MODELING NEGATIVE CAMPAIGNING

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Negative campaigning is an important aspect of campaign competition but plays little or no role in existing models of campaigns. Within the context of plurality elections for a single office we model the incentives that affect the use of negative campaigning. Under simplifying but still quite general assumptions we show a number of results, including the following key conclusions: (1) for two-candidate competition the front-runner will engage in more positive and less negative campaigning than the opponent; (2) in a three-candidate contest with one candidate clearly trailing by a large margin and playing mainly a spoiler role, that candidate will only engage in positive campaigning; and (3) in any three-candidate contest, no candidate engages in negative campaigning against the weaker of his two opponents, so that to the extent there is negative campaigning, it will be directed against the front-runner or it will come from the front-runner. These results have direct empirical applications to multicandidate primaries and nonpartisan contests and can provide insight into recent general elections as well.

We present a model of negative campaigning, an important aspect of campaign competition that has received a great amount of journalist attention. Negative campaigning has been a focus of political communication studies (see, e.g., Garramone 1984; Johnson-Cartee and Copeland 1989, 1991; Pfau and Kenski 1990), as well as some important experimental research (e.g., Ansolabehere and Iyengar 1993, Roddy and Garramone 1988). However, it has largely been neglected by formal modelers. (Harrington and Hess 1993 and Thomas 1990 are notable exceptions). Adapting terminology from Surlin and Gordon, we use the term *negative campaigning* to refer generally to that which "attacks the other candidate personally, the issues for which the other candidate stands, or the party of the other candidate" (1977, 93).

While there is dispute about the prevalence and intensity of negative political campaigning today as compared to some earlier periods of American history—when, according to one noted historian, "no accusation was too coarse or too vulgar to be made" (Wood 1978, 109; see also Beiler 1992)—most students of contemporary politics in the United States are in agreement that the level of negativity in campaigns has, in general, been rising considerably in recent decades. For example, Pfau and Kenski state that "the 1980s experienced an explosive growth in attack politics" (1990, 13), while Young asserts that "today one of two political ads are negative; twenty years ago only about one in five were" (1987, 66). Moreover, the consensus among political journalists and consultants, even if not backed by evidence that is fully compelling in social science terms, is that "[while] there is room for argument about whether negative ads will damage the system in the long run, there is no argument about their short-term impact, [that] they work and they win elections. Voters pay attention to them" (Ehrenhalt 1985, 2560).[2]

Ansolabehere and Iyengar note that until recently, academic research has suggested that "campaign communication in general, and advertising in particular, have little persuasive impact on voters" and go on to comment that "the argument that campaign advertising has only minimal effects is difficult to reconcile with candidate's actual behavior. All serious campaigns invest heavily in advertising" (1993, 1–2). The increasingly larger proportions of campaign budgets that are devoted to the media, survey studies that show the striking memorability of some political ads, especially "attack" ads (e.g., Johnson-Cartee and Copeland 1989), experimental work by scholars such as Ansolabehere, Iyengar, and Kinder showing the strong effects of political advertising and political rhetoric (e.g., Ansolabehere et al. 1994; Iyengar, Peters, and Kinder 1982), and the recent work of Bartels (1993) showing measurement error effects in using survey data to judge the impact of campaign events on voter choice all point toward paying far more attention to campaign-specific processes of political persuasion than has often been the case in political science models of voter choice based on party identification, issue proximity, or the impact of economic conditions on incumbent performance evaluations.

We are here interested in specifying how key factors such as the closeness of the race and the number of candidates can be expected to affect the division of each candidate's efforts between positive and negative campaigning. We model a limited but important part of the big picture of political competition that is omitted in most standard Downsian modeling, namely, the decisions by candidates to inform voters about (alleged) negative aspects of their opponent's character, history, and issue stands in addition to informing voters about themselves and their positions.

Our model builds on various commonsense attributes of present-day politics (e.g., that candidates know where they stand in the polls but do not know future vote shares with certainty, that some voters are undecided, that voters can change their mind over the course of the campaign and even move from support to a position of undecided or vice versa, and that campaigning can matter) and one key assumption about political competition, namely, that negative campaigning reduces the support level of the candidate who is attacked and may also reduce, at least in the short run, the attacking candidate's own support because of voters who are disgruntled by negative campaigning. The generally alienating effect of negative campaigning on voters has been identified in survey data (Garramone 1984) and in experiments (Ansolabehere et al. 1994). For example, Garramone reports that over three-quarters of the survey respondents expressed disapproval in their evaluation of negative advertising. As for the losses accruing to the attacker, Garramone (1984) has found strong support and refers to the effect as the "boomerang effect."[3] To allow for a candidate rationally to engage in negative campaigning, we maintain that the negative effect (vote loss) must be greater for the candidate being attacked than for the attacker.[4]

Under a simple but still quite general formulation (e.g., positing a function that describes what happens to each candidate's expected vote share as a result of any allocation of effort to negative and positive campaigning), in which we model a campaign as a series of snapshots of an evolving dynamic process,[5] we show a number of interesting results, including the following key conclusions:

Ceteris paribus, (1) for two-candidate competition there will often be a mix of positive and negative campaigning, but the front-runner can be expected to engage in more positive, and less negative, campaigning than his opponent; (2) in two-candidate competition, a candidate's negative campaigning is often greater the stronger the opponent's support; (3) in a three-candidate contest with one candidate clearly trailing by a large margin and playing mainly a spoiler role, that candidate will only engage in positive campaigning; and (4) in any three-candidate contest, no candidate engages in negative campaigning against the weaker of his two opponents, so that any negative campaigning will be directed against the front-runner or will come from the front-runner.

We shall consider at the end how well these conclusions fit evidence on negative campaigning. Now we turn to an exposition of the model.

TWO-CANDIDATE COMPETITION

There are two candidates, labeled 1 and 2. Each candidate has one unit of time or resource that can be allocated between positive campaigning, denoted y_i, and negative campaigning, x_i, so that

$$1 = x_i + y_i \quad i = 1, 2. \tag{1}$$

We suppose that the initial support for each candidate, r_1 for 1 and r_2 for 2, is common knowledge. The same is, therefore, true for the fraction of remaining undecided voters $R = 1 - r_1 - r_2$. Because undecided voters can be expected to be both more active seekers of information about the candidates and more liable to be convinced by one of them compared to the voters who are already decided, we suppose that positive campaigning attracts undecided voters. In particular, for any given pair of positive campaigning by the two candidates, (y_1, y_2), let $q^i(y_1, y_2)$ denote the share of undecided voters received by candidate $i (= 1, 2)$. We assume the following properties for these *positive campaigning sharing functions:*

Assumption 1. $q^1(y_1, y_2)$ *is increasing in* y_1 *and decreasing in* y_2; $q^2(y_1, y_2)$ *is decreasing in* y_1 *and increasing in* y_2.

Assumption 2. $q^1(y_1, y_2) = q^2(y_2, y_1)$ *for all* (y_1, y_2).

Assumption 3. $q^1(y_1, y_2) + q^2(y_1, y_2) = 1$ *for all* (y_1, y_2).

Assumption 1 says that the share of undecided voters received by each candidate is increasing in the amount of positive campaigning undertaken by the candidate himself and decreasing in the amount of the opponent's positive campaigning. Assumption 2 is a symmetry property. It implies that if both candidates undertake the same amount of positive campaigning, then they attract equal shares of the undecided voters. Finally, assumption 3 states that each of today's undecided voters decides to support one of the two candidates tomorrow. We adopt this property for convenience; our results would follow through with more realistic, but more complicated, alternatives.[6] Given assumptions 1–3, it can be shown that there is a symmetric function $q(\cdot, \cdot)$, increasing in its first argument, decreasing in its second argument, and such that $q^1(y_1, y_2) = q(y_1, y_2)$ and $q^2(y_1, y_2) = q(y_2, y_1) = 1 - q(y_1, y_2)$.[7] From now on, we shall use the convention of denoting $q^1(y_1, y_2) = q(y_1, y_2)$ by q and thus denoting $q^2(y_1, y_2) = 1 - q(y_1, y_2)$ by $1 - q$. A general class of functions satisfying assumptions 1–3 is

Assumption 4. $q^i(y_1, y_2) = f(y_i)/[f(y_1) + f(y_2)]$, *where* $f(\cdot)$ *is an increasing nonnegative function.*

Whereas positive campaigning attracts undecided voters in the way we have just specified, negative campaigning is undertaken in order to reduce the support of one's opponent by turning a fraction of the opponent's current supporters into undecided voters. Negative campaigning, however, is not just harmful for the recipient of the negative attack. It harms the initiator as well, through the "boomerang effect," by turning off some of the initiator's current supporters and turning them into undecided voters but at a lower rate than for the recipient of negative campaigning. As discussed in the introduction, incorporating the "boomerang effect" makes our model consistent with some strong empirical evidence. In its absence, however, our qualitative results would be unaffected, in the sense that candidates would engage in more negative campaigning compared to the case with the "boomerang effect" but the relative magnitudes and other characterization results would be unaffected. In particular, for any given pair (x_1, x_2) of negative campaign allocations by the two candidates, the reduction in support for candidates 1 and 2 are assumed to take the form

$$B(x_1 + Ax_2) \cdot r_1 \quad \text{and} \quad B(x_2 + Ax_1) \cdot r_2, \tag{2}$$

respectively, where A (>1) measures by how much more negative campaigning hurts the victim relative to the instigator of a negative attack. The parameter B is an overall measure of the effectiveness of negative campaigning; a higher B implies a higher sensitivity of voters to negative campaigning.

