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Dollarization Traps

The paper analyzes dollarization in the sense of *asset substitution*, where a foreign currency competes with local assets, especially domestic capital, as a store of value, the impact of dollarization on capital accumulation and output, and why economies remain dollarized long after a successful inflation stabilization. We relate this dollarization hysteresis to a financial intermediation failure that happens during high inflation. We show that in dollarized countries, inflation stabilization policies may not have any effect on domestic capital accumulation, thus preventing such policies from stimulating growth—i.e. dollarized economies are vulnerable to "dollarization traps."

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UNOFFICIAL "DOLLARIZATION" HAS become a pervasive phenomenon in many emerging market economies. Discussions of the dollarization phenomenon have often focused either on *official dollarization*, where a government abandons the domestic currency and replaces it with a "hard" foreign currency (such as the U.S. dollar), or on unofficial *currency substitution*, i.e. the competition between U.S. dollars and the domestic currency as a *medium of exchange*. In this paper, we focus on unofficial dollarization in the sense of *asset substitution*, where a foreign currency competes with local assets, especially domestic capital, as a

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store of value. The paper assesses the impact of dollarization on capital accumulation, a topic that has been neglected in the dollarization literature.

It is well understood and documented that economies become dollarized during episodes of high inflation. However, disinflations are not necessarily followed by *de*dollarization. In particular, Argentina, Bolivia, Peru, Romania, Russia, Ukraine, and other countries have remained highly dollarized long after the inflation rate was brought down to single digits.

This paper presents a new explanation of the dollarization hysteresis paradox. We relate it to the underdevelopment of the financial system or a financial intermediation failure that happens during a period of high inflation. The link between financial underdevelopment and dollarization has been noted in several descriptive papers, but it has never been modeled explicitly. In our model high inflation undermines financial intermediation, leading to the adoption of a less efficient production technology, which in turn makes a dollarization trap possible. Two technologies and a fixed cost of operating the more efficient technology are key to generating a dollarization trap in our model. In this trap arbitrage equates the return on productive capital and dollars. Hence the exogenously given return on dollars pins down the return on productive capital, thus making the capital stock and output independent of inflation. A disinflation increases holdings of dollars rather than the capital stock. *Rising* dollarization despite *falling* inflation is a counterintuitive result which is nevertheless consistent with empirical evidence of several Latin American and transition economies.[1] The only way to exit from such a trap is to reduce inflation below a threshold level.

The rest of the paper is organized as follows. The next section discusses related literature. Section 2 describes the model. Section 3 discusses production technologies. Section 4 discusses steady-state equilibria and transitional dynamics. Section 5 considers the effects of inflation and disinflation on the equilibrium. Section 6 offers concluding remarks. All proofs are relegated to the Appendix.

1. RELATED LITERATURE

As noted above, discussions of the dollarization phenomenon have focused in the first instance on official dollarization, where a government abandons the domestic currency and replaces it with a "hard" foreign currency (such as the U.S. dollar).[2] In the second instance, discussions of dollarization have focused on unofficial currency substitution, i.e. competition between U.S. dollars and the domestic currency as a medium of exchange. The literature on currency substitution has been concerned with issues of real money demand, optimal money growth, the inflation tax, and real exchange rate movements in the context of endowment, infinitely lived

1. See, e.g., Savastano (1996) for Latin America and Sahay and Vegh (1996) for transition economies.

2. Berg and Borensztein (2000) discuss the pros and cons of full official dollarization. Edwards (2001) and Edwards and Magendzo (2003) examine empirically the effects of official dollarization for economic growth.

representative agent models. Different authors adopt different money demand specifications.[3] Vegh (1995) and Sturzenegger (1997) are the only ones who explicitly model the production side of the economy. However, in their models labor is the only input, and so there is no substitution between productive capital and dollars.

We focus on unofficial dollarization in the sense of asset substitution, where a foreign currency competes with local assets, especially domestic capital, as a store of value. Our approach to dollarization is consistent with Calvo's definition; he defines currency substitution as "the use of foreign currency as a means of exchange" and dollarization as "the use of foreign currency in any of its three functions: unit of account, means of exchange, *and, in particular, store of value*" (Calvo 1996, p. 153; emphasis added).

It is well understood and documented that economies become dollarized during episodes of high inflation, but disinflations are not necessarily followed by *de*dollarization.[4] The dollarization hysteresis paradox has been addressed using three different approaches.

The first approach is to modify existing currency substitution models to include adjustment costs or network externalities. Oomes (2003), Cuddington and Garcia (2002), Guidotti and Rodriguez (1992), and Uribe (1997) develop models in which the cost of using a foreign currency for transactions negatively depends on the share of market participants who use this currency. Once the economy gets dollarized, there is no benefit for an individual market participant to switch back to using domestic currency as long as other participants continue to use dollars. The limitation of this approach is that it explains the use of a foreign currency as a medium of exchange but not as a store of value. However, it is the store-of-value function of money and not the medium of exchange role that accounts for the billions of dollars kept "under the mattress" in Latin America and Eastern Europe. Furthermore, in most of these countries firms and proprietors are obliged by law to accept only domestic currency, which limits the use of dollars as a medium of exchange to nonmarket exchanges and the shadow economy.

The second approach is to explain dollarization hysteresis as arising from a lack of confidence in domestic monetary assets, resulting from past inflations, devaluations, or bank failures (Feige 2003). This approach hinges on the assumption that economic agents possibly make systematic mistakes by holding dollars "under the mattress" and forfeiting a higher return on domestic monetary instruments. The "peso problem" as a potential explanation of dollarization hysteresis is inconsistent with

3. For example, Calvo (1985), Canzoneri and Diba (1992), and Imrohoroglu (1996) put home and foreign currency in the utility function of the representative agent, Guidotti (1993) and Agenor and Khan (1996) use a cash-in-advance framework where individuals are required to use domestic currency to purchase domestic goods and foreign currency to purchase foreign goods. Engineer (2000) utilizes a Townsend (1980) turnpike framework. Recently several search theoretic models of money have incorporated dual (or multiple) currencies. These contributions include Camera, Craig, and Waller (2004), Craig and Waller (2004), and Head and Shi (2003).

4. Feige (2003), Guidotti and Rodriguez (1992), Havrylyshyn and Beddies (2003), Kamin and Ericsson (2003), Sahay and Vegh (1996), Savastano (1996), and Van Aarle and Budina (1996) describe the dollarization experience of various Latin American and East European countries.

the very strong macroeconomic fundamentals in several of these countries (including Peru and Russia in the early 2000s).

The third approach, "financial dollarization" due to Ize and Levy Yeyati (2003), explains dollarization of the balance sheets of domestic banks as an optimal response to exchange rate and inflation risk. In Ize and Levy Yeyati's asset market model, currency choice is determined on both sides of a bank's balance sheet by the need to hedge against inflation and foreign exchange risk. They find that the equilibrium gravitates toward the minimum variance portfolio allocation, i.e. they explain dollarization using the second moments (i.e., volatility) of inflation and real exchange rate depreciation, rather than using the first moments (i.e., expected inflation and depreciation).[5] Dollarization levels can remain high, in spite of disinflationary policies, if the expected volatility of the inflation rate is high in relation to the volatility of the real exchange rate.

There are some limitations to this financial dollarization literature. First, it takes the banking system as given and is therefore unable to compare the degree of dollarization in economies with different levels of financial development. Second, it does not incorporate the banking system into a general equilibrium framework and does not study the impact of financial dollarization on the broad macroeconomy. Finally, dollarization hysteresis is observed in several countries with high real exchange rate volatility, notably, Russia.

