Heterogeneous Agent Modeling: Experimental Evidence*

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Abstract

We report on experimental evidence rationalizing the use of heterogeneous agent models. We provide compelling evidence that subjects in laboratory experiments often behave in ways that depart from the rational choice ideal. Further, these subjects' heuristic approaches often differ from one another in distinct, classifiable ways. It follows that models of heterogeneous, boundedly rational agents can often deliver predictions that are a better fit to the experimental data at both the micro and the macro levels of analysis than can rational-choice, single-actor models. Our focus in this chapter is on experimental studies developed to address questions in macroeconomics and finance.

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1 Introduction

The world consists of heterogeneous agents who differ from one another in numerous respects. Modeling such heterogeneity presents the economics profession with a number of challenges. For instance, which dimensions of heterogeneity are the most empirically relevant? What range of heterogeneity should be allowed? Do agents simply differ in their preferences or do they also depart in various predictable degrees from the rational choice framework? In this chapter we show how experimental evidence on agent-type heterogeneity can be used to answer these questions and how experimental evidence has been used to construct parsimonious yet rich heterogeneous agent models. We further demonstrate how such experimentally validated heterogeneous agent models can explain a number of important economic phenomena that would be difficult to explain using the standard homogeneous, rational actor approach.

As a motivating example, consider the pricing of assets subject to uncertain dividend realizations. Experimental tests, beginning with Smith et al. (1988), have consistently found that individual subjects over-price such assets relative to the asset's rational expectations fundamental value (see Palan (2013) for a survey). On the other hand, once a group of subjects has experienced a price "bubble," they are less prone to exhibit mis-pricing in repeated interactions. Thus, inexperience and experience provide one dimension of heterogeneity that can matter for the incidence of price bubbles, as documented by Dufwenberg et al. (2005). An alternative dimension on which to consider heterogeneous agents is in terms of the cognitive abilities of the subjects themselves. Bosch-Rosa, Meissner and Bosch-Domenech (2018) and Hanaki et al. (2017) report that the mixture of cognitive abilities in a population of agents, as measured by simple tests, matters for the incidence of asset price bubbles. In particular, they find that bubbles are less likely among more cognitively sophisticated subjects and more likely among groups with mixed cognitive abilities.

The development of heterogeneous agent models has come about as the result of the failure of homogeneous, representative agent models to adequately capture micro-level properties of macroeconomic and financial time series data. A further reason is development of advanced computing power that enabled the use of computational algorithms to solve the more complicated economic models with heterogeneous agents beginning in the second half of the 1990s, e.g., with Campbell (1998), Den Haan (1996) and Krusell and Smith (1998). These researchers and others in the large literature on heterogeneous agent models that has blossomed since (see surveys, e.g., by Heathcoate et al. (2009) Krueger et al. (2016), and Ragot (2018)) have sought to match *distributional* data on wealth, employment, wage earnings, and educational status, among other factors, using models where agents are allowed to differ in these dimensions and others and where markets are incomplete. At the same time, data on certain features of these heterogeneous-agent models, for instance data on individual's cognitive abilities, or their expectations about future variables, are not generally available, and so modelers have often used the short-cut assumption that agents are unboundedly rational and possess rational expectations. Nevertheless, agents *can* differ in the boundedness of their rationality and in their forecast specifications and these differences are often important, micro-level building blocks for heterogeneous-agent representations, e.g., in the literature on learning in macroeconomics (Sargent (1993), Brock and Hommes (1997, 1998), Grandmont (1998) and Evans and Honkaphoja (2001)). Perhaps as a consequence, some researchers have began to conduct controlled experiments addressing expectation formation and the extent of bounded rationality in the laboratory. A further use of laboratory experiments has been to address questions of equilibrium selection in settings, e.g., bank runs, where there can be multiple rational expectations equilibria, and where theory is silent about the conditions under which a particular equilibrium is selected.

The use of controlled laboratory experimental evidence to validate as well as to provide evidence for heterogeneous agent models is a relatively recent methodology, but it has also spawned the development of the literature in agent-based modeling (see Duffy (2006) for a survey). Once a laboratory experiment has been programmed, it is a simple matter to automate the responses of the human subjects with robot players. Some laboratory experiments involve interactions between human subjects and robot players in an effort to reduce noise, e.g., Hommes et al. (2005), discussed below. Gode and Sunder (1993) took the logical step of completely replacing the human subjects with robot players in their exploration of behavior in continuous double oral auctions, and this work was influential in the blossoming of the agent-based approach to social science research. Many agent-based modelers use experimental data to calibrate or validate their heterogeneous agents, but most do not, as they find the constraints of laboratory environments too confining. In this chapter we discuss the development of heterogeneous agent models that *were* conditioned on experimental data, or that were used to validate experimental findings. In some instances (e.g., Arifovic et al. (2017) discussed in section 4.2.2) heterogeneous agent models are used to help design experiments.

The experimental evidence we discuss comes primarily from studies involving the "convenience sample" of university student subjects. This population has been dismissed by some with the dubious claim that students "are not real people" (List (2011, p. 4)). Field data studies involving "real actors" might seem to be more empirically valid, but these studies often involve a considerable loss of control relative to laboratory studies that can make causal inference more difficult, if not impossible (Falk and Heckman (2009)). In defense of the use of undergraduate subjects, we note that economic models are often so general and parsimonious that there do not exist any real-world "professionals" who might be expected to outperform student subjects. Indeed, Frechette (2015) reports that among 13 experimental studies that have compared the performance of student subjects with professionals, the professionals do no better than the student subjects in 9 of these studies. Among the other 4 studies, professionals are closer than students to theoretical predictions in just 1 study, while students are closer than professionals in the other 3 studies! Thus, it seems that not much is lost, and greater control is gained by examining data from laboratory experiments with student subjects. Indeed, in several instances there are striking parallels between the heterogeneity observed in the laboratory and that found in studies using non-experimental field data.

This survey adds to, builds upon and extends several prior reviews of the experimental evidence for heterogeneity and the use of such data as a rationale for heterogeneous agent modeling. Duffy (2006) provides a survey detailing the complementarities between human subject experiments and agent-based models, which necessarily involve heterogeneous, interacting agents possessing various degrees of rationality. Many heterogeneous agent models are derived in the context of macroeconomic or financial market settings, which we also focus on here.[1] Prior surveys of experiments related to macroeconomics and finance can be found in Assenza et al. (2014), Duffy (2006, 2010, 2016) and Hommes (2011, 2013). In addition to providing an update to these surveys, we emphasize the role of heterogeneity in agent or subject types for better understanding aggregate phenomena.

The rest of the chapter is organized as follows. Section 2 discusses heterogeneity and bounded rationality in both optimization and forecasting tasks. This section discusses introduces a distinction between learning to optimize and learning to forecast experiments. Section 3 discusses the consequences of heterogeneous types for monetary policy. Finally, section 4 discusses evidence for heterogeneity in equilibrium selection among models of interest to macro economists, including bank run models and network economy models admitting multiple types of payments.

2 Heterogeneity and Bounded Rationality in Decision Making

In this section we discuss evidence that experimental subjects' choices depart in several different ways from the homogeneous rational actor model. Specifically we focus on the dynamics of group decision making, individual intertemporal optimization and expectation formation. We wish to emphasize at the outset that we view the homogeneous rational actor model as an important benchmark for economic analysis. Indeed, without this benchmark, we would not be able to characterize the many ways in which agents exhibit heterogeneous

[1]For heterogeneous agent models derived from games or microeconomic settings, see Mauersberger and Nagel (2018) appearing in this volume.

behavior.

2.1 Group Decisions on Provision of a Public Good

One of the simplest group decision-making experiments that reliably reveals heterogeneity in decision-making among members of a group is the linear public goods game, as first studied by Isaac and Walker (1988). This is a N-player game and while it is typically played in the laboratory with small numbers of subjects, in principle, it can be played with any population of size N, and so we include it here in our discussion of macroeconomic experiments.[2] In this game, the N players repeatedly decide whether or not to contribute some portion of their (typically common) endowment, w, to a public account with the remainder going to their private account. Contributions to the public account yield a public good benefit to all N players. Denote the amount that agent i contributes to the public account by c_i. In the public good game, player i's payoff is given by:

$$\pi_i = w - c_i + M \sum_{j=1}^{N} c_j \tag{1}$$

The key assumption made in this game is that $1/N < M < 1$. While it *would* be efficient for all to set $c_i = w$, in which case $\pi_i = MNw > w$ by the assumption that $M > 1/N$, since the marginal per capita return (MPCR) to contributing to the public good, $M < 1$, it is in fact a dominant strategy for each *individual* agent to contribute nothing to the public good, i.e., to set $c_i = 0$. Thus, this public good game is essentially an N-player prisoner's dilemma game.

By contrast with the rational choice prediction of no contribution, subjects in experimental public good games generally *do* contribute positive amounts to the public good, though these amounts decline with repetition of the game. Figure 1 shows a typical pattern of giving to the public account (public good) over 12 periods when $N = 4$ and $M = 0.4$. Subjects were incentivized to maximize the payoff function π_i in every period of the experiment. As the figure shows, on average subjects contribute about 1/2 of their endowment to the public good in the first period, but this contribution declines to nearly zero by the 12th and final period.

The pattern of over-contribution and gradual decay in contributions to public goods can be accounted for by *heterogeneity* in the choices of subject participants. Evidence for such

[2] Indeed, Isaac, Walker and Williams (1992) report experiments with large group sizes of $N = 40$ and $N = 100$. More generally, we regard games with $N \leq 2$ to be the domain of game theory or decision theory (and the subject of the chapter by Mauersberger and Nagel (2018), while games with $N > 3$ present aggregation issues and games with large N tend to approximate the competitive market conditions that are often assumed to hold in macroeconomic and finance settings.

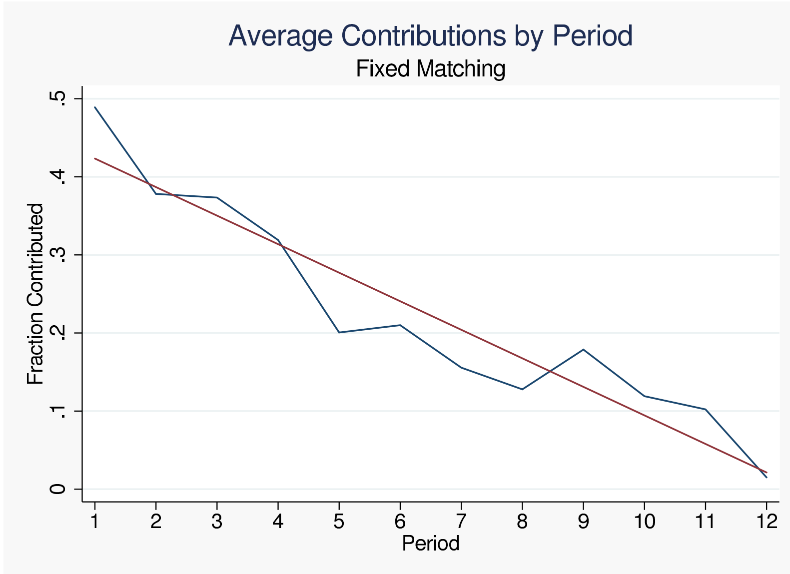

Figure 1: Fraction of endowment contributed to the public good in 4-player public good games with $M = 0.4$ under fixed matchings. Averages from eight 4-player groups. Source: Duffy and Lafky (2016).

heterogeneity has been produced by Fischbacher et al. (2001) using a simple "strategy-method" design.[3] In this design, subjects are presented with the payoff function of the public good game with $N = 4$, $w = 20$ and $M = 0.4$ and were asked to make two types of decisions. The first decision was to indicate how much they would contribute. The second decision involved completion of a contribution schedule showing how much each subject would contribute conditional on the average contribution of the other 3 subjects in their group. Specifically, for each (integer) average contribution amount the other 3 subjects, 0,1,2,...20 of their 20 token endowment, each subject supplied 21 conditional responses indicating how much they would conditionally contribute. Finally, subjects were matched in groups of four. One, randomly chosen subject's decision was made according to his contribution table while the other three subjects' choices were made according to their unconditional contribution decision; since the determination of which decision was chosen was random, subjects were incentivized to seriously consider both decisions. Then subjects were paid according to the outcome of their group's total contribution. The results of this experiment are nicely summarized in Figure 2 which reports on the average contribution schedule amounts of selected classes of subjects. The contribution schedules of 22 of the 44 subjects (i.e. 50%) are consistent with reciprocity or conditional cooperation as indicated by the close alignment of the conditional cooperation amounts with the 45 degree line. Thirteen subjects (i.e. about 30%)

[3] In a strategy method experiment, one elicits each subject's complete contingent strategy as opposed to simply asking subjects to make a choice. The strategy is then typically played on the subjects' behalf, thus incentivizing subjects to truthfully reveal their strategy.

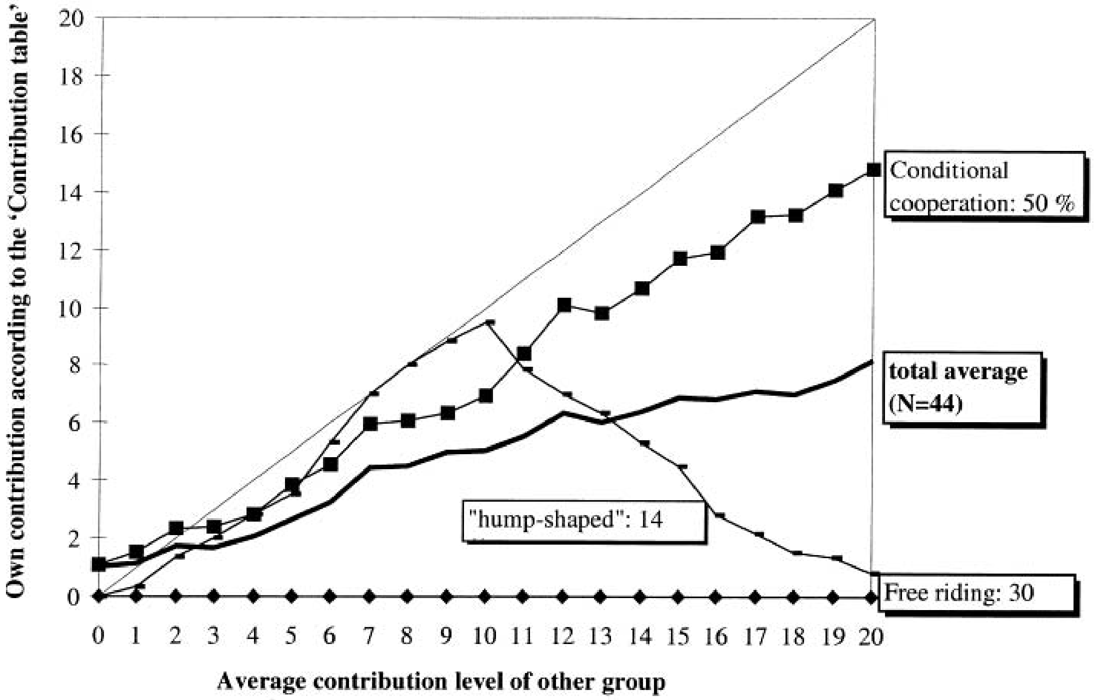

Figure 2: Type heterogeneity in a one-shot public good game. Source: Fischbacher et al. 2001

are classified as purely selfish (perfectly rational) as they submitted a contribution schedule with 0 for all 21 entries. Six subjects (or 14%) are hybrids between conditional cooperators and selfish types exhibiting the hump-shaped pattern; they are conditionally cooperative up to point (around 1/2 of ω) and then are more free-riding. The remaining 6 percent are not easily classifiable.