Recapitulating the different pieces of the model thus far, the candidates choose how much effort to allocate between positive and negative campaigning (see equation 1); the positive campaigning efforts determine the relative shares of the undecided voters received by the two candidates; and negative campaigning determines the support subtracted from each candidate according to equation 2 with the lost voters becoming new undecided voters.

Putting these pieces together, the support for each candidate in the subsequent period becomes:

$$r_1^0 = r_1 + q(y_1, y_2)R - B(1 - y_1 + A(1 - y_2))r_1 + \varepsilon_1$$

$$r_2^0 = r_2 + (1 - q(y_1, y_2))R - B(1 - y_2 + A(1 - y_1))r_2 + \varepsilon_2. \tag{3}$$

The ε_is are zero-mean error terms subject to the restriction $r_i^0 + r_j^0 \leq 1$. Note that by the use of equation 1, we have eliminated the x_is, and the new support levels in equation 3 can be considered just functions of the y_is; this makes the analysis easier later on, but remember that $1 - y_i = x_i$ for both i.

We take as each candidate's objective the maximization of the difference between expected support and the expected opponent's support (i.e., $V^i = E(r_i^o) - E(r_j^o)$, where $i \neq j$ and $E(\cdot)$ is the expectation operator) which, given equation 3, yields the following payoff functions:[8]

$$V^1(y_1, y_2) = (2q(y_1, y_2) - 1)R - B(1 - y_1 + A(1 - y_2))r_1 + B(1 - y_2 + A(1 - y_1))r_2 + r_1 - r_2$$

$$V^2(y_1, y_2) = (1 - 2q(y_1, y_2))R - B(1 - y_2 + A(1 - y_1))r_2 + B(1 - y_1 + A(1 - y_2))r_1 + r_2 - r_1. \tag{4}$$

These two payoff functions along with the set of possible positive campaigning pairs (y_1, y_2) define a game in strategic (normal) form. We shall employ the concept of noncooperative (Nash) equilibrium as a solution concept. A strategy pair (y_1^*, y_2^*) is an *equilibrium* if

$$V^1(y_1^*, y_2^*) \geq V^1(y_1, y_2^*) \quad \text{for all } y_1$$

and

$$V^2(y_1^*, y_2^*) \geq V^2(y_1^*, y_2) \quad \text{for all } y_2. \tag{5}$$

To guarantee that such an equilibrium exists we need to introduce an additional property on the sharing function $q(\cdot, \cdot)$:

ASSUMPTION 5. $q_{11} \equiv \partial^2 q(y_1, y_2)/\partial y_1^2 \leq 0$ *(which, by symmetry, is equivalent to* $q_{22} \equiv \partial^2 q(y_1, y_2)/\partial y_2^2 \geq 0$)

According to assumption 5, a candidate's share of the undecided voters is a concave function of the candidate's own positive campaigning effort. In other words, other things equal, there are diminishing returns to each candidate's positive campaigning; the more positive campaigning undertaken, the lower the extra support received. We think this property is empirically plausible, but it could prove that some campaigns do not conform to it. In such a case a pure-strategy equilibrium might not exist. A mixed-strategy equilibrium, however, would exist; and although it would be technically very difficult to characterize such an equilibrium, there is no a priori reason for a reversal of the various results we obtain later on.

THEOREM 1. *In two-candidate competition make assumptions 1–3 and 5. Then the payoff function of each candidate is concave in the candidate's own strategy, and a pure-strategy equilibrium exists.*

(For the proofs of theorem 1, as well as of other results not found in the main body of the paper, see the Appendix.)

For the remainder of this section, we assume that assumptions 1–3 and 5 are satisfied. We now proceed with the characterization of equilibrium strategies and outcomes. Let $V_1^1 \equiv \partial V^1(y_1, y_2)/\partial y_1$ and let V_2^2 be similarly defined. From equation 4 it follows that

$$V_1^1(y_1, y_2) = 2q_1(y_1, y_2)R - B(Ar_2 - r_1) \tag{6}$$

$$V_2^2(y_1, y_2) = -2q_2(y_1, y_2)R - B(Ar_1 - r_2), \tag{7}$$

(where $q_i = \partial q/\partial y_i$). The first term of each of these derivatives represents the marginal benefit of putting an infinitesimally small extra effort into positive campaigning, whereas the second term represents its marginal cost, which also represents the marginal benefit of negative campaigning. If in equilibrium a candidate were to put positive amounts of effort into both positive and negative campaigning (in symbols, $0 < y_i^* < 1$), then the marginal benefit and the marginal cost must be equated and the derivative in equations 6–7 would be set equal to zero for that candidate. (Otherwise, the candidate would have an incentive to change strategy, thus contradicting the definition of equilibrium in equation 5.) There is no reason, however, that in every possible circumstance the equilibrium must satisfy this condition and in fact it does not. Cases in which one or both candidates choose to put all their effort into either positive or negative campaigning (y_i^* is equal to 0 or 1 for at least one $i = 1, 2$) are interesting and make intuitive sense, as we shall see.

Suppose, for specificity, that $Ar_2 \leq r_1$, which means, given that $A > 1$, that the initial support for candidate 2 is lower than that for candidate 1 and, depending on how large A is, it can be much lower. Then, since by equation 6 q_1 is positive, it must be the case, for all possible values of (y_1, y_2), that

$$V_1^1(y_1, y_2) > 0 \quad (\text{where } r_1 \geq Ar_2). \tag{8}$$

Since, by theorem 1, V^1 is concave in y_1, the optimal choice of y_1 must equal one (all effort put into positive campaigning) regardless of candidate 2's choice of y_2. It then follows that in equilibrium candidate 1 would put all effort into positive campaigning. Obviously, when $r_1 \geq Ar_2$ we cannot simultaneously have $r_2 \geq Ar_1$; thus we cannot use the same argument to establish that candidate 2 puts all effort into positive campaigning whenever candidate 1 does so. In fact, an inspection of equation 7 in this case reveals that when $r_1 \geq Ar_2$, for candidate 2 to put all effort into positive campaigning becomes less likely, though not impossible. In the absence of a condition like $r_1 \geq Ar_2$, equation 8 could be true if B (the relative *effectiveness* of negative campaigning) is small enough or A (the harm of negative campaigning to the receiver relative to the harm done to the attacker) is low enough. We summarize our result thus far as a proposition:

PROPOSITION 1. *If a candidate's initial support is sufficiently high relative to that of the opponent, that candidate will not engage in negative campaigning. If negative campaigning is sufficiently ineffective or sufficiently harmful to the attacker, negative campaigning is less likely.*

Next consider a sharing function $q(\cdot, \cdot)$ such that the derivative $q_1(0, y_2)$ (or, symmetrically, $q_2(y_1, 0)$) is finite. Then for B and A sufficiently high or r_1 sufficiently low, the following inequality may be true regardless of the value of y_2:

$$V_1^1(0, y_2) = 2q_1(0, y_2)R - B(Ar_2 - r_1) \leq 0. \tag{9}$$

In this case, candidate 1 would always choose to put all effort into negative campaigning ($y_1^* = 0$). Again, we summarize this result below.

