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Limitations of the spatial model*

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Abstract. One approach to avoiding the implications of Arrow's paradox is to impose restrictions on the preferences of voters. A restriction often assumed in the literature is that voters' preferences can be represented by a choice among points in a space, where voters' preferences are convex. We claim that these assumptions will often be unjustified. Government must often address issues of externalities or public goods, which means that the possibility frontier will not be concave. This in turn means that voters' preferences over *feasible* policies will not be convex.

Ever since the publication of Arrow's classic work on *Social Choice and Individual Values*, the literature on social choice has followed two paths. One continues in Arrow's tradition and demonstrates that politics inevitably involves cycling, manipulation, and other paradoxes (see Gibbard, 1973; Satterthwaite, 1975; McKelvey, 1979). The other path is to impose restrictions on the preferences of voters so as to avoid such problems. The most common approaches involve convex preferences (as in Weingast and Shepsle, 1984), convex preferences with a satiation point (Enelow and Hinich, 1984; Kramer, 1977) and one-dimensional issues with single peakedness (as in Downs, 1957); the last two assumptions correspond to spatial voting models. Though we have great sympathy for these attempts at making public choice a predictive science, we find ourselves in the unhappy position of arguing that assumptions that may at first appear reasonable are unlikely to accurately reflect people's preferences in the political arena. In particular, preferences need not be convex.

Consider first a spatial model which supposes that a voter's preferences can be represented by closed indifference curves surrounding a bliss point. Along any ray emanating from his bliss point (or ideal point), the voter prefers one point over another if it is closer to his bliss point. The model has been specialized to suppose that indifference curves are circular, or that utility is a function of the generalized Euclidean distance from the bliss point; though we shall at times make such simplifications for expository purposes, they are not essen-

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tial for our arguments. We shall suppose that each point in the space represents a policy, which can be interpreted as the quantities of various goods that will be produced or distributed.

Spatial voting models are inspired by economic models that suppose a consumer prefers to shop at the nearest store (see Hotelling, 1929). Though this shopping problem may appear similar to a voter's decision to support that candidate whose positions are closest to his own, the analogy is flawed. Consider the decision of where to locate a school. All locations are deemed feasible, so that no constraints need be considered. It may be perfectly reasonable to believe that given a choice among a set of existing schools parents wish to send their children to the school closest to them. But that does not at all mean that given a choice of where to build a new school parents as voters would prefer that the school be placed ten yards away from them rather than half a mile away. Schools can be a noisy nuisance, and the decision of where to locate a school is not the same as the decision of where to go to school. A voter's preference about where to build a school cannot be represented by indifference curves surrounding a bliss point; the optimum locations may lie along a circle some distance from the voter's home, and points too far, as well as those too close, will be judged inferior. A voter's preferences then will not be convex. For example, in Figure 1 let a voter's residence be at point H, and let the most preferred location of a school be anywhere that is half a mile from his house. The circle containing points a and b in Figure 1 represents one indifference curve. Convexity would require that if points a and b lie on an indifference curve, then the voter prefers any point, such as H, that lies on the segment ab to point a or b. Clearly, however, that condition is violated in this example. The voter's preferences are not convex.

The reader may believe that the example is atypical, and that many or even most political decisions involve choices where people's preferences are well-behaved. That is not so. The point made here relies crucially on the observation that not all policies are feasible, and that for the possibility sets that likely apply to governmental decisions, the preferences of voters over feasible policies will not be convex.

Consider the choice between two goods, goods 1 and 2, produced in quantities x_1 and x_2 respectively, which we may call highways (constructed by government), and apples (produced by the private sector). (See Fig. 2). Government policy may explicitly involve only determining the number of highways to be built. But since taxes must be raised to build highways, any increase in their construction must inevitably cause a reduction in the consumption of apples. We are thus justified in viewing governmental policies as affecting the consumption of both private and public goods.

