STABILITY AND CENTRALITY OF LEGISLATIVE CHOICE IN THE SPATIAL CONTEXT

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Majority-rule spatial voting games lacking a core still always present a "near-core" outcome, more commonly known as the Copeland winner. *This is the alternative that defeats or ties the greatest number of alternatives in the space. Previous research has not tested the Copeland winner as a solution concept for spatial voting games without a core, lacking a way to calculate where the Copeland winner was with an infinite number of alternatives. We provide a straightforward algorithm to find the Copeland winner and show that it corresponds well to experimental outcomes in an important set of experimental legislative voting games. We also provide an intuitive motivation for why legislative outcomes in the spatial context may be expected to lie close to the Copeland winner. Finally, we show a connection between the Copeland winner and the Shapley value and provide a simple but powerful algorithm to calculate the Copeland scores of all points in the space in terms of the (modified) power values of each of the voters and their locations in the space.*

It is well known that, under majority rule in a world with more than one issue dimension, a majority winner does not, in general, exist. That is, there is no one alternative that can defeat each and every other alternative in paired contest (McKelvey 1976, 1979; Schofield 1978). Furthermore, we can expect that all or virtually all the alternatives will cycle with one another. One common interpretation of those results is that institutions that use majority rule ought not to work: since choices are cyclical, losers should always be able to find some alternative they like better that could defeat the present status quos, and so on ad infinitum. Thus, all legislatures should be in constant turmoil as losers try to reverse decisions they do not like. Another implication of the results of what are often called "chaos" theorems is that expected outcomes of legislative majority-rule decision processes could range over the entire issue space, depending upon exactly how the agenda is specified; hence, political decision making should lack any clear central tendency. These instability results have led to considerable skepticism about the usefulness of the standard populist view of democratic decision making as a clear choice from among a set of alternatives in accord with the will of the majority (see review in Riker 1980, 1982).

When we look at the world, whether it be the real world of legislative decision making or the parallels to that real world that have been generated by carefully constructed experimental voting games such as those of Fiorina and Plott (1978), we find that outcomes are actually quite stable and that voters in a committee or legislative setting are able to pick an alternative and stick to it. Indeed, in legislatures, once a bill is enacted, changes from a given status quo are apt to be only incremental, even over a period of years. Also, outcomes are far from randomly distributed over the issue space. Rather, votes seem to track some central tendency in the group. In particular, in experimental legislative spatial voting games where, in principle, *any* outcome would be feasible, we observe that different committees rapidly make choices that are remarkably alike (see, e.g., Fiorina and Plott 1978).

Investigating why the world does not behave as the theoretical models of majority rule in more than one dimension say it should has been a booming industry in the past several years (Tullock 1981). Various approaches have been offered to account for an observed stability in legislative majority-rule processes beyond what could be expected by the various "chaos" theorems and impossibility results. Among these is the view that various institutional features simplify or delimit the domain of choice so as to make a majority winner if not certain, at least much more likely (Grofman and Feld 1986; Shepsle 1979; Shepsle and Weingast 1982). Also, even though there may not be a unique majority winner and even though the top cycle set (i.e., the minimal set of alternatives that cycle with one another) may be very large, there may still be a probability distribution over expected outcomes that is well behaved, for example, with probability mass concentrated in some central area of the Pareto set (Ferejohn, Fiorina, and Packel 1980; Ferejohn, McKelvey, and Packel 1984). Moreover, even if prior to the actual decision there were alternatives that could have defeated it in pairwise contest, a number of postdecisional mechanisms (both cognitive and institutional) act to reinforce the legitimacy and stability of whatever choice the society may have made (Grofman and Uhlaner 1985).

A variety of other ideas have been proposed to account for stability. For example, it has been suggested that the de facto decision process may be supramajoritarian and this may lead to stability even in situations where the majority rule is unstable (Schofield, Grofman, and Feld 1985). Kramer (1977) argues that repeated two-party competition will drive outcomes into the minimax set in the interior of the Pareto set. Wuffle, Feld, Owen, and Grofman (n.d.) argue the attractiveness of what they label the "finagle point" in spatial games of two-candidate competition. A number of authors claim that coalition-formation processes will induce stability, proposing a plethora of competing solution concepts as to where outcomes will lie (see, e.g., McKelvey, Ordeshook, and Winer 1978; Owen 1982; Straffin and Grofman 1984). It has also been argued that stability is more likely if actual choices more nearly resemble a yes-no referendum on whether existing policies will be maintained than if choices are made among a numerous and almost certainly cyclic set of alternatives (Riker 1982). Finally, Miller (1980) and others (Banks 1985; Feld, Grofman, and Miller 1985; McKelvey 1986; Miller, Grofman, and Feld 1985; Moulin 1984; Shepsle and Weingast 1984) have suggested that, even when all alternatives cycle with one another, there may be sufficient internal structure to the majority-preference relation for a legislative or polity that the only likely outcomes either will be few or will lie in a small area central in the issue space, within the set of outcomes that has been called the "uncovered set."

In exploring the reasons for stability of legislative choice, we focus on the properties in the spatial context of an old solution concept, the *Copeland winner.* The *Copeland winner* (which we shall also refer to as the *strong point*) is simply that alternative which is majority-preferred to the highest proportion of other alternatives in the space. The Copeland winner is identical to the core if a majority winner exists. The core is the most theoretically powerful and empirically predictive of known solution concepts. The Copeland winner has never before been applied to try to make sense of multidimensional legislative voting because it was not known to calculate where the Copeland winner was in a multidimensional issue space with an infinite number of alternatives.

