All-Pay Auctions Versus Lotteries as Provisional Fixed-Prize Fundraising Mechanisms: Theory and Evidence*

John Duffy† Alexander Matros‡

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Abstract

We compare two fixed-prize mechanisms for funding public goods, an all-pay auction and a lottery, where public good provision can only occur if the participants' contributions equal or exceed the fixed-prize value. We show that the provisional nature of the fixed-prize means that efficiency and endowment conditions must both be satisfied to assure positive public good provision. Our main finding is that provisional fixed-prize lotteries can outperform provisional fixed-prize all-pay auctions in terms of public good provision in certain cases where efficiency holds and endowments are large relative to prize values. We test these predictions in a laboratory experiment where we vary the number of participants, the marginal per capita return (mpcr) on the public good, and the mechanism for awarding the prize, either a lottery or an all-pay auction. Consistent with the theory, we find that the mpcr matters for contribution amounts under the lottery mechanism. However, inconsistent with the theory, bids are significantly higher than predicted and there is no significant difference in the level of public good provision under either provisional, fixed-prize mechanism. We consider several different modifications to our framework that might help to explain these departures from theoretical predictions.

Keywords: All-pay auction, lottery, public goods, fixed-prize mechanisms, fundraising, experiment.
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†Department of Economics, University of California, Irvine. Email: duffy@uci.edu.

‡Moore School of Business, University of South Carolina and Lancaster University Management School. Email: alexander.matros@gmail.com

1 Introduction

How should a public good be financed? Since Morgan (2000) first demonstrated that fixed-prize charitable lotteries could yield greater contributions to a public good than the voluntary contribution mechanism (VCM), several papers have sought to confirm this result in the laboratory and the field, see, e.g., Morgan and Sefton (2000), Lange et al. (2007), Orzen (2008), Schram and Onderstal (2009), Corazzini et al. (2010), and Onderstal et al. (2013) among others. Moreover, several studies, see Orzen (2008), Schram and Onderstal (2009), Corazzini et al. (2010), and Onderstal et al. (2013) compare public good provision under fixed-prize lotteries and various types of fixed-prize auctions. A common, (though not universal) finding in this literature is that fixed-prize fundraising mechanisms generally outperform the VCM in terms of raising funds, though the various studies do not all agree on which mechanism for awarding the fixed-prize works best.

In this paper, we compare the performance of two provisional fixed-prize fundraising mechanisms – a lottery and an all-pay auction – where the value of the fixed prize must be financed by the fundraising mechanism itself. Under the provisional, self-financing fixed prize mechanisms we study, if endowments are not sufficiently large or if the mechanism does not raise funds sufficient to cover the amount of the fixed prize, then there is no public good provision and any contributions are refunded. Such provisional, self-financing fixed-prize mechanisms are attractive, as the fundraiser does not bear any risk as to whether the cost of the prize awarded (e.g., a car) can be covered by the contributions of participants.[1] Indeed, we do not observe non-provisional fixed-prize charitable fundraising mechanisms in the field unless the prize itself is a charitable donation. For instance, many charitable fundraising mechanisms have the structure of parimutuel-betting systems in that the prize amount is endogenously determined and equal to some fixed percentage of the total amount raised by the mechanism (typically a lottery). For example, in the U.S. many small charitable organizations sell tickets to "50-50" lotteries where the endogenously determined monetary prize is 50 percent of the total value of all tickets sold; the remaining 50 percent goes to the charity.

However, as Morgan (2000) has clearly shown, pure parimutuel prized-based lottery mechanisms have the disadvantage that they generate contribution incentives equivalent to those of the VCM, that is, the equilibrium public good provision under a parimutuel-prize lottery mechanism is the same as under the VCM. Morgan shows that an alternative mechanism that avoids this problem is a provisional but *fixed*-prize fundraising mechanism and this type of mechanism is the focus of our paper. Provisional fixed-prize fundraising mechanisms are also found in the field. A typical setting is a fundraising drive by a local charitable organization that asks for donations from a small, finite population of n potential contributors and offers a fixed cash prize or a prize of fixed value (e.g., a car) provided that funds are raised that are sufficient to cover the value of the prize. For example, the Rotary Club of Pawtucket, Rhode Island holds an annual fundraiser in the form of a raffle. In the May 2011 raffle[2] the grand prize was fixed at $10,000 cash. Three hundred tickets were offered at $100 each and all ticketholders were also invited to a luncheon. The contest rules clearly state that "If less than 150 tickets are sold, all ticket money will be refunded and the drawing will

[1] Indeed, these provisional fixed-prize mechanisms are similar to non-prize-based provision-point public good mechanisms (e.g. Bagnoli and Lipman, 1989) in which the public good is not provided unless contributions exceed a certain threshold.

[2] Details available at http://pawtucketrotaryfundraiser.eventbrite.com/
Rotary clubs are charitable organizations of business and professional leaders that are devoted to providing health and education services and to alleviating poverty.

not take place." In this contest, 276 tickets were sold so the raffle was held. After subtracting the \$10,000 prize, the charity netted \$17,600 less the cost of the luncheon. This is an example of the kind of provisional fixed-prize lottery mechanism that we study in this paper.

Our main focus is on whether lottery or all-pay auction provisional fixed-prize mechanisms are better for public good provision in settings where all players are ex-ante identical and have equal budget constraints. In this setting we provide an overview of theoretical results showing that a necessary condition for positive public good provision under both provisional fixed-prize mechanisms is that the endowments of the n contributors must be sufficiently high to finance the prize and that public good provision must be efficient. If public good provision is not efficient or if the endowment conditions are not satisfied, then equilibrium contributions to the public good will be zero under *both* provisional fixed-prize mechanisms (lottery or all-pay auction).[3] We further calculate the prize value that maximizes public good contributions under both provisional fixed prize mechanism. Finally, we demonstrate that if public good provision *is* efficient and if individual endowments are very large relative to the provisional fixed-prize amount, then any pure or mixed strategy equilibrium under the all-pay auction mechanism will involve *zero* public good provision while the unique symmetric pure strategy equilibrium under the lottery mechanism under these same endowment and efficiency conditions will always yield *positive* public good provision. That is, our main theoretical finding is that under certain empirically plausible endowment conditions, the use of a provisional-fixed prize lottery mechanism can outperform the use of a provisional fixed prize all-pay auction mechanism in terms of public good provision. This finding is new to the literature. It stands in contrast to results (discussed in section 2) showing that all-pay auctions for awarding *exogenously* given fixed prizes generate greater public good provision than do lotteries. The key reason for our different finding is the assumption that the fixed-prize is *not* exogenously given, but is only provisionally provided if contributions equal or exceed the value of the fixed-prize. Based on our theoretical findings, we argue that when endowments are large relative to prize values, which in many instances may be a reasonable assumption (e.g., if endowments are viewed as liquid assets), then lotteries might be preferred to all-pay auctions as fundraising mechanisms in the provisional, fixed-prize environment that we study.

In addition to pointing out some theoretical differences between self-financed (provisional) and exogenous (non-provisional) fixed-prize fundraising mechanisms, we have also conducted an experiment to test some of the comparative statics implications of the theory that we developed for provisional, fixed-prize fundraising mechanisms. We use a $2 \times 2 \times 2$ experimental design where the treatment variables are: (1) all-pay auction or lottery rules to determine the prize winner; (2) group size $n = 2$ or $n = 10$ and (3) marginal per capita return (*mpcr*) on the public good, $\beta = .25$ or $\beta = .75$. A novelty of our study over existing experimental studies is that we vary both the group size, n, and the mpcr, β, in addition to comparing the two different provisional, fixed prize fundraising mechanisms and we also consider the role of efficiency.

Our theoretical findings indicate that, under certain conditions, public good provision can be greater under the provisional, fixed-prize lottery mechanism than under the corresponding all-pay auction mechanism for the same number of participants, n, and marginal per capita return (*mpcr*) on the public good. Under other conditions, the reverse is true – see the discussion in sections 3-5. We focus on the environment where the lottery is predicted to outperform the all-pay auction.

[3] Under an alternative assumption of exogenously given (non-provisional) fixed prizes, the use of lotteries or all-pay auctions to award the exogenously given fixed prize does not require that public good provision be efficient for there to exist an equilibrium with positive public good provision.

Our experiment has yielded the following findings. First, consistent with the theory, contributions to the public good increase with the *mpcr* under the provisional fixed-prize lottery mechanism. However, opposite to the theory, contributions to the public good increase with the *mpcr* in the all-pay auction mechanism and, for a given *mpcr*, the average amount bid by each participant increases with the total number of participants, n, under both fixed-prize mechanisms. Consequently, public good provision also increases with n, a finding that *is* consistent with theoretical predictions for the lottery mechanism but is *in*consistent with theoretical predictions for the all-pay auction mechanism; in the latter, for a given *mpcr*, expected public good provision should remain at zero as n increases. Further, for most values of n and the *mpcr* considered in this experiment, the amounts bid are significantly greater than theoretical predictions, though there is some decline in bids as subjects gain experience. Finally, and most significantly, for any given *mpcr* and n, we find *no significant difference* in the amount of public good provision under either provisional, fixed-prize mechanism. Thus, despite a theoretical prediction of zero contributions under the parameterization of the all-pay auction mechanism that we study, we find that contributions under that mechanism are significantly positive and no different from those under the lottery mechanism where positive contributions are predicted. We offer several explanations for the excessive contributions that we observe and for the absence of any difference in bidding behavior across the two mechanisms. These include learning, risk aversion, joy-of-winning preferences, departures from expected utility maximization and biased judgments about the game being played.

The rest of this paper is organized as follows. Section 2 situates our paper in the theoretical and experimental literature. Sections 3-5 present the theory. Section 6 describes our experimental design and section 7 reports our main experimental findings. Section 8 discusses the relationship between our experimental findings and the theoretical predictions, and how differences might be resolved, and section 9 offers a summary and conclusions. Proofs of some propositions are found in Appendix A. Appendices B-C provide some additional theoretical results. Sample experimental instructions are provided in Appendix D.

2 Related Literature

There are several prior theoretical and experimental studies of lotteries and/or auctions as fundraising mechanisms that we build upon or that are related to this paper. Morgan (2000) initiated the literature by exploring the performance of both provisional and non-provisional fixed-prize and parimutuel lottery mechanisms and he provides conditions under which fixed-prize lotteries outperform the VCM in public good provision. Morgan and Sefton (2000) provide experimental evidence in support of Morgan's (2000) theoretical predictions. Importantly, Morgan and Sefton (2000) adopt an experimental design where the fixed prize offered is *not* provisional on contributions being sufficient to cover that prize amount. They note (in footnote 6, p. 787) that "while this assumption [of a non-provisional prize] is patently unrealistic, the results are unchanged by more realistically allowing the raffle [lottery] to be called off and the bets returned in the event that insufficient wagers are made." While it is possible that results are unchanged by making the fixed-prize provisional, this is not a general result. Morgan (2000)'s results clearly require that public good provision be efficient for the fixed-prize mechanism to generate positive public good provision. Here we emphasize the importance of this efficiency condition, showing that it plays a critical role in public good provision under provisional fixed-prize mechanisms, and in our experiment, we also consider treatments where public good provision is or is not efficient.

Goeree et al. (2005) compare lotteries with auctions in the case where bidders have *independent private values* for a prize object and where all proceeds from the fundraising mechanism accrue to a public good (charity) for which all bidders derive some benefit. In their setting, the prize object is exogenously given (e.g., a donated good) and thus not provisional on the amounts bid.[4] They observe that while lotteries may be preferred to *winner-pay* auctions, lotteries are always inefficient and may generate less revenue when compared with k-price *all–pay* auctions, where the winner is the individual submitting the highest bid, the k highest bidders pay the k-th highest bid and all other (lower) bidders pay their bids. Goeree et al. provide conditions under which the lowest-price all pay auction is the optimal fundraising mechanism in that it generates the most revenue and assures that the prize is awarded to the individual with the highest valuation. Schram and Onderstal (2009) experimentally compare lotteries, winner-pay and all-pay auctions where, as in Goeree et al. (2005), participants have independent private values for a non-provisional prize good. Schram and Onderstal report that all-pay auctions outperform the other two mechanisms in charitable fundraising. By contrast, our paper compares lotteries and first-price all–pay auctions under *complete* information, where all participants are certain of the value that others assign to winning the provisional fixed prize. This setting, while simple, allows us to derive equilibrium predictions for bidding strategies and expected public good provision levels as functions of the number of bidders, n, and the marginal per capita return on the public good, β.

Orzen (2008) theoretically and experimentally compares the VCM with non-provisional lotteries and all-pay auctions. His theoretical findings for the all-pay auction are different from ours as he allows negative public good provision, i.e., if the prize is not covered by contributions, the public good provided can be negative. Moreover, the setting of Orzen's experiment is also different. He considers a single parameterization of the model, $\beta = 0.5$ and $n = 4$ and compares how different fundraising mechanisms perform for those parameters. Like us, he finds no significant difference between lotteries and all-pay auctions for the parameterization that he studies. Corazzini et al. (2010) also compare the VCM with non-provisional lotteries and all-pay auctions when $\beta = 0.5$ and $n = 4$, but they are more interested in the effect of heterogeneity in individual endowments. They report that in their setting contributions are significantly higher under the non-provisional fixed-prize lottery than under the non-provisional fixed-prize all-pay auction. Faravelli and Stanca (2014) compare non-provisional lotteries and all–pay auctions with or without a public good component. They find (as we do) that the difference in bidding behavior between the two mechanisms is greatly reduced when bidding is for a public good that enters into players' payoff functions relative to the other case they study of pure rent-seeking contests (no public good). Damianov and Peeters (2018) report on an experiment similar to Orzen's, but in a setting where there are some bystander bidders who have no budget to contribute to the public good but who benefit from it nonetheless. In this setting they show theoretically that the lowest-bid, all pay auction maximizes expected revenue relative to a lottery or an own-bid all pay auction and they provide experimental evidence in support of these predictions.

In all of these prior experimental studies the prizes are *exogenously* given (non-provisional). The importance of making the fixed prize provisional is demonstrated by the findings of Landry et al. (2006) and Lange et al. (2007). In a field experiment, Landry et al. (2006) report that total individual donations were less than the non-provisional fixed-prize value; for instance, their $1,000

[4] The assumption of an exogenously given prize (auction object) that is already in the possession of the charity (seller) follows the tradition in the auction literature. However, in a charity auction this assumption is not necessarily appropriate, for instance, if the charity does not initially have a prize to offer or must buy a prize.

fixed-prize lottery treatment raised just $688 in total contributions! A similar finding is reported in Lange et al. (2007) who use $\beta = 0.3$ and $n = 4$ in their experiments. They find that under a non-provisional fixed-prize lottery mechanism, total contributions were insufficient to finance the non-provisional fixed prize (total contributions averaged about one-half of the prize value). These findings point to the importance of considering *provisional* fixed-prize mechanisms. Indeed, the main difference in our experimental design from all prior *experimental* studies in the literature on prize-based mechanisms is that we award the fixed prize *only if* contributions to the public good equal or exceed the amount of the fixed prize.

Among other related papers, Giebe and Schweinzer (2014) devise lottery schemes that use taxes on private goods to finance efficient allocations to a public good in a manner that does not distort consumption of the private good. Davis et al. (2006) compare lotteries to English (winner-pay) auctions. They find that lotteries generally outperform English auctions in public good provision despite the fact that revenues are predicted to be the same under these two mechanisms. Gneezy and Smorodinsky (2006) consider all-pay auctions without a public good but with a prize of common value as in our design. They report (as we do) that subjects over-bid and that the auctioneer's revenues are two to three times greater than the value of the prize, even after the auction was repeated several times. Carpenter et al. (2008) compare first price all-pay charity auctions with first- and second-price charity auctions in a field experiment and report that the first-price charity auction revenue dominates the all-pay charity auction counter to theoretical predictions. They attribute their finding to greater participation in the first-price as opposed to the all-pay auction formats by the bidders in their study who (unlike this study) could decide whether or not to participate in charity auctions. Carpenter and Matthews (2017) generalize the Morgan (2000) model replacing the linear raffle contest success function with a generalized Tullock (1980) contest success function and, in a field experiment, vary whether the chances of winning accrue faster (convex case) or slower (concave case) as bidders bid more. They find that revenues are highest in the convex case and this design, as convexity increases is, in the limit an all-pay auction. Onderstal et al. (2013) compare the VCM, fixed-prize private value lottery and fixed-prize private value all-pay auction mechanisms in a field experiment. They report that the VCM raises the most money, the fixed prize lottery raises the next greatest amount and the all-pay fixed prize auction raises the least. They suggest that the prize they offered (a Nintendo DS game console and software valued at €169) in their lottery and all-pay auction treatments may have crowded out intrinsic, pro-social motivations for giving among the 300 participants in each contest. Finally, we note that Dechenaux et al. (2015) provide a survey of this experimental literature in the context of the broader contest literature.

3 Theory

We consider how two different n-player fixed-prize mechanisms affect public good provision. We first explore the case where the fixed prize, V, is awarded to the mechanism-determined winner regardless of whether total contributions equal or exceed V. Formally, in this "exogenous" or "non-provisional" fixed-prize case, player i maximizes her expected payoff, which is given by:

$$u_i\left(x_i, x_{-i}; n, \beta, M\right) = \left(e - x_i\right) + \beta\left(\sum_{j=1}^{n} x_j\right) + M\left(x_i, x_{-i}; n\right)V, \tag{1}$$

by choosing her contribution level $x_i \leq e$, where $e > 0$ is player i's endowment (e is assumed to be the same for all n participants), $0 < \beta < 1$ is the marginal per capita return on the public good (also the same for all participants), and $V > 0$ is the fixed-prize amount awarded to the winner under the mechanism, M, which is either a lottery or an all-pay auction. We will call problem (1) the exogenously given (or non-provisional) fixed-prize mechanism.

We next consider the self-financing, provisional fixed-prize mechanism that is the focus of this paper. We assume that player i maximizes her expected payoff which is given by:

$$\pi_i\left(x_i, x_{-i}; n, \beta, M\right) =$$

$$\left\{ \begin{array}{cl} (e - x_i) + \beta\left(\sum_{j=1}^{n} x_j - V\right) + M\left(x_i, x_{-i}; n\right)V, & \text{if } \sum_{j=1}^{n} x_j \geq V, \\ e, & \text{if } \sum_{j=1}^{n} x_j < V, \end{array} \right. \tag{2}$$

by choosing her contribution level $x_i \leq e$, and V is the fixed-prize amount provisionally awarded to the winner under the self-financing mechanism M. Note that the threshold level for public good provision is V. If this threshold level is not reached, then the fixed prize amount V is not awarded and all contributions are refunded so that each player's payoff is equal to her endowment, e.[5] If the threshold level is reached, $\sum_{j=1}^{n} x_j \geq V$, then the prize, V, is financed first and awarded to the winner and the remaining amount, $\left(\sum_{j=1}^{n} x_j - V\right)$, goes toward public good provision. We will call problem (2) the self-financed, provisional fixed-prize mechanism.

4 Lottery

If the mechanism, M, for awarding the non-provisional fixed prize is a lottery, then (1) becomes

$$u_i\left(x_i, x_{-i}; n, \beta, \text{Lottery}\right) = (e - x_i) + \beta\left(\sum_{j=1}^{n} x_j\right) + \left(\frac{x_i}{\sum_{j=1}^{n} x_j}\right)V. \tag{3}$$

It is well-known, see Morgan and Sefton (2000) and Orzen (2008), that there exists a unique symmetric pure-strategy equilibrium where each player spends

$$x^* = \min\left\{\frac{(n-1)}{n^2}\frac{V}{(1-\beta)}, e\right\}, \tag{4}$$

and public good provision is equal to

$$G = nx^* = \min\left\{\frac{(n-1)}{n}\frac{V}{(1-\beta)}, ne\right\}.$$

Now consider the self-financing, provisional fixed-prize lottery mechanism. In that case, (2) becomes

$$\pi_i\left(x_i, x_{-i}; n, \beta, \text{Lottery}\right) =$$

$$\left\{ \begin{array}{cl} (e - x_i) + \beta\left(\sum_{j=1}^{n} x_j - V\right) + \left(\frac{x_i}{\sum_{j=1}^{n} x_j}\right)V, & \text{if } \sum_{j=1}^{n} x_j \geq V, \\ e, & \text{if } \sum_{j=1}^{n} x_j < V. \end{array} \right. \tag{5}$$

[5] We assume there are no credibility issues with respect to whether the prize amount is raised or not, which is analogous to the standard assumption that there is no credibility issue with whether the public good is provided.

4.1 No Public Good Provision

First note that under the provisional, fixed-prize mechanism, the endowment size and the fixed-prize value become important considerations in whether the public good is provided or not. If the symmetric endowment amount e, is too small, or alternatively, if the fixed prize value, V, is too large then players cannot collect enough funds to finance the fixed-prize (even if they spend their entire endowment). In such situations, the public good is never provided. This problem does *not* arise under the non-provisional fixed prize mechanism because in that case the prize is exogenously given and *any* positive contribution leads to public good provision. Formally, consider the following endowment condition:

$$0 < e \leq \frac{V}{n}. \tag{6}$$

If condition (6) holds, then the prize value is so high that players do not have enough resources to finance the prize and as a result the public good is never provided in any equilibrium in this case.

Proposition 1 *Consider the self-financing provisional fixed-prize lottery mechanism. Suppose that endowment condition* (6) *holds. Then, any strategy profile* $(x_1, ..., x_n)$ *is a Nash equilibrium. The public good is not provided in all of these equilibria.*

The proof is straightforward and is thus omitted.

Consider next the case where $V < ne$ so that the fixed-prize value can be covered and public good provision becomes a possibility. In this case there remains what can be termed a "public-good-game" effect. The typical public good game has an equilibrium where each player contributes nothing. It turns out that the provisional fixed-prize lottery mechanism has this property as well.

