Compulsory versus Voluntary Voting
An Experimental Study*

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Abstract

We report on an experiment comparing compulsory and voluntary voting institutions in a voting game with common preferences. Rational choice theory predicts sharp differences in voter behavior between these two institutions. If voting is compulsory, then voters may find it rational to vote insincerely, i.e., against their private information. If voting is voluntary so that abstention is allowed, then sincere voting in accordance with a voter's private information is always rational while participation may become strategic. We find strong support for these theoretical predictions in our experimental data. Moreover, voters adapt their decisions to the voting institution in place in such a way as to make the group decision accuracy differences between the two voting institutions negligible. The latter finding may serve to rationalize the co-existence of compulsory and voluntary voting institutions in nature.

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1 Introduction

Committees and juries often decide on the matters before them by resorting to a vote. In some settings, voting by all members is compulsory, for example, in U.S. federal court, juror abstention in a criminal trial is not allowed and the court can poll each juror about their vote after the verdict has been rendered (Rule 31, U.S. Federal Rules of Criminal Procedure). In other settings, voting is voluntary in that abstention is allowed, for example in certain U.S. state court civil proceedings where unanimity is not required.[1]

The goal of this paper is to experimentally examine whether voters adapt their voting and participation decisions to the voting institution that is in place: compulsory or voluntary voting. The "rational choice" theory of voting posits that the particular voting institution determines the "rules of the game" and that individuals take into account such rules and others' behavior when deciding their vote. The theory predicts sharp qualitative differences in voting behavior between these two institutions, and by and large, our laboratory results are consistent with the theoretical predictions.

The environment we study involves repeated play of an abstract group decision-making task. All group members have identical (common) preferences which yield them a positive payoff only if they correctly identify, via the voting outcome, the unknown, binary state of the world, e.g., jury members wish to convict the guilty and acquit the innocent, or committee members want to choose the option that is most appropriate to the current state of the world. Prior to voting on the state of the world, each group member receives a private noisy but informative signal regarding the unknown state of the world, e.g., "guilty" or "innocent." This is the environment of the Condorcet Jury Theorem (Condorcet (1785)) which is frequently used to address the efficiency of various compulsory voting mechanisms in aggregating decentralized information. Condorcet assumed that voters would always vote sincerely, i.e., according to their private signal.

The validity of Condorcet's assumption was first questioned by Austen-Smith and Banks (1996) who showed that if voting is compulsory, then rational voters may have incentives to vote *strategically*, i.e., sometimes voting *against* their private information (see also Feddersen and Pesendorfer (1996, 1997, 1998), Myerson (1998)). On the other hand, Krishna and Morgan (2012, henceforth K-M) have recently shown that if voting is voluntary so that abstention is possible, then in the same common preference, Condorcet jury model, *sincere* voting, i.e., voting in accordance with one's private information, is always rational when voters face private costs of voting. In K-M's voluntary voting framework, participation decisions become strategic and will depend on the private costs of voting (if there are such costs).[2]

[1] While less applicable to the small group size environment that we study here, there are also differences in voting requirements for larger-scale elections for political offices. For instance, 29 countries, representing one-quarter of all democracies currently compel their citizens to vote (more accurately, to show up to vote) in political elections, while in most democracies voluntary voting is the norm (Birch 2009).

[2] Börgers (2004) compares compulsory versus voluntary voting under majority rule in a costly voting model with *private* values; as noted earlier, we study a common values framework. Börgers argues that voters ignore a negative externality generated by their own decision to vote: by voting they decrease the likelihood that other voters are pivotal. Consequently there is over-participation when voting is voluntary; making voting *compulsory* only serves to reduce welfare even further.

Here we fix the voting rule – majority rule – while using the Condorcet Jury environment to study the extent of sincere versus strategic voting when voter participation is either voluntary or compulsory. Specifically, we examine whether voluntary voting (allowing for abstention) with or without voting costs suffices to induce *sincere* voting behavior relative to the case of compulsory voting, where some *insincere* (strategic) voting is predicted to occur. We further explore the information aggregation consequences of these different voting mechanisms with the aim of understanding both how and why compulsory and voluntary voting mechanisms coexist in nature.

Under the "rational choice" framework, voters are supposed to perfectly account for the consequences of their voting and/or participation decisions on voting outcomes and the voting decisions of others. The empirical relevance of the rational choice approach to actual voting behavior has been questioned, largely on the basis of field or survey data. Green and Shapiro (1994) were among the earliest to question the empirical relevance of rational choice theory. In a detailed analysis of several election datasets, Blais (2000) shows that existing rational choice theories have only limited power to explain turnout. Matsusaka and Palda (1999) reach similar conclusions in their extensive study of both survey and aggregate data and suggest that turnout decisions appear to be random. Achen and Bartels (2002) show that voting behavior is affected by unrelated events like shark attacks. In another paper (Achen and Bartels (2006)), the same authors contend that "voters adopt issue positions, adjust their candidate perceptions and invent facts to rationalize decisions they have already made." Drawing extensively on the Survey of Americans and Economists on the Economy, Caplan (2007) demonstrates that voter behavior is driven by systematically biased beliefs.