As noted above, this paper presents a fourth, alternative explanation of the dollarization hysteresis paradox that is built on a financial intermediation failure that happens during the period of high inflation. The link between financial underdevelopment and dollarization has been noted in several descriptive papers, but it has never been modeled explicitly. For example, Miguel Savastano argues that "the relative importance of foreign currency as an inflation hedge will be inversely related to the economy's level of financial development. An economy with a well-developed financial market is, in principle, capable of adapting rapidly to a high inflation environment by offering a rich set of fairly liquid, high-yield instruments denominated in domestic currency ('near monies') that preserve the real value of the public's portfolio" (Savastano 1996, p.226). Chile and, especially, Brazil are examples of countries that went through periods of high inflation in the 1970s–1990s, but avoided dollarization. These two economies arguably have the most sophisticated banking systems in the South America. Furthermore, Feige (2003) observes that in economies in transition all measures of dollarization are negatively correlated with the EBRD index of banking reform, which is an indirect measure of financial development.

Besides addressing the dollarization hysteresis paradox, our paper overcomes several limitations of the existing literature on dollarization. There is very little

5. See also Broda and Levy Yeyati (2003), who focus on the liability side of the bank balance sheet and show that banks may have an incentive to attract dollar deposits above the socially optimal level. In their model the currency mismatch is the only source of bank default (all loans are denominated in pesos, the domestic currency, and banks default in the case of large devaluation shock), but the dollar depositors share the burden of default with peso depositors. Hence the banks have a preference for dollar liabilities, because the peso-dollar spread priced by risk-neutral depositors exceeds the effective relative cost of dollar liabilities for the bank.

research on substitution between dollar denominated assets and domestic assets other than money. This is surprising, since the use of foreign currency (dollars) as a store of value usually precedes the use of foreign currency as a medium of exchange (Calvo and Vegh, 1992, Heyman and Leijonhufvud, 1995). Another serious limitation of the current dollarization literature is that it neglects the *real* effects of dollarization. Specifically, most existing models analyze dollarization in the context of endowment economies; none of them studies the interaction between dollarization and physical capital accumulation.

2. THE ENVIRONMENT

We use a three-period overlapping generations model in which agents save during the first two periods and consume in the final period. In every period t a continuum of agents of measure 1 is born. There is no population growth. In the first period, agents are endowed with one unit of labor which they supply inelastically on the labor market receiving in exchange the perfectly competitive market wage, w_t. They derive utility from consumption of the single, perishable good in the last (the third) period of life only. The only restriction we impose on the utility function is that it depends positively on consumption. There exist three kinds of assets to transfer wealth across time: domestic currency, productive capital (which depreciates fully after used in production), and dollars. Agents regard dollars and capital as perfect substitutes. Henceforth we will refer to them both as capital market assets (CMA). We assume that there is no inflation of dollars and that purchasing power parity holds. Therefore, the gross return from holding dollars is always unity. We restrict our attention to equilibria in which the return on domestic currency is lower than the return on dollars and capital. In other words, we abstract from deflation and liquidity traps here.

To motivate positive demand for domestic currency, we assume that domestic currency must be held for at least one period prior to the purchase of CMA.[6] Therefore, agents hold currency between the first and the second period of life, and invest in CMA at the end of the second period of life. This assumption is analogous to a cash-in-advance constraint.[7]

6. There exist a variety of alternative setups with qualitatively identical implications. An example is a model in which positive demand for domestic currency exists due to spatial separation and limited communication, like in Champ, Smith, and Williamson (1996) and Schreft and Smith (1997, 1998). A version of the paper with this alternative setup is available from the authors upon request. The disadvantage of the spatial-separation-and-limited-communication framework is that it requires more restrictions on the utility function and on the allocation of newly printed fiat money than the framework adopted in this version of the paper.

7. Indeed, our requirement that the young hold their savings in domestic money can be motivated using the same arguments given in support of the standard cash-in-advance assumption, i.e., that consumers have to convert their assets into money (cash or checkable bank accounts) prior to purchasing consumption goods. Analogously, we can think of workers receiving their wages in the form of domestic money transfers to their bank accounts. It takes at least some time (one period in our model) before they are able convert these wages into other assets. This constraint is, admittedly, a "short-cut;" it would be useful to endogenize this friction—a task we leave to future research.

In our model it is the purchase of capital market assets (rather than consumption goods) that are subject to a cash-in-advance constraint, as in Stockman (1981). The motivation for this assumption is that it generates a negative relationship between the inflation rate and the steady-state capital stock, consistent with empirical evidence.[8]

There exist two productive technologies. The "efficient" technology requires some kind of financial intermediation—henceforth called a "financial center"—that can be operated only at a fixed cost. The financial center can be thought of as a stock market, a bank, or any other technology (e.g. computer center, power plant, infrastructure), that enables an economy to operate at the frontiers of its production possibilities. The efficient technology yields output in per capita terms, $y = Ak^{\alpha} - \phi$, where k is the capital stock per worker, $A > 0$ and $\alpha \in (0,0.5)$ are technology parameters, and ϕ is the cost to operate the financial center.

The "primitive" technology is costless to operate and does not require a financial center (i.e., the agents can use it directly), but it is less productive than the efficient technology; the primitive technology yields output in per capita terms equal to $Y = A\gamma k^{\alpha}$, where $\gamma < 1$.

The motivation for the fixed cost associated with operating the efficient technology is that it avoids a problematic "arbitrage problem" that arises in standard models—problematic in the sense that standard models tend to generate a negative relationship between dollarization and inflation. Specifically, as long as the production function exhibits diminishing marginal returns, a reduction in the capital stock leads to a rise in the real return to domestic capital, which in turn results in a shift away from investment in dollars and toward capital. Thus, in an environment where inflation reduces capital accumulation, inflation, and dollarization will tend to be negatively related. This arbitrage problem clearly contradicts the empirical evidence on dollarization in high inflation economies; i.e. high inflation tends to raise the demand for dollars. The non-convexity in production overcomes this problem: when the aggregate capital stock is sufficiently high, the scale of production allows for an efficient, modern financial intermediation technology with a marginal rate of return to capital that is higher than in the non-intermediated "primitive" technology that prevails under lower capital stocks. Further, the non-convexity also underpins the possibility of dollarization hysteresis.

Finally, we note that there is a government that prints money (a.k.a. domestic currency) at the beginning of every period and gives the newly printed bills as lump-sum transfers to domestic agents ("helicopter drops"). We assume that the share of the young agents (in the aggregate transfer) is $\tau_1 \geq 0$, the share of the middle-age agents is $\tau_2 \geq 0$, and the share of the old is $1 - \tau_1 - \tau_2 \geq 0$. The gross growth rate of nominal money supply is constant each period and is denoted ρ. Hence $M_t = \rho M_{t-1}$.

8. See Barro (1996), Chari, Jones, and Manuelli (1995), Fischer (1993), Ghosh and Phillips (1998), Khan and Senhadji (2001), and Sarel (1996).

3. THE FINANCIAL CENTER

We assume that the financial center is a joint-stock company owned by prospective investors, the middle-aged agents. It is run as an independent profit-maximizing firm, and hence it operates only if it makes a non-negative profit.[9] The profit is rebated to the owners in the following period, i.e. when the investment matures.

We make two assumptions about the efficient production technology that are important to the financial center:

Assumption 1: $A\alpha^2 k^{\alpha-1} - 1 > 0$.
Assumption 2: $2\phi > A\alpha(A\alpha^2)^{\alpha/(1-\alpha)} - (A\alpha^2)^{1/(1-\alpha)} > \phi$.