PROPOSITION 2. *If candidate* i*'s initial support is sufficiently low or attacking one's opponent is sufficiently effective and does not do much harm to the attacker, then candidate* i *may put all of his effort into negative campaigning.*

We now move away from the polar cases we just examined and concentrate on equilibria (y_1^*, y_2^*) in which both candidates choose to do both positive and negative campaigning ($0 < y_1^* < 1$ and $0 < y_2^* < 1$). Note that for this to happen, it is necessary to have both $Ar_1 > r_2$ and $Ar_2 > r_1$. Clearly at such an equilibrium the derivatives in equations 6–7 are equal to zero, or

$$2q_1(y_1^*, y_2^*)R - B(Ar_2 - r_1) = 0$$
$$-2q_2(y_1^*, y_2^*)R - B(Ar_1 - r_2) = 0, \tag{10}$$

which imply

$$q_1(y_1^*, y_2^*) = B(Ar_2 - r_1)/(2R)$$

and

$$-q_2(y_1^*, y_2^*) = B(Ar_1 - r_2)/(2R),$$

respectively. Suppose, for specificity, that candidate 1 has more initial support than candidate 2, so that $r_1 > r_2$. This implies $Ar_1 - r_2 > Ar_2 - r_1$, which, used in equation 10, yields $q_1(y_1^*, y_2^*) < -q_2(y_1^*, y_2^*)$. By Lemma I in the Appendix (derived under assumptions 1–5) and, to guarantee strict inequalities, a minor technical condition), this inequality is equivalent to $q(y_1^*, y_2^*) > 1/2$, which is, in turn, equivalent to $y_1^* > y_2^*$. Therefore, we have shown the following result.

PROPOSITION 3. *The front-runner (i.e., the candidate with higher initial voter support) engages in more positive and less negative campaigning than his opponent.*

To facilitate a more detailed analysis, we now examine equilibria under the following functional form of $q(\cdot, \cdot)$ (note that this function is not defined at $(y_1, y_2) = (0, 0)$):

ASSUMPTION 6. $q(y_1, y_2) = \dfrac{y_1}{y_1 + y_2}$.

Under assumption 6, the equilibrium conditions (equation 10) become

$$\frac{2y_2}{(y_1 + y_2)^2} R - B(Ar_2 - r_1) = 0 \tag{11}$$

$$\frac{2y_1}{(y_1 + y_2)^2} R - B(Ar_1 - r_2) = 0. \tag{12}$$

One way of getting a better sense of the strategic interactions is to derive the reaction functions of the two candidates using equations 11–12.[9] Denote by $\rho_1(y_2)$ the reaction function of candidate 1 and by $\rho_2(y_2)$ the reaction function of candidate 2. Then it can be shown, by using equations 11–12, that

$$\rho_1(y_2) = y_2^{1/2}\left[\frac{2R}{(Ar_2 - r_1)B}\right]^{1/2} - y_2$$
$$\rho_2(y_1) = y_1^{1/2}\left[\frac{2R}{(Ar_1 - r_2)B}\right]^{1/2} - y_1. \tag{13}$$

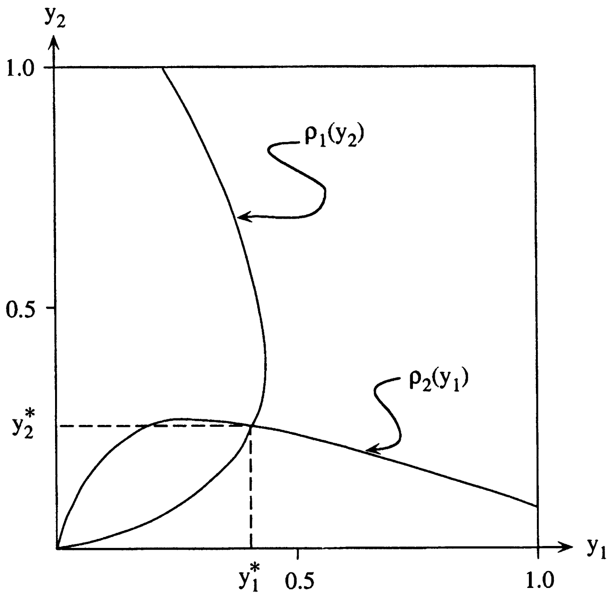
To find the equilibrium, we can set $y_1 = \rho_1(y_2)$ and $y_2 = \rho_2(y_1)$ and then solve equation 13 for y_1 and y_2. A typical diagrammatic configuration of equation 13 is shown in Figure 1. An equilibrium is at an intersection of the two graphs. Since assumption 6 is undefined at $(y_1, y_2) = (0, 0)$, the relevant intersection of ρ_1 and ρ_2 is at (y_1^*, y_2^*), as indicated in the figure.

It is straightforward, though tedious, to find a general closed-form solution for the interior equilibrium (y_1^*, y_2^*):

$$y_1^* = \frac{2R(Ar_1 - r_2)}{B(Ar_1 - r_2 + Ar_2 - r_1)^2} = \frac{2(1 - r_1 - r_2)(Ar_1 - r_2)}{B(r_1 + r_2)^2(A - 1)^2} \tag{14}$$

FIGURE 1

Reaction Functions with A = 2, B = 1, r_1 = .4, and r_2 = .35



$$y_2^* = \frac{2R(Ar_2 - r_1)}{B(Ar_1 - r_2 + Ar_2 - r_1)^2} = \frac{2(1 - r_1 - r_2)(Ar_2 - r_1)}{B(r_1 + r_2)^2(A - 1)^2} \qquad \textbf{(15)}$$

Based on equations 14–15 we have the following comparative static results:

PROPOSITION 4. *Suppose* $q(\cdot, \cdot)$ *satisfies assumption 6. Then:*

a. *Both candidates increase their positive campaigning when the fraction of undecided voters increases* ($\partial y_i^*/\partial R >$ *for both* i = 1, 2).
b. *Both candidates reduce their positive campaigning when the overall effectiveness of negative campaigning increases* ($\partial y_i^*/\partial B < 0$ *for both* i = 1, 2).
c. *The front-runner will always reduce positive campaigning when* A *increases (i.e., when the harm to the recipient of negative attacks increases). The follower will also reduce positive campaigning when* A *increases, unless the follower is sufficiently far behind the front-runner* ($\partial y_1^*/\partial A \lesseqgtr 0$ *as*

$$\frac{r_1}{r_2} \gtrless \frac{2}{A+1} \quad \textit{and} \quad \partial y_2^*/\partial A \lesseqgtr 0 \quad \textit{as} \quad \frac{r_2}{r_1} \gtrless \frac{2}{A+1}\Bigg).$$

d. *Negative campaigning is increasing in the opponent's initial support* ($\partial y_i^*/\partial r_j < 0$, i ≠ j).
e. *The effect of a candidate's own support level on equilibrium allocation of effort between positive and negative campaigning is ambiguous. In particular, we have*

$$\partial y_i^*/\partial r_i \gtrless 0 \quad \textit{as} \quad r_i \lesseqgtr r_j \frac{(A+1)R+1}{A} \quad \textit{where } i \neq j.$$

THREE-CANDIDATE COMPETITION: TWO MAIN CANDIDATES AND A SPOILER

Now consider candidate competition with three candidates, labeled 1, 2, and 3. We shall examine a special case in which candidate 3 has sufficiently low initial support that candidates 1 and 2 do not consider 3 a threat. These two main candidates have similar payoff functions to those in the two-candidate case ($U^1 = r_1^0 - r_2^0$ and $U^2 = r_2^0 - r_1^0$), whereas candidate 3, the spoiler, maximizes just his or her own percentage support ($U^3 = r_3^0$).

Candidates 1 and 2 allocate their effort between positive campaigning just as they do in the two-candidate cases and described in equation 1; there is no issue of attacking candidate 3, since they do not consider 3 a threat.[10] Moreover, because negative campaigning harms the attacker and candidate 3 cares only about his or her own support, candidate 3 would not want to engage in negative campaigning and would put all effort into positive campaigning (i.e., $y_3 = 1$ regardless of the choices of the two other candidates). Thus the first three-candidate model is the two-candidate model with the difference that some currently undecided voters will go to a third candidate. Allowing for this difference requires the specification of a sharing function among three, instead of just two, candidates. For any given triple (y_1, y_2, y_3) of positive campaigning efforts, let $q^i(y_1, y_2, y_3)$ denote the share of undecided voters received by candidate i (= 1, 2, 3). We maintain the following properties:

ASSUMPTION 7. $q^i(y_1, y_2, y_3)$ *is increasing in* y_i *and decreasing in* y_j *for* i ≠ j *for all* i.

