Pergamon

Electoral Studies 20 (2001) 299–303

Electoral Studies

www.elsevier.com/locate/electstud

Short communication

# A note of caution in interpreting the threshold of exclusion

Bernard Grofman *

*School of Social Sciences, Department of Politics, University of California, Irvine, CA 92697, USA*

**Abstract**

For any electoral system, threshold of exclusion of that system has been used as a measure of the degree to which parties and candidates have an incentive to "narrow-cast" their appeals to a limited constituency. However, we must adjust the threshold of exclusion to take into account the fact that multimember constituencies will, *ceteris paribus*, be larger than single member constituencies. 

*Keywords:* Threshold of exclusion; Proportional representation; Electoral systems

The *threshold of representation* is the *minimum* support necessary to earn a party its parliamentary seat, based on the most favorable case scenario in terms of how the other parties divide up their votes;[1] the *threshold of exclusion* on the other hand, is the *maximum* support that can be attained by a party while still failing to win even one seat. (Rae et al., 1971; see also Rae, 1971; Rokkan, 1968; Loosemore and Hanby, 1971; Grofman, 1975; Lijphart and Gibberd, 1977).[2] The threshold of representation provides a necessary condition for parliamentary representation, the

* Tel.: +1-949-824-6394; fax: +1-949-824-8762.

*E-mail address:* bgrofman@uci.edu (B. Grofman).

[1] "(T)his analysis...is optimistic (Panglossian), for it presumes that established parties are obliging enough not to form alliances against an emergent party and even go so far as to divide their votes to its best advantage" (Rae et al., 1971, pp. 479–480).

[2] Rae et al., (1971, p. 480) observe that calculation of the threshold of exclusion is simplified by the fact that a small party's opponents "have no better strategy than either to (a) let one of their number stand alone against the party in each district, or (b) form a wholesale electoral alliance to oppose it in each district." Thus, we may calculate $T_E$ by supposing that a party with vote share $v_i$ faces a single adversary with a vote proportion of $1-v_i$.

0261-3794/01/$ - see front matter 
PII: S0261-3794(00)00029-9

threshold of exclusion provides a sufficient condition for it.[3] In general, the threshold of representation is lower than the threshold of exclusion. Since these concepts were first defined over thirty years ago, thresholds of exclusion and thresholds of representation have been used to provide an indicator of the relative proportionality of electoral systems, and the threshold of exclusion has been used as a measure of the incentives for "narrowcasting" of candidate/party appeals to a small set of voters. Since the lower the threshold of exclusion the lower the vote share a party needs to guarantee electing one of its candidates, it might appear that the lower an electoral system's threshold of exclusion, the lower the incentive for parties in that system to expand their electoral base.

The aim of this brief research note is to correct a common misperception of the implications of thresholds of exclusion for the expected incentives for narrowcasting.

Let $m$=number of members being elected from a given district; $n$=number of parties contesting the election in some given district; $k$=number of votes per voter in a limited voting system $v_i$=Vote share for party $i$ in a given constituency.

We show in Table 1 values of the threshold of exclusion ($T_E$) for STV and for three other well-known electoral systems (SMD plurality, limited voting/SNTV, and list PR under d'Hondt.[4] Results shown are based on the (standard) assumption that each party runs a full slate.

Rae et al. (1971) observe that, for each of the multimember voting schemes, thresholds of exclusion are inverse functions of district size ($m$); thus, the thresholds

Table 1
Threshold of exclusion and number of votes to win for various electoral systems[a]

| | STV | SNTV | Limited voting ($k>1$) | List PR (d'Hondt) | SMD plurality |
|---|---|---|---|---|---|
| $T_E$ | $1/(m+1)$ | $1/(m+1)$ | $k/(m+k)$ | $1/(m+1)$ | 1/2 |
| Number of votes to win | $qm/(m+1)$ | $qm/(m+1)$ | $qkm/(m+k)$ | $qm/(m+1)$ | $q/2$ |

[a] Here $m$ is the number of seats to be filled and $q$ is the number of voters per district. Row 1 is adapted and expanded from Grofman (1975: Table 1, p. 313). As in that table we assume that each party runs a full slate of candidates for multi-seat offices.

[3] In the analysis that follows I neglect exclusion rules such as those that deny representation to parties with less than a minimum percentage of the national vote. In actual electoral politics, such rules can be very important for denying representation to small parties, usually to the benefit of the largest and second largest parties in the system, but are best considered separately from electoral system type, per se.

