Comment on "Adaptive Learning and Monetary Policy Design" *

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The original motivation for investigating the stability of rational expectations equilibria (REE) under learning was to provide a microfoundation for how agents might come to hold rational expectations. 'Learnability' (or expectational stability) of REE was later proposed as an equilibrium selection criterion in environments with multiple equilibria. In this paper, Evans and Honkapohja survey and address frontier issues in a very promising new application of learning analysis: the design of monetary policy rules.

This new application has been made possible by the introduction of a simple, linear, microfounded "New Keynesian" model (described, e.g., in Clarida et al. (1999)), where monetary policy plays a critical role in determining the nominal interest rate. The model of the private sector consists of two equations:

$$x_t = -\varphi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t, \tag{1}$$

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t. \tag{2}$$

The "IS" equation (1) has the output gap, x_t, depending on the nominal interest rate, i_t as well as on expectations of future inflation $E\pi_{t+1}$, the future output gap Ex_{t+1}, and a demand shock, g_t. The price setting equation (2) derives from the staggard sticky price literature and has current inflation, π_t, dependent on the current output gap, expected future inflation and a supply shock u_t. The model is closed by the addition of a policy rule that the monetary authority uses to set the nominal interest rate, i_t.

The presence in this model of expectations of future inflation and output leads naturally to questions regarding the determinacy or indeterminacy of the REE. But it leads just as naturally to questions concerning the stability of REE under adaptive learning behavior. Evans and Honkapohja rightfully place learnability of REE at the same level of importance as determinacy, though the two are independent criteria.

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Satisfaction of both criteria is highly desirable. Determinacy means that a REE is locally unique so that standard comparative statics exercises can be conducted. Learnability implies that agents need not initially know the REE so long as their model specification –their "perceived law of motion"– nests the REE solution as a special case. If the parameter updating procedure leads agents to the REE, that equilibrium is pronounced learnable, otherwise it is not learnable (it is expectationally unstable). Learnability of REE under a particular monetary policy rule can be further distinguished from *transparency* of monetary policy; the latter is concerned with how well the private sector understands the goals and instruments of monetary policy. By contrast, in the learning analysis of Evans and Honkapohja, private sector agents need not be aware that a monetary authority even exists so long as their perceived laws of motion are of the same form as the reduced form equations of the model environment.

Evans and Honkapohja primarily focus on determinacy and learnability of REE in the New Keynesian model under *optimal* monetary policy rules, i.e. rules derived from minimization of a policy loss function over an infinite horizon with discounting, where the loss function in period t is $L_t = (\pi_t - \bar{\pi})^2 + \alpha x_t^2$, and $\bar{\pi}$ is a target level of inflation. They also examine an interest rate rule introduced by McCallum and Nelson (2000) that seeks to approximate optimal monetary policy – which they call the "approximate targeting" (AT) rule. By contrast, Bullard and Mitra (2002) consider the determinacy and learnability of REE under *instrument rules*, consisting of versions of Taylor's rule. Optimal policy rules are derived under assumptions that the policy maker can or cannot commit to maintenance of the policy in future periods. In either case, optimal policy rules are shown to come in two varieties, "fundamentals based" (FB) and "expectations based" (EB); the EB rule nests the FB rule, but adds private sector expectations of future inflation and output. Evans and Honkapohja find that FB rules do not always lead to determinate REE and furthermore, these REE are never expectationally stable. The latter finding means that if agents did not initially possess knowledge of the REE and monetary policy followed the FB rule, then agents' parameter updating process would, over time, lead them further away from the REE, an undesirable outcome. On the other hand, the EB rules, as well as McCallum and Nelson's AT rule yield REE that are both learnable and determinate assuming the data necessary to implement these rules are observable. The rest of the paper is devoted to frontier issues concerning the robustness of these findings using different versions of the EB and AT rules, in the case of commitment, and comparing the welfare properties of the various versions of these two rules. My comments pertain more generally to this new application of learning and the path Evans and Honkapohja have taken (Evans and Honkapohja (2002,2003)), and are less concerned with the specific frontier issues tackled at the end of this paper.

