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REVIEW ESSAY:

CONDORCET MODELS, AVENUES FOR FUTURE RESEARCH

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1. INTRODUCTION

The study of information pooling and group decision making is not confined to a single discipline. There are a number of streams of research which deal with the same questions, sometimes in ignorance of related or even identical work in cognate disciplines. The inspiration for much recent research (see review in Grofman, Owen, and Feld, 1983) is the classic work of Condorcet (1785) and Poisson (1837), but other important traditions include work on Bayesian and neo-Bayesian models of individual and group information pooling (see Bordley, Diaconis and Zabell, and Schum essays, this volume), work on cognitive heuristics (see Batchelder essay, this volume), work on the social psychology of group (especially jury) decision processes and on group problem solving (see Hastie essay, this volume), work on interactive methods for pooling expert judgments (see, e.g., Dalkey and Helmer, 1963; MacKinnon and Anderson, 1976; Gustafson et al., 1973), work on expectations based on modeling in economics (see, e.g., Grossman, 1976; Friedman, 1979; Verrecchia, 1980; Frydman, 1982; Mayshar, 1983), and work on optimum delegation-decentralization of multicomponent decision making (see Radner essay, this volume, for an economic approach; Kochen and Deutsch, 1980, for a political science approach; and Roby and Lanzetta, 1961, for a social psychological approach using graph-theoretic tools).

In this essay we shall not attempt the formidable task of synthesis—a task which we believe to be premature in a field which has only recently begun to take shape. Rather, we shall identify three recent results in the Condorcet-Poisson tradition that we believe to be of considerable theoretical importance and then ten topics in the area of information pooling and group judgment that we believe deserve further investigation. The three results which we wish to single out are the Generalized Condorcet Jury Theorem, the Bayesian Optimal Group Decision Rule, and Young's reconciliation of the seemingly incompatible approaches of Borda and Condorcet to the selection of an optimal group decision procedure (the Borda rule versus the Condorcet criterion).

Some notation will be useful: Let us look at a group of size N confronting a dichotomous choice. Let us posit that each individual has a certain probability, p_i, $(0 \leq p_i \leq 1)$, of choosing that alternative which is the preferred choice with respect to some single designated criterion by which alternatives are to be evaluated. If the decision rule is ordinary majority, m [for simplicity we let N be odd and hence $m = (N+1)/2$], for a homogeneous group (i.e., one with $p_i = p$ for all i), we have the Condorcet Jury Theorem, first proved in 1785 by the French mathematician and philosopher Nicolas Caritat de Condorcet. Let P_N be the majority judgmental accuracy of a group of size N, that is, the probability that the group majority will, in a pairwise comparison, pick the alternative which scores higher on the designated criterion variable.

CONDORCET JURY THEOREM. If voter choices are mutually independent, then

$$P_N = \sum_{h=m}^{N} \binom{N}{h} p^h (1-p)^{N-h}; \tag{1}$$

and if $1 > p > \frac{1}{2}$, then P_N is monotonically increasing in N and $\lim_{N\to\infty} P_N = 1$; if $0 < p < \frac{1}{2}$, then P_N is monotonically decreasing in N and $\lim_{N\to\infty} P_N = 0$; while if $p = \frac{1}{2}$, then $P_N = \frac{1}{2}$ for all N.

If $p > \frac{1}{2}$, for independent decision makers, the group judgmental accuracy under majority voting approaches infallibility as the group size grows larger. Moreover, the rate of convergence is quite rapid. For example, if $p = .8$, $p_{13} > .99$. (For exact values and various approximation formulas see Grofman et al., 1983; also Grofman, Feld, and Owen, 1984.)

2. GENERALIZED CONDORCET JURY THEOREM

The Condorcet Jury Theorem has been extended in various ways. Owen et al. (1983) have recently generalized it to apply to any distribution of p_i values. Let $\bar{p} = \sum p_i/N$, that is, let $\bar{p}$ be the mean value of p_i.

GENERALIZED CONDORCET JURY THEOREM. If voter choices are mutually independent, then if $p > .5$, $\lim_{N\to\infty} P_N = 1$; if $p < .5$, $\lim_{N\to\infty} P_N = 0$; while if $\bar{p} = .5$, then

$$1 - e^{-1/2} > \lim_{N\to\infty} P_N > e^{-1/2}. \tag{2}$$

The generalized Condorcet Jury Theorem gives us a great deal of confidence in majority decision making in large groups. When the assumption of mutually independent choice is satisfied, all that we require is that the average voter be more likely than chance to pick the better of the available alternatives.[1]

