Inertia and Determinism
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ABSTRACT

Suppose all of the particles in the universe should happen to come to rest at the same time, in positions so arranged that all of the forces on every particle balance to zero at that time. What would happen next? Or rather, what does Newtonian mechanics say will happen next?

1 Preface

When I refer in this article to 'the Law of Inertia' and 'the Second Law', I do not mean to refer to these statements as Newton himself understood them or formulated them in the *Principia*. I am using the terms in the way contemporary physicists (such as myself at least) understand them.

Thus, when I refer to 'the Second Law', I mean the statement $\mathbf{F}_{net,external} = \frac{d}{dt}\mathbf{p}_{system}$ (or, in its simplest and most familiar formulation, the statement $\mathbf{F} = m\mathbf{a}$).

And when I refer to 'the Law of Inertia', I mean more or less the following statement: A body at rest will remain at rest, and a body in motion will remain in motion, in a straight line at a constant speed, unless acted upon by a net external force; and conversely, if a body remains at rest over time, or moves in a straight line at a constant speed, then no net external force is acting on the body.

2 Inertia and Stasis

2.1 Stating the Law of Inertia more precisely

The Second Law is usually stated in mathematical symbols, whereas the Law of Inertia usually remains in verbal form (often as a convoluted sentence, like my own effort above). One reason for this difference in presentation may be that it is surprisingly difficult to express the Law of Inertia in precise mathematical terms. But to do so is worthwhile for our purposes. To this end, let us imagine a simple Newtonian universe full of Newtonian point particles with positive masses $m_1, \ldots, m_N$ exerting Newtonian action-at-a-distance forces on one another. Particle j occupies position $\mathbf{r}_j(t)$ at time t. The instantaneous velocity and acceleration of particle j at time t are $\dot{\mathbf{r}}_j(t)$ and $\ddot{\mathbf{r}}_j(t)$, respectively. The instantaneous force $\mathbf{F}_j(t)$ acting on particle j at time t is the vector sum of all the forces exerted at time t by all the other particles in the universe. For simplicity, we shall exclude velocity-dependent forces from our model universe.[1] Then the force on particle j is of the form

$$\mathbf{F}_j(t) = \mathbf{F}_j(\mathbf{r}_1(t), \ldots, \mathbf{r}_N(t)). \tag{1}$$

In this context, the Second Law takes the form of a coupled system of autonomous differential equations:

$$\ddot{\mathbf{r}}_j(t) = \frac{1}{m_j}\mathbf{F}_j(\mathbf{r}_1(t), \ldots, \mathbf{r}_N(t)) \quad (j = 1, \ldots, N). \tag{2}$$

And the Law of Inertia can be expressed mathematically—or at least, with greater than usual precision—as follows:

(a) The position of particle j over a range of times (t_1, t_2) will be of the form $\mathbf{r}_j(t) = \mathbf{b} + \mathbf{c}t$ for some constant vectors $\mathbf{b}$ and $\mathbf{c}$ if and only if $\mathbf{F}_j(t) = 0$ for all times in this range.

This statement follows immediately from the Second Law in the form of Equation (2).

2.2 The stasis scenario

Let us now consider the following hypothetical scenario, understood to take place in a particular inertial frame of reference:

> Suppose all of the particles in the universe should happen to come to rest at the same time, in positions so arranged that all of the forces on every particle balance to zero at that time.

What would happen next? Or rather, what does Newtonian mechanics say will happen next?

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[1] The deterministic status of Newtonian mechanics in the presence of velocity-dependent forces has already been questioned; see Hutchison ([1993]). Here we limit our discussion to forces that are functions of position, so that we can focus on a different set of questions about determinacy.

A natural response to this question would be to extend the Law of Inertia beyond Clause (a) to include an additional clause such as the following:

(b) If at time t_0 all of the particles in the universe are at rest, and if at this time the forces $\mathbf{F}_j(t)$ are all zero, then all of the particles in the universe will remain at rest for all $t > t_0$.