Proposition 2 *Consider the self-financing provisional fixed-prize lottery mechanism. Suppose that the endowment condition*

$$\frac{V}{n} < e \tag{7}$$

holds. Then, there exists a set of pure-strategy Nash equilibria where the public good is not provided.[6] *This set can be characterized in the following way*

$$\left\{(x_1, ..., x_n) : \max_i X_{-i} \leq (1-\beta)\, V \text{ and } x_i + X_{-i} \leq V\right\},$$

where $X_{-i} = \sum_{j \neq i} x_j$.

Proof: See Appendix A.

Given the contributions of the other players $-i$, player i is indifferent between contributing nothing or contributing the amount $(V - X_{-i})$ which is exactly enough to finance the prize in any equilibrium in the set of equilibria. This indifference property holds for all players in all equilibria in Proposition 2. Contributing more than $(V - X_{-i})$ is dominated by contributing $(V - X_{-i})$.

[6] Of course, there is a continuum of mixed-strategy equilibria, where each player randomizes among her strategies in this equilibrium set. There is no public good provision in all mixed-strategy equilibria.

Proposition 2 contrasts with the equilibrium of the non-provisional fixed-prize lottery mechanism. Recall that under the latter mechanism there is a unique equilibrium where the public good is always provided and there is no public-good-game effect. The intuition for this result is easy to comprehend: since the prize is given, if all other players contribute nothing, a player prefers to contribute a small amount and therefore to win the prize and to benefit from public good provision instead of making a zero contribution.[7]

4.2 Public Good Provision

It turns out that the provisional fixed-prize lottery mechanism *does* have a unique, symmetric pure-strategy equilibrium where the public good *is* provided. A necessary condition for such an equilibrium is the efficiency condition:

$$\beta n > 1. \tag{8}$$

Note that the efficiency condition (8) does not play any role under the exogenously given fixed-prize lottery mechanism – see expression (4). The following result is a version of Morgan (2000).[8]

Proposition 3 *Consider the self-financing provisional fixed-prize lottery mechanism. The endowment condition (7) and the efficiency condition (8) both hold if and only if there exists a unique pure-strategy Nash Equilibrium (NE) $(x_1, ..., x_n)$, where the public good is provided and*

$$x_i = \min\left\{\frac{(n-1)}{n^2}\frac{V}{(1-\beta)}, e\right\}. \tag{9}$$

Public good provision (less the fixed-prize value V) is given by:

$$\widetilde{G} = nx^* - V = \min\left\{\frac{(\beta n - 1)}{n(1-\beta)}V, ne - V\right\} > 0. \tag{10}$$

From (10) we see that in the efficient case, public good provision is increasing in both n and β. However, in the inefficient case, where condition (8) does not hold, the public good is not provided under the provisional fixed-prize lottery mechanism though it would be provided if the prize were exogenously given. This difference is intuitive. If the prize is exogenously given, then players can not only win the prize, (as they do in a standard non-public good lottery), but they can also receive benefits from the public good. As a result, players find it optimal to bid positive amounts and the public good is provided. However, if the fixed prize is provisional and self-financed, then total spending has to be high enough to finance the fixed prize which can only happen if it is efficient to provide the public good in the first place. Note that the endowment also plays an important role in the success of the self-financing mechanism: the endowment must be large enough to allow players to finance the prize. The endowment size does not matter in the case of an exogenously given prize.

[7] Morgan (2000) assumes that "wealth constraints are ... to be non-binding for all consumers" and analyzes when public good is provided in an equilibrium. See his footnote 9.

[8] Morgan (2000) as well as Orzen (2008) only focus on the efficient case when condition (8) holds. Note that if the efficiency condition (8) does not hold, then all contributions are refunded and public good provision is equal to zero under a provisional, self-financed lottery. Public good provision is negative in Morgan and Sefton (2000) if the efficiency condition (8) does not hold.

Proposition 3 shows that there is a unique symmetric pure-strategy equilibrium under the provisional fixed-prize lottery mechanism involving public good provision for any fixed-prize value $V < ne$ in the efficient case. Morgan (2000) discusses the optimal prize value in the case of quasi-linear preferences. He shows that the higher is the prize, the closer is the outcome to the first-best. An obvious question in our setting with budget constraint is: given the endowment e, and assuming that efficiency holds, what prize value, V, maximizes public good provision? By contrast with Morgan's case without budget constraints, there are two competing effects in determination of the optimal prize value. First, a higher prize encourages higher individual contributions; this is captured by the term $\frac{(\beta n-1)}{n(1-\beta)}V$ in (10). Second, a higher prize value means that less will be spent for public good provision as individuals' contributions have to finance that higher prize value first; this is captured by the term $ne - V$ in (10). Since public good provision is the smaller of these two terms, the optimal prize makes these two competing effects equal. The following corollary of Proposition 3 answers the question.

Corollary 4 *Consider the self-financing, provisional fixed-prize lottery mechanism. Suppose that the endowment condition (7) and the efficiency condition (8) both hold. Then, for a given number of participants, n, and endowment e,*

$$V^L = \frac{n^2}{(n-1)}(1-\beta)\,e$$

is the prize value that maximizes public good provision. The highest possible public good provision (less the fixed-prize V) is:

$$\widetilde{G^L} = \frac{n}{(n-1)}(\beta n - 1)\,e.$$

Proof. See Appendix A.

5 All-Pay Auction

If the mechanism M is an all-pay auction[9], then (1) becomes

$$\pi_i\left(b_i, b_{-i}; n, \beta, \text{All-pay auction}\right) =$$

$$\begin{cases} V + (e - b_i) + \beta\left(\sum_{j=1}^{n} b_j\right), & \text{if } b_i > b_j \text{ for any } j \neq i, \\ \frac{V}{K} + (e - b_i) + \beta\left(\sum_{j=1}^{n} b_j\right), & \text{if } i \text{ ties } (K-1) \text{ others for the high bid,} \\ (e - b_i) + \beta\left(\sum_{j=1}^{n} b_j\right), & \text{if } b_i < b_j \text{ for some } j \neq i. \end{cases} \tag{11}$$

Note that we use b rather than x to denote "bids" under the all-pay auction mechanism. In this case, Orzen (2008) proves[10] that if

$$0 < e \leq \frac{V}{(1-\beta)\,n}, \tag{12}$$

[9] See Baye et al. (1996) for a complete characterization of equilibria in the complete information version of the first price all-pay auction without a public good component.

[10] Orzen considers self-financing mechanisms, see his equation (1). However, his results are correct only in the case where the fixed prize is exogenously given. Therefore, we view Orzen (2008) as studying the case of non-provisional fixed prizes, i.e., the setting with the payoff function (11).

then there exists a symmetric pure-strategy equilibrium where each player bids his entire endowment, e. He also demonstrates that if

$$e \geq \frac{V}{(1-\beta)}, \tag{13}$$

then there exists a symmetric mixed-strategy equilibrium where each player bids in the interval $\left[0, \frac{V}{1-\beta}\right]$ according to the following distribution function:

$$F(b) = \left((1-\beta)\frac{b}{V}\right)^{\frac{1}{n-1}}. \tag{14}$$

Note that expected public good provision in the latter case is positive and is given by:

$$EG = n\int_0^{\frac{V}{1-\beta}} b dF(b) = \frac{V}{1-\beta}.$$

Finally, if

$$\frac{V}{(1-\beta)n} < e < \frac{V}{(1-\beta)}, \tag{15}$$

then there exists a symmetric mixed-strategy equilibrium where each player bids his entire endowment, e, with probability p and bids in the interval $\left[0, \frac{(1-p)^{n-1}V}{1-\beta}\right]$ according to the distribution function F in (14) with probability $(1-p)$, where probability $0 < p < 1$ is a unique positive solution of the following equation:

$$\frac{1-(1-p)^n}{p} = \frac{(1-\beta)n}{V}e.$$

We now turn to the self-financing provisional fixed-prize all-pay auction mechanism that is the focus of this paper. In that case, (2) becomes

$$\pi_i(b_i, b_{-i}; n, \beta, \text{All-pay auction}) = \tag{16}$$

$$\begin{cases} V + (e-b_i) + \beta\left(\sum_{j=1}^n b_j - V\right), & \text{if } b_i > b_j \text{ for any } j \neq i \text{ and } \sum_{j=1}^n b_j \geq V, \\ \frac{V}{K} + (e-b_i) + \beta\left(\sum_{j=1}^n b_j - V\right), & \text{if } i \text{ ties } (K-1) \text{ others for the high bid and } \sum_{j=1}^n b_j \geq V, \\ (e-b_i) + \beta\left(\sum_{j=1}^n b_j - V\right), & \text{if } b_i < b_j \text{ for some } j \neq i \text{ and } \sum_{j=1}^n b_j \geq V, \\ e, & \text{if } \sum_{j=1}^n b_j < V. \end{cases}$$

5.1 No Public Good Provision

As in our analysis of the provisional fixed-prize lottery, we proceed in a series of steps resulting in a number of propositions for the provisional fixed prize all-pay auction mechanism. We first consider the role of the endowment, e, relative to the fixed prize value, V. If condition (6) holds, then the prize value is so high that players do not have enough resources to finance the prize and, as a result, the public good is never provided in any equilibrium in this case.

Proposition 5 *Consider the self-financing provisional fixed-prize all-pay auction mechanism. Suppose that condition* (6) *holds. Then, any strategy profile* $(x_1, ..., x_n)$ *is a Nash equilibrium. The public good is not provided in all of these equilibria.*

The proof is straightforward and is thus omitted.

Second, we consider the public-good-game effect. It turns out that the self-financed provisional fixed-prize all-pay auction also has this property. The intuition is clear: if all other players contribute nothing or very little, then a player has to finance the prize (almost) alone in order to get it. Formally,

Proposition 6 *Consider the self-financing provisional fixed-prize all-pay auction mechanism. Then,* $(x_1, ..., x_n) = (0, ..., 0)$ *is a pure-strategy NE where the public good is not provided.*

The proof is straightforward and is thus omitted.

It turns out that the public-good-game effect is even stronger if individual endowments are small enough.

Proposition 7 *Suppose that the endowment condition*

$$\frac{V}{n} < e < V$$

holds. Then, there exists a set of pure-strategy Nash equilibria where public good is not provided.[11] *This set can be characterized in the following way*

$$\left\{ (x_1, ..., x_n) : \max_i X_{-i} < V - e \right\},$$

where $X_{-i} = \sum_{j \neq i} x_j$.

The proof is straightforward and is thus omitted.

Given the contributions of the other players, $-i$, player i does not have enough resources (her endowment is too small) to finance the prize and, therefore, she is indifferent among all her choices in any equilibrium in the set of equilibria. This indifference property holds for all players in all equilibria in Proposition 7. Note that the endowment has to satisfy the condition $e < V$ in Proposition 7.

By contrast, the non-provisional fixed-prize all-pay auction does not have a public-good-game effect. Again, the intuition is clear: since the prize is given, if all other players contribute nothing, a player prefers to contribute a small amount and therefore to win the prize and to benefit from public good provision instead of making a zero contribution.

5.2 Public Good Provision

The provisional fixed-prize all-pay auction can also have equilibria where the public good *is* provided. As in the provisional fixed-prize lottery case, a necessary condition for such an equilibrium under the all-pay auction mechanism is that the efficiency condition (8) holds. Note that the efficiency condition (8) does not play any role in the exogenously given fixed-prize all-pay auction.

[11] Of course, there is a continuum of mixed-strategy equilibria, where each player randomizes among her strategies in this equilibrium set. There is no public good provision in all mixed-strategy equilibria.

Proposition 8 *Consider the self-financing provisional fixed-prize all-pay auction. Suppose that the efficiency condition (8) and the following endowment condition*

$$\frac{V}{n} < e < \frac{V}{(1-\beta)\,n} \tag{17}$$

both hold. Then, there exists a pure-strategy NE, $(x_1, ..., x_n) = (e, ..., e)$, where the public good is provided. Public good provision (less the fixed-prize value V) is given by:

$$\widetilde{G} = ne - V > 0.$$

Suppose that the efficiency condition (8) does not hold. Then, the public good is not provided in any pure-strategy equilibrium.

Proof: See Appendix A.

It turns out that if $e = \frac{V}{(1-\beta)n}$, then $(x_1, ..., x_n) = (e, ..., e)$ is a pure-strategy NE, where the public good is provided. However, $(x_1, ..., x_n) = (0, e, ..., e)$ might also be a pure-strategy NE, where the public good is provided, depending on parameters β and n. We consider the case of such asymmetric pure-strategy NE in the following proposition.

Proposition 9 *Consider the self-financing provisional fixed-prize all-pay auction. Suppose that there exists $k \in \{1, ..., n-2\}$ such that the following endowment condition*

$$\frac{V}{(1-\beta)\,(n-k+1)} < e \leq \frac{V}{(1-\beta)\,(n-k)} \tag{18}$$

holds. If the following efficiency condition

$$\beta\,(n-k) > 1 \tag{19}$$

holds, then there exist $C_n^k = \frac{n!}{k!(n-k)!}$ asymmetric pure-strategy NE, where $(n-k)$ players bid their entire endowment, e, while the other k players bid zero. Public good provision (less the fixed-prize value V) is given by:

$$\widetilde{G} = (n-k)\,e - V > 0.$$

Suppose that $\frac{V}{2(1-\beta)} < e$, then the public good is not provided in any pure- or mixed-strategy equilibrium.

Proof: See Appendix A.

Figure 1 illustrates the endowment and efficiency conditions for the set of equilibria characterized in Propositions 8 and 9.

Propositions 5, 8, and 9 characterize the range of parameters under which the provisional, fixed-prize all-pay auction mechanism yields public good provision. First, if the endowment is too small, i.e., if $0 < e \leq \frac{V}{n}$, then the public good is never provided, from Proposition 5. Second, if the endowment is too large, $e > \frac{V}{2(1-\beta)}$, then the public good is again never provided, from Proposition 9. This means that the public good can be provided only if the endowment is in the intermediate range, $\frac{V}{n} < e \leq \frac{V}{2(1-\beta)}$. We shall refer to endowments in this range as a being

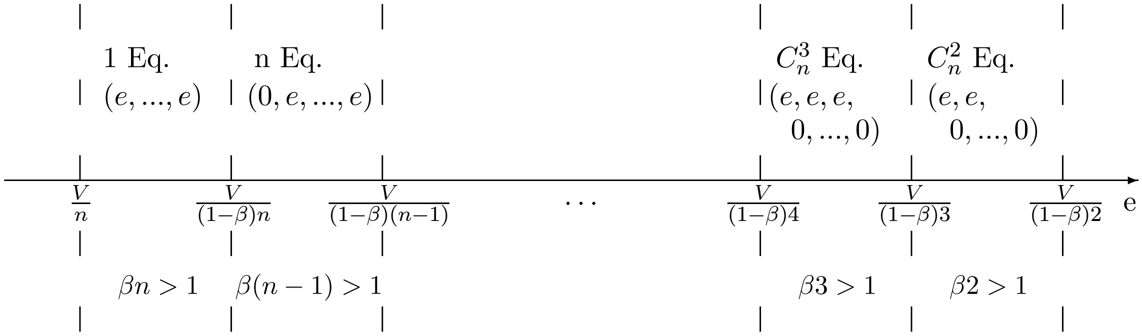

Figure 1: Illustration of Equilibrium Possibilities involving Positive Public Good Provision under the Provisional Fixed Prize All-Pay Auction Mechanism

"medium" endowments. Moreover, Propositions 8 and 9 demonstrate that public good provision depends on efficiency as well. If it is efficient to provide the public good even for just two players, i.e., if $2\beta > 1$, then for any endowment in the range $\left(\frac{V}{n}, \frac{V}{2(1-\beta)}\right]$ the public good is provided in a pure-strategy equilibrium, from Propositions 8 and 9. If it is efficient to provide the public good for three or more players, $3\beta > 1 > 2\beta$, then for any endowment in the smaller range $\left(\frac{V}{n}, \frac{V}{3(1-\beta)}\right]$ the public good is provided in a pure-strategy equilibrium, and so on. Finally, if it is efficient to provide the public good only for all n players, $n\beta > 1 > (n-1)\beta$, then for any endowment in the smallest range $\left(\frac{V}{n}, \frac{V}{n(1-\beta)}\right]$ the public good is provided in a pure-strategy equilibrium, from Proposition 8. Note that if it is inefficient to provide the public good even for all n players, i.e., if $n\beta < 1$, then for any endowment, the public good is not provided in any equilibrium.

Proposition 9 describes asymmetric pure-strategy equilibria with public good provision for endowments that are larger than those allowed in Proposition 8 for the case of symmetric pure-strategy equilibria. The following two corollaries help to further clarify Proposition 9. Corollary 10 describes asymmetric, pure-strategy equilibria with public good provision that involve $(n-1)$ active players and the corresponding restriction that must be imposed on endowments. Corollary 11 describes the largest endowment for which asymmetric pure-strategy NE with positive public good provision exist.

Corollary 10 *Consider the self-financing provisional fixed-prize all-pay auction. Suppose that the following efficiency condition*

$$\beta\,(n-1) > 1$$

and the following endowment condition

$$\frac{V}{(1-\beta)\,n} < e \leq \frac{V}{(1-\beta)\,(n-1)} \tag{20}$$

both hold. Then, there exist n asymmetric pure-strategy NE, (x_i, x_{-i}), where $x_i = 0$ and $x_j = e$ if $j \neq i$, and the public good is provided. Public good provision (less the fixed-prize value V) is given by:

$$\widetilde{G} = (n-1)\,e - V > 0.$$

Corollary 11 *Consider the self-financing provisional fixed-prize all-pay auction. Suppose that the following efficiency condition*

$$2\beta > 1$$

holds. If there exists $k \in \{1, ..., n-2\}$ *such that the following endowment condition*

$$\frac{V}{(1-\beta)(n-k+1)} < e \leq \frac{V}{(1-\beta)(n-k)} \tag{21}$$

holds, then there exist $C_n^k = \frac{n!}{(k)!(n-k)!}$ *asymmetric pure-strategy NE, where* $(n-k)$ *players bid their entire endowment,* e*, while the other* k *players bid zero. Public good provision (less the fixed-prize value* V*) is given by:*

$$\widetilde{G} = (n-k)\,e - V > 0.$$

Given our findings of asymmetric pure-strategy equilibria where players bid either zero or their entire endowment, it is natural to expect multiple mixed-strategy equilibria where the public good is provided. Since the endowment condition

$$\frac{V}{n} < e < \frac{V}{2(1-\beta)}$$

has to hold in all such mixed-strategy equilibria, we do not describe these equilibria here because we already know that the public good can be provided for this endowment condition under a pure strategy NE.[12]

In Appendix C we provide a complete characterization of the equilibria that are possible under our two provisional fixed–prize mechanisms for the $n = 2$ case for all possible endowments.

We find that if the endowment is very large relative to the value of the prize – the case we study in our experiment – then the unique (symmetric) equilibrium prediction is for all players to contribute zero, resulting in zero public good provision. By sharp contrast, under the exogenously given fixed-prize all-pay auction mechanism where the endowment is again very large relative to value of the prize, specifically for any e satisfying condition (13), there continues to exist a symmetric mixed strategy equilibrium where expected public good provision is positive. In this mixed-strategy equilibrium, players submit bids from a bounded, continuous interval from 0 on up to some upper bound. This upper bound is fixed at $\frac{V}{1-\beta}$ for any $e > \frac{V}{1-\beta}$ and players are indifferent among all bids in the interval $[0, \frac{V}{1-\beta}]$.

The latter type of equilibrium does not exist when the fixed-prize is provisional on contributions equaling or exceeding V. For some intuition, consider a "small" bid – a bid between 0 and $\frac{V}{n}$. A bid of zero will dominate any small bid: If a player wins with a small bid, the prize is not provided since, from our definition of a small bid, the provisional threshold will not have been reached. If a player loses with a small bid, he has to pay that small bid and is worse off relative to a bid of zero. Suppose next that participants consider bidding on a bounded continuous interval strictly greater than 0. In that case, bids at the left boundary always lose under an all-pay auction and such bids are costly to the bidder as they are strictly greater than zero. Essentially there is no left boundary such that individuals are indifferent among bids over some bounded continuous interval. The only exception arises if players randomize between a zero bid and a bid of their entire endowment. However, if the endowment is sufficiently large relative to the prize, then even this mixed equilibrium ceases to exist

[12] In Appendix B, we characterize such mixed strategy equilibria for the $n = 2$ case.

and the unique equilibrium prediction calls for zero contributions by all participants in the all-pay auction with a provisional fixed-prize. In that case, the use of the all-pay auction mechanism to award a provisional but fixed prize transforms the game so that its incentives are equivalent to those of the VCM where the dominant equilibrium strategy is for all to contribute nothing.

Proposition 8 shows that there exists a symmetric pure-strategy equilibrium involving positive public good provision for any prize value, V, that satisfies condition (17). As in the case of the lottery mechanism, we can again ask: given endowments e, what prize value, V, maximizes public good provision under the provisional, fixed-prize all-pay auction mechanism? We answer this question in the following proposition.

Proposition 12 *Consider the self-financing provisional fixed-prize all-pay auction mechanism. Suppose that efficiency condition* (8) *and the endowment condition* (17) *both hold. Then, for a given number of participants, n, and endowment e,*

$$V^A = (1 - \beta)\, ne$$

is the prize value that maximizes public good provision. The highest possible public good provision (less the prize V) is:

$$\widetilde{G^A} = ne - V^A = \beta ne.$$

Proof. See Appendix A.