Suppose that a person's preferences over combinations of x_1 and x_2 (feasible as well as infeasible combinations) can be represented by convex indiffer-

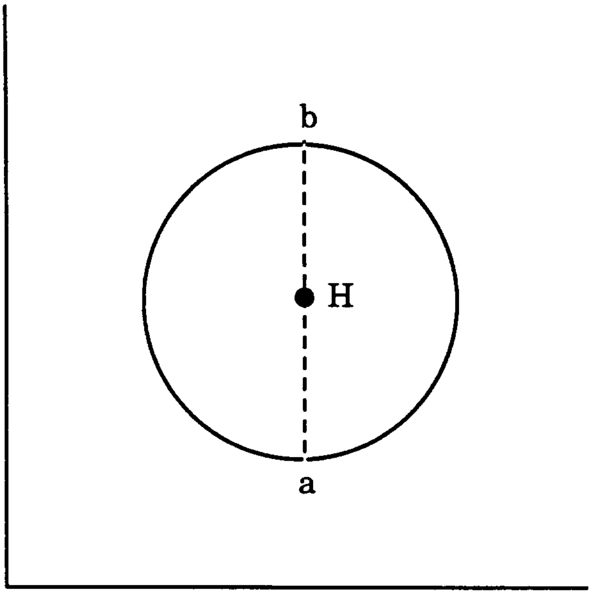

Figure 1.

ence curves. That is, the indifference curves satisfy the "more is better rule" (so that each indifference curve is downward sloping), and the slope of each curve becomes flatter as the quantity of $x_1$increases. (For negatively valued goods, like pollution, we merely use the notational device of expressing them in positive terms, for example as pollution-control).

Not all such combinations, however, are feasible. And if we consider, say, an election in which different candidates propose different policies, we should examine voters' preferences only over the feasible policies. The set of feasible policies can be represented by a possibility frontier; Figure 2 shows a hypothetical frontier, PP. (See McCubbins and Schwartz, 1985, for a different application of possibility frontiers in the study of politics.)

Two cases must be distinguished: either the possibility frontier is concave or else it is not. If the first case holds there is good justification for supposing that voters' preferences are convex. Otherwise, as will be seen, that assumption should not be made.

When the possibility frontier is concave, which is equivalent to saying that the possibility set is convex, there exists a point on the possibility frontier, such as point T in Figure 2, which lies on the voter's highest indifference curve. In that sense, point T can be interpreted as a bliss point, and then the set of feasible policies preferred over any given feasible policy will be convex. Since the possibility frontier defines the maximum amount of x_2 for any given amount of x_1, we can summarize a voter's preferences by considering the utility he ob-

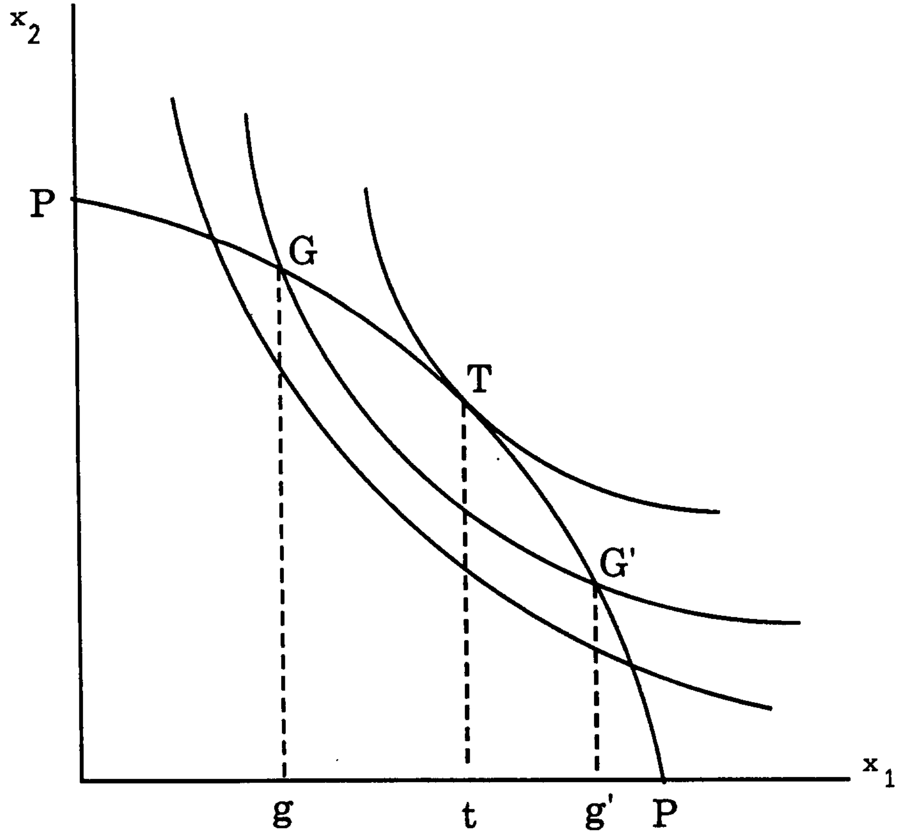

Figure 2.