By contrast, our approach is to test the comparative statics implications of the rational voter theory using laboratory experiments which have several advantages over field studies for addressing the empirical relevance of rational voter model predictions. First, in the laboratory, we can carefully control the information that subjects receive prior to making their participation or voting decisions; such control is generally not possible using field data. Thus we can accurately determine if voters are voting sincerely, i.e., in accordance with their private information, or if they are voting insincerely, i.e., against their private information. Second, in the laboratory we can carefully control and directly observe voting costs which is more difficult to do in the field. Third, in the laboratory, we can implement the theoretical assumption that subjects have common preferences by inducing them to hold such preferences via the payoff function that determines their monetary earnings.[3] Thus by minimizing confounding and extraneous factors, the laboratory environment we adopt provides vacuum tube-like conditions for assessing the rational choice view of how voting behavior should respond to the voting rules in place. If the theoretical predictions do not hold in the sparse and controlled environment of the laboratory, then it seems unlikely that they will hold in the more complex and noisy environment of the real world.

Within the Condorcet jury set-up that we study, the noisy private signals are informative: a guilty (innocent) signal is more likely to be observed in the guilty (innocent) state. However, the two signals have asymmetric precisions. This asymmetry in signal precisions implies that the

[3]Outside of the controlled conditions of the laboratory, preferences might differ greatly across voters; for example, jury members might have differing "thresholds of doubt," so that each requires a varying amount of evidence before s/he could vote to convict. Such a scenario can be modeled as each voter incurring a different magnitude of utility loss from an incorrect decision (as in Feddersen and Pesendorfer (1998, 1999b)).

likelihood of the state being, e.g., innocent, conditional on having received an innocent signal is larger than the likelihood of the other state, e.g., guilty, conditional on having received a guilty signal. In other words, the two signals are differently informative about the two states of the world.

In this environment we study three voting mechanisms. In the compulsory voting mechanism, abstention is not allowed and there is no cost to voting.[4] Under the majority voting rule and given the asymmetry in signal precisions, the unique symmetric, compulsory voting equilibrium prediction is that voters with the more informative signal vote sincerely, according to their signal, while those whose signal is less informative vote against their signal with positive probability. We refer to the latter behavior as *strategic* or *insincere* voting. Under the voluntary mechanism, we consider cases where voting is costless or costly. If voting is voluntary and costly, then the unique symmetric equilibrium prediction is that voters vote sincerely, in accordance with their signal, conditional on choosing to vote (not abstaining). If voting is voluntary and costless, then there exist two symmetric informative equilibria, but in the Pareto superior equilibrium, conditional on choosing to vote, all voters again vote sincerely (as in the voluntary but costly voting case).[5]

We further examine comparative statics predictions regarding participation rates under the two voluntary voting mechanisms. Regardless of whether or not there are voting costs, the voter with the more informative signal participates in voting with a higher frequency than does the voter with the less informative signal. Moreover, we expect higher participation in voting under costless than under costly (and voluntary) voting. Thus our design enables us to test the effects of voting mechanisms on the two important strategic dimensions (voting and participation) of the theory.

Finally, we also assess how the voting institutions impacts on the accuracy of group decisions. For our parameterization of the model, the theory suggests that the voluntary but costless voting mechanism results in the most accurate group decision-making followed by the compulsory mechanism and then by the voluntary but costly mechanism.

We report the following experimental findings. First, consistent with theoretical predictions, there is significantly more strategic voting under the compulsory voting mechanism than under either of the two voluntary voting mechanisms; under the latter two mechanisms, nearly all subjects are voting sincerely. Second, and also consistent with theoretical predictions, we find that under the voluntary and costless voting mechanism subjects with the more informative signal participate with a higher frequency than do subjects with the less informative signal; when voting costs are added to the voluntary voting mechanism these participation rates remain asymmetric, but become lower as theory predicts. We do find that subjects over-participate in voting relative to equilibrium predictions but these over-participation rates nevertheless respect the comparative statics predictions of the theory. Finally, under both compulsory and voluntary voting mechanisms, we find that groups achieve the correct outcome between 85 and 90 percent of the time and that the ranking of the three mechanisms in terms of the accuracy of group decisions is in line with the

[4] One could add a voting cost to the compulsory voting mechanism but the addition of such a cost would not change the equilibrium prediction in any way.

[5] The other, less efficient equilibrium under the voluntary but costless voting mechanism is the same equilibrium that obtains under the compulsory mechanism; in this equilibrium there is full participation by both signal types, but those whose signal is less informative vote insincerely against their signal with exactly the same frequency as in the compulsory voting equilibrium (while the other signal type always votes sincerely). Thus under the voluntary but costless voting mechanism there is an interesting equilibrium selection issue that our experiment can address.

theoretical predictions. However, we find that the theoretical group decision accuracy differences across the three mechanisms are small (given our parameterization of the model) and indeed, the observed differences in decision accuracy across the three voting mechanisms in our experimental data are not statistically significant from one another. Taken together, our findings suggest that there is strong support for the rational choice prediction that individuals adapt their behavior to the particular voting institution that is in place, thus providing a possible explanation for why compulsory and voluntary voting mechanisms are observed to coexist in nature.