Clause (b) seems generally consistent with the idea that 'a body at rest will remain at rest unless acted upon by an outside force.'[2] But other possibilities are allowed by Second Law dynamical evolution. For example, one might decide that the universe will immediately 'back out' of the stasis by the time-reverse of the trajectory that brought it there. Or one might assert that the universe always pauses in stasis for a fixed period of time τ before coming back to life in some way; in this approach, τ is a fundamental constant of nature. One could even assert that the universe will pause for a *fundamentally random* length of time before coming back to life; in this approach, Newtonian mechanics becomes indeterministic, and we can raise the question of whether there exist any 'classical' completions of it! Like Clause (b) itself, all of these extensions of the Law of Inertia are consistent with the Second Law, which in itself privileges none of them.[3]

Before we go too much further, we should note that the stasis scenario considered above cannot occur in finite time under Second Law evolution unless there are non-Lipschitz forces at work in the model universe.[4–6]

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[2] Newton himself seems to have favored something like Clause (b); see the Second Letter to Bentley (Cohen [1958], pp. 292–3): 'And much harder it is to suppose that all the particles in an infinite space should be so accurately poised one among another as to stand still in a perfect equilibrium. For I reckon this as hard as to make not one needle only but an infinite number of them (so many as there are particles in an infinite space) stand accurately poised upon their points. Yet I grant it possible, at least by a divine power; and if they were once so placed I agree with you that they would continue in that posture without motion for ever, unless put into new motion by the same power.'

[3] Of all the possibilities outlined in this paragraph, I think most contemporary students of the subject would lean towards Clause (b) as the prescription that 'feels right'. Indeed, the traditional verbal formulation of the Law of Inertia could even be read as incorporating Clause (b). If so, then the 'right' mathematicization of the traditional, verbal form of the Law of Inertia might consist of Clause (a) and Clause (b) together.

[4] The Lipschitz condition is roughly intermediate in strength between continuity and differentiability. Specifically, a function $f : E \to \mathbf{R}^n$, where E is an open set, is said to satisfy a Lipschitz condition on E if there is a positive constant K such that for all $x, y \in E$, $|f(x) - f(y)| \leq K|x - y|$. For example, the function $f : \mathbf{R} \to \mathbf{R}$ given by $f(x) = \sqrt{|x|}$ is continuous but does not satisfy a Lipschitz condition on a neighborhood of the origin.

[5] A classic reference for the theory of ordinary differential equations is Coddington and Levinson ([1955]). A quick summary of some key results:
- The standard existence theorem for ordinary differential equations is that at least one local solution to the initial value problem $\frac{dx}{dt} = f(t, x)$, $x(0) = a$ exists for all initial conditions if f is a continuous function (this is Peano's theorem, 1886).
- The standard uniqueness theorem is that *exactly* one local solution exists for all initial conditions if f satisfies a Lipschitz condition. This is Picard's theorem, also called the Picard–Lindelöf theorem, 1890–1894.

[6] If the universe evolves to static equilibrium at time t_0, then the Second Law allows multiple future evolutions from this time forward. (For example, everything could remain static, or the

Beginning in the nineteenth century with Ernst Mach[7] and Rouse Ball (Mach [1919], Ball [1972]), and continuing on to more recent times (Hanson [1963], Earman and Friedman [1973]),[8,9] commentators on Newtonian mechanics have universally asserted that the Law of Inertia follows immediately from the Second Law. Does it? If we are willing to insist, as an axiom on a par with the Laws of Motion themselves, that non-Lipschitz forces do not belong to the theory, then the answer is yes.[10] But if we do not wish to make this *a priori* restriction on the kinds of forces that may appear in the theory, then we can say two things. First, the Law of Inertia itself stands incomplete, or at least ambiguous, until a specific approach to the stasis scenario is selected. And, second, whatever approach is selected, the completed Law of Inertia will no longer follow mathematically from the Second Law. The Law of Inertia would instead act as a boundary condition, selecting the physical trajectory from among the many mathematical solutions to the Second Law differential equation.