Importantly, this heterogeneity in contribution decisions can be used to account for the pattern of initial over-contribution to the public good and the subsequent decay in such contributions over time. For instance, Ambrus and Pathak (2011) show that the presence of two player types, selfish and reciprocating, conditional cooperators alone suffices to generate this pattern. The initial positive contribution levels are all due to the conditional cooperators who start out contributing positive amounts. By contrast, the selfish types contribute nothing. At the end of each round, individual payoffs, π_i, are realized and the group payoff component, $M\sum_{i=1}^{N} c_i$ can be inferred (or more typically, it is simply revealed). The conditional cooperators learn that the group average is less than would obtain if all others were contributing levels similar to their own contribution levels and so these types conditionally adjust their giving downward in the next round, given this new information. This downward adjustment explains the decline in contributions over time. Note that this pattern of behavior requires some measure of both of these two distinct, heterogeneous player types, a theme that we will see again and again in this chapter.

In addition to the over-contribution and decay pattern in public good experiments, there are other features of this environment pointing to heterogeneity in subject behavior. For instance, while average contributions in public goods games begin at around 50% of the total endowment, typically more than half of subjects begin by equally splitting their endowment between the private and public accounts. There is also considerable variation in individual contributions as a percentage of endowment. Individual contributions show no consistent monotonic pattern over time. Some increase, some decrease, and some have a zig-zag pattern. Thus, subject behavior in this environment is best described as being 'persistently' heterogeneous. Further, increases in the MPCR lead to an increase in the average rate of contributions, especially in small group sizes. Additionally, increases in the size of the group also lead to an increase in the average rate of contributions. This is particularly true in later repetitions and for small values of the MPCR. Finally, there is a "restart effect"; that is, if after 10 periods the subjects are told the game is restarting, then contributions in period 11 increase relative to those in period 10.

Standard theory provides little help in understanding this degree of heterogeneity in choices and behavior over time. But this behavior can be accounted for by individual evolutionary learning model (IEL) proposed by Arifovic and Ledyard (2012). The IEL model is based on an evolutionary process which is individual, and not social. In IEL, each agent is assumed to carry a collection of possible strategies in their memory. These remembered strategies are continually evaluated and the better ones are used with higher probability. IEL is particularly well-suited to repeated games with large strategy spaces such as subsets of the real line. We discuss the details of IEL implementation and other applications where it is used for validation of experimental data in section 5. Here, we focus on the results related to public good games.

Arifovic and Ledyard (2012), extend the payoff function (equation 1) used in the public goods experiments to include altruism and envy considerations and assume that the extent of these other-regarding preferences vary across agents. In other words, some agents can be completely selfish, some more or less altruistic as well as more or less envious. This corresponds to the interpretation that human subjects walk into the lab with given type of other regarding preference.

In order to incorporate other regarding preferences into the IEL learning, Arifovic and Ledyard used the following utility (payoff function):

$$u^i(c) = \pi^i(c) + \beta^i \bar{\pi}(c) - \gamma^i \max\{0, \bar{\pi}(c) - \pi^i(c)\}. \quad (2)$$

where $\pi^i(c)$ is the standard public good payoff as given in equation 1, $\bar{\pi}(c)$ is the average payoff of all N players (revealed at the end of each round), $\beta^i \geq 0$ indicates how altruistic player i is[4], and $\gamma^i \geq 0$ defines how 'spiteful' or 'envious' agent i is. That is, i loses utility when i's payoff is below the average payoff in this group. Arifovic and Ledyard model the heterogeneity by assuming that each subject i comes to the lab endowed with a value of (β^i, γ^i) which is distributed, independently and identically, in the population according to a distribution $F(\beta, \gamma)$.

With linear other-regarding preferences (ORP), for given (N,M) and heterogeneity across (β, γ), there are only three types of Nash Equilibrium strategies: free riding or selfishness $(c^i = 0)$, fully contributing or altruism $(c^i = w)$, and contributing an amount equal to the average or fair-minded behavior $(c^i = \bar{c} = (\sum_i c^i)/N)$. Thus, the introduction of ORP adds the possibility of different equilibrium levels of contributions. However, the theory is still static, i.e., it predicts constant levels of contribution, and thus, cannot account for the observed patterns of PG experimental behavior.

Arifovic and Ledyard further assume a specific distribution of types, $F(\cdot)$. Some agents, P% of the population, are purely "selfish"; that is, they have the type (0,0). The rest, (1-P)% of the population, have other-regarding preferences where the (β^i, γ^i) are distributed identically, independently, and uniformly on $[0, B] \times [0, G]$.

They use Isaac and Walker data (IW) to find the 'good' values for (P, B, G).[5] For each IW treatment, (M, N), and for each parameter triple, (P, B, G), they conducted 40 trials. Each trial involves a draw of a new type from $F(\cdot)$ for each agent. Those types were selected randomly as follows. Each agent became selfish with probability P. If her type turned out to be selfish, then her utility was based only on her own payoff. That is, $\beta^i = \gamma^i = 0$. If the agent did not become selfish, then a set of values of β^i and γ^i was drawn uniformly and independently from the ranges $[0, B]$, and $[0, G]$ respectively. Arifovic and Ledyard conducted a grid search over the values of P, B, and G to minimize the NMSE between the simulation and experimental data. The data they used was average contribution over all ten periods and average contribution over the last three periods.

Figure 3 shows the simulated and experimental data for $N = 4$ and for two values of M, a low of 0.3 and a high of 0.75 and Figure 4 shows the same for $N = 10$.

Further, Arifovic and Ledyard check how their model performs, using the parameter values estimated from IW, when transferred to a different experimental data set. Here, we illustrate the exercise by presenting data related to Andreoni (1995). He used $(N, M) = (5, 0.5)$ and conducted two treatments, *Regular* and *Rank*. [6] In Figure 5, we present the pattern of average contributions for Andreoni's data and for the IEL simulations.

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[4]Here altruism refers to a preference for higher values of the average payoff to all agents.

[5]In their approach to learning in general, AL's methodology is to find parameters that provide a 'loose' fit for one set of data, and then use these parameters to generate simulation data for other experiments.

[6]In the *Regular* treatment the subjects' payoffs were the same as in IW. In the *Rank* treatment they were paid according to their rank.

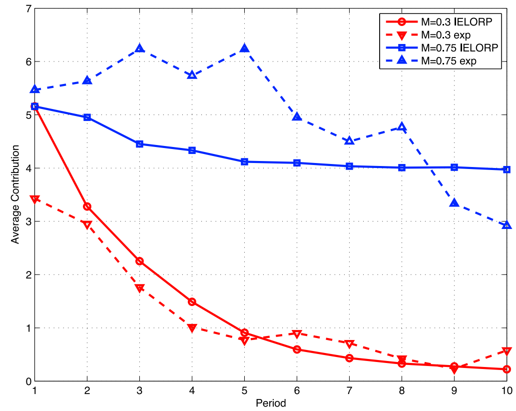


Figure 3: IEL and experimental data for $N = 4$

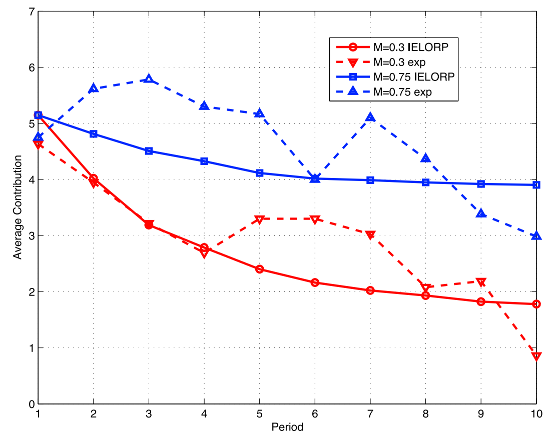


Figure 4: IEL and experimental data for $N = 10$

Finally, Arifovic and Ledyard were also able to 'generate' the *restart* effect, which has been a huge puzzle for both theory and simulation models. Andreoni (1988) was the first to present the *restart* effect results (later replicated by Croson (1996)). The key finding was that average contributions increased after the restart but then began to decline again. Figure 6. Arifovic and Ledyard demonstrate the robustness of the IELORP to a wide range of changes in its parameter values.[7] In general, the IEL model is robust to changes in its parameter values across different applications.

2.2 Intertemporal Optimization

A workhorse framework for intertemporal optimization is the lifecycle model of consumption and savings due to Modigliani and Brumberg (1954) and Ando and Modigliani (1963). An

[7]Robustness in terms of changes in the parameter values of a learning/adaptive model is not common feature of the behavioral models. Usually, they are parameterized to fit a specific experiment, and, usually, are not robust to slight parameter changes. However, this is not the case with IEL.

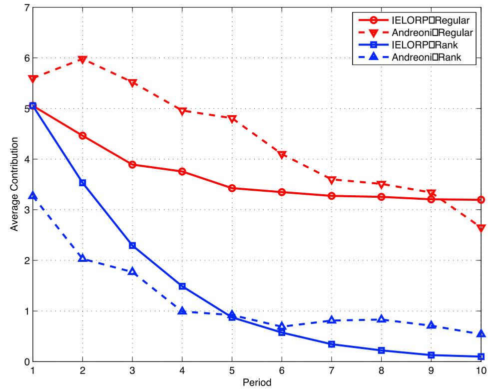


Figure 5: IEL and Andreoni's Regular and Rank treatment

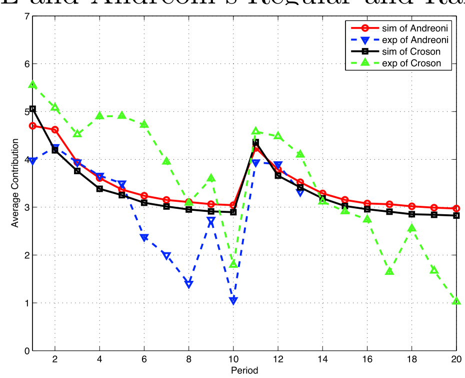


Figure 6: Restart Effect - IEL compared to Andreoni (1988) and Croson (1996)

experimental evaluation of this model was first performed by Johnson et al. (2001). In their experiment, subjects were asked to suppose that they had just turned 35 and would live for 40 more periods (years) before dying at age 75. They would get an annual salary of $25,000 until retirement at age 65, after which they would get 0. Individuals could borrow or save at a known, constant 4 percent interest rate and it was also known that there was no inflation, discounting, taxes or other expenses to be paid nor any uncertainty. Consumption choices were made once per year (period) and what was not consumed was automatically saved. In one treatment using this environment, subjects were asked to make 40 choices as to how much they would consume in every period (year), and they could condition these choices on information about their accumulated asset position (from savings). Subjects were paid a flat fee for their answers, as there was no objective right or wrong answer to these decisions. However, in a later treatment, the same subjects were asked to reconsider the exact same lifecycle problem and rank their *preferences* over five different financially feasible lifetime consumption profiles, each of which exhausted their resources in the final 75th period of life. The five consumption profile choices were:

1. $21,841 per year, every year, i.e., constant consumption.

Profile Number	1(0%)	2(2%)	3(4%)	4(-2%)	5(Step)
First Choice	.23	.31	.25	.13	.08
Second Choice	.23	.44	.15	.11	.07

Table 1: Frequency of Subjects Choosing Alternative Profiles

2. \$16,008 at age 35, growing by 2% per year thereafter.

3. \$11,240 at age 35, growing by 4% per year thereafter.

4. \$28,592 at age 35. falling by 2% per year thereafter.

5. \$23,420 from age 35 until age 65, then \$10,921 from 65 to 75.

The distribution of subjects' consumption profile choices is shown in Table 1

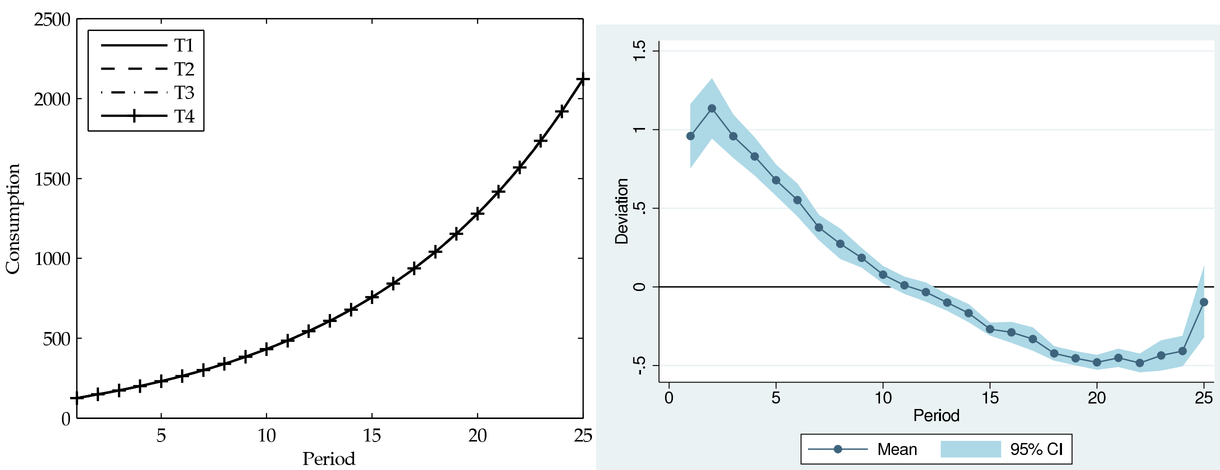
First, as is evident from Table 1, subjects clearly exhibit *heterogeneous* preferences with regard to lifecycle consumption profiles, with a majority of subjects favoring positive growth in their consumption profile over time. Second, Johnson et al. examined the relationship between subjects' consumption choices over 40 periods (from age 35-75) in the first treatment to their expressed first choice consumption profile in the later treatment. Specifically, they calculate the average annual absolute percentage deviation between each subject's most preferred consumption profile and his/her actual consumption choices. For those whose preferred profile was the constant consumption profile, the mean deviation is 15 percent; for those preferring the 2 percent, 4 percent, -2 percent or the step function profile, the mean deviations were 21, 25, 37 and 46 percent, respectively. Thus, not only are subjects heterogeneous in their preferences, they are also heterogeneous in the bounded rationality of their decision-making.