The aim of this paper is threefold. First and foremost, we show that the Copeland winner, known in the finite case to have a number of desirable axiomatic characteristics (Copeland 1951; Straffin 1980), has a number of additional properties that make it a particularly attractive solution concept in the spatial context. In the spatial context, the Copeland winner (1) always exists, will in general be unique, and can be located relatively easily; (2) once found, turns out to predict outcomes in the Fiorina and Plott (1978) experimental voting games better than do more than one dozen previously proposed models; and (3) can be given an intuitive motivation that seems to explain why voters in a legislative situation would end up with outcomes at or near the Copeland winner.

Second, we show that, in the spatial context, the Copeland winner can be expressed in terms of the (modified) Shapley value, a key game-theoretic measure of power. This connection is a surprising one and important in that it shows a deep mathematical link between two hitherto largely unrelated areas of research.

Finally, we show that the Copeland values (defined as the proportion of all other points to which a given point is majority-preferred) fall off monotonically with distance from the Copeland winner along any ray. Moreover, the strong point is always in the uncovered set[1] and (roughly speaking) a central point in the Pareto set. Hence, the most "popular" alternatives in the space are all spatially clustered near a central area of the Pareto set.

We believe that most legislative agenda processes can be expected to lead voters to outcomes near the strong point,[2] especially in legislative voting games played noncooperatively (i.e., without coalitional alliances) and without party organizations to structure voting.

Locating the Strong Point

Finding the strong point requires finding a point x such that the area (measure) of the set of points that are majority-preferred to it is minimal. We shall present the basic results for two dimensions: for Euclidean preferences; and for n, the number of voters, odd.

THEOREM 1. *For voter ideal points located on a convex polygon, the strong point, x, is given by*

$$x = \frac{\sum_{j=1}^{n} c_j P_j}{\pi} \qquad (1)$$

where the P_j are the voter ideal points, and the c_j correspond to the "star" angles of the polygon (measured in radians).

A complete proof of this result is given in Shapley and Owen 1985. We illustrate the calculations for $n = 5$ (see Figure 1).

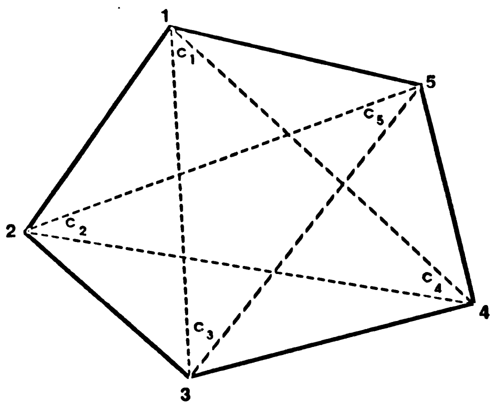


Figure 1. The Star Angles c_1 through c_5 of a Convex Pentagon

The star angles of the pentagon shown in Figure 1 are

$$\begin{aligned} c_1 &= \measuredangle\, 413 = .4636 \\ c_2 &= \measuredangle\, 524 = .8663 \\ c_3 &= \measuredangle\, 135 = .8330 \\ c_4 &= \measuredangle\, 241 = .2408 \\ c_5 &= \measuredangle\, 352 = .7378 \end{aligned}$$

The sum of the star angles, $\sum_{j=1}^{n} c_j$, equals π.

The P_i values (locations of voter ideal points) in Figure 1 are given by

$$\begin{aligned} P_1 &= (80, 40) \\ P_2 &= (100, 40) \\ P_3 &= (140, 80) \\ P_4 &= (120, 110) \\ P_5 &= (60, 100) \end{aligned}$$

Substituting these values in Equation 1, we obtain $x = (99.79, 70.06)$.

When not all voter ideal points are on a convex polygon, the minimization formula is identical, but the specification of the angles changes. For example, for the five-voter game with one voter, E, inside the convex hull (see Figure 2), the strong point is given by the same formula as in Equation 1, but now $A = (30, 52)$, $B = (25, 72)$, $C = (62, 109)$, $D = (165, 32)$, $E = (51, 59)$. Then

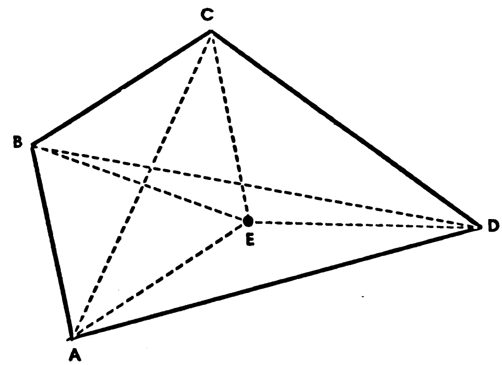


Figure 2. Five-Voter Game with One Ideal Point, E, inside the Convex Hull (E Also Being the Strong Point)

$$\begin{aligned} c_1 = c_A &= \measuredangle\, CAE = .7375 \\ c_2 = c_B &= \measuredangle\, EBD = .1853 \\ c_3 = c_C &= \measuredangle\, ACE = .2950 \\ c_4 = c_D &= \measuredangle\, BDE = .0457 \\ c_5 = c_E &= \measuredangle\, BEC + \pi \\ &\quad - \measuredangle\, AED = 1.878 \end{aligned}$$

Thus, $x = (47.23, 62.43)$.