6 Experimental Design and Predictions

We have designed an experiment to test some of the comparative statics implications of the theory developed in the previous sections. In our experiment the commonly known fixed prize, V, is *always provisional* and must be financed by subject contributions. Thus, the payoff function for each subject i is given by an equation of the form (2). We use a $2 \times 2 \times 2$ experimental design where our treatment variables are (1) the mechanism, M – either the all-pay auction or lottery rules determine the prize winner; (2) the group size, $n = 2$ or $n = 10$; and (3) the marginal per capita return (mpcr) on the public good, $\beta = .25$ or $\beta = .75$. We chose to vary n and β so as to also test some comparative statics predictions of the theory under the two different provisional prize mechanisms, particularly under the lottery mechanism. We chose values for n and β that are found in the existing literature.[13] All other model parameters, i.e., the fixed prize value of $V = 100$ and the individual endowment of $e = 400$ were kept fixed across all rounds of all experimental sessions of our experiment so as to focus attention on the role played by the three treatment variables.[14]

We chose to set $e = 4V$ as we think it is realistic to assume that for the small charitable fund-drives we have in mind, the value of the prize offered is considerably less than any single participant's endowment.[15] Further, with these choices for e and V, and for all four $\{n, \beta\}$ treatment pairs that

[13] For instance, Isaac et. al. (1984) study a VCM using a 2×2 design with $n = 4$ or 10 and $\beta = .3$ or .75.

[14] In future research it would be of interest to set V equal to the values that maximize public good provision (for given e, n and β) as stated in Propositions 4 and 12. However as these two values for V differ from one another (varying also with e, n and β) and the main focus of this paper is on the effect of the two different provisional prize mechanisms (as well as variations in n and β) on contributions to a public good, we chose to keep V (and e) fixed across all treatment conditions.

[15] This is a reasonable assumption if the endowment is viewed as the participant's liquid assets.

we consider, it is always the case that the unique equilibrium prediction under the provisional fixed-prize all-pay auction mechanism is for all players to bid zero and thus there should be zero public good provision. The reason is that e is too large relative to V to satisfy any of the conditions for positive public good provision given in the propositions of section 5.2. In particular, according to Proposition 9, if $\frac{V}{2(1-\beta)} < e$, as is the case in for all four treatment pairs under our fixed-prize all-pay auction mechanism, then there does not exist an equilibrium with positive public good provision. By contrast, for three of the four $\{n, \beta\}$ treatment pairs in our experimental design, the provisional fixed-prize lottery mechanism has a symmetric pure strategy equilibrium involving positive bids and positive public good provision (in addition to the symmetric zero-bid/zero-provision equilibrium as well). Thus, our experimental design provides a stark predicted difference between the two provisional fixed-prize mechanisms in terms of bidding behavior and public good provision. We also explore whether the efficiency condition (8) plays the important role that is predicted by the theory. For three of our four treatment pairs for $\{n, \beta\}$, our experimental parameterization corresponds to the efficient case, where condition (8) holds; these are the same three cases where the provisional fixed-prize lottery mechanism has a symmetric pure strategy equilibrium involving positive bids and public good provision. In the one treatment where $n = 2$ and $\beta = .25$, the efficiency condition (8) does not hold and so it is inefficient to provide the public good under either the provisional fixed-prize lottery or all-pay auction mechanisms. Thus, the latter treatment allows us to examine whether the efficiency condition matters for public good provision under the fixed-prize lottery mechanism, as the theory predicts.

Given our parameter choices, equilibrium bid and public good predictions for our experimental design are given in Table 1. For the provisional fixed-prize lottery mechanism we focus on the symmetric pure-strategy Nash equilibrium where the public good is provided if such equilibria exist under the conditions detailed in Proposition 3. Recall from Proposition 2 that there will always exist a set of Nash equilibria where the public good is not provided under the provisional fixed-prize lottery mechanism for all treatment conditions of our experiment. Note that in one of our four provisional fixed-prize lottery mechanism treatments, the one where $n = 2$ and $\beta = .25$, the efficiency condition that $\beta n > 1$ is not satisfied so that Proposition 3 does not apply. Thus, for this one treatment, consistent with Proposition 2, there exist many pure strategy equilibria where all n players bid an amount $x_i \in [0, 75]$ and where $x_1 + x_2 \leq V = 100$. However, in all of these equilibria, the NE prediction for public good provision net of the prize amount, $\widetilde{G}$, is unambiguously 0, as total bids can never exceed $V = 100$. Notice finally that for the provisional fixed-prize all-pay auction mechanism, the equilibrium bid and public good provision levels are always zero regardless of the different treatment values for n and β. As noted above, this prediction arises because condition $\frac{V}{2(1-\beta)} < e$ always holds in our experimental design, i.e., the endowment e, is too large relative to the prize value V and we know from Proposition 9 that in this case there are no equilibria with public good provision.

It is instructive to compare the equilibrium predictions for the self-financing, provisional fixed-prize model we study as reported in Table 1 with the non-provisional model where the fixed prize is exogenously given (model (1)). Table 2 provides these predictions. In the non-provisional exogenous fixed prize case, the all-pay auction is predicted to result in positive bids and public good provision under all treatment conditions and at levels that in expectation exceed the bids and public good provision generated by the non-provisional fixed prize lottery mechanism. Further, with a non-provisional exogenous fixed prize, the lottery mechanism is predicted to yield positive public good provision in all cases, even the inefficient case where $n = 2$ and $\beta = .25$.

$\beta = .25$	Lottery		All-Pay Auction	
	$x_i = \min\left\{\frac{n-1}{n^2}\frac{V}{1-\beta}, e\right\}^\dagger$	$\tilde{G} = \min\left\{\frac{(\beta n-1)}{n(1-\beta)}V, ne - V\right\}^\dagger$	$x_i = 0$	$G = 0$
$n = 2$	$x_i \in [0, 75]^\ddagger$	0	0	0
$n = 10$	12	20	0	0
$\beta = .75$	$x_i = \min\left\{\frac{n-1}{n^2}\frac{V}{1-\beta}, e\right\}^\dagger$	$\tilde{G} = \min\left\{\frac{(\beta n-1)}{n(1-\beta)}V, ne - V\right\}^\dagger$	$x_i = 0$	$G = 0$
$n = 2$	100	100	0	0
$n = 10$	36	260	0	0

† If $\beta n > 1$. There will also exist a set of equilibrium where the public good is not provided.
‡ Since $\beta n < 1$, according to Proposition 2, there exists a set of pure-strategy equilibria where $x_i \in [0, (1-\beta)V]$ and $x_1 + x_2 \leq V = 100$ so that public good provision is 0.

Table 1: Equilibrium predictions under the provisional fixed-prize model (2) for the parameterization where $e = 400$ and $V = 100$

$\beta = .25$	Lottery		All-Pay Auction	
	$x_i = \min\left\{\frac{n-1}{n^2}\frac{V}{1-\beta}, e\right\}$	$G = \min\left\{\frac{n-1}{n}\frac{V}{1-\beta}, ne\right\}$	$F(b) = \left((1-\beta)\frac{b}{V}\right)^{\frac{1}{n-1}}$	$EG = \frac{V}{1-\beta}$
$n = 2$	$\frac{100}{3}$	$\frac{200}{3}$	$\left(\frac{3b}{400}\right)$	$\frac{400}{3}$
$n = 10$	12	120	$\left(\frac{3b}{400}\right)^{\frac{1}{9}}$	$\frac{400}{3}$
$\beta = .75$	$x_i = \min\left\{\frac{n-1}{n^2}\frac{V}{1-\beta}, e\right\}$	$G = \min\left\{\frac{n-1}{n}\frac{V}{1-\beta}, ne\right\}$	$F(b) = \left((1-\beta)\frac{b}{V}\right)^{\frac{1}{n-1}}$	$EG = \frac{V}{1-\beta}$
$n = 2$	100	200	$\left(\frac{b}{400}\right)$	400
$n = 10$	36	360	$\left(\frac{b}{400}\right)^{\frac{1}{9}}$	400

Table 2: Equilibrium predictions under the non-provisional exogenous fixed-prize model (1) for the parameterization where $e = 400$ and $V = 100$

We report data from 16 experimental sessions, each involving 20 subjects, for a total of 320 subjects. Each session involves a within-subject design where either the all-pay auction or the lottery mechanism was used to determine the prize winner in each group of size $n = 2$ or 10 over the first 15 rounds.[16] Over the remaining 15 rounds, the other mechanism was used to determine the prize winner in each group of size n. The change in the mechanism was *not* announced in advance. For each session, we used a single fixed parameter set for $\{n, \beta\}$, either $\{2, .25\}$, $\{2, .75\}$, $\{10, .25\}$, or $\{10, .75\}$ for all 30 rounds (i.e., under both mechanisms). Thus the single, one-time treatment change within a session was only in the mechanism used to determine the prize winner. To minimize the consequences of possible order effects, we reversed the order of the mechanisms used in the first and last 15 rounds across sessions involving the same $\{n, \beta\}$ treatment conditions. Specifically, two sessions of each $\{n, \beta\}$ treatment used the lottery mechanism in the first 15 rounds followed by 15 rounds of the all-pay auction mechanism. The other two sessions use the all-pay auction mechanism in the first 15 rounds followed by 15 rounds of the lottery mechanism.

[16] We chose a within-subject design because such designs are statistically more powerful than between-subject designs; in a within-subject design, each subject serves as their own control, thereby minimizing the effects of individual differences (see, e.g., Camerer 2003 pp. 41-42).

A summary of the session characteristics is provided in Table 3.

	Lottery First 15 Rounds Auction Last 15 Rounds	Auction First 15 Rounds Lottery Last 15 Rounds	Total Sessions
$n = 2$, $\beta = .25$	2 Sessions	2 Sessions	4
$n = 2$, $\beta = .75$	2 Sessions	2 Sessions	4
$n = 10$, $\beta = .25$	2 Sessions	2 Sessions	4
$n = 10$, $\beta = .75$	2 Sessions	2 Sessions	4

Table 3: Experimental Design

In the $n = 2$ treatment, the 20 subjects were randomly paired at the start of each of the 30 rounds to form 10 groups of size 2. In the $n = 10$ treatment, the 20 subjects were randomly matched into two groups of size 10 at the start of each of the 30 rounds so that there were 2 groups of size 10. We used random matching each round so as to avoid any repeated game effects.

Subjects were University of Pittsburgh undergraduates with no prior experience with our experiment. No subject participated in more than one experimental session. Subject interactions and decision-making were anonymous and were conducted using networked PCs in the Pittsburgh Experimental Economics Laboratory. Prior to the first round of play, subjects were given written instructions that were also read aloud in an effort to induce common knowledge of endowments, the prize, the mechanism for winning the prize and the value of tokens in terms of dollars. Following 15 rounds of play, continuation instructions were provided and read aloud; these continuation instructions explained that the only change relative to the first 15 rounds of play would be in the mechanism used to determine the prize winner and that this new mechanism would be in effect for the final 15 rounds of play. The instructions used in $n = 10$, $\beta = .75$ treatment, where the lottery mechanism was used in the first 15 rounds and the all-pay auction mechanism was used in the final 15 rounds, are provided in Appendix D; other instructions are similar, differing only in the values for n and β or in the order of the two mechanisms.

The sequence of play of each round of a session was as follows. Each subject was endowed with 400 tokens. They were instructed that they could bid any number of these tokens for a provisional, fixed prize of 100 tokens. The winner in their group of size n (2 or 10) was determined according to the mechanism that was in place for that round. Specifically, subjects were instructed that when the all-pay auction was the mechanism, the winner was the player who bid the most tokens and that in the event of a tie, the winning bidder would be randomly chosen from among all those who bid the most tokens. When the lottery was the mechanism, subjects were instructed that the winner was chosen randomly from all members of their n member group who bid at least 1 token and that each bidder's chance of winning was set equal to the ratio of their bid to the total tokens bid by all n members of their group in that round.[17] Importantly, subjects were instructed that if the total amount bid for the prize by all n members of their group did not equal or exceed the fixed prize value of 100 tokens, then the prize would not be offered. In that case, all bids were returned and subjects ended the round with their endowment, e, of 400 tokens. If the total amount bid for the prize by all n group members equaled 100 or more, then the prize was awarded according to the mechanism that was in place for that round. Finally, subjects were instructed that

[17] We avoided use of the terms "lottery" and "auction" and simply explained to subjects how the two different mechanisms determined a prize winner in each round.

amounts bid in excess of the $V = 100$ token prize would be placed in a "group account". Subjects were informed that all members of their group of size n, even those who did not bid any tokens toward the 100 token prize, could earn additional tokens based on the total number of tokens in their group's account. Specifically, subjects were informed that the amount of additional tokens each group member could receive from the group account was determined by the amount of tokens remaining in the group account after the prize was paid out, $\left(\sum_{j=1}^{n} b_j - 100\right)$, and by the mpcr, β, and was given by $\beta\left(\sum_{j=1}^{n} b_j - 100\right)$ if $\sum_{j=1}^{n} b_j \geq 100$ and was 0 otherwise. (Here b_j denotes the amount bid by individual j in a group of size n in either the lottery or all-pay auction treatments). Subjects were also given a table showing how many additional tokens each group member could earn if their group account reached various token levels of 100 or more. Subjects were instructed that their earnings in each round were the sum of three numbers: 1) the amount of tokens remaining in their "private account," i.e., their endowment of 400 tokens for the round less any tokens they bid in that round (provided that $\sum_{j=1}^{n} b_j \geq 100$); 2) their prize winnings of $V = 100$ tokens if (and only if) they were the prize winner in their n member group for that round and 3) their payoff in tokens from their n-member group account for that round.

At the end of each session, two rounds were randomly chosen, one from the first 15 rounds of the session and one from the last 15 rounds of the session, as these sets of rounds involved two different mechanisms. Subjects' total token amounts from the 2 randomly chosen rounds were converted into dollars at the known and fixed rate of 1 token = \$0.01 (1 cent). In addition, subjects were guaranteed \$5.00 for showing up on time. Each subject's total earnings for this 90 minute experiment depended on the treatment. For each of the $\{n, \beta\}$ pairs average total earnings per subject (across all sessions of that treatment pair) were as follows: $\{2, .25\}$: \$12.82; $\{2, .75\}$: \$14.47; $\{10, .25\}$: \$15.45; and $\{10, .75\}$: \$39.20. Notice that by not bidding in any round of any treatment, a subject could guarantee him/herself a payment of at least \$13.00 and possibly more depending on whether others placed bids and the prize threshold was met.[18]

7 Experimental Findings

We first consider whether and how behavior varies across the treatments of our $2\times2\times2$ experimental design. For each session of each treatment, we calculated the average amount bid, denoted by 'Avg. Bid', and the average amount of public good provision less the prize (if awarded) which we refer to as 'Avg. G' separately for each provisional fixed-prize mechanism (lottery, all-pay auction). The variable Avg. Bid is calculated by taking the average amount bid in each group of size n and then calculating the average of all such group averages for a given session over various lengths of $T \leq 15$ rounds. This 'Avg. Bid' number can be compared with the theoretical predictions for x_i as given in Table 1.[19] The session-level average contribution to the public good, Avg. G, *excludes the prize amount of* $V = 100$ if the prize was awarded and is equal to 0 if the prize was not awarded or if

[18]By not bidding in the first and last 15 rounds, subjects earn their endowment of 400 tokens twice, or \$8.00 at the .01 conversion rate plus a \$5.00 show-up payment for a total of \$13.00. In addition, subjects could get additional earnings from the public good if it was provided.

[19]The average bid actually paid may be less than Avg. Bid, depending on whether contributions exceeded the prize level, $V = 100$, or not. However, the theoretical predictions in Table 1 for x_i are for this 'ex-ante' bid amount, the same amount that is reported in Tables 4, 5 and 6 below. By contrast, the Avg. G measure, described below takes account of whether each group of n players met the prize threshold or not, again consistent with theoretical predictions for $\widetilde{G}$ or G as given in Table 1.

the sum of a group's bids was exactly equal to 100. Specifically, the average public good provision less the prize in each session over an interval of $T \leq 15$ rounds was calculated as follows:

$$\text{Avg. } G = \frac{1}{T} \sum_{t=1}^{T} \frac{n}{20} \sum_{k=1}^{20/n} \max \left\{ \sum_{j=1}^{n} b_{j,t}^{k} - 100, 0 \right\},$$

where $n = 2$ or 10, k indexes membership in the group of size n and t indexes the round number. This 'Avg. G' number can be compared with the theoretical predictions for $\widetilde{G}$ or G as given in Table 1.

Table 4 provides a simple overview of our main findings by reporting the average amounts bid, Avg. Bid, and the average amount of public good provision using pooled data from all four sessions of each treatment, $\{M, n, \beta\}$ over the $T = 15$ rounds of each mechanism (averages of four session-level observations per treatment). For convenience, Table 4 reports the Nash equilibrium (NE) bid and public good provision less the prize predictions under the headings, 'NE Bid' and 'NE G', respectively, which were reported earlier in Table 1 for the model parameterization of our experiment. Finally, Table 4 also indicates the frequency with which the public good was provided ('Prov.'), i.e., the frequency with which the group total bids equaled or exceeded 100, and the NE prediction regarding public good provision ('NE Prov.'). Tables 5–8 provide a more disaggregated view of our data, reporting on the four *session-level* observations for Avg. Bid and Avg. G less the 100 token prize for each treatment, $\{M, n, \beta\}$, over various intervals of time, T, specifically over all rounds, $1-15$ (as in Table 4) but also for round 1 only, for rounds $1-5$, $6-10$, $11-15$ and for the final round 15 only.

Lottery							
n	β	Avg. Bid	NE Bid	Avg. G	NE G	Prov.	NE Prov. %
2	.25	59.3	$x_i \in [0, 75]$	36.4	0.0	0.63	0.00
2	.75	148.6	100.0	207.9	100.0	0.80	1.00
10	.25	86.6	12.0	767.6	20.0	0.98	1.00
10	.75	201.8	36.0	1918.0	260.0	1.00	1.00
All-Pay Auction							
n	β	Avg. Bid	NE Bid	Avg. G	NE G	Prov.	NE Prov. %
2	.25	67.9	0.0	56.2	0.0	0.69	0.00
2	.75	159.3	0.0	230.3	0.0	0.82	0.00
10	.25	87.0	0.0	772.5	0.0	0.98	0.00
10	.75	210.8	0.0	2008.0	0.0	1.00	0.00

Table 4: Average Bids and Public Good Provision Across Treatments Relative to Theoretical Predictions, Averages From All Periods of All Sessions of Each Treatment

Based on the numbers shown in these tables we can report a number of different findings.

Finding 1 *Average bids and public good provision (less the prize amount) are generally greater than theoretical predictions.*

Support for this finding can be found in Table 4 which suggests that both Avg. Bid and Avg. G are generally greater than the Nash equilibrium predicted values (NE Bid and NE G, respectively)

Lottery Mechanism	Average Amount Bid in Round Number(s):					
Session No., $\{n, \beta\}$, Order	1-15	1	1-5	6-10	11-15	15
1 {2, .25} Lottery, Auction	68.7	93.7	83.3	72.2	50.4	45.7
2 {2, .25} Auction, Lottery	53.9	63.4	59.8	49.9	52.1	48.8
3 {2, .25} Lottery, Auction	66.3	77.7	76.5	69.6	52.9	51.4
4 {2, .25} Auction, Lottery	48.3	51.0	54.6	46.4	43.8	46.2
Average of Sessions 1-4	59.3	71.4	68.6	59.5	49.8	48.0
1 {2, .75} Lottery, Auction	165.0	126.0	183.8	180.0	131.3	98.5
2 {2, .75} Auction, Lottery	135.6	143.7	133.9	129.1	124.0	248.0
3 {2, .75} Lottery, Auction	212.4	212.4	250.6	236.2	177.6	154.3
4 {2, .75} Auction, Lottery	81.3	91.7	95.4	80.4	68.0	81.2
Average of Sessions 1-4	148.6	143.4	165.9	156.4	125.2	145.5
1 {10, .25} Lottery, Auction	119.6	124.8	154.3	124.0	80.6	96.2
2 {10, .25} Auction, Lottery	50.2	78.9	70.9	46.4	33.1	17.4
3 {10, .25} Lottery, Auction	135.3	140.5	167.1	133.3	105.6	96.5
4 {10, .25} Auction, Lottery	41.3	114.9	69.3	29.5	25.1	45.8
Average of Sessions 1-4	86.6	114.8	115.4	83.3	61.1	64.0
1 {10, .75} Auction, Lottery	252.5	287.1	258.6	268.2	230.7	222.1
2 {10, .75} Lottery, Auction	235.2	170.6	233.5	251.4	220.6	240.5
3 {10, .75} Auction, Lottery	165.8	199.0	197.4	177.8	122.3	137.0
4 {10, .75} Lottery, Auction	153.6	102.7	146.5	158.4	156.0	171.6
Average of Sessions 1-4	201.8	189.8	209.0	213.9	182.4	192.8

Table 5: Session-Level Average Bids Under the Lottery Mechanism Over Various Intervals of Time

under both self-financed provisional fixed-prize mechanisms. Further support for this finding at the session-level is provided in Tables 5–8.

Consider first the $n = 2$, $\beta = .25$ treatment of the provisional fixed prize lottery mechanism. In that treatment, the efficiency condition ($\beta n > 1$) is not satisfied so the theoretical prediction (from Proposition 2) is that bids should be strictly less than 75, the sum of the two players' bids should not exceed 100 and as a consequence there should be 0 public good provision as the prize value, $V = 100$, cannot be covered by two such bids. Table 4 indicates that bids over all rounds in this treatment averages 59.3. Further, the public good provision threshold of 100 is met and the public good is provided 63% of the time so that Avg. G for this treatment is 36.4. Using the four session-level observations for Avg. Bid over rounds 1-15 for the $\{2, .25\}$ treatment as reported in the second column of Table 5 a one-sample upper-tailed Wilcoxon signed rank test indicates that we cannot reject the null hypothesis that the median of the mean bids is less than or equal to 75 ($p = .98$) in favor of the alternative that it is strictly greater than 75. Still, we observe that Avg. G over all 15 rounds (or even the last 5 rounds) is strictly greater than 0 in all four sessions of the $\{2, .25\}$ treatment as indicated in the second (or sixth) column of Table 7. We do note that there is evidence of a steep decline in Avg. G from 59.7 in the first round to 18 in the final 15th round, which suggests that subjects may be learning (albeit slowly) the equilibrium prediction of zero public good provision for this treatment.