3. BAYESIAN OPTIMAL DECISION RULE

For a specified set of p_i values, even if all p_i values are greater than $\frac{1}{2}$, assigning equal weights to all players (and then using a majority voting rule) will in general not be the optimal decision procedure. For a group of voters whose choices are mutually independent and where the alternatives are a priori equally likely, the decision rule which maximizes group accuracy (i.e., maximizes the likelihood the group will make the better of the two choices open to it) is any weighted majority voting rule which assigns weights w_i:

$$w_i \propto \log \frac{p_i}{1 - p_i}. \tag{3}$$

This result is closely related to Bayes's Theorem, and we shall refer to it as the Bayesian optimal group decision rule. The result was proved by Shapley in 1979 (Shapley and Grofman, 1984) and independently by Nitzan and Paroush (1982); we have also found the mathematically identical result in a number of sources where the substantive context is quite different (see, e.g., Pierce, 1961; Minsky and Papert, 1971; Duda and Hart, 1973).

Note that the weight assignment given to an individual in Eq. (3) is a function purely of his competence and is *independent* of the competence of the other members of the group. Note also that the *power* of any given individual to cast a decisive vote will vary with the values of the weights assigned to the other group members. Thus, at the extreme, in some groups an individual's weights can allow him to be dictator (outvoting all the other group members); while in other groups he may (in the language of game theory) be a *dummy*, with no decisive power whatsoever.[2]

Where voter competences are known, the Shapley-Grofman-Nitzan-Paroush result answers the question: "What is the optimal group decision procedure?"

4. YOUNG–BORDA MAXIMUM LIKELIHOOD ESTIMATE OF THE CONDORCET WINNER

The eighteenth-century mathematicians Jean Charles de Borda and the marquis de Condorcet were rivals. Each proposed his own solution to the question of how to specify a system of voting which would lead to a majority which could reasonably be regarded as the genuine will of a majority of a group. "To people who have not looked into the problem this seems a foolish inquiry; it seems obvious that a majority is a majority and that is that. In reality, the problem is a most difficult one" (Black, 1958, p. 56).

Consider a group of five members choosing among four alternatives, three of whom have preferences $a_1a_2a_4a_3$ and two with preferences $a_3a_2a_4a_1$. We may represent these preferences in matrix form in Figure 1.

Borda (1781) proposed to pick the alternative whose row sum was maximal, in this case a_2. This element can be thought of as the one which is, on average, highest-ranked. Condorcet (1785) proposed to pick the alternative which is preferred by a majority to each and every other alternative. In terms of the matrix in the figure, this is that alternative a_i such that for all j, $a_{ij} - a_{ji} > 0$. If such an alternative exists it must be unique, but no such alternative may exist. Following Black (1958), such an alternative is commonly called the Condorcet winner. In the example above the Condorcet winner is a_1. In this example the Condorcet winner and the Borda winner do not coincide.[3] Each method has much to recommend it (Riker, 1982; Grofman, 1987, forthcoming, Chap. 1), although, like most contemporary

$$
\begin{array}{c c c}
 & \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} & \\
\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} &
\begin{bmatrix} x & 3 & 3 & 3 \\ 2 & x & 3 & 5 \\ 2 & 2 & x & 2 \\ 2 & 0 & 3 & x \end{bmatrix} &
\begin{array}{c} 9 \\ 10 \\ 6 \\ 5 \end{array}
\end{array}
$$

Figure 1. Matrix representation of voter preferences in a hypothetical committee (N = 5, m = 4).

scholars, we hold the view that the Condorcet winner ought to be chosen if it exists.

Condorcet advocated his voting method of "simple raisonnement" because he was unable to solve the problem he posed for himself, to wit: if voters could be characterized by competencies p_i of choosing the better alternative from any pair, what rule should be used to aggregate voter preferences so as to pick the alternative most likely to be best (Black, 1958, Appendix)? Young (this volume) has solved Condorcet's problem and in the process elegantly reconciled the seemingly incompatible approaches of Borda and Condorcet by showing that, if $p_i = p$ for all i, the alternative which is most likely to be the "best" choice is given by the Borda rule. In other words, the Borda winner is also the maximally likely Condorcet winner when we take into account the fact that observed preferences are only an imperfect indicator of the "true" pairwise rankings. Young (this volume) also shows that, in general, the procedure which picks the maximally likely Condorcet winner is an analogue to the Borda rule in which each voter's choices are weighted by $\log[p_i/(1-p_i)]$. Thus Young's work not only reconciles Borda and Condorcet but also integrates preference-based models of voter choice–social welfare with research on Bayesian maximum likelihood estimates.

5. TOPICS FOR FURTHER RESEARCH

5.1. Optimal Group Size and Type I and Type II Trade-offs

In almost all cases the feasible decision rules will formally be of the "one person, one vote" variety. If adding group members is costly, we would like to be able to specify trade-offs between group size and group accuracy.