3 Indeterministic Examples

People are sometimes surprised to learn that the Second Law may permit multiple solutions from the same initial data, and it is helpful to have some concrete examples in view. We begin below with an abstract problem, so that we can isolate the mathematical issues at stake. But beyond the purely mathematical point in question, there is also some interest in providing (more or less) 'physical' examples exhibiting indeterminism. John D. Norton ([2006]) has devised perhaps the simplest of such examples, namely the problem known as 'the dome', which involves a particle resting at the apex of a curiously shaped dome; the particle may remain at rest at the apex or else at any moment begin sliding down the

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universe could evolve via the time-reverse of the approach to equilibrium.) Since there are multiple solutions, the Lipschitz hypothesis of the uniqueness theorem must be violated.

[7] 'If a force determine, not position, not velocity, but acceleration, *change* of velocity, it stands to reason that where there is no force there will be no change of velocity. It is not necessary to enunciate this in independent form' (Mach [1919], p. 143). At this point in his historical exposition, Mach is discussing Galileo. But his point is the essentially same throughout the discussion of Newton also; see (Mach [1919], pp. 241–3).

[8] 'As is well known, once one relates the ideas of force and acceleration as the Second Law does, the Law of Inertia tumbles out as a special case. . . .' (Hanson [1963], p. 120).

[9] '. . . [O]n all of the above formulations [the Law of Inertia] is an empirical law . . . but it is superfluous since it is a consequence of the Second Law.' (Earman and Friedman [1973], p. 337.)

[10] The conventional wisdom among physicists is that even if the Law of Inertia appears 'on paper' to be mathematically a special case of the Second Law, nevertheless one still needs the Law of Inertia as a separate part of the theory because the Law of Inertia 'defines what we mean by an inertial reference frame.' I find this doctrine bizarre, inasmuch as the words 'reference' and 'frame' do not appear in the Law of Inertia, which makes its status as a definition of these terms dubious. I do of course grant that the Laws of Motion don't make much sense unless a certain amount of metaphysical 'setup' has been taken care of first. See Earman and Friedman ([1973], pp. 337–8) for more on this point.
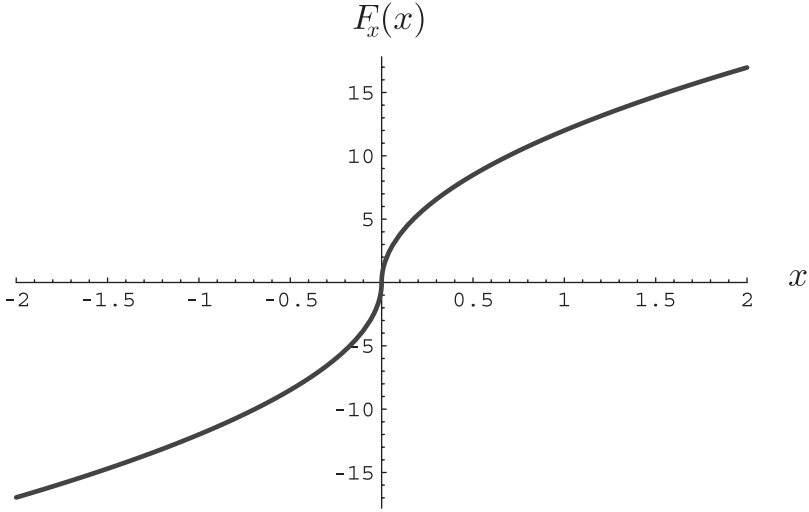
$$F_x(x)$$

Figure 1. A graph of the non-Lipschitz function $F_x(x) = 12\,\mathrm{sgn}(x)\sqrt{|x|}$.

surface under the influence of gravity. Further on below, we offer and discuss another example, this one involving electrostatic forces.