ASSUMPTION 8. *Interchanging the positive campaigning efforts of two candidates is equivalent to an interchange of their shares of undecided voters.*

ASSUMPTION 9. $q^1(y_1, y_2, y_3) + q^2(y_1, y_2, y_3) + q^3(y_1, y_2, y_3) = 1$.

Note that these properties mirror assumptions 1–3 of the two-candidate case, with assumptions 8 and 9 implying $q^i(y, y, y) = q^j(y, y, y) = 1/3$ for all i, j and for all y. (In words: If all three candidates put the same amount of effort, the shares of undecided voters received are the same for all the candidates).

With $y_3 = 1$, the payoff functions of the three candidates are

$$\begin{aligned}
U^1(y_1, y_2, 1) &= [q^1(y_1, y_2, 1) - q^2(y_1, y_2, 1)]R - B[1 - y_1 \\
&\quad + A(1 - y_2)]r_1 + B[1 - y_2 + A(1 - y_1)]r_2 + r_1 - r_2 \\
U^2(y_1, y_2, 1) &= [q^2(y_1, y_2, 1) - q^1(y_1, y_2, 1)]R - B[1 - y_2 \\
&\quad + A(1 - y_1)]r_2 + B[1 - y_1 + A(1 - y_2)]r_1 + r_2 - r_1 \\
U^3(y_1, y_2, 1) &= q^3(y_1, y_2, 1)R + r_3. \qquad \textbf{(16)}
\end{aligned}$$

An equilibrium for the strategic form game with these payoff functions is defined similarly to equation

5. Such an equilibrium is shown to exist, under a condition similar to assumption 5, in theorem 2. For comparison purposes here, we examine the equilibrium under the following adaptation of assumption 6:

ASSUMPTION 10. $q^i(y_1, y_2, y_3) =$

$$\frac{y_i}{y_1 + y_2 + y_3} \quad \textit{for all } i = 1, 2, 3.$$

In an interior equilibrium $(\bar{y}_1, \bar{y}_2)$, we have

$$U_1^1(\bar{y}_1, \bar{y}_2, 1) = 0 \quad \text{and} \quad U_2^2(\bar{y}_1, \bar{y}_2, 1) = 0,$$

which, under assumption 10, become

$$\frac{2\bar{y}_2 + 1}{\bar{y}_1 + \bar{y}_2 + 1} R = B(Ar_2 - r_1) \qquad \textbf{(17)}$$

$$\frac{2\bar{y}_1 + 1}{\bar{y}_1 + \bar{y}_2 + 1} R = B(Ar_1 - r_2), \qquad \textbf{(18)}$$

respectively. In turn, dividing equation 17 by equation 18 yields

$$\frac{2\bar{y}_2 + 1}{2\bar{y}_1 + 1} = \frac{Ar_2 - r_1}{Ar_1 - r_2}.$$

Without loss of generality suppose $r_1 > r_2$ (candidate 1 is the front-runner). Then, since $Ar_2 - r_1 < Ar_1 - r_2$, the above equation implies $\bar{y}_1 > \bar{y}_2$. Thus, as in proposition 3, *the front runner continues to engage in more positive campaigning than the main opponent.*

Solving equations 17–18 yields

$$\bar{y}_1 = \frac{2(1 - r_1 - r_2 - r_3)(Ar_1 - r_2)}{B(r_1 + r_2)^2 - (A - 1)^2} - 1/2 \qquad \textbf{(19)}$$

$$\bar{y}_2 = \frac{2(1 - r_1 - r_2 - r_3)(Ar_2 - r_1)}{B(r_1 + r_2)^2(A - 1)^2} - 1/2. \qquad \textbf{(20)}$$

Note that the first terms of equations 19 and 20 with $r_3 = 0$ are equal to y_1^* and y_2^*, respectively—the equilibrium positive campaigning efforts reported in equations 14–15 for the case of pure two-candidate competition. Consequently, the presence of the third spoiler candidate reduces positive campaigning of both main candidates. The reason for this lower positive campaigning for a given share of undecided voters is that its marginal return is reduced for the two main candidates, since candidate 3 puts all his effort into positive campaigning and takes a part of the undecided votes, thus reducing the total number of undecided voters left to be captured by candidates 1 and 2. The two main candidates then find negative campaigning relatively more profitable. Note also from equations 19–20 that for the two main candidates to put at least some effort into positive campaigning $(\bar{y}_1, \bar{y}_2 > 0)$, y_1^* and y_2^* must be greater than 1/2. That is, the conditions for an interior equilibrium are more stringent in this case. Finally, since $\bar{y}_1$ and $\bar{y}_2$ differ from y_1^* and y_2^*, respectively, by only a constant when $R\ (= 1 - r_1 - r_2 - r_3)$ is fixed, the comparative static results reported in proposition 4 are also valid here. We summarize these results as proposition 5:

PROPOSITION 5. *Suppose a long-shot third candidate enters the competition between the two main candidates, with the payoff functions described in equation 16, and make assumption 6. Then:*

a. The long-shot candidate only engages in positive campaigning, whereas the two main candidates intensify their negative campaigning.
b. All the results reported in proposition 4 continue to hold for the two main candidates.

We should emphasize that although the entrance of the long-shot candidate increases the two main candidates' negative campaigning, the overall ratio of negative to positive campaigning could well decrease since the long-shot candidate engages solely in positive campaigning. In addition, the results reported in proposition 5 do not have to be valid in general three-candidate contests to which we now turn.

THREE-CANDIDATE COMPETITION: A GENERAL MODEL

In some elections, especially primaries, there are more than two candidates who at the outset have a realistic chance of winning. In the course of such campaigns, the phenomenon of "ganging on the front-runner"—all the other candidates attacking the front-runner— is often observed. We shall examine a general three-candidate model that includes the model of the previous section as a special case and that helps us understand those concerted attacks against front-runners. The model and the results can easily be extended to contests with any number of candidates, but at additional notational burden.

Again, we label the candidates, 1, 2, and 3. Each candidate i allocates effort among positive campaigning (y_i), negative campaigning against candidate j (x_i^j), and negative campaigning against candidate k (x_i^k), where $i \neq j \neq k$. Give that each candidate has one unit of resource, the constraint faced by each one of them is

$$1 = x_i^j + x_i^k + y_i \quad \text{for all} \quad i = 1, 2, 3 \quad \text{and} \quad i \neq j, k. \qquad \textbf{(21)}$$

As before, positive campaigning determines the share of undecided voters won over by each candidate, with $q^i(y_1, y_2, y_3)$ denoting the share of candidate i for any positive campaigning triple (y_1, y_2, y_3). We assume the q^i's satisfy assumptions 7–9. A vector of negative campaigning efforts $(x_1^2, x_1^3, x_2^1, x_2^3, x_3^1, x_3^2) \equiv x$, reduces the support of candidates by turning some supporters into undecided voters with the percentage reduction in support for candidate i equaling $B[x_i^j + x_i^k + A(x_j^i + x_k^i)]r_i$. Thus the new support level of candidate i becomes

$$r_i^0 = r_i + q^i(y_1, y_2, y_3)R - B[x_i^j + x_i^k + A(x_j^i + x_k^i)]r_i. \qquad \textbf{(22)}$$