I note first that, while the New Keynesian model used is an important benchmark model, and its forward–looking nature is what makes analysis of learning behavior possible, the model gives primary importance to monetary policy in the determination of interest rates. The scope for policy to affect interest rates is likely to be more limited in a model, unlike the New Keynesian one, that also included private sector capital accumulation, as arbitrage considerations would require that real interest rates were equal to the marginal product of capital. It would be useful to consider the robustness of the learning and determinacy findings spelled out in this paper to such richer environments before taking too seriously the recommendations that follow from studying learning in the simple New Keynesian model, e.g. that monetary policy should condition on private sector expectations. To do that, one would have to include capital in the representative household's maximization problem, and derive a new loss function objective that approximated the expected utility function of the household in this new environment.

Second, while it is straightforward to check the eigenvalue conditions associated with determinacy or stability of REE under learning, the *economic intuition* for why certain types of rules – the EB and AT rules – satisfy these conditions – and other rules – the FB rules – do not is less clear. Consider the issue of indeterminacy. As noted earlier, indeterminacy becomes an issue in environments, such as the New Keynesian model, where current realizations of variables depend on their expected future values. Using this same New Keynesian model, Bernanke and Woodford (1997) argued that if policy rules conditioned on private sector expectations, the likelihood of indeterminacy should *increase* as expectations are given even more weight in the determination of the REE. That logic seems quite intuitive. But for the optimal policy rules examined in this paper, that logic is undone; the EB rules, where the policy maker conditions on private sector expectations, are the *only* optimal rules that are both determinate and learnable. There are important differences between Evans and Honkapohja' setup and that of Bernanke and Woodford, that might account for the differences. By contrast with Bernanke and Woodford's setup, the policymaker in the Evans and Honkapohja environment is using an optimally derived policy rule and is not trying to learn something about fundamentals from the private sector forecasts. Instead, the policy maker *knows* the fundamentals - the shocks u_t and g_t and is conditioning both on these fundamentals and private sector forecasts; therein may lie the explanation for the different findings of the two papers, though again, the precise mechanism leading to the dramatically different findings is unclear. Similarly surprising is the conclusion that whether optimal policy is derived under commitment or discretion is unimportant for the learning results (though not for the determinacy results); the FB rules are never expectationally stable while the EB rules are always expectationally stable, regardless of whether or not the central bank operates under commitment. Intuition would suggest that the optimal policy rule under commitment, with its inclusion of a lagged value of the output gap, would aid in the learning of the REE; the lagged output gap would serve as a brake on variations in the nominal interest rate arising from current period shocks; this intuition does not always hold, e.g in the case of FB rules, and it is not so clear why.

Third, optimal monetary policy rules, by contrast with instrument rules such as Taylor Rules, are derived via minimization of a loss function objective that is an approximation to the same expected utility function that the representative household uses to solve its maximization problem and which leads to the linearized "structural equations" of the model. It is hard to object to policy rules thus derived. But one suspects that the popularity of *instrument rules*, such as Taylor rules, derives in no small part from policymaker's uncertainty about the model environment in which they operate. In the environment of Evans and Honkapohja, the monetary authority knows the true model and it is the public that must learn the REE of this model. It would be interesting to examine the robustness of the learning results to situations where policy makers face model uncertainty, while staying within the optimal policy rule framework. One step in this direction, which is pursued in this paper, is to suppose that policymakers must learn the structural parameters λ and φ. But one could go further and imagine that policymakers are uncertain about the specification of the structural equations of the model. For example, policymakers might be uncertain whether the true model involves forward– or backward– looking behavior; they might perceive the structural equations to be of the more general form suggested by Clarida et al. (1999)

$$\begin{aligned} x_t &= -\varphi(i_t - E_t\pi_{t+1}) + \theta x_{t-1} + (1-\theta)E_t x_{t+1} + g_t, \\ \pi_t &= \lambda x_t + \phi\pi_{t-1} + (1-\phi)\beta E_t\pi_{t+1} + u_t, \end{aligned}$$

and have to learn the true specification, i.e. that $\theta = \phi = 0$.

