Measuring Compactness and the Role of a Compactness Standard in a Test for Partisan and Racial Gerrymandering

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As we move into the 1990s, compactness of legislative districts is likely to take on greater importance because of its relevance to questions of racial and partisan discrimination. We show that at least two distinct components of shape—dispersion and perimeter length—are necessary elements of any reasonable compactness measure and that compactness may be based on geography or on population. An appropriate strategy, therefore, is simultaneously to consider multiple measures that collectively define compactness. We identify and evaluate multiple operationalizations and compare them both theoretically and empirically. Data from five states provide abundant evidence that the major types of measures sometimes vary widely in their evaluation of the compactness of a given *district*. There is some support, however, for the hypothesis that multiple measures yield similar assessments of districting *plans*. We conclude with a discussion of how compactness might be used in legislative districting.

Compactness of legislative districts is a long-sought-after goal. It is implicit, for example, in most criticisms of the original "Gerry-mander" as well as of Rep. Burton's carving-up of California in the 1980s (e.g., Griffith 1907; Congressional Quarterly 1982, Part I, 141–67); it is explicit in proposals by Common Cause (Adams 1977) and in constitutional and statutory requirements of half of the states (Eig and Seitzinger 1981). It is also the subject of

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numerous scholarly papers proposing an array of quantitative measures of the concept. Nevertheless, compactness has seldom been assessed, and when it is considered at all, it continues to be "measured" primarily by casual visual inspection.

As we move into the 1990s, compactness, as well as efforts to measure it quantitatively, are likely to be taken more seriously. For nearly two decades after the Supreme Court decisions in *Baker v. Carr*, 369 U.S. 186 (1962) and *Reynolds v. Sims*, 377 U.S. 533 (1964), equality of district populations was the chief concern, and questions about compactness, use of existing political boundaries, and so on, could be conveniently ignored. But as questions of discrimination became more prominent in the 1970s and 1980s, concerns other than population equality—compactness among them—again became relevant.

In the most prominent case of the 1980s involving minority rights and districting, the Supreme Court ruled that for multimember districts to be judged discriminatory the minority group must be "sufficiently large *and geographically compact* to constitute a majority in a single-member district" (*Thornburg v. Gingles*, 478 U.S. at 50, 1986, emphasis added). On the same day, the Court also ruled that *political* discrimination was justiciable, and Justices Powell and Stevens cited compactness as a major determinant of partisan gerrymandering (*Davis v. Bandemer*, 478 U.S. at 173, 1986) while Justices White, Brennan, Marshall, and Blackmun (at 2815) described it as a subsidiary but still useful criterion.

Scholars, meanwhile, have been of two minds about compactness. There are some who dismiss it as outdated, irrelevant, or even as a positive nuisance (Cain 1984, chap. 3), and as biased toward one party (Lowenstein and Steinberg 1985, 23–27). Some also see *Davis* as setting such impossibly high standards for a finding of discriminatory vote dilution that questions about compactness and other such standards are rendered moot (Lowenstein, 1990). On the other hand, compactness is supported by a number of scholars (e.g., Morrill 1981; Baker 1986, 1990) and by many in the general public (e.g., Horn et al. n.d.) as a prime defense against gerrymandering.

In any event, compactness as a goal will not go away easily. Lower courts are beginning to hear arguments about the compactness requirement as set forth in *Thornburg* and about compactness as it relates to partisan discrimination. Criticisms continue to be made in the public media about the shape of districts and what is implied by horrendously shaped figures (e.g., in stories about the importance of gubernatorial and state legislative elections in 1990). Proposals for a more active use of compactness continue to be introduced (e.g., Horn 1978). And, as noted earlier, compactness remains a constitutional or statutory requirement in numerous states and localities.[1]

[1]Various lower court citations can be found in Grofman (1985, 85–86). See also *Dillard v.*

As for quantification specifically, more precise measures are sure to be used as scholars and courts pay more attention to the problem. It is already the case, for example, that expert witnesses have presented conflicting claims about how compact particular plans or districts are (e.g., *Burns v. Holmes*, R.I. 1983; *U.S. v. County of Los Angeles Board of Supervisors*, D. Cal. 1988).

There is a major problem, however, in this move toward quantification, namely, that little attention has been paid to theoretical characteristics of proposed compactness measures or to empirical comparisons among them. Overlap among definitions (some are mathematical transformations of others), the characteristics assessed by the various measures, and the kinds of distortions each measure taps best have only recently begun to be explored (e.g., Young 1988). Similarly, correlations among different measures, practical matters about methods of calculation, and even descriptive data for single states, are almost nonexistent.

In this paper we present the first comprehensive study of compactness measures. We draw on Young's (1988) fine illustrations of the pitfalls of specific definitions, but we go well beyond his work by expanding the scope of coverage and, most importantly, by identifying and classifying the components of compactness that the various measures are designed to capture. In doing so, we provide a theoretical basis and strong empirical support for the simultaneous use of multiple compactness scores.[2]

We also provide empirical evidence from five states about the compactness of specific districts and about the relative compactness of competing districting plans that were actually used or proposed for use in these states. Though the analysis is necessarily exploratory, there is abundant evidence that three major types of measures sometimes vary widely in their evaluation of the compactness of a given *district*. There is some support, however, for the hypothesis that multiple measures often yield similar assessments of districting *plans*.

We leave for the end a discussion of how compactness might best be used in litigation concerning partisan and racial gerrymandering. Our analysis leads us to argue strongly against its use in a mechanical way, and we suggest a variety of other "rules" as well. As to the value of compactness as such, we believe that an adequate assessment cannot be made until we have a firm understanding of the computability and comparability of its multiple definitions. The primary goal of this paper is to provide the theoretical and empirical underpinnings to that understanding.

Baldwin County Board of Education (No. 87-T-1158-N, M.D. AL, April 8, 1988, 15) and *McDaniels v. Mehfoud* (No. 88-0020-R, E.D. VA, Dec. 30, 1988, 7). A useful review of the "sufficiently large and compact" requirement is found in Karlan (1989); Karlan argues, and we agree, that in this context the Court may have used the word "compactness" to mean little more than contiguity.

[2]Another recent comparison of multiple measures is Horn, Hampton, and Vandenberg (n.d.).

Definition and Measurement of Compactness

The Ingredients of Compactness Measures

Compactness, at a simple intuitive level, conforms to a standard dictionary definition: a figure is compact if it is "packed into . . . a relatively small space" or if its parts are "closely . . . packed together" (American Heritage). By way of contrast, a figure is not compact to the degree that it is "spread out." Thus, we think of circles and squares as compact and long, narrow forms, areas with protruding arms or fingers, and "odd" shapes like salamanders, as not compact.