[4] A more comprehensive list of threshold values for both $T_E$ and $T_R$ is given in Lijphart and Gibberd (1977). They also correct results in Grofman (1975).



decrease at a decreasing rate as $m$ increases.[5] Of the four systems we consider, the threshold of exclusion is lowest for STV, SNTV, and d'Hondt list PR, and highest for plurality.[6] But STV also makes it relatively easy for a minority voting bloc to gain some representation if its members are able to agree on who to vote for; if the bloc has support equal to a Droop quota then it cannot be denied representation.[7]

A further look at the entries in Table 1 makes it clear that SNTV and plurality may be taken as two ends of a continuum, with limited voting providing the middle ground.[8] For limited voting, if $k=m$ then limited voting becomes plurality/plurality bloc voting. If $k=1$, and $m>1$, then we have SNTV.[9] Since the values of $T_E$ are the same for plurality bloc voting (i.e., plurality voting in multimember districts) as they are for plurality voting in single member districts as we vary $k$ between 2 and $m$, we obtain values of $T_E$ that are intermediate for limited voting between those for plurality systems and those for SNTV (Grofman, 1975).

## A common error in interpreting $T_E$

While we usually focus on $T_E$ as an indicator of how large a party's voting strength must be in order to have a realistic chance to gain representation, we can also think of it as providing a measure of the incentives to develop a narrow as opposed to a broad-based constituency. Even under the "worst" of circumstances, it is not necessary to try to gain more votes than represented by the threshold of exclusion to achieve initial electoral success. Clearly, the smaller $T_E$, the more "narrow-cast" can be a party's appeals.[10]

But we must be careful not to misinterpret $T_E$, in that the actual number of *voters*

[5] This suggests that, since gains in increasing representation for small voting blocs diminish with increasing $m$, we can gain a good portion of the benefits of increased magnitude with "medium-sized" districts. This argument is strengthened if there are independent reasons to keeping constituencies from being too large, such as a desire to enhance legislator-constituency ties, or a desire to prevent "narrowcasting" of a party's appeals.

[6] For $m\geq 3$ and $n\geq m$, the threshold of representation is lowest for SNTV and highest under plurality.

[7] See also the discussion of the maximum/minimum seats/votes curve in Grofman (1975, pp. 318–319), based on ideas in Dahl (1956).

[8] Charles Dodgson (a.k.a. Lewis Carroll, author of Alice in Wonderland) was apparently the first to investigate the properties of the limited vote. He wrote almost a century ago (Dodgson, 1884), but his work remained unknown or misunderstood until the economist Duncan Black, who was both an authority on voting methods and an authority on Carroll, restated Carroll's arguments and calculations in a clearer form (Black, 1967).

[9] Roughly speaking, we may take $(m-k)/m$ as an indicator of how likely a limited vote system is to be proportional in its effects; the closer that index is to one, the more proportional is the system. The limited vote was made use of for parliamentary elections in Great Britain from 1967 to 1884 for some constituencies; voters had two votes in the twelve three-member constituencies (Berrington, 1975). It has also been used for election to the Senate in Spain (Lijphart et al., 1986), various local elections in several states in the United States (Grofman, 1982), and, in its SNTV variant, for various national and local elections in Japan, Korea and Taiwan (Grofman et al., 1999).

[10] Game-theoretic arguments about incentives for narrow-casting are found in Cox (1990); Myerson (1993a,b); see also Carey and Shugart (1995).

needed to win victory will also be a function of district size. Thus, in looking, for example, at comparisons between STV or d'Hondt list PR and SMD systems, it is sometimes said that, since under STV or d'Hondt list PR with, say, $m$=3, a candidate only needs 25% of the vote to win ($T_E$=1/4), while in a single-member district a candidate needs 50%+ of the vote to win ($T_E$=1/2+), it is much easier (indeed, twice as easy) to win election under STV or d'Hondt list PR than under SMD in terms of the number of voters to whom one must appeal. *This is a quite misleading calculation*!