(For brevity, we omit error terms.) The payoffs of the three candidates as functions of the new support levels have the following form:

$$W^1 = r_1^0 - \alpha_2 r_2^0 - \alpha_3 r_3^0$$

$$W^2 = r_2^0 - \beta_1 r_1^0 - \beta_3 r_3^0$$

$$W^3 = r_3^0 - \gamma_1 r_1^0 - \gamma_2 r_2^0. \tag{23}$$

The parameters can take values between 0 and 1, and they represent weights attached by each candidate to the expected support levels of their opponents. That is, the parameters reflect the extent to which a candidate might be more concerned with the vote share and the likely success of one candidate as opposed to another. The model of the previous section is obtained from equation 23 by setting $\alpha_2 = \beta_1 = 1$ and $\alpha_3 = \beta_3 = \gamma_1 = \gamma_2 = 0$ (i.e., the two main candidates care symmetrically about each other's vote shares, whereas the third candidate cares only about how well he or she personally does.) In some cases, the weights could be interpreted as probabilities. For example, with $\alpha_2 = 1 - \alpha_3 = \alpha$, $\beta_1 = 1 - \beta_3 = \beta$, and $\gamma_1 = 1 - \gamma_2 = \gamma$, we can interpret equation 23 as follows: all candidates believe $\text{Prob}(r_2^0 > r_3^0) = \alpha$, $\text{Prob}(r_1^0 > r_3^0) = \beta$, and $\text{Prob}(r_1^0 > r_2^0) = \gamma$, with candidate 1 maximizing $W^1 = \alpha(r_1^0 - r_2^0) + (1 - \alpha)(r_1^0 - r_3^0)$ and similarly for the two other candidates.[11] By substituting equation 22 into equation 23 and taking into account equation 21, the payoffs can be expressed as functions of $x = (x_1^1, x_1^3, x_2^1, x_2^3 x_3^1, x_3^2)$ [we let $y \equiv (1 - x_1^2 - x_1^3, 1 - x_2^1 - x_2^3, 1 - x_3^1 - x_3^2)$]:

$$\begin{aligned}W^1(x) = {} & r_1 - \alpha_2 r_2 - \alpha_3 r_3 + [q^1(y) - \alpha_2 q^2(y) - \alpha_3 q^3(y)]R \\ & + B\{[x_2^1 + x_x^3 + A(x_1^2 + x_3^2)]a_2 r_2 + [x_3^1 + x_3^2 + A(x_1^3 \\ & + x_2^3)]\alpha_3 r_3 - [x_1^2 + x_1^3 + A(x_2^1 + x_3^1)]r_1\} \\ W^2(x) = {} & r_2 - \beta_1 r_1 - \beta_3 r_3 + [q^2(y) - \beta_1 q^1(y) - \beta_3 q^3(y)]R \\ & + B\{[x_1^2 + x_1^3 + A(x_2^1 + x_3^1)]\beta_1 r_1 + [x_3^1 + x_3^2 + A(x_1^3 \\ & + x_2^3)]\beta_3 r_3 - [x_2^1 + x_2^3 + A(x_1^2 + x_3^2)]r_2\} \\ W^3(x) = {} & r_3 - \gamma_1 r_1 - \gamma_2 r_2 + [q^3(y) - \gamma_1 q^1(y) - \gamma_2 q^2(y)]R \\ & + B\{[x_1^2 + x_1^3 + A(x_2^1 + x_3^1)]\gamma_1 r_1 + [x_2^1 + x_2^3 + A(x_1^2 \\ & + x_3^2)]\gamma_2 r_2 - [x_3^1 + x_3^2 + A(x_1^3 + x_2^3)]r_3\}.\end{aligned} \tag{24}$$

To guarantee a pure-strategy equilibrium, we introduce the following three-candidate analogue to assumption 5 (i.e., a candidate's marginal return to positive campaigning is diminishing as the candidate engages in more negative campaigning):

ASSUMPTION 11. *$q^i(y_1, y_2, y_3)$ is a twice-differentiable concave function of y_i for all $i = 1, 2, 3$.*

THEOREM 2. *For three-candidate competition, make assumptions 7–9 and 11. Then the payoff function of each candidate is concave in the candidate's own strategy vector and a pure-strategy equilibrium exists.*

For the remainder of this section and without loss of generality, suppose $r_1 > r_2 > r_3$ and consider the following property:

ASSUMPTION 12. *$\gamma_1 r_1 > \gamma_2 r_2$, $\beta_1 r_1 > \beta_3 r_3$, and $\alpha_2 r_2 > \alpha_3 r_3$.*

Consider the first inequality in assumption 12. The two parameters, γ_1 and γ_2, are weights attached by candidate 3 to the initial support levels of candidates 1 and 2, respectively. Given the convention of $r_1 > r_2$, the inequality states that the weight attached by candidate 3 to the stronger opponent (γ_1) should be sufficiently high relative to the weight attached to the weaker opponent. In particular, rewriting the inequality as $\gamma_1/\gamma_2 > r_2/r_1$, it states that the ratio of weights should be greater than the inverse ratio of initial supports. A candidate who weighs the stronger supporter more than the weaker one always satisfies this property. The two other inequalities in assumption 12 have a similar interpretation.

PROPOSITION 6. *Make assumptions 7–9 and 11–12 and consider any equilibrium strategy combination x^*. Then, no candidate engages in negative campaigning against the weaker of two opponents (i.e., under the convention $r_1 > r_2 > r_3$, we have $x_1^{3*} = x_2^{3*} = x_3^{2*} = 0$).*

An immediate implication of this proposition is that if the two weaker candidates were to engage in negative campaigning they would both attack the front-runner, the phenomenon frequently observed in primary multicandidate elections. If the front-runner were to engage in negative campaigning, it would be directed against the stronger opponent.

To see the logic of this proposition, note that for a candidate to engage in negative campaigning against one opponent, the marginal cost of the last extra unit of negative campaigning must be equated to its marginal benefit. In equilibrium, the marginal cost of negative campaigning against either of a candidate's opponents is the same: it is the marginal return to positive campaigning. On the other hand, the marginal benefits of negative campaigning are different because they take into account the initial support level of each opponent and the weight attached to the support of that opponent by the candidate in question. Given the constancy of the damage inflicted by negative campaigning (the parameters B and A), it then always pays to put all negative campaigning effort, if any, against the opponent with higher marginal benefits, that is, the stronger opponent. Thus if every candidate were to engage in at least some negative campaigning, candidates 2 and 3 would concentrate their attacks against candidate 1 and candidate 1 would attack candidate 2; nobody would attack the weakest candidate 3.

These stark results reported in proposition 6 would not always survive changes in the specification of our model. Most obviously, if a candidate were more vulnerable to negative campaigning on some aspect of his history or issue positions than others, then we could have negative attacks against a weaker candidate with a comparatively large vulnerability.

We could model the different vulnerabilities by, for example, introducing a different B for attacks against each different candidate. Consider, for example, the limiting case with $B = 0$ in equation 22 for candidate 1 (i.e., he is completely invulnerable to attacks), but with $B > 0$ for candidates 2 and 3. Then, by appropriately modifying the payoff functions in equation 24 and performing a similar exercise to that in the proof of proposition 6, it can be easily shown that candidate 1 is never attacked by the other two (weaker candidates). Depending on the values of the parameters in their payoff functions, candidates 2 and 3 might or might not engage in negative campaigning against each other. In other words, as with other results in the paper, proposition 6 should not be interpreted as saying that attacks against weaker candidates should never occur in practice but, rather, that there is a tendency for not attacking weaker opponents in three-way contests.

DISCUSSION

We see our work as a contribution to two different research traditions. On the one hand, there is a considerable body of formal deductive modeling, largely inspired by rational choice ideas, that models campaign competition as a (sequential) process of decision making by candidates about questions such as what policy positions to offer and on how best to spend campaign funds.[12] The present effort is directly complementary to recent work in that tradition, most importantly to that of Harrington and Hess (1993).[13] On the other hand, we also see our endeavor as a contribution to the (largely empirical) literature on public opinion and political persuasion (e.g., Bartels 1993; Franklin 1991; Patterson and McClure 1973; Popkin 1991; Zaller 1992), including important recent experimental work on campaign advertising and rhetoric (Ansolabehere and Iyengar 1991, 1993; Ansolabehere et al. 1994; Iyengar and Kinder 1978; Roddy and Garramone 1988), to which we see ourselves as contributing theoretically significant and testable hypotheses. The original inspiration for the models developed herein derives from two sources, one theoretical and one empirical.

The theoretical source was a seemingly paradoxical result in game theory concerning three-way duels, or *truels*.[14] Often the most accurate duelist has a lower probability of survival than the second-best (or even third-best) shooter. The reason for this result is that the optimal strategy of either of the other duelists is to shoot at the duelist who is the best shot. We began with a trueling model but quickly realized that the assumptions that were peculiar to campaign competition led to results about three-candidate competition in single-member plurality elections that were only tangentially related to earlier results about three-way duels. Nevertheless, our general three-candidate model yields insights similar to that in truels: to the extent there is negative campaigning, the two weaker candidates attack the front-runner and not each other.