Assumption 1 ensures a positive relationship between the profit of the financial center and its scale of operations. The corollary of Assumption 1 is that the efficient production technology is dynamically efficient, i.e., $A\alpha k^{\alpha-1} > 1$. The purpose of this assumption is simply that, when the efficient technology is used, the return on capital is greater than the return on dollars and thus the financial center never invests in dollars. We view this as a weak restriction on the efficient production technology.

The purpose of the second assumption is to rule out the possibility of more than one financial center from arising—i.e. the financial center is a monopoly.[10] This is purely a simplifying assumption. We emphasize that the monopoly position of the financial center has *no role* in the main results; economywide savings and capital accumulation are not affected by the market structure or pricing decisions of the financial center. To see why, note first that the financial center decides on the rate of remuneration for depositors (agents in their second period of life). However, the level of saving in both the first and second period of life is not affected by the return on saving—because agents consume only in their third period of life and thus savings is completely inelastic—unaffected by the monopolistic position of the financial center. Hence, the capital stock is also unaffected by the pricing decisions of the financial center.[11]

The financial center attracts the deposits of middle-aged agents, and pays these agents just the amount they would earn on their own using only the primitive technology and dollars. We assume that the agents resolve their indifference between using the primitive technology and dollars or the financial center by investing all of their savings (CMA) with the financial center whenever the financial center is in operation. If the financial center does not operate, individuals must decide how to allocate their CMA between the primitive technology and dollars. We refer to the portfolio

9. The size of an individual agent's share in the center is negligible. Therefore, the agent cannot influence the actions of the center. He can only decide whether to become a shareholder of the center or not.

10. Alternatively we could assume that any agent could operate the financial center as a private monopoly firm. All the predictions of the model would hold, but we would have an indeterminacy as to which of the agents sets up the financial center.

11. In fact, agents' wealth is also unaffected by the monopolistic financial center because they have equal ownership of the center, if it operates.

of CMA that agents consider investing in the primitive technology and dollars as their "autarkic portfolio;" the amount of this portfolio will be denoted by p_t.

Consider the return on the autarkic portfolio—the return that individuals could earn on their own regardless of whether the financial center operates or not. There are two cases, corresponding to whether the return on capital using the primitive technology is greater than or equal to the return on dollars. If capital dominates dollars in rate of return, dollars are not present in the autarkic portfolio. Otherwise, both capital and dollars are included in the portfolio, and the return on both assets is equalized by arbitrage. Next we consider each of these cases in turn.

3.1. Return on Capital Equals the Return on Dollars

In the case where the return on capital equals the return on dollars and the financial center does not operate, agents hold both capital and dollars in the autarkic portfolio. The autarkic portfolio involves investment in capital up to the point where the gross marginal product of capital is unity, the same as the marginal return on dollars:

$$\gamma A\alpha(k^{TR})^{\alpha-1} = 1$$

where k^{TR} is the capital stock per worker. We denote the capital stock as k^{TR}, because this is the level of the capital stock in the dollarization trap (TR stands for "trap"). Specifically, as will become clear below, in the dollarization trap the return on capital and dollars is equalized; k^{TR} is the level of capital in such a case. Solving for k^{TR} yields:

$$k^{TR} = (A\alpha\gamma)^{1/(1-\alpha)} . \tag{1}$$

Thus, the case where the agents' autarkic portfolio of CMA consists of both capital and dollars arises whenever $p > k^{TR}$; the amount of dollars in the portfolio is the residual amount, $p - k^{TR}$. The total return on autarkic portfolio p is:

$$A\alpha\gamma(k^{TR})^{\alpha-1}k^{TR} + (p - k^{TR})*1 = p ,$$

and hence the gross rate of return on this portfolio is unity.

In the case where the return on capital equals the return on dollars and the financial center is in operation, agents deposit their savings with the financial center, which in turn pays the agents a gross rate of return equal to 1. The cost of setting up the financial center is ϕ (in per capita terms). Figure 1 illustrates the revenue and the expenses of the financial center. The total return on capital, i.e., the revenue of the center is given by $A\alpha k^{\alpha}$, while $k + \phi$ are the expenditures of the center. Therefore, given that the financial center invests only in capital (this follows from Assumption 1), the center's profit in per capita terms is:

$$\Pi^d = A\alpha k^{\alpha} - k - \phi . \tag{2}$$

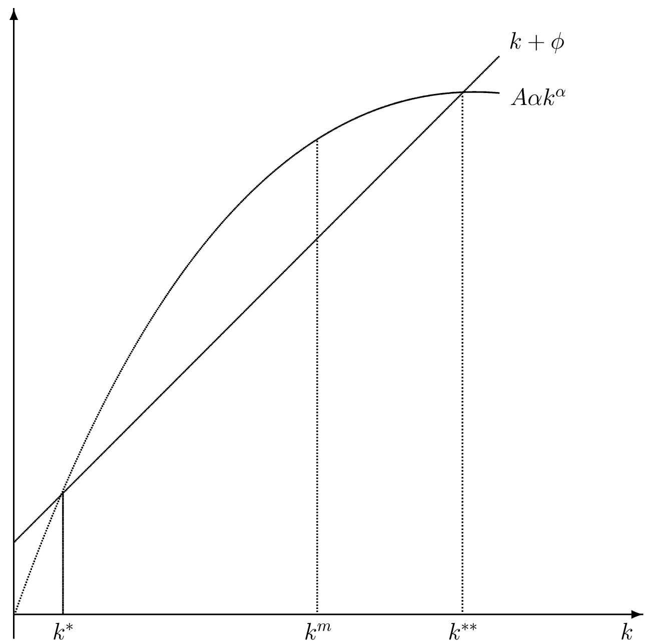

FIG. 1. Revenue, Expenditures and Profit of the Financial Center. The concave curve shows the revenue of the financial center. The revenue equals the share of the national income that capital earns $= f'(k)*k = A\alpha k^{\alpha-1}*k = A\alpha k^{\alpha}$. The upward-sloping straight line shows the expenditures of the financial center, the fixed cost ϕ, plus the cost of capital k. The financial center operates only if it makes a non-negative profit, i.e., its revenue exceeds the expenditures. Financial center operates if $k^* \leq k \leq k^{**}$. The profit attains the maximum at $k = k^m$.

The financial center operates so long as $\Pi^d \geq 0$, and shuts down whenever Π^d becomes negative. Profit is non-negative when revenue is no less than the expenditures, i.e., $k^* \leq k \leq k^{**}$, where k^* and k^{**} are the smaller and the larger roots of the equation $A\alpha k^{\alpha} = k + \phi$, respectively. Finally, $k^m \equiv (A\alpha^2)^{1/(1-\alpha)}$ is the point where profit is maximized. Note that Assumption 1 implies $k < k^m$.

Assumption 2 can be rewritten as:

$$2\phi > A\alpha(k^m)^{\alpha} - k^m > \phi.$$

This ensures that two (or more) financial centers are not able to make a profit in the economy. However, the profit of a single financial center is positive at its maximum point k^m. Therefore, the equation $A\alpha k^{\alpha} - k - \phi = 0$ has two positive real roots, and $\Pi^d \geq 0$ for $k^* \leq k \leq k^m = (A\alpha^2)^{1/(1-\alpha)}$. Hence these two assumptions ensure that the financial center makes a non-negative profit and operates only when the capital stock is sufficiently high (and when the inflation rate is low, see Section 3), and shuts down when the capital stock is too low.

Therefore, the financial center operates, if $p \geq k^*$, and shuts down if $p < k^*$.