It is to be expected, then, that "spread" or "dispersion" is an essential ingredient of many quantitative compactness measures. To put the matter on a more formal footing, one can show that another potential measure, namely perimeter length, is by itself insufficient for measuring compactness. To do so, one begins with a fixed perimeter length and observes that one can create numerous shapes that differ widely in their spread. Measures of dispersion are thus essential to distinguish these shapes. What quantitative measures do is to compare each shape to some standard. Since a circle encompasses the greatest area for a given perimeter length, the circle is usually chosen as the standard. It is obvious, of course, that circles cannot be fitted together so as to form multiple, fully compact districts, so hexagons are sometimes suggested as the norm. But whatever shape is chosen as the standard, the theoretical point remains: dispersion must be considered.[3]

In some contexts, dispersion might be a sufficient measure of compactness. In the districting context, however, dispersion measures alone are inadequate precisely because they are insensitive to perimeter irregularities, especially to the kinds of irregularities that may signal gerrymandering. A nearly circular district, for example—though almost ideal in its dispersion—would raise considerable suspicion about the designer's intent and about its likely consequences if its boundary were a jagged border of small but inexplicable twists and turns. More generally, it is possible to construct figures with a constant dispersion value but which score as low as we like in perimeter terms. Thus, perimeter length must also be considered if one is adequately to assess district compactness.[4]

[3] Manninen (1973, 63) makes the same point with two shapes that have the same area as well as perimeter and therefore the same compactness according to conventional perimeter measures. One is elongated and would generally be regarded as much less compact, as indeed it is according to the most common dispersion measures.

[4] Consider a square circumscribed by a circle and a dispersion measure defined as the ratio of the district area to the area of the smallest circumscribing circle. If we add protuberances that do not extend beyond the circle and compensate for the area of the protuberances by indentations into the square, dispersion is unchanged while perimeter length increases indefinitely. Such a result exemplifies a well-known result in topology on "space-filling" curves (Guillermo Owen, personal communication, April 1988).

To make the situation more complicated, it is sometime argued that it is the distribution of voters and not land that is important, as area as such is not represented. A district is not compact from this perspective if it looks like a square but most of the population lives on a thin line running diagonally across it. Conversely, a district is compact if most of the population lives in a perfect square even if its land area is that of a tortured camel. While this is an important point, population is not a component of shape as such; the argument here is over what to consider the shape of—land or population. In either case, dispersion and perimeter are distinct elements of shape, neither one of which fully encompasses our intuitive notions of compactness.[5]

In short, there are at least two components (dispersion and perimeter length) and arguably three (if one counts population dispersion, perimeter length, and geographic dispersion) to any satisfactory measure of compactness. No single-variable definition is adequate.[6] The search for *the* measure of compactness is illusory. The problem is not, however, that compactness is ill-defined but that it is "multidimensional," referring to geographic dispersion, perimeter, and population characteristics of a district, no one of which is adequate to give a proper assessment of compactness.[7]

Given the impossibility of defining a single-variable measure, an appropriate strategy is simultaneously to consider multiple measures that collectively define the concept. The utility of the measures—and ultimately of the concept itself—depends in part on their relative agreement on the compactness of competing districting plans. Assessing this agreement requires empirical analyses of a sort that we shall introduce for the first time below.

Specific Measures

Having determined that compactness can be properly measured only by multiple attributes, we now turn to the task of measuring each of the components. We shall find that just as there are multiple characteristics to measure, there is more than one measure of each characteristic. In this case, however, there are good reasons to adopt certain definitions as opposed to others.

An initial consideration is whether a measure of compactness should be independent of scale or should reflect the absolute size of a district. Should a

[5] All of the population measures that have been proposed so far are dispersion measures. However, if one were to discard the notion of geographic shape, population measures involving perimeters would soon follow.

[6] As Manninen (1973, 40) put it, even without considering population, "the development of a single number which can adequately express the two-dimensional quality of shape is impossible." Or, as Lee and Sallee (1970, 555) said more formally, "there exists no continuous one-to-one function from S, the set of all plane shapes, into R, the set of real numbers."

[7] Of course one can combine multiple indicators into a single score, but combining them requires a decision about their relative importance. Thus, for example, simply averaging a dispersion and perimeter measure implicitly assigns them equal importance.

square district have the same compactness score whether it is two miles or 50 miles on a side? Is a large circular district more compact than a small but irregularly-shaped one?

A few researchers have argued that size is indeed relevant (Harris 1964, 221; Theobald 1970, 132–33), but the arguments for incorporating size into a definition of compactness are not compelling, and few have taken the idea seriously. First, the argument against large districts harkens back to the idea that transportation and communication are the primary reasons for compactness. This argument was perhaps especially significant in the past, but it loses some of its strength in an age of telephones, computer links, superhighways, and airplanes. Second, a measure sensitive to size would favor combinations of urban and rural areas instead of "intermixing extremely small urban districts with rural districts hundreds of miles of long" (Theobald 1970, 132). Whether districts should be homogeneous or combine different types of areas (whether urban and rural, rich and poor, etc.) is another matter entirely and should not be determined merely as a side effect of a compactness measure. Thus, with one exception, we will ignore measures that are dependent on the absolute size of districts.[8]

Assuming independence of scale, interpretation is greatly simplified if all measures vary between 0 and 1, with 0 being least and 1 being most compact so that increasing values indicate increasing compactness. Whenever possible, we shall convert measures to this form; where there are multiple measures that are transformations of one another, we shall rely on the 0–1 version and simply note the others in passing.

In table 1 we list nearly two dozen proposed compactness measures. The measures were originally proposed for a variety of applications; the oldest are the result of efforts to measure the shape of drainage ditches and grains of sand. As is noted, some are simply transformations of one another. They are grouped under the headings of dispersion (geographic), perimeter (geographic), population, and "other" measures and then further grouped within those four categories. We shall comment briefly on each of the major classes.

Dispersion Measures. Dispersion measures assess how tightly packed or spread out the geography of a district is. Underlying all of these measures is the notion that a perfect district is a regular, simple shape, usually a circle but sometimes a hexagon or a square. Differences arise chiefly because there are multiple ways of measuring deviations from the perfect shape.

Width/length measures are attractive because of their extreme simplicity that nonetheless seems to capture the most essential element of dispersion. Squares (as well as circles) are regarded as good, while stretched-out or irregular shapes, especially if they contain fingers that stick out from the main body, are given low marks. This class of measures can be criticized, however,

[8]If other things are equal (which they rarely are), or in unusual circumstances, one might opt for smaller deviations in absolute size.