3.1 Abstract example

Consider a point particle with unit mass moving in one dimension along the x-axis under the influence of an external, time-independent field of force, as described by (see Figure 1)

$$F_x(x) = 12\,\mathrm{sgn}(x)\sqrt{|x|}. \tag{3}$$

Suppose the particle is initially located at $x(0) = 1$ and is initially moving to the left with velocity component $\dot{x}(0) = -4$. It is required to find the subsequent motion $x(t)$ of the particle for times in the interval $[0, \infty)$. We therefore solve the Second Law differential equation

$$\ddot{x} = 12\,\mathrm{sgn}(x)\sqrt{|x|} \tag{4}$$

on the domain $t \in [0, \infty)$ subject to the initial conditions $x(0) = 1$ and $\dot{x}(0) = -4$. There are infinitely many different functions $x(t)$ satisfying Equation (4) on the domain $t \in [0, \infty)$ and satisfying the given initial conditions. These solutions are given by the following family of trajectories (see Figure 2 for some representative graphs):

$$x_\tau^{\pm}(t) = \begin{cases} (1-t)^4 & 0 \le t < 1 \\ 0 & 1 \le t < 1+\tau \\ \pm(1+\tau-t)^4 & t \ge 1+\tau. \end{cases} \tag{5}$$
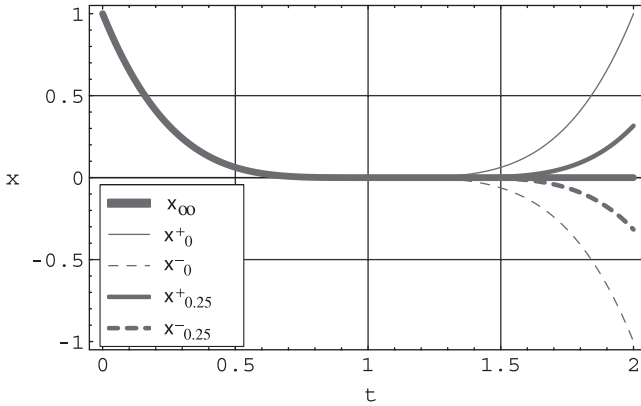
Figure 2. Five solutions of $\ddot{x} = 12\,\mathrm{sgn}(x)\sqrt{|x|}$.

The motion is easier to describe in words than it is to present in symbols: the particle reaches the origin, stops for some arbitrary length of time $\tau \geq 0$, and then exits along either the positive or negative x-axis. Note the special case $\tau = \infty$: In trajectory x_∞, the particle reaches the origin and remains there forever (the Clause (b) solution).

Because all of the trajectories in Equation (5) satisfy Equation (4) (as the reader may verify by careful differentiation), all of these motions are permitted by the Second Law.

A key feature of this example is that the particle in question reaches a state of rest, at a stable point, in *finite time*. This is by contrast to the example of, say, rolling a ball up a spherical hill against the force of gravity. Technically, if the ball starts below the top of the hill with mechanical energy E just sufficient to exist at the top of the hill, then the ball will approach the peak forever but will never quite reach it. This phase trajectory shares no phase point in common with the phase 'trajectory' of a ball with the same energy sitting at rest atop the hill. (The non-intersection of phase trajectories is of course what ensures determinacy in traditional formulations of classical mechanics.[11])

Before we dismiss the field of force $F_x(x)$ defined in Equation (3) as unphysical, let us recognize here at least an interesting mathematical role for the Law of Inertia: that of a boundary condition. If we adopt Clause (b), or some other rule, to complete the Law of Inertia, then our choice could be seen as selecting the physical trajectory from among the various mathematical solutions to the Second Law differential equation. On this point of view, the Law of Inertia is not a special case of the Second Law, but rather a principle that regulates the Second Law.