3.2. Return on Capital Exceeds Return on Dollars

When $p \leq k^{TR}$, the autarkic portfolio of CMA contains no dollars, as the gross return on capital using the primitive technology, $A\alpha\gamma k^{\alpha-1}$, is greater than or equal to unity. The financial center, if it operates, invests in capital only. Its revenue is the return on capital using the efficient technology, $A\alpha k^{\alpha}$, and its expenditures equal the sum of the return on the autarkic portfolio (paid to depositors) and the cost of operating the financial center, $A\alpha\gamma k^{\alpha} + \phi$. Therefore, the profit of the financial center is:

$$\Pi = A\alpha k^{\alpha} - (A\alpha\gamma k^{\alpha} + \phi) = (1 - \gamma)A\alpha k^{\alpha} - \phi \,. \tag{3}$$

The financial center operates if and only if $\Pi \geq 0$, or

$$p \geq \tilde{k}^* \equiv \phi^{1/\alpha}[(1 - \gamma)A\alpha]^{-1/\alpha} \,. \tag{4}$$

Comparison among k^*, k^{TR} and $\tilde{k}^*$ yields the following three possibilities:

Lemma 1 *A) If $k^{TR} < k^*$, then $k^{TR} < \tilde{k}^* < k^*$.*
B) If $k^{TR} > k^*$, then $k^* < \tilde{k}^* < k^{TR}$.
C) If $k^{TR} = k^*$, then $k^{TR} = k^* = \tilde{k}^*$.

Only in case A can the autarkic portfolio contain dollars, and thus only in this case is a dollarization trap a possibility. If $p < k^*$, the financial center shuts down, but dollars are held in the autarkic portfolio as long as $p > k^{TR}$. In cases B and C, i.e. when $k^* \leq k^{TR}$, the financial center shuts down only if $p < \tilde{k}^* < k^{TR}$. In that case the autarkic portfolio contains productive capital only.

The findings of this subsection are summarized as follows:

Proposition 1: *If $k^* > k^{TR}$, the financial center operates if and only if $p \geq k^*$. If $k^* \leq k^{TR}$, the financial center operates if and only if $p \geq \tilde{k}^*$.*

4. COMPETITIVE EQUILIBRIUM AND STEADY STATES

This section identifies three possible equilibria that can arise in our model: (1) the financial center operates using the efficient technology and does not invest in dollars, (2) the financial center does not operate and agents invest using the primitive technology but do not hold dollars, and (3) the financial center does not operate and agents invest using the primitive technology and dollars.

4.1. Equilibrium with the Efficient Technology

Six equations determine the competitive equilibrium of this economy when the efficient technology is used:

$$w_t = (1 - \alpha)Ak_t^{\alpha} \tag{5}$$

$$S_t = \frac{(1 - \frac{1}{\rho})w_t}{1 - (1 - \frac{1}{\rho})\tau_1} \tag{6}$$

$$m_t = w_t + \tau_1 S_t \tag{7}$$

$$c_t = \alpha A k_t^{\alpha} + (1 - \tau_1 - \tau_2)S_t - \phi \tag{8}$$

$$k_{t+1} = Ak_t^{\alpha} - c_t - \phi \tag{9}$$

$$\pi_t = \frac{m_{t-1}}{k_{t+1} - \tau_2 S_t} \tag{10}$$

where c_t is the consumption of the old (third period of life) agent in period t[12], π_t is the gross inflation factor between periods $t - 1$ and t, and m_t is real money holdings by young agents. Six equations (5)–(10) determine six endogenous variables, w_t, S_t, m_t, c_t, k_{t+1}, and π_t. Variables k_t and m_{t-1} are the state variables determined in period $t - 1$.

Equation (6) is derived from the fact that the real value of the newly printed money in period t is:

$$S_t = \frac{M_t - M_{t-1}}{P_t} = \left(1 - \frac{1}{\rho}\right)\frac{M_t}{P_t} = \left(1 - \frac{1}{\rho}\right) m_t = \left(1 - \frac{1}{\rho}\right)(w_t + \tau_1 S_t) \tag{11}$$

where P_t is the price level in period t, $m_t = \frac{M_t}{P_T}$ is real money demand equal to real money supply in equilibrium. The last equality follows from our assumption that young agents' savings of wage income, w_t, and transfers, $\tau_1 S_t$, are held in domestic money until their second period of life. Equation (8) takes advantage of the fact that the old agents split the return on capital and consume all their revenue net of the intermediation cost ϕ. Moreover, an old agent gets $(1 - \tau_1 - \tau_2)$ share of the real value of newly printed fiat money, which is also consumed. Equation (9) is the aggregate resource constraint. Equation (10) is the transformed budget constraint of the middle-aged agent,

$$k_{t+1} = \frac{m_{t-1}}{\pi_t} + \tau_2 S_t \, .$$

12. Note that old agents have three sources of income: the return on autarkic portfolio, the profit of the financial center, and their share of lump-sum transfers of newly printed fiat money.

After substituting Equation (5) for w_t, Equation (6) for S_t, and inserting Equation (8) into Equation (9), we get a single first-order difference equation:

$$k_{t+1} = Ak_t^{\alpha} - \alpha Ak_t^{\alpha} - (1 - \tau_1 - \tau_2)\frac{\left(1 - \frac{1}{\rho}\right)(1 - \alpha)Ak_t^{\alpha}}{1 - \left(1 - \frac{1}{\rho}\right)\tau_1}$$

or, after some simplification,

$$k_{t+1} = A(1 - \alpha)\frac{1 - \tau_2 + \tau_2\rho}{\rho - (\rho - 1)\tau_1}k_t^{\alpha}\,. \tag{12}$$

Define $\psi \equiv A(1 - \alpha)(1 - \tau_2 + \tau_2\rho)/(\rho - (\rho - 1)\tau_1)$. For a given value of ρ, ψ is a positive constant. Hence the dynamical properties of Equation (12) are the same as the properties of the standard Diamond (1965) model.

LEMMA 2: *The dynamical equation (12) has a unique positive steady state, $k = \psi^{1/(1-\alpha)}$. This steady state is stable.*

The following proposition establishes the main comparative statics result of this subsection.

PROPOSITION 2: *If the financial center operates, an increase in the steady-state money supply growth rate reduces the per capita capital stock.*

The comparative statics result is illustrated in Figure 2. An increase in ρ reduces the constant ψ and hence shifts the graph of Equation (12) down. Therefore, it intersects the 45 degree line at a lower level of k. This negative relationship between the steady-state inflation rate and the capital stock established in Proposition 2 is often termed the reverse Mundell-Tobin effect.

The efficient technology is used if and only if the financial center can make a profit. From Proposition 1 this happens whenever $p \geq k^*$, if $k^* > k^{TR}$, and whenever $p \geq \tilde{k}^*$, if $k^* \leq k^{TR}$. The use of the financial center implies that $p = k$, as agents deposit all their middle-age wealth with the center. Hence $k \geq k^*$ is the necessary and sufficient condition for the operation of the financial center in the case $k^* > k^{TR}$, and $k \geq \tilde{k}^*$ is the necessary and sufficient condition in the case $k^* \leq k^{TR}$.