A criticism of the Johnson et al. experiment is that the payoffs subjects faced were hypothetical and their true preferences were unknown. Subsequent experiments testing intertemporal consumption-savings policies have sought to remedy this problem by inducing subjects to hold specific (concave) preferences over consumption so that the subjects can be rewarded on the basis of how close their lifecycle consumption profile is to the theoretically optimal consumption path.[8] See for example, Carbone and Hey (2004), Carbone (2006), Ballinger et al. (2003, 2011), Carbone and Duffy (2014) and Meissner (2016). As in Johnson et al., there is no uncertainty about how long agents will live and the lifecycle income process is known, and possibly stochastic. These studies also report that in certain environments, subject's consumption choices deviate substantially from the unconditional optimal path or

[8]In these experiments, the utility function over consumption in period t, $u(c_t)$ converts a subject's consumption choice into money earnings using the induced mapping u, which is typically a concave function.

even the conditionally optimal path that conditions on a subject's current asset position (wealth). Carbone and Hey (2004) identify four player types, (i) those who understand the basic nature of the problem and behave in a near optimal-manner, ii) those who are pre-occupied with present consumption and discount the future heavily; (iii) those who seem to like to accumulate wealth and build up large asset positions and (iv) those who engage in consumption bingeing, building up stocks of wealth over cycles of around 4 or 5 periods and then consuming all of that wealth. Ballinger et al. (2001) report that cognitive abilities, as measured by performance in two non-verbal, visually oriented tests (the Beta III test and a working-memory span test) are correlated with performance in lifecycle consumption planning. Those with high cognitive abilities, perform better (closer to the optimal path) than those with lower cognitive abilities, controlling for demographic and other non-cognitive characteristics.

Even within the same incentivized lifecycle consumption-savings problem, subjects exhibit heterogeneity with respect to their departures from the optimal or conditionally optimal consumption path. For instance, Duffy and Li (2017) study a lifecycle consumption/savings problem with no uncertainty or discounting, where subjects have induced logarithmic preferences over consumption and face a known lifecycle income profile, and a known fixed interest rate on savings. The optimal consumption path in this environment is increasing over the lifecycle (due to the positive interest rate and lack of any discounting), as shown in panel a of Figure 7, but the behavior of subjects is at odds with this prediction; the mean percentage by which subjects deviate from the conditionally optimal consumption path (together with a 95% confidence interval) is shown in panel b of Figure 7.



(a) Optimal Consumption

(b) Percent Deviation from the Optimal Path

Figure 7: Optimal consumption path and percent deviation from optimal path in the experiment of Duffy and Li (2017)

As Figure 7(b) clearly reveals, on average, individuals consume more than the optimal amount in the first 17 periods (equivalent to 39 years in model time) and, as a consequence, they have less savings later in life so that in those later periods they under-consume relative to the optimal consumption path. Duffy and Li report that this aggregate, over-consumption followed by under-consumption pattern can be explained by heterogeneity in subject's life-cycle consumption and savings decisions. They report that nearly 50 percent of subjects can be regarded as hand-to-mouth consumers, consuming all (or nearly all of their income) in each period while the remaining subjects can be characterized as conditionally optimal consumers, who make calculation mistakes and who can be viewed as re-optimizing their consumption/savings plan at each new period of the lifecycle, conditional on current asset holdings. The presence of the two types explains both the over and under consumption phenomenon and its magnitude. When income is greater than the conditionally optimal path, as in the early periods of life, the presence of hand-to mouth consumers means that average consumption is greater than optimal. When income is less than the conditionally optimal amount in the later (retirement) periods of life, the presence of the hand-to-mouth consumers means that average consumption is below the conditionally optimal level. Interestingly, Campbell and Mankiw (1989) used a similar, two-type model with 50 percent hand-to-mouth consumers and 50 percent rational consumers to explain volatility in aggregate U.S. quarterly consumption data.

2.3 Expectation Formation

Heterogeneity in expectation formation is also well documented using laboratory experiments. In many models in macroeconomics and finance, expectations matter for optimizing choices, and the results of those choices in turn determine the realizations of the variables that agents were forecasting. This belief–outcome interaction can be complicated for subjects (not to mention theorists), and so typically experimentalists have asked subjects to choose actions with expectations implicit, or to form expectations only and to be paid on the basis of the accuracy of those expectations, see, e.g., Schmalensee (1976), Dywer et al. (1983), Smith Suchanek and Williams (1988) and Kelley and Friedman (2002).

For instance, in one early experiment, Hey (1994) asked subjects to forecast a random time series variable, X_t. The law of motion for X_t was autoregressive, i.e., $X_t = \mu + \rho X_{t-1} + \epsilon_t$, where μ, and ρ are fixed parameters and ϵ_t is a mean zero noise term with variance σ^2, but subjects were *not* aware of this data generating process. They could choose a prior history of $k \leq 50$ past values for X_t and after observing this history, they were asked to form forecasts. They were incentivized to form accurate forecasts in each of 20 periods, t, as the payoff function for each subject, i, in each period t, was a quadratic scoring rule of the form $\pi_i = \max\left[a - b(X^e_{i,t} - X_t)^2, 0\right]$, where $X^e_{i,t}$ denotes subject i's time t expectation of X_t; the actual value of X_t was revealed ex-post and then earnings were determined. Hey reports that few subjects, just 2 out of 48 (4%), formed the rational expectation, $EX = \frac{\mu}{1-\rho}$, while 2/3 (66%) of subjects could be characterized as adaptive learners. The remaining 30% were not characterizable using either rational or adaptive expectations!

Asset Pricing Experiments

Hey's experiment has an exogenous data generating process. To capture endogenous belief-outcome interaction, it is necessary for expectations to matter for the variables that subjects are forecasting. A simple framework in which this is the case is an asset pricing model studied experimentally by Hommes et al. (2005). In this model there is a 1 period, risk free bond paying a known, fixed return of r per period and long-lived risky asset (e.g., a stock) paying stochastic, i.i.d. dividends each period with a mean value of $\bar{d}$. The co-existence of these two types of assets requires, via arbitrage, that the price of the risky asset is given by:

$$p_t = \frac{1}{1+r}(p^e_{t+1} + \bar{d}). \tag{3}$$

where p^e_{t+1} is the time expectation of the risky asset at time $t+1$. Under rational expectations, $p^e_{t+1} = p_t = p^f \equiv \frac{\bar{d}}{r}$, so that the rational expectation equilibrium (REE) price path should be $p_t = p^f$. Hommes et al. (2005) study whether subjects can price the asset consistent with this rational expectations prediction in a laboratory experiment. However, they study a slightly altered version of this pricing equation wherein the price of the risky asset is generated by:

$$p_t = \frac{1}{1+r}\left((1-\eta_t)p^e_{t+1} + \eta_t p^f + \bar{d} + \epsilon_t\right),$$

where ϵ_t is a mean zero noise term and $\eta_t \in (0,1)$ is a time-varying weight assigned to fundamental traders' price forecasts, i.e., those who correctly price the asset according to its fundamental value, p^f. The remaining weight, $(1-\eta_t)$, is given to non-fundamental, adaptive learning agents. In the Hommes et al. (2005) experiment, the fundamental traders are robot players, and the weight assigned to their rational expectations price forecast, η_t, diminishes as the system converges. The human subjects, six per group, comprise the non-fundamental forecasters. In each period t, each human subject i forms a forecast of the price in period $t+1$, $p^e_{i,t}$, and each is paid according to the ex-post accuracy of their own forecast using the same quadratic scoring rule as in Hey' study. Differently from Hey, the experiment is a group-decision making task since p^e_{t+1} in the equation used to determine p_t is taken to be the average of the six human subjects' forecasts, i.e., $p^e_{t+1} = \frac{1}{6}\sum_{i=1}^{6} p^e_{i,t+1}$. Notice further that expectations now matter for actual price realizations so that there is belief-outcome interaction. Subjects in the experiment were asked to forecast a price for the asset in the interval $[0, 100]$ and were not told any details of the equation determining the realization of p_t, though they know r and $\bar{d}$ and, in principle, could compute $p^f = \frac{\bar{d}}{r}$. In contrast to Hey (1994), subjects in the Hommes et al. design have the incentive to coordinate their expectations over time, so heterogeneity should now only be transitory, and may disappear with experience; the presence of the robot fundamental traders helps in this regard. A further difference is that subjects in the Hommes et al. experiment make forecasts of the price in period $t+1$ in period t, but the price they forecast in period t is not revealed until period $t+2$, so, in effect, they are forecasting two periods ahead.

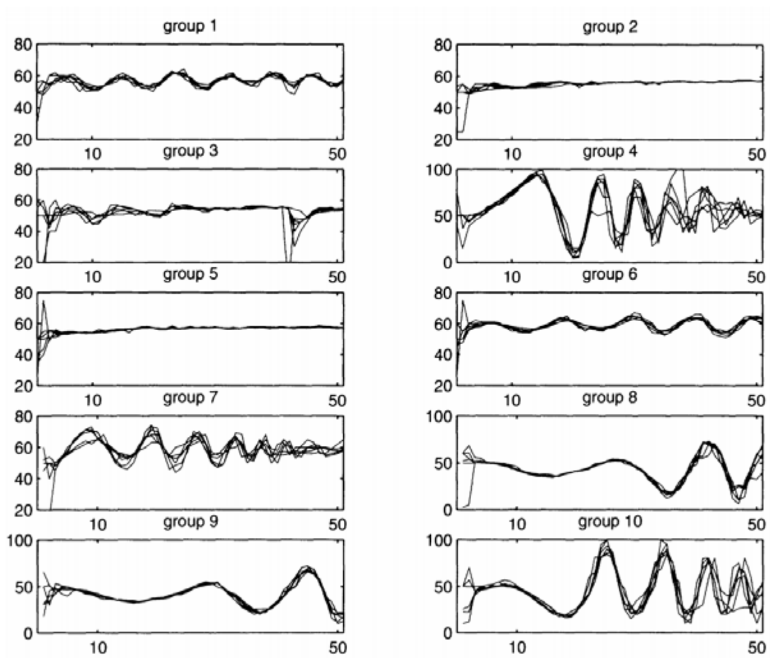


Figure 8: Individual price forecasts (p^e_{t+1}) over time, from Hommes et al. 2005

Figure 8 shows the time series of price forecasts over 50 periods from ten different groups of six forecasters. Several observations are evident. First, only a few groups (2,5) have converged to the REE price, equal to 60 in this experiment by the end of 50 periods. These groups were found to be comprised of individuals whose forecast rules were of an AR(1) nature, including naive best response, $p^e_{t+1} = p_{t-1}$ or past averaging $p^e_{t+1} = \frac{1}{t-1}\sum_{j=1}^{t-1} p_{t-j}$. Second, there is tremendous coordination on price forecasts *within* each group, a kind of group specific expectation norm. Third, many groups' expectations result in an oscillatory path for prices that sometimes appears to be convergent (groups 4,7,10) and sometimes not (groups 1,6,8,9). Both sets of groups are found to be employing an AR(2) expectation formation process of the form: $p^e_{t+1} = \alpha + \beta p_{t-1} + \gamma\,(p_{t-1} - p_{t-2})$. The estimated value of γ is found to be positive, indicating that if subjects see a positive (negative) trend in the past two prices, they expect prices to continue to increase (decrease). This trend-extrapolation behavior explains the oscillatory pattern for prices in these groups. Finally, some groups' expectation rule, e.g. group 3, are simply not well understood. Hommes et al. conclude that 75 percent of their subjects can be classified using linear adaptive rules that depart from the rational expectations equilibrium prediction.

Hommes et al. (2008) studied a slightly different version of this same experiment where $p^f = 60$ as before but where the robot players and the noise term were eliminated and the price forecast interval was enlarged by a factor of 10 to $[0, 1000]$. Their aim was to explore the possibility of rational bubbles, which can be derived as follows. Using the law of iterated expectations, we can expand the price equation as: $p_t = \sum_{i=1}^{n}(1+r)^{-i}\bar{d} + (1+r)^{-n}E_t(p_{t+n})$. Taking the limit as n goes to infinity, $p_t = \sum_{i=1}^{\infty}(1+r)^{-i}\bar{d} + \lim_{n\to\infty}(1+r)^{-n}E_t(p_{t+n})$. Assuming a limit exists, denote the last term by b_t, so that the rational expectations solution consists of a fundamental and a bubble term $p_t = p^f + b_t$. To be a rational bubble, the bubble term must grow at rate r. Hommes et al. did not find evidence for rational bubbles in this strict sense, but they did report that in 5 of their six experiments, prices periodically hit the upper bound of 1000 – more than 15 times fundamentals– before trending down again. They show that this pattern in again driven by positive feedback, trend-following expectation formation rules. Hüsler et al. (2013), using Hommes' data, showed that for the groups in which bubbles arose, the bubble growth rate was "super-exponential". In particular, the rate of change of prices is well approximated by an equation of the form $\log\left(\frac{p_t}{p_{t-1}}\right) = r + \gamma p_{t-1}$, where $\gamma > 0$ is the anchoring weight placed on the more recent price; the positive weight on the latter means that the prices grows at a rate greater than r (super-exponential). Hüsler et al. further show that alternative functional forms for the growth rate of prices (exponential growth or anchoring on lagged returns as opposed to lagged prices) do not perform as well in explaining the path of price bubbles.