Thus, in two dimensions, locating the strong point is quite straightforward.

If there is a core, it is well known that with an odd number of voters the core must be one of the voter ideal points

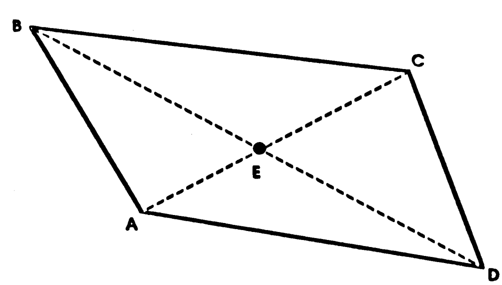


Figure 3. Five-Voter Game with a Core

(McKelvey and Wendell 1976). Plott (1967) provides a condition sufficient for a core: the existence of an (interior) point, E, such that every line through that ideal point passes through the same number of voter ideal points on each side of E. We show in Figure 3 a five-voter situation in which the ideal points satisfy the Plott conditions. Here, c_1 through c_4 would be zero and $\measuredangle BEC = \measuredangle AED$; that is, $c_5 = c_E = \Pi$, and thus $x = P_5 = P_E$. Hence, x is a core point.

The formula given in Equation 1 was derived for the case where all points in the space are feasible alternatives. However, this strong assumption is not required. Moreover, an adaptation of the methodology that gave rise to Equation 1 can be used to find the strong point for choice over any feasible set of alternatives, although the algebra may become more complex and numerical approximation techniques may be necessary. As long as the feasible set includes the points in a circle containing the central portion of the Pareto set, the formula given in Equation 1 will hold.[3] We subsequently show how to calculate the point with highest Copeland value for an important special case, that in which the feasible set of outcomes is restricted to the set of points that can defeat the status quo.

The Spatial Distribution of Copeland Values: Simulation and Results and Implications for Agenda Processes

Simulation Results

We use a simulation to determine the Copeland values for each of the points in the space for some simple five-voter games in two-dimensional space (games investigated by Ferejohn, Fiorina, and Packel 1980; and Fiorina and Plott 1978), one with a core and one without. Inspection of the resulting graphs shows the rate at which Copeland values drop off as we move away from the strong point.[4] Shapley and Owen (1985) have demonstrated that Copeland values will fall off monotonically with distance from the strong point along any ray. In particular, the isovalue curves form a family of concentric circles with the equation

$$(x - x^*)^2 + (y - y^*)^2 = k \qquad (2)$$

where (x^*, y^*) is the strong point.

Our first example is one of the three five-voter games used in Fiorina and Plott 1978, game 1, with voter ideal points at (39, 68), (30, 52), (25, 72), (62, 109), and (165, 32). The core is at (39, 68) (see Figure 4). As we see, Copeland scores are very high near the core point. The experimental outcomes found by Fiorina and Plott in their high-payoff-with-communication condition had a mean of (37, 68); in their high-payoff-no-communication condition, the mean was (38, 69). In both conditions, but especially the former, agreed-upon outcomes were *very* tightly clustered near the mean. The core (and thus the strong point, since in this situation the two are identical) predicted quite well in this game.

Our second example is the third of the three five-voter games used in Fiorina and Plott 1978; this one has voter ideal points at (51, 59), (30, 52), (25, 72), (62, 109), and (165, 32). This game is without a core. We show Copeland values for the points in this game in Figure 5.

Fiorina and Plott (1978) consider sixteen different solution concepts generating eight distinct predictions for game 3. For this game, which lacks a core, none of the solution concepts was of much value. The observed outcomes were rather far from any of the points predicted by any of the solution concepts.

Nonetheless, in this game, observed outcomes were still tightly clustered highly around a mean observed outcome of (45, 62), albeit not quite as tightly as in the games with a core (a standard devia-

Figure 4. Approximate Distribution of Copeland Values in a Five-Voter Game with a Core, with Voter Ideal Points at (39, 68), (30, 52), (25, 72), (62, 109) and (165,32)