Consider both cases separately. First suppose that $k^* > k^{TR}$. The inverse relationship between k and ρ, dynamical Equation (12), and the condition $k \geq k^*$ allows us to calculate values of ρ compatible with the efficient-technology in the steady-state equilibrium. The equilibrium exists if and only if:

$$\rho \in [1,\rho^*]\,,$$

where:

$$\rho^* = \frac{1 - \tau_2 - \tau_1\frac{(k^*)^{1-\alpha}}{A(1-\alpha)}}{\frac{(k^*)^{1-\alpha}}{A(1-\alpha)}(1-\tau_1) - \tau_2}\,. \tag{13}$$

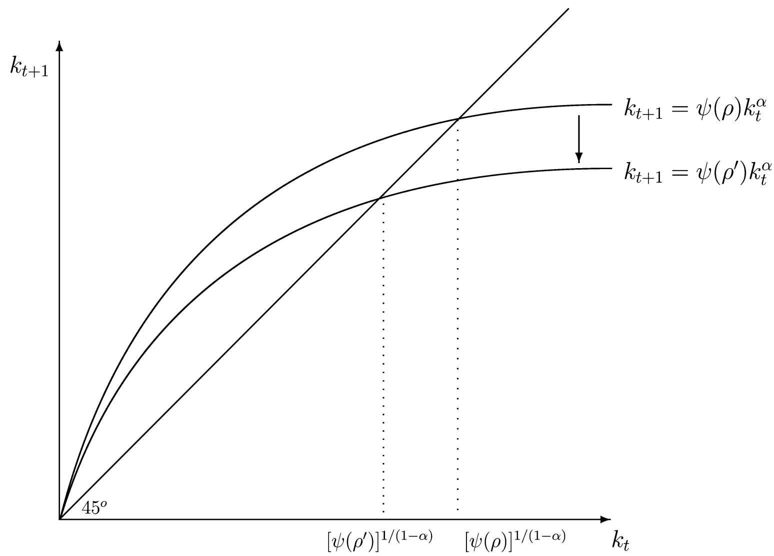

FIG. 2. The Effects of an Increase in Money Growth. An increase in the money growth rate from ρ to ρ' shifts the graph of the dynamical equation $k_{t+1} = \psi(\rho)k_t^\alpha$ down. The steady-state value of the capital stock falls from $[\psi(\rho)]^{1/(1-\alpha)}$ to $[\psi(\rho')]^{1/(1-\alpha)}$.

The right-hand side of Equation (13) is obtained by solving the steady-state version of Equation (12) for ρ and substituting k^* for k.

Second, suppose $k^* \leq k^{TR}$. The equilibrium exists if and only if:

$$\rho \in [1, \tilde{\rho}^*] ,$$

where

$$\tilde{\rho}^* = \frac{1 - \tau_2 - \tau_1 \frac{(\tilde{k}^*)^{1-\alpha}}{A(1-\alpha)}}{\frac{(\tilde{k}^*)^{1-\alpha}}{A(1-\alpha)}(1 - \tau_1) - \tau_2} . \tag{14}$$

Note that whether k^* is greater than or less than k^{TR} is a parametric restriction and thus only one of the sets $[1, \rho^*]$ and $[1, \tilde{\rho}^*]$ is relevant for any given parameterization.

4.2. Equilibrium with the Primitive Technology and No Dollars

Six equations determine the competitive equilibrium of this economy when the inefficient technology is used:

$$w_t = (1 - \alpha)A\gamma k_t^\alpha \tag{15}$$

$$S_t = \frac{(1 - \frac{1}{\rho})w_t}{1 - (1 - \frac{1}{\rho})\tau_1} \tag{16}$$

$$m_t = w_t + \tau_1 S_t \tag{17}$$

$$c_t = \alpha A \gamma k_t^{\alpha} + (1 - \tau_1 - \tau_2) S_t \tag{18}$$

$$k_{t+1} = A \gamma k_t^{\alpha} - c_t \tag{19}$$

$$\pi_t = \frac{m_{t-1}}{k_{t+1} - \tau_2 S_t} . \tag{20}$$

The System (15)–(20) is almost identical to the System (5)–(10). Given that the inefficient technology is used, Equations (15), (18), and (19) contain the coefficient γ. The other difference is the absence of the intermediation cost ϕ in the consumption Equation (18) and the capital accumulation Equation (19).

After substituting Equation (15) for w_t, Equation (16) for S_t, and inserting Equation (18) into Equation (19), we get a single first-order difference equation:

$$k_{t+1} = A\gamma k_t^{\alpha} - \alpha A \gamma k_t^{\alpha} - (1 - \tau_1 - \tau_2) \frac{(1 - \frac{1}{\rho}) A \gamma (1 - \alpha) k_t^{\alpha}}{1 - (1 - \frac{1}{\rho}) \tau_1}$$

or, after some simplification,

$$k_{t+1} = A\gamma(1 - \alpha) \frac{1 - \tau_2 + \tau_2 \rho}{\rho - (\rho - 1)\tau_1} k_t^{\alpha} . \tag{21}$$

LEMMA 3: *The dynamical system (21) has a unique, positive steady state,* $k = \left[A\gamma(1 - \alpha)(1 - \tau_2 + \tau_2\rho)/\{\rho - (\rho - 1)\tau_1\}\right]^{1/(1-\alpha)}$. *This steady state is stable.*

The following proposition establishes the main comparative statics result of this subsection.

PROPOSITION 3: *If the financial center does not operate, and dollars are not used, an increase in the steady-state money supply growth rate reduces the capital stock per worker.*

The primitive technology is used if and only if the financial center would operate with a loss. From Proposition 1 this happens whenever $p < k^*$, if $k^* > k^{TR}$, and whenever $p < \tilde{k}^*$, if $k^* \leq k^{TR}$. Dollars are not present in the autarkic portfolio, if and only if $p \leq k^{TR}$. Hence, a steady-state equilibrium with the primitive technology and no dollars occurs if $p = k \leq k^{TR}$ in case A ($k^* > k^{TR}$), and if $p = k < \tilde{k}^*$ in cases B–C ($k^* \leq k^{TR}$).

Consider both cases separately. If $k^* > k^{TR}$ (case A), the inverse relationship between k and ρ, dynamical Equation (21) and the condition $k \leq k^{TR}$ allows us to calculate values of ρ compatible with the primitive-technology-no-dollars steady-state equilibrium. The equilibrium exists if and only if:

$$\rho \geq \rho_1 ,$$

where:

$$\rho_1 = \frac{1 - \tau_2 - \tau_1 \frac{(k^{TR})^{1-\alpha}}{A(1-\alpha)\gamma}}{\frac{(k^{TR})^{1-\alpha}}{A(1-\alpha)\gamma}(1 - \tau_1) - \tau_2}. \tag{22}$$

The right-hand side of Equation (22) is obtained by solving the steady-state version of Equation (21) for ρ and substituting k^{TR} for k.

If $k^* \leq k^{TR}$ (cases B–C), the primitive-technology-no-dollars steady-state equilibrium exists if and only if:

$$\rho > \tilde{\rho}_2^*,$$

where

$$\tilde{\rho}_2^* = \frac{1 - \tau_2 - \tau_1 \frac{(\tilde{k}^*)^{1-\alpha}}{A(1-\alpha)}}{\frac{(\tilde{k}^*)^{1-\alpha}}{A(1-\alpha)}(1 - \tau_1) - \tau_2}. \tag{23}$$

4.3. Equilibrium with the Primitive Technology and Dollars

A steady-state equilibrium with dollars can exist only if $k^{TR} < k^*$. Otherwise, the financial center starts operating at a capital stock below k^{TR}. The dynamics of the model are described by the following equations:

$$w_t = (1 - \alpha)A\gamma(k_t)^\alpha \tag{24}$$

$$S_t = \frac{(1 - \frac{1}{\rho})w_t}{1 - (1 - \frac{1}{\rho})\tau_1} \tag{25}$$

$$m_t = w_t + \tau_1 S_t \tag{26}$$

$$c_t = A\gamma(k_t)^\alpha - k_t \tag{27}$$

$$c_t = (k_t + d_{t-1})*1 + (1 - \tau_1 - \tau_2)S_t \tag{28}$$

$$k_{t+1} + d_t = \frac{m_{t-1}}{\pi_t} + \tau_2 S_t \tag{29}$$

where d_t is the stock of dollars accumulated in period t. Equation (28) is the budget constraint of the old agent. It states that he has two sources of revenue to finance his consumption: the return on CMA (gross return equal to one) and the lump-sum transfers of newly printed fiat money. Equation (29) is the asset accumulation equation (the budget constraint of the middle-aged agent) when dollars are held. The right-hand side is the real value of the wealth of middle-aged agents invested in CMA, and the left-hand side is holdings of capital and dollars.