Table 1

A Typology of Compactness Measures

I. Dispersion Measures

Length Versus Width

Dis_1—W/L, where L is the longest axis[a] and W is the maximum length[b] perpendicular to the longest axis (Harris 1964).

Dis_2—W/L, where W and L are that of the circumscribing rectangle with minimum perimeter.

Dis_3—W/L, where W and L are that of the rectangle enclosing the district and touching it on all four sides for which the ratio of length to width is a maximum (Young 1988).[c]

Dis_4—W/L, where L is the longest axis and W and L are that of a rectangle enclosing the district and touching it on all four sides.

Dis_5—L-W, where L and W are measured on north-south and east-west axes, respectively (Eig and Setizinger 1981, 55–56).

Dis_6—L-W, where L and W are defined as in Dis_1 (Harris 1964).

District Area Compared with Area of Compact Figure

Dis_7—ratio of the district area to the area of the minimum circumscribing circle (Reock 1961).

Dis_8—ratio of the district area to the area of the minimum circumscribing regular hexagon (Geisler 1985).

Dis_9—ratio of the district area to the area of the minimum convex figure that completely contains the district.

Dis_{10}—ratio of the district area to the area of the circle with diameter equal to the district's longest axis (Horton 1932; Gibbs 1961). $Dis_{10} = A/[\pi(L/2)^2]$, where A is the area of the district and L is the longest axis.

Moment-of-inertia

Dis_{11}—moment of inertia, i.e., the variance of the distances from all points in the district to the district's areal center of gravity (Schwartzberg 1966; Kaiser 1966). Adjusted to range from 0 to 1, $Dis_{10} = A/(2\int_D \int r^2 dD)^{1/2}$, where D is a district with area A.

Dis_{12}—average distance from the district's areal center to the point on the district perimeter reached by a set of equally spaced radial lines (Boyce and Clark 1964).

II. Perimeter Measures

Perimeter Only

Per_1—sum of the district perimeters (Wells 1982; Eig and Seitzinger 1981, 14; Adams 1977).[d]

Perimeter-Area Comparisons

Per_2—ratio of the district area to the area of a circle with the same perimeter (Cox 1927). $Per_2 = (4\pi A)/P^2$, where A is the area and P is the perimeter of the district.

Per_3—$1 - 2(\pi A)^{1/2}/P$ (Attneave and Arnoult 1956). $Per_3 = 1 - Per_2^{1/2}$.

Per_4—ratio of the perimeter of the district to the perimeter of a circle with an equal area (Horton 1932; Schwartzberg 1966). $Per_4 = P/[2(\pi A)^{1/2}] = (1/Per_2)^{1/2}$.

Per_5—perimeter of a district as a percentage of the minimum perimeter enclosing that area (Pounds 1972). $Per_5 = 100(Per_4)$.

TABLE 1 (*continued*)

III. Population Measures

District Population Compared with Population of Compact Figure

Pop$_1$—ratio of the district population to the population of the minimum convex figure that completely contains the district (Hofeller and Grofman 1990).
Pop$_2$—ratio of the district population to the population in the minimum circumscribing circle (Hofeller and Grofman 1990).

Moment-of-inertia

Pop$_3$—population moment of inertia, which can be normalized to vary from 0 to 1 (Weaver and Hess 1963).

IV. Other Measures

Theobald (1970)—absolute deviation from average area.
Papayanopoulos (1973)—sum of all pairwise distances between centers of subunits of the legislative district, weighted by subunit population.
Taylor (1973)—$(N - R)/(N + R)$, where N is the number of nonreflexive interior angles, R is the number of reflexive interior angles.
Iowa Reapportionment Statute (Eig and Seitzinger 1981, 56)—"combined" areal, perimeter, and population measures such as the ratio of the dispersion of population about the district's population center to the dispersion of population about the district's geographic center.

[a]The longest axis is the greatest distance between any two points in the district. The line defining that distance may go outside the district.

[b]Though it has not been clearly specified, we would interpret this to be the maximum *within* the district.

[c]Young specifies *L/W*, but the inverse ranges from 0 to 1.

[d]For comparisons across a set of districts, this measure can be normalized to vary from 0 to 1. Take the inverse of each perimeter length and divide by that of the shortest perimeter. The shortest perimeter thus becomes 1, and other perimeters retain their original proportions (except that longer lengths, which express less compactness, become lower numbers).

Note: Measures of compactness have been invented and reinvented over time and across disciplines. We have emphasized citations made in the context of legislative districting. A useful analysis, and the source of some of our citations, is Manninen (1973). A brief summary of "shape analysis" is Austin (1984).

as being too dependent on extreme points; an otherwise perfectly compact district can be given an extremely low score because of a single projection, and significant changes in apparent compactness can be made without altering the width/length comparisons. They can also give very high ratings to "devious, unnatural figures" (Young 1988, 109)[9]

[9]The figure Young cites is horseshoe-shaped. We note *Dis*$_5$ and *Dis*$_6$ for completeness only. The *difference* between length and width is dependent on scale, and the difference between north-south and east-west axes is dependent on scale and incorporates the meaningless requirement that districts be oriented in a particular direction.

Measures comparing the area of a district with the area of a compact figure are attractive in that they suggest comparisons with a single, precisely specified model. Multiple measures exist because of differences in choosing the standard shape. Typically the circle is regarded as ideal because it maximizes the area within a given perimeter. It is pointed out, however, that no conceivable *set* of districts could be perfect by this standard because nonoverlapping circles cannot fill an entire area; squares or hexagons are suggested as an alternative. This objection perhaps misses the point because there is no real circumstance in which a state could be divided into perfectly hexagonal or square districts anyway, especially if population equality or any other standard were to be adhered to as well. Nonetheless, varying the standard shape could lead to sets of districts being ranked differently, so the measures are theoretically distinct.

Assuming a circle is the standard, there is the additional question of which circle to use. The minimum circumscribing circle is perhaps the most obvious. However, another possibility is the circle whose diameter is equal to the longest axis of the district. Typically these two are the same. When they are not, as in figure 1, the longest axis circle has the advantage of being considerably easier to calculate.