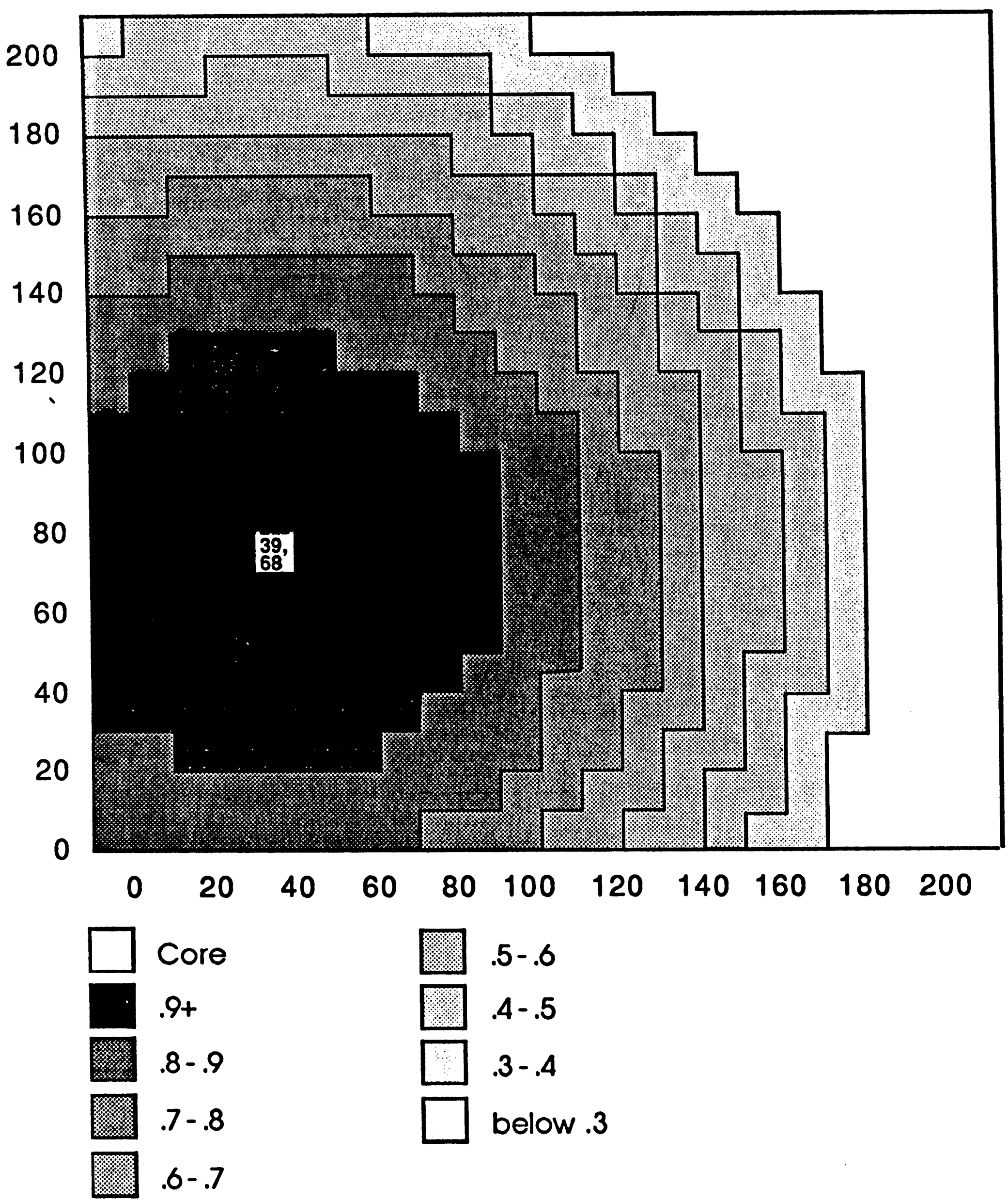
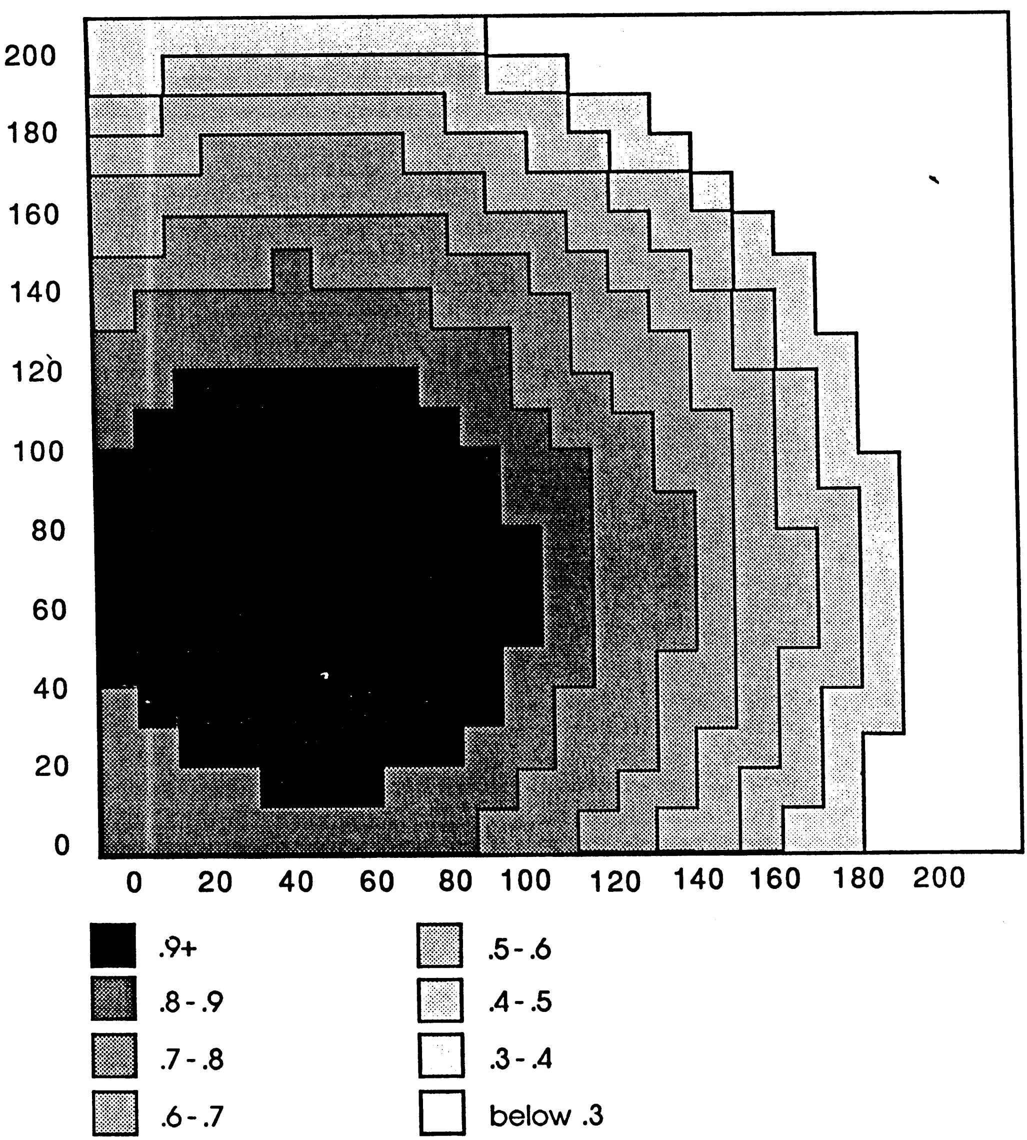