Finally, it would be interesting to consider the consequences of *policy–smoothing* for the learnability and determinacy of REE. Policy-smoothing involves purposeful dampening of the interest rate target by the monetary authority which might serve to promote learnability of the REE. In the case of Taylor-type instrument rules, policy–smoothing is typically modeled by incorporating a lagged nominal interest rate to the policy rule, e.g.

$$i_t = \rho i_{t-1} + (1-\rho)\left(\chi_\pi\pi_t + \chi_x x_t\right),$$

where $0 < \rho < 1$. Bullard and Mitra (2001) consider whether instrument rules of this type lead to REE that are learnable and determinate. In the optimal policy rule framework that Evans and Honkapohja consider, such an analysis has not been conducted. To do it, one must introduce an alternative policy loss function that, according to Woodford (1999), is an equally good approximation to the expected utility of the household. This alternative loss function incorporates an interest–rate smoothing objective. Specifically, suppose the policymaker's objective function is:

$$E_t\left\{\sum_{t=0}^{\infty}\beta^t\left[(\pi_t - \bar{\pi})^2 + \alpha_x x_t^2 + \alpha_i(i_t - \bar{i})^2\right]\right\}, \tag{3}$$

where $\bar{i}$ is a target nominal interest rate, and $\alpha_x > 0$, $\alpha_i > 0$ represent relative weights. With this objective function, the optimality condition will be altered. Consider first the simplest case where there is no commitment, and set $\bar{\pi} = 0$. In this case, the policy maker maximizes (equivalently minimizes) (3) subject to versions of equations (1-2) modified for the case of no commitment:

$$x_t = -\varphi i_t, \tag{4}$$

$$\pi_t = \lambda x_t. \tag{5}$$

The first order conditions with respect to π_t, x_t and i_t are:

$$\begin{aligned} 2\pi_t + \eta_t &= 0, \\ 2\alpha_x x_t + \omega_t - \lambda\eta_t &= 0, \\ 2\alpha_i(i_t - \bar{i}) + \varphi\omega_t &= 0, \end{aligned}$$

where ω_t and η_t are the Lagrange multipliers associated with the constraints (4) and (5), respectively. Combining these first order conditions, the optimality condition is now written as

$$\lambda\pi_t + \alpha_x x_t - \alpha_i\varphi^{-1}(i_t - \bar{i}) = 0, \tag{6}$$

which differs from equation (7) in Evans and Honkapohja's paper. The result is a different class of optimal policy rules. In particular, one can rearrange (6) to get the rule:

$$i_t = \bar{i} + \frac{\varphi\lambda}{\alpha_i}\pi_t + \frac{\varphi\alpha_x}{\alpha_i}x_t, \tag{7}$$

which looks a lot like Taylor's rule, though here it is *optimally* derived. As the interest rate rule (7) depends on only on the variables π_t and x_t, which are assumed to be known at date t, and not on expected future values of these variables, this rule can lay claim to being a "fundamentals –based" rule. The optimal rule (7), differs from Evans and Honkapohja's FB rule which, in the case of discretionary policy, depends only on the known shocks g_t and u_t. While (7) is similar to McCallum and Nelson's AT rule, it differs from the latter in that it is optimally derived.

The private sector's perceived law of motion can be written as:

$$y_t = a + cv_t,$$

where

$$y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}, \quad v_t = \begin{pmatrix} g_t \\ u_t \end{pmatrix}, a = \begin{pmatrix} a_x \\ a_\pi \end{pmatrix}, \quad \text{and } c = \begin{pmatrix} b_x & d_x \\ b_\pi & d_\pi \end{pmatrix},$$

In Evans and Honkapohja (2003) $b_x = b_\pi = 0$ but in the case considered here, these coefficients will be nonzero.