Consider next the provisional prize lottery mechanism in the three treatment conditions where

AP Auction Mechanism	Average Amount Bid in Round Number(s):					
Session No., $\{n, \beta\}$, Order	1-15	1	1-5	6-10	11-15	15
1 $\{2, .25\}$ Lottery, Auction	51.1	62.9	56.7	55.7	40.8	29.0
2 $\{2, .25\}$ Auction, Lottery	82.2	112.4	112.1	82.7	52.0	43.5
3 $\{2, .25\}$ Lottery, Auction	58.3	76.5	67.7	60.0	47.1	42.0
4 $\{2, .25\}$ Auction, Lottery	80.2	112.0	102.3	79.1	59.2	79.5
Average of Sessions 1-4	67.9	90.9	84.7	69.4	49.8	41.8
1 $\{2, .75\}$ Lottery, Auction	171.7	172.2	170.2	184.5	160.3	129.5
2 $\{2, .75\}$ Auction, Lottery	221.4	212.4	250.6	236.2	177.6	154.3
3 $\{2, .75\}$ Lottery, Auction	135.7	209.3	145.2	144.5	117.3	96.4
4 $\{2, .75\}$ Auction, Lottery	108.5	133.9	119.6	103.1	102.9	104.4
Average of Sessions 1-4	159.3	181.9	171.4	167.1	139.5	121.1
1 $\{10, .25\}$ Lottery, Auction	72.3	160.7	107.8	73.1	36.0	31.5
2 $\{10, .25\}$ Auction, Lottery	99.8	107.8	124.8	107.5	67.2	87.1
3 $\{10, .25\}$ Lottery, Auction	88.7	117.7	106.9	92.5	66.8	52.9
4 $\{10, .25\}$ Auction, Lottery	87.3	114.9	140.2	82.4	39.3	30.0
Average of Sessions 1-4	87.0	125.3	119.9	88.9	52.3	50.3
1 $\{10, .75\}$ Auction, Lottery	254.7	274.0	265.8	260.1	238.0	263.6
2 $\{10, .75\}$ Lottery, Auction	229.5	223.1	228.8	240.5	219.1	221.9
3 $\{10, .75\}$ Auction, Lottery	209.7	131.9	210.4	227.6	191.2	216.0
4 $\{10, .75\}$ Lottery, Auction	149.4	170.3	161.5	149.0	137.8	125.3
Average of Sessions 1-4	210.8	199.8	216.6	219.3	196.5	206.7

Table 6: Session-Level Average Bids Under the All-Pay Auction Mechanism Over Various Intervals of Time

it is efficient to provide the public good ($\beta n > 1$). In that case there exists a symmetric Nash equilibrium with positive bids and positive public good provision as reported in the second column of Table 5 (as well as a symmetric equilibrium with zero bids and zero public good provision), i.e., the treatments where $\{n, \beta\} = \{2, .75\}, \{10, .25\}$, or $\{10, .75\}$. Using the four session-level observations on Avg. Bid over rounds 1-15 for each of these treatments as reported in the second column of Table 5 a one-sample two-tailed Wilcoxon signed rank test allows us to reject the null hypothesis that the median of the Avg. Bids is equal to the positive NE Bid as reported in Table 4 ($p < .10$) for two of the three treatment conditions (3 tests); the sole exception is for the $n = 2, \beta = .75$ treatment where the NE Bid prediction is 100; in that case, for three of the four session-level observations, bid averages are greater than the NE prediction of 100 while one session-level observations is less than 100 (session 3, Avg. Bid= 81.3) so that we cannot reject the null hypothesis of no difference from the NE Bid prediction of 100 ($p = .14$). Not surprisingly, a similar finding holds if we use the four session level observations on Avg. G over rounds 1-15 as reported in the second column of Table 7 and test the null hypothesis that the median of the Avg. G observations equals the NE G prediction for these same three treatments. These same conclusions would remain unchanged if we used as our session level observations the values of Avg. Bid or Avg. G over just the final 5 rounds, 11-15, of the lottery mechanism (as reported in the sixth column of Tables 5 and 7). We further observe that public good provision is high in these

Lottery Mechanism	Average G Less Prize in Round Number(s):					
Session No., $\{n, \beta\}$, Order	1-15	1	1-5	6-10	11-15	15
1 $\{2, .25\}$ Lottery, Auction	49.4	95.2	72.2	51.8	24.2	10.2
2 $\{2, .25\}$ Auction, Lottery	31.7	55.5	41.9	27.3	26.0	22.6
3 $\{2, .25\}$ Lottery, Auction	42.4	63.2	57.0	47.5	22.8	16.2
4 $\{2, .25\}$ Auction, Lottery	22.0	24.8	28.5	23.0	14.7	22.9
Average of Sessions 1-4	36.4	59.7	49.9	37.4	21.9	18.0
1 $\{2, .75\}$ Lottery, Auction	233.1	151.9	267.5	262.2	169.6	102.4
2 $\{2, .75\}$ Auction, Lottery	182.9	215.4	199.6	178.3	170.9	171.3
3 $\{2, .75\}$ Lottery, Auction	326.7	341.9	398.5	306.0	275.7	353.9
4 $\{2, .75\}$ Auction, Lottery	89.0	116.2	113.3	80.8	72.9	105.1
Average of Sessions 1-4	207.9	205.6	236.6	204.9	180.3	179.2
1 $\{10, .25\}$ Lottery, Auction	1096.3	1147.5	1443.4	1139.5	706.0	862.0
2 $\{10, .25\}$ Auction, Lottery	401.6	689.0	609.4	364.3	231.0	74.0
3 $\{10, .25\}$ Lottery, Auction	1253.3	1305.0	1569.0	1232.6	956.0	865.0
4 $\{10, .25\}$ Auction, Lottery	319.2	1048.5	592.9	204.6	160.2	357.8
Average of Sessions 1-4	767.6	1047.5	1053.7	735.3	513.3	539.7
1 $\{10, .75\}$ Auction, Lottery	2425.0	2770.5	2486.1	2582.2	2073.4	2121.0
2 $\{10, .75\}$ Lottery, Auction	2251.8	1606.0	2235.1	2414.3	2106.1	2305.0
3 $\{10, .75\}$ Auction, Lottery	1558.2	1889.5	1874.0	1677.5	1123.1	1270.0
4 $\{10, .75\}$ Lottery, Auction	1436.0	926.7	1364.7	1483.8	1420.5	1615.5
Average of Sessions 1-4	1918.0	1798.2	1990.0	2039.5	1680.8	1827.9

Table 7: Session-Level Average Group Contribution Less Prize Under the Lottery Mechanism Over Various Intervals of Time

three lottery treatments at 82% for $\{2, .75\}$ treatment and close to or at 100% for the $\{10, .25\}$ and $\{10, .75\}$ treatments.

Consider next the provisional fixed prize all-pay auction mechanism. We cannot similarly test the NE prediction that Avg. Bid = Avg. $G = 0$ under the all-pay auction mechanism as such predictions lie at the boundary of feasible bids and public good provision levels. Nevertheless, it seems clear from Tables 6 and 8 that the experimental evidence runs counter to the prediction of 0 bids and public good provision as both Avg. Bid and Avg. G are, for all $\{n, \beta\}$ treatments and all sessions of the all-pay auction mechanism, substantially greater than zero. We note further, as revealed in Table 4, that the frequency with which the public good is actually provided (i.e., the frequency with which contributions exceed the prize level) averages 69 percent or higher in the all-pay auction treatments and, as was found under the lottery mechanism, is generally increasing in n or β; in the case where $n = 10$, the frequency of public good provision is very close or equal to 100 percent, even though it is predicted to be zero. Summarizing, we find that in most cases bids and public good provision differ from NE predictions under the provisional fixed prize lottery mechanism and are strictly greater than the NE prediction of zero under the provisional fixed prize all-pay auction mechanism.

We note that there is some evidence of treatment order effects in bids and public good provision net of the prize. In particular, as Tables 5-8 reveal, mean bids are often (but not always) higher

AP Auction Mechanism	Average G Less Prize in Round Number(s):					
Session No., $\{n, \beta\}$, Order	1-15	1	1-5	6-10	11-15	15
1 $\{2, .25\}$ Lottery, Auction	27.5	45.7	33.0	36.1	13.5	1.2
2 $\{2, .25\}$ Auction, Lottery	82.6	124.8	131.1	80.2	36.6	26.0
3 $\{2, .25\}$ Lottery, Auction	35.1	57.0	48.0	35.4	21.8	20.1
4 $\{2, .25\}$ Auction, Lottery	79.5	144.0	114.1	74.0	50.5	88.1
Average of Sessions 1-4	56.2	92.9	81.6	56.4	30.6	33.9
1 $\{2, .75\}$ Lottery, Auction	246.6	249.2	247.3	270.6	222.0	163.9
2 $\{2, .75\}$ Auction, Lottery	348.4	324.8	403.2	378.3	262.9	221.1
3 $\{2, .75\}$ Lottery, Auction	187.6	335.1	208.0	203.3	151.5	112.2
4 $\{2, .75\}$ Auction, Lottery	138.6	183.1	157.4	131.8	126.6	128.7
Average of Sessions 1-4	230.3	273.1	254.0	246.0	190.8	156.5
1 $\{10, .25\}$ Lottery, Auction	626.0	1507.0	977.7	631.0	269.5	264.5
2 $\{10, .25\}$ Auction, Lottery	900.4	977.6	1148.2	975.3	577.7	800.5
3 $\{10, .25\}$ Lottery, Auction	787.3	1077.0	969.0	824.7	568.4	429.0
4 $\{10, .25\}$ Auction, Lottery	776.1	1049.0	1302.0	723.5	302.7	199.5
Average of Sessions 1-4	772.5	1152.7	1099.2	788.6	429.6	423.4
1 $\{10, .75\}$ Auction, Lottery	2446.5	2639.5	2558.4	2501.1	2280.1	2536.0
2 $\{10, .75\}$ Lottery, Auction	2194.8	2130.5	2188.4	2304.7	2091.4	2118.5
3 $\{10, .75\}$ Auction, Lottery	1997.4	1218.5	2003.5	2176.2	1812.4	2060.0
4 $\{10, .75\}$ Lottery, Auction	1394.0	1603.0	1514.5	1389.7	1277.7	1152.5
Average of Sessions 1-4	2008.0	1897.9	2066.2	2092.9	1865.4	1966.8

Table 8: Session-Level Average Group Contribution Less Prize Under the All-Pay Auction Mechanism Over Various Intervals of Time

for the lottery (all-pay auction) treatment if the lottery (all-pay auction) treatment was played in the first 15 rounds rather than in the last 15 rounds. This finding is a natural consequence of our within-subject design and the common pattern of decay over time in public good contributions by a fixed cohort of increasingly experienced subjects. In an effort to mitigate the role of possible order effects on our aggregate analysis of bids and public good provision, we did vary the order of the two treatments so that the lottery mechanism was played in the first 15 rounds of half of all sessions of our four treatments and the all pay auction was played in the first 15 rounds of the other half of all sessions of our four treatments.

Our next finding concerns a test of one of the comparative statics predictions of our provisional fixed prize theory. Specifically, if we hold the group size, n, fixed and focus on the symmetric NE with positive public good provision, then a larger β should result in higher individual bids and greater public good provision under the provisional lottery mechanism and indeed this is the case. By contrast, under the provisional all-pay auction mechanism if we hold n fixed then a larger β should not result in higher individual bids or greater public good provision. Note that the latter comparative static prediction for the all-pay auction would also hold if the fixed prize was exogenously given (see Table 2) and public good provision was predicted to be non-zero. Inconsistent with this theoretical prediction, we find that for fixed n, an increase in β leads to higher individual bids and greater public good provision under the all-pay auction mechanism just

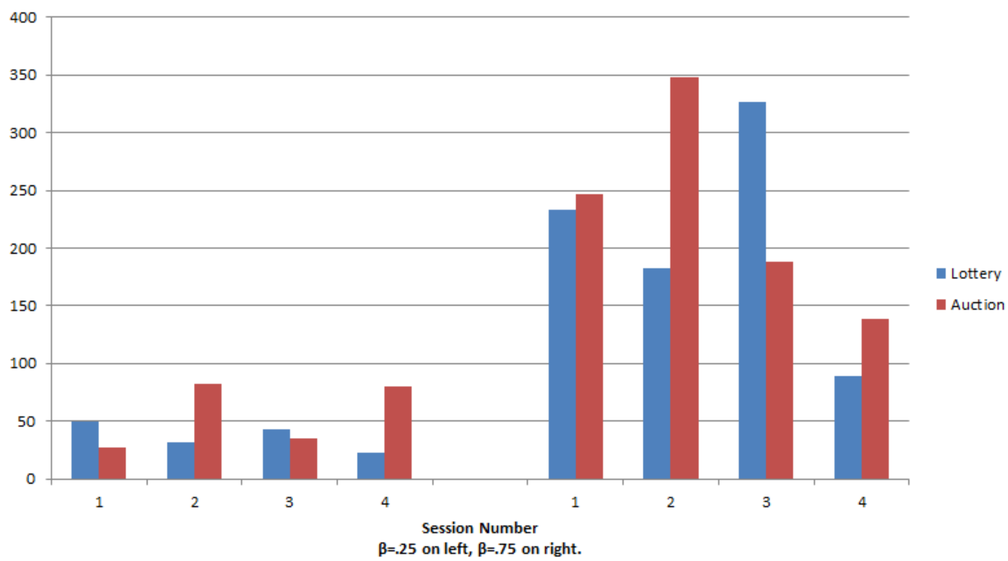

Figure 2: Public good provision under the lottery and all-pay auction mechanisms when $n = 2$: $\beta = .25$ versus $\beta = .75$

as we found for the case of the lottery mechanism. We summarize this as follows:

Finding 2 *For a fixed group size n, larger β leads to higher individual bids and to higher public good provision under both mechanisms.*

Support for Finding 2 is found in Tables 4-8 and in Figures 2 and 3 which illustrate session-level observations on Avg. G over all 15 rounds of each treatment. When $n = 2$, a switch from $\beta = .25$ to $\beta = .75$ causes G to increase, on average, by a factor of 5.7 in the lottery treatment and by a factor of 4.1 in the all-pay auction treatment. Similarly, when $n = 10$, a switch from $\beta = .25$ to $\beta = .75$ leads G to increase, on average, by a factor of 2.5 in the lottery treatment and by a factor of 2.6 in the auction treatment. For each mechanism, if we fix n at 2 or 10, the amount of G is always significantly higher when $\beta = .75$ as compared with when $\beta = .25$ according to non-parametric Mann-Whitney tests of the null hypothesis of no difference using the session-level observations that are illustrated in Figures 2 and 3 ($p = .02$ – lowest possible p-value – for all four tests $n = 2$ and $n = 10$; lottery, all-pay auction).

Our third finding considers the other comparative statics prediction, namely the impact of group size, n, holding β fixed. According to the theory (see again Table 1), for fixed β, a larger group size n should lead to: 1) lower individual bids but higher public good provision under the lottery mechanism (focusing on the symmetric NE with positive public good provision) and 2) zero individual bids and zero public good provision under the all-pay auction mechanism. With regard to these predictions we have:

Finding 3 *For a fixed β, a larger n does not reduce the individual amounts bid but it does result in higher public good provision under both mechanisms.*

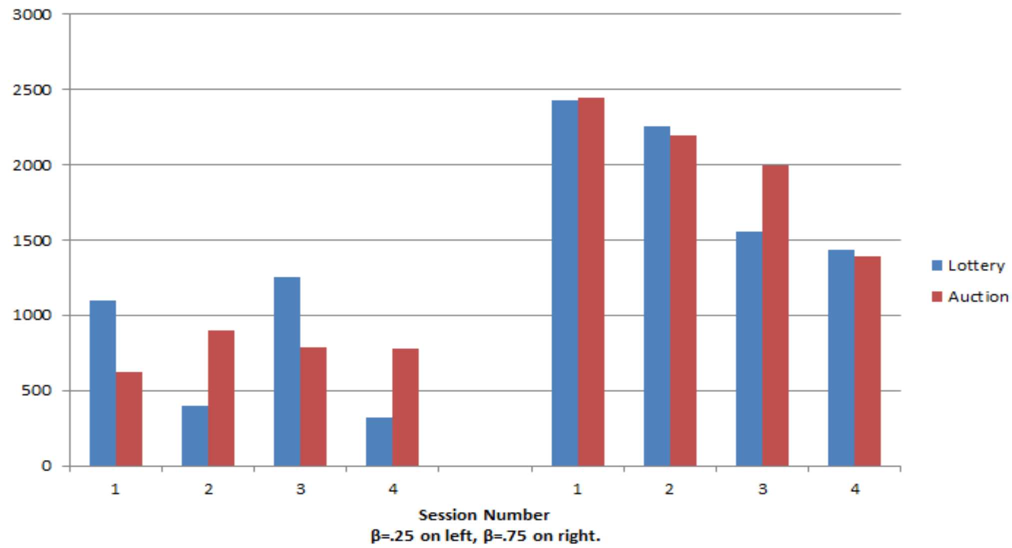

Figure 3: Public good provision under the lottery and all-pay auction mechanisms when $n = 10$: $\beta = .25$ versus $\beta = .75$

Support for Finding 3 is again found in Tables 4-8 and in Figures 2-3. Consider first the session-level observations for Avg. Bid over rounds 1-15 as reported in the second columns of Tables 5 and 6. Suppose we hold β fixed at either .25 or .75 and we consider whether average bids are different when $n = 2$ as compared with when $n = 10$. For three of the four treatment conditions, lottery with $\beta = .25$, lottery with $\beta = .75$, and all-pay auction with $\beta = .75$, non-parametric Mann-Whitney tests indicate that we cannot reject the null hypothesis of no difference in average bids across the two different values for n ($p > .10$ for all three tests). For the all-pay auction with $\beta = .25$, we can reject the null hypothesis of no difference in favor of the alternative that bids are significantly higher when $n = 10$ than when $n = 2$ ($p = .08$).

While average bids should decrease as n increases holding β fixed, under the lottery mechanism, the NE prediction calls for G to nevertheless *increase* in this same scenario. This aspect of the lottery theory does find support in the experimental data. Intuitively, as we found no decrease in the individual amounts bid as n increases from 2 to 10, it should come as no surprise that G is higher under both mechanisms as n is increased from 2 to 10.

Confirmation again comes from non-parametric Mann-Whitney tests on the session-level observations that are illustrated in Figures 2-3 (see also column 2 of Tables 7 and 8). Fixing $\beta = .25$ or .75, an increase in n from 2 to 10 leads to significantly higher public good provision G under both mechanisms (the null hypothesis of no difference in G is rejected, $p = .02$ – lowest possible p-value – for all four tests, $\beta = .25$, $\beta = .75$; lottery, all-pay auction).

We note further that non-provision of the public good is predicted under the lottery mechanism when $n = 2$ and $\beta = .25$, however as Table 4 indicates, public good provision actually occurred an average of 63 percent of the time in this treatment. Indeed, holding β fixed, provision of the public good increased as n is increased under both mechanisms. Recall from Finding 3 that individual bid amounts did not generally change for fixed β as n was increased. Nevertheless, having

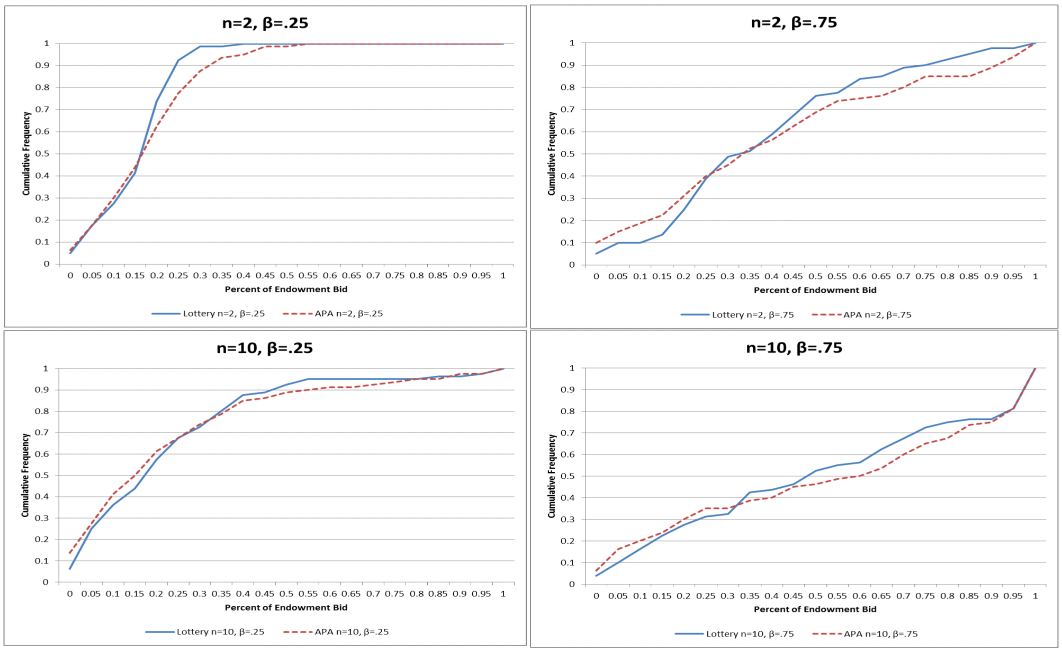

Figure 4: Cumulative distribution of individual bids as a percentage of endowment, lottery versus all-pay auction under each treatment condition

more individual bidders (larger n) bidding similar amounts ensures that the provision point where contributions exceeded the prize level of $V = 100$ is more likely to be achieved as n is increased.