Figure 5. Approximate Distribution of Copeland Values in a Five-Voter Game without a Core, with Voter Ideal Points at (51, 59), (30, 52), (25, 72), (62, 109), and (165, 32)



tion of 10.3 in this game versus standard deviations of 5.2, 7.3, and 8.3 in various high-payoff experimental conditions in the two games with a core). Fiorina and Plott (1978, 590) contrast this finding of tightly clustered outcomes with the "chaos" suggested by McKelvey's (1976) result that "from any point in the space one can construct a sequence of alternative policies which under sincere voting lead to any arbitrarily selected point." They also note that they "did not notice any behavioral differences" and that "subjects in experiments on Game 3 appeared to have no greater difficulty in reaching a decision than did those in Game 1." Commenting about games without a core but whose outcomes were tightly clustered in a fashion inexplicable by any of the numerous theories they looked at, Fiorina and Plott (1978, 590) state "[We] wonder whether some unidentified theory is waiting to be discovered and used," and they go on to suggest that "if some as yet undeveloped theory is driving the . . . experiments (with game 3) it had better specialize to the . . . core when the latter exists."

We claim that the hypothesis that outcomes will be clustered at or near the strong point is just such a theory—reducing to the core when one exists and capable of accounting for observed outcomes in majority voting games without a core such as those in game 3 in Fiorina and Plott (1978). The strong point of game 3 is (47, 62), a value obtained from use of Equation 1, as solved earlier and confirmed by our simulation. It predicts the mean outcome in that game (45, 62) almost perfectly. The fit of the strong point in game 3 is as good as the fit of the core to the Fiorina and Plott (1978) experimental outcomes in the two games they used that had cores.

Of course, we are not claiming that the strong point can account for voting outcomes in all majority-rule games. Although the core is far and away the best single predictor of outcomes in games where a core exists (and the strong point reduces to the core when a core exists), it may not be chosen if the group focuses on ethical judgments related to equity norms (Eavey and Miller 1982). Also, it is well known that institutional features of choice (e.g., veto power, see Wilson and Herzberg 1984) or restrictions on feasible amendments such as a germaneness rule (Shepsle 1979) can significantly affect outcomes. In such cases we would need to modify our model.[5]

Often such modifications can be done simply. For example, one common rule for legislative decision making is to require a final vote against the status quo (Black 1958; Farquharson 1969).[6] For this rule, the modifications to Equation 1 needed to find the point with the highest Copeland value among the *feasible* outcomes (those in the win set of the status quo) are quite straightforward. We merely find the closest tangency of the family of circles defined in Equation 2 (itself, in turn, derived from the strong point defined by Equation 1) to the win set of the status quo; that is, we find the point in the win set of the status quo that is closest to the overall strong point in the space. The fact that Copeland values decline monotonically from the overall strong point provides the justification for this point of tangency's being the point in the win set of the status quo with the highest Copeland value. However, this point is not necessarily the one that beats the maximum number of points in the win set itself.

Implication for Agenda Processes

Consider an open-agenda process in which alternatives are proposed with equal likelihood and that has some deterministic stop rule (e.g., stop if an alternative defeats k successive challengers). Then the likelihood that an outcome will be chosen is a function of its Copeland value; the probability density for the open-agenda process with such a stop rule will be even more thickly massed around the strong point than will the distribution of Copeland values because points that defeat few other points will be eliminated quickly. The open-agenda process with a k-defeats stop rule is a relatively unconstrained one. In more constrained (and more realistic) agenda models (e.g., one in which the likelihood that a point will be proposed is a function of the number of majority coalitions that prefer it to the status quo [Ferejohn, Fiorina and Packel 1980]), outcome probabilities will be even more sharply massed around the strong point than is true in the open-agenda process. The same is true, we believe, if we were to compare the open-agenda process to one in which proposed policies must be elements of the uncovered set (Shepsle and Weingast 1984, 65 n. 17), at least as long as there is some sort of stochastic stop rule. Thus, to establish that Copeland values fall off monotonically with distance from the strong point guarantees us that most reasonable agenda processes will, with very high probability, result in outcomes at or near the strong point.