The following lemma shows that the model does not have any transitional dynamics in this "dollarization trap" equilibrium.

LEMMA 4: *For a given rate of the money supply growth, the capital stock, k, the wage rate, w, lump-sum transfers, S, the inflation rate, π, the consumption, c, the real fiat money holdings, m, and dollar holdings, d, are all time invariant and equal to their steady-state values.*

The capital stock is pinned down by the arbitrage condition in a neighborhood of the steady state (not only in the steady state). Values of $k_t = k^{TR}$ and ρ uniquely determine the values of all other variables of the model.

Proposition 4 presents the main comparative statics result of this subsection.

PROPOSITION 4: *In the dollarization trap equilibrium, disinflation increases dollar holdings.*

A disinflation (a reduction in ρ) raises the right-hand side of Equation (29), because the increase in the real value of the money holdings of the middle-aged, m/π, offsets the reduction in the transfer from the government, $\tau_2 S$. Hence the left-hand side rises as well, and therefore the dollar holdings, d, also go up.

Next we calculate the values of ρ compatible with this dollarization trap equilibrium. The lowest possible money growth rate—denoted as ρ_2—makes the right-hand side of Equation (29) equal to k^*, i.e., saving is sufficiently high that $p = k^*$ which induces a switch back to the efficient technology. Define $w^{TR} \equiv (1-\alpha)A\gamma(k^{TR})^{\alpha}$. Using this notation, ρ_2 is defined implicitly by the following modification of Equation (29):

$$k^* = w^{TR}\left\{\frac{1}{\rho_2} + \frac{\tau_1(1-\frac{1}{\rho_2})}{\rho_2[1-(1-\frac{1}{\rho_2})\tau_1]} + \frac{\tau_2(1-\frac{1}{\rho_2})}{1-(1-\frac{1}{\rho_2})\tau_1}\right\}.$$

Solving for ρ_2 yields:

$$\rho_2 = \frac{w^{TR}(1-\tau_2) - \tau_1 k^*}{k^* - \tau_1 k^* - \tau_2 w^{TR}}. \tag{30}$$

Similarly, the highest possible money growth rate—denoted as ρ_1—consistent with this equilibrium makes the right-hand side of Equation (29) equal to k^{TR} (zero demand for dollars). Algebraically,

$$k^{TR} = w^{TR}\left\{\frac{1}{\rho_1} + \frac{\tau_1(1-\frac{1}{\rho_1})}{\rho_1[1-(1-\frac{1}{\rho_1})\tau_1]} + \frac{\tau_2(1-\frac{1}{\rho_2})}{1-(1-\frac{1}{\rho_1})\tau_1}\right\}.$$

Solving for ρ_1 yields:

$$\rho_1 = \frac{w^{TR}(1-\tau_2) - \tau_1 k^{TR}}{k^{TR} - \tau_1 k^{TR} - \tau_2 w^{TR}}. \tag{31}$$

We know that $\rho_1 > \rho_2$, because the right-hand side of Equation (29) is monotonically decreasing in ρ. Hence it attains the lower value, k^{TR}, at a higher level of the steady-state inflation rate, than the higher value, k^*. We conclude the subsection with the following lemma.

LEMMA 5: $\rho^* > \rho_1 > 1$.

Lemma 5 ensures that the dollarization trap occurs in the relevant range of values of ρ, that is, dollarization traps are only possible at sufficiently high rates of growth of the money stock and inflation.[13]

4.4. Summary

There are two different cases depending on whether k^{TR} is greater than, less than, or equal to, k^*.

Case A. $k^* > k^{TR}$. All three equilibria are feasible, and there may be multiple equilibria for a range of the money supply growth rate. Specifically, if $\rho \in (1,\rho_2)$, only the efficient-technology equilibrium exists. If $\rho \in [\rho_2,\rho_1)$ both the efficient-technology steady state and the steady state with the inefficient technology and dollars are possible. For $\rho \in [\rho_1,\rho^*)$ there exist two steady equilibria: the efficient-technology equilibrium and the inefficient-technology-no-dollars equilibrium. Finally, for $\rho \geq \rho^*$ the only equilibrium is with the inefficient technology and no dollars. Thus, the range of the gross inflation factor values $[\rho_2,\rho^*]$ is compatible with two different steady-state equilibria. Figure 3 illustrates this case.

Case B–C. $k^{TR} \geq k^*$. The inefficient-technology-and-dollars equilibrium is not feasible. If $\rho \leq \tilde{\rho}^*$, the efficient-technology equilibrium exists. If $\rho \geq \tilde{\rho}_2$, the primitive-technology-no-dollars equilibrium exists. Therefore the range of the gross inflation factor values $[\tilde{\rho}_2,\rho^*]$ is compatible with two different steady-state equilibria, one of which uses the efficient technology and one which uses the inefficient technology (and no dollars). Figure 4 illustrates this case.

5. DYNAMICS OF INFLATION AND DISINFLATION

5.1. Qualitative Features of a Dollarization Trap

A dollarization trap arises only if $k^* > k^{TR}$. In this case, assume that inflation is low, but rising. The economy starts at point A and gradually moves to point B as shown in Figure 3. Along the way the efficient technology is used, but the capital stock is falling due to higher inflation. Any temporary deviation from the steady-state equilibrium dies out, because steady states are stable.

If the inflation rate exceeds ρ^*, a bifurcation takes place. The financial center shuts down, and agents have to use the primitive technology instead. The economy

13. ρ_2 can be less than 1. In that case the relevant range of values of ρ is $(1, \rho_1)$.

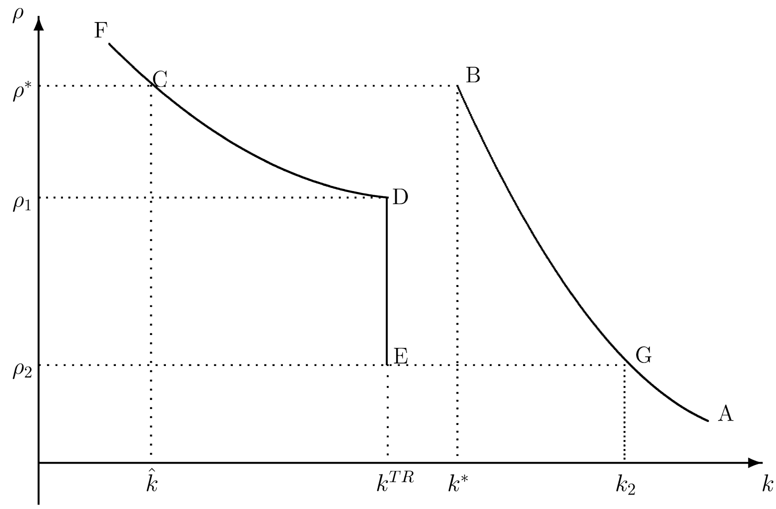

FIG. 3. The Relationship Between Steady-state Inflation and the Capital Stock when $k^* > k^{TR}$. In the steady state, the gross rate of growth of the nominal money supply, ρ, equals the gross inflation factor. Suppose the economy starts at point A. As long as ρ rises, the economy moves north-west. Along the trajectory A–B the efficient technology is used, but the capital stock k falls. If the inflation rate exceeds ρ*, the financial center shuts down, and the agents switch to the primitive technology. The economy 'jumps' from B to C. Along the trajectory D–F, there is a negative relationship between k and ρ. If ρ falls below ρ_1, the economy is stuck in the dollarization trap, where k is pinned down by the return on dollars, $k = k^{TR}$. When the inflation rate falls to ρ_2, the financial center resumes operations, and the capital stock 'jumps' from k^{TR} to k_2.