To assess determinacy and stability of the REE under learning, we follow Evans and Honkapohja and combine the interest rate rule (7) with (1) and (2) to obtain the reduced form:

$$y_t = A + ME_t y_{t+1} + Pv_t,$$

where y_t and v_t are the 2×1 vectors defined above, and A, M, and P are conformable vectors and matrices with elements that are combinations of structural parameters.

As noted by Evans and Honkapohja, determinacy of the REE obtains if both eigenvalues of M are less than unity. In this case, we have

$$M = \frac{1}{\xi}\begin{bmatrix} \alpha_i & \varphi(\alpha_i - \lambda\varphi\beta) \\ \lambda\alpha_i & \varphi(\lambda\alpha_i + \beta\varphi\alpha_x) + \beta\alpha_i \end{bmatrix},$$

where $\xi = \alpha_i + \varphi^2(\alpha_x + \lambda^2)$. As the eigenvalues of this matrix do not yield clear analytical results, we investigate them numerically. We adopt Woodford's (1999) calibration, which is the only calibration that gives values for both α_x and α_i, specifically:

$$\beta = .99, \varphi = (0.157)^{-1}, \lambda = .024, \alpha_x = .047, \alpha_i = .233, \text{ and } \mu = \rho = 0.35.$$

Using this calibration we find that both eigenvalues of M are less than unity so the REE is determinate. Evans and Honkapohja note that in the case of the reduced form studied here, determinacy of the REE also implies that the REE is expectationally stable. The E-stability conditions are that (i) all eigenvalues of M have real parts less than unity and (ii) all products of eigenvalues of M times eigenvalues of F have real parts less than unity. The matrix F, as seen in equation (3) of Evans and Honkapohja's paper, is just the matrix of autoregressive parameters for the shock processes and has eigenvalues equal to μ and ρ which satisfy $0 < |\mu| < 1$, $0 < |\rho| < 1$ by assumption. If all eigenvalues of M are less than unity it follows that all products of eigenvalues of M times eigenvalues of F have real parts less than unity. Hence, we have found an example of an optimally derived, fundamentals-based rule that results in a determinate and learnable REE; in particular, the rule does not require the monetary authority to condition on private sector expectations.

The case of commitment remains to be examined. That case is more difficult, as maximization of (3) subject to (1-2) under rational expectations yields the optimality condition

$$\lambda\pi_t + \alpha_x(x_t - x_{t-1}) - \lambda\alpha_i\beta^{-1}\bar{i} - \alpha_i\varphi^{-1}i_t + \alpha_i(\beta\varphi)^{-1}(\beta + \lambda\varphi + 1)i_{t-1} - \alpha_i(\beta\varphi)^{-1}i_{t-2} = 0,$$

which differs from the commitment optimality condition (c.f. equation (8) in Evans and Honkapohja's paper) by the inclusion of the nominal interest rate i at three different dates! Rearranging, we have the optimal policy rule studied by Giannoni and Woodford (2003):

$$i_t = -\frac{\lambda\varphi}{\beta}\bar{i} + \frac{\lambda\varphi}{\alpha_i}\pi_t + \frac{\varphi\alpha_x}{\alpha_i}(x_t - x_{t-1}) + \left(\frac{\beta + \lambda\varphi + 1}{\beta}\right)i_{t-1} - \frac{1}{\beta}i_{t-2}. \tag{8}$$

We leave the stability of the REE under learning for this more complicated optimal policy rule as an exercise for future research. Additionally, it would be of interest to examine variations on the rules (7) and (8) that did not require observations on contemporaneous x_t and π_t.

As these comments suggest, I am intrigued by the findings from this new application of learning theory and think several extensions are worth pursuing. Regarding the extensions that Evans and Honkapohja have pursued in the second half of their paper, I think they are both necessary and well–done. I congratulate the authors on an informative survey and extension of the literature and would urge policymakers, in particular, to give careful attention to the important issues raised in this paper.

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