We now turn to the important question of whether bidding behavior was different between the two mechanisms for the same $\{n, \beta\}$ treatment conditions. We explore this issue at both the individual and aggregate level. We start by looking at the distribution of *individual* bids as a percentage of individual endowments over all 15 rounds of the lottery and all-pay auction mechanisms under each of our four treatment conditions. Figure 4 shows the cumulative distribution functions of bids as a percentage of each individual's endowment for each $\{n, \beta\}$ pair using pooled data from all sessions of the four treatments. This figure reveals that the distribution of bids between the lottery and all-pay auction mechanisms are very similar to one another under all four treatment conditions. We summarize this result as:

Finding 4 *Holding n and β constant, there is little difference in the distribution of individual bids between the two provisional fixed-prize mechanisms.*

We next address whether there is an *aggregate* difference in the level of public good provision (net of the prize if awarded) between the two mechanisms for the same $\{n, \beta\}$ pair. According to the theory, for fixed n and β, the lottery design may result in higher public good provision because the all-pay auction mechanism should result in zero public good provision under all parameterization of our experimental environment. However, as we have already seen in the individual analysis of bidding behavior, there is no difference in bid distributions between the two mechanisms for fixed n and β. Perhaps not surprisingly, we can report that:

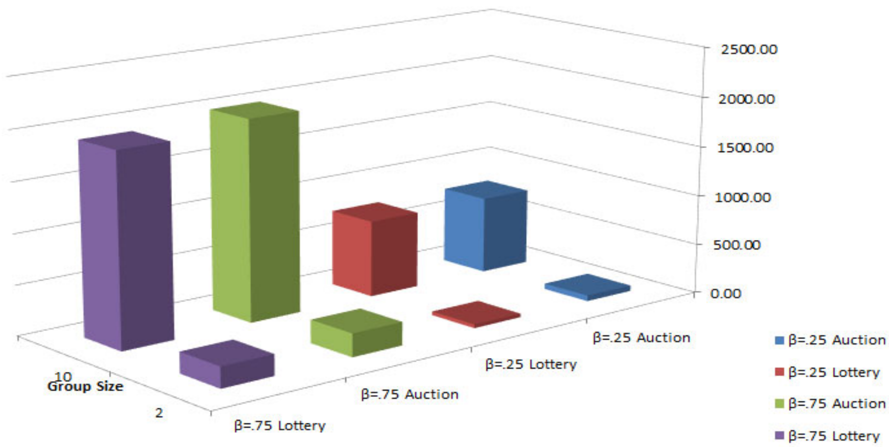

Figure 5: Average Group Contribution Less 100 Token Prize, All Data form All Sessions of All Treatments

Finding 5 *For fixed n and β, public good provision (net of the prize, if awarded) is insignificantly different between the two provisional fixed-prize mechanisms.*

Support for finding 5 is shown in Figure 5. Statistical support for finding 5 is found by conducting four Mann-Whitney tests using the four session-level observations for Avg. G for each treatment (auction or lottery) for the same values of $\{n, \beta\}$. Using the session-level observations for Avg. G over all 15 rounds as reported in the second column of Tables 7 and 8 a Mann-Whitney test indicates that we cannot reject the null hypothesis of no difference in public good provision in any pairwise test between the lottery and all-pay auction mechanisms for the same $\{n, \beta\}$ values ($p > .10$ for all four tests). This same conclusion of no difference in Avg. G between mechanisms would continue to hold if we used as our session-level observations the values of Avg. G over the final five rounds, 11-15, of each session as reported in the sixth column of Tables 7 and 8.

A typical pattern of behavior in public good games is a decline in contributions over time as individuals learn to give less. We also find evidence of such learning behavior in our experimental data. Tables 5-6 report average bids over all rounds 1-15, in round 1 alone, over rounds 1-5, 6-10, 11-15, and in the final round 15 alone. Tables 7-8 do the same for public good provision levels (Avg. G). Finally, Figure 6 illustrates the average value of G (net of the prize, if awarded) over rounds 1,2,...15 as a percentage of total endowment using all data from all sessions of each treatment, $\{M, n, \beta\}$.

The Tables and Figure 6 provide evidence that average bids and public good provision are declining with experience in nearly all sessions of all treatments. Since overall average bids (over all rounds 1-15) as reported in Table 4 were found to have generally exceeded Nash equilibrium predictions, this decay in bids and public good provision over time serves to bring behavior closer to equilibrium predictions by the final rounds of each session of each treatment. We summarize this finding as follows:

Finding 6 *We observe a decline in both the average individual amount bid and in public good provision in the last 5 rounds as compared with the first five rounds under both mechanisms.*

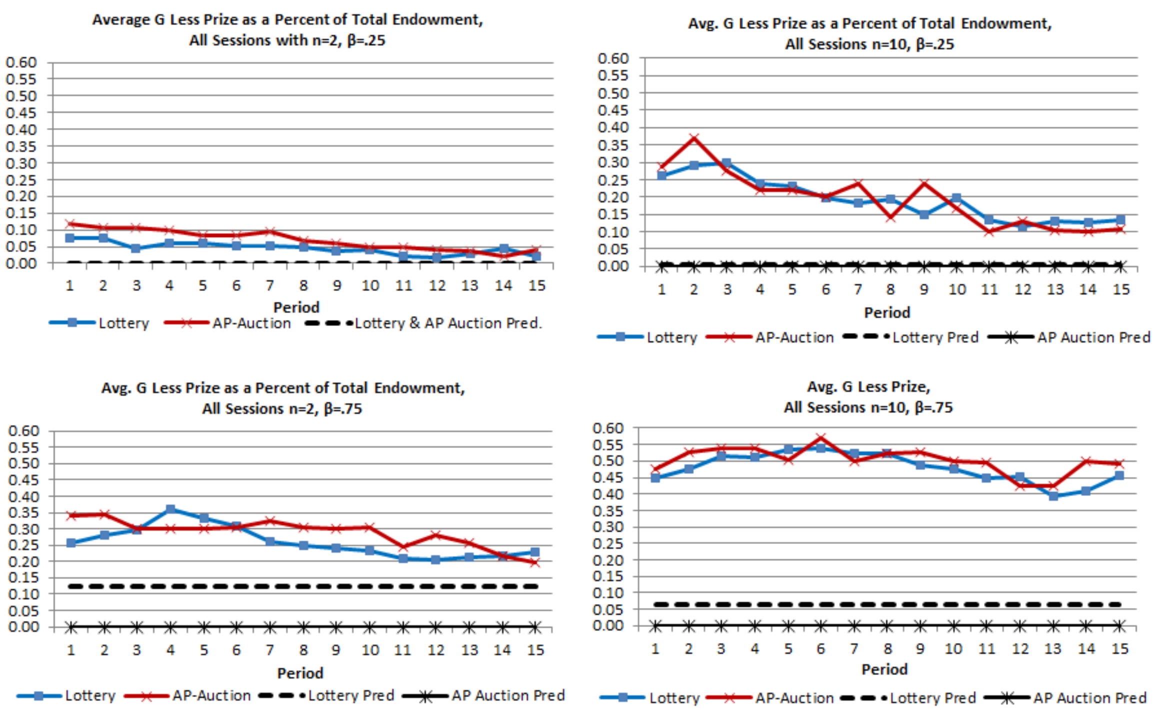

Figure 6: Average Public Good Provision G Less Prize as a Percent of Total Endowment Over Rounds 1-15. Averages are from all 4 Sessions of Each Treatment (M,n, β).

Consider first the session-level average individual bids in rounds 1-5 versus rounds 11-15 under the various treatments of the lottery or auction mechanisms, as reported in columns 4 and 6 of Tables 5 and 6. Using a Wilcoxon signed-rank test for matched pairs on the four session-level observations per treatment we can reject the null hypothesis of no difference in average bids over rounds 1-5 as compared with average bids over rounds 11-15 in the same session in favor of the alternative that average bids are *lower* in the final 5 rounds as compared with the first five rounds in seven of our eight treatments ($p < .10$ in seven tests): 1) lottery, $n = 2$, $\beta = .25$; 2) lottery $n = 2$, $\beta = .75$; 3) lottery $n = 10$, $\beta = .25$; 4) auction $n = 2$, $\beta = .25$; 5) auction $n = 2$, $\beta = .75$; 6) auction $n = 10$, $\beta = .25$; 7) auction $n = 10$, $\beta = .75$. The sole exception is the lottery $n = 10$, $\beta = .75$ treatment where we cannot reject the null hypothesis of no difference ($p = .14$). However, the exception arises because in just one session of that treatment (session 4), average bids in the last 5 rounds were higher than in the first 5 rounds while in the other sessions of that treatment they were lower.

Consider next the session-level average amounts of public good provision in rounds 1-5 versus rounds 11-15 under the various treatments of the lottery or auction mechanisms, columns 4 and 6 of Tables 7 and 8. Here the results mirror those found for average individual bids. There is a statistically significant decrease in public good provision G less the 100 token prize from the first 5 to the last 5 rounds of all sessions of all treatments ($p < .10$) except in the lottery $n = 10$, $\beta = .75$ treatment where we cannot reject the null hypothesis of no difference ($p = .14$).

While the decline over time in average individual bids and public good provision brings behavior more in line with NE predictions, there remains, in the last 5 rounds, substantial over-bidding and over-provision of the public good relative to equilibrium predictions. Indeed recall that Finding 1 remains unchanged if attention is restricted to average individual bids and public good provision G less the prize in the last five rounds 11-15 instead of over all rounds 1-15. The two exceptions are for the provisional lottery mechanism where $n = 2$ and $\beta = .25$ or .75. In these two treatments we cannot reject the null hypothesis of no difference between average behavior and the NE bid predictions in the final 5 rounds using the Wilcoxon signed-ranks test ($p > .10$ for all tests). Recall that for the all-pay auction mechanism, we cannot perform a statistical test as the NE bid and public good provision are at the zero lower bound. However it is apparent from Tables 6 and 8 and Figure 6 that bids and public good provision generally remain greater than 0 even in the final 5 rounds of all of the all-pay auction sessions.

8 Discussion

Our experimental findings are inconsistent with theoretical predictions on two related dimensions. First, contributions to the public good are excessive relative to theoretical predictions. Second, there is no difference in contributions between the two self-financing fixed prize mechanisms despite the theoretical prediction of zero contributions under the parameterization we adopt for the all-pay auction mechanism implemented in the laboratory. In this section we offer some possible explanations for these anomalous findings.

8.1 Excessive contributions

The pattern of excessive contributions to the public good under both mechanisms and the decline in such contributions as participants gain experience (see Figure 6), is a pattern commonly found in the literature on VCM games (see, e.g., Isaac et al. (1984)), all-pay auctions (see, e.g., Gneezy and Smorodinsky (2006)) and in the literature on non-provisional fixed-prize fundraising mechanisms as well (see, e.g., Corazzini et al. (2010)). Excess contributions might be explained by a variety of factors including bounded rationality and learning, non-risk neutral preferences, or non-expected utility maximization.

We have already seen evidence favoring bounded rationality and learning as an explanation in our observation that mean bids do decline over time in the direction of NE bids and public good provision levels as subjects gain experience. However, with the exception of the two $n = 2$ lottery treatments (where $\beta = .25$ or $.75$) this decline does not begin to approximate Nash equilibrium predictions by the end of our treatments. We speculate that in larger lottery groups of size 10 or in the less familiar all-pay auction setting this equilibration process might take a longer period of time than was allowed in our laboratory setting.

To address this possibility, we conducted two additional "long" sessions of the $n = 10$, $\beta = .25$ version of the lottery and all-pay auction treatments. Specifically, we eliminated the within-subject design, where we switched from one prize mechanism to the other after 15 rounds of play and instead conducted a full, 30 round of play under a single mechanism/treatment condition, i.e. either lottery or all-pay auction. There were no other changes in the design: in each session, 20 subjects were randomly matched at the start of each of the 30 rounds to form 2 groups of size 10, the model was parameterized as before ($e = 400$ and $V = 100$) and subjects were paid for two randomly chosen rounds from the first and from the last 15 rounds, so that payoff incentives were similar to the prior, shorter sessions. The results from these two long sessions are reported in Table 9 and illustrated in Figure 7. The results indicate that doubling subjects' experience with a particular mechanism

Treatment Cond.	One Long Session			All Short Sessions			Nash Eq.		
$n = 10$, $\beta = .25$	Avg. Bid	Avg. G	Prov.	Avg. Bid	Avg. G	Prov.	Bid	G	Prov.
Lottery	134.8	1248.1	1.0	86.6	767.6	0.98	12.0	20.0	0.0
All-Pay Auction	137.7	1276.7	1.0	87.0	772.5	0.98	0.0	0.0	0.0

Table 9: Average Bids, Public Good Less Prize (G) and Provision Frequencies (Prov.) in One Long (30 Round) Session of the $n = 10$, $\beta = .25$ Lottery and All-Pay Auction Treatments as Compared with the Shorter (15 Round) Sessions of the Same Treatments.

does not move behavior closer to NE predictions; as Table 9 reveals, subjects' bids and public good provision levels in the Long, 30 Round sessions are, on average, *further away* from NE predictions. Nevertheless, as Figure 7 makes clear, there is a pronounced decline in those average bids over time so that behavior in the final, 30th round of the Long sessions is similar to behavior in the final, 15th round of the Short sessions. Indeed, it appears as though subjects are adjusting the speed with which they approach the equilibrium outcome to compensate for the longer horizon-length of the Long treatment sessions.

A second possible explanation is that players do not have risk neutral preferences with respect to uncertain money amounts as the theory supposes. To explore whether deviations from risk neutral preferences played a role in over-bidding, we consider the possibility that agents attach utility $u(m)$ to monetary earnings m, so that the maximization problem under the Lottery mechanism now becomes:

$$\max_{x_i} \pi_i \left(x_i, x_{-i}; n, \beta, \text{Lottery}\right) =$$

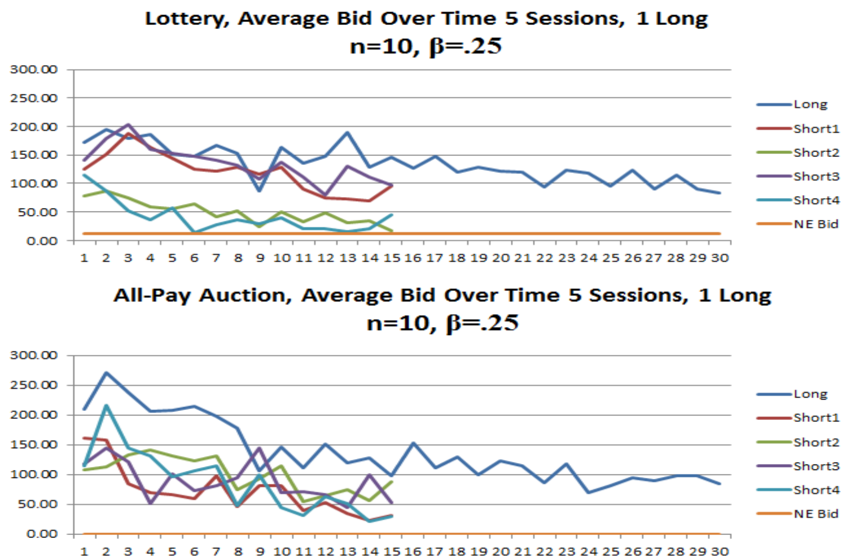

Figure 7: Mean Bids Over Time in One Long (30 Round) Session and Four Short (15 Round) Sessions of the $n = 10$, $\beta = .25$ Lottery and All-Pay Auction Treatments.

$$\begin{cases} \left(\frac{x_i}{\sum_{j=1}^n x_j}\right) u\left[(e - x_i) + \beta\left(\sum_{j=1}^n x_j - V\right) + V\right] + \\ \left(1 - \left(\frac{x_i}{\sum_{j=1}^n x_j}\right)\right) u\left[(e - x_i) + \beta\left(\sum_{j=1}^n x_j - V\right)\right], & \text{if } \sum_{j=1}^n x_j \geq V, \\ u(e), & \text{if } \sum_{j=1}^n x_j < V. \end{cases} \tag{22}$$

Consider the case of a symmetric pure strategy equilibrium where the fixed prize amount is raised and the public good is provided $\left(\sum_{j=1}^n x_j > V\right)$. Combining the first order condition from the above maximization problem with the symmetry condition $x_1 = ... = x_n = x$, we have:

$$\begin{aligned} & \frac{n-1}{n^2 x} u\left[(e - x) + \beta(nx - V) + V\right] + (\beta - 1)\left(\frac{1}{n}\right) u'\left[(e - x) + \beta(nx - V) + V\right] \\ = & \frac{n-1}{n^2 x} u\left[(e - x) + \beta(nx - V)\right] - (\beta - 1)\left(\frac{n-1}{n}\right) u'\left[(e - x) + \beta(nx - V)\right]. \end{aligned} \tag{23}$$

Now suppose that we choose the standard, constant relative risk aversion specification for u, i.e., $u(m) = \frac{m^{1-r}}{1-r}$ so that $u'(m) = m^{-r}$. The risk neutral agents of the theory have $r = 0$, but we now allow for risk-loving ($r < 0$) or risk-averse ($r > 0$) subjects.

The question we ask is: can any of the four average bid amounts from our four treatments be consistent with a symmetric equilibrium of the model for some choice of r, the coefficient of relative risk aversion? That is, suppose we substitute each of the four average bid amounts from our four treatments, as reproduced in Table 10 into equation (23) in place of x and attempt to numerically solve for the value of r that would satisfy equation (23).

The answer is that for each of the four $\{\beta, n\}$ Lottery mechanism treatment conditions, there is no value for r satisfying equation (23). This means that there does not exist a utility function of the form $u(m) = \frac{m^{1-r}}{1-r}$ that could rationalize the high average bids we observe in the lottery treatments

Lottery	$\beta = .25$	$\beta = .75$
$n = 2$	59.3	148.6
$n = 10$	86.6	201.8

Table 10: Average Bids in the Four Treatments of the Lottery Mechanism (as reported earlier in Table 4)

as being consistent with symmetric equilibrium behavior. Intuitively, one might think that some value for $r < 0$, implying risk-loving behavior, might work to rationalize the observed over-bidding but the average bids in the experiment are so large and the expected payoffs are so negative that it is not possible to vary r in a way that can rationalize the bids we observe as corresponding to any symmetric equilibrium.

Since we cannot rationalize observed bidding behavior as a symmetric equilibrium of an expected utility maximization problem, we next turn to a third possibility that subjects have a utility for winning above and beyond the monetary prize value.[20] In this case, we return again to the case of linear utility (risk neutrality). Consider the self-financing provisional fixed-prize all-pay auction mechanism in particular (a similar argument can be applied to the lottery mechanism). Let us assume that players who win the prize receive the monetary utility value of the prize V, as well as some "emotional" or "joy-of-winning", W utility from winning the prize. Note that another interpretation of W can be that players have to raise V in order to get a prize of value $(V + W)$, where the extra W is a matching donation. In this case, (2) becomes

$$\pi_i \left(b_i, b_{-i}; n, \beta, \text{All-pay auction}\right) =$$

$$\begin{cases} (V+W) + (e - b_i) + \beta \left(\sum_{j=1}^n b_j - V \right), & \text{if } b_i > b_j \text{ for any } j \neq i \text{ and } \sum_{j=1}^n b_j \geq V, \\ \frac{(V+W)}{K} + (e - b_i) + \beta \left(\sum_{j=1}^n b_j - V \right), & \text{if } i \text{ ties } (K-1) \text{ others for the high bid and } \sum_{j=1}^n b_j \geq V, \\ (e - b_i) + \beta \left(\sum_{j=1}^n b_j - V \right), & \text{if } b_i < b_j \text{ for some } j \neq i \text{ and } \sum_{j=1}^n b_j \geq V, \\ e, & \text{if } \sum_{j=1}^n b_j < V. \end{cases}$$

As we have already seen in section 5.2, Propositions 8 and 9 that there are a variety of different pure-strategy (symmetric or asymmetric) equilibria involving positive bids and public good provision under the all-pay auction mechanism provided that an efficiency condition holds and that the endowment is not too large, i.e. $\frac{V}{n} \leq e \leq \frac{V}{2(1-\beta)}$.[21] With the addition of joy of winning motivations, the latter endowment condition changes to $\frac{V+W}{n} \leq e \leq \frac{V+W}{2(1-\beta)}$. Given our parameterization of the experiment, where $V = 100$ and $e = 400$, we can estimate the minimum value of W necessary for the existence of pure-strategy equilibria involving positive bids and public good provision. For example, in our treatment where $n = 2$ and $\beta = .75$, in order to obtain positive bids in the interval $[0, 400]$, we must have that $e = 400 \leq \frac{100+W}{2(1-.75)}$ or that $W \geq 100$. Similarly, for $n = 10$ and $\beta = .75$, there exist pure-strategy asymmetric equilibria with two active players if $W \geq 100$, while for $n = 10$ and $\beta = .25$, the same type of asymmetric equilibria would require that $e = 400 \leq \frac{100+W}{2(1-.25)}$, or that

[20] Indeed, a number of studies have linked over-bidding in auctions (Goeree et al. 2002; Cooper and Fang, 2008) and contests (Parco et al., 2005; Sheremeta 2010) to such "joy–of–winning" motivations.

[21] There also exist mixed strategy equilibria as well, see in particular Proposition 13 of Appendix B.

$W \geq 500$. Thus, in principle one can rationalize the positive bidding behavior that we observe in our all-pay auction treatment with a sufficiently high value for W.

A difficulty with this rationalization is that the bidding behavior in the symmetric equilibria involves all players bidding their endowment amount $e = 400$. In the asymmetric equilibria, the active players bid $e = 400$ while the inactive players bid 0. The evidence for such equilibrium behavior (symmetric or asymmetric) is mixed. For instance in the $n = 2, \beta = .75$ all-pay auction treatment, the symmetric equilibrium strategy where both players bid $e = 400$ is played just 5.67 percent of the time, while the asymmetric equilibrium strategy where one player bids 400 while the other bids 0 is played 9.33 percent of the time. Thus, for the $n = 2, \beta = .75$ treatment, we could potentially explain an additional 15 percent of the observed outcomes by adding a joy-of-winning motivation. By contrast, for this same treatment, we observe that the symmetric equilibrium where *both* players bid 0 occurs just 5.67 percent of the time; of course the latter equilibrium exists regardless of whether or not there is a joy-of-winning motivation. Thus, in total, it is possible to account for 20.67 percent of the observed outcomes in the $n = 2, \beta = .75$ all-pay auction treatment by adding the joy-of-winning motivation.