Gelfand and Solomon (1973, 1974, 1975, 1977a) following Poisson (1837) have developed a two-parameter model for jury decision making in which individuals may have a different competence in perceiving the guilt of the guilty than in perceiving the innocence of the innocent. They estimate Type I and Type II accuracies for juries of size 6 and size 12 under a variety of assumptions as to the nature of the group decision mechanism. Similar work has been done by Nagel and Neef (1975) and Grofman (1980, 1981b).

Grofman (1975b) uses a normal approximation to the binomial to develop a simple expression for the trade-offs between the accuracy of a group of size N whose members have competence $p + x$ and a group of size $N + y$ whose members have competence p. This formula permits an easy way to calculate the advantages of, say, blue-ribbon juries over larger but less competent decision bodies. Grofman (1979a) has (somewhat tongue in cheek) looked at Abraham Lincoln's famous dictum as a function of group size and the number of opportunities for deceit offered. Grofman, Feld and Owen (1984) show how to calculate optimal group size for the case where the addition of an $(N+1)$th member to a group of size N has a known cost function and the value of a correct group decision can be specified.

Closely related to the group size question are issues of optimal inclusion and delegation.

5.2. Optimal Inclusion

Consider a group operating under simple majority rule which is debating whether or not to add new members, with the only desideratum being the effect on group accuracy. Grofman (1975b) and Margolis (1976) show conditions under which adding members who lower the mean competence of a group can actually raise the judgmental competence (P_N value) of the group. Feld and Grofman (1984) show when it can be beneficial to two groups (e.g., two stock investment clubs which manage a common pool of funds based on the majority vote of their members) to merge membership.

5.3. Optimal Delegation

In many cases groups will wish, because of time and manpower constraints, to delegate decision making to small subcommittees with authority to make binding commitments (perhaps subject to some sort of review process). One way to increase accuracy may be to identify the most competent members of the group and allow them to decide. Grofman (1978) provides some estimates of the likelihood that a group majority will be more accurate than the group's most accurate member. Another way of increasing the accuracy of group decision making, at the cost of decreasing its decisiveness, is to require more than simple majority agreement (e.g., unanimity). We might use a small committee operating under a unanimity rule but convene a new group if the previous group fails to reach unanimity. This is the U.S. procedure for dealing with hung juries. Consider a group whose members each have competence p. If we convene a k-member group and require it to reach unanimity, we will require an expected k/p^k committee members before we reach consensus, and the group will be right $p^k/[p^k + (1-p)^k]$ proportion of the time. For given p, by increasing k, we can reach any desired level of accuracy. If $N = 9$ and $p = .8$, $P_N = .9804$; but if we let $k = 3$, $E(N) = 5.86$, while our expected accuracy is .9846. Thus, we are better off delegating. Grofman, Feld, and Owen (1982) conjecture that it may always be possible to specify a process of sequential decentralized decision making that is more desirable in terms of both accuracy and *expected* manpower use than simple majority decision making by the full group.

An alternative to sequential "batch" sampling until a group achieves the necessary consensus is sequential sampling until the difference between the number of individuals who favor a given alternative and the number who favor any other alternative exceeds a prespecified number; that is, we do not throw out the results of earlier deadlocked group deliberations but keep a running tally. Again, for given p, we can find a margin sufficient to guarantee any desired level of group accuracy. This procedure is more efficient, but applying this procedure to the jury case violates the expectation that a *new* trial should be just that—with the verdict preferences of the previous hung jury irrelevant.

5.4. *Optimal Allocation of Effort*

Related to the question of optimal delegation is the question of optimally allocating limited resources on a multicomponent decision task. Owen and Grofman (1983) examine this question for a single decision maker under various simplifying assumptions. Still unresolved is the case of optimal group composition where tasks are factorizable into subcomponents and actors can be thought of as having the task of optimally partitioning a group into r subsets to deal with n simultaneous tasks.

5.5. *Decomposition*

Certain decisions can be thought of as requiring a conjunctive judgment (e.g., in law, to find a conspiracy requires that a number of different conditions all be met), while others can be thought of as requiring a disjunctive judgment (e.g., in law, to find a violation of the Voting Rights Act *either* discriminatory intent *or* discriminatory effect is sufficient). We can ask whether it is better to examine composite propositions as wholes or whether we should decompose them into their atomic components and examine each of these singly. Very preliminary results on this question (for groups using simple majority) are given in Grofman (1987, forthcoming).

