by external pressures or events.

However, in the case of legislation which is expiring, the status quo would be no bill at all. No legislation at all is very unlikely to yield a position on the issue continuum near the ideal point of the median legislator. Hence, under the above assumptions:

⁹⁾ If legislators vote sincerely, once the median alternative is entered, under standard amendment procedure, it will beat all other alternatives to it in pairwise competition [*Black*; *Farquharson*].

¹⁰⁾ This observation might also be true for certain Supreme Court decision arenas where the court is constrained by force of public opinion and the realities of political power to move with a deliberate speed that resembles the pace of a snail. For example, the overthrow of the "separate but equal" doctrine in *Brown v. Board of Education* was not an overnight revolution. The Court had steadily chipped away at this doctrine in the decade prior to the *Brown* decision. In such cases, where alternatives are constrained we would not expect outcomes to always be at the ideal point of the median jurist. In such cases, hence, we might expect decision coalitions to be often supra-minimal.

Result 8: Under standard amendment procedure in which the final vote is a vote between the amended bill and the status quo, we would expect that the final vote on a bill to replace a piece of legislation which is due to expire would not give rise to a minimal winning majority.

Similarly, we would expect that votes in a new issue area would not be minimal winning, since the median point may not yet be easily located.

Let us now briefly consider the implications for coalition size of an incremental approach to policy making. Such a norm of incremental change might be expected to apply to a variety of issue domains confronting legislators, e.g., budgetary questions. [For discussion of related points, see *Wildavsky*.] We believe that, when choice of alternatives is constrained by a norm of incremental change from the status quo, under the above assumptions, such a constraint would make it even more likely that the winning coalitions would be larger than minimal winning. This argument can be clarified by an example. Consider again the seven-member legislature discussed above. If the only feasible choices are q vs. f or q vs. g , then coalitions larger than minimal will result. Only when q is close to the ideal point of the median legislator could we expect close to minimal winning coalitions, and if we began with a status quo at variance with the median preference it would require a number of incremental shifts before coalitions would be anywhere near the minimal winning size.

4. Conclusions

While our models are clearly intended as simplifications of a complex reality, we hope nonetheless to have demonstrated that the most common sorts of decisions facing a legislature are not ones which give rise to zero-sum conflicts, and we need not expect winning voting coalitions to be necessarily of minimal winning size.¹¹⁾

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¹¹⁾ For other models of coalitions involving policy/ideological dimensions which also do not necessarily give rise to minimal winning coalitions, see *Grofman* [1982], *Owen/Grofman* [this volume].

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