For the two $n = 10$ treatments, the symmetric equilibrium wherein all players bid $e = 400$ is *never* observed. In these treatments there are many more pure-strategy asymmetric equilibria to consider, but we don't typically observe (perfect) play of such equilibria either. Asymmetric equilibria where k active players bid $e = 400$ while $n - k$ inactive players bid 0 are observed just 1.67 percent of the time in the $n = 10$, $\beta = .75$ treatment and just 0.83 percent of the time in the $n = 10$, $\beta = .25$ treatment. A more typical outcome involves many players bidding their endowment, many bidding 0 and some bidding intermediate amounts. For example, consider the (15th) round of one of our all-pay auction sessions (#3 of the $n = 10, \beta = .75$ treatment). For that round, in one group of size 10 we find that 5 players bid their endowment of 400, 3 players bid 0, 1 bid 200 while another bid 20. In the other group of size 10, we observe that 4 players bid their endowment of 400, 3 players bid 0, 1 bid 300 and 2 bid 100. This distribution of bids, which is typical of the end of our all-pay auction experiments does not precisely match the asymmetric equilibria of Proposition 9 which require bids of $e = 400$ by active players and bids of 0 by inactive players, though it is not far off from the asymmetric equilibrium. We must nevertheless conclude that for the two $n = 10$ treatments, we cannot explain the positive bids and public good provision using a joy-of-winning motivation.

A fourth possibly for overbidding is that agents acted as non-expected utility maximizers. A standard feature of many non-expected utility models including Prospect Theory and rank-dependent utility models is that individuals use a distorted probability weighting function, $\omega(p)$, of the true probabilities, p, in making their choices under uncertainty. In particular, there is considerable behavioral evidence that the distorted probability weighting function has an "inverted S-shape": (see, e.g., Prelec (1998) for an axiomatization of this type of probability weighting function) for low probability events such as winning a lottery, individuals overweight the probability of winning, i.e., $\omega(p_l) > p_l$, while for higher probability events, they under-weight the probability of winning, i.e. $\omega(p_h) < p_h$.

Let us consider application of the probability weighting approach to our provisional, fixed-prize lottery fundraising mechanism with the aim of understanding how it can explain observed overbidding. Specifically, suppose that subjects used some continuous, increasing, and differentiable function, $\omega(p)$, to weight the probabilities of winning the fixed prize under the lottery mechanism; we will not impose any further restrictions on $\omega(p)$. In that case the individual maximization problem is altered as follows:

$$\max_{x_i} \pi_i \left(x_i, x_{-i}; n, \beta, \text{Lottery,}\omega\right) =$$

$$\begin{cases} (e - x_i) + \beta \left(\sum_{j=1}^{n} x_j - V\right) + \omega \left(\frac{x_i}{\sum_{j=1}^{n} x_j}\right) V, & \text{if } \sum_{j=1}^{n} x_j \geq V, \\ e, & \text{if } \sum_{j=1}^{n} x_j < V. \end{cases} \tag{24}$$

Focusing again on the case of symmetric pure strategy equilibrium where the fixed prize amount is raised and the public good is provided $\left(\sum_{j=1}^{n} x_j > V\right)$, the first order condition combined with symmetry constraint, $x_1 = ... = x_n = x^*$ is:

$$\beta - 1 + \omega' \left(\frac{1}{n}\right) \frac{(n-1)}{n^2 x^*} V = 0,$$

or

$$x^* = \min \left\{ \omega' \left(\frac{1}{n}\right) \frac{(n-1)}{n^2} \frac{V}{(1-\beta)}, e \right\}. \tag{25}$$

We can regard the case where $n = 10$ as representing the relatively lower probability of winning the prize (in the symmetric equilibrium) relative to the $n = 2$ case where there is the highest possible probability of winning the prize.

We again use each of the four average bids from the Lottery treatments as reported above in Table 10 and the fact that $V = 100$ to make some inferences about $\omega'\left(\frac{1}{n}\right)$. Consider first the case where $n = 2$. If $\beta = .25$ then

$$\omega' \left(\frac{1}{2}\right) \times \frac{100}{3} = 59.3, \text{ or } \omega' \left(\frac{1}{2}\right) = 1.779,$$

while if $\beta = .75$ then

$$\omega' \left(\frac{1}{2}\right) \times 100 = 148.6, \text{ or } \omega' \left(\frac{1}{2}\right) = 1.486.$$

Notice that both values for $\omega'\left(\frac{1}{2}\right)$ are relatively close to one another.

Consider next the case where $n = 10$. If $\beta = .25$ then

$$\omega' \left(\frac{1}{10}\right) \times 12 = 86.6, \text{ or } \omega' \left(\frac{1}{10}\right) = 7.217,$$

while if $\beta = .75$ then

$$\omega' \left(\frac{1}{10}\right) \times 36 = 201.8, \text{ or } \omega' \left(\frac{1}{10}\right) = 5.606.$$

Notice that these last two values for $\omega'\left(\frac{1}{10}\right)$ are not so close to one another. However, in all four cases the implied slope of the probability weighting function is greater than 1, which indicates that there is always *over*-weighting of the objective probability of winning the prize in all of our lottery treatments. This over-weighting of the probability of winning can explain why bids are in excess of equilibrium predicted levels. Furthermore, the comparative statics implication of a change in n is also consistent with the inverted S-shape form of the probability weighting function in that for either value of β we have

$$\omega' \left(\frac{1}{10}\right) > \omega' \left(\frac{1}{2}\right),$$

which means that individuals overestimate smaller probabilities of winning the prize much more than they overestimate higher probabilities of wining the prize under the lottery mechanism. This last insight can explain why bids do not decline as n increases from 2 to 10, despite the objective decrease in the probability of winning the fixed prize in any symmetric equilibrium as n increases.

We note that our approach to probability weighting is quite general and does not require that we specify a particular form of the weighting function $\omega(p)$; our data are consistent with any inverted S-shape probability weighting function. We note that Baharad and Nitzan (2008) also analyze equilibrium in contests under distorted probabilities, but they consider a specific probability weighting function of the form

$$\varpi(p) = \frac{p^{\alpha}}{[p^{\alpha} + (1-p)^{\alpha}]^{(1/\alpha)}}$$

and use that function to describe rent dissipation in their standard contest environment (without public goods).

As our mechanisms also have a public good element, it is possible that the probability weighting function also depends on the marginal per capital return β, i.e., $\omega(p, \beta)$. Indeed, our experimental findings as discussed above suggest that $\omega_{\beta}(p, \beta) < 0$; as β increases, the public good component of players' payoffs increases and so there may be less concern for winning the fixed prize and consequently less over-weighting of the probability of winning that prize.

8.2 No difference in bidding behavior across mechanisms

The absence of any difference in bids or public good provision levels across the two mechanisms (lottery, auction) for the same (β, n) parameterization suggests that subjects are not considering how the provisional nature of the prize should alter their strategic best response in the all-pay auction relative to the case where the prize is exogenously given. Recall from Table 2 that in the case where there is an exogenously given fixed prize, the expected bid amounts under the all-pay auction are always strictly positive and not very different (though strictly greater) than the predicted bid amounts under the lottery mechanism. Recall also that for the lottery mechanism, whether the fixed prize is exogenously given or is provisional on contributions equaling or exceeding the prize value is of no consequence for theoretical predictions regarding bids.

The apparent disregard of subjects for the strategic consequences of making the fixed-prize provisional under the all-pay auction mechanism is consistent with the notion, advanced by Leininger (2000), that players view first price all-pay auctions as a kind of binary lottery. Since in the complete information all-pay auction that we study all bids are paid and the prize (if offered) has a known value V, each bidder's expected payoff does not depend on information held by other bidders. From the individual bidder's perspective, this version of an all–pay auction is effectively an opportunity to buy, at a fixed price equivalent to the bidder's bid b_i, a *lottery* with a known payoff of V with some probability $\tilde{p}$ and a payoff of 0 with probability $1 - \tilde{p}$. Viewed this way, there is not much difference, from the player's perspective, between our first price all-pay auction and the lottery mechanism and it seems plausible that subjects in our experiment may have adopted this view. In that case, it is less surprising that there is little difference in bidding behavior across the two mechanisms.

9 Conclusions

We have studied two fixed-prize mechanisms for raising charitable contributions, a lottery and an all-pay auction. We study these two mechanisms in a complete information setting with a known fixed prize of common value V where all bidders are ex-ante identical and have equal budget constraints. We focus on fixed-prize mechanisms that are *self–financing*, where the prize is not exogenously given. Specifically, we require that the total endowment of the n players must be sufficient to finance the prize and that total contributions must equal or exceed the prize level, V, for the fixed prize to be awarded. Public good provision is thus the net value $(G - V)$ if positive and 0 otherwise. We emphasize that the existence of equilibria with positive public good provision under both self-financing fixed-prize mechanisms requires that certain endowment conditions are satisfied and that public good provision is efficient, i.e., that $\beta n \geq 1$. For both provisional fixed-prize mechanisms, we also calculate the value of the provisional fixed-prize, V, that maximizes contributions to the public good. Our main finding is that, under certain plausible conditions for the value of the fixed-prize relative to endowments, the provisional fixed-prize lottery mechanism can generate greater public good provision than the provisional fixed-prize all-pay auction mechanism. The latter result is new to the literature on public good fixed-prize fundraising mechanisms.

We have also conducted an experimental test of some of our theoretical predictions, implementing for the first time in the laboratory, two provisional fixed-prize mechanisms for raising contributions to a public good. The main innovation of our experimental design is that we have varied both the group size, n, and the marginal per capital return on the public good, β, which enables us to consider some of the important comparative statics implications of the Nash equilibrium predictions, e.g., the role of efficiency, which have not previously been addressed in the literature. Relative to theoretical predictions, we find over-bidding and over-provision of the public good under both provisional fixed-prize mechanisms. However, there is also evidence that individuals learn over time to bid less under all treatment conditions so that over-bidding and over-provision decline with experience. Still, final amounts bid and levels of public good provision remain substantially greater than theoretical predictions in most of our treatments.

In contrast to our theoretical prediction of zero bids and zero public good provision we find that bids are always large and positive under the provisional prize all-pay auction mechanism. Under the lottery mechanism we find more consistency with our theory: for fixed n, bids and public good provision increase as β increases and for fixed β public good provision increases as n increases. However we also observe that for fixed β, bids do not decrease as n increases a finding that is inconsistent with the theory. Perhaps most importantly, despite the prediction that the lottery will yield greater public good provision given our choices for e, V, and for all $\{n, \beta\}$ treatment conditions, we find absolutely no difference in public good provision between the lottery and the all-pay auction provisional fixed-prize mechanisms.

Toward an explanation of why our findings depart from theoretical predictions, we have considered the possibility that subjects have risk preferences that depart from risk neutrality or that they have joy-of-winning motivations or that they are non-expected utility maximizers. We find some limited support in our data for the latter two explanations. We further suggest that subjects do not properly account for the provisional nature of the prize in their bidding behavior and view the all-pay auction mechanism as just another type of binary lottery; this perspective may account for our finding of no difference in public good provision between the two provisional fixed-prize mechanisms.

Finally, we note that our findings have important implications for designers of prize-based fundraising mechanisms. First, our theoretical findings suggest that both the efficiency of public good provision and the size of the fixed prize relative to participants' endowments are important considerations if a *provisional* fixed-prize mechanism is adopted. If public good provision is efficient and if the prize value is small relative to endowments, then there is a theoretical argument in favor of adopting a lottery mechanism for awarding the provisional, fixed-prize. We also characterize the optimal prize value given the environment. Second, while our experimental findings indicate no statistical difference in contributions or public good provision between the two provisional fixed-prize mechanisms, these provisional mechanisms do fix the negative public good provision problem described in Landry et al. (2006) and Lange et al. (2007). Under the provisional mechanisms we study the prize was never paid out in those instances (particularly in the $n = 2$ and $\beta = .25$ treatment) where contributions were insufficient to fund the prize. Third, our experimental finding of no difference in behavior under the two fundraising mechanisms should be treated with great caution as we are considering the case where participation in the two mechanisms is required. As Carpenter et al. (2008) report in their field study that allowed for endogenous participation, participants are significantly less likely to participate in an all-pay charity auction than in a first-price charity auction. If participation were endogenous and if the provisional lottery mechanism led to greater participation than the provisional all-pay auction mechanism, then our result showing that public good provision is increasing in the group size, n, would imply that a provisional fixed-prize lottery mechanism would be the preferred choice. We leave to future research the important questions of endogenous participation and the performance of provisional fixed-prize mechanisms in the field.

References

[1] Bagnoli, M. and B. Lipman (1989), "Provision of public goods: fully implementing the core through voluntary contributions," *Review of Economic Studies* 56, 583–601.

[2] Baharad, E. and S. Nitzan (2008), "Contest efforts in light of behavioural considerations," *Economic Journal* 118, 2047–2059.

[3] Baye, M., D. Kovenock, and C. de Vries (1996), "The all–pay auction with complete information," *Economic Theory* 8, 291-305.

[4] Camerer, C.F. (2003), *Behavioral Game Theory: Experiments in Strategic Interaction.* Princeton: Princeton University Press.

[5] Carpenter, J., J. Holmes, and P.H. Matthews (2008), "Charity auctions: A field experiment," *Economic Journal* 118, 92-113.

[6] Carpenter, J. and P.H. Matthews (2017), "Using raffles to fund public goods: Lessons from a field experiment," *Journal of Public Economics* 150, 30-38.

[7] Cooper, D.J. and H. Fang (2008), "Understanding Overbidding in Second Price Auctions: An Experimental Study," *Economic Journal*, 118, 1572-1595.

[8] Corazzini, L., M. Faravelli, and L. Stanca (2010), "A prize to give for: An experiment on public good funding mechanisms," *Economic Journal* 120, 944-967.

[9] Davis, D., L. Razzolini, R. Reilly, and B. Wilson (2006), "Raising revenues for charity: Auctions versus lotteries," in: D. Davis and R. Isaac (eds.), *Research in Experimental Economics* Vol. 11, JAI Press, Greenwich, pp. 47-91.

[10] Damianov, D.S. and R. Peeters, (2018), "Prize-based mechanisms for fund-raising: theory and evidence," *Economic Inquiry*, 56, 1562-1584.

[11] Dechenaux, E., D. Kovenock, and R. Sheremeta (2015), "A survey of experimental research on contests, all-pay auctions and tournaments," *Experimental Economics* 18, 609-669.

[12] Faravelli, M. and L. Stanca (2014), "Economic incentives and social preferences: Causal evidence of non-separability," *Journal of Economic Behavior and Organization* 108, 273–289.

[13] Giebe, T. and P. Schweinzer (2014), "Consuming your way to efficiency: Public goods provision through non-distortionary tax lotteries," *European Journal of Political Economy* 36, 1-12.

[14] Gneezy, U. and R. Smorodinsky (2006), "All-pay auctions – an experimental study," *Journal of Economic Behavior and Organization* 61, 255-275.

[15] Goeree, J., C. Holt, and T.R. Palfrey (2002), "Quantal response equilibrium and overbidding in private-value auctions," *Journal of Economic Theory* 104, 247–272.

[16] Goeree, J., E. Maasland, S. Onderstal, and J. Turner (2005), "How (not) to raise money," *Journal of Political Economy* 113, 897-918.

[17] Isaac, R., J. Walker, and S. Thomas (1984), "Divergent evidence on free riding: An experimental examination of possible explanations," *Public Choice* 43, 113-149.

[18] Landry, C., A. Lange, J. List, M. Price, and N. Rupp (2006), "Toward an understanding of the economics of charity: Evidence from a field experiment," *Quarterly Journal of Economics* 121, 747-782.

[19] Lange, A., J. List, and M. Price (2007), "Using lotteries to finance public goods: Theory and experimental evidence," *International Economic Review* 48, 901–927.

[20] Leininger, W. (2000), "Auction theory from an all-pay view: Buying binary lotteries," working paper, University of Dortmund.

[21] Morgan, J. (2000), "Financing public goods by means of lotteries," *Review of Economic Studies* 67, 761-784.

[22] Morgan, J. and M. Sefton (2000), "Funding public goods with lotteries: Experimental evidence," *Review of Economic Studies* 67, 785-810.

[23] Onderstal, S., A. Schram, and A.R. Soetevent (2013), "Bidding to give in the field," *Journal of Public Economics* 105, 72–85.

[24] Orzen, H. (2008), "Fundraising through competition: Evidence from the lab," Discussion paper no. 2008-11, The Centre for Decision Research and Experimental Economics, School of Economics, University of Nottingham.

[25] Parco, J., A. Rapoport and W. Amaldoss (2005), "Two-stage contests with budget constraints: An experimental study. *Journal of Mathematical Psychology* 49, 320-338.

[26] Prelec, D. (1998), "The probability weighting function," *Econometrica* 66, 497-527.

[27] Schram, A. and S. Onderstal (2009), "Bidding to give: An experimental comparison of auctions for charity," *International Economic Review* 50, 431-457.

[28] Sheremeta, R. (2010), "Experimental comparison of multi-stage and one-stage contests," *Games and Economic Behavior* 68, 731-747.

[29] Sheremeta, R. (2011), "Contest design: An experimental study," *Economic Inquiry* 49, 573-590.

Appendix A

Proof of Proposition 2.

Consider player i's best reply correspondence when her opponents spend $\sum_{j\neq i} x_j = X_{-i}$:

$$BR_i\left(X_{-i}\right) = \begin{cases} \left[0, \min\left\{V - X_{-i}, e\right\}\right], & \text{if } 0 \leq X_{-i} \leq (1-\beta)V, \\ \min\left\{\sqrt{\frac{V}{(1-\beta)}X_{-i}} - X_{-i}, e\right\}, & \text{if } (1-\beta)V < X_{-i} \leq \frac{V}{(1-\beta)}, \\ 0, & \text{if } X_{-i} > \frac{V}{(1-\beta)}. \end{cases}$$

Suppose that $X_{-i} \leq (1-\beta)V$. Then, player i's best reply is the whole interval $[0, \min\{V - X_{-i}, e\}]$. Therefore, any pure strategy profile $(x_1, ..., x_n)$, such that $0 \leq x_i \leq \min\{(1-\beta)V, e\}$, $\max_i X_{-i} \leq (1-\beta)V$, and $\sum_{i=1}^{n} x_i \leq V$, is a Nash equilibrium. ■

Proof of Corollary 4.

From Proposition 3, in particular expression (10), we have to consider the following maximization problem in order to find the prize value that maximizes public good provision:

$$\max_{V\in[0,ne]} \left(\min\left\{\frac{(\beta n - 1)}{n(1-\beta)}V, ne - V\right\}\right).$$

Hence, we must have

$$\frac{(\beta n - 1)}{n(1-\beta)}V = ne - V,$$

or

$$V = \frac{n^2}{(n-1)}(1-\beta)e.$$

The highest public good provision (less the prize, V) is thus:

$$\widetilde{G^*} = \min\left\{\frac{(\beta n - 1)}{n(1-\beta)}V, ne - V\right\} = \frac{n}{(n-1)}(\beta n - 1)e.$$

■

Proof of Proposition 8.

Suppose that there exists a pure-strategy equilibrium where the public good is provided.

Suppose that $(b_1, ..., b_n)$ is such an equilibrium and

$$\max_j b_j = \overline{b} < e. \tag{26}$$

Then, there exist players l and k such that $b_l = \overline{b} \geq b_k$. First, suppose that $b_k = \overline{b}$. Then, consider player k's payoff, if she bids $\overline{b} + \epsilon < e$:

$$\pi_k\left(\overline{b} + \epsilon, B_{-k}\right) = \left(e - \left[\overline{b} + \epsilon\right]\right) + V + \beta(\left[\overline{b} + \epsilon + B_{-k}\right] - V),$$

where ϵ is "a very small amount" and $B_{-k} = b_1 + ... + b_{i-1} + b_{i+1} + ... + b_n$. Note that $\pi_k\left(\overline{b} + \epsilon, B_{-k}\right) > \pi_k\left(\overline{b}, B_{-k}\right)$. Second, suppose that $0 < b_k < \overline{b}$, then $\pi_k(0, B_{-k}) > \pi_k(b_k, B_{-k})$. Third, suppose that $b_k = 0$. Then, $\pi_k(0, B_{-k}) = e + \beta(B_{-k} - V)$. However,

$$\pi_k\left(\overline{b} + \epsilon, B_{-k}\right) - \pi_k(0, B_{-k}) =$$

$$V - (1 - \beta)\left[\overline{b} + \epsilon\right] > V - (1 - \beta)\, e \geq \frac{n-1}{n} V > 0.$$

Therefore, condition (26) cannot hold in a pure-strategy equilibrium where the public good is provided.

Suppose that $(b_1, ..., b_n)$ is a pure-strategy equilibrium where the public good is provided and

$$\max_j b_j = e. \tag{27}$$

Then, there exist players l and k such that $b_l = e \geq b_k$.

First, suppose that $0 < b_k < e$, then $\pi_k(0, B_{-k}) > \pi_k(b_k, B_{-k})$.