"jumps" from point B to point C in Figure 3. Along the trajectory D–F, there is still a negative relationship between the inflation rate and the capital stock. Therefore, a disinflation raises the capital stock above $\hat{k}$, the level of capital stock at C. However, if the inflation rate falls below ρ_1, the investment and output recovery halts. The economy is stuck in a trap, where the level of the capital stock is completely pinned down by the return on dollars. Disinflation translates not into a larger capital investment, but into larger dollar holdings. Hence our model is consistent with the empirical evidence that falling inflation sometimes coexists with a *rising* dollarization. Only when the inflation rate falls to a sufficiently low level, ρ_2, does another bifurcation take place, the financial center resumes operations, and the capital stock "jumps" from k^{TR} to k_2.

It is important to note that because of the multiplicity and stability of equilibria, the level of the capital stock (and output) during the initial period of disinflation is lower than the comparable level during rising inflation (for the same inflation rate). Moreover, as the disinflation progresses (as the economy moves from D to E), the gap rises and reaches its maximum when the inflation rate falls to ρ_2.

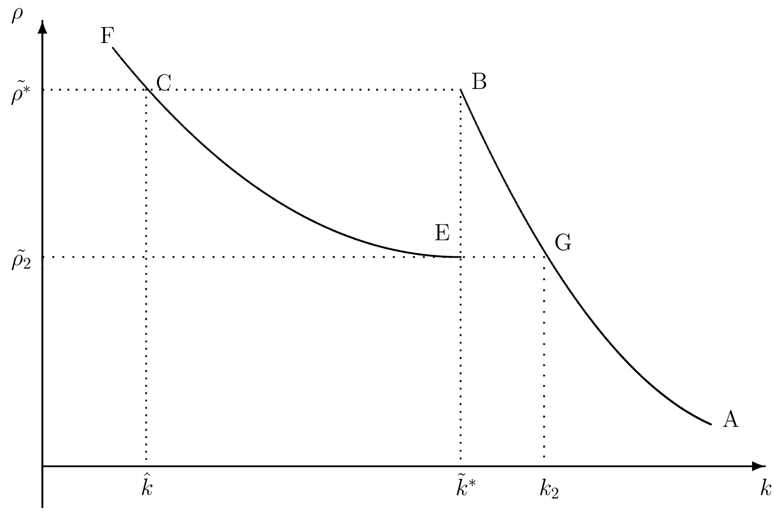

FIG. 4. The Relationship Between Steady-state Inflation and the Capital Stock when $k^* \leq k^{TR}$. If $k^* < k^{TR}$, recall from Lemma 1 that $\hat{k}$, while if $k^* = k^{TR}$, then $k^* < \tilde{k}^* < k^{TR}$. Suppose the economy starts at point A. So long as ρ is rising, the economy moves north-west. Along this trajectory the efficient technology is used, but the capital stock k is falling. If the inflation rate exceeds $k^{TR} = k^* = \tilde{k}^*$, the financial center shuts down, and the agents switch to the primitive technology. The economy 'jumps' from B to C. Along the trajectory E–F, there is a negative relationship between k and ρ. When the inflation rate falls to $\tilde{\rho}^*$, the financial center resumes operations, and the capital stock 'jumps' from $\tilde{\rho}_2$ to k_2. Dollars are never used, and the economy never gets stuck in the dollarization trap.

5.2. Dollarization Traps and Financial Development

What factors affect the likelihood of a dollarization trap? The trap arises only if $k^* > k^{TR}$, therefore anything that serves to reduce the value of k^* relative to k^{TR} makes the trap less likely, and indeed if k^* falls below k^{TR} the trap is impossible. It is straightforward to show that a reduction in the per capita intermediation cost ϕ lowers k^*, but it does not affect k^{TR}.

In Figure 1, a reduction in ϕ shifts the expenditure line of the financial center, $k + \phi$, downward and hence the point of intersection of this line with the revenue curve, $A\alpha k^{\alpha}$, shifts to the left. As k^* falls, the condition $k^* > k^{TR}$ is less likely to be satisfied. In other words, the likelihood of a dollarization trap is reduced. This finding is very intuitive. Intermediation costs, ϕ are inversely related to the level of financial development. Therefore, a more developed financial system makes a dollarization trap less likely. As $\phi \to 0$, we have $k^* \to 0$ as well, and the dollarization trap becomes impossible.

6. CONCLUSION

This paper studies the link between inflation, the demand for foreign currency, or "dollars," as a store of value, and capital accumulation. Our principal aim in the paper is to identify circumstances which can explain key empirical facts in dollarized countries. These empirical facts are that inflation is a main cause of dollarization, that dollarization coincides with adverse real economic performance, and that both the level of dollarization and the performance of the real economy may be very slow to reverse following a stabilization of inflation.

The key assumption in the paper is that the efficiency of the production technology depends positively on the level of capital accumulation. This assumption appears in slightly different form in various endogenous growth models. In our model, this assumption is critical for explaining dollarization hysteresis. The reason is that, to be consistent with the stylized facts in dollarized economies, a sufficiently high inflation rate must reduce the marginal product of capital, decrease capital accumulation, and induce a higher demand for dollars. This is impossible if there is a single neoclassical production technology.

There are three main implications of the model. First, there exist steady states with a relatively high capital stock, low dollarization, and low inflation, as well as steady states with a relatively lower capital stock, higher inflation, and substantial dollarization. Second, for a range of intermediate inflation rates, a steady-state equilibrium in which the efficient technology (and no dollars) are used coexists with a steady-state equilibrium in which the inefficient technology and dollars are used. Third, in the equilibrium where the inefficient technology and dollars are used, the link between the rate of inflation and capital accumulation is severed. This implies that hysteresis in dollarization ratios, capital accumulation, and output is a central prediction of the model. It is possible for economies to become stuck—for a range of inflation rates—in low output, technology induced "development traps," where the net marginal product of capital is the same as the return from holding dollars. The only way to exit from such an equilibrium is to reduce inflation below a threshold level.

APPENDIX

Proof of Lemma 1.

It is easy to verify that $\Pi^d > \Pi$ if and only if $k < k^{TR}$, $\Pi^d < \Pi$ if and only if $k > k^{TR}$, and $\Pi^d = \Pi$ if and only if $k = k^{TR}$.

First, consider the case when both k^* and $\tilde{k}^*$ are smaller than k^{TR}. We know that $\Pi^d > \Pi$ for $k < k^{TR}$. Therefore, $\Pi^d(\tilde{k}^*) > \Pi(\tilde{k}^*) = 0$. Given that $\Pi^d(k)$ is increasing in k, the root of the equation $\Pi^d(k) = 0$, defined as k^*, is smaller than $\tilde{k}^*$. Hence $k^* < \tilde{k}^* < k^{TR}$.