A little thought will reveal that, if district magnitude is proportional to district population, then the three-member constituency has three times as many voters as the one-member constituency! Thus, under the assumptions above, if there are $q$ voters per representative, it will take $qm/(m+1)$ voters in one's electoral support group to be sure of winning election under STV or SNTV or d'Hondt list PR in $m$-member districts, and only $q/2$ voters to be sure of winning election under SMD. Note that it actually takes more voters to be sure of winning under STV than under SMD. Indeed, in the limit, as $m$ tends towards infinity, it takes twice as many voters to be sure of winning under SNTV or d'Hondt as under SMD!

Of course, the implications of this last calculation also need to be carefully thought through. As constituencies get more populous, groups that were not sufficiently geographically concentrated enough to make up the majority in any geographically compact single-member district, may have sufficient numbers in a multi-member district to achieve representation. For example, if a party's voting strength is uniformly distributed across a given geographic area at, say, 35% of the votes, then, against a single opponent, it will never win a majority, but it will always win at least one seat under d'Hondt list PR in two-seat districts. Thus, in order to determine whether candidates will be forced to "cast their nets more broadly" in multimember district constituencies than in single-member districts, we need to be attentive to what kinds of interests might be represented and to the exact geographic distribution of those interests. Too simplistic a reliance on the threshold of exclusion can be quite misleading.

## Acknowledgements

I am indebted to Clover Behrend for bibliographic assistance. This research has been partially supported by NSF Grant #SBR–97–30578 to Bernard Grofman and Anthony Marley.

## References

Berrington, H., 1975. Summary of electoral systems and formulae. In: Finer, S.E. (Ed.), Adversary Politics and Electoral Reform. Anthony Wigram, London.

Black, D., 1967. The central argument in Lewis Carroll's the principles of parliamentary representation. Papers on Non-Market Decision Making 3, 1–17.



Carey, J.M., Shugart, M., 1995. Incentives to cultivate a personal vote: a rank ordering of electoral formulas. Electoral Studies 14 (4), 417–440.

Cox, G.W., 1990. Centripetal and centrifugal incentives in electoral systems. American Journal of Political Science 31, 82–108.

Dahl, R.A., 1956. A Preface to Democratic Theory. University of Chicago Press, Chicago (IL).

Dodgson, C.L., 1884. The Principles of Parliamentary Representation. Harrison and Sons, London.

Grofman, B., 1975. A review of macro-election systems. In: Wildenmann, R. (Ed.) German Political Yearbook (Sozialwissenschaftliches Jahrbuch fur Politik), vol. 4. Verlag, Munich, Germany, pp. 303–352.

Grofman, B., 1982. Alternatives to single member plurality districts: legal and empirical issues. In: Grofman, B., Lijphart, A., McKay, R., Scarrow, H. (Eds.), Representation and Redistricting Issues. D.C. Heath, Lexington, Massachusetts, pp. 107–128.

Grofman, B., Lee, S.-C., Winckler, E., Woodall, B. (Eds.), 1999. Elections in Japan, Korea and Taiwan under the Single Non-Transferable Vote: The Comparative Study of an Embedded Institution. University of Michigan Press, Ann Arbor (MI).

Lijphart, A., Gibberd, R.W., 1977. Thresholds and payoffs in list systems of proportional representation. European Journal of Political Research 5, 219–244.

Lijphart, A., Lopez Pintor, R., Sone, Y., 1986. The limited vote and the single nontransferable vote: lessons from the Japanese and Spanish examples. In: Grofman, B., Lijphart, A. (Eds.), Electoral Laws and Their Political Consequences. Agathon Press, New York, pp. 154–169.

Loosemore, J., Hanby, V.J., 1971. The theoretical limits of maximum distortion: some analytic expressions for electoral systems. British Journal of Political Science 1, 467–477.

Myerson, R.B., 1993a. Incentives to cultivate favored minorities under alternative electoral systems. American Political Science Review 87 (4), 856–869.

Myerson, R.B., 1993b. Effectiveness of electoral systems for reducing Government corruption: a game theoretic analysis. Games and Economic Behavior 5, 118–132.

Rae, D., 1971. The political consequences of electoral laws, 2nd ed. Yale University Press, New Haven, CT.

Rae, D., Hanby, V., Loosemore, J., 1971. Thresholds of representation and thresholds of exclusion: an analytic note on electoral systems. Comparative Political Studies 3, 479–488.

Rokkan, S., 1968. Elections: Electoral Systems. International Encyclopedia of the Social Sciences. Crowell-Collier, MacMillan, New York.