Second, suppose that $b_k = 0$. Then, $\pi_k(0, B_{-k}) = e + \beta(B_{-k} - V)$. However,

$$\pi_k(e, B_{-k}) - \pi_k(0, B_{-k}) \geq \frac{V}{n} - (1 - \beta)\, e > 0.$$

Hence, $(e, ..., e)$ is the only candidate for the pure-strategy NE. If $(e, ..., e)$ is indeed a NE, then a "zero" deviation does not increase a player's payoff, or

$$\begin{cases} e + \beta((n-1)e - V) \leq (V/n) + \beta(ne - V), & \text{if } (n-1)e \geq V, \\ e \leq (V/n) + \beta(ne - V), & \text{if } (n-1)e < V. \end{cases}$$

Therefore,

$$\begin{cases} e \leq \frac{V}{n(1-\beta)}, & \text{if } e \geq \frac{V}{(n-1)}, \\ (\beta n - 1)\, e \geq \frac{(\beta n - 1)}{n} V, & \text{if } e < \frac{V}{(n-1)}. \end{cases} \tag{28}$$

Note that the first condition in (28) implies that:

$$\frac{V}{(n-1)} \leq e \leq \frac{V}{n(1-\beta)}, \tag{29}$$

or

$$\beta n > 1,$$

which means that it is efficient to provide the public good.

Similarly, consider the second inequality in (28). Then,

$$\frac{V}{n} \leq e < \frac{V}{(n-1)}, \text{ if } \beta n > 1, \tag{30}$$

and

$$e < \min\left\{\frac{V}{(n-1)}, \frac{V}{n}\right\} = \frac{V}{n}, \text{ if } \beta n < 1. \tag{31}$$

The endowment in (31) is too small for there to be public good provision. Therefore, we have to consider only cases (29) and (30). Note that in both cases the efficiency condition (8) holds. Hence, there exists a symmetric pure strategy equilibrium where each player bids his entire endowment, $b^* = e$, if inequalities (29) and (30) hold, which together imply that:

$$\frac{V}{n} \leq e \leq \frac{V}{(1-\beta)\,n}. \tag{32}$$

■

Proof of Proposition 9.

Suppose that the following efficiency condition

$$(n-1)\,\beta > 1 \tag{33}$$

and the following endowment condition

$$\frac{V}{n\,(1-\beta)} \leq e \leq \frac{V}{(n-1)\,(1-\beta)} \tag{34}$$

both hold. This is the case of $k = 1$ in the statement of the proposition. Then, $(0, e, ..., e)$ is a pure-strategy NE if no player has a profitable deviation. In particular, a "zero" deviation for any player except player 1 does not increase her payoff[22], or

$$\begin{cases} e + \beta((n-2)e - V) \leq (V/\,(n-1)) + \beta((n-1)\,e - V), & \text{if } (n-2)e \geq V, \\ e \leq (V/\,(n-1)) + \beta((n-1)\,e - V), & \text{if } (n-2)e < V. \end{cases}$$

Therefore,

$$\begin{cases} e \leq \frac{V}{(n-1)(1-\beta)}, & \text{if } e \geq \frac{V}{(n-2)}, \\ e \geq \frac{V}{(n-1)}, & \text{if } e < \frac{V}{(n-2)}. \end{cases} \tag{35}$$

Note that the first condition in (35) implies that:

$$\frac{V}{(n-2)} \leq e \leq \frac{V}{(n-1)\,(1-\beta)}. \tag{36}$$

Similarly, consider the second inequality in (35). Then,

$$\frac{V}{n-1} \leq e < \frac{V}{(n-2)}. \tag{37}$$

[22] "Zero" deviation is the most profitable deviation. See the proof of Proposition 8 for the silimar argument.

Note that in both cases (36) and (37) efficiency condition (33) has to hold. From (36) and (37) we get

$$\frac{V}{(n-1)} \leq e \leq \frac{V}{(n-1)(1-\beta)}. \tag{38}$$

If $(0, e, ..., e)$ is a pure-strategy NE, then a "e" deviation for player 1 does not increase her payoff, or

$$\begin{cases} e + \beta((n-1)e - V) \geq (V/n) + \beta(ne - V), & \text{if } (n-1)e \geq V, \\ e \geq (V/n) + \beta(ne - V), & \text{if } (n-1)e < V. \end{cases}$$

Therefore,

$$\begin{cases} e \geq \frac{V}{n(1-\beta)}, & \text{if } e \geq \frac{V}{(n-1)}, \\ e \leq \frac{V}{n}, & \text{if } e < \frac{V}{(n-1)}. \end{cases} \tag{39}$$

Note that the first condition in (39) implies that:

$$e \geq \max\left\{\frac{V}{(n-1)}, \frac{V}{n(1-\beta)}\right\} = \frac{V}{n(1-\beta)}. \tag{40}$$

Similarly, consider the second inequality in (39). Then,

$$e < \min\left\{\frac{V}{(n-1)}, \frac{V}{n}\right\} = \frac{V}{n}. \tag{41}$$

This means that the endowment is too small and the public good is not provided.

Hence, from (38) and (40) we get

$$\frac{V}{n(1-\beta)} \leq e \leq \frac{V}{(n-1)(1-\beta)}.$$

Hence, if efficiency condition (33) and endowment condition (34) both hold, then there exists n pure-strategy equilibria where $(n-1)$ players bid their entire endowment, and one player bids zero.

Suppose that conditions (18) and (19) hold, then, analogously to the case of $k = 1$, it is straightforward to check that if $(n-k)$ players bid their entire endowment, e, while the other k players bid zero, then we get an asymmetric pure-strategy NE where the public good is provided.

Suppose that $e > \frac{V}{2(1-\beta)}$, then $(e, ..., e)$ is the only candidate for a pure-strategy NE where public good is provided. The logic here is the same as it is in the proof of Proposition 8. However, it is easy to check that given that all other players bid their entire endowment, e, the best choice is to bid zero instead of e. Hence, there is no pure-strategy NE where the public good is provided.

Suppose that there exists a mixed-strategy equilibrium where the public good is provided and $e > \frac{V}{2(1-\beta)}$.

First, note that there cannot be any positive bid mass points below the endowment amount e: each player has an incentive to increase his bid by $\varepsilon > 0$ at the mass point.

Second, note that the winning bid must be at least $\frac{V}{n}$ because we assume that the public good will be provided. It follows that no player bids in the interval $\left(0, \frac{V}{n}\right)$ in the equilibrium.

Third, suppose that players randomize on an interval $\left[\underline{b}, \overline{b}\right]$ in a mixed-strategy equilibrium where the lower bound satisfies $\frac{V}{n} \leq \underline{b} < e$.

Consider bid $\underline{b} > 0$. We assume here that if there are several intervals where players randomize, then $\underline{b}$ is the lowest bound of all such intervals. Such a bid can only be a winning bid if all others bid zero as there are no positive bid mass points from the first observation. Therefore any player would prefer to bid zero instead of $\underline{b} > 0$.

Next consider the case where players randomize on the non-closed interval $\left(\underline{b}, \overline{b}\right]$ in a mixed-strategy equilibrium, where the lower bound again satisfies $\frac{V}{n} \leq \underline{b} < e$, and if there are several intervals where players randomize, then $\underline{b}$ is the lowest bound of all such intervals. Consider a bid of $\underline{b} + \delta < \overline{b}$. Using (16) we have:

$$\pi_i\left(\underline{b} + \delta, b_{-i}; n, \beta, \text{All-pay auction}\right) =$$

$$\begin{cases} V + (e - (\underline{b} + \delta)) + \beta\left(\sum_{j=1}^{n} b_j - V\right), & \text{if } \underline{b} + \delta > b_j \text{ for any } j \neq i \text{ and } \sum_{j=1}^{n} b_j \geq V, \\ \frac{V}{K} + (e - (\underline{b} + \delta)) + \beta\left(\sum_{j=1}^{n} b_j - V\right), & \text{if } i \text{ ties } (K-1) \text{ others for the high bid and } \sum_{j=1}^{n} b_j \geq V, \\ (e - (\underline{b} + \delta)) + \beta\left(\sum_{j=1}^{n} b_j - V\right), & \text{if } b_i < b_j \text{ for some } j \neq i \text{ and } \sum_{j=1}^{n} b_j \geq V, \\ e, & \text{if } \sum_{j=1}^{n} b_j < V. \end{cases} \tag{42}$$

Note that the probability of winning the prize or the probability of the event

$$\left[\underline{b} + \delta > b_j \text{ for any } j \neq i \text{ and } \sum_{j=1}^{n} b_j \geq V\right]$$

is proportional to δ^{n-1}. This is the probability to attach to the first two elements of the payoff function (42). At the same time, a probability of losing (not winning the prize) or the probability of the event

$$\left[\underline{b} + \delta < b_j \text{ for some } j \neq i \text{ and } \sum_{j=1}^{n} b_j \geq V\right]$$

is proportional to $\left(1 - \delta^{n-1}\right)$. This is the probability to attach to the third element of the payoff function (42). Thus, via continuity, there exists a $\delta_0 > 0$ such that for any bid $\underline{b} + \delta$, where $0 < \delta < \delta_0$, we will have:

$$\pi_i\left(0, b_{-i}; n, \beta, \text{All-pay auction}\right) > \pi_i\left(\underline{b} + \delta, b_{-i}; n, \beta, \text{All-pay auction}\right),$$

since

$$\pi_i\left(0, b_{-i}; n, \beta, \text{All-pay auction}\right) = \begin{cases} e + \beta\left(\sum_{j=1}^{n} b_j - V\right), & \text{if } \sum_{j=1}^{n} b_j \geq V, \\ e, & \text{if } \sum_{j=1}^{n} b_j < V. \end{cases}$$

That is, taking into account expected payoffs, a bid of zero outperforms a bid of $\underline{b} + \delta$. Hence, players cannot randomize on continuous intervals in any mixed-strategy equilibrium.

It follows that each player can only randomize between two bids: $b = 0$ and $b = e$. Since $e > \frac{V}{2(1-\beta)}$, each player bids her dominant choice: zero and we obtain the equilibrium described in Proposition 6. ■

Proof of Proposition 12.

From Proposition 8, it follows that we have to consider the following maximization problem in order to find the prize value that maximizes public good provision:

$$\max_{V \text{ is such that (17) holds}} (ne - V).$$

In particular we are looking for the minimal value of V which satisfies inequalities (17). Then, from (17), we get

$$(1-\beta)\, ne \le V < ne.$$

Hence, the optimal prize value is

$$V^A = (1-\beta)\, ne,$$

and the highest possible public good provision (less the prize V) is given by:

$$\widetilde{G^A} = ne - V^A = \beta ne.$$

■

Appendix B: Symmetric Mixed-strategy Equilibria under the All-Pay Auction

In this Appendix we describe conditions under which there exists a unique, symmetric mixed strategy Nash equilibrium under the provisional, fixed prize auction in the case where $n = 2$.

Suppose that $n = 2$. If the endowment is medium, i.e., if $V \le e \le \frac{1}{2}\frac{V}{(1-\beta)}$, *and* it is efficient to provide the public good, then there exists a symmetric mixed-strategy equilibrium where the public good is provided with a positive probability.

Suppose that each player bids $b = e$ with probability $p \in (0,1)$ and bids $b = 0$ with probability $(1-p)$. Then, a player must be indifferent between both choices, or his expected payoffs are the same in both cases.

Case 1. If a player submits a zero bid, his payoff becomes

$$\begin{cases} (1-p)\, e + p\, [e + \beta(e-V)], & \text{if } e \ge V, \\ e, & \text{if } V/2 < e < V. \end{cases}$$

Case 2. If a player bids $b = e$, his payoff becomes

$$\begin{cases} (1-p)\, [V + \beta(e-V)] + p\left[\frac{1}{2}V + \beta(2e-V)\right], & \text{if } e \ge V, \\ (1-p)\, e + p\left[\frac{1}{2}V + \beta(2e-V)\right], & \text{if } V/2 < e < V. \end{cases}$$

Suppose that $e \ge V$, then, in the equilibrium, we have

$$(1-p)\, e + p\, [e + \beta(e-V)] = (1-p)\, [V + \beta(e-V)] + p\left[\frac{1}{2}V + \beta(2e-V)\right],$$

or

$$e = \left(1 - \frac{1-2\beta}{2\,(1-\beta)}p\right) V.$$

Since $e \ge V$, then it must be

$$2\beta \ge 1.$$

Hence, if $e \geq V$ and it is efficient to provide public good, $2\beta \geq 1$, then we get a symmetric mixed strategy equilibrium for

$$e = \left(1 - \frac{1 - 2\beta}{2(1-\beta)} p\right) V > V$$

and some

$$0 < p < 1.$$

This implies that for any endowment, e, such that:

$$V < e < \left(1 + \frac{2\beta - 1}{2(1-\beta)}\right) V = \frac{V}{2(1-\beta)},$$

there exists a unique $p \in (0, 1)$ such that we have a mixed-strategy equilibrium.

Suppose alternatively that $V/2 \leq e < V$, then, we have

$$e = (1-p)\, e + p\left[\frac{1}{2}V + \beta(2e - V)\right] = \frac{(1-2\beta)}{2(1-2\beta)} V = \frac{V}{2}.$$

This implies that for

$$e = \frac{V}{2},$$

any $p \in (0, 1)$ will constitute a symmetric mixed-strategy equilibrium.

We summarize our findings regarding symmetric mixed-strategy equilibria under the provisional fixed-prize all-pay auction mechanism when $n = 2$ as follows:

Proposition 13 *Consider the provisional, fixed prize all-pay auction where $n = 2$. Suppose that efficiency condition* (8) *holds. Then, for any endowment, e, that satisfies condition*

$$V < e < \frac{V}{2(1-\beta)}, \tag{43}$$

there exists a unique symmetric mixed-strategy equilibrium. In this equilibrium, every player bids his entire endowment, e, with a positive probability $p \in (0, 1)$ and submits a zero bid with the complementary probability $(1 - p)$, where

$$p = \frac{2(1-\beta)}{2\beta - 1} \frac{e - V}{V}.$$

Appendix C: Characterization of all Nash equilibria in the $n = 2$ case (Not Intended for Publication)

This appendix analyzes the $n = 2$ case in some detail. We will assume that $e > V/2$ so that the public good can be provided. The best reply correspondences are used to characterize all of the Nash equilibria in the $n = 2$ case.

C.1 Provisional fixed-prize lottery

The best reply correspondence for player i is used to characterize all Nash equilibria and is given as follows:

$$BR_i(x_{-i}) = \begin{cases} [0, \min\{V - x_{-i}, e\}], & \text{if } 0 \leq x_{-i} \leq (1-\beta)V, \\ \min\left\{\sqrt{\frac{V}{(1-\beta)}x_{-i}} - x_{-i}, e\right\}, & \text{if } (1-\beta)V < x_{-i} \leq \frac{V}{(1-\beta)}, \\ 0, & \text{if } x_{-i} > \frac{V}{(1-\beta)}. \end{cases}$$

As we know from Propositions 2 and 3, there are multiple pure-strategy equilibria. First, there exists a set of Nash equilibria where the public good is not provided. This set can be characterized as follows:[23]

$$\{(x_1, x_2) : 0 \leq x_{-i} \leq \min\{(1-\beta)V, e\} \text{ and } x_1 + x_2 \leq V\}.$$

Second, if public good provision is efficient, or if $2\beta > 1$, then there exists one more pure strategy equilibrium where the public good is provided. In this equilibrium,

$$x_1^e = x_2^e = \min\left\{\frac{1}{4}\frac{V}{(1-\beta)}, e\right\}.$$

If public good provision is inefficient, or if $2\beta \leq 1$, then $(x_1, x_2) = \left(\frac{1}{4}\frac{V}{(1-\beta)}, \frac{1}{4}\frac{V}{(1-\beta)}\right)$ is also a NE. However, in the inefficient case it is the case that $\frac{1}{4}\frac{V}{(1-\beta)} \leq (1-\beta)V$, which means that this NE belongs to the set of NE without public good provision. Figure 8 illustrates the best reply correspondences, the set of NE without public good provision, and the unique NE with public good provision which only obtains in the efficient case.

C.2 Provisional fixed-prize all-pay auction

The best reply correspondence depends on whether public good provision is efficient or not. We will consider both cases.

[23] Of course, there is a continuum of mixed-strategy equilibria, where each player randomizes among her strategies in this equilibrium set. There is no public good provision in all such mixed-strategy equilibria.

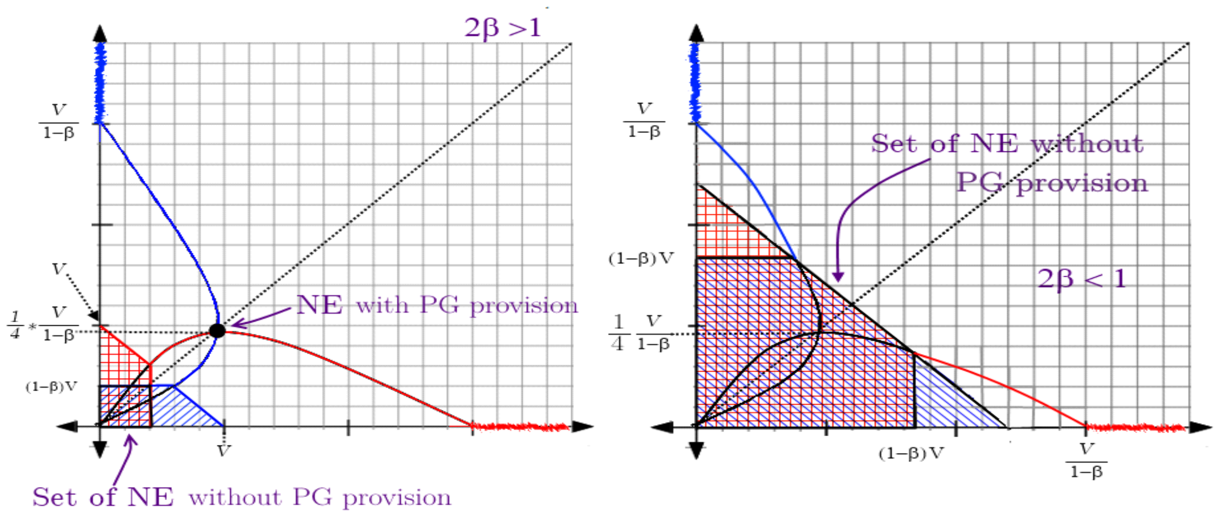

Figure 8: Best response correspondences and Nash equilibria (NE) with or without public good (PG) provision under the provisional fixed-prize lottery mechanism when $n = 2$. The left panel shows the efficient case where $2\beta > 1$; the right panel shows the inefficient case where $2\beta \leq 1$.

Efficient case: $2\beta > 1$. In this case the best reply correspondence for player i is:

$$BR_i\left(b_{-i}\right) = \begin{cases} \begin{cases} [0, e]\,, & \text{if } 0 \leq b_{-i} < V - e, \\ V - b_{-i}, & \text{if } V - e \leq b_{-i} < \frac{V}{2}, \\ \text{not defined}, & \text{if } \frac{V}{2} \leq b_{-i} < e, \\ e, & \text{if } b_{-i} = e, \end{cases} & \text{if } e < V \quad \text{(case E1)}, \\ \begin{cases} [0, V]\,, & \text{if } b_{-i} = 0, \\ V - b_{-i}, & \text{if } 0 < b_{-i} < \frac{V}{2}, \\ \text{not defined}, & \text{if } \frac{V}{2} \leq b_{-i} < e, \\ e, & \text{if } b_{-i} = e < \frac{1}{2}\frac{V}{(1-\beta)}, \end{cases} & \text{if } V \leq e < \frac{1}{2}\frac{V}{(1-\beta)} \quad \text{(case E2)}, \\ \begin{cases} [0, V]\,, & \text{if } b_{-i} = 0, \\ V - b_{-i}, & \text{if } 0 < b_{-i} < \frac{V}{2}, \\ \text{not defined}, & \text{if } \frac{V}{2} \leq b_{-i} < e, \\ \{0, e\}\,, & \text{if } b_{-i} = e, \end{cases} & \text{if } e = \frac{1}{2}\frac{V}{(1-\beta)} \quad \text{(case E3)}, \\ \begin{cases} [0, V]\,, & \text{if } b_{-i} = 0, \\ V - b_{-i}, & \text{if } 0 < b_{-i} < \frac{V}{2}, \\ \text{not defined}, & \text{if } \frac{V}{2} \leq b_{-i} < \min\left\{\frac{V}{(1-\beta)}, e\right\}, \\ 0, & \text{if } b_{-i} \geq \min\left\{\frac{V}{(1-\beta)}, e\right\}. \end{cases} & \text{if } e > \frac{1}{2}\frac{V}{(1-\beta)} \quad \text{(case E4)}. \end{cases}$$

There are three situations. First, if the endowment is small, $e < V$, then there are multiple pure-strategy equilibria. There exists a set of Nash equilibria where the public good is not provided.

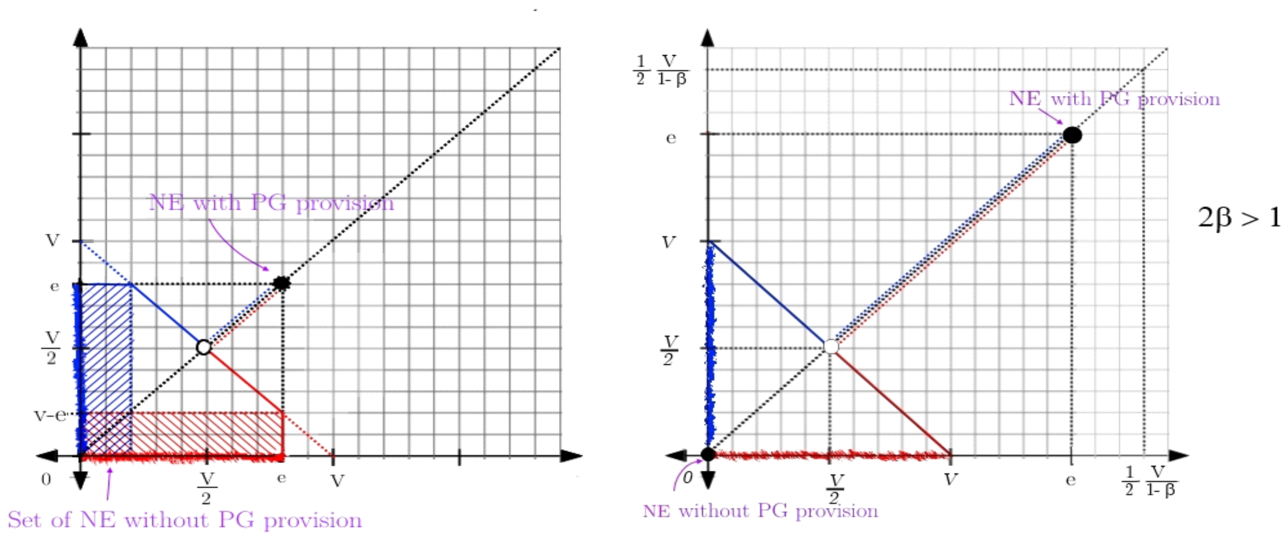

Figure 9: Best response correspondences and Nash equilibria (NE) with or without public good (PG) provision under the provisional fixed-prize all-pay auction mechanism with $n = 2$ and $2\beta > 1$. Left panel shows the case where $e < V$; right panel shows the case where $V \leq e \leq \frac{1}{2}\frac{V}{(1-\beta)}$.