By utilizing the area of the entire shape, this subclass of measures relies on more than just a few extreme points and thus represents an improvement over width/length measures. However, these measures can be criticized for serious failures to reflect our intuition. A triangle, for example, can have a low rating while a coiled snake can have a high score (Young 1988, 106). And despite their improvement on width/length measures, they are still rather dependent on extreme points.[10]

Moment-of-inertia measures try to improve upon width/length and area-comparison measures by giving appropriate weight to all or many points in the district. The moment-of-inertia itself is intended to assess the average (squared) distance of all points in the district from the center of the district. Thus, extreme points are considered, but only according to their frequency; if there are few extreme points, as when a single, narrow finger mars an otherwise circular district, the compactness score will be relatively high. In this respect moment-of-inertia scores are an improvement over other dispersion measures. However, they are considerably more difficult to calculate and are far less interpretable for most individuals. Moreover, they can be criticized for giving high scores to districts that "meander within a confined area" (Young 1988, 111).[11]

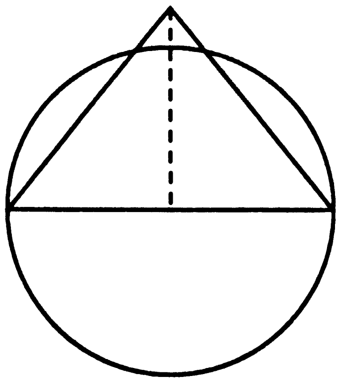
In short, each type of dispersion measure has advantages and disadvan-

[10] For example, the city of Rochester is relatively circular with its northern-most point a few miles south of Lake Ontario—except that a narrow corridor along the Genessee River extends the city's boundary to the lake. Because of this extension, a circumscribing or longest axis circle would be very large and the city therefore judged noncompact.

[11] We reject Dis_{12} as too dependent on how the radial lines are chosen. If one can imagine

FIGURE 1

FIGURE IN WHICH CIRCLE WITH DIAMETER EQUAL TO THE FIGURE'S LONGEST AXIS IS NOT LARGE ENOUGH TO BE CIRCUMSCRIBING



$Dis_7 = .371$
$Dis_{10} = .401$

tages. We have rejected some specific measures, but we are left with three different types and multiple individual measures.

Perimeter Measures. There are two types of perimeter measures. The first consists of a single measure, the sum of district perimeters. The sum of perimeters is attractive because of its striking simplicity and interpretability. Perhaps for this reason it has been advocated by Common Cause (Adams 1977, 874–75), introduced into Colorado's constitution (Eig and Seitzinger 1981, 14), and cited by the late Robert Dixon (1980–1981, 847) as if it were *the* measure of compactness. Accordingly, we will include it in the empirical comparisons below even though it is dependent on scale. The measure has been criticized on the grounds that it might allow gerrymandering in urban areas because lengthened borders there could be balanced by slight changes in the borders of rural districts (Young 1988, 111–12).[12]

The other type of perimeter measure compares the perimeter and area of a shape and is based on the fact that the ratio of perimeter to area is smallest for a circle. This type of comparison has been proposed independently by a number of authors and in alternative forms that sound quite distinct. How-

calculating the measure with every possible radial line, it becomes very similar in spirit to the moment-of-inertia, and the latter would seem to have the more natural interpretation.

[12]The sum of perimeters is different from other measures in that it is intended almost exclusively for comparing entire plans. There have been a few suggestions for applying it to subsets of districts, but they have not received serious attention (Adams 1977, 875; Walker 1976, 475).

ever, the different versions are in fact identical or are simple transformations of one another (see table 1). Thus we need to consider only one form, Per_2, which is bounded by a limiting value of 0 (as the shape approaches a straight line, which by definition has no width and therefore zero area) and 1 (a circle).

Population Measures. The two classes of population measures are analogous to those of geographic dispersion. The first compares the population of a district to the population in a compact figure such as the minimum circumscribing circle. Questions about whether to use a circle or some other figure as a standard apply equally to the population and geographic measures. The second class is a population moment of inertia. Like its geographic counterpart, this measure assesses all points and their distances from the (population) center, but it weights the points by their population. Thus, the importance of extreme points depends on both their frequency and the population associated with them.

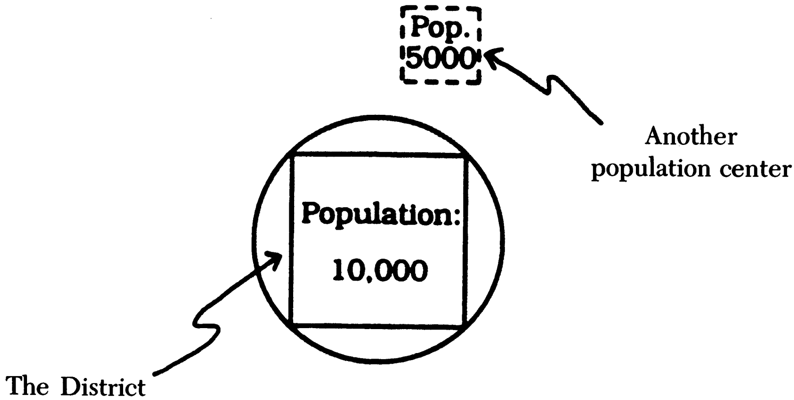
Population measures are relatively new and therefore have not been subject to the same kind of analysis and criticism as other measures. However, simple examples show that problematic results are just as possible as with geographic measures. In figure 2a, for example, the district is perfectly compact by Pop_2. In figure 2b the population of the district and even its distribution are unchanged; one might hope that a population measure would be sensitive to this point and also remain the same. Yet adding a completely unpopulated area to the district can dramatically change the compactness score.

Population measures are also more difficult to calculate than geographic measures. The population of a district is naturally available, but establishing the population of a circumscribing area requires a major operation. A special problem for the population moment-of-inertia is its complexity. Other compactness scores may be moderately difficult to calculate, but their basic components—areas, distances, perimeters—are all very familiar, and most of the definitions are straightforward. Moments-of-inertia are not so simple, and the problem of understandability is exacerbated when it is based on population. Finally, measures of geographic compactness generally coincide well with our visual impressions. Population compactness, on the other hand, may violate our notion of spatial compactness and therefore be less easy to grasp and more difficult to justify.