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[11] We may remark here that of course Norton's ([2006]) dome surface is *not* spherical!
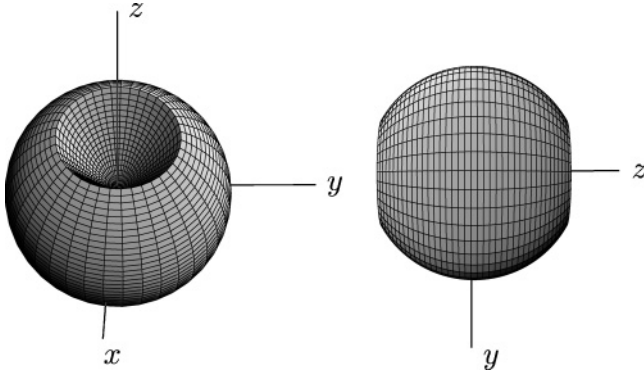
Figure 3. Two views of the fixed, azimuthally symmetric, charged body.

3.2 Second example

As a more 'physical' example, we consider an essentially Newtonian problem within the domain of quasistatic electrodynamics.[12] There are two objects in the problem. The first object is an azimuthally symmetric body, fixed in place with its center at the origin, and carrying an electric charge distribution $\rho(\mathbf{r})$ described in spherical coordinates by

$$\rho(r, \theta) = \begin{cases} r^{3/2} & \text{if } r \leq 1 \text{ and } |\cos\theta| \leq \cos\alpha \\ 0 & \text{otherwise.} \end{cases} \qquad (6)$$

Here $0 < \alpha < \pi/2$. See Figure 3, which illustrates $\alpha = \pi/6$.

The function $\rho(\mathbf{r})$ is non-analytic, but generally this problem is quite well behaved. To be specific: ρ is everywhere finite; ρ integrates to finite total charge; the resulting electrostatic field is everywhere finite; the electrostatic field is continuous; and the electrostatic field has finite total field energy.

The second object in the problem is a point quadrupole Q of unit mass that moves in the external electrostatic field created by the fixed body. The Cartesian components of Q are $Q_{ij} - \delta_{ij}(-\delta_{i1} - \delta_{i2} + 2\delta_{i3})$. The quadrupole can be visualized as the limiting case of a linear configuration of three point charges located on the z-axis. The central negative charge is $-q < 0$; the two remaining point charges, each of charge $q/2$, are located at a distance δ on either side of the central point charge. The limit ($q \to \infty$, $\delta \to 0$) with $q\delta^2$ held constant at the value unity leads to the point quadrupole Q_{ij}.

The quadrupole is initially located on the positive z-axis, with an initial velocity directed towards the center of the charge distribution. The motion

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[12] This problem doesn't belong to the model universe described in Section 2.1, but one reason we present it is that it shows how one may avoid some of the more 'unphysical' aspects of the usual indeterministic electrostatic problems (see, e.g., footnote 5 on p. 12 of (Norton [2006]), which discusses a ball of charge with a divergent charge density).

of the quadrupole is governed by the electrostatic field created by the charge distribution. An integration over the charge distribution Equation (6) leads to the electric potential at points on the z-axis for $|z| \leq 1$. With ϵ_0 put to unity, the result is

$$\Phi(z) = -g|z|^{7/2} + \sum_{\ell=0,\,\text{even}}^{\infty} \frac{b_\ell(\alpha)}{7 - 2\ell} z^\ell, \tag{7}$$

where $b_\ell(\alpha) \equiv \int_{-\cos\alpha}^{\cos\alpha} P_\ell(x)\,dx$, P_ℓ is the ℓth Legendre polynomial, and $g \equiv \sum_{\ell=0}^{\infty} b_\ell(\alpha) \left(\frac{1}{7-2\ell} - \frac{1}{9+2\ell} \right)$. Henceforth we take $\alpha = \pi/6$, for which $g \approx 0.1285$.