The empirical trigger for our work was the discussion by journalists and media pundits of the use and nonuse of negative campaigning in the 1992 presidential campaign, in particular their attempts to explain why both Bush and Clinton appeared reluctant to attack Perot. Journalists who have analyzed the 1992 election offered as a key explanation of the unwillingness of Bush and Clinton to attack Perot personally that neither front-runner wished to be the first to attack Perot because they wished to let sleeping dogs lie and feared that Perot was the kind of person who would retaliate with direct attacks on the candidate who first attacked him even if that cost Perot support. While we do not dispute that this explanation may have been relevant to the calculations of the Democratic and Republican presidential nominees, we are suspicious of explanations that rely *largely* on idiosyncratic features of one of the candidates to account for the key features of the overall dynamic of the campaign when there are regularities that may apply across campaigns when we identify relevant variables and model their impact.

We believe that the model we have proposed (although it is clearly simplified) can nonetheless help us to account for important parts of the 1992 campaign dynamic without requiring us to incorporate features of the election tied to Perot's unique campaign style and personality. Our model calls attention to the very different strategies in a three-way race that makes sense for front-runners and near front-runners, as opposed to strategies that make sense for the candidate who is a distant third.[15] In particular, our three-candidate model with a "spoiler" candidate can help account for (1) why Bush and Clinton as front-runners directed their attacks primarily against each other and not against Perot,[16] (2) why Perot's reentry into the race raised the negativity of the front-runners' campaigning (at least initially),[17] and (3) why it made sense for Perot to concentrate so much on positive campaigning.

But our model has specific empirical implications that are broader than any single campaign. The results are compatible with commonsense intuitions and consistent with a considerable body of evidence that incumbents (who are often front-runners) are much less likely to engage in negative campaigning than are challengers—such as the finding by Kaid and Davidson (1986), in a study of 1982 contests for the U.S. Senate, that only 10% of incumbent ads were negative but 46% of the challenger were negative.[18] Similarly, our results help us account for a pattern, frequently been observed by journalists, of increases in negativity of campaign tactics in two-candidate contests by a candidate whose support is eroding—as occurred in the 1988 presidential campaign when Dukakis' lead shrank as a result of Bush attack ads and in the 1992 Senatorial election in California, when Barbara Boxer became more negative in her ads as Bruce Herschensohn narrowed her initially very large lead. In addition, our results in the general three-candidate model (which can easily be extended to a greater number of candidates) are consistent with the "ganging up on the front-runner" pattern found in numerous primaries and nonpartisan elections such as the anti-Dukakis coalition of 1988 and the anti-(front-runner) Michael Woo stance of many candidates in the 1992 Los Angeles mayoral competition that allowed Richard Riordan to emerge into the runoff relatively unscathed.

Of course, in no way are we claiming that our modeling of the factors influencing negative campaigning is the last word, and we have treated the actual psychological processes of political persuasion as a "black box." Moreover, while the model seems to capture many of the critical elements of the incentive structure for negative campaigning, there are limitations on its ability to capture important subleties. In particular, if we treat *any* evidence to the contrary as disconfirmation even if the overall pattern is one that generally supports the results of the model, there are some predictions of our model that are falsified in the 1992 election. For example, according to a strict interpretation of our model, there should have been no negative campaigning directed against Perot by either Bush or Clinton in the last phase of the campaign when Perot was clearly running third. Yet at the very end of the campaign Bush did attack Perot, referring to him (along with Clinton) as a "bozo." This might have been precipitated by Perot's reporting of stories of plots and threats (such as the one allegedly centered around his daughter's wedding) which damaged his own credibility, almost certainly made him more vulnerable to negative campaigning, and also reduced the probable voter backlash against a front-runner who might choose to attack him. As we indicated, such peculiarities of a campaign could be captured by assigning different vulnerability parameters to different candidates. Although we believe individual candidate characteristics are important in actual campaigns, we have here largely abstracted from them in order to concentrate on tendencies and regularities in negative campaigning that hold independently of individual candidate attributes.

A number of ideas for possible modification to our assumptions have occurred to us. One is to allow for certain "last period" effects, where a candidate who is running second will take desperation measures (since, with loss of the election expected, he has little to lose by strategic experimentation) or will engage in more negative attacks because there is no time for rebuttal. As the model now stands, the last period of a campaign is treated the same way as any other period. Modifying this assumption, however, by allowing different ways of determining the votes of the undecided voters in the last period does not appear to change the qualitative results of two-candidate competition or that of thee candidates with a "spoiler" candidate. Nor, we conjecture, would our results change in the general three-candidate contest even if we were to include last-period effects. We plan to examine this in a subsequent paper.

Another assumption that could be modified has to do with the function that assigns expected gains and losses of various campaign strategies. The model posits that the expected consequences of an attack do not depend upon whether or not it is seen as an unprovoked first strike, do not allow the consequences of an attack to vary with the number of attacks that have previously been made, and do not include effects on voter turnout. Moreover (as noted), we have not incorporated candidate-specific vulnerabilities, although we can do so straightforwardly with predictable results.[19]

However, just as Downs's work on spatial models of political competition launched a body of research that has enhanced our understanding of the electoral process, so we hope that the present paper will lead to more interest by political scientists in describing and modeling negative campaigning and other aspects of political debate. We believe that political persuasion is an important issue that has been little studied by those working in the rational choice tradition. We should note that Downs himself, in perhaps the least cited aspect of his classic work emphasized the importance of political persuasion (1957, 83, 84; see also Grofman and Withers 1993; Weatherford 1993). Nonetheless, the simple median voter story of candidate competition that passes for Downs's view in some American government textbooks has the candidates offering issue positions among which voters choose, with voter choice based simply on voter's relative proximity to platforms proposed by the candidates. Missing from that story are at least four critical elements of politics: (1) the information conveyed by candidates is not only about putative issue positions but also about candidate attributes such as competence and trustworthiness; (2) almost invariably, candidates not only describe themselves and their own policy positions but also seek to (mis)characterize their opponent and their opponent's policy positions as well; (3) in a world of multidimensional issue competition, candidates not only seek to convey the positions they wish to be attributed to themselves and to their opponent but also often seek to persuade voters that some dimensions (some issues) are more important than others; and (4) voters do not believe all of what they are told. Here, we have sought to develop an approach that recognizes that fact that *competing* stories are being told, not just a story each candidate tells about his own policy positions or his own office-worthiness. We see the present essay as a beginning of such modeling efforts for the understanding of one important aspect of political persuasion—the consequences of negative campaigning and the incentives for its use.

APPENDIX A: SYMBOLS

x_i	positive campaigning by candidate i
y_i	negative campaigning by candidate i
r_i	initial support for candidate i (in fractions)

$R = 1 - r_1 - r_2$	share of undecided voters
$q^i(y_1, y_2)$	positive campaigning sharing function for candidate i
$q(\cdot, \cdot)$	positive campaigning sharing function satisfying assumptions 1–3
B	overall effectiveness of negative campaigning
A	relative effectiveness of negative campaigning
r_i^0	new support for candidate i (in fractions)
V^i	payoff function of candidate i in two-candidate competition
V_j^i	$\equiv \partial V^i(y_1, y_2)/\partial y_j$
q_i	$\equiv \partial q(y_1, y_2)/\partial y_i$
$\rho i(y)$	reaction function of candidate i
U^i	payoff function of candidate i in competition with two main candidates and a spoiler
W^i	payoff function of candidate i in general three-candidate competition
x_i^j	negative campaigning of candidate i against candidate j (three-candidate competition)

APPENDIX B: PROOFS

Proof of Theorem 1. To show existence of a pure-strategy equilibrium, it is sufficient to show that the payoff function of each candidate is concave in the candidate's own strategy, that is, we just need to show that V^1 is concave in y_1 and V^2 is concave in y_2. Let $V_1^1 \equiv \partial V^1(y_1, y_2)/\partial y_1$ and let V_2^2 be similarly defined. From the payoff functions in equation 4, it is straightforward to derive the following expressions:

$$V_1^1 = 2q_1 R - B(Ar_2 - r_1)$$

$$V_2^2 = -2q_2 R - B(Ar_1 - r_2),$$

(where $q_i = \partial q/\partial y_i$). Defining V_{11}^1 and V_{22}^2 similarly, we have $V_{11}^1 = 2q_{11}R$ and $V_{22}^2 = -2q_{22}R$. By assumption 5, V_{11}^1 and V_{22}^2 are both nonpositive, and so V^1 and V^2 are concave as required. Q.E.D.

LEMMA 1. *Make assumptions 1–3 and 5 and* $q_{12}(w, z) \neq 0$ *when* $w \neq z$. *Then* $q_1(y_1, y_2) < -q_2(y_1, y_2) \leftrightarrow q(y_1, y_2) > 1/2$.