Overall, then, there are multiple components of compactness and multiple measures of each component. Each component and each measure has advantages and disadvantages, and for every measure there are shapes that yield counterintuitive results and for which another measure seems better. Evaluation of alternative measures must therefore go beyond identification and comparison of their theoretical properties to include comparisons of

FIGURE 2

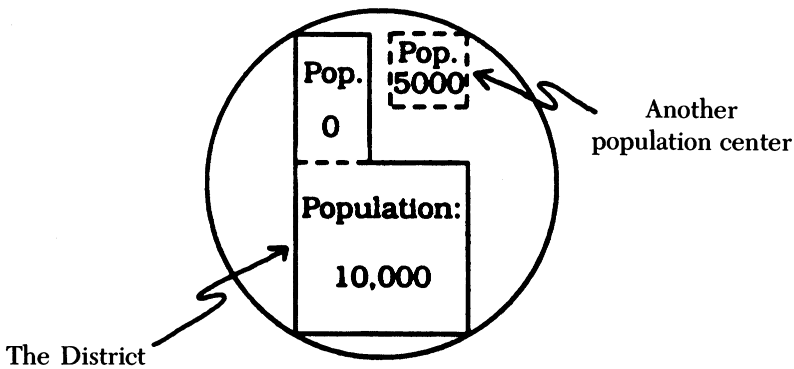
FIGURE IN WHICH ADDING AN AREA WITH ZERO POPULATION SIGNIFICANTLY CHANGES POPULATION COMPACTNESS



(a)

Population in circle and outside of district is zero

$Pop_2 = 1.0$



(b)

Population in circle and outside of district is zero except for rectangle indicated

$Pop_2 = .667$

compactness scores in real situations. We now turn to the task of estimating and analyzing compactness in actual districting plans.

Empirical Comparisons of Compactness

In an analysis of actual plans, it is important to know what kinds of comparisons are appropriate to make. Differences across states, for example, are inevitable and usually inconsequential because of differing initial shapes. Rectangular states are likely to have higher compactness scores than those with irregular shapes; states with long shorelines and coastlines are likely to have lower perimeter scores than others. This problem is overcome by limiting our comparisons to one state at a time. Second, some measures may be consistently higher than others—e.g., perimeter measures mostly higher than dispersion measures or one specific dispersion measure higher than another. Absolute size, however, is less important than the correlation among compactness scores and the relative rankings of alternative districts. Third, while there will surely be variations in the ratings of individual districts, what is most important is the degree to which multiple measures pick out the same *plan* as most compact.

The plans that we will compare were chosen in part because of data availability. They are, however, real plans proposed by actual political protagonists rather than plans drawn by computer simulation remote from real politics; some were implemented and all were implementable. The cases were also chosen because of their diversity as to type—they include congressional districts (Colorado, New York, California), state legislative districts (Rhode Island, Indiana) states with distinctly different geographies, states with one-party control (Indiana, California), and states with split leadership (Colorado, New York, Rhode Island). Furthermore, they include Indiana and California, the states said to have the most egregious gerrymanders in the 1980s. Because of what is available, some comparisons are among alternative plans for the same time period while others are between 1970s and 1980s districts for a given state.

The data requirements for computing compactness are demanding, and the means for making the computations are time-consuming and expensive (see appendix). As a consequence, we were not able to calculate all of the measures for all plans. Our analysis is sufficient, however, for drawing certain conclusions, and it serves as a basis for future, more extensive analyses.

Our first comparison, in Rhode Island, is simple because we have only two measures for each plan. As shown in table 2, the state legislative districting plan in force during most of the 1970s was more compact than the 1980s plan whether one judges by the sum of district perimeters (Per_1) or the "area/perimeter" (Per_2) measure. This result is perhaps unsurprising because both are perimeter measures, but it is a gratifying start.[13]

[13] We present and discuss means only. The medians are typically very similar to the mean; we will note the one instance in which the two statistics differ meaningfully.

TABLE 2

COMPACTNESS OF STATE HOUSE DISTRICTS FOR RHODE ISLAND (100 DISTRICTS)

	Per_1	Per_2 Range	Per_2 Mean (s.d.)
1974 Legislative plan[a]	1.00	.23–.97	.59 (.19)
1982 Legislative plan[b]	.89	.22–.82	.47 (.14)

[a]Legislative plan used from 1974–1980.
[b]Legislative plan upheld by the Rhode Island Supreme Court against a challenge on grounds of racial and political gerrymandering.

TABLE 3

COMPACTNESS OF CONGRESSIONAL DISTRICTS FOR UPSTATE NEW YORK (15 DISTRICTS)

	Dis_{10} Range	Dis_{10} Mean (s.d.)	Per_1	Per_2 Range	Per_2 Mean (s.d.)
Court plan[a]	.24–.61	.35 (.10)	1.00	.24–.67	.41 (.13)
Legislative plan[b]	.14–.45	.27 (.09)	.88	.19–.59	.35 (.13)

Correlations between measures (Pearson r/Spearman rho):

	Court Plan Per_2	Legislative Plan Per_2
Dis_{10}	.68/.65	.61/.44

[a]Back-up plan for the 1980s drawn by a Special Master with the aid of a consulting firm and with no political or incumbent concerns. Special attention to compactness (exact definition not reported).
[b]Adopted by a divided state legislature (Republican Senate, Democratic Assembly) and became law. Favorable to incumbents of both parties, preserves whole municipalities.

Our next comparison, for upstate New York, is more informative because the addition of a dispersion measure allows us to look at the correlation between dispersion and perimeter scores of individual districts within each plan (table 3).[14] The magnitude of these correlations suggests some sizable differences in rankings of districts, especially for the legislative plan. These discrepancies tell us what kind of noncompactness exists. For example, dis-

[14]Because Per_1 is intended for whole-plan comparisons, it is not included in the correlations.

trict 25 under the legislative plan has a lengthy longest axis, but because of an extension that takes in the cities of Utica and Rome the area is also large—making the district reasonably compact by Dis_{10} (tied for fourth). The perimeter, however, is very long, in part owing to the extension, making the district the 12th-most compact by Per_2. Conversely, the 34th district's straight border along Pennsylvania gives it a high rank by Per_2 (tied for fourth), but because it is relatively long and narrow it has a low rank on Dis_{10} (tied for 10th). Moreover, the differences should not obscure the fact that some districts are judged almost identically by both measures; districts 23 and 26 are ranked first and second (tied), while district 28 is ranked last and second to last.

A comparison *across* plans yields the same result as in Rhode Island: one of the plans is more compact according to all of the measures we were able to calculate (table 3). The minimum and especially the maximum of Dis_{10} and Per_2 are higher for the court plan, and the mean values are 20% to 30% higher; the sum of perimeters measure is 14% higher.

Our next comparison involves the 1980s districting plan for the Indiana state House of Representatives—the plan that was declared a gerrymander by the Indiana District Court in *Bandemer v. Davis* (603 F. Supp. 1479, 1984). We contrast the 1980s districts with the 1970s districts for the same body. We have three measures, but for the first time we have two dispersion measures, including a width/length measure. The results are shown in table 4.