If a known status quo will be in the final pairing, then, as noted above, the strong point can be defined relative to the set of alternatives that defeats the status quo. If the final vote always involves the previously defined status quo, then legislators have an incentive to propose alternatives with a high probability of beating those against which they might subsequently be paired. Again, the probabilities of choice will be highest for alternatives near the strong point because these are the ones whose "viability" is greatest.[7]

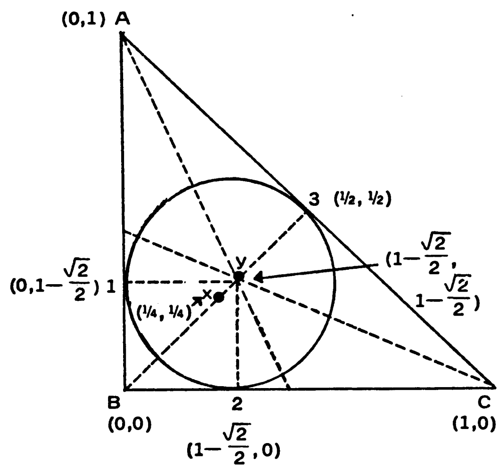
Even if legislators try to manipulate the agenda by introducing alternatives so as to eliminate other alternatives (a situation not true for the Fiorina and Plott games), under standard amendment procedure, as long as legislators cannot be certain that any given vote will be the final vote (or the next-to-final vote), then alternatives that defeat others are likely to be proposed and chosen.[8] Moreover, when voters are sophisticated, the outcomes of the usual legislative procedures will lie within the uncovered set (Shepsle and Weingast 1984), in which the strong point is a central point.

The Strong Point and the Center of the Yolk

Our results are closely related to an important recent body of work whose full implications have not yet been absorbed into the established wisdom of social choice. A group of scholars (Ferejohn, Fiorina, and Packel 1980; Ferejohn, McKelvey, and Packel 1984; Hoffman and Packel 1981; Packel 1981) have shown that even in the absence of a core, certain plausible sincere majority-rule processes are well behaved in the sense that there exist stochastic limiting distributions that will characterize the set of expected outcomes and that these distributions can be expected (almost always) to confine outcomes to a relatively small section of the space. The area in which outcomes will cluster is centered around what they call the *yolk* of the voting game.[9]

It would be nice if the strong point corresponded to the center of the yolk. Unfortunately it does not, except when there is a core—although it does appear that the center of the yolk and the strong point are not very far away from each other in the examples we have looked at. In particular, in the Fiorina and Plott game without a core shown in Figure 5, the center of the yolk is approximately (46, 64). For that game we have previously found the strong point to be (47, 62). To aid the reader in understanding the connection between the center of the yolk and the strong point, we present a simple example. Consider the right triangle with ver-

Figure 6. Comparison of the Location of the Center of the Yolk, *y*, with the Location of the Strong Point, *x*, in a Simple Three-Voter Game



tices (0, 0), (0, 1), and (1, 0) shown in Figure 6.

Using Equation 1, the strong point of that triangle can be shown to be (1/4, 1/4). The *yolk* (Ferejohn, McKelvey, and Packel 1984) is the minimum sphere that intersects all median hyperplanes. In two-dimensional space with three voters, the median hyperplanes are simply the lines through the vertices of the triangle that pass within or through the edges of the triangle. Hence, in Figure 6, the center of the yolk is simply the center of the largest circle that can be inscribed within the triangle. Some simple geometry shows that this point is $(1 - \sqrt{2}/2, 1 - \sqrt{2}/2) = (.293, .293)$.

It seems apparent that with a more symmetric distribution of ideal points, the center of the yolk would be even closer to the strong point. If there is a core, of course, the two concepts are identical because the yolk collapses to a point, that is, collapses to the core (which is the strong point). Moreover, very preliminary work suggests that the strong point will also usually lie close to other recently proposed solution concepts.

The Strong Point and the Modified Shapley Value

Owen (1971) introduced a generalization of the Shapley value based on coalitional probabilities that were a function of ideological proximity. Shapley (1977) modified the Owen (1971) model to provide a further nonsymmetric generalization of the Shapley value. These papers developed a new type of Shapley value applicable to voting games where plays are located in some Euclidean "ideological" space (see also Owen 1972; Straffin 1977).

THEOREM 2. *The strong point, x, can be expressed as*

$$x = \sum_{j=1}^{n} \phi_j P_j \tag{3}$$

where the P_j are, as before, the locations of voter ideal points, and the ϕ_j are the modified Shapley values as defined in Shapley 1977.

While the formula of Equation 3, like that of Equation 1, is quite simple, the proof of Theorem 2 (Shapley and Owen 1985) is very complex.

The usual Shapley value can best be explained by saying that voter i's power is the probability that, in a randomly chosen ordering of the n-players (with all orderings having equal probability), voter i will be in pivot position, that is, the coalition of all members preceding i in the order loses, but it wins with i's cooperation (Straffin 1980; Owen 1982). The modified Shapley values of both Owen and Shapley treat players as points in Euclidean space of some dimension; the power value is then defined as the probability that a given player will be pivotal in a randomly chosen ordering, the difference lying in the probability distributions that underlie this ordering. In Owen's scheme, a point in the space is chosen via a uniform distribution; orderings are taken in terms of the players' increasing distances from this randomly chosen point; while in Shapley's scheme, a direction in space (all points in the dual space) is chosen via a uniform distribution; orderings are then taken in terms of the players' locations along each direction.

