Representation and Redistricting Issues

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5 For Single-Member Districts Random Is Not Equal

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Senator Danforth:	I don't know how many black members there are in the Congress. Maybe you do.
Mr. Wells:	I don't know the actual figure, no.
Senator Danforth:	But it is roughly fifteen, something like that, whereas it is about 10 or 11 percent of the population of this country is black. So, they are about one third of what their representation would be (15/435 = 3.4 percent).
Senator Levin:	Suppose it could be shown that to use what amounts to the chance method is going to result in even a lower black representation. I don't see how that is possible.
Mr. Wells:	I agree with you, Senator. I don't see how it is possible. Even if it is possible in one particular district, it may not be in the next one. That is the beauty of operating on a chance pattern.
	In the long run, if I understand mathematical logic, a chance pattern will, over the long haul, operate in such a way as to make the percentage of the population and the percentage of representation more or less equal. It may not do that in any given redistricting arrangement. But, in looking at it over a series of years, it should accomplish that (Wells 1979, p. 529).

Dave Wells's assertion that, for single-member districts, "a chance pattern will, over the long haul, operate in such a way as to make the percentage of the population and the percentage of representation more or less equal" is a view I suspect is widely held. For example, David Cohen of Common Cause in his testimony on S.596 also calls attention to the discrepancy between black and Hispanic population figures and the percentages of black/Hispanic congressional representatives as evidence of nonrandom (and discriminatory) districting. Unfortunately, except under very special circumstances, unlikely to be ever achieved in practice, random districting will not yield proportionality between a group's vote percentage and the percentage of seats it wins. We show in table 5-1 potential expected average long

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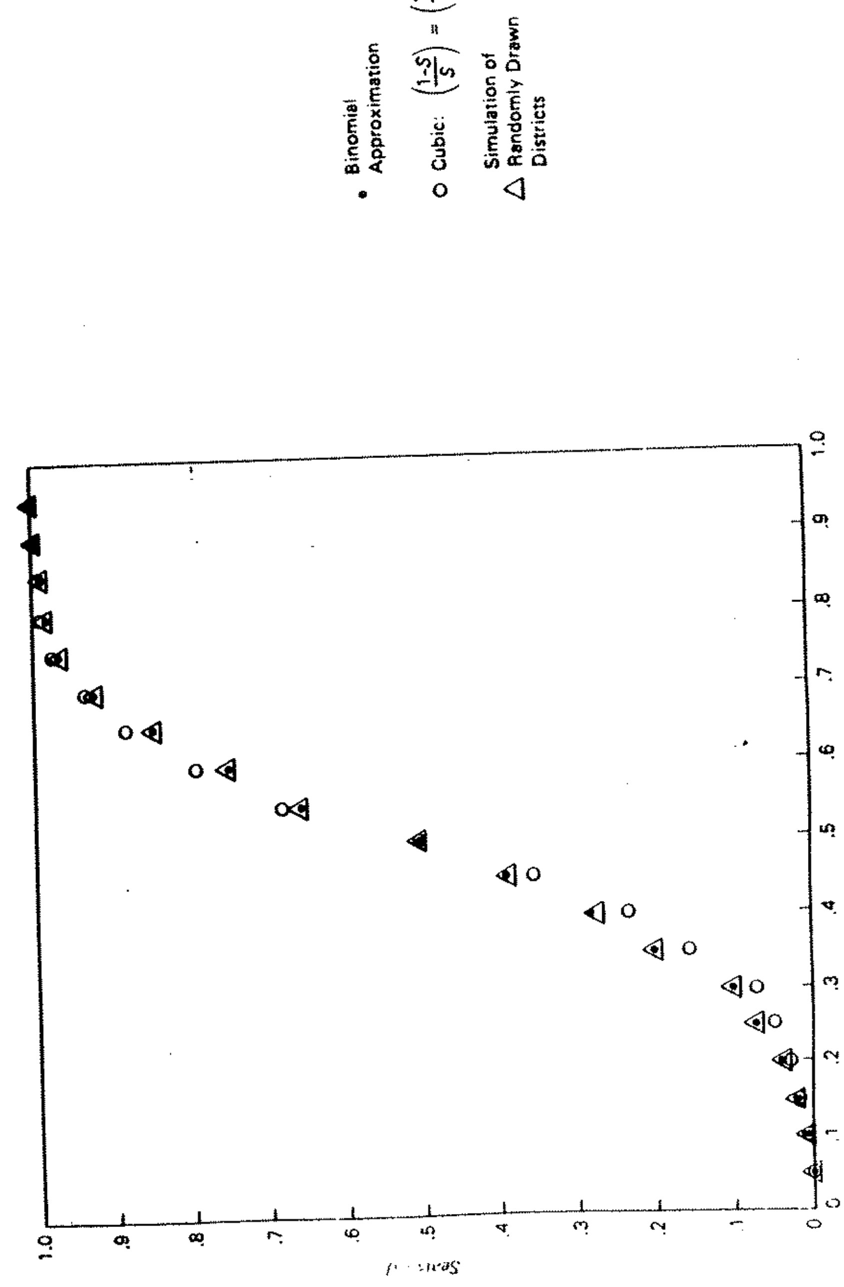
relationships between aggregate vote percentages and seat percent- s in a single-member election system with a large number of randomly wn equal population districts under the assumption that there are two inct groups competing in each district so that whichever seats are lost by one group will be won by the other. The parameter k is an index of pro- tionality (Theil, 1969, 1970; Taagepera, 1973; Tufte, 1973; Grofman, 5; Niemi and Deegan, 1978). Only for $k = 1$ will the percentage of seats n equal the percentage of votes received.