The correlations among district scores range from low to high. As in New York, where the two measures diverge they call attention to characteristics of specific districts. District 29 (1982), for example, is ranked second on Dis_{10} but tied for 41st on Per_2; district 52 is ranked third and 30th, respectively. Neither district is grossly noncompact, but to the extent that questions are raised, it is about their perimeters and not their dispersion. Similarly, despite the discrepancies, there are districts that are especially compact or noncompact by all available measures. The "worst case" districts, so-called by the District Court, have mean compactness scores of .55, .29, and .24 for Dis_4, Dis_{10}, and Per_2, respectively.

Fittingly, the correlations between the two dispersion measures are higher than the correlations between dispersion and perimeter measures. Although there are instances in which the two dispersion scores diverge substantially, it is difficult to see what, if any, insights are added by the width/length measure. For example, district 4 looks like a square with a bite taken out of one side and ranks about in the middle (36th) in terms of Dis_{10}. To note that its width and length are nearly identical, so that it is ranked second-highest on Dis_2, does not seem particularly useful. Thus, the within-plan comparisons in Indiana seem to reinforce doubts about width/length measures and emphasize the contrasting aspects of shape tapped by Dis_{10} and Per_2.

In any event, the 1970s districts are more compact than the 1980s districts by all three measures. The differences are moderate—from 8% to 12%—and

TABLE 4

COMPACTNESS OF STATE HOUSE DISTRICTS FOR INDIANA
(1972: 73 DISTRICTS; 1982: 77 DISTRICTS)

	Dis_4		Dis_{10}		Per_2	
	Range	Mean (s.d.)	Range	Mean (s.d.)	Range	Mean (s.d.)
1972 legislative plan[a]	.44–.98	.71 (.14)	.20–.62	.45 (.11)	.19–.70	.44 (.13)
1982 legislative plan[b]	.27–.97	.66 (.16)	.16–.66	.40 (.11)	.18–.73	.40 (.13)

Correlations between measures (Pearson *r*/Spearman *rho*):

	1972 Plan		1982 Plan	
	Dis_{10}	Per_2	Dis_{10}	Per_2
Dis_4	.85/.86	.43/.46	.86/.85	.30/.31
Dis_{10}		.73/.73		.66/.67

[a] Plan in force throughout the 1970s.
[b] Plan in force from 1982–1990. This is the alleged gerrymander litigated in *Davis v. Bandemer*, 478 U.S. 109 (1986).

for two of the measures the single-most compact district occurs in the 1982 plan. Separate analysis shows that the multimember districts and Marion County districts—cited by the District Court as especially suspect—are relatively noncompact according to the perimeter measure although not according to the dispersion measure (Niemi and Wilkerson 1990). The most significant point, however, is the consistency across the three measures in the rating of the two plans.

Our fourth case is the 1980s Congressional districting in Colorado. The results in this instance are much less accurate than those analyzed so far; the perimeter measure is especially problematic because we did not have detailed maps of the Denver area. Nevertheless, the case contributes a new perspective for two reasons. First, there are only six Congressional districts in Colorado, so an unusually compact or ill-compact district has a considerable effect on mean scores. There should also be considerable variability of within-plan correlations, and it is unlikely that there will be complete agreement across measures on the ranking of entire plans. Second, we have results for 17 different plans. This also makes it unlikely that all plans will be identically ranked by all available measures.

Given this perspective, the results show more agreement than one might have expected (table 5). The within-plan correlations are indeed variable, with occasional instances in which individual districts are ranked identically (plan 1, 6, and 13 on Dis_4 and Dis_{10}) and others in which they are ranked in distinctly opposing orders (especially plans 7 and 11 for Dis_4 and Per_2 and plan 10 for Dis_{10} and Per_2). Still, the correlations are mostly positive (five of 51 rank order correlations are negative) and mostly above .60. Thus, as in previous cases, multiple measures point to distinctive characteristics of specific districts while at the same time indicating that some districts are consistently judged more compact than others.

Comparisons across districts indicate that width/length scores are uniformly higher than either of the other measures. This is not surprising, at least for area/circle comparisons; the width/length measure is based on extreme points, and it is likely that the circular area defined by the extreme points is not all part of the district. Still, the magnitude of the differences was unexpected; several times the minimum width/length score is greater than the maximum of Dis_{10}. No single district is most compact by every measure. Plan 16 is most compact on the two dispersion measures, but it fares rather badly on the perimeter score. Plan 9 is the opposite, ranking highest in terms of its perimeter but very low with respect to dispersion. Plan 4 is perhaps the best overall, though it ranks only seventh on the two perimeter measures (first or second on dispersion).[15] Thus, in some instances, we will not find one plan that is uniformly judged most compact.

[15] Using median scores, district 4 is the most compact on all but Per_1.

TABLE 5

COMPACTNESS OF CONGRESSIONAL DISTRICTS FOR COLORADO (6 DISTRICTS)

	Dis_4		Dis_{10}			Per_2	
	Range	Mean	Range	Mean	Per_1	Range	Mean
Plan 1[a]	.68–.83	.76	.39–.58	.46	.80	.35–.59	.48
Plan 2	.56–.77	.69	.25–.49	.41	.75	.24–.70	.47
Plan 3	.50–.92	.67	.35–.58	.43	.96	.43–.71	.55
Plan 4	.51–.94	.80	.32–.65	.49	.91	.24–.69	.53
Plan 5	.50–.96	.69	.29–.68	.44	.89	.26–.63	.49
Plan 6	.55–.91	.72	.33–.65	.46	.92	.40–.78	.56
Plan 7	.59–.86	.75	.36–.56	.46	.83	.29–.79	.50
Plan 8	.44–.77	.63	.27–.49	.39	1.00	.46–.68	.53
Plan 9	.44–.86	.62	.32–.52	.42	1.00	.47–.75	.60
Plan 10	.53–.89	.74	.31–.54	.42	.84	.36–.58	.47
Plan 11	.56–.91	.72	.35–.61	.43	.82	.38–.68	.48
Plan 12	.42–.81	.71	.33–.58	.46	.89	.30–.73	.53
Plan 13	.45–.94	.75	.24–.62	.47	.90	.44–.70	.55
Plan 14	.26–.84	.59	.19–.53	.37	.88	.35–.55	.46
Plan 15	.48–.91	.68	.32–.70	.46	.98	.46–.84	.59
Plan 16	.51–.99	.81	.31–.60	.49	.82	.34–.64	.48
Plan 17	.67–.85	.78	.38–.52	.46	.99	.32–.63	.51