To prove Theorem 2, Shapley and Owen (1985) make use of the fact that both the modified Shapley values and the Copeland values can be expressed in terms of $P_j(\Theta)$, where $P_j(\Theta)$ is the ideal point of the voter, j, who is pivotal when the ordering is in the Θ direction, that is, j is the median voter for voter projections onto all lines that make an angle of Θ with the origin (which may be arbitrarily chosen). In particular, if we let $A(q)$ be the Copeland value of voter q, then

$$A(q) = 2 \int_0^\pi \{U(\Theta) \bullet [P_q(\Theta) - q]^2\} d\Theta \quad (4)$$

where $U(\Theta)$ = the unit vector in the direction of $\Theta = (\cos \Theta, \sin \Theta)$. Equation 4 gives us the area of the set of points that beats q.

On the other hand,

$$\phi_q = 1/\pi \int_0^\pi P_q(\Theta) d(\Theta),$$

that is, the power of a player (as defined by his or her modified Shapley value) is simply the measure of the set of angles for which he or she is pivotal.

Conclusions

We believe that the strong point is a powerful solution concept that provides a plausible explanation for the stability and centrality of committee and legislative choices. The strong point is well defined and well behaved, easy to calculate, reduces to the core when there is a core, lies within the uncovered set, is responsive to individual differences in preferences, is centrally located in the Pareto set (thus guaranteeing that outcomes will not be too far from what most voters want), and explains hitherto inexplicable regularities observed in experimental settings. By specifying the Copeland winner in the spatial context and by showing its deep mathematical connection to the (modified) Shapley value, we also provide theoretically useful linkages between several distinct areas of positive political theory: social choice theory (in which the Copeland winner has long been known to have various desirable axiomatic properties), game theory, and spatial modelling.

As previously noted, it is our view that the search for a single ultimate solution concept for all political games is a futile one. There simply is no reason to believe that, say, coalitional models of situations involving politics among voting blocs will give the same answers as models based on, say, the institutional structure of two-candidate party-based competition.[10] The solution concept we have proposed, the Copeland winner in the spatial context, was reviewed in the context of legislative voting games in a pure majority-rule setting under standard amendment procedure. Its usefulness in other settings is a matter for further research, but we believe the strong point has already been shown to be a powerful solution concept in one important institutional setting, that of the legislature.[11] However, we would note that other recently proposed solution concepts such as the center of the yolk (Feld, Grofman, and Miller 1985; McKelvey 1986) are likely to lie near the strong point and thus that it may be quite difficult to determine which model offers the best fit to experimental results.

Notes

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1. McKelvey (1986) and Moulin (1984) have shown that the Copeland winner must lie within the uncovered set. The uncovered set is the set of points that can defeat all other points on the space either directly or at one remove (Miller 1980). A wide variety of plausible sincere processes for agenda formation considered in Ferejohn, McKelvey, and Packel 1984, have been shown to lead voters to points in or near the uncovered set; while Miller (1980), McKelvey (1986) and Shepsle and Weingast (1984) have shown that sophisticated voting under standard amendment procedure leads to outcomes within the uncovered set. Our simulations have shown the strong point to be near the center of the uncovered set, at least for three-voter and five-voter games.

2. See n. 1.

3. We can state this result more precisely. Let the *yolk* be the minimum circle (sphere) that touches all median lines (hyperplanes), and let r be the radius of the yolk. Then it is known that the uncovered set lies within $4r$ of the center of the yolk (McKelvey 1986) and may be even closer than that; and it is also known that the strong point is in the uncovered set (McKelvey 1986; Moulin 1984). Furthermore, it is known that no point x that defeats some point y can be more than $2r$ further from the center of the yolk than is y itself (Feld, Grofman, and Miller 1985). Thus, even in the most extreme case, the only points that could defeat the strong point would lie within 6 radii of the center of the yolk. When the number of voters is large, a circle of radius $6r$ will be well within the Pareto set. We do not need to worry about points outside this central circle because they will not defeat the strong point and thus are irrelevant in defining the area of the strong point's win set. The $6r$ bound is, moreover, a very extreme one; for example, if the strong point were at the center of the yolk, then the only relevant points would be in a circle of radius $2r$. Because the strong point is likely to be close to the center of the yolk (see discussion later in the text), points within just slightly more than $2r$ of the yolk are the only ones with which we will need to concern ourselves in calculating the location of the strong point—the feasibility or infeasibility of other points is irrelevant. Thus, in absence of peculiar conditions that rule out the central region of the space as feasible, we may use Equation 1 to calculate the location of the strong point.

In experimental games (see, e.g., Fiorina and Plott 1978) and in the real world (where the only proposals to be taken seriously will be in some limited domain of the policy space), it is quite common for players to be restricted to alternatives from some specified domain of the space. It is also common for imposed utility schedules to be nonmonotonic with distance, for example, ellipsoidal or even kinked indifference curves.

In the Fiorina and Plott games, the imposed utility functions were not monotonic with distance and there were some important zones of discontinuity (payoff breaks). Margolis (1982, 114-98) argues that these discontinuities made the core points or points near them "prominent" outcomes because they fell in the set of "fair" alternatives defined by the domain in which all voters got some "reasonable" amount of utility. Thus, Margolis attributes corelike outcomes to processes quite far removed from purely self-interested calculations. He further argues that the protocols support such a "group-payoff" interpretation.