The heterogeneity of expectation formation rules means that a single model of adaptive behavior will not suffice to explain the experimental data from these asset pricing experiments. Anufriev and Hommes (2012) therefore propose the use of a heuristic switching model, based on Brock and Hommes (1997) to explain the experimental data of Hommes et al. (2005 and 2008). In this simulation model, the price forecasts of several heuristic models, $p^e_{h,t+1}$, indexed by h get aggregated up with weights $n_{h,t}$ to generate the mean price expectation that enters the data generating equation of the model:

$$\bar{p}^e_{t+1} = \sum_{h=1}^{H} n_{h,t} p^e_{h,t+1}$$

The fitness of rule h is updated every period based on past performance,

$$U_{h,t-1} = \mu U_{h,t-2} - (p_{t-1} - p^e_{t-1})^2$$

where $\mu \in (0, 1)$ denotes a memory term. Finally the fitness of each heuristic is used to

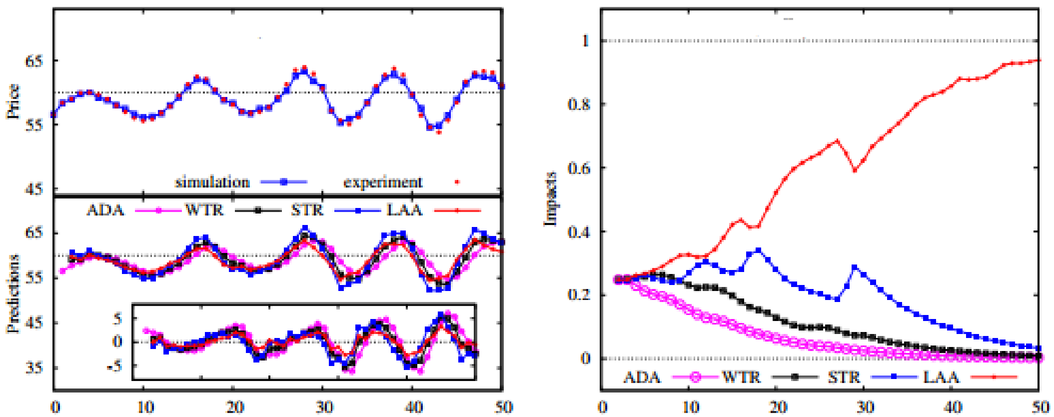


Figure 9: Experimental Data versus One-Step Ahead Prediction from Anufriev and Hommes's HSM for Group 6 of Hommes et al. 2005

determine the weight given to it in the expectation equation for $\bar{p}^e_{t+1}$:

$$n_{h,t} = \lambda n_{h,t-1} + (1-\lambda)\frac{e^{\beta U_{h,t-1}}}{\sum_{h=1}^{H} e^{\beta U_{h,t-1}}}.$$

The four heuristic rules for expectation formation were:

1. Adaptive Expectations (ADA): $p^e_{t+1} = 0.65p_{t-1} + 0.35p^e_t$
2. Weak Trend Following (WTR): $p^e_{t+1} = p_{t-1} + 0.4(p_{t-1} - p_{t-2})$
3. Strong Trend Following (STR): $p^e_{t+1} = p_{t-1} + 1.3(p_{t-1} - p_{t-2})$
4. Anchoring and Adjustment (LAA): $p^e_{t+1} = 0.5((t-1)^{-1}\sum_{j=0}^{t-1} p_j + p_{t-1}) + (p_{t-1} - p_{t-2})$

These rules were motivated by the estimation results of subjects' individual forecasts, where similare learning rules with only a few significant lags were quite common.

The first rule is a standard adaptive expectations rule while the second and third rules are "trend" following rules, that seek to exploit trends in the most two recent price realizations (between dates $t-1$ and $t-2$). The final rule is a flexible anchoring and adjustment rule which puts weight on the hypothesized fundamental price, approximated by the sample average, $(t-1)^{-1}\sum_{j=0}^{t-1} p_j$, and also gives some weight to price trends.

The model is initialized with equal weights on all four heuristics and a specific choice of parameter values, $\beta = 0.4$, $\eta = 0.7$ and $\delta = 0.9$. In- and out-of sample simulations provide a good fit to the experimental data. An example is shown in Figure 9, which reports simulation results for group 6 in the asset pricing experiment, which followed an oscillatory path around the REE. As the figure reveals, all four rules in the heuristic switching model initially have some weight, but ultimately the more flexible anchoring and adjustment rule (LAA) becomes dominant in explaining the persistent fluctuations in prices; the adaptive (ADA) and the two trend following rules (WTR and STR) miss the turning points in prices and are thus given less weight over time. In other groups, especially those that converge to the REE, all rules have similar forecasting success and thus all four continue to have weight in the heuristic switching model over time.

Cobweb model experiments

A second experimental environment in which there is belief-outcome interaction and evidence for heterogeneous expectations is the Cobweb model of Ezekiel (1938) which is the framework first used by Muth (1972) to formulate the rational expectations hypothesis. This model consists of equations for the demand and supply of a single, perishable good. Demand is a decreasing function of the period t, market price, $D(p_t)$, while supply, which must be chosen one period in advance, is an increasing function of the price expected to prevail at time t, $S(p_t^e)$, based on information available through period $t-1$. Hommes et al. (2007) elicit price forecasts from i=1,2...,6 subjects, and given these forecasts they optimally solve for the supply that each forecaster i would bring to the market. Thus aggregate supply is given by $\sum_{i=1}^{6} S(p_{i,t}^e)$ and, since the demand side is exogenously given, market clearing implies that the equilibrium price is given by:

$$p_t = D^{-1}\left(\sum_{i=1}^{6} S(p_{i,t}^e)\right) + \epsilon_t \tag{4}$$

where ϵ_t is an added i.i.d. mean 0 noise term reflecting fundamental uncertainty. Subjects were incentivized to chose $p_{i,t}^e$ as close to p_t as possible as their payoff function was again determined by a quadratic-loss scoring rule. The main treatment variable was a supply function parameter that varied the amount of nonlinearity in supply, and so affected whether the steady state price, p^*, was stable, unstable or strongly unstable under Ezekiel's benchmark choice of naive expectations, $p_t^e = p_{t-1}$, though for more general adaptive learning specifications, the steady state could be stable and the limit of the learning process. The main issue they examined was the validity of the RE prediction that $p_t^e = p_t^* + \epsilon$. They found that the REH found some support in the case where the steady state was stable under naive expectations, but that it did not predict well in the unstable or strongly unstable environments. More precisely, while the mean realized price over the sample of 50 periods was always close to p^* the sample variance was larger and more persistent the greater was the instability of the system under the naive expectations benchmark. Off-the-shelf adaptive learning processes such as past averaging of prices or error-correction approaches were also not as useful in explaining experimental data from the Cobweb model as these models led to too regular fluctuations and predictable autocorrelation patterns not found in the data.

Indeed, an explanation of these findings also requires an explicitly heterogeneous-agent model. Arifovic (1994) was the first to use a *genetic algorithm* to explain experimental findings for the cobweb model similar to those reported by Hommes et al. (2007). Genetic algorithms are computer programs that mimic naturally occurring evolutionary processes: selection based on relative fitness, reproduction and mutation on a population of candidate solutions to an optimization problem. These algorithms, first developed by Holland (1975) (see also Goldberg (1989)), have been shown to be ideal function optimizers for “rugged landscapes” as the population basis of the search and the evolution of new strategies over time avoids the possibility that the algorithm gets stuck at (prematurely converges to) local optima. In a typical Genetic Algorithm, there exists a population of candidate solutions or “chromosomes” coded in some manner, typically a binary encoding. There is a also a fitness criterion, e.g., a profit, utility or payoff function that is the objective of the optimization problem and that is used to evaluate the chromosomes. Initial populations of solutions (chromosomes) are typically randomly generated, over some reasonable domain for the solution space. Then solutions are evaluated for their fitness. The most fit solutions are probablisitically more likely to survive into the next “generation” of candidate strategies in a reproduction step to the algorithm. This reproduction step is followed by a crossover step, where pairs of solution strings are randomly matched, a cut-point is randomly determined and the genetic material (binary encodings to one side of the cut point) are swapped in a process mimicking genetic recombination. Finally, encodings are subject to some mutation as well, with some small probability, e.g., a bit is flipped from a 0 to a 1 or vice versa. This process then repeats over multiple generations until some convergence criterion is met or a maximum number of generations has passed. Along this path, genetic algorithms thus consist of very heterogeneous populations of candidates solutions or strategies. In Arifovic's (1994) application to the Cobweb model, the population of chromosomes represented a population of different firms' decision rules as to how much quantity to bring to the market in a given period (demand was automated). Aggregate quantity together with exogenous market demand determined the market price, which was used to evaluate each firm's profits, the fitness criterion. Arifovic found that genetic algorithm simulations, like the experiments of Hommes et al. converged to the rational expectations solution in the stable case and to a neighborhood of the REE in the unstable case, and that the volatility of the heterogeneous agent genetic algorithm was a good approximation to the heterogeneity observed in experimental data. While Arifovic's simulations considered the optimal quantity decision of firms, Hommes and Lux (2013) used a genetic algorithm model on populations of price forecasts in order to address the price volatility in the Hommes et al. (2007) Cobweb model experiments as the system was made more unstable. Like Arifovic (1994), they found that simulations of the genetic algorithm for price forecasts yielded a good match to the experimental data. This match to the experimental data relies upon the genetic algorithm's use of past fitness, its heterogeneity of different solutions, and the genetic operators, which allow for the development of new strategies.

More recently, Arifovic and co-authors have begun working with multiple population genetic algorithms, one population for each decision-maker. These algorithms, which Arifovic refers to as "individual evolutionary learning" (IEL) algorithms are close cousins to genetic algorithms. In addition to having different populations of strategies (different GAs) for each decision-maker, IEL avoids the use of the crossover operation of the genetic algorithm (mutation alone often suffices for experimentation) and it allows for more dynamic specifications of the strategy space, permitting the evolution of conditional strategies. The IEL algorithm was described earlier in connection with the public good game (sees section 2.1) and will also be discussed later in connection with experimental studies of equilibrium selection (bank runs (3.1) and adoption of new payment systems (3.3)).

Positive versus negative feedback

The asset market experiment and the cobweb model experiments differ in two important dimensions. First, in the Cobweb model, (equation (4)), $p_t = f(p_t^e)$, requiring one step-ahead-forecasts for prices. By contrast in the asset market experiment, (equation (3)) $p_t = f(p_{t+1}^e)$, requiring two step-ahead forecasts for prices. Second, and more importantly, in the asset market experiment, there is *positive* feedback between price expectations and price realizations, i.e., $\partial f/\partial p_{t+1}^e > 0$ while in the Cobweb model there is negative feedback, i.e., $\partial f/\partial p_t^e < 0$. Positive feedback mechanisms, as in the asset pricing model are associated with strategic complementarities; an increase on one agent's price forecast causes others to choose higher price forecasts as well. By contrast, negative feedback systems are associated with strategic substitutability; a higher price forecast (higher expected demand) by one oligopolist provides incentives for the another oligopolist to lower his or her price forecast.[9] It turns out that this difference matters for the speed of convergence to the rational expectations equilibrium, as shown in experimental research by Fehr and Tyran (2001, 2008), Potters and Suetens (2009), Sutan and Willinger (2009), Heemeijer et al. (2009) and Cooper et al. (2017). Here we focus on the work of Fehr and Tyran, Heemeijer et al. and Cooper et al.

In Fehr and Tyran (2001), human subjects play a 4-player "price-setting" game. In each of $2T$ periods, subject i chooses a price P_i and earns a real payoff that is a function of the time t average price chosen by other players, $P_{-i,t}$ and the time t nominal money supply M_t:

$$\pi_{i,t} = f\left(P_i, \bar{P}_{-i,t}, M_t\right)$$

The function f yields a unique, dominance-solvable equilibrium for every value of M, is homogeneous of degree 0 in all arguments, and $f_{P_{-i,t}} \geq 0$, so there is a weak, strategic

[9]See, e.g., Haltiwanger and Waldman (1989).

complementarity in price-setting. They conduct a treatment where subjects' earnings are paid according to the above, real payoff function. In addition, there is also a *nominal* payoff treatment where subjects' earnings are reported to them in nominal terms, $P_{-i,t}\pi_i$. In both of these treatments, there is a nominal shock: the nominal money supply is known to be a constant level M for the first T periods and then to decline to a permanently lower level λM , $\lambda < 1$ for the last T periods. The question is then whether subjects will adjust their prices downward at date T from P to λP .

Their results show that, in the *real* payoff treatment, the adjustment to a new, equilibrium nominal price, λP occurred almost instantaneously. However, the adjustment in the second, *nominal* payoff treatment was very sluggish, with nominal prices adjusting slowly to the new equilibrium. Fehr and Tyran characterize this as *money illusion* that depends on whether subjects are paid in real, price adjusted or in nominal terms. This adjustment is more difficult in the normal payoff function treatment where subjects have to correctly deflate their nominal payoff function. In addition, arising from the strategic complementarity in price settings, the failure of some subjects to adjust to the nominal shocks may make it a best response for others who are not subject to money illusion to only partially adjust to the shock.

In order to isolate the latter possibility, Fehr and Tyran (2001) also conduct individual-decision making experiments under both the real and nominal payoff functions where the other $n-1$ players are known to the human subjects to be robot players that are not subject to money illusion and that adjust prices downward proportional too the shock, at the time of the shock. Their results show that, in this treatment, the extent of price sluggishness is greatly reduced.

Fehr and Tyran (2008) consider not only the prior case where there is a strategic complementarity in price setting, but also the case where there is a strategic substitutability in price setting, i.e. $f_{P_{-i,t}} \leq 0$. Their results show that money illusion and the resulting nominal inertia in response to a fully anticipated monetary shock is greatly reduced in the case of strategic *substitutes* relative to the case of strategic *complements*. This leads to much faster adjustment toward the post-shock equilibrium.

Heemeijer et al. (2009) studied two simple linear models for price determination:

$$p_t = \frac{20}{21}(123 - p_t^e) + \epsilon_t \quad \text{(negative feedback)} \quad p_t = \tfrac{20}{21}(3 + p_t^e) + \epsilon_t \quad \text{(positive feedback)}$$

where p_t^e was taken to be the average of 6 subject forecasts for p_t. Both models have the same rational expectations equilibrium prediction, namely that $p_t = 60 + \epsilon_t$. Nevertheless, that path by which groups of 6 subjects learned this equilibrium was very different depending on whether they were in the positive or negative feedback treatment. Under negative feedback, convergence obtained rather quickly, within 5 periods, on average. However, under positive

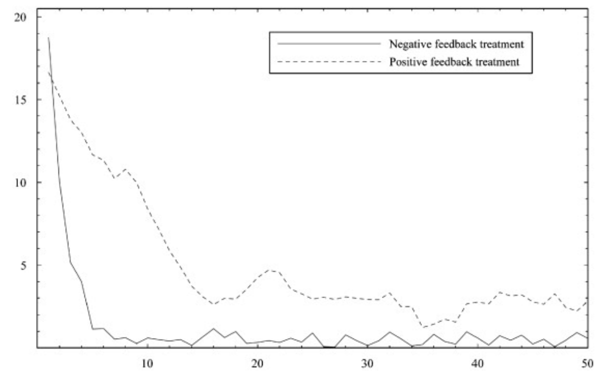


Figure 10: Absolute Deviation of Median Market Price from the REE Price Over 50 Rounds, Positive versus Negative Feedback Treatments, Pooled Data: Source Heemeijer et. al. (2009)

feedback, there was only a very slow oscillatory movement toward the equilibrium price of 60. Indeed, average prices and volatility under positive feedback were significantly greater at the 5% significance level as compared with the negative feedback case. Figure 10 illustrates the absolute difference between the median market price and the equilibrium price over all groups and the 50 rounds of Heemeijer et al.'s experiment.