Correlations between measures (Pearson *r*/Spearman *rho*):

	Plan 1		Plan 2		Plan 3		Plan 4	
	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2
Dis_4	.87/1.0	.64/.54	.87/.77	.64/.54	.77/.71	.21/.14	.74/.90	.27/.50
Dis_{10}		.65/.54		.71/.66		.73/.77		.80/.60
	Plan 5		Plan 6		Plan 7		Plan 8	
	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2
Dis_4	.91/.80	.38/.50	.95/1.0	.93/.83	.26/.37	−.12/−.37	.97/.90	.59/.67
Dis_{10}		.69/.70		.82/.83		.83/.60		.56/.66
	Plan 9		Plan 10		Plan 11		Plan 12	
	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2
Dis_4	.77/.83	.15/.37	.72/.71	−.01/−.14	.86/.77	−.27/−.31	.61/.66	.07/.09
Dis_{10}		.73/.77		−.15/−.31		.03/.26		.71/.66
	Plan 13		Plan 14		Plan 15		Plan 16	
	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2	Dis_{10}	Per_2
Dis_4	.99/1.0	.80/.89	.95/.83	.72/.37	.93/.94	.83/.89	.93/.77	.50/.31
Dis_{10}		.80/.89		.74/.66		.96/.94		.39/.14
	Plan 17							
	Dis_{10}	Per_2						
Dis_4	−.04/−.09	−.19/.31						
Dis_{10}		.67/.60						

[a] Plans for the 1980s received by a federal court when a Republican legislature and Democratic governor could not agree. Plans came from individual legislators of both parties, the governor's office, and various groups.

While no one plan is singled out as the most compact, there is nonetheless a large gap between the best and the worst of the 17 plans. In a case such as this, compactness would be more or less useful depending on the plans being compared. If, on other grounds, one favored plans such as 5 and 11, compactness would be of little value in choosing between them. On the other hand, if plans 2 and 7 were being compared, there would be little doubt that the latter is more compact. In short, quantitative measures will often answer the question of which districting plan is most compact; in other instances they will confirm that none of the plans under consideration can be said to be uniquely most compact.

Finally, we turn to California, where we compare congressional districts in the 1970s and two "Burton" plans for the 1980s. Again we introduce a new element—a compactness measure based on population. In addition, we have the dispersion measure based on a circumscribing circle (Dis_7) rather than the longest-axis circle (Dis_{10}).

That the population measure taps something different from areal measures is obvious from the low correlations between Pop_1 and both Dis_7 and Per_1 (table 6) and from the scores for individual districts. A startling example is district 2 in the Master's plan. According to Pop_1 it is the most compact district of all, with a near-perfect score of .967; its geography, however, rates it second from the bottom on both Dis_7 and Per_2, with scores of .18 and .23, respectively. But unpopulated land, which accounts for the results in district 2, is not the only factor at work. District 15 is relatively compact by Dis_7, tied for 10th place among the 43 districts of the 1970s; nearby population concentrations (Fresno) were excluded from the district, leaving it tied with two others for the 38th spot on Pop_1. Nor does the population exclusion in district 15 reflect an especially jagged perimeter, making Pop_1 and Per_2 redundant; the district is ranked 15th by the perimeter measure.

Overall comparisons for California show that scores on the population measure, as on the width/length measure in Indiana, tend to be considerably greater than those for area/circle and perimeter measures (table 6). In addition to higher averages, nearly every individual district has a higher score on Pop_1 than on both Dis_7 and Per_2. What is most significant about the overall comparisons, however, is the extraordinarily low perimeter scores for the Burton I and Burton II plans. Especially significant is that the high end of the range is so truncated, suggesting that the perimeter of every district was carefully constructed.

As in most of our other cases, the results in California are consistent across measures. Using any of the four scores (although use of the length of perimeter measure is questionable because of the differing numbers of districts), the Master's Plan was more compact than either Burton I or II. But this is the first instance in which one measure points so dramatically to the specific feature of a plan that makes it ill-compact. The difference between the Mas-

TABLE 6

COMPACTNESS OF CONGRESSIONAL DISTRICTS FOR CALIFORNIA
(1970S: 43 DISTRICTS; 1980S: 45 DISTRICTS)

	Dis_7			Per_2		Pop_1	
	Range	Mean (s.d.)	Per_1	Range	Mean (s.d.)	Range	Mean (s.d.)
Master's[a]	.13–.60	.40 (.11)	1.00	.15–.72	.38 (.11)	.40–.97	.72 (.15)
Burton I[b]	.15–.57	.34 (.10)	.64	.05–.40	.16 (.09)	.26–.87	.61 (.17)
Burton II[c]	.13–.54	.34 (.09)	.68	.06–.39	.20 (.09)	.30–.93	.62 (.17)

Correlations between measures (Pearson r/Spearman *rho*):

	Master's		Burton I		Burton II	
	Per_2	Pop_1	Per_2	Pop_1	Per_2	Pop_1
Dis_7	.76/.72	.28/.29	.54/.51	.30/.32	.53/.50	.13/.11
Per_2		.37/.37		.36/.39		.31/.37

[a] Drawn by Special Master for the State Supreme Court. Used 1974–1980.

[b] Drawn under direction of U.S. Rep. Philip Burton. Generally regarded as one of the most carefully crafted gerrymanders in American electoral history. Used 1982.

[c] Redrawing by Rep. Burton of Burton I. Slightly less grotesque than Burton I, but still regarded as a Democratic gerrymander. Used 1984–1990.

ter's Plan and Burton I and II is "only" 16%-18% for Dis_7 and Pop_1; for Per_2 it is a whopping 90%-111%. As for the two Burton plans themselves, one measure shows no change at all, one shows a tiny increase in compactness, and the third shows a 25% improvement over the very low base.

DISCUSSION

Our primary goal has been to clarify the concept and measurement of compactness, with an emphasis on understanding the numerous quantitative measures that have been proposed over the past 25 years. Our theoretical analysis showed that compactness has multiple components and that no single measure can adequately assess all of them. That led directly to our empirical analysis, in which we compared compactness scores of alternative districting plans. We found that the major types of measures sometimes vary widely in their evaluation of the compactness of given districts but that there is some support for the hypothesis that multiple measures yield similar assessments of districting plans. Along the way we also gained further insight into the strengths and weakness of particular compactness measures and of their theoretical and empirical relationships to one another.

Given these results, especially that compactness has multiple, distinct components, how can compactness be used as a districting criterion? We suggest four guidelines.