tains from any specified level of x_1 (which implicitly defines a level of x_2). Thus, points G and G′ lie on the same indifference curve in Figure 2, so that the voter obtains the same utility from g units of x_1 as from g′ units. In general let g and g′ represent any two points which give the voter the same utility. It is evident that points g and g′ will lie on opposite sides of point T (that level of x_1 which gives the voter the greatest utility). This also means that any level of x_1 intermediate between g and g′ will lie closer to point t than either g or g′, and thereby represents in this example a higher level of utility. In other words, the voter's preference set is convex.

More generally, if the voter's utility is a function of n variables, and some feasible policies are restricted to lie along a possibility frontier, then the voter's preferences over feasible policies can be represented along (n – 1) dimensions: we find the intersection of each indifference curve with the possibility frontier and project it onto a plane of dimension (n – 1). This will generate closed indifference curves, with a bliss point defined by the tangency point between the n-dimensional indifference curves and the n-dimensional possibility frontier. Each of these curves will enclose a convex region, representing points preferred to any point on the indifference curve. If these conditions are met, the researcher has some grounds for assuming that preferences are convex and that voters have bliss points.

Unfortunately, the analysis does not apply if the possibility frontier is not concave. Most importantly, non-concave production possibility frontiers, or non-convex production sets, are likely to arise when public policy involves externalities (see Starret, 1972). Examples of issues that may generate non-convexities are what to do about air and water pollution, how much national defense to provide, and should a highway be built that improves transportation.

An example will illustrate that in the presence of public goods or of externalities, the production possibility set need not be convex. Let the total amount of labor in the economy be L, let L_1 be the amount of labor used in the production of good 1, and let L_2 be the amount of labor used in the production of good 2. Good 1 (say highways) is produced according to the production function $x_1 = L_1$. The production function for good 2 is given by

$$\begin{aligned} x_2 &= \sqrt{(1/2)\, L_2}, \quad \text{for } x_1 < 1/2 \\ x_2 &= \sqrt{x_1 L_2}, \quad \text{for } x_1 > 1/2. \end{aligned} \tag{1}$$

This says that once highway capacity is above a certain level (so that highways reach apple growers, or congestion is not too high), apple production is more efficient the greater the number of highways. Highways are a public good in this example because their use as an input in the production of x_2 does not diminish the utility highways directly provide consumers. Note that neither of these production functions are unusual; they exhibit, for example, constant or diminishing returns to scale, and diminishing marginal product of labor.

Suppose that L = 1, and recall that the constraint on the amount of labor in the economy requires that $L_1 + L_2$ is no greater than L. The production possibility frontier is shown schematically in Figure 3; it is composed of the curve PP′ (where at P′ the value of x_1 is 1/2) and the curve P′P′′. Note that the possibility frontier is not concave, or equivalently that the slope of the possibility frontier does not become everywhere steeper as we move from left to right. To see this, substitute $1 - x_1$ for L_2 in equations (1), and take the derivative with respect to x_1. When evaluated at $x_1 = 1/2$ we find that immediately to the left of point P′ the value of dx_2/dx_1 is $-1/2$, while immediately to its right the value of dx_2/dx_1 is 0, so that as we move from left to right in the vicinity of point P′ the frontier becomes flatter rather than steeper.