Proof of Lemma 1. ($\leftarrow$) Suppose $q(y_1, y_2) > 1/2$ ($\leftrightarrow y_1 > y_2$). (Note that we have used assumptions 1–3 in expressing $q^1(y_1, y_2)$ as $q(y_1, y_2)$ and $q^2(y_1, y_2)$ as $q(y_2, y_1)$.) Then by the concavity of the sharing function in the candidate's own positive campaigning, assumption 5 implies

$$q_1(y_1, y_2) \leq q_1(y_2, y_2) \quad \text{and} \tag{25}$$

$$-q_2(y_1, y_2) \geq -q_2(y_1, y_1). \tag{26}$$

Given $q_{12}(w, z) \neq 0$ when $w \neq z$ (and since $y_1 \neq y_2$), we examine the following two cases:

Case 1: $q_{12}(y_1, y_2) > 0$.
Then, given $y_1 > y_2$, we have $q_2(y_1, y_2) < q_2(y_2, y_2)$ which implies $-q_2(y_1, y_2) > -q_2(y_2, y_2)$. Since $-q_2(y_2, y_2) = q_1(y_2, y_2)$, by equation 25 we then have $-q_2(y_1, y_2) > q_1(y_1, y_2)$.

Case 2: $q_{12}(y_1, y_2) < 0$.
This case implies that $q_1(y_1, y_2) < q_1(y_1, y_1) = -q_2(y_1, y_1)$ which by equation 26 is less than or equal to $-q_2(y_1, y_2)$, or that $q_1(y_1, y_2) < -q_2(y_1, y_2)$ as stated in the lemma.

($\rightarrow$) This can be shown, using the same steps as above, that $q(y_1, y_2) \leq 1/2$ implies $q_1(y_1, y_2) \geq -q_2(y_1, y_2)$. Q.E.D.

Proof of Proposition 4. Parts a, b, and d are straightforward. To show part c, use equations 14–15 to derive

$$\partial y_1^*/\partial A = \frac{2Rr_1}{B(r_1 + r_2)^2(A-1)^2} - \frac{4R(Ar_1 - r_2)}{B(r_1 + r_2)^2(A-1)^3}$$

$$= \frac{2Rr_1(A-1)}{B(r_1 + r_2)^2(A-1)^3} - \frac{4R(Ar_1 - r_2)}{B(r_1 + r_2)^2(A-1)^3}$$

$$= \frac{2R[r_1(A-1) - 2(Ar_1 - r_2)]}{B(r_1 + r_2)^2(A-1)^3}$$

$$= \frac{2R[2r_2 - (A+1)r_1]}{B(r_1 + r_2)^2(A-1)^3}.$$

From the last expression it follows that $\partial y_1^*/\partial A \lesseqgtr 0$ as $2r_2 - (A+1)r_1 \lesseqgtr$ or, as $r_1 / r_2 \gtreqless 2/(A+1)$. Showing the expression for y_2^* follows similar steps, a process we will not repeat here.

As for part e, from equation 14 we have

$$\partial y_1^*/\partial r_1 = \frac{-2(Ar_1 - r_2) + 2A(1 - r_1 - r_2)}{B(r_1 + r_2)^2(A-1)^2} - \frac{4(1 - r_1 - r_2)(Ar_1 - r_2)}{B(r_1 + r_2)^3(A-1)^2}$$

$$= \frac{2}{B(r_1 + r_2)^3(A-1)^2}[-(Ar_1 - r_2)(r_1 + r_2) + A(1 - r_1 - r_2)(r_1 + r_2) - 2(Ar_1 - r_2)(1 - r_1 - r_2)].$$

It follows that $\partial y_1^*/\partial r_1 \gtrless 0$ as the last expression inside the brackets is positive or negative. After some tedious algebra, this expression simplifies into

$$(A+1)r_2(1 - r_1 - r_2) + r_2 - Ar_1 = (A+1)r_2 R + r_2 - Ar_1.$$

Thus $\partial y_1^*/\partial r_1 \gtrless 0$ as $(A+1)r_2R + r_2 - Ar_1 \gtrless 0$ or as

$$r_1 \lesseqgtr r_2 \frac{(A+1)R + 1}{A}.$$

The equivalent expression for $\partial y_2^*/\partial r_2$ is similarly derived. Q.E.D.

Proof of Theorem 2. Candidate's i strategy vector is (x_i^j, x_i^k). Concavity of candidate i's payoff function in that vector implies existence of pure-strategy equilibrium; thus this is what we need to show. For specificity, consider candidate 1's payoff function and let $W^1_{23} \equiv \partial^2 W^1/\partial x_1^2 \partial x_1^3$, $W^1_{22} \equiv \partial^2 W^1/(\partial x_1^2)^2$, $W^1_{33} \equiv \partial^2 W^1/(\partial x_1^3)^2$. It is straightforward to show that

$$W^1_{23} = W^1_{22} = W^1_{33} = (q^1_{11} - \alpha_2 q^2_{11} - \alpha_3 q^3_{11})R,$$

where $q^i_{11} = \partial^2 q^1(y_1, y_2, y_3)/\partial y_1^2$ for $1 = 1, 2, 3$. By assumption 11, the value of any of these second derivatives is nonpositive. Then, the following quadratic form

$$[z_1 \quad z_2]\begin{bmatrix} W^1_{22} & W^1_{23} \\ W^1_{23} & W^1_{33} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

equals $(q^1_{11} - \alpha_2 q^2_{11} - \alpha_3 q^3_{11})R(z_1 + z_2)^2$, which is nonpositive for all vectors (z_1, z_2). Thus, the Hessian (the matrix of second derivatives) of W^1 with respect to candidate 1's strategy vector is negative semidefinite, which implies that concavity of W^1 is candidate in his or her strategy. Showing the same result for candidates 2 and 3 follows exactly the same steps.

Q.E.D.

Proof of Proposition 6. We consider only the case of candidate 1; the proof of the two other cases follows the same steps. First, suppose $x_1^{2*} \in [0, 1)$. Then, the following condition must hold:

$$\frac{\partial W^1(x^*)}{\partial x_1^2} = -[q_1^1(y^*) - \alpha_2 q_1^2(y^*) - \alpha_3 q_1^3(y^*)]R + B[A\alpha_2 r_2 - r_1] \le 0, \quad \textbf{(27)}$$

where $y^* \equiv (1 - x_1^{2*} - x_1^{3*}, 1 - x_2^{1*} - x_2^{3*}, 1 - x_3^{1*} - x_3^{2*})$. The derivative of candidate 1's payoff function with respect to negative campaigning against the weaker candidate 3 is

$$\frac{\partial W^1(x^*)}{\partial x^3} = -[q_1^1(y^*) - \alpha_2 q_1^2(y^*) - \alpha_3 q_1^3(y^*)]R + B[A\alpha_3 r_3 - r_1] \le 0. \quad \textbf{(28)}$$

The first terms of equations 27 and 28 are identical; and since, by assumption 12, $\alpha_2 r_2 > \alpha_3 r_3$, the second term of equation 28 is smaller than the second term of equation 27, or $B(A\alpha_3 r_3 - r_1) < B(A\alpha_2 r_2 - r_1)$. Thus we have

$$\frac{\partial W^1(x^*)}{\partial x_1^3} < \frac{\partial W^1(x^*)}{\partial x_1^2} \le 0,$$

which, given the concavity of $W^1(x)$ in x_1^3, implies $x_1^{3*} = 0$.

Finally, suppose $x_1^{2*} = 1$. Obviously, given the constraint in equation 21, we cannot have anything else but $x_1^{3*} = 0$. Therefore, regardless of the value of x_1^{2*}, we have $x_1^{3*} = 0$ as stated in the proposition.

Q.E.D.

Notes

We would like to thank Joe Harrington, and participants in the Public Choice study group at the University of California, Irvine for helpful comments. Skaperdas thanks the National Science Foundation for financial support.