At one time it was thought that $k = 3$ was the most likely k value. This jecture is known as the *cube law of politics* (Kendall and Stuart, 1950). ent work (see esp. Tufte, 1973) has found a wide variation in k, with only liamentary elections in Great Britain approximating the magic value of Estimated values range from .71 (U.S. congressional elections in the iod 1966-1970; Tufte, 1973) to 4.4 (the U.S. Electoral College, 8-1968; Taagepera, 1973). Most of the fitted values are, however, in the ge 2.2 ± .8. For example, Tufte finds k for U.S. congressional elections 00-1970) to be 2.2. The value $k = 3$, does, however, provide a *quite good* proximation to the situation where partisan/group strength is randomly ributed across districts. We show in figure 5-1 the results of a simulation a 99-district election. The cube law fits the data remarkably well.

As is apparent from table 5-1, the higher the k value the less successful be the minority party/group in terms of translating its votes into seats. reover, for $k > 1$, the smaller the minority, the less well it does in ieving a proportionality between its vote share and its seat share. For ex- ple, for $k = 2$, 10 percent of the vote achieves only an expected 1.2 per- t of the seats (1.2/10 = 12 percent) on average but 20 percent of the vote ieves an expected 5.8 percent of the seats (5.8/20 = 29 percent) on rage, while 40 percent of the vote achieves an expected 30.8 percent of seats (30.8/40 = 77 percent) on average.

le 5-1
erage Expected Seat Percentage (S) as a Function of Vote Percentage (V)
$k\left(\frac{1-S}{S}\right)=\left(\frac{1-V}{V}\right)^k$

)/S(%)	K=.75	K=1	K=1.25	K=1.5	K=2	K=2.5	K=3	K=3.5
	16.1	10	6.0	3.5	1.2	.4	.1	.04
	21.3	15	10.3	6.9	3.0	1.2	.5	.2
	26.1	20	15.0	11.1	5.8	3.0	1.5	.8
	30.4	25	20.2	16.1	10.0	6.0	3.6	2.1
	34.6	30	25.8	21.9	15.6	10.8	7.3	4.9
	38.6	35	31.5	28.3	22.4	17.4	13.4	10.2
	42.4	40	37.6	35.2	30.8	26.6	22.9	19.5
	46.3	45	43.8	42.5	40.2	37.8	35.5	33.3
	50.0	50	50.0	50.0	50.0	50.0	50.0	50.0



Note: The author is indebted to Nick Noviello for programming selection simulation reported here.

Figure 5-1. Relationship between Vote Percentage and Seat Percentage for a Single-Member District Legislature Whose Equipopulation Districts Have Been Randomly Drawn

In any actual election system, the value of k will depend upon the spatial ribution of partisan/group support across districts. Roughly speaking, more the distribution of partisan/group strength is similar in all districts is, the lower the variance), the higher will be k. (See Johnston 1976; ni and Deegan 1978, for a discussion of related issues.) Wildgen and strom (1980) note that, in general, minorities will do better if their ngth is geographically concentrated; while majority seat share is max-ed the more evenly distributed is its vote strength.

Backstrom, Robins, and Eller (1978) and Engstrom and Wildgen (1977) proposed to measure fairness of apportionment (that is, proportionality veen vote share and seat share), as relative to that which would be stically expected under a random drawing of (equal population) district *given the actual pattern of minority concentration*. It is possible to blish confidence limits around predicted values at any desired level, for nple, 95 percent. Observations that fall outside these limits can be med to be due to deliberate gerrymandering.

As we see from table 5-1, for any given level of minority voting ngth, what would be a reasonable minority seat representation depends much on k, which in turn depends upon the spatial distribution of ority strength. For example, under a random drawing of district bound-s, a minority with a 30 percent vote strength could expect on average percent of the seats if $k = 1.5$ but only 7.3 percent of the seats if $k =$ n the U.S. Congress, were $k = 2$, then, an 11 percent black population ority could only *expect* to get 1.5 percent of the seats on average, while a rcent Hispanic minority could only *expect* to get .9 percent of the seats average, even were districting completely unbiased. That blacks win e congressional seats than Hispanics (3.4 percent versus about 1 per-) relative to what might be expected is, in our view, at least in part due e fact that blacks are somewhat more geographically concentrated than Hispanics. Only where minority strength is at least 45 percent is seat e relatively insensitive to variation in k.

In sum, blind districting is extremely unlikely to give rise to propor-al results, except when minority and majority are near equal in size.