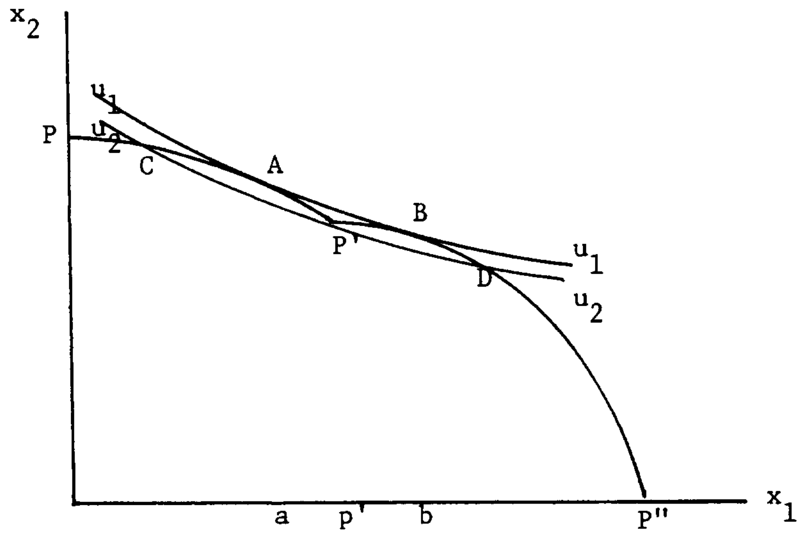

Figure 3.

The implications of such nonconcavity are also illustrated in Figure 3. Consider a voter with indifference curve u_1u_1. There are two bliss points, at A and at B, where indifference curve u_1u_1 is tangent to the possibility frontier. This means that as we move along the possibility frontier from left to right, the voter's utility first increases and then decreases in the neighborhood of point A; it then increases and decreases again in the neighborhood of point B. The implication is that preferences are not convex.

To see this, consider again a voter's preferences in terms of good 1 alone (where the quantity of good 2 is determined by the possibility frontier). Points a and b on the horizontal axis correspond to points A and B on the possibility frontier. We observe that points a and b give the voter the same utility, but that point p′ (which corresponds to point P′ on the possibility frontier and which lies between points a and b) gives less utility than either point a or point b. It is evident that the utility function in terms of good 1 is not concave. Consider further all points on the possibility frontier that lie on the indifference curve u_2u_2 (that is, points C, P′, and D). As we move from left to right along the horizontal axis, we find that utility first increases (from C to A), then decreases (from A to P′), then increases again (from P′ to B) and finally decreases again (from B to D).

Thus, for the type of decisions considered by government, the preferences of voters need not be convex or even single-peaked. Instead, the standard spatial voting model can be justified only as an application of multidimensional unfolding (Coombs, 1964). If voters must choose among a finite number of alternatives, say m, Coombs' results can be applied to mean that voters' preferences can be represented by a spatial voting model in (m – 1) dimensions. These issue dimensions may not, however, have anything to do with the problem as voters usually view it. Thus, suppose the issue is where to locate a school, a problem one normally thinks of in two dimensions, say latitude and longitude. The unfolding theorem would have voters consider the problem not in these terms, but in terms of dozens, or even hundreds, of issue dimensions which only the sophisticated would be able to translate into terms they could understand. We do not believe such a convoluted, technical, interpretation is what most researchers have in mind. The researcher should be aware that assumptions which appear plausible when applied to the preferences of consumers over private goods need not be applicable to preferences over governmental policies.

References

Arrow, K. (1963). *Social choice and individual values*. New York: Wiley.

Coombs, C.L. (1964). *A theory of data*. New York: Wiley.

Downs, A. (1957). *An economic theory of democracy*. New York: Harper and Row.

Enelow, J., and Hinich, M. (1984). *The spatial theory of voting: An introduction*. New York: Cambridge University Press.

Gibbard, A. (1973). Manipulation of voting schemes: A general result. *Econometrica* 41: 587–601.

Hotelling, H. (1929). Stability in competition. *Economic Journal* 29: 41–57.

McCubbins, M., and Schwartz, T. (1985). The politics of flatland. *Public Choice* 46: 45–60.

McKelvey, R. (1979). General conditions for global intransitivities in formal voting models. *Econometrica* 47: 1085–1111.

Satterthwaite, M.A. (1975). Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory* 10: 187–217.

Shepsle, K., and Weingast, B. (1984). Uncovered sets and sophisticated voting outcomes with implications for agenda institutions. *American Journal of Political Science* 28: 49–74.

Starret, D. (1972). Fundamental nonconvexities in the theory of externalities. *Journal of Economic Theory* 4: 130–199.