The explanation for this difference again lies in the value of different forecasting rules in the two environments. When there is positive feedback, if enough agents use trend-following rules, other forecasters find that trend following is profitable and prices often deviate substantially from fundamentals. By contrast, in markets with negative feedback, as the number of individuals adopting trend following rules becomes sufficiently great, the incentives for contrarian, fundamental-based strategies become greater and so trend-following strategies do not survive very long in such negative feedback systems. Bao et al. (2012) consider the same two models, but examine how subjects react to unanticipated shocks to the fundamental value of the price. They report that under the negative feedback system, there is a rapid adjustment to the new, post-shock equilibrium, but under the positive feedback system, the adjustment to the shock is slow; initially the price under-reacts to the change, but over time there is over-reaction. Bao et al. use a heuristic switching model to explain this under- and over-reaction pattern.

Within a given type of feedback (positive or negative), players can also be heterogeneous with regard to their depths of reasoning, i.e., their beliefs about others —see the chapter by Mauersberger and Nagel (2018) for further details in the context of the Beauty Contest Game, (Nagel (1995)). They can also be heterogeneous in terms of their knowledge of the data generating process or the history of play by others. For instance, Cooper et al. (2017) induce heterogeneity about the history of play by periodically replacing players in an N-player beauty contest game with new players, coincident with changing the (interior) equilibrium period of the game. They find that the periodic addition of a single, inexperienced player (and removal of an experienced player) in a population of size 4, can have large effects on the speed of convergence to the new equilibrium when the environment is characterized by strategic complementarities (positive feedback) but not when it is characterized by strategic substitutability (positive feedback). This pattern follows from the manner in which the experienced subjects react to the play of the inexperienced subjects; in the substitutes case, too high (or too low) a choice by the inexperienced subject can be counteracted by lower (higher) choices by the experienced subjects but in the complements (positive feedback) environment, the experienced subjects find it more difficult to counteract the errant behavior of the inexperienced subject and thus convergence to the new equilibrium occurs more slowly. These findings show how even a small amount of heterogeneity can nevertheless have large, differential impacts on outcomes.

2.4 Learning to Forecast vs. Learning to Optimize Experimental Designs

The expectation formation experiments discussed in the previous section decouple the expectation formation problem from optimization decisions. This is a common practice in the macroeconomic learning literature as well (see, e.g. Evans and Honkapohja (2001)).[10] The maintained assumption is that while agents may be boundedly rational in expectation formation, they have no trouble optimizing given their incorrect forecasts. The experiments that have followed this approach are known as "learning to forecast" experiments, since subjects' only aim to get the expectation of future variables correct; indeed, subjects usually have no other knowledge of the system they are operating in (a topic we shall return to shortly). The trading decisions necessary to assure arbitrage between the two assets in the asset market or profit maximization in the cobweb model commodity market are simply automated for subjects. Indeed, in the experiments of Hommes et al. (2005, 2008), subjects are simply instructed that they are forecast "advisors" to some firm.

The use of such learning to forecast experiments dates back to the work of Marimon and Sunder (1993, 1994) who studied savings behavior in two period overlapping generations models. They found that in old age (period 2) subjects were often failing to spend all of their savings, so they chose to automate the problem so that subjects, in the first, young period of their lives only had to forecast the price level that would prevail in the second and final period of their lives when they were old. Given this price forecast, the optimal saving and consumption decisions was automatically computed for subjects.

[10] See the chapter by Branch and McGough (2018) for modeling bounded rationality in forecasting as well as bounded rationality in decision-making in macroeconomic models.

However as we have seen in the public good and intertemporal consumption/savings experiments, some subjects are not very good at solving optimization problems (or applying backward induction). In those "learning to optimize" experiments, subject forecasts are not explicitly elicited but are rather implicit in subjects' optimizing decisions. Nevertheless, it is instructive to understand the extent to which subjects can both form expectations *and* optimize conditional on those expectations. Such an exercise was considered by Bao, Hommes and Duffy (2013) who compared the performance of subjects in learning to forecast and learning to optimize treatments of the cobweb model experiment (described above) as well as additional treatments where subjects did both tasks (forecast and optimize) or in the case where subjects are simply forecasting prices, their payoffs were determined according to a quadratic loss scoring rule between their forecasted price and the actual price resulting from the average forecasts of a group of 6 subjects. In the learning to optimize treatment, the six subjects were in the role of supplier choosing a quantity of the good in period t, $q_{i,t}$, to bring to the market. The payoff to each of the 6 subjects was given by the net profit resulting from this quantity choice, i.e., $\pi_{i,t} = p_t q_{i,t} - c(q_{i,t})$, where p_t is the actual market price determined via the market clearing condition and c was an induced (and known) cost function. In a third treatment subjects were asked to form both forecasts and choose quantities while in a fourth treatment, subjects were matched in teams, where the forecasting task was performed by one team member and the optimizing by another, and they shared the equal weighted payoff from both activities.

Bao et al. (2013) report that the speed of convergence to the RE price is fastest when subjects only have to forecast prices, that is, in the learning-to-forecast experiment. In second place, is the treatment where one subject is specialized in forecasting and the other is specialized in optimizing. In third place was the treatment where subjects were asked to make optimal quantity decisions only (learning to optimize), followed by the treatment where subjects were asked to both forecast and optimize, which had the slowest time to convergence. Heterogeneity in price forecasts and quantity decisions was also significantly greatest in this last treatment where subjects had to perform both forecasting and optimizing roles. The elicitation of the two different decisions reveals that a majority of subjects 59.5% were not making quantity decisions that were optimal given their price forecasts. By contrast, in the team setting, quantity decisions were optimal given price forecasts for a majority of subject pairs (69.4%), suggesting that heterogeneity in price forecast/quantity outcomes may be reduced in teams relative to individuals. Bao et al. further report that subjects' forecast rules in this negative feedback environment were a mix of adaptive and contrarian rules, with more than 50 percent of subjects being classified as adaptive learners.[11]

[11] Bao et al. (2017) study learning to forecast versus learning to optimize in a positive feedback system. They find that asset bubbles are a robust feature and even larger in magnitude under the learning-to-optimize

2.5 Adaptive versus Eductive Learning

The studies of expectation formation discussed in the prior sections presume that agents have limited or no knowledge of the data generating process for the variables they are seeking to forecast. It is natural to think that agents in such settings would adapt their expectations of the variables they are forecasting over time in an inductive manner based on past outcomes, to develop new forecasts that better approximate the data generating process. A second approach to modeling learning asks not whether an equilibrium is eventually reached under adaptive learning, but whether the equilibrium is eductively stable, in the sense of Guesnerie (1992, 2002). Eductive learning departs from adaptive learning in that all agents perfectly know the data generating process, and assuming common knowledge of the rationality of other actors can *deduce* that the equilibrium path associated with that process is associated with a particular REE, thereby facilitating coordination on that REE. Eductive stability can be illustrated with respect to the same Cobweb model used to study adaptive learning. In a linear demand and supply model, the market clearing price in a Cobweb economy is given by:

$$p_t = \mu + \alpha \bar{p}_t^e + \epsilon_t$$

where $\bar{p}_{t+1}^e$ denotes the average of all supplier's forecasts and ϵ_t is again an i.i.d. mean 0 noise term. Assume now that μ and α are constants known to all forecasters; under the adaptive expectations view, these parameters are unknown and have to be learned. The equation for p_t yields a unique rational expectations equilibrium in which:

$$p_t^* = \bar{p}_t^{e,*} + \epsilon_t, \quad \text{where } \bar{p}_t^{e,*} = \frac{\mu}{1-\alpha}$$

The REE can be shown to be eductively stable provided that $|\alpha| < 1$. Intuitively, if this condition is satisfied, then starting from any initial price, an iterative reasoning process, performed in notional or mental time, leads agents to the REE. By contrast, as Evans (2001) has pointed out, the REE is stable under adaptive learning so long as $\alpha < 1$, which is a less restrictive condition. Bao and Duffy (2016) exploit this difference to assess the extent to which subjects can be characterized as either adaptive or eductive learners. To conduct this test, they had to put the two theories on an informationally equivalent footing; since the eductive approach assumes complete knowledge of the data generating process while adaptive learning does not, they chose to inform subjects of the equation for prices including the values they chose for $\mu = 60$ and $\alpha \in \{-.5, -.9, -2, -4\}$. In one treatment, subjects were matched in groups of three and the average forecast of the three players was used to determine $\bar{p}_t^e$ in each of 50 periods; subjects were again paid according to a proper scoring rule. Figure 11 shows the evolution of these average forecasts over time for 10 different cohorts in each of the four treatments where $\alpha = .5$, $.9$, -2, and -4, respectively.

treatments.

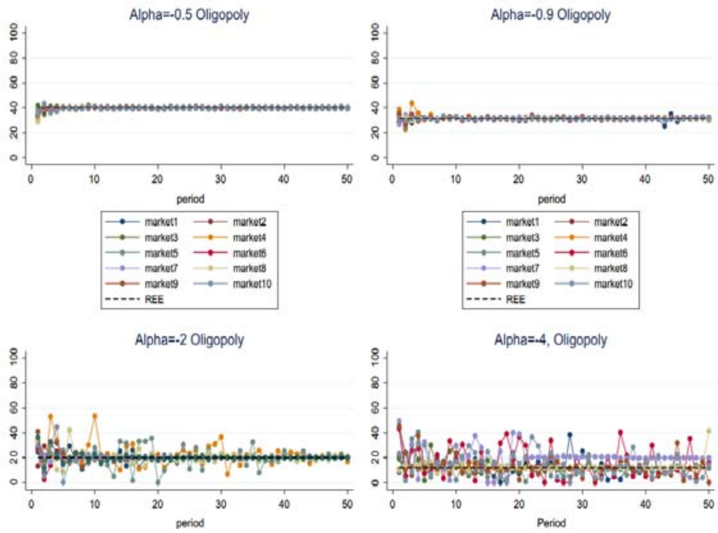


Figure 11: Average expectations of prices over time in the Oligopoly treatment of Bao and Duffy, four different values for α: 0.5, 0.9, -2.0 and -4.0

When $|\alpha| < 1$, consistent with both the adaptive and eductive learning hypotheses, expectations converged to REE. However, in the two treatments where $\alpha < -1$, convergence was slower or did not occur within the time frame of the experiment. Bao and Duffy show that these differing results are due to the presence of roughly equal numbers of adaptive and eductive type players (as a well as a sizable fraction of subjects who are not classifiable). The presence of the two types and the relative influence of each matters for whether the system converges or does not converge to the REE, evidence once again that type heterogeneity matters.

3 Heterogeneity and Monetary Policy

We then turn to a more complicated setting, the New Keynesian model of monetary policy, where coordination problems can arise if the equilibrium of the model is not determinate or locally unique (indeterminate). Generally speaking, central banks wish to avoid such settings, in favor of those where the equilibrium is unique, and there is a large theoretical and experimental literature addressing the extent to which policy can play such a coordinating role.

3.1 New Keynesian Experiments

A number of recent papers has focused on studying the effects of monetary policy in *forward-looking* versions of the sticky price, New Keynesian model (as developed in Woodford, 2003):

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right) \tag{5}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{6}$$

$$i_t = f(E_t \pi_{t+1}, E_t x_{t+1}) \tag{7}$$

$$r_t^n = \rho r_{t-1}^n + \epsilon_t \ . \tag{8}$$

The first equation for the output gap, x_t, is the expectational IS curve, with σ representing the intertemporal elasticity of substitution. The second equation for the inflation, π_t, is the New Keynesian Phillips curve, with β equal to the period discount factor, and κ is a parameter that captures the stickiness of the prices. Finally, the third equation represents the specification of the central bank's policy rule for the nominal interest rate i_t, and with the assumption of rational expectations. Expectations of future inflation and output gap play a crucial role in the dynamics of the system. The central bank manages these expectations through its choice of an interest rate (policy) rule.

A number of recent papers have studied the behavior of the above system in the context of learning-to-forecast experiments. In these experiments, subjects are asked to forecast next period's inflation rate and output gap and then, subjects' mean or median expectations for inflation and the output gap are substituted into equations (5) and (6) in place of rational expectations. The experimental studies investigate issues related to the stability of economies under different interest rate policy rules, the role of the expectational channel in the reduction of the variance of output and inflation, and the role of different monetary policies in the management of the economies at the zero-lower bound. In these types of settings, there is heterogeneity of subjects' expectations which play a crucial role in the dynamics and stability of these economies over time.

For example, Pfajfar and Zakelj (2014) simplify the model by using x_{t-1} as the naive expectation for $E_t x_{t+1}$, and study only expectations of inflation in a laboratory environment. Thus, in these experiments subjects are asked to forecast just inflation, knowing only qualitative features of the underlying model and seeing historical time series on inflation, output gap and interest rates. In addition, they add AR(1) shocks to (5) and (6), respectively. The average of 9 subjects' inflation forecasts is used to replace $E_t \pi_{t+1}$. They study two kinds of policy rules of the form:

$$i_t = \gamma(\bar{\pi} - \bar{\pi}_t) + \bar{\pi} \tag{9}$$

were $\bar{\pi}$ is the central bank's target inflation and $\bar{\pi}_t$ is either actual inflation at time t or time t expectation of the inflation rate at $t+1$. Thus, the first one is a contemporaneous rule that requires the central bank to respond to current deviations from the inflation target and the second one is a forward looking rule where the central bank reacts to deviations of expected inflation from the target. The other treatment variable is γ that takes 3 different values, 1.35, 1.5, and 4. The Taylor principle is satisfied for all of the values which implies, that, under rational expectations, the equilibrium is determinate (locally) and stable, again locally, under a certain specification of adaptive learning. When policy conditions on expectations of $(t+1)$ inflation, the policy with higher γ is much more effective in reducing the standard deviations of inflation expectations. Overall, the policy that conditions on current π_t rather than on future $E_t\pi_{t+1}$ delivers best performance in terms of stabilization of the experimental economies both in terms of inflation variability and output fluctuations. Essentially, the π_t rule is reducing the weight that subjects' expectations play.