1. Similar or identical definitions of negative campaigning have been used by campaign consultants (Tarrance 1982) and in survey-based studies of negative campaigning (Garramone 1984). We would, however, follow Johnson-Cartee and Copeland in also including within the rubric of negative ads those in which the negativity is left to inference, such as Johnson's famous "Daisy" ad in 1964 (1991, 17). Other definitions of negative campaigning that have been proposed are narrower. For example, Merritt asserts that only ads that focus "primarily on degrading perceptions of the rival" should be classified as negative (1984, 27). We have deliberately chosen a more general definition of negative campaigning so as to include both policy aspects of political communication and aspects that deal with candidate attributes (e.g., attacks directed at an opposing candidate's personality, morality of record) that are only incidentally relevant to identifying areas of policy difference between a candidate and an opponent. Even the physical appearance of a candidate can become a feature that affects voters' judgments and that enters into negative campaigning (Rosenberg et al. 1986). Although some aspects of a candidate's history or personality will be seen by (some) voters as relevant to whom they should choose even though such candidate features cannot be directly tied to particular policy positions (e.g., claims about, say, what Ted Kennedy did or did not do at Chappaquidick), in general it is difficult to distinguish between attacks against character that have policy implications and those that do not. Thus we use a simple dichotomy between statements in praise of one's past history and present policy positions (positive campaigning) versus attacks directed in some fashion *against* one's opponent or his policy positions (negative campaigning). Alternatively, following Harrington and Hess (1993) we might model political competition as having two dimensions: a policy dimension and a "valence" dimension where the personal attributes of candidates are evaluated.

2. See discussion in Pfau and Kenski (1990, 2–3 and chap. 2). Psychologists have generally found that "not only is negative information more heavily weighted than positive information in the initial formation of impressions, but negative information exhibits a greater capacity to alter already existing impressions" (Kellerman 1984, 37–38). Lau (1985) has examined the differential impact of negative and positive information as it pertains to politics. Pfau and Kenski take negative campaigning very seriously; indeed, the central focus of the latter part of their book is on how to mitigate the more pernicious effects of negative campaigning through the use of "inoculation" strategies.

3. However, the existence of the boomerang effect is not unchallenged. Johnson-Cartee and Copeland note that political consultants tend to dismiss the boomerang effect and claim that even if there is a fall off in support for the candidate engaging in attack ads, that fall off is very short-run (1991, 14–15). All the qualitative results we present below do not depend on incorporating the boomerang effect in our model. In its absence we would just obtain higher levels of negative campaigning.

4. While Garramone (1985) finds that political advertising had a strong negative impact on respondent's evaluation of the sponsor of the negative ads they remember and only a slight negative influence on the evaluation of the target of the ads, this is just one study and we believe that the assumption that we have made is more compatible with the prevalence of negative advertising. If it were true that negative campaigning generally hurt the sponsor more than the victim, it is hard to see how negative ads with attributed candidate sponsorship would ever be used. Of course, some negative ads may hurt the sponsor more than the target. And certain types of attack may be delegated or otherwise left to organizations not

directly linked to the candidate or be left to lesser spokespersons, and thus remain disavowable.

5. Candidate decision making can be seen as part of a complex ongoing game in which candidates are seeking not only to influence voters directly but also to affect the decisions being taken by activists and interest groups about their levels of campaign participation and contributions, and the decisions of journalists as to the nature and extent of campaign coverage—decisions that can have a tremendous (indirect) impact on the candidate's election chances. Thus certain campaign decisions may be taken for reasons that are linked in quite complex ways to a candidate's ultimate goal of influencing decisions by the voters as to whether or not to vote and about which candidate to support if they do vote. Here, to make the analysis tractable, we abstract away from such institutional richness of detail.

6. A more general alternative is $q^1(y_1, y_2) + q^2(y_1, y_2) + \bar{q}(y_1, y_2) = 1$, where $\bar{q}(y_1, y_2)$ denotes the fraction of undecided voters remaining undecided. A reasonable property of $\bar{q}(\cdot)$ is that it is decreasing (and symmetric) in its arguments (i.e., the more positive campaigning there is, the smaller the proportion of voters remaining undecided). Our results would be unaffected with this modification and the companion property to assumption 5 introduced later on.

7. For a derivation of this result in another context, see Skaperdas 1994, theor. 1. Functions satisfying assumptions 1–3 have been used in various areas of several social science disciplines, including voting (Coughlin 1986), rent seeking (Tullock 1980), and conflict (Skaperdas 1992). For a general axiomatic development see Luce and Suppes 1965 or Suppes et al. 1989. For a comparison of the properties of two functional forms, see Hirshleifer (1989). In all of these cases, the functions have a probabilistic interpretation. We can also interpret here these functions as probabilistic at the individual voter level but maintain a deterministic interpretation at the aggregate level by assuming a large number of voters.

8. Given equation 3, $V^i = E(r_i^0) - E(r_j^0)$ is equivalent to maximizing $\text{Prob}(r_i^0 > r_j^0)$ when the error terms are independent of the strategies of the candidates (see Aranson, Hinich, and Ordeshook 1974, theor. 4). See also Hinich and Ordeshook 1970 for an earlier treatment of the problem of payoff-function equivalence.

9. The reaction function (or the best-reply or best-response function) of a candidate describes the optimal strategy of that candidate for any given strategy of the opponent. Candidate 1's reaction function is derived from equation 11 by expressing y_1 as a function of y_2 and the other parameters. Candidate 2's reaction function is similarly derived from equation 12.

10. In fact, we shall show the optimality of never engaging in negative campaigning against the weaker of your two opponents in a generalized three-candidate model, which includes the present model as a special case.

11. However, we do not think we can derive this objective function from more fundamental assumptions on the error terms, as can be done with the payoff functions in two-candidate competition (see n. 8).

12. In addition to the vast body of work on electoral competition in a single or multidimensional issue space springing from the work of Anthony Downs (1957; see reviews in Enelow and Hinich 1984, 1990 and Grofman 1993), there is work specifically on how decisions about campaign contributions are made (e.g., Ben Zion and Eytan 1975; Bental and Ben Zion 1975; Crain and Tollison 1976); on the optimal allocation of campaign appearances and campaign funds in space and time (e.g., Aldrich 1980; Bartels 1988; Brams and Davis, 1974; Colantoni, Levesque, and Ordeshook 1975; Lave and March 1975); and on the consequence of campaign finance regulation (e.g., Aranson and Hinich 1979). Two more recent contributions are Thomas 1990 and Hammond and Humes 1993. Thomas posits that candidates do cost–benefit calculations of various types of campaigning; and in the Hammond and Humes model, each candidate emphasize the issues in which he or she does best.

13. Harrington and Hess (1993) distinguish between policy and nonpolicy dimensions, with campaigning taking place along the policy dimension. Positive campaigning influences the candidate's own position—negative campaigning, the opponent's position. One main finding of Harrington and Hess, complementing ours on the effect of candidates' voter support, is that candidates weaker in the nonpolicy dimension engage in more negative campaigning than their opponents.

14. In the original version of the truel, each duelist has a certain number of bullets and a certain probability of hitting his target, and only one opponent can be fired at in any round. At each round, each duelist simultaneously chooses whether or not to fire and, if so, on whom to fire. (For several other variations, see Dresher 1981.) With respect to truels, the questions that can be asked are (1) What is each duelist's optimal strategy, that is, when would they shoot and at whom? and (2) If each duelist follows an optimal strategy, how does the probability of survival depend upon the duelists' relative shooting accuracies? In particular, is the most accurate duelist most likely to survive? See Shubik 1954.

15. We believe that treating the 1992 contest as a plurality election is a reasonable approximation. While the actual mode of selection of the president is by a complex process that includes the weighted voting rule that determines the electoral college vote and the rule for how to select a president if no candidate receives a majority of the electoral college vote, voter attention in 1992 was focused heavily on the national popular vote.

16. In general, in 1992, neither major party candidate sought to distinguish their policy stands from those of Perot except on peripheral issues such as the gasoline tax. Journalists asserted that the reason for this reluctance to distinguish their own issue positions from those of Perot was that neither Clinton nor Bush wanted to alienate Perot supporters, whom they hoped to eventually woo to their side. However, this does not satisfactorily explain why there was repeatedly expressed policy divergence between the front-runners Clinton and Bush. We believe that a sensible rational choice story can be told about relative policy position and divergence in three-candidate contests, but this is a story that we cannot pursue here.

17. Impressionistically, it seems to us that voter backlash against negative campaigning was not constant but increased in the last phases of the campaign, as signaled by the question asked by a member of the audience early in the second debate that was prefaced by a reprimand to the candidates for having engaged in too much negative campaigning.

18. See discussion of this point in Kern 1989 and Johnson-Cartee and Copeland 1991.

19. For the importance of the latter part, see Ansolabehere et al. (n.d.).

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