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Conflict of Interest Between Faculty, Students, and Administrators: Consequences of the Class Size Paradox

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The 1960s saw a great concern about the rise of the multiversity and the dehumanization of education. A common complaint was that universities were becoming factories whose huge classes left no opportunity for human interaction. Although part of the problem may be the increase in average class size, another part of the problem may be the fact that the average class size experienced by students is substantially greater than the average class size experienced by faculty. At one moderately large eastern university, the State University of New York at Stony Brook, the students experienced an average class size of 147 while the faculty experienced an average class size of 40. This paradox is the subject of this note. We shall explain how this class size paradox is a necessary consequence of variation in class size, and we shall show how the inconsistent experiences of students and decision makers leads them to a conflict of interest, where students would prefer to minimize the variance in class size while faculty are more inclined to maximize that variance.

The logic of the class size paradox results from the fact that classes of different size are experienced by a different number of students. Large classes are experienced by a large number of students in each class, while small classes are only taken by a few students. Consider the comparison between two

situations: (a) two classes with 40 students each, or (b) one class with 60 students and one with 20 students. There are 80 students being taught in two classes in both situations and the faculty will experience the same average class size of 40 in both situations. However, in the first situation all students experience a class of size 40 so this is their average experience; while in the second situation, 60 experience a class size of 60 while only 20 experience a class size of 20, with the result being an average of 50 for the 80 students.

With the following notation, we can explicate the logic of the class size paradox. Let

s_i = size of class i
μ = average class size for the school
n = number of classes offered
σ^2 = The variance in class size for the school
P_f = Faculty perceived average class size
P_s = Student perceived average class size
R = Perceptual ratio of student/faculty perceptions of class size ($= P_S / P_P$)
T = Total enrollment $\left(= \sum_{i=1}^{n} s_i \right)$

It is easy to see that the mean class size experienced by faculty is exactly the same as the average class size for the entire school, ie.

$$P_f = \sum_{i=1}^{n} s_i/n = \mu$$

The above result holds whenever all faculty members teach some fixed number of classes and/or if average class size is uncorrelated with the number of classes taught.

The average class size experienced by students can be found by summing the s_i class size experienced by s_i students in each class and dividing by the total enrollment, ie.

$$P_s = \sum_{i=1}^{n} s_i^2/T = \sum_{i-1}^{N} s_i^2 / \sum_{i=1}^{n} s_i$$

The above results hold whenever all students take a fixed number of classes and/or if average class size is uncorrelated with course load. The perceptual ratio between students and faculty is thus defined as:

$$R = P_s/P_f = n \sum_{i=1}^{n} s_i^2 / \left(\sum_{i=1}^{n} s_i \right)^2$$

Some slight manipulations of this expression can produce an expression for the perceptual ratio in terms of the mean, μ and the variance, σ, of the sizes of classes in the school as follows:

$$R = 1 + \sigma^2/\mu^2$$

Note that the perceptual ratio is one plus the squared coefficient of variation. Unequal class size is responsible for the discrepancy in student and faculty class size perceptions, and the class size paradox. If all classes are of equal size, then $P_s = P_f$; but so long as there is any variations in class sizes, then typical student perception of average class size must be greater than the typical faculty perception of average class size, ie. *the perceptual ratio must be greater than 1.*

At Stony Brook the mean class size in the Spring semester of 1973 was 40.5, and the variance was 4330. Consequently, students experienced a class size average of 147 and the perceptual ratio was 3.6. Presumably, it would be desirable if this discrepancy in perceptions should be recognized by faculty and administrators who are concerned for the welfare of all.[1] However, students and faculty find themselves on opposite sides of preferences for variance in class size, while the faculty retain the power.

To see the conflict between faculty and student preference for class size variance, let us assume the following:

1) Increases in class size exhibit diminishing marginal costs for both students and faculty, ie. larger classes are worse, but an extra student has lower cost in an already large class.

2) Marginal cost of increases in class size is not decreasing so rapidly for students as to make variation in class size desirable given the class size paradox. This assumption is mathematically specified in the appendix.

We take the first assumption as a truism based upon our own experiences and buttressed by expressions of preference by students and faculty whose views we have surveyed.[2] The second assumption requires some justification. Consider the following alternative ways to allocate 300 students across 6 classes: (a) six classes of 50 students apiece, or (b) five classes with 30 students apiece and one class with 150 students.

According to our first assumption, the faculty would clearly prefer the latter alternative, since both involve an identical average class size (50) and, because of diminishing marginal cost, the advantages to faculty of the smaller classes more than outweigh the increased burden of the one larger class. However, students see the situation differently. They see the choice as between all students taking classes of size 50 or half the students taking a class of size 150 and half taking a class of size 30, with an average perceived class size of *90.* We surveyed 117 students in three classes with this choice, and most of them (60%) preferred equal-sized classes, alternative (a). Students and faculty are on opposite sides of this particular decision, and we expect that they would be on opposite sides of most similar decisions.

How student/faculty conflict as to the optimum distribution of class sizes is settled will be based on the relative power of the faculty and the students. In research oriented universities, the faculty will be powerful and, caeteris paribus, there should be higher variance in class size than in, say, community colleges, where the faculty will be less powerful.