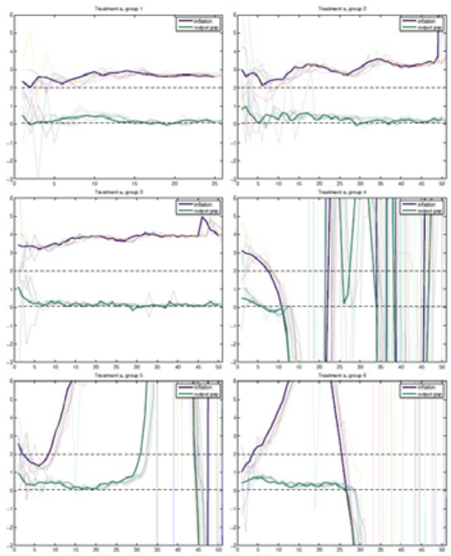
In addition, the paper analyzes the heterogeneity of subjects expectations by considering different expectational models. Besides rational expectations, the authors also consider 12 alternative expectation formation models studied in the literature. To characterize subjects' expectation formation process, a new test for rationality based on the difference between their perceived law of motion and the actual law of motion is also introduced in a heterogeneous expectations environment. Overall, only 30% to 45% of subjects conform to rational expectations while other subjects' behavior can be explained by trend extrapolation, adaptive learning or sticky information. More importantly, switching between different models fits the experimental data far better than a single model.

Assenza et al. (2014) discuss how monetary policy can manage self-organization of heterogeneous expectations in a lab experiment. Their experimental design uses the contemporaneous inflation targeting rule (without output gap) and small IID shocks instead of autocorrelated shocks that have been implemented in other experiments. The 3 main treatments feature different Taylor rule reaction coefficients and inflation targets. In treatment (a), monetary policy is weakly responsive (the reaction coefficient equals 1) and the inflation target is 2%. Treatment (b) includes aggressive monetary policy (the reaction coefficient equals 1.5) and the same 2% benchmark inflation target. Treatment (c) is implemented by increasing the inflation target to 3.5% while preserving the aggressive nature of monetary policy.

Two types of aggregate patterns are observed in treatment (a): convergence to a non-fundamental steady state and exploding inflation and output (see figure 12). All the sessions in treatment (b) exhibit nice convergence to the fundamental steady state (see figure 13). Both convergence to the fundamental steady state and persistent oscillations emerge in treatment (c) (see figure 14). The stabilizing effects of monetary policy come from the aggressiveness of the policy rule, not the inflation target.

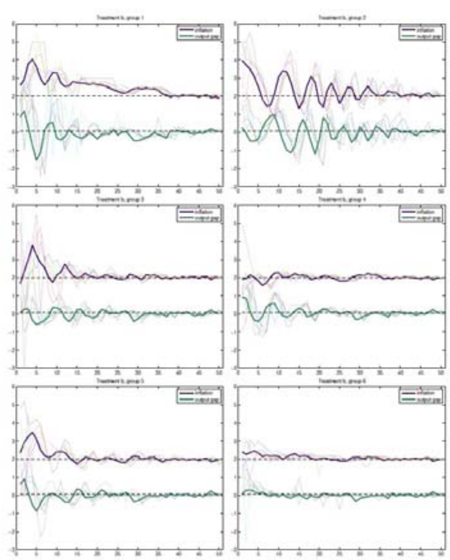
The authors also adopt a heuristics switching model developed by Anufriev and Hommes

Figure 12: Experimental Results in Assenza et al. (2014): Weakly Responsive Monetary Policy



Blue thick line: realized inflation; green thick line: realized output gap; thin lines: individual forecasts for inflation and the output gap.

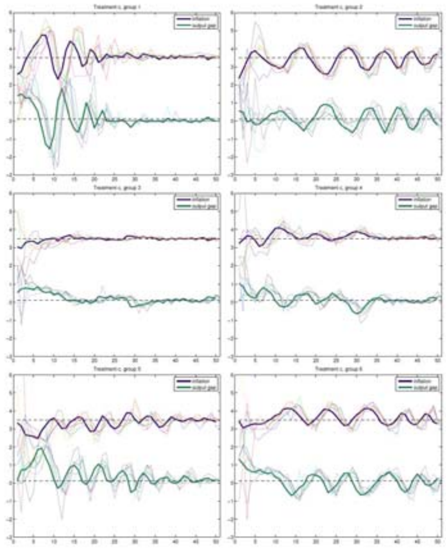
Figure 13: Experimental Results in Assenza et al. (2014): Aggressive Monetary Policy



Blue thick line: realized inflation; green thick line: realized output gap; thin lines: individual forecasts for inflation and the output gap.

(2012) in which agents switch between 4 different forecasting rules by evaluating their relative past performance (described earlier in the chapter). Simulation results demonstrate that all types of aggregate patterns observed in the experiment can be replicated by the heuristics switching model. Hommes et al. (2017) replicate the experimental results from the

Figure 14: Experimental Results in Assenza et al. (2014): Aggressive Monetary Policy and High Inflation Target



Bluethick line: realized inflation; green thick line: realized output gap; thin lines: individual forecasts for inflation and the output gap.

Assenza et al. learning-to-forecast experiment using a genetic algorithm over price forecasts. The genetic algorithm simulations match the experimental data in terms of the response to different policy rules.

Mauersberger (2018) introduces a new, learning-to-forecast experimental design within a micro-founded NK framework, where subjects forecast individual, rather than aggregate, outcomes in an economy based on the linearized heterogeneous expectations New Keynesian model (Woodford (2013)). Unlike Woodford (2013) which features an exogenous preference shock and a mark-up shock, there are no exogenous shocks in the Mauersberger's setup.

Subjects are randomly assigned to be either household advisors or firm advisors. Households, firms and the central bank are computerized. Household advisors are asked to forecast the deviation of household's real expenditure from its long run, steady state level (called 'usual level' in the experiment). Firm advisors need to submit forecasts of the deviation of a firm's optimal price in the next period from the current general price level. Aggregate outcomes are computed using medians of subjects' expectations and choices made by computerized households and firms that are based on these expectations.

Like other experiments, Mauersberger studies the effects of monetary policies that differ in the degree of aggressiveness. He uses a contemporaneous, inflation targeting rule with a zero lower bound, i.e. :

$$i_t = \max(0, \bar{i} + \phi_\pi(\pi_t - \bar{\pi})) \tag{10}$$

where $\bar{i}$ is a steady state level of the nominal interest rate.

However, his results differ from the previous experimental literature as he finds that a much stronger Taylor rule is required for stability and convergence of the experimental economies within 50 periods, i.e., the convergence to the rational expectations steady state can only be obtained when the Taylor rule reaction coefficient is equal to 3. At the group level, heterogeneity is rather pronounced for lower Taylor rule coefficients (0.5 or 1.5), but it vanishes under a more aggressive monetary policy. [12]

Mauersberger uses Thompson sampling algorithm (Thompson (1933)) to explain expectation formation in his experiment. This is the algorithm where agents update their beliefs in a Bayesian manner. However, agents do not choose the optimal action implied by the posterior. Instead, they make a random draw from the posterior each time that an action must be taken and best respond to this random draw. Using the method of *simulated paths* where two initial experimental observations are used to update the algorithm, and, afterwards, the dynamic paths of the economies are solely determined by the algorithms' choices and outcomes, Mauresberger shows that Thompson sampling performs well in capturing the features of the experimental data. It generates quite a bit of heterogeneity and volatility for ϕ_π equal to 0.5 and 1.5. However, these are substantially reduced in case of $\phi_\pi = 3$.

He then compares the performance of Thompson sampling to a number of learning/adaptive models that have been standardly used to investigate expectations in macroeconomic environments: two forms of adaptive learning, least-square learning and constant gain learning (Evans and Honkapohja (2001)), and the heuristic-switching model (Anufriev and Hommes (2012)) which was discussed earlier in the chapter. In addition to the mean-squared error calculated over 50 periods, he uses other statistical measures to better assess the behavior of the models, such as the first and second moments, the mean squared distance from the REE and an index of intra-period dispersion. The heuristic-switching model results in a relatively fast convergence to the rational expectations steady state even when the Taylor rule coefficient is 1.5, which is at odds with Mauersberger's experimental data. Comparing Thompson sampling and the three other benchmark models, Thompson sampling provides a good fit to the data along a number of dimensions (convergence, volatility patterns, individual dispersion etc.).

3.2 New Keynesian Experiments at the Zero Lower Bound

Further evidence for the need for heterogeneous agent models comes from experiments that study the effects of monetary and fiscal policy on subjects' expectations in the New Keynesian model when interest rates are near the zero lower bound, as they were in much of the developed world in the period following the 2007-08 global financial crisis. These experiments

[12]Note that, for example, Assenza et al. (2014) experimental results indicate convergence for the Taylor rule inflation coefficient of 1.5.

reveal a variety of reactions to policy efforts to stimulate the economy when interest rates cannot be driven lower. Arifovic and Petersen (2017) study the effects of monetary and fiscal policy on subjects' expectations in learning-to-forecast experiment using a linearized New Keynesian framework described in (5), (6), and (8).

Compared with the earlier experiments that we discussed, Arifovic and Petersen introduce a binding constraint on nominal interest rates[13]. The nominal interest rate is $i^* = 75$ basis points in the steady state, and cannot be reduced below zero in the event of sufficiently low inflation or output gap. Thus, the Taylor rule in their experimental economy is given by:

$$i_t = \begin{cases} i^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t & \text{if } i_t \geq 0 \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

They selected their shock sequences using a social evolutionary learning algorithm in Arifovic et al. (2012) and the same set of shock sequences is used in all treatments. In order to push the laboratory economies towards the zero lower bound, large exogenous demand shocks are introduced in the middle of the shock sequences.

Arifovic and Petersen conduct four different treatments. In their baseline treatment, the central bank implements a standard Taylor rule and a constant inflation target (*Constant*). The state-dependent target treatment and the directional state-dependent treatment are designed to test the effectiveness of quantitative and qualitative forward guidance through explicitly announcing either the state-dependent inflation target (quantitative forward guidance, *SD*), or the direction in which the target is moving (qualitative forward guidance, *Dir. SD*). Finally, by adding an expansionary Ricardian fiscal policy to the baseline treatment, the fiscal policy treatment (*FP*) allows the authors to evaluate the effects of both monetary and fiscal policy near the zero lower bound.

They find that the state-dependent inflation target (*SD* and *Dir. SD*) treatments do not bring significantly greater stability than the standard Taylor rule (*Constant*) treatment. This is more pronounced when fundamentals improve slowly. Arifovic and Petersen argue that poorer performance of the state-dependent inflation target policies is due to a loss of confidence in the central bank's ability to stabilize the economy. If the confidence in inflation targeting is lost during the *crisis*, expectations diverge further and further away from the target inflation values. In these experimental economies, the central bank is fully committed to its state-dependent policy to keep interest rates low even after the economy starts recovering. Unlike the rational expectations framework, where credibility and commitment are equated, in the experimental economies, subjects do not necessarily perceive the central bank's announcements as credible. As Arifovic and Petersen put it, "subjects need to *see it*

[13] Note that Mauersberger (2018) also implements the same constraint. However, the focus of his study is not on the policies related to zero lower bound.

to believe it".

On the other hand, anticipated fiscal policy intervention (*FP* treatment) results in significantly faster and more stable recoveries. The explanation for this behavior is most likely related to the fact that state-dependent inflation targeting monetary policies provide a promise of future recovery when future is uncertain while anticipated expansionary fiscal policy stimulates demand with certainty.

Hommes et al. (2015) also design a learning-to-forecast experiment to evaluate the effects of monetary and fiscal policy at the zero lower bound. By adopting a non-linear New Keynesian framework with multiple equilibria (a low inflation steady state and a targeted high inflation steady state), the authors can test the implications of adaptive learning models near the zero lower bound. More specifically, with aggressive monetary policy and a fixed amount of government spending, only the targeted steady state is locally stable under learning. The local instability of the low inflation steady state makes deflationary spirals possible under large pessimistic shocks. Another theoretical prediction is that the targeted steady state is globally stable under learning if both aggressive monetary policy and a fiscal switching rule that further increases government spending at the zero lower bound are implemented.

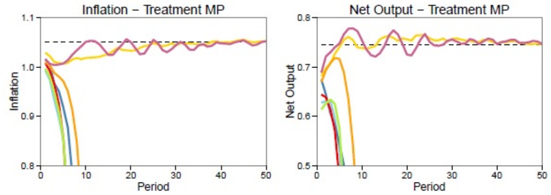
The four treatments are constructed based on variations along two dimensions. The first dimension is a type of policy regime: the policy regime M is a combination of aggressive monetary policy and constant government spending while the policy regime F replaces constant government spending with the fiscal switching rule. The second dimension is related to two types of expectational shocks: In scenario P, pessimistic expectations are induced by making "historical" information about inflation and net output accessible to subjects at the beginning of the experiment; in scenario S, late expectational shocks are realized in the form of "bad" newspaper reports.

In treatment MP, 5 out of 7 groups succumb to deflationary spirals because monetary policy cannot eliminate the adverse effects of initial pessimism. In treatment MS, monetary policy seems to be effective as 3 out of 7 groups converge to the targeted steady state despite the onset of late expectational shocks (see figure 15). In the two treatments with the fiscal switching rule, neither form of expectational shocks is destructive enough to generate deflationary spirals as all groups converge to the targeted steady state eventually (see figure 16). The slow convergences in some groups are due to the fact that the fiscal switching rule can only anchor subjects' inflation expectations when net output recovers.

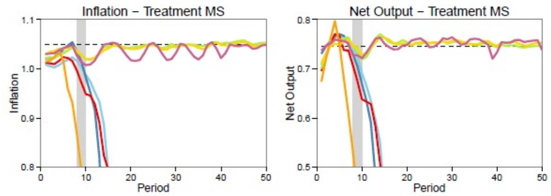
4 Heterogeneity in Equilibrium Selection

Models that admit a multiplicity of equilibria are commonplace in economics, but they pose a real challenge as they invalidate the use of standard comparative statics analysis; if one

Figure 15: Hommes et al. (2015): Aggressive Monetary Policy Only, for pessimistic expectations (top panels) and "bad news" shocks (bottom panels)



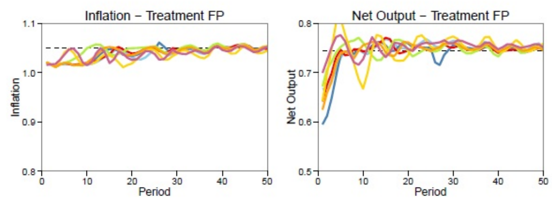
(a) Treatment MP



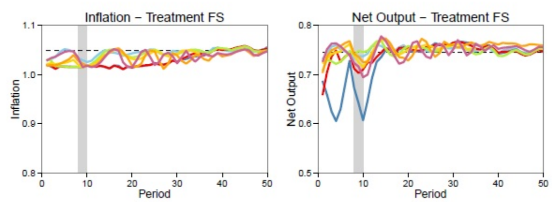
(b) Treatment MS

Overview of experimental results of the 4 treatments, 7 groups each. Left panels: realized inflation. Right panels: realized net output. Dashed lines depict targeted equilibrium levels. Shaded areas indicate expectational news shocks.