This set can be characterized in the following way[24]

$$\{(b_1, b_2) : 0 \leq b_{-i} < V - e\}.$$

There also exists one more pure-strategy equilibrium, where the public good is provided. In this equilibrium,

$$b_1^e = b_2^e = e.$$

Second, if the endowment is medium, specifically if $V \leq e \leq \frac{1}{2}\frac{V}{(1-\beta)}$, then there are two pure-strategy equilibria. The first, where $b_1 = b_2 = 0$, is a NE without public good provision. There also exists a second pure-strategy equilibrium where the public good is provided. In this equilibrium,

$$b_1^e = b_2^e = e.$$

Figure 9 illustrates the best reply correspondences, the set of NE without public good provision, and the NE with public good provision, if the endowment is small, $e < V$ (case E1) and medium, $V \leq e < \frac{1}{2}\frac{V}{(1-\beta)}$ (case E2)

Third, if the endowment is large, i.e., if $e > \frac{1}{2}\frac{V}{(1-\beta)}$, then there exists a unique pure-strategy equilibrium. In this equilibrium $b_1 = b_2 = 0$, so the NE is one without public good provision. Figure 10 illustrates the best reply correspondences, and the NE with and without public good provision for the case where the endowment is exactly $e = \frac{1}{2}\frac{V}{(1-\beta)}$ (case E3) and where it is larger, $e > \frac{1}{2}\frac{V}{(1-\beta)}$ (case E4).

[24] Of course, there is a continuum of mixed-strategy equilibria, where each player randomizes among her strategies in this equilibrium set. There is no public good provision in all mixed-strategy equilibria.

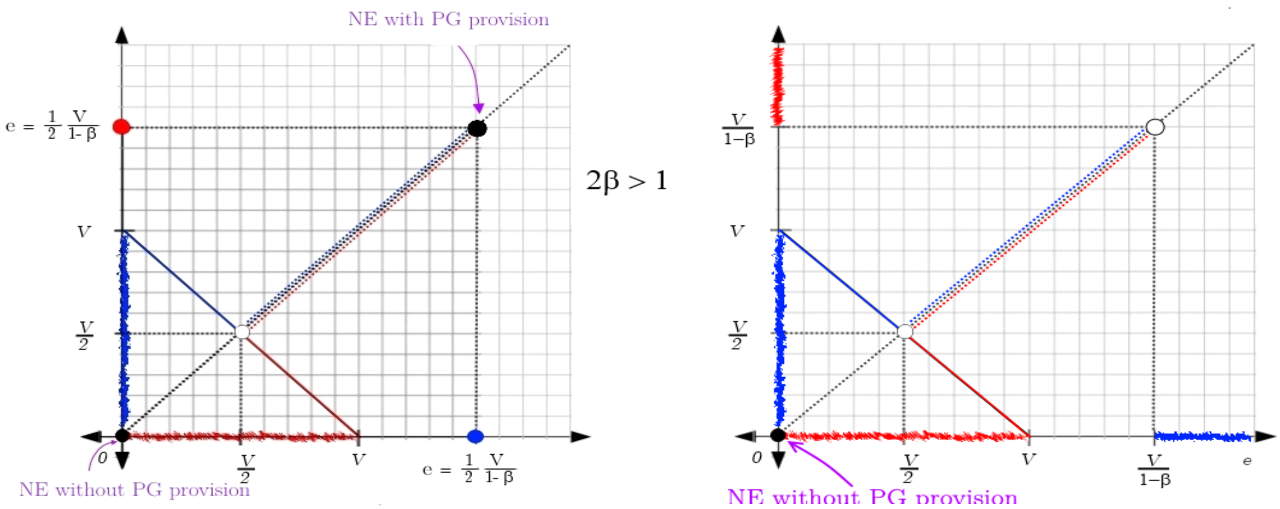

Figure 10: Best response correspondences and Nash equilibria (NE) with or without public good (PG) provision under the provisional fixed-prize all-pay auction mechanism with $n = 2$ and $2\beta > 1$. Left panel shows the case where $e = \frac{1}{2}\frac{V}{(1-\beta)}$; right panel shows the case where $e > \frac{1}{2}\frac{V}{(1-\beta)}$.

Inefficient case: $2\beta \leq 1$. In this case, the best reply correspondence for player i is

$$BR_i(b_{-i}) = \begin{cases} \begin{cases} [0,e], & \text{if } 0 \leq b_{-i} < V - e, \\ V - b_{-i}, & \text{if } V - e \leq b_{-i} < \frac{V}{2}, \\ \text{not defined}, & \text{if } \frac{V}{2} \leq b_{-i} < e, \\ 0, & \text{if } b_{-i} = e, \end{cases} & \text{if } e < V \text{ (case I1)}, \\ \begin{cases} [0,V], & \text{if } b_{-i} = 0, \\ V - b_{-i}, & \text{if } 0 < b_{-i} < \frac{V}{2}, \\ \text{not defined}, & \text{if } \frac{V}{2} \leq b_{-i} < e, \\ 0, & \text{if } b_{-i} = e. \end{cases} & \text{if } V \leq e \leq \frac{1}{2}\frac{V}{(1-\beta)} \text{ (case I2)}, \\ \begin{cases} [0,V], & \text{if } b_{-i} = 0, \\ V - b_{-i}, & \text{if } 0 < b_{-i} < \frac{V}{2}, \\ \text{not defined}, & \text{if } \frac{V}{2} \leq b_{-i} < \min\left\{\frac{V}{(1-\beta)}, e\right\}, \\ 0, & \text{if } b_{-i} \geq \min\left\{\frac{V}{(1-\beta)}, e\right\}. \end{cases} & \text{if } e > \frac{1}{2}\frac{V}{(1-\beta)} \text{ (case I3)}. \end{cases}$$

There are again three situations to consider with respect to the endowment. First, if the endowment is small, $e < V$, then there exists a set of Nash equilibria and two pure-strategy NE where the public good is not provided. This set can be characterized as follows:

$$\{(b_1, b_2) : 0 \leq b_{-i} < V - e\}.$$

Note that $(b_1 = e,\ b_2 = 0)$ and $(b_1 = 0,\ b_2 = e)$ are NE without public good provision. Figure 11 illustrates the best reply correspondences and all pure-strategy NE without public good provision, if the endowment is small, $e < V$ (case I1).

Second, if the endowment is medium, i.e., if $V \leq e \leq \frac{1}{2}\frac{V}{(1-\beta)}$, then there exists a unique pure-strategy equilibrium, $b_1 = b_2 = 0$ without public good provision. Third, if the endowment is large,

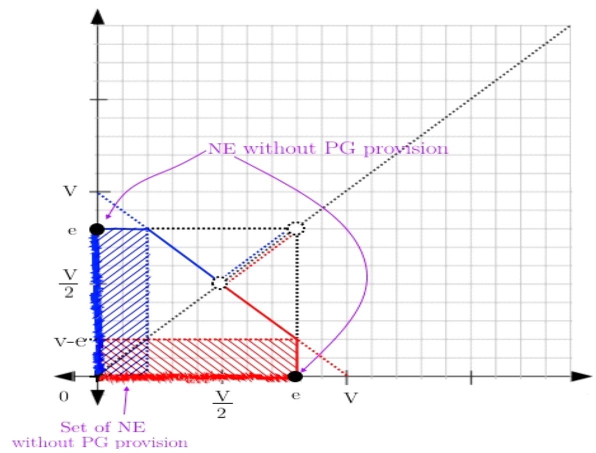

Figure 11: Best response correspondences and Nash equilibria (NE) without public good (PG) provision under the provisional fixed-prize all-pay auction mechanism with $n = 2$, $2\beta \leq 1$ and $e < V$.

$e > \frac{1}{2}\frac{V}{(1-\beta)}$, then there exists a unique pure-strategy equilibrium $b_1 = b_2 = 0$ without public good provision. Figure 12 illustrates the best reply correspondences and the unique NE without public good provision, if the endowment is medium, $V \leq e \leq \frac{1}{2}\frac{V}{(1-\beta)}$ (case I2) and large, $e > \frac{1}{2}\frac{V}{(1-\beta)}$ (case I3), respectively.

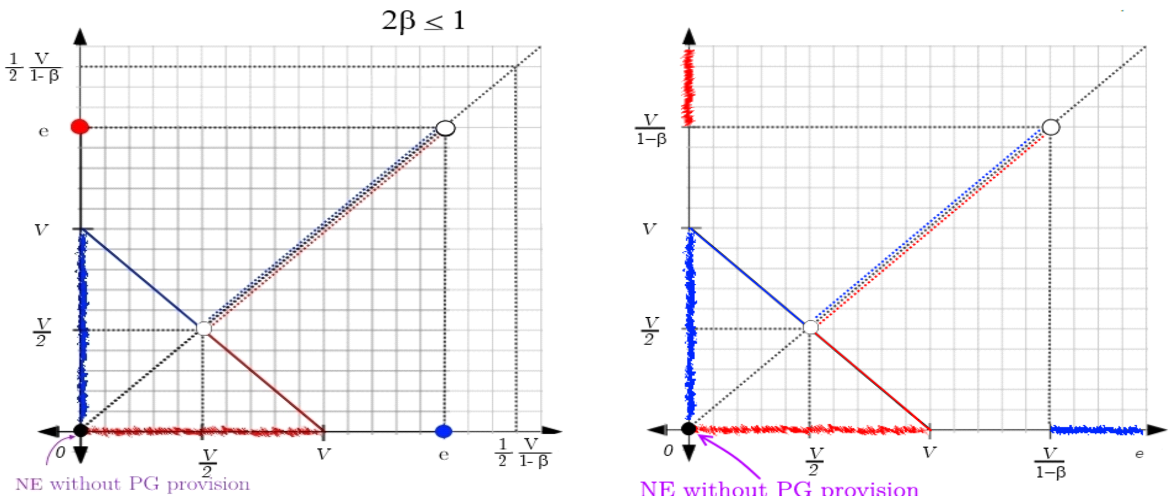

Figure 12: Best response correspondences and Nash equilibria (NE) without public good (PG) provision under the provisional fixed-prize all-pay auction mechanism with $n = 2$ and $2\beta \leq 1$. Left panel shows the case where $V \leq e \leq \frac{1}{2}\frac{V}{(1-\beta)}$; right panel shows the case where $e > \frac{1}{2}\frac{V}{(1-\beta)}$.

Appendix D: Instructions Used in the Auction-Lottery Treatment, n=10, β=.25 (Not Intended for Publication)

Note: Other instructions are similar, changing only the order of the two mechanisms and/or the values of n, β.

Overview

Welcome to this experiment in the economics of decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's session. There is no talking for the duration of this experiment. If you have a question, please raise your hand.

Today's experiment is divided into two parts, each consisting of 15 rounds. In each round of the first part, you participate in a simple decision-making game that is described below. You will receive instruction for the second part of the experiment following the conclusion of the first part. You will make your decisions using the computer workstation, which will also provide you with feedback about the outcomes of those decisions.

There are 20 participants in today's experiment. At the beginning of each round, you will be assigned randomly to one of two groups of 10 participants, either group 1 or 2. The group to which you are randomly assigned each round is indicated on your screen. While you may be assigned to the same group number (1 or 2) more than once in succession, the composition of participants in the group to which you are assigned will vary from round to round. You will play each round only with the members of your group of size 10. You will not be told the identity of any member of your group, nor will any of them know your identity even after the session is over. Your earnings will depend on the choices you make as well as the choices made by the other participants in your group.

The Game

At the start of each round, each member of your group including yourself is endowed with 400 tokens. You are asked whether you would like to contribute any number of your endowment of tokens toward the possibility of winning a prize of 100 tokens. Your token contribution decision is made anonymously; no participant can associate you with your decision.

Specifically, on the decision screen for each round you are asked: How many of your 400 tokens would you like to contribute? In the input box, type in the number of tokens you want to contribute, any number between 0 and 400, inclusive. You can change your mind anytime prior to clicking the OK button. When you are satisfied with your choice, click the OK button.

After all participants have clicked the OK button, the computer program will calculate the total number of tokens that all members of your group of size 10 (including you) have contributed. Let us call this number X.

If X<100, then the 100 token prize is not awarded to any member of your group. Each group member gets back any of his/her 400 tokens contributed toward winning the prize. Earnings for the round are 400 tokens for each subject.

If $X \geq 100$, then the 100 token prize is won by the member of your group who contributed the most tokens toward winning the prize. If there is a tie, then one of the individuals contributing the most tokens toward winning the prize will be randomly selected and awarded the 100 token prize. The more tokens you contribute, the greater is your chance of winning the 100 token prize. Tokens that you do not contribute toward winning the 100 token prize remain in your "private" account.

If $X \geq 100$, then the amount X-100 of tokens will be placed in a "group" account. All 10 members of your group, even those who did not contribute any tokens toward winning the 100 token prize will earn additional tokens based on the number of tokens in the group account.

Specifically, if $X \geq 100$, each member of the group will earn $.25 \times (X-100)$ tokens. These tokens are in addition to the tokens that remain in your private account (400-c) or the 100 token prize awarded to the winner. The table below gives you a non-exhaustive list of your possible earnings from the group account.

If X is	then (X-100) is	and the Tokens Earned by Each Member of the Group is
100	0	0
150	50	12.5
200	100	25
250	150	37.5
300	200	50
350	250	62.5
400	300	75
450	350	87.5
500	400	100
1000	900	225
1500	1400	350
2000	1900	475
2500	2400	600
3000	2900	725
3500	3400	850
4000	3900	975

If $X<100$, no tokens are placed in the group account.

<u>Earnings</u>

Your total tokens for each round are the sum of three items.

1. The number of tokens that remain in your private account. If $X<100$, the number of tokens in your private account will be set equal to your endowment of 400 tokens. Otherwise, if $X \geq 100$, the number of tokens in your private account is 400-c, where c is the number of tokens that *you* contributed toward winning the 100 token prize.
2. If $X \geq 100$, AND you are the prize winner, then you receive an additional 100 token prize for that round.
3. If $X \geq 100$, you and every other member of your group earns an additional $.25 \times (X-100)$ tokens based on the number of tokens (X-100) in the group account.

At the end of today's experimental session, the computer program will randomly select two rounds: one from the first part (15 rounds) of today's session and one from the second part. Your token total for the rounds selected will be converted into dollars at a rate of 1 token = \$0.01 (1 cent).

Feedback

At the end of each period your computer screen will report back to you:

- The number of tokens you offered toward winning the prize, c
- The total number of tokens submitted by all members of your group including you, X.
- Whether the prize was awarded (if $X \geq 100$) or not (if $X < 100$).
- If $X \geq 100$, the number of tokens offered by the winner of the prize.
- If $X \geq 100$, whether you won or lost the 100 token prize.

You will also be shown the calculation of your total token earnings for the round. Specifically, you will learn:

1. The number of tokens that remain in your private account, 400-c.
2. Prize tokens: 100 if you won the prize, 0 otherwise.

If $X \geq 100$, you will be told the number of tokens in the group account, X-100. This number is used to calculate:

3. The tokens you (and everyone in your group) earns from the group account, equal to $.25 \times (X-100)$ tokens.

- Finally, you will be told your total tokens earned for the round, which is the sum of the token amounts in items 1-3 above.

Record Sheets

Please record the information reported to you on the outcome of each round on your record sheet under the appropriate headings. Be sure also to indicate on your record sheet your ID number.

For your convenience, the history of information reported back to you at the end of each round will appear at the bottom of your first decision screen.

Questions

Are there any questions before we begin?

Instructions Part Two

You are about to begin the second part of the experiment. This part also consists of 15 rounds.

As in the first part of today's experiment, at the start of each round in this second part, the computer program will again randomly divide you up into two groups of 10 participants -- group 1 or group 2. While you may be assigned to the same group number (1 or 2) more than once in succession, the composition of participants in the group to which you are assigned will vary from round to round. You will play each round only with the members of your group of size 10. You will not be told the identity of any member of your group, nor will any of them know your identity even after the session is over. Your earnings will depend on the choices you make as well as the choices made by the other participants in your group.

The game is similar to the one played in the first part. The only difference is that the 100 token prize, if offered, ($X \geq 100$), can be won by *any* member of the group who contributes more than 0 tokens toward winning the prize. Your chance of winning the 100 token prize is equal to the number of tokens you contributed – call this c—divided by the total number of tokens contributed, X, that is, you have a c/X percent chance of winning the 100 token prize. Thus, unlike the first part, it is no longer the case that the group member who contributes the most tokens automatically wins the prize; now *anyone who contributes more than 0 tokens has some chance of winning the 100 token prize.* Notice that the more tokens you contribute, c, relative to the total number of tokens contributed by all 10 members of your group (including you), X, the greater is your chance of winning the 100 token prize.

Every other aspect of the decision-making game is the same as before.

Specifically, on the decision screen for each round you are asked: How many of your 400 tokens would you like to contribute? In the input box, type in the number of tokens you want to contribute, any number between 0 and 400, inclusive. You can change your mind anytime prior to clicking the OK button. When you are satisfied with your choice, click the OK button

After all participants have clicked the OK button, the computer program will calculate the total number of tokens that all members of your group of size 10 (including you) have contributed. Let us call this number X.

If $X<100$, then the 100 token prize is not awarded to any member of your group. Each group member gets back any of his/her 400 tokens contributed toward winning the prize. Earnings for the round are 400 tokens for each subject.

If $X \geq 100$, then the 100 token prize is randomly awarded to one (and only one) member of your group who contributed more than zero tokens toward winning the prize. While the computer program randomly selects one member of your group contributing more than zero tokens as the prize winner, your chance of winning the 100 token prize in this random selection is equal to the number of tokens you contributed toward winning the prize, c, divided by the total number of tokens contributed by all members of your group including you, X. That is, you have a c/X chance of winning the 100 token prize. The more tokens you contribute, c, relative to the total X, the greater is your chance of winning the 100 token prize. But notice that each member of your 10-person group who contributes more than 0 tokens toward winning the prize has some chance of winning the prize. Tokens that you do not contribute toward winning the 100 token prize remain in your "private" account.

If X≥100, then the amount X-100 of tokens will be placed in a "group" account. All 10 members of your group, even those who did not contribute any tokens toward winning the 100 token prize will earn additional tokens based on the number of tokens in the group account.

Specifically, if X≥100, each member of the group will earn .25 ×(X-100) tokens. These tokens are in addition to the tokens that remain in your private account or the 100 token prize awarded to the winner. The table below gives you a non-exhaustive list of your possible earnings from the group account.

If X Is	then (X-100) is	and the Tokens Earned by Each Member of the Group is
100	0	0
150	50	12.5
200	100	25
250	150	37.5
300	200	50
350	250	62.5
400	300	75
450	350	87.5
500	400	100
1000	900	225
1500	1400	350
2000	1900	475
2500	2400	600
3000	2900	725
3500	3400	850
4000	3900	975

If X<100, no tokens are placed in the group account.

<u>Total Earnings</u>

Your total tokens for each round are the sum of three items.

1. The number of tokens that remain in your private account. If X<100, the number of tokens in your private account will be set equal to your endowment of 400 tokens. Otherwise, if X≥100, the number of tokens in your private account is 400-c, where c is the number of tokens that *you* contributed toward winning the 100 token prize.
2. If X≥100, AND you are the prize winner, then you receive an additional 100 token prize for that round.
3. If X≥100, you and every other member of your group earns an additional .25 ×(X-100) tokens based on the number of tokens in the group account.

At the end of the experimental session, the computer program will randomly select two rounds: one from the first part (first 15 rounds) and one from the second part (last 15 rounds). The sum of tokens earned in these two rounds will be your total token earnings for the experiment. At the end of the experiment your total token earnings will be converted into cash earnings at the rate of <u>1 token = $0.01 (1 cent)</u>.

Feedback

At the end of each period your computer screen will report back to you:

- The number of tokens you contributed toward winning the prize, c
- The total number of tokens submitted by all members of your group including you, X.
- Whether the prize was awarded, yes (if $X \geq 100$) or no (if $X < 100$).
- If $X \geq 100$, your percent chance of winning the prize, c/X (up to five decimal places).
- If $X > 100$, whether you won or lost the 100 token prize.

You will also be shown the calculation of your total token earnings for the round. Specifically, you will learn:

1. The number of tokens that remain in your private account, 400-c.
2. Prize tokens: 100 if you won the prize, 0 otherwise.

If $X \geq 100$, you will be told the number of tokens in the group account, X-100. This number is used to calculate:

3. The tokens you (and everyone else in your 10 member group) earns from the group account, equal to $.25 \times (X-100)$ tokens.

- Finally, you will be told your total tokens earned for the round, which is the sum of the token amounts in items 1-3 above.

Record Sheets

Please record the information reported to you on the outcome of each round on your record sheet under the appropriate headings. Be sure also to indicate on your record sheet your player ID number.

For your convenience, the history of information reported back to you at the end of each round will appear at the bottom of your first decision screen.

Questions

Are there any questions before we begin?