Administrators may also wish to manipulate the variance in class size for reasons of their own, in such a way as to put them into conflict with faculty preferences for lighter teaching loads. Consider a student body of 99 students, each taking two courses and 11 faculty members, each teaching exactly one course. If all classes had 18 students then, of course, both students and faculty would experience a perceived average class size of 18. However, let us assume that there are actually 10 classes with 10 students each and one class with 98 students. Each student experiences an average class size of 54 while faculty members still experience an average class size of 18.

Now consider a student body of 198 students, each taking exactly two courses, and 11 faculty members each teaching exactly one course. If each class has 36 students, the average class size is 36, and the average student perceived class size is also 36. Thus, compared to the above example, in students' eyes class size has been cut by 50%, while from the faculty perspective class size has been doubled. Clearly, an administrator bent on satisfying state legislators by increasing student FTEs without provoking student protest against "factory" education should take this example to heart.

A second conflict of interest is worthy of mention, the conflict between departments within the same university. Departments exercise considerable control over what classes are offered, and may thus directly or indirectly affect the variation in class sizes. Over and above the preferences of individual faculty members, departments are frequently given an indirect but powerful incentive by school administrators to increase the variance in class sizes of their courses. Departments are frequently allocated resources by the school administration proportional to the number of students taught. If departments offer all bigdraw courses like sex, (Psychology 217) propaganda (Political Science 258) and rock music (Music 109) then they recieve a larger share of university resources. Of course, large classes are generally undesirable to faculty, but if a few gigantic service courses are offered, then the faculty in a department are in a position to offer a number of more specialized smaller classes for their own majors. Thus, the student majors in the department are on the side of the faculty in preferring large introductory level courses for non-majors and small upper level courses for majors. Unfortunately for all, this is a prisoner's dilemma situation between departments. As each department comes to behave in the same way, each student comes to spend more of his time in large introductory courses. Furthermore, in order to preserve the specialized *small* courses, the departments have an incentive to maximize the number of students in the introductory courses and *minimize* the number of their own majors who will be trying to take their small upper division courses.

Although our predictions about the nature of conflicts of interest between students and faculty as to the optimal distribution of class sizes have only been tested in a preliminary fashion in our survey of student and faculty class size preferences, and our hypotheses as to class size variance between different types of schools have not yet been tested, we hope to have shown that the class size paradox is of considerable potential theoretical significance in accounting for important elements of educational choice and educational conflict.

APPENDIX

In this appendix, we shall explicitly show the conditions that we have assumed to underlie the conflict of interest between faculty and students under which equal class sizes produces a maximum cost for faculty and a minimum cost for students. For simplicity, we first consider the situation where there are two classes of sizes a and b with mean μ, ie. $a + b = 2\mu$. Let us consider cost as a function of a.

Faculty cost: let the cost of a class size x be f(x), so the cost of the two classes is $C = f(a) + f(b)$. To show that equal class size produces a maximum cost, we first take the first derivative. This gives us $C' = f'(a) + f'(b)\,b'$. Since $b = 2\mu - a$, $b' = -1$. Hence $C' = f'(a) - f'(b)$. It is clear that there is a point of inflection where $a = b = \mu$. To show that this a maximum, we take the second derivative: $C'' = f''(a)\,b'f''(b) = f''(a) + f''(b)$. Since cost is assumed to be increasing at a marginally decreasing rate, $C'' = 2f''(\mu) < 0$ at equal class sizes. Hence for faculty, cost is maximum where a =b.

Student cost: let the cost of a class of size x be g(x). For students, $C = ag(a) + bg(b)$. Looking at the first derivative we find $C' = ag'(a) + g(a) + bg'(b)b' + g(b)b' = ag'(a) + g(a) - bg'(b) - g(b)$. It is clear that this expression is zero when $a = b = \mu$. To determine where this experession reaches a minimum, we take the second derivative. We find $C'' = ag''(a) + bg''(b) + 2g'(b)$. Thus costs will be minimized for classes of equal size whenever this second derivative is greater than zero at $a = b = \mu$, ie. whenever $C'' = 2\mu g''(\mu) + 4g'(\mu) > 0$. Thus, equal class size generates minimum cost whenever $g''(\mu) > -2g'(\mu)/\mu$. In other words, students will prefer equal class sizes so long as marginal cost is decreasing "suffiently" slowly, and we normally expect this second order condition to be satisfied.

The results given above can be readily generalized to the case where there are more than two classes by holding constant the size of all but two classes and showing that the partial derivative of the cost function is maximum (minimum) when these two classes are of equal size. By continuing this process, the general result follows.

FOOTNOTES

[1]Elsewhere, we have discussed how variance in class size not only contributes to discrepancies in perception, but aslo lowers student attendance rate, while at the same time (somwhat paradoxically) increasing student classroom participation. (Feld and Grofman, 1977)

[2]We make the assumption that the marginal cost of class size is positive at least where class size is greater than 30. We have some evidence based on a survey of Stony Brook students conducted by the senior author, that many students do not desire very small classes (eg. less than ten students).

REFERENCES

Feld, Scott L. and Bernard Grofman. 1977. "Variation in Class Size, the Class Sixe Paradox, and Some Consequences for Students." Research in Higher Education. Vol. VI.