Figure 16: Hommes et al. (2015): Aggressive Monetary Policy and Fiscal Switching Rule, for pessimistic expectations (top panels) and "bad news" shocks (bottom panels)



(c) Treatment FP



(d) Treatment FS

Overview of experimental results of the 4 treatments, 7 groups each. Left panels: realized inflation. Right panels: realized net output. Dashed lines depict targeted equilibrium levels. Shaded areas indicate expectational news shocks.

does not know which equilibrium to focus on, it is not possible to determine how changes in the exogenous variables or parameters of the model affect the endogenous variables of the model. A potential solution to this problem is to use experimental evidence to validate a focus on a particular equilibrium. In certain cases, this approach works well, but in other cases, as we shall see, there is also heterogeneity in the equilibrium that groups of subjects coordinate upon, and heterogeneous agent models help to understand this phenomenon. We first consider the case of bank run models, where there generally exist two equilibria, one where all depositors keep their money in the bank and another where all run on the bank to withdraw their deposits Second, we consider coordination problems and equilibrium selection in the adoption of new payment methods. In these two applications, we show how heterogeneity in agent types and behaviors is a main driver of the equilibrium selection process.

4.1 Bank Runs

The experimental literature on bank runs demonstrates how heterogeneity of subjects' expectations influences outcomes and equilibrium selection using the canonical bank run model of Diamond and Dybvig (1983). This intertemporal model involves just three periods. In period 0, all depositors deposit their endowments of money into a bank, which has exclusive access to a long-term investment opportunity. The depositors are willing to deposit their funds with the bank because the contract the bank offers the depositors provides the depositors with insurance against uncertain liquidity shocks. In period 1, some fraction of depositors learn that they have immediate liquidity needs (are *impatient*) and must withdraw their deposits early. The remaining fraction learn they are patient and can wait to withdraw their deposit in the final period 2. The bank uses its knowledge of these fractions in optimally deriving the deposit contract, which stipulates that depositors may withdraw the whole of their unit endowment at date 1 while those who wait until period 2 to withdraw can earn $R > 1$. While there exists a separating, Pareto efficient equilibrium where impatient types withdraw early in period 1 and patient types wait until the final period 2, there also exists an inefficient pooling equilibrium where uncertainty about the behavior of other patient types causes all patient types to withdraw their deposits in period 1 which results in a *bank run*. In this case, the bank has to liquidate its long-term investment in period 1 and depending on the liquidation value of this investment, it may have insufficient funds to honor its deposit contract in period 1.

The possibility of this bank-run equilibrium is the focus of a number of experimental studies including Madiés (2006), Garratt and Keister (2009), Schotter and Yorulmazer (2009), Arifovic et al. (2013), Arifovic and Jiang (2016), Kiss et al. (2012, 2016), and Brown et al. (2016). Here we will focus on the papers by Arifovic et al. (2013) and Arifovic and Jiang (2017) as the experimental results are followed by modelling of their dynamics using evolutionary algorithms, but we briefly summarize the other experimental studies of bank runs.

These studies typically dispense with the non-strategic impatient types (or model them using robot players) and consider n-player coordination games were all players are patient, or strategic, i.e. they can choose to withdraw in period 1 or 2. Madiés was the first to demonstrate that for certain parameterizations of the model, an inefficient run equilibrium can be selected by laboratory subjects, though less than full (partial runs) are more common. Further he showed that a suspension of convertibility or full (but not partial) deposit insurance can work to prevent such runs. Similarly, Garratt and Keister (2009) showed that inefficient run equilibrium were not selected unless some subjects faced stochastic liquidity demand shocks causing them to withdraw early. Schotter and Yorulmazer (2009) consider the case where the bank run is already in progress and consider how insider information about the solvency of the bank matters for the speed and severity of the panic. Kiss et al. (2012) examine whether observability of prior depositors withdrawal decisions in a sequential move game or lack of such observability in a simultaneous move game together with varying rates of and deposit insurance affects the incidence of bank runs. They find that without observability, both full and partial deposit insurance are effective in decreasing the incidence of bank runs, while with observability, neither level of deposit insurance coverage makes much difference. Kiss et al. (2016) focus further on observability, examining the precise social network structure of which depositors decisions are observed by subsequent depositors. They report that the social network structure matters for the incidence of bank runs. Chakravarty et al. (2016), Duffy et al. (2017) and Choi et al. (2017) have recently reported experiments examining how deposit withdrawals at one bank can have contagious effects on withdrawal decisions at other connected banks, (2, 4 and 6-15 banks respectively), using a variety of different interbank network structures; the main takeaway from this research is that details of the network structure matter for the incidence of such contagions.

Arifovic et al. (2013) use the 2-period version of the DD model with $N = 10$ *patient* subjects who play the role of depositors. They start with one unit of money deposited with the bank and choose to withdraw early or wait. If all subjects choose to withdraw early in the first period, the payment is fixed at 1, and if all choose to withdraw later, in period 2, they all receive a payoff of $R = 2$. Arifovic et al. vary the rate of return, r, to withdrawing early ($r < R$). This rate determines the value of the coordination parameter η which measures the minimum fraction of depositors required to wait so that waiting entails a higher payoff than withdrawing. In other words, if the fraction of subjects who choose to wait is greater than η, those who choose to wait receive a higher payoff than those who choose to withdraw. Otherwise, if the fraction of those who choose to wait falls below the value of η, then these *patient* subjects receive a lower payoff. Thus, the payoff to the depositor who chooses to withdraw is

$$\pi_1 = \min\left\{r, \frac{N}{e}\right\}, \tag{12}$$

and the payoff for those who choose to wait is

$$\pi_2 = \max\left\{0, \frac{N - er}{N - e}R\right\}. \tag{13}$$

Note that if $e > \hat{e} \equiv N/r$, the bank will not have enough money to pay all early withdrawers the promised rate r, and those who choose to all those who decide to withdraw, and those who choose to wait will receive zero payoff.

Arifovic et al. ran experimental sessions with 7 or 9 phases where each phase corresponded to one value of η, and lasted for 10 experimental periods. The values of η changed in the ascending, descending and randomized order to control for the 'order' effect. They find that whether coordination-based bank runs occur depends on the value of the coordination parameter, η. In particular, the value of the coordination parameter can be divided into three regions: "run", "no-run" and "indeterminacy", characterized respectively by high (with $\eta > 0.8$), low (with $\eta < 0.5$) and intermediate (with η between 0.6 and 0.7) values of the parameter. When the coordination parameter lies in the run (no-run) region, strategic uncertainty is low: subjects are almost unanimous in their choices, and all experimental economies stay close or converge to the run (no-run) equilibrium. In games with the coordination parameter located in the indeterminacy region, subjects are much less certain as to what the 'right' choice is; as a result, the outcomes of the experimental economies vary widely and become difficult to predict.

Figure 17 shows their results for the values of η between 0.1 and 0.9, for sessions 9 - 20. The values of η are given on top of each panel, and the number of subjects (out of 10) who were not withdrawing, on the left side of each panel.

In order to capture the learning effect in the experimental data, Arifovic et al. combine the evolutionary algorithm (Temzelides, 1997) with logit regression models to estimate the rate of experimentation from the experimental data. [14] The evolutionary algorithm consists of two elements. The first is myopic best response, which, in the context of the DD model, is "withdraw" if the number of depositors choosing to wait in the previous period, z_{t-1}, (i.e. $N - e_t$) is $\leq z^*$, the number of depositors who choose to wait that equated the payoffs associated with 'wait' or 'withdraw'. Otherwise, the best response is to wait. In the context of this model, experimentation means to flip one's strategy from "withdraw" to "wait" or vice versa.

In the experiments, subjects have information about r (and subjects can deduce η from the payoff tables throughout the experiment). Subjects are not directly informed about z_{t-1}, but most of the time, subjects can refer to the payoff table to deduce z_{t-1} from their own payoffs in the past period. For the evolutionary algorithm, Arifovic et al. assume that if a subject cannot deduce whether $z_{t-1} > z^*$, she skips the first part of the algorithm and does not update her strategy; otherwise, she updates her strategy to the best response. For experimentation, we assume that if a subject can deduce the exact value of z_{t-1}, her experimentation rate depends on z_{t-1} and η; otherwise, her experimentation rate depends

[14] Temzelides (1997) proves a limiting case that as the probability of experimentation approaches zero the economy stays at the no-run equilibrium with probability 1 when $\eta < 0.5$.

only on η. The value of the rate of experimentation is to be estimated from the experimental data with logit models. They sort all the observations on the subjects' individual behavior depending on whether or not they were able to observe or deduce z_{t-1} and best respond to it. Then, they estimate rates of experimentation for different groups using logit regressions. Using the estimated rates of experimentation, they evaluate the performance of the algorithm in two ways. The first one is to use the algorithm's simulated paths and compare the outcomes with the experimental economies. The second is to use one set of data, the Chinese data, to estimate the rate of experimentation, and apply the algorithm to predict the action choice in each of the Canadian observations. [15] The algorithm performed well in terms of predictive power along both dimensions.

Given the above described results, in a follow-up paper, Arifovic and Jiang (2017) introduce an extrinsic "sunspot" variable to the above model and investigate how it affects the coordination outcome. Their hypothesis was that the power of the sunspot as a coordination device increases with the extent of strategic uncertainty. To test this hypothesis, they chose three experimental treatments with three values of the coordination parameter – 0.2, 0.9 and 0.7. Under their hypothesis, the power of the extrinsic signal is likely to be very weak when $\eta = 0.2$, weak when $\eta = 0.9$, and strong when $\eta = 0.7$. [16] Their results show that the randomly generated 'sunspot' announcements did no affect the outcomes of the sessions that were conducted for either $\eta = 0.2$ that all converged to the 'no-run' equilibrium or for $\eta = 0.9$ where they all converged to the run equilibrium. Where the announcement 'evolved' into serving as a coordination device was in the sessions conducted with $\eta = 0.7$. The results observed in these sessions are shown in Figure 18. The circles on the upper edges of the panels represent periods when 'optimistic' announcements were broadcasted, and the circles on the lower edges represent periods when 'pessimistic' announcements were broadcasted. (These data were not presented to the subjects on their graphical interface.) The solid lines present the number of subjects who decided to wait in each period (out of 10). The figure illustrates that in 4 out of 6 sessions the subjects coordinated on following the extrinsic variable.

4.2 Adoption of a New Payment System

To conclude our chapter on the role of heterogeneity of expectations we describe the work of Arifovic, Duffy and Jiang (2017), ADJ, on the adoption of a new payment system. This

[15] They ran experiments at one location in China, and two locations in Canada.

[16] The announcements that all subjects receive had the following wording :'The forecast is that e^* or more people will choose to withdraw,' or 'The forecast is that e^* or fewer people will choose to withdraw.' A specific integer number that equalizes the payoffs of 'withdraw' and 'wait' for a given η was substituted for e^* in the experimental instructions.

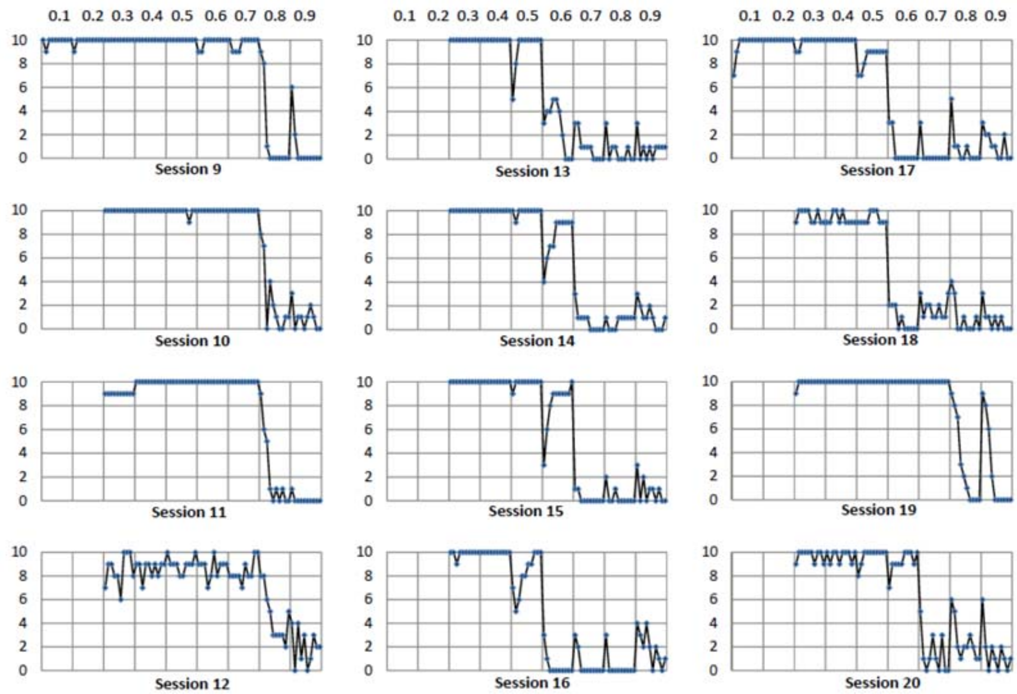


Figure 17: Arifovic et al. (2013) results for different values of η, values of η are given on top of each panel, and number of subjects who chose to 'wait' on the y-axis

is another important coordination problem, where heterogeneity in agent behavior plays an important role in equilibirum selection.

ADJ develop a model of the introduction of a new payment instrument, "e-money," that competes with the existing payment method, "cash". The new payment method is more efficient for both buyers and sellers in terms of per transaction costs. Such cost-saving motive lies behind the various attempts to introduce a new payment method.

In their theoretical environment, there are a large number of buyers (consumers) and sellers (firms) in the market, each of unit measure. Each seller $i \in [0, 1]$ is endowed with 1 unit of good i. The seller derives zero utility from consuming her own good and instead tries to sell his good to buyers. The price of the good is fixed at one. Each buyer $j \in [0, 1]$ is endowed with 1 unit of money. In each period, the buyer visits all sellers in a random order. The buyer would like to consume one and only one unit of good from each seller, and the utility from consuming each good is $u > 1$.

There are two payment methods: cash and e-money. Each cash transaction incurs a cost, τ_b, to buyers, and a cost, τ_s, to sellers. The per transaction costs for e-money are τ_b^e and τ_s^e for buyers and sellers, respectively. Sellers have to pay an up-front cost, $F > 0$, that enables them to accept e-money payments.[17]

[17]For example, to rent or purchase a terminal to process e-money transactions.