5.6. *Interdependencies*

Most of the research in the Condorcet-Poisson tradition uses a Bernoulli independent trials model. Clearly, however, in the real world individual judgments are not independent, and it is not clear whether deference paid by individuals to the views of others helps or hurts group accuracy. Gelfand and Solomon (1975) and Klevorick and Rothschild (1979) look at juries in which a form of conformity to the group majority is likely; Owen (Grofman, Owen, and Feld, 1983, p. 273) looks at what happens if we have a "guru" (opinion leader) to whom a bloc of voters pay heed. Owen (this volume) extends this result to look at the case where various blocs of like-minded voters exist; Shapley and Grofman (1984) also look at interdependencies, but this area of research is still undeveloped.[4] Among the results in Shapley and Grofman (1984) are the identification of certain special cases where interdependencies may give rise to a nonmonotonic relationship between group consensus and group accuracy.

5.7. Accuracy of Expectations

There is a rapidly growing literature in economics on rational expectations and stability of macroeconomic forecasts. In the models in the literature of which we are aware, individuals may differ in the information they have but not in their competence (i.e., ability to make use of information). A natural point of intersection between work on the Condorcet tradition and work on rational expectations would appear to lie in examination of the Keynesian beauty contest. Keynes (1936) proposed the idea of a beauty contest in which judges sought to judge not who was most beautiful but who would be thought most beautiful. (Cf. "The value of a stock is what people think it's worth.") Very preliminary work along these lines has been done by Grofman (1983).

5.8. Test Theory

There is a natural parallel between various results on group decision making and the literature on test theory. Some of these linkages are shown by Batchelder and Romney in this volume. Feld and Grofman (1983) show that under certain circumstances it is possible to score a true–false (or multiple-choice) exam without an answer key. It would be desirable to further generalize the Condorcet model to permit variation in competence with test item difficulty (cf. Lord, 1980).

5.9. Evaluating Competence

The Shapley–Grofman–Nitzan–Paroush result tells how groups should decide when p_i values are known; but how can p_i values be estimated? Grofman and Feld (this volume) and Batchelder and Romney (this volume) discuss ways in which that question might be answered. Gelfand and Solomon (1977a) and Grofman (1980a) consider how to estimate mean juror competence from data on observed jury verdicts.

5.10. Second-Best Solutions

The Shapley-Grofman-Nitzan-Paroush result and the Young result provide optimal decision procedures. A natural question is how much accuracy is lost if other (perhaps simpler) group decision procedures are used. Pinkham and Urken (1982) provide some preliminary simulation results on this question, and an example where simple majority rule is almost as good as Bayesian optimal weights is discussed in Grofman, Feld and Owen (1982), but this important area of research is still almost completely unexplored.

6. CONCLUSIONS

We believe that the three basic theorems and the other work we mention provide a necessary complement to the standard Arrow framework for social choice. The latter emphasizes individual preferences and preference aggregation mechanisms; the former emphasize individual competence and group judgmental accuracy. In our view, although the question of what choice is most faithful to the popular will is important, so, too, is the question of what choice is best. What is particularly exciting about Young's work (and also the Shapley-Grofman-Nitzan-Paroush work) are the insights provided into just how closely linked these two questions are.[5] If we are correct about the importance of this body of research, then a far greater debt is owed to the marquis de Condorcet than contemporary social science has yet acknowledged.

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NOTES

1. For small numbers perverse results are possible—e.g., $\bar{p} > \frac{1}{2}$ but $P_N < \frac{1}{2}$, but such results are unlikely. Boundary conditions (based on N) are given in Grofman et al. (1983.)

2. In any particular group, optimal weight assignments are not unique. Different weight assignments may yield identical power scores. (Shapley and Grofman, 1984.)

3. Simulation research suggests that in practice the Condorcet winner and the Borda winner will coincide quite often (Chamberlain and Cohen, 1978).

4. See also Batchelder and Rommey (this volume).

5. We are not arguing that all decisions fit the model of an underlying unidimensional criterion of judgment in which individuals differ only in their competence or rank alternatives

with respect to that agreed-upon criterion. Interests *are* real, and individuals do differ in their criteria of evaluation; what is sauce for the goose need not be sauce for the gander. Nonetheless, on the one hand, many judgments (e.g., stock market projections) are "interest-free"; on the other hand, where individuals are acting within an organizational setting a common interest (e.g., profit maximizing) can, in principle, be identified. In such contexts, the competence of individuals and groups and the impact of decision rules on group accuracy are important questions. Of course, such questions ideally should be examined in the broader context of institutional design—a context in which potential conflicts between individual incentives and the group good are recognized (see Radner essay, this volume). Moreover, the extent to which different people see the world differently (but share common perceptions with large numbers of others) is itself an important issue in the sociology of knowledge.