First, comparisons should almost always be limited to the state or other jurisdiction being districted. Because of different initial shapes, along with rivers, coasts, and other "natural" boundaries, Maryland and Montana, Wisconsin and Wyoming, New York and Nebraska, etc., are unlikely to achieve comparable degrees of compactness.

Second, and for largely the same reason, quantitative scores should be used to make comparisons, not to eliminate plans or districts that fail to meet a predetermined level.[16] There is no score for any one measure, much less for all of them, that on the face of it indicates unsatisfactory compactness; characteristics of the area being districted make identification of such levels impossible. The fact that compactness is a relative measure does not render it meaningless. We deal with many such concepts every day. There is no precise temperature, for example, that marks the transition from cold to hot. Yet 10 degrees is rather uniformly regarded as cold and 90 degrees as hot.

Third, compactness should be one of a number of districting criteria and not a sole criterion after population equality and racial fairness. A more important point follows: one should not mechanically choose from among alternative plans the one with the greatest compactness. Ill-compact districts, in

[16] While we emphasize comparisons of entire plans, there may be instances in which the compactness of specific districts is at issue (e.g., when it is alleged that specific districts and not others were gerrymandered).

Justice Stevens' words, "are a signal that something may be amiss" (*Karcher v. Daggett*, 462 U.S. at 758, 1983). They call for justification of district boundaries but not necessarily revision. Sometimes justification may be as simple as pointing out for specific districts that rivers and other natural boundaries account for low scores. At other times it may take the form of arguments about which goal should take precedence, such as when greater compactness would require splitting existing towns or counties. But the point is that compactness alone does not make a districting plan good.

Fourth, multiple measures should be used whenever possible. Areal dispersion, perimeter length, and population dispersion are not substitutable. When multiple measures coalesce in support of a single plan, the evidence in its favor is very strong. Our analysis suggests that that will often be the case, especially when the number of plans has been narrowed down to two or three. When there is lack of complete agreement, it is still likely that certain plans will be identified as especially compact or noncompact. There will, of course, be situations in which use of multiple measures indicates "no decision," that no single plan is most compact. But that is inherent in the concept, not a defect in the measurements as such. When multiple measures show genuine ambiguity, it is precisely because no one district, or no single plan, has all of the characteristics of compactness.

Which particular measures should be used? On that point we reached less closure, though we still made considerable progress. We narrowed the number of measures by identifying those that are transformations of one another; we strongly argued for normalized measures rather than those that are dependent on scale; we identified a 0–1 form for all measures to ease interpretation; we found that width/length measures add little to other measures of areal dispersion; we pointed out that in most circumstances two dispersion measures (Dis_7 and Dis_{10}) are equivalent. This still leaves multiple measures of each type (Dis_7-Dis_{10}, Per_1-Per_2, Pop_1-Pop_3), and while measures within a given type are relatively similar, there will no doubt be instances in which they will rank districts and even entire plans differently.[17] Yet as we noted, some disagreement among measures is inherent in the concept of compactness, and conflict rather than coincidence of rankings may be useful.[18]

As we move into the 1990s round of districting controversies, disputes about compactness will be numerous. We noted at the outset that there are those who would dismiss it outright as well as those who believe in it passionately. Whatever turns out to be its utility as a districting standard, we

[17] Additional questions will arise in some applications. For example, is only mean compactness relevant, or is it appropriate to ask which plan has the district with minimum compactness?

[18] If desired, a legislature would seem to be within its right to adopt one kind of measure, or even one specific measure, as the binding definition of compactness in its jurisdiction. If compactness is to be more than an empty concept, some precise definition in the law would be useful.

hope that we have sufficiently clarified the concept so as to stimulate more rational, enlightened discussion of its merits and faults as well as further study of its supposed effects.

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Appendix

Computations were done at different times and places, so that the means of calculating compactness and the accuracy of the computations varies. The calculations for New York and California are extremely precise, those for Rhode Island relatively so. Those for Indiana and Colorado are crude but sufficiently accurate for present purposes.

Precise specification of district boundaries and accurate measurement of features such as perimeter and area is an expensive, time-consuming operation. Geographic information systems generally treat districts as polygons, closed shapes consisting of connected line segments representing the boundaries of the district. In most cases these polygons are created by the process known as "digitizing," the conversion of line data on maps to digital information stored in a computer-readable format.

Part of this process includes "linear simplification" through the elimination of unwanted detail. In creating simple graphics, such as state road maps, it is obvious that many details of the actual borders of areas cannot be included. Considerable simplification must be made of the irregular natural boundaries that characterize coastlines and waterways and the sharp twists and turns of highway networks and property lines. Simplification must also be made in computerized mapping, but large-scale maps can be used to capture far more detail than is visible to the casual state or city map user.

In computerized mapping, the elimination of detail takes place either during the input or digitizing process (when lines are "traced" into the computer) or during processing (when the number of points indicating the intersection of line segments is reduced). In either case, the result is smoother curves that require less resolution in plotting, less storage, and less plotting time. Display maps produced by simplification algorithms may use as few as 10% of the intersection points in the actual boundary.

The compactness values for New York are based on highly accurate calculations taken from digital files containing detailed geography for the actual borders of the congressional districts. These files were digitized from U.S. Geological Survey maps by the New York State Department of Transportation and the New York State Legislative Task Force on Demographic Research and Reapportionment. Special computer programs were then used to calculate areas and perimeters of the districts.

A technical problem is how to treat districts including bodies of water—e.g., national and international borders that fall within bodies of water. In New York we used official state boundary descriptions. In calculating area, however, we sometimes eliminated from the districts portions of bodies of water and uninhabited small off-shore islands that are officially part of a district. Including a large portion of water area in a district can affect compactness measures significantly, and a clearer picture of the compactness of the major land portions of a district may often be achieved if these areas are simply omitted.

The level of precision in our other calculations varies greatly. Relatively detailed information was used in California. Population-based measures of compactness require a centroid-based mapping scheme. With each element of census (or other) geography we associate a point at its center of gravity and treat all population in the geographic unit as concentrated at that point. Data files that contain such information are available for public use from the Census Bureau. Computations were done by the Division of Computer Services of the Republican National Committee.

Large-scale maps were analyzed by hand in Rhode Island by a firm of civil engineers hired by the state and acting under Grofman's supervision. Data from Colorado and Indiana are based on 8 1/2″ x 11″ state maps (with some more detailed maps for urban areas) with districts overlaid by hand. Digitization was carried out at the Institute of Optics of the University of Rochester using an Omnicon Image Scanner. A part of the scanner system was a program to compute areas, perimeters, and other features needed to calculate compactness scores.

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