For simplicity, we consider the dynamics in the limit in which the initial position is close to the origin; this approximation allows us to write down simple exact solutions in closed form without changing the essential features of the example. In the neighborhood of the origin, the potential Φ can be expanded as

$$\Phi(z) = \frac{\sqrt{3}}{7} - \frac{z^2}{8\sqrt{3}} - g\,|z|^{7/2} + o(z^4). \tag{8}$$

The force acting on the quadrupole is $\mathbf{F} = -\frac{1}{2}\frac{d^3\Phi}{dz^3}\hat{\mathbf{z}}$. Taking the required derivatives of Equation (8), we obtain the Second Law equation of motion for the quadrupole, valid near the origin:

$$\ddot{z} = a\,\text{sgn}(z)\sqrt{|z|}, \tag{9}$$

where $a \equiv 105g/16 \approx 0.843$. (Compare Equation (9) to Equation (4).)

The function $\zeta(t)$ defined by

$$\zeta(t) = \frac{a^2}{144}(T - t)^4 \tag{10}$$

with $T \equiv \frac{144^{1/4}}{a^{1/2}}\epsilon^{1/4}$ satisfies Equation (9) on the interval $[0, T]$ with initial conditions $\zeta(0) = \epsilon$ and $\dot{\zeta}(0) = -2\sqrt{a/3}\,\epsilon^{3/4}$. At the end of the interval, we have $\zeta(T) = 0$, i.e., the quadrupole reaches the origin at time T. Note that if we take the mass density of the azimuthally symmetric body to be proportional to the charge density, then since $\rho = 0$ everywhere on the z-axis, the quadrupole executes the entire motion without bumping into anything. (This was the reason for the complication of the conical holes.)

Now, if we ask for the trajectory on the longer interval $[0, \infty)$, there will be infinitely many solutions, just as in the case of the trajectories $x_\tau^\pm$ given in Equation (5). Thus the Second Law permits a number of behaviors, all of the following kind: the quadrupole reaches the origin, stops for some arbitrary length of time $\tau \geq 0$, and then exits along either the positive or negative z-axis.

Symbolically, the solutions are of the form

$$z_\tau^\pm(t) = \begin{cases} \zeta(t) & 0 \le t < T \\ 0 & T \le t < T + \tau \\ \pm\zeta(T + \tau - t) & t \ge T + \tau. \end{cases} \tag{11}$$

Note once again the special case $\tau = \infty$: In trajectory z_∞, the particle reaches the origin and remains there forever.

4 Non-Lipschitz Forces and Determinism

How interesting is the quadrupole example physically? Perhaps not terribly interesting: For example, consider the $r^{3/2}$ charge density. How could one assemble such a distribution? Or, where are such distributions to be found in nature? Have $r^{3/2}$ distributions ever proved useful in explaining specific phenomena? (Answers: I don't know, I don't know, and not that I know of.)

On the other hand, the quadrupole problem is no more idealized or abstract than many problems which physicists consider on a daily basis, whether as models for phenomena, as predictions for experimental outcomes, or as explanations of principles. Certainly, the problem would not stand out as peculiar in a graduate textbook on electrostatics. Indeed, I do not know of any principle one could invoke to exclude this problem from the subject of Newtonian mechanics, other than a simple *horror nonlipschitzi*.

Axiomatically excluding non-Lipschitz forces from Newtonian mechanics would mean excluding the quadrupole problem (not to mention 'the dome' and similar examples). But what else might it mean? From the standpoint of physics, is anything actually at stake here?