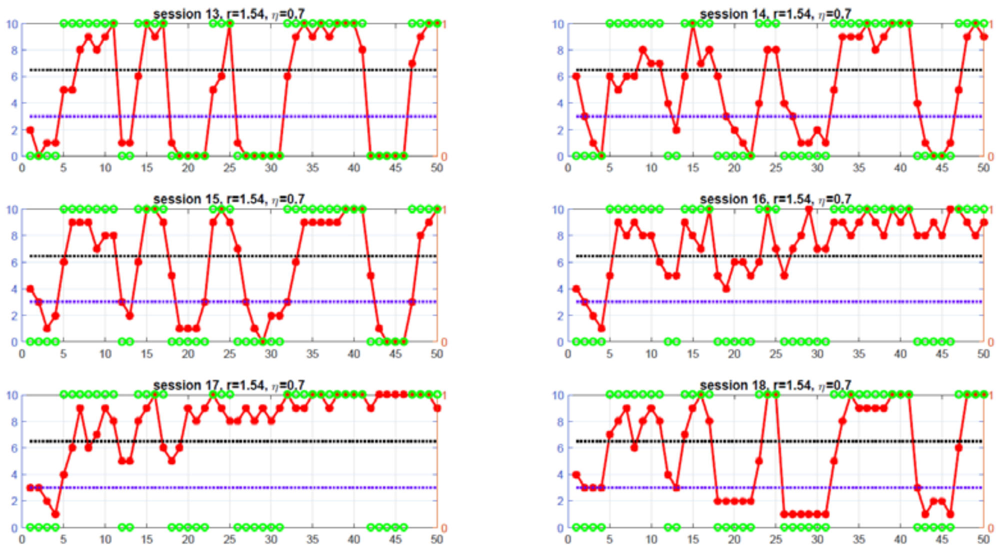


Figure 18: Arifovic and Jiang (2016) results for the value of the coordination parameter $\eta = 0.7$, $N = 10$

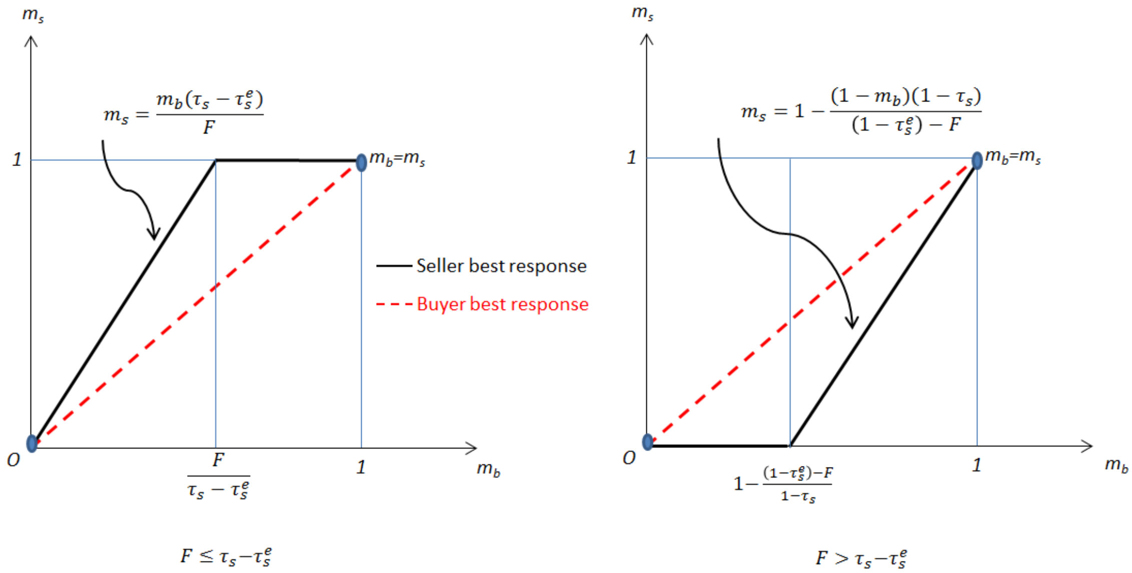
In the beginning of each trading period, sellers decide whether to accept e-money at the one-time fixed cost of F or not. Cash, being the traditional (and legally recognized) payment method, is universally accepted by all sellers. Simultaneous with the sellers' decision, buyers make a portfolio choice as to how to divide their money holdings between cash and e-money. After sellers have made acceptance decisions and buyers have made portfolio decisions, the buyers then go shopping, visiting all of the stores in a random order. When a buyer enters store i, she buys one unit of good i if the means of payment are compatible. Otherwise, there is no trade.

Under the certain assumptions about the cost structure of the model, such that e-money saves on per transaction costs for both buyers and sellers; that buyers prefer cash trading to no trading; the net benefit to the society of carrying all transactions in e-money is positive; and that $F \leq 1 - \tau_s^e$, there are exist at least two symmetric, pure strategy equilibria. See Figure (19).

In one of these equilibria, $m_b = m_s = 1$: all sellers accept e-money, and all buyers allocate all of their endowment to e-money – call this the all-e-money equilibrium (this equilibrium always exists provided that $F \leq 1 - \tau_s^e$). There is a second symmetric pure strategy equilibrium where $m_b = m_s = 0$ and e-money is not accepted by any seller or held by any buyer – call this the all- cash equilibrium. In both equilibria, there is no payment mismatch, and the number of transactions is maximized at 1. In the case where $F = \tau_s - \tau_s^e$, there exists a continuum of possible equilibria in which $m_s \in (0, 1)$ and $m_b = m_s$.

The e-money equilibrium is socially optimal as it minimizes total transactions cost. Note

Figure 19: Symmetric Equilibria



that buyers are always better off in the all- e-money equilibrium relative to the all cash equilibrium. The seller's relative payoff in the two equilibria, however, depends on the fixed cost, F, and on the savings on per transaction costs from the use of payment 2. If $F = \tau_s - \tau_s^e$, then the seller's payoff is the same in the cash and e-money equilibria; if $F < \tau_s - \tau_s^e$, then the seller's payoff is higher in the e-money equilibrium than in the cash equilibrium; finally, if $F > \tau_s - \tau_s^e$, then the seller's payoff is lower in the e-money equilibrium than in the cash equilibrium

4.2.1 Experimental Design and Results

The experimental set-up was designed to match the model as closely as possible, but without the continuum of buyers and sellers of unit mass. They conduct experiment with 14 subjects in each session, who are randomly and equally divided into roles of sellers (7) and buyers (7). The roles remained fixed during a session. The subjects played a repeated, market game, that consisted of 20 markets per session.

Each market consists of two stages. The first stage is a payment choice stage. Each buyer was endowed with seven experimental money (EM) units and had to decide how to allocate his/her seven EM between the two payment methods. At the same time, sellers decide whether or not to accept the new payment method; they always accept the old payment method, cash in the experiment.

During the second stage, each buyer visits each seller in a randomly determined order. Depending on whether or not a seller accepts payment 2 and what a buyer has in their portfolio the transition may or may not take place.[18]

In addition to making payment choices in the first stage, subjects were also asked to forecast other participants' payment choices for that market.

They main treatment variable is the fixed cost to accept the new payment method, T, which a seller has to pay at the beginning of each period if she accepts payment two. They use three different values, $T = 1.6$, 2.8, and 3.5, and conduct 4 sessions for each treatment. In all treatments $\tau_s^e = \tau_b^e = 0.1$, and $\tau_s = \tau_b = 0.5$

Their results show that the new payment method will take off if the fixed cost is low so that both sides benefit by switching to the new payment method. If the fixed cost is high such that the seller endures a loss in the equilibrium where the new payment method is used relative to the equilibrium where it is not accepted, some sellers nevertheless respond by accepting the new payment method initially, fearing to lose business, but they mostly eventually learn over time to resist the new payment method and pull the economy back to the old payment method. If neither side displays much will-power to move behavior toward one equilibrium or the other, then the economy may linger in the middle ground between the two equilibria.

4.2.2 Learning and Prediction

The theoretical model described above is static and thus, cannot account for diverse patterns of behavior observed in experimental economies. It also has multiple equilibria, and the theory does not provide guidance as to which equilibrium will be selected. On the other hand the adoption of a payment method is inherently a dynamic process. ADJ experiment suggests that this dynamic process involves some learning over the repeated markets of our design. The heterogeneity of subjects' expectations and subsequent action, their adaptation and interaction results in selection of different outcomes.

In order to model this dynamic environment inhabited by agents with heterogeneous expectation, ADJ implement the IEL in order to emulate their experimental environment. Thus, each of the seven artificial buyers and sellers in their evolutionary model has as a set of J rules; each rule consists of a single number. For buyer i, $i \in \{1, 2, ..., 7\}$, a rule $m^i_{b,j}(t) \in \{0, 1, ..., 7\}$, where $j \in \{1, 2, ..., J\}$, represents the number of EM units the buyer places in e-money in market t. For seller $i \in \{1, 2, ..., 7\}$, a rule $m^i_{s,j}(t) \in [0, 1]$, where $j \in \{1, 2, ..., J\}$ and J is the total number of rules, represents the probability that the seller accepts e-money.

The updating of the algorithm is done in a usual way (the pseudo code is given in the Appendix), and involves *experimentation*, *computation of hypothetical payoffs*, *replication*, and the selection of the action that is actually used in a given market. The way the hypothetical payoffs are computed is key to the algorithms good performance in capturing the

[18]If seller accepts both payments, then the transaction will always take place.

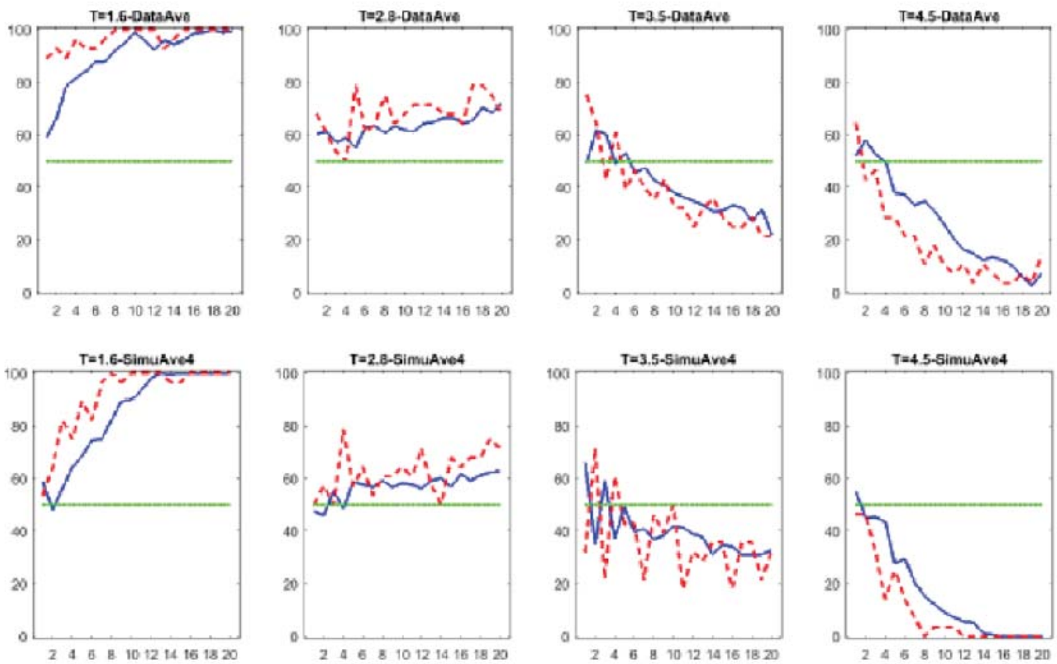


Figure 20: Experimental Data and Simulated Data for Four Treatments of Arifovic et al. (2017).

dynamics observed in the experimental sessions. [19]

They first simulated the model for the values of T that were used in the experiments. The simile tons showed a really good fit to the experimental data. Given the model's good performance, ADJ used it to investigate what minimum value of T has to be to have the IEL converge to the payment 1 equilibrium. They found that this happens when $T = 4$. Then, they tested the model in the new experimental treatment with $T = 4.5$. The experimental sessions converge very close to the all-payment-1 equilibrium.

Figure (20) presents the time path of payment choice averaged across the experimental sessions and across the simulated sessions. The first row of panels represent the averages over 4 sessions for each T, the second row of the panels are the averages over the selected number of 4 simulations. The third row represents averages over 50 IEL simulations for each T. the same number of simulations for each T over.

In figure 20 we present data from our experiments and our IEL simulations, for four values of T.

[19]Space limitations do not allow us to describe the algorithm in full detail, but interested readers can look at ADJ, 2017.

5 Conclusion

Heterogeneity in agents' expectations and decision-making is well documented in the experimental literature. The evidence we have surveyed in this chapter makes a solid case for the use of heterogeneous agent models to better comprehend such questions as lifecycle consumption and savings, contributions to public goods and expectation formation. Heterogeneous agent models are also important for understanding how agents approach coordination problems and predicting which equilibrium they may choose to coordinate upon. Two computational models of heterogeneous agent behavior, the individual evolutionary model and the heuristic switching models are shown to do well in terms of their fitting to experimental data.

Nevertheless, these heterogeneous-agent models have their limitations. The heterogeneity they capture is all within an individual. For instance, the heuristic switching model is essentially a homogeneous agent model where different heuristics are applied probabilistically. The IEL model imposes heterogeneity exogenously according to the genetic operators. Future work on the modeling of heterogeneous agents could explore how heterogeneity takes place both *between* and within individuals and how such heterogeneity might arise endogenously rather than being exogenously imposed.

The evidence we provide on heterogeneity also stands in contrast to much agent-based modeling and macroeconomic researchers solving heterogeneous agent models using computational methods. The latter seek to replicate heterogeneity observed in data at the *aggregate level*, e.g., wealth distributions, or the distributions of city sizes. In this effort, the empirical validity comes from the aggregate distributional or emergent outcomes that their models are able to generate and *not* from the empirical validity of individual-level heterogeneous agent characteristics. In our view, the more micro-level validation of individual behavior that we focused on in this survey is an important check on the internal validity of heterogeneous agent models and should be viewed as complementary to the more aggregate-level validation.

We conclude by emphasizing that heterogeneous agent behavior is commonplace and not something that needs to be fixed. Sometimes heterogeneity is a persistent phenomenon, as in differences in cognitive abilities or types, but at other times heterogeneity may be a more transient phenomenon, as in the process by which agents come to learn a strongly stable rational expectations equilibrium. If one's concern is with long run behavior in stationary environments that are not subject to shocks, then heterogeneous agent models might not hold much appeal. But in a world that is frequently buffeted by shocks, the short-to-medium-run reactions of agents might be more the phenomena to study, and such data can be collected in the laboratory and used to construct and validate heterogeneous agent models.

Even in the stationary environments that are not subject to shocks, in cases where there is a strong positive feedback (near unit root), heterogeneity of expectations matters, and coordination on boom and bust cycles may arise and may be explained by heterogeneous agent models.

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