4. In our simulation, a delimited set of feasible points was used to calculate Copeland values. In finding the strong point analytically, we have measured area relative to a surface that—by the nature of the geometric construction—is confined either near to or within the convex hull (see Figure 1 in Shapley and Owen 1985). In our simulations, we permitted comparisons with points in a much larger area of the space. Clearly, as we enlarge the space of points with which comparisons are to be made, the absolute Copeland values will change. As we move away from the hull, the points exterior to the hull are largely Pareto-inferior to those at or in the hull; any point in the Pareto set will beat almost all the points in the space. Thus, how far out we go for our comparisons will affect the Copeland scores in absolute terms. If the convex hull is asymmetrically located with respect to the boundaries of the space whose points we are using for our comparison, extending the boundaries of the space may even marginally affect relative Copeland values and thus the location of the strong point, but, for reasons discussed in n. 3, adding points distant from the center of the yolk will not affect the win set of the strong point. Thus, if we are either "close in" enough or go far enough "out," such asymmetries will be irrelevant. In any case, the effects appear to be minor at worst.

If the yolk is small relative to the Pareto set, we conjecture that the exact shapes of voter indifference curves (e.g., circular vs. elliptical) is of minor importance in determining the strong point (Copeland winner). Even though the induced indifference curves in the experimental games studied by Fiorina and Plott (1978) are not circular, the values we have determined for the strong point in those games fit the observed outcomes in these games quite well.

5. More generally, we believe that processes of choice that involve committees or legislatures voting over sets of alternatives (see, e.g., Miller 1980; Miller, Grofman, and Feld 1985) may lead to different outcomes than those that involve choosing among competing coalitions or protocoalitions (see, e.g., Grofman 1982; McKelvey, Ordeshook, and Winer 1978) or between *two* competing political parties/candidates.

6. In standard amendment procedure (SAP, see Farquharson 1969; Grofman 1969), in order to win, an alternative (e.g., a main motion as amended) must also defeat the status quo.

7. Then, under SAP, the strong point will only be chosen if it is among those points that can defeat the status quo. Of course, if we treat the status quo as a random variable, then the strong point, since it defeats most points in the space, will also probably defeat the status quo.

8. We use the term *agenda* in the technical sense of a set of alternatives to a given motion to be voted on in a given sequence by a group such as a legislature (see, e.g., Farquharson 1969; Grofman 1969; Miller 1977), rather than in the broad sense of Cobb and Elder (1972) as the set of issue domains that are considered appropriate for legislative action.

9. See n. 3.

10. Cf. Ferejohn, Fiorina, and Weisberg (1978, 160–61). See also n. 5.

11. Glazer, Grofman, and Owen (1985) show that the strong point is a natural choice for an incumbent in two-candidate electoral competition if we assume that, due to electoral uncertainties, the challenger position can be thought of as a uniformly distributed random variable. Even if we do not make such an assumption, Glazer, Grofman, and Owen (1985) note that the strong point is a desirable point for candidates if there is a stochastic stop rule in an iterated sequence of candidate locational choices because the strong point beats most points in the space.

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Forthcoming in September

The following articles, controversies, and research notes have been tentatively scheduled for publication in the September 1987 issue:

David Austen-Smith and William Riker. "Asymmetric Information and the Coherence of Legislation."

Francis A. Beer, Alice F. Healy, Grant P. Sinclair, and Lyle E. Bourne, Jr. "War Cues and Conflictual Foreign Policy Acts: A Laboratory Experiment."

Jonathan Bendor, Serge Taylor, and Roland van Gaalen. "Stacking the Deck: Bureaucratic Missions and the Search for Alternatives."

Steven J. Brams and D. Marc Kilgour. "Threat Escalation and Crisis Stability: A Game-Theoretic Analysis."

Robert S. Erikson, John P. McIver, and Gerald C. Wright. "Explorations in Political Geography: State Culture and Political Attitudes."

Michael R. Hawthorne and John E. Jackson. "The Individual Political Economy of Federal Tax Policy."

Gregory J. Kasza. "Bureaucratic Politics in Radical Military Regimes."

Keith Krehbiel, Kenneth A. Shepsle, and Barry Weingast. "Institutional Constraints on Congressional Committee Power." A Controversy.

John Langton and Mary G. Dietz. "Trapping or Teaching the Prince." A Controversy.

Stuart Elaine MacDonald and George Rabinowitz. "The Dynamics of Structural Realignment."

Neil Mitchell, Rhode E. Howard, and Jack Donnelly. "Liberalism and Human Rights." A Controversy.

Helmut Norpoth. "Guns and Butter and Government Popularity in Britain." A Research Note.

Robert Powell. "Crisis Bargaining, Escalation, and MAD."

Lawrence A. Scaff. "Fleeing the Iron Cage: Politics and Culture in the Thought of Max Weber."

Laura L. Vertz, John P. Frendreis, and James L. Gibson. "Nationalization of the American Electorate: A Multi-Office Perspective." A Research Note.

Erratum

Bruce Bueno de Mesquita and David Lalman, "Reason and War" (December 1986, 1113–29): On page 1116, column 1, the first equation should read

$$P^i(Esc_i) = (E^i(U_{ij})+3)/6$$