As one sort of affirmative answer, let us imagine a contemporary of Newton—we'll call him Robert Tooke—who objects to Newton's law of universal gravitation ($F_{\text{Newton}} = GMm/r^2$) not on the basis of its qualities of action at a distance *per se*, but rather on the basis that this action extends infinitely far from the source object. Our Dr. Tooke proposes the following remedy (see Figure 4):

$$F_{\text{Tooke}} = \begin{cases} G\,Mm\sqrt{r^{-4} - L^{-4}} & 0 < r \le L \\ 0 & r \ge L. \end{cases} \tag{12}$$

The parameter L, a length scale, appears to be an arbitrary fudge-factor; but then again, a scientist of Newton's time might very well have reasoned that if God may set the strength of the gravitational interaction (G), then He might very well set the reach of it also (L).

Now I have two questions. First, in our scenario, was Dr. Tooke making a valid scientific proposal? I think the answer is yes, at least in the sense that observational data can be used to place limits on the value of L. For example,
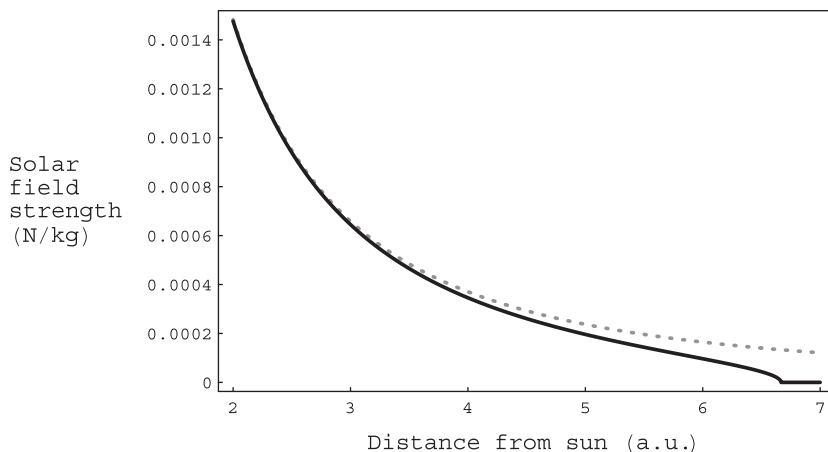
Figure 4. Comparison of the gravitational theories of Newton (dotted curve) and the fictional Dr. Tooke (solid curve). The curves illustrate the decreasing gravitational field strength of the sun as a function of distance in the solar system. Here the L parameter has the value 10^{12} m.

the fact that Saturn revolves around the sun would appear to rule out the value of L shown in Figure 4. (The orbital semimajor axis of Saturn is over 9 a.u.)

Second, if some mathematical savant had pointed out that Tooke's proposal would have allowed for indeterminism in the universe, would this insight have sufficed on its own to demolish the proposal? It would seem not; only observational facts about celestial motions could have done that.

Within the community of mathematicians and physicists, it is often taken for granted that Newtonian mechanics has a deterministic structure. At times, this is even made a matter of definition. Arnold ([1998], p. 1) *defines* classical mechanics as the study of 'the motion of systems whose past and future are uniquely determined by the initial positions and initial velocities of all particles of the system.' Landau and Lifshitz ([1960], p. 1) go beyond mere definition, claiming that determinism is in fact an *observed* feature of classical systems. But such observations could only apply to the small class of non-chaotic systems. And for that matter, to the best of our knowledge, our world is not, in fact, deterministic; so the claim that determinism has ever been observed is open to dispute.

Mathematicians like Arnold probably want to impose smoothness conditions because doing so makes theorems easier to prove. Then, having imposed the smoothness condition, they don't want to feel that they are leaving out any interesting behaviors; so they define classical mechanics to *be* the very mathematical object they are studying.

Maybe the mathematicians are correct that their theorems are not leaving out any interesting behaviors. But I don't think they can be correct in saying that the smoothness conditions of their treatises are mandated by observed facts about determinism.

5 Beyond the Stasis Scenario

The view of Newtonian mechanics as a deterministic theory is firmly entrenched, at least among physicists and mathematicians. But the Lipschitz condition required to support this view is usually left unstated, and the grounds for imposing this condition are unclear. Assuming for the moment that we wish to allow non-Lipschitz forces into the theory—or at any rate, assuming that we do not wish categorically to exclude them—then it is interesting to return to a discussion of the stasis scenario of Section 2.2.

If we like, we can adopt something like Clause (b) to handle the stasis scenario. But our job will not be finished. There will be other indeterministic situations not resolved by Clause (b). For example, a point particle of unit mass might be subject to a field of force given in Cartesian coordinates by

$$\mathbf{F}(x, y, z) = (2, 0, 12\,\mathrm{sgn}(z)\sqrt{|z|}). \tag{13}$$

With initial conditions $x(0) = y(0) = z(0) = \dot{x}(0) = \dot{y}(0) = \dot{z}(0) = 0$, the Second Law permits multiple trajectories for the point particle, including these two:

$$(x(t), y(t), z(t)) = (t^2, 0, 0), \tag{14}$$

$$(x(t), y(t), z(t)) = (t^2, 0, t^4). \tag{15}$$

The key feature of this example, apart from its indeterminacy, is that at $t = 0$, the force acting on the particle is non-zero. (At $t = 0$, we have $\mathbf{F}(x(0), y(0), z(0)) = (2, 0, 0)$, a non-zero force vector pointing along the x-axis.) Because the force on the particle does not vanish at $t = 0$, Clause (b) does not apply.[13]

One response to this example might be to attempt to repair Clause (b), or to strengthen it, leading to a conception of the Law of Inertia so strong that it ensures determinism in all possible situations. But it is unclear whether such a program is mathematically possible. Another approach would be to give up the attempt to complete the Laws of Motion, and simply conclude—despite the prejudices of history—that Newtonian mechanics is, and has always been, an indeterministic picture of the universe.

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[13] This example is interesting mathematically because it shows that integral curves of Hamiltonian flows can cross at *non-singular* points. I am not aware of any other example like this in the existing literature on classical dynamics or ordinary differential equations.

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References

Arnold, V. I. [1998]: *Ordinary Differential Equations*, R. A. Silverman (trans.), Cambridge, MA: MIT Press.

Ball, W. W. R. [1972]: *An Essay on Newton's 'Principia'*, first published 1893, reprint with an introduction by I. B. Cohen, New York and London: Johnson Reprint Corp.

Cohen, I. B. assisted by Schofield R. E. [1958]: *Isaac Newton's Papers and Letters on Natural Philosophy and Related Documents*, Cambridge: Cambridge University Press.

Coddington, E. A. and Levinson, N. [1955]: *Theory of Ordinary Differential Equations*, New York: McGraw-Hill.

Earman, J. and Friedman, M. [1973]: 'The Meaning and Status of Newton's Law of Inertia and the Nature of Gravitational Forces', *Philosophy of Science*, **40**(3), pp. 329–59.

Hanson, N. R. [1963]: 'The Law of Inertia: A Philosopher's Touchstone', *Philosophy of Science*, **30**(2), pp. 107–21.

Hutchison, K. [1993]: 'Is Classical Mechanics Really Time-Reversible and Deterministic?', *British Journal for the Philosophy of Science*, **44**, pp. 307–23.

Landau, L. D. and Lifshitz, E. M. [1960]: *Classical Mechanics*, J. B. Sykes and J. S. Bell (trans.), Oxford: Pergamon Press.

Mach, E. [1919]: *The Science of Mechanics: A Critical and Historical Account of Its Development*, Fourth Edition, Chicago and London: Open Court.

Norton, J. D. [2006]: 'The Dome: An Unexpectedly Simple Failure of Determinism,' manuscript prepared for Symposium *The Vagaries of Determinism and Indeterminism*, PSA 2006: Philosophy of Science Association 20th Biennial Conference, Vancouver, November 2006. Available at <philsci-archive.pitt.edu/archive/00002943>.