Second, consider the case when both k^* and $\tilde{k}^*$ are greater than k^{TR}. We know that $\Pi > \Pi^d$ for $k <$ k^{TR}. Therefore, $\Pi^d(\tilde{k}^*) < \Pi(\tilde{k}^*) = 0$. Given that $\Pi^d(k)$ is increasing in k, the root of the equation $\Pi^d(k) = 0$, defined as k^*, is greater than $\tilde{k}^*$. Hence $k^{TR} < \tilde{k}^* < k^*$.

Third, we show the impossibility of the case $k^* < k^{TR} < \tilde{k}^*$. If this inequality were true, then $\Pi^d(\tilde{k}^*) < 0$ (because $\Pi^d(k) < \Pi(k)$ for $k > k^{TR}$ and $\tilde{k}^*$ is defined as the root of $\Pi(k) = 0$). Therefore, $k^* > \tilde{k}^*$. This is a contradiction. The impossibility of the case $\tilde{k}^* < k^{TR} < k^*$ can be shown in a similar way.

Fourth, we show that if $\tilde{k}^* = k^*$, then $\tilde{k}^* = k^* = k^{TR}$. If $k = \tilde{k}^* = k^*$, then $\Pi(k) = \Pi^d(k) = 0$. Solving $\Pi(k) = \Pi^d(k)$ for k yields $k = k^{TR}$.

Finally, we show that if k^{TR} equals either k^*, or $\tilde{k}^*$, then it equals the other critical value as well. Suppose that $k^{TR} = k^*$ (the case when $k^{TR} = \tilde{k}^*$ can be proved in a similar way). Then, $\Pi^d(k^{TR}) = 0$. However, we already know that for $k = k^{TR}$, $\Pi^d(k) = \Pi(k)$. Hence, $\Pi(k^{TR}) = 0$. Therefore, k^{TR} is the root of $\Pi(k) = 0$, and $k^{TR} = \tilde{k}^*$. Q.E.D.

PROOF OF LEMMA 2.

In the steady state $k = \psi k^{\alpha}$. This equation has two roots: $k = 0$ and $k = \psi^{1/(1-\alpha)} > 0$.

To prove stability, we can show that $\partial k_{t+1}/\partial k_t$ evaluated at the steady state is smaller than unity by absolute value.

$$\frac{\partial k_{t+1}}{\partial k_t} = \psi\alpha k^{\alpha-1} = \psi\alpha(\psi^{1/(1-\alpha)})^{\alpha-1} = \alpha < 1 \ .$$

Q.E.D.

PROOF OF PROPOSITION 2.

$$\frac{\partial k}{\partial \rho} = \frac{\partial k}{\partial \psi}\frac{\partial \psi}{\partial \rho};$$

$$\frac{\partial k}{\partial \psi} = \frac{1}{1-\alpha}\psi^{1/(1-\alpha)-1} > 0$$

$$\begin{aligned}\frac{\partial \psi}{\partial \rho} &= A(1-\alpha)\frac{\tau_2[\rho - (\rho-1)\tau_1] - (1-\tau_1)(1-\tau_2+\tau_2\rho)}{[\rho-(\rho-1)\tau_1]^2} \\ &= -A(1-\alpha)\frac{1-\tau_1-\tau_2}{[\rho-(\rho-1)\tau_1]^2} < 0 \ .\end{aligned}$$

Therefore,

$$\frac{\partial k}{\partial \rho} < 0.$$

Q.E.D.

Proofs of Lemma 3 and Proposition 3 mirror the proofs of Lemma 2 and Proposition 2 and are omitted.

Proof of Lemma 4.

Arbitrage between productive capital and dollars ensures that the capital stock per worker equals k^{TR} not only in the steady state, but also in a neighborhood of the steady state. By Equations (24) and (27), the wage rate and per capita consumption are also equal to their steady-state values. Equations (25) and (26) ensure the same for the lump-sum transfers of the government, S_t, and the money holdings, m_t, as long as the wage rate is at its steady-state level. Equation (28) guarantees the equality of the dollar holdings to their steady-state value. Finally, Equation (29) ensures that the gross inflation factor equals the gross growth rate of money supply, as long as the dollar holdings, money holdings, and lump-sum money transfers are at their respective steady-state levels. Q.E.D.

Proof of Proposition 4.

After substituting Equation (24) for w_t, Equation (25) for S_t, and inserting Equation (26) into Equation (29), the steady-state version of Equation (29) after some manipulations becomes:

$$d = A\gamma(1 - \alpha)\frac{1 - \tau_2 + \tau_2\rho}{\rho - (\rho - 1)\tau_1}(k^{TR})^{\alpha} - k^{TR}.$$

Differentiating with respect to ρ yields:

$$\frac{\partial d}{\partial \rho} = A(1 - \alpha)\frac{\tau_2[\rho - (\rho - 1)\tau_1] - (1 - \tau_1)(1 - \tau_2 + \tau_2\rho)}{[\rho - (\rho - 1)\tau_1]^2}(k^{TR})^{\alpha}$$

$$= - A(1 - \alpha)\frac{1 - \tau_1 - \tau_2}{[\rho - (\rho - 1)\tau_1]^2}(k^{TR})^{\alpha} < 0.$$

Q.E.D.

Proof of Lemma 5.

Part 1. Proof that $\rho_1 < \rho^*$.

We will prove it by contradiction. Let's assume that $\rho_1 \geq \rho^*$.

Using the notation $\psi(\rho) \equiv A(1 - \alpha)\frac{1 - \tau_2 + \tau_2\rho}{\rho - (\rho - 1)\tau_1}$, the steady-state version of Equation (29) can be written for $\rho = \rho_1$,

$$k^{TR} = \psi(\rho_1)\gamma(k^{TR})^{\alpha}$$

or, taking into account Equation (11),

$$\alpha \mathrm{A} = \psi(\rho_1) \; . \tag{32}$$

On the other hand, the steady-state version of Equation (12) for $\rho = \rho^*$, can be written as:

$$(k^*)^{1-\alpha} = \psi(\rho^*).$$

Hence,

$$(k^*)^{\alpha-1} = \frac{1}{\psi(\rho^*)},$$

or,

$$A\alpha(k^*)^{\alpha-1} = \frac{A\alpha}{\psi(\rho^*)} = \frac{\psi(\rho_1)}{\psi(\rho^*)}.$$

In the proof of Proposition 2 we proved that

$$\frac{\partial\psi}{\partial\rho} < 0\,.$$

Hence the assumption $\rho_1 > \rho^*$ implies that

$$\frac{\psi(\rho_1)}{\psi(\rho^*)} < 1\,.$$

Therefore,

$$A\alpha(k^*)^{\alpha-1} < 1. \tag{33}$$

This result contradicts the assumption of dynamic efficiency of the efficient technology.
Q.E.D.

Part 2. Proof that $\rho_1 > 1$.
By assumption $\alpha < 0.5$. Therefore, $1 - \alpha > \alpha$. Hence,

$$w^{TR} = (1-\alpha)A\gamma(k^{TR})^{\alpha} > \alpha A\gamma(k^{TR})^{\alpha} = k^{TR}\alpha A\gamma(k^{TR})^{\alpha-1} = k^{TR}\,.$$

Hence,

$$\frac{W^{TR}}{k^{TR}} > 1$$

and

$$\rho_1 = \frac{w^{TR}(1-\tau_2) - \tau_1 k^{TR}}{k^{TR} - \tau_1 k^{TR} - \tau_2 w^{TR}} > 1\,.$$

Q.E.D.

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