2 Related Literature

Palfrey (2009) provides a survey of experimental studies of voting behavior. Guarnaschelli, McKelvey and Palfrey (2000) is the earliest experimental study reporting evidence of strategic (insincere) voting in the context of the same Condorcet jury model. They found that, under the unanimity rule, a significant percentage (between 30% and 50%) of subjects were observed to be voting against their signal, which is consistent with the equilibrium predictions of Feddersen and Pesendorfer (1998) for the model parameterization they studied. Guarnaschelli et al. (2000) also study behavior under a majority voting rule as we do in this paper. However under majority rule and the *symmetric* signal precision environment that Guarnaschelli et al. study, voters should always vote sincerely, according to their signal. By contrast, in the compulsory voting majority rule set-up that we study involving *asymmetric* signal precisions, the equilibrium prediction calls for some insincere voting; that is, insincere voting behavior in our model is driven by the asymmetry of signal precisions and not by super-majority (asymmetric) voting rules.

Ladha et al. (2003) report on an experiment involving the Condorcet jury model set-up where they consider group choices versus individual choices. They find that sincere voting is higher in the case of compulsory group decision-making under majority rule than in the case of compulsory individual decision-making in the same environment.

Goeree and Yariv (2011) also report on an experiment using the Condorcet jury model where subjects are compelled to vote but where various voting rules are considered, preferences are varied so that jurors do not always have a common interest and most significantly, subjects are able to freely communicate with one another prior to voting. They report that absent communication, there is evidence that subjects vote strategically in accordance with equilibrium predictions under various voting rules, but that these institutional differences are diminished and efficiency is increased when subjects can communicate (deliberate) prior to voting. As with our study, the work of Goeree and Yariv provides further evidence that voters adapt their behavior to institutions, in this case, through the use of communication.

Importantly, none of these prior experimental studies allow for abstention– they only study compulsory and costless voting mechanisms. If instead we allow voters to make participation decisions which can either be costless or costly prior to making their voting decisions as in K-M (2012), we can change the incentive structure of strategic voting decisions in such a way that sincere voting in the Condorcet Jury model no longer contradicts rationality.

There are two papers that compare compulsory and voluntary voting institutions using laboratory experiments. The focus of these studies is on the effect of voting institutions on information acquisition as opposed to our focus on the sincerity of voting behavior. Seebauer and Großer (2006) examine how compulsory or voluntary voting institutions matter for costly information acquisition in a Condorcet jury model game. In their environment, conditional on acquiring a costly signal, voting should always be sincere under both voluntary and compulsory voting mechanisms and they find this to be the case. Shineman (2010) reports on an experiment comparing voluntary and compulsory voting institutions and finds that, consistent with her model, compulsory institutions increase incentives for informed voting relative to voluntary institutions. However Shineman's model is decision–theoretic in the sense that voters do not take into account the effect of their decisions on other voters.

A second, related experimental voting literature studies the team participation game model of voter turnout due to Palfrey and Rosenthal (1983, 1985); see, e.g., Schram and Sonnemans (1996), Cason and Mui (2005), Großer and Schram (2006), Levine and Palfrey (2007) and Duffy and Tavits (2008). In this voluntary and costly voting game, two teams of players compete to win an election; for instance under majority rule, the team with the most votes wins. Experimental studies of this environment have typically involved no private information and have supposed that voters face homogeneous costs to voting (abstention is free). Levine and Palfrey (2007) have designed experiments with heterogeneous voting costs to test several of the comparative static predictions of the Palfrey and Rosenthal (1985) model. By contrast, the Condorcet jury environment that we study does not involve team competition but does have private information (regarding the true state of the world) and we adopt Levine and Palfrey's (2007) design of having heterogeneous voting costs in our voluntary but costly voting treatment. Further, we make the important comparison between the voluntary voting mechanism of the team participation game set-up and the compulsory voting mechanism that is more typically used in the Condorcet jury model. Thus, our paper provides an important bridge between these two approaches.

Finally, our analysis of voluntary and costless voting is related to experiments by Battaglini, Morton and Palfrey (2010) and Morton and Tyran (2011) that test the "swing voter's curse" theory of Feddersen and Pesendorfer (1996). According to that theory, if preferences are identical, less informed voters will rationally delegate the decision to more informed voters by abstaining. Battaglini, Morton and Palfrey (2010) study an environment where each voter can be one of two types drawn randomly: perfectly informed or perfectly uninformed of the state of the world.[6] In Morton and Tyran (2011) there are two pre-defined voter groups: experts and non-experts. Both groups get noisy signals, but the experts' signals are more precise than those of the non-experts. In both of these experimental studies, the less informed voters abstain, delegating their decision to the more informed voters. In fact, Morton and Tyran find that there is "too much delegation" in the sense that the non-expert group sometimes under-participates compared to the equilibrium prediction. Our voluntary costless voting treatment with asymmetric signal precisions can be viewed as a more stringent test of the "swing voter's curse" theory since in our set-up, the fact that one signal is more informative than the other has to be *inferred from* Bayesian updating. By contrast, in Battaglini, Morton and Palfrey (2010), a voter knows whether she is

[6] They also consider a case where there are *partisans* whose preferences are independent of the state of the world.

perfectly informed or uninformed and in Morton and Tyran (2011), experts and non-experts are clearly identified ex-ante. A further subtlety that makes our test more stringent is that in our framework the two signal groups will always have opposed preferences over the alternatives that are induced ex-post by the signals received. In the equilibrium of the voluntary and costless voting environment that we study, the less informed (signal) group of voters plays a mixed strategy with respect to whether they vote or abstain, while the more informed group always votes according to their signal. The delegation of decision-making by the less informed to the more informed group thus involves a more subtle weighing of the tradeoff between expressing one's true preference, i.e., voting according to the noisy but informative signal received, and avoiding being pivotal to the outcome by abstaining and delegating the decision to the more informed group which has exactly opposed ex-post preference. Finally we note that unlike Battaglini, Morton and Palfrey (2010) or Morton and Tyran (2011) we also consider a voluntary voting treatment where voting is costly and we show that similar conclusions hold even when the more informed voting group abstains with a positive probability. Moreover, an interesting contrast with Morton and Tyran (2011) is that while they find under-participation, we observe over-participation in both our costless and costly voluntary voting treatments.

3 Model

The experimental design implements the standard Condorcet Jury setup. We consider three different voting mechanisms: 1) compulsory and costless voting (C); 2) voluntary and costless voting (VN); 3) voluntary and costly voting (VC). In all three cases, a group consisting of an odd number, N, of individuals faces a choice between two alternatives labeled R (Red) and B (Blue). The group's choice is made in an election decided by simple majority rule.[7] There are two equally likely states of nature, ρ and β. Alternative R is the better choice in state ρ while alternative B is the better choice in state β. Specifically, in state ρ each group member earns a payoff $M(>0)$ if R is the alternative chosen by the group and 0 if B is the chosen alternative. In state β the payoffs from R and B are reversed. Formally, we have

$$\begin{aligned} U(R|\rho) &= U(B|\beta) = M, \\ U(R|\beta) &= U(B|\rho) = 0. \end{aligned}$$

Prior to the voting decision, each individual receives a private signal regarding the true state of nature. The signal can take one of two values, r or b. The probability of receiving a particular signal depends on the true state of nature. Specifically, each subject receives a conditionally independent signal where

$$\Pr[r|\rho] = x_\rho \quad \text{and} \quad \Pr[b|\beta] = x_\beta.$$

We suppose that both x_ρ and x_β are greater than $\frac{1}{2}$ but less than 1 so that the signals are informative but noisy. Thus, the signal r is associated with state ρ while the signal b is associated with state

[7] A plurality rule is the obvious benchmark to use in the voluntary voting case where abstention is allowed as it selects the alternative receiving the highest number of votes. In order to ensure comparability between voluntary and compulsory voting mechanisms we use the simple majority rule (rather than supermajority) for both mechanisms.

β (we may say that r is the correct signal in state ρ while b is the correct signal in state β).

We further assume that $x_\rho > x_\beta$, i.e., that the correct signal is more accurate in state ρ than in state β. We make this assumption of asymmetric signal precisions for several reasons. First, as noted earlier, under the majority rule some asymmetry in the signal precisions is required for there to be some insincere voting under the compulsory voting mechanism; in the case of symmetric signal precisions we would obtain sincere voting behavior under the majority rule compulsory voting mechanism. As we wish to use the same majority voting rule to facilitate comparisons between the compulsory and voluntary voting mechanisms and we further wish to highlight the possibility of insincere voting under the compulsory mechanism, we must have some asymmetry in the signal precisions.[8] Second, *symmetric* signal precisions can be viewed as a knife-edge case (Austen-Smith and Banks (1996)). Indeed, there are many empirically plausible scenarios where signal precisions can be thought to vary across states. For instance, in the canonical jury model set-up, in the "guilty" state, there may be material evidence that can be construed as a clear signal of guilt while in the "innocent" state, material evidence signaling the absence of guilt might be more circumstantial (e.g., alibis) which could be construed as being less clear. Such differences in the nature of the evidence signaling guilt or innocence would be captured by asymmetric signal precisions as we assume here. Finally, we note that asymmetric signal precisions make the rational voter equilibrium predictions harder for subjects to compute, thereby stacking the deck against rational voter model predictions. If we find (as we do) that observed behavior nevertheless follows the equilibrium predictions, then our results should provide an even stronger case for the empirical relevance of the rational voter model.

The posterior probabilities of the states after signals have been received are:

$$q(\rho|r) = \frac{x_\rho}{x_\rho + (1 - x_\beta)} \quad \text{and} \quad q(\beta|b) = \frac{x_\beta}{x_\beta + (1 - x_\rho)}.$$

Since $x_\rho > x_\beta$, we have $q(\rho|r) < q(\beta|b)$. Thus, b is a stronger, more informative signal in favor of state β than r is in favor of state ρ. The latter observation is a critical inference that individuals must make if they are to make rational voting decisions.

Having specified the preferences and information structure of the model, we discuss in the next three subsections, the strategies, equilibrium conditions and equilibrium predictions for each of the three voting mechanisms that we explore in our experiment. We restrict attention to symmetric, informative equilibria as these are the most relevant equilibrium predictions given the information that is available to subjects in our experiment.[9] In particular, we require that in equilibrium all voters of the same signal type play the same strategies. In what follows we only discuss the equilibrium predictions and the conditions under which they are valid; a derivation of these solutions is presented in Appendix A (Supplementary Material), and a complete characterization of equilibria

[8] An alternative possibility is to change the voting rule under compulsory voting to be a super-majority rule (or even unanimity as in Guarnaschelli et al. (2000)); such rules can lead to insincere voting under symmetric signal precisions.

[9] There always exists "uninformative" equilibria where every voter ignores their signal and votes for a fixed alternative. In the case of voluntary and *costly* voting mechanism – one of the three mechanisms we study – individuals would not be willing to pay the cost for voting uninformed for any fixed alternative.

under the three voting mechanisms studied in this paper is provided in Appendix B (Supplementary Material).

3.1 Compulsory voting

When voting is compulsory, the strategy of a voter is a specification of two probabilities $\{v_r, v_b\}$ where v_r is the probability of voting for alternative R given an r signal and v_b is the probability of voting for alternative B given a b signal (that is, v_s is the probability of voting according to one's signal s, or voting *sincerely*). Under the compulsory voting mechanism there exists a unique symmetric informative equilibrium - see Appendix B (Supplementary Materials) for a complete characterization. If $x_\rho > x_\beta$, then for a large set of parameters including those of our experimental design, voters with signal b (i.e., signal type-b) always vote for B (i.e., $v_b^* = 1$) while voters with signal r (i.e., signal type-r) mix between the two alternatives (i.e., $v_r^* \in (0,1)$).

Such mixing requires that the voter obtaining signal r be indifferent between voting for R or B conditioning on a tied vote (given play of equilibrium strategies by the other players), which gives the following equilibrium condition

$$U(R|r) - U(B|r) \equiv M\{q(\rho|r)\Pr[Piv|\rho] - q(\beta|r)\Pr[Piv|\beta]\} = 0,$$

where $U(A|s)$ is the payoff that a voter gets when alternative $A \in \{R, B\}$ is chosen and her signal (type) is $s \in \{r, b\}$; and $\Pr[Piv|\omega]$ is the probability that a vote is pivotal in state $\omega \in \{\rho, \beta\}$. Since voting is compulsory and N is chosen to be an odd number, a vote is pivotal only when exactly half of the other $N-1$ voters have voted for R and the other half have voted for B. Since the pivot probabilities depend on v_r, the above indifference condition determines v_r^*. Moreover, given this value for v_r^* and the fact that type-b voters strictly prefer to vote sincerely in equilibrium, we must have

$$U(B|b) - U(R|b) \equiv M\{q(\beta|b)\Pr[Piv|\beta] - q(\rho|b)\Pr[Piv|\rho]\} > 0.$$

The intuition for why type-b voters vote sincerely and type-r voters mix is as follows. If everyone votes her signal, the event of a tied vote among the other $N-1$ voters implies that there are an equal number of r and b signals. Since signals are less accurate in state β (i.e., $x_\rho > x_\beta$), an equal number of r and b signals is more likely to occur in state β than in state ρ. Conditioning on pivotality, the likelihood of state β is large enough that it swamps the information about states contained in the private signal and the best response to a strategy profile with sincere voting is to vote for B irrespective of the signal. If, on the other hand, some type-r voters vote against their signals while all type-b voters vote sincerely, an equal number of votes for R and B implies a larger number of r signals than b signals: in particular, the information contained in the pivotal event is not strong enough to make the private signal irrelevant. In fact, the mixing probability is chosen in such a way that a private signal of r leads to the posterior likelihood of the two states being equal (conditioning on pivotality), thereby preserving the incentive to mix on obtaining an r signal. Clearly, a b signal leads to an inference of state β being more likely than state ρ in the event of a tie, and so the best response for a type-b voter is therefore to always vote for B (i.e., to always vote sincerely).

3.2 Voluntary and costless voting

When voting is voluntary, the action space includes three choices: a vote for R, a vote for B, or abstention, which we denote by ϕ. Thus, a voter's (mixed) strategy is a mapping from the signal type space $\{r, b\}$ to the set of all probability distributions over $\{R, B, \phi\}$. Since abstention is allowed, it now becomes possible that the two alternatives get an equal number of votes: in such cases the winner is chosen randomly. This set-up is exactly the same as K-M except that we have a fixed number, N, of voters (as this is easier to explain to subjects) while in K-M the number of voters is randomly drawn from a Poisson distribution.[10] In the K-M setting, all equilibria entail sincere voting: conditional on voting, type-b voters vote B and type-r voters vote R (K-M Theorem 1). This result does not automatically generalize to a set-up with fixed N; for arbitrary values of N there may be other kinds of equilibrium. Indeed, for any N, the unique symmetric equilibrium of the compulsory voting model where there is full participation (no abstention) and type-b voters always vote sincerely while type-r voters mix with probability $v_r^* \in (0, 1)$ will also be an equilibrium under the voluntary and costless voting mechanism. Once we make voluntary voting costly, the latter insincere voting equilibrium disappears under the voluntary voting mechanism and, as discussed in the next section, we will have a unique symmetric *sincere* voting equilibrium.[11] To be consistent with K-M, we focus our attention in this section on the sincere voting equilibrium.

Given the restriction to sincere voting, the strategy of a voter simplifies to two participation rates $\{p_r, p_b\}$, one for each signal type. In this case, full participation (i.e., $p_r = p_b = 1$) cannot be an equilibrium for the same reason that sincere voting is not an equilibrium under the compulsory voting mechanism. In fact, following Lemma 1 in K-M, we can show that under voluntary and costless voting, we must have $p_b > p_r$ in any equilibrium with sincere voting[12]. In our discussion of the unique symmetric equilibrium under compulsory voting, we observed that, in order to preserve the incentive for informative voting, the event where there is a tied vote among the other $N - 1$ players (i.e., an equal number of votes for R and B) must indicate a signal profile where there are more r signals than b signals. Under sincere voting, this is achieved only if type-b voters vote with a higher probability than type-r voters. Therefore, while the compulsory voting mechanism addresses the pivotality concern by having type-r voters sometimes vote against their signal, under the voluntary voting mechanism the same concern is addressed by having type-r voters abstain from voting with a higher probability.

In the case with costless voting, in the equilibrium that involves sincere voting, we should have $p_b^* = 1$ and $p_r^* \in (0, 1)$, i.e., type-b voters always participate and vote for B while type-r voters mix between abstaining and voting for R. The participation rate for type-r voters is determined by making the type-r voter indifferent between voting for R and abstaining, specifically by setting

$$U(R|r) - U(\phi|r) \equiv M\{q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta]\} = 0,$$

[10] K-M show that any difference between these two approaches disappears when the group size, N, is sufficiently large.

[11] A proof of the existence of two symmetric informative equilibria under the voluntary and costless voting mechanism is provided in Appendix B (Supplementary Material).

[12] The statement and proof of Lemma 1 in K-M can be shown to apply to the fixed N environment that we study with only minor modifications.

where $\Pr[Piv_R|\rho]$ denotes, for example, the probability that a vote for R is pivotal in state ρ and this pivot probability is a function of the participation rate, p_r, of type-r.[13] Under our parameter specification, the above indifference condition identifies a unique value of p_r^*. Moreover, given p_r^*, since a type-b voter strictly prefers to vote for B rather than abstain, we must have that

$$U(B|b) - U(\phi|b) \equiv M\{q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho]\} > 0.$$

Additionally, sincere voting by type-r voters requires that given equilibrium participation rates we must have

$$\begin{aligned} & U(R|r) - U(B|r) \geq 0 \\ \Leftrightarrow\quad & U(R|r) - U(\phi|r) \geq U(B|r) - U(\phi|r) \\ \Leftrightarrow\quad & q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta] \geq q(\beta|r)\Pr[Piv_B|\beta] - q(\rho|r)\Pr[Piv_B|\rho]. \end{aligned}$$

Similarly, sincere voting by type-b voters requires that

$$\begin{aligned} & U(B|b) - U(R|b) \geq 0 \\ \Leftrightarrow\quad & q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho] \geq q(\rho|b)\Pr[Piv_R|\rho] - q(\beta|b)\Pr[Piv_R|\beta]. \end{aligned}$$

These two conditions require that voting *sincerely* be incentive compatible. We check (in Appendix A, Supplementary Material) that both conditions hold given our solutions for p_r^* and p_b^*.

3.3 Voluntary and costly voting

Under the voluntary but costly voting mechanism, each voter faces a cost, c, to voting so that her overall utility is $U(A|\omega) - c$ if she votes and $U(A|\omega)$ if she abstains, where $A \in \{R, B\}$ is the winning alternative and $\omega \in \{\rho, \beta\}$ is the state. The voting cost is a random variable drawn independently across individuals from a set $\mathcal{C} = [0, \bar{c}]$, $\bar{c} > 0$, according to an atomless distribution, F. We further assume that voting costs are drawn independently of signals. After observing their voting cost and signal, voters decide whether to vote or to abstain. Thus, in this setting a player type consists of both a signal and a cost of voting. Generally, the (mixed) strategy of a voter is a mapping from the type space $\{r, b\} \times \mathcal{C}$ to the space of probability distributions over $\{R, B, \phi\}$.

Under certain conditions (that are satisfied by the parameters chosen in our experimental design), we show in Appendix B (Supplementary Material) that under costly voluntary voting, the insincere voting equilibrium of the compulsory voting mechanism can no longer be an equilibrium. Indeed, we show that for our parameterization of the model the *unique* symmetric equilibrium under the voluntary but costly voting mechanism involves *sincere* voting by all player types. Therefore, the choice faced by each voter under the voluntary and costly voting mechanism is whether to vote

[13] Since we allow abstention under the voluntary voting mechanisms, a vote can either make or break a tie. If we denote by T, T_{-1}, and T_{+1} the events that the number of votes for R is the same as, one less than, and one more than the number of votes for B, respectively, then for each $\omega \in \{\rho, \beta\}$,

$$\Pr[Piv_R|\omega] = \Pr[T|\omega] + \Pr[T_{-1}|\omega] \qquad \text{and} \qquad \Pr[Piv_B|\omega] = \Pr[T|\omega] + \Pr[T_{+1}|\omega],$$

where the pivot probabilities depend on the participation rate, p_r.

sincerely or to abstain. If voting is costly, then there exists a positive threshold cost, c_s^*, for each signal $s \in \{r, b\}$ such that a voter whose signal is s votes only if her realized cost is below the threshold c_s^*. The equilibrium participation rate for each signal, $p_s^* = F(c_s^*)$, $s \in \{r, b\}$, is determined by the cost threshold at which a voter with signal s is indifferent between voting sincerely and abstaining, specifically

$$\begin{aligned}
U(R|r) - U(\phi|r) &\equiv M\{q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta]\} = F^{-1}(p_r), \\
U(B|b) - U(\phi|b) &\equiv M\{q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho]\} = F^{-1}(p_b).
\end{aligned}$$

These two equations require that the expected benefit from sincere voting must equal the cutoff cost c_s^* given that all other voters adopt the same cutoff costs for participation and that all those choosing to participate also choose to vote sincerely. Here, the pivot probabilities are again functions of both types' participation rates (p_r, p_b).

The two equations above identify the equilibrium participation rates $\{p_r^*, p_b^*\}$ simultaneously (and uniquely for our parameter values and uniform cost distribution over $\mathcal{C}$). By the same logic used for the voluntary and costless voting mechanism, we must have $p_b^* > p_r^*$ to preserve the incentives for informative voting. In other words, we must have $c_b^* > c_r^*$. Furthermore, given the equilibrium participation rates, each participating voter must prefer to vote sincerely. Therefore, just as in the case with costless voluntary voting, we must have

$$\begin{aligned}
& U(R|r) - c \geq U(B|r) - c \\
\Leftrightarrow \quad & q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta] \geq q(\beta|r)\Pr[Piv_B|\beta] - q(\rho|r)\Pr[Piv_B|\rho]; \\
& U(B|b) - c \geq U(R|b) - c \\
\Leftrightarrow \quad & q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho] \geq q(\rho|b)\Pr[Piv_R|\rho] - q(\beta|b)\Pr[Piv_R|\beta].
\end{aligned}$$

We can again show (see Appendix A, Supplementary Material) that both of these inequalities hold given our solutions for p_r^* and p_b^*.

4 Experimental Design

We consider two treatment variables: 1) the voting mechanism, compulsory or voluntary, and within the voluntary treatment alone we further consider 2) whether voting is costless or costly. We adopt a between subjects design so that in each session subjects only make decisions under one set of treatment conditions. Across the three treatments - compulsory (C), voluntary and costless (VN), and voluntary and costly (VC) voting - of our experiment all parameters of the voting model and all other dimensions of the experimental design, e.g., the group size, the number of repetitions, the history of play, the payoff function, etc., are held constant.

The experiment was presented to subjects as an abstract group decision–making task using neutral language that avoided any direct reference to voting, jury deliberation, etc. so as not to trigger other motivations for voting that we want to abstract away from (e.g., civic duty, the sanction of peers, etc.).

Each session consists of a group of 18 inexperienced subjects and 20 rounds. At the start of each round, the 18 subjects were randomly assigned to one of two groups of size $N = 9$ subjects.

One group is assigned to the red jar (state ρ) and the other group is assigned to the blue jar (state β) with equal probability, thus fixing the true state of nature for each group. No subject knows which group she has been assigned to and group assignments are determined randomly at the start of each new round so as to avoid possible repeated game dynamics. Subjects *do* know that it is equally likely that their group is assigned to the red jar or to the blue jar at the start of each round.

The red jar contained fraction x_ρ red balls (signal r) and fraction $1 - x_\rho$ blue balls (signal b) while the blue jar contained fraction x_β blue balls and fraction $1 - x_\beta$ red balls. We set $x_\rho = .9$ and $x_\beta = .6$, across all sessions of our experiment, and these signal precisions were made public knowledge in the written instructions, which were also read aloud at the start of each session.[14] Our signal precision choices were made subject to the following constraints: 1) a signal is indicative of a distinct state, i.e., $x_\omega > 0.5$, 2) signals are not perfectly informative, i.e., $x_\omega < 1$ and 3) signal precisions are in multiples of 0.1 (as we presented subjects with a choice among 10 balls). Given these constraints, under the compulsory voting mechanism our signal precision choices maximize the extent of insincere voting by those receiving an r-signal. In the case of voluntary voting, the same signal precision choices maximize the difference in participation rates between the two signal groups. Thus, given our constraints, our signal precision choices provide the starkest possible differences in equilibrium predictions across our three treatments which serves to facilitate identification of treatment differences in the (possibly noisy) experimental data.

The sequence of play in a round was as follows. First, each subject blindly and simultaneously draws a ball (with replacement) from her group's (randomly assigned) jar. This is done virtually in our computerized experiment; subjects click on one of 10 balls on their decision screen and the color of their chosen ball is revealed.[15] While the subject observes the color of the ball she has drawn, she does not observe the color of any other subject's selections or the color of the jar from which she has drawn a ball. A group's common and publicly known objective is to correctly determine the jar, "red" or "blue", that has been assigned to their group.

In the two treatments without voting costs, after subjects have drawn a ball (signal) and observed its color, they next make a voting decision. In the compulsory voting treatment (C), they must make a "choice" (i.e., vote) between "red" or "blue", with the understanding that their group's decision, either red or blue, will correspond to that of the majority of the 9 group members' choices and that the group's aim is to correctly assess the jar (red or blue) that was assigned to the group. In the voluntary but costless voting treatment (VN), the only difference from the compulsory treatment is that subjects must make a "choice" between "red", "blue" or "no choice" (abstention). The group's decision in this case, "red" or "blue," will correspond to that of the majority of the group members who made a choice between "red" or "blue", i.e., who participated in voting. In the voluntary treatments (but not in the compulsory treatment) there is the possibility of ties in the voting outcome, i.e., equal numbers of votes for red and blue (including also the possibility that no one chooses to vote). In the event of a tie, the group's decision is labeled "indeterminate", otherwise it is labeled "red" or "blue" according to the majority choice of those who participated

[14] A sample of the written instructions used in the experiment is provided in Appendix C (Supplementary Material).

[15] For each round and for each subject, the assignment of colors to the 10 ball choices the subject faced was made randomly according to whether the jar the subject was drawing from was the red jar (in which case percentage x_ρ of the balls were red) or the blue jar (in which case percentage x_β balls were blue).

in voting.

In the voluntary but costly voting treatment (VC), after each subject i has drawn a ball, each gets a private draw of their cost of voting for that round, c_i, that is revealed to them *before* they face a voting/participation decision. After privately observing both the color of the ball they drew and their cost of voting, each group member had to privately decide whether to vote for the red jar, the blue jar or to abstain ("no choice") as in the case where voting is voluntary and costless. The group's decision is again made by majority rule among all group members who do not abstain and the color chosen by the majority is the group's decision. A tie is again regarded as an "indeterminate" outcome.

Payoffs each round are determined as follows. If the group's decision according to the majority rule is correct, i.e., if a majority of the group members chose red (blue) and the color of the jar assigned to that group is in fact red (blue), then each of the 9 members of the group, even those who abstained in the two voluntary voting treatments, receive 100 points (i.e., we set $M = 100$). If the group's decision is incorrect, then each of the 9 members of the group receive 0 points. If the group's decision was "indeterminate" i.e., if there is a tied vote for "red" or "blue", then each of the 9 members of the group receive 50 points. This payoff function is the same across all three treatments.

In the voluntary and costly voting (VC) treatment only, the cost of voting is implemented using an "NC-bonus" payment where "NC" stands for "no choice". Thus, in the VC treatment, subject i gets $100 + c_i$ points if she abstains and her group's decision is correct while she gets c_i points if she abstains but her group's decision is incorrect. Similarly, she gets $50 + c_i$ points if she abstains and her group's decision is indeterminate. A decision by subject i to vote in a round of the VC treatment means that she loses the NC-bonus for that round, receiving a payoff of either 100, 0 or 50 depending on whether her group's decision is correct, incorrect or indeterminate, respectively. Subjects are informed that the NC-bonus for each round (c_i) is an i.i.d. uniform random draw from the set $\{0, 1, ..., 10\}$[16] for each subject i and applies only to that round.[17]

Each session consisted of 20 rounds of play. Subjects were instructed that their point totals from all 20 rounds of play would be converted into dollars at the fixed and known rate of 1 point = \$0.01 and that these dollar earnings would be paid to them in cash at the end of the session. In addition, subjects were given a \$5 cash show–up payment. Thus, it was possible for each member of each group (red or blue) to earn up to \$1 in each of the 20 rounds of play and in the VC treatment only, subjects could earn or forego an additional NC bonus of up to \$0.10 per round. Average earnings for this 1-hour experiment (including the \$5 show-up payment) were \$22.51.

Table 1 summarizes our experimental design, which involved four sessions of each of our three treatments. As we have 18 subjects per session, we have collected data from a total of $4\times3\times18 = 216$ subjects. Subjects were recruited from the undergraduate population of the University of Pittsburgh and the experiment was conducted in the Pittsburgh Experimental Economics Laboratory. No subject participated in more than one session of this experiment.

[16] The upper bound for c_i could have been set higher, up to 100, but we chose a low value to encourage voter participation.

[17] Our implementation of voting cost follows that of Levine and Palfrey (2007) and has the nature of an opportunity cost.

Session Numbers	No. of subjects per session	No. of rounds per session	Voting Mechanism	Voting Costly?
C1-4	18	20	compulsory	no
VN1-4	18	20	voluntary	no
VC1-4	18	20	voluntary	yes

Table 1: The Experimental Design

5 Research Hypotheses

We first consider the equilibrium predictions for the compulsory voting mechanism (C). For our parameter values, there exists a unique symmetric informative equilibrium in which subjects with signal b always vote for Blue (vote sincerely), i.e., the equilibrium probability of voting sincerely, given a blue signal, $v_b^* = 1$ while those with signal r vote against their signal (vote for Blue) with strictly positive probability (i.e., $v_r^* < 1$ so there is some insincere or strategic voting). More precisely under our parameterization of the model, voters in the C treatment who receive a red (r) signal are predicted to play a mixed strategy where they vote insincerely against their r-signal (they vote Blue) 15.6% of the time and they vote sincerely according to their r-signal (they vote Red) the remaining 84.4% of the time, i.e., $v_r^* = .844$. Equivalently, we predict that an average of 15.6% of signal type-r subjects will vote against their signal each round.

As noted in section 3.2 under the voluntary mechanism without voting costs (VN) there is an equilibrium involving sincere voting by both signal types. However, in this same environment, the compulsory voting equilibrium involving insincere voting by r-signal types but full participation by both signal types represents a second equilibrium possibility. These two equilibria are the only symmetric informative equilibria under the voluntary and costless voting mechanism (see Appendix B, Supplementary Material for the details). Thus the VN environment provides us with an interesting equilibrium selection question that our experiment can address. In the sincere voting equilibrium of the voluntary and costless voting mechanism, both signal types vote sincerely, i.e., $v_r^* = v_b^* = 1$ but participation rates may be less than one and dependent on the signal received, red (r) or blue (b). We denote these equilibrium participation rates by p_r^* and p_b^*. By contrast in the insincere equilibrium of the voluntary and costless voting mechanism (as in the compulsory voting mechanism) $p_r^* = p_b^* = 1$, $v_b^* = 1$ but $v_r^* \in (0, 1)$. We note that in the VN environment it is easily shown that the insincere voting equilibrium is Pareto-dominated by the sincere voting equilibrium.

The sincere voting equilibrium is also predicted under the voluntary but costly voting mechanism (VC), but in the latter case the sincere equilibrium is unique in the class of symmetric equilibria (again see Appendix B, Supplementary Material for the details). Further, the equilibrium predictions for the VC treatment can be alternatively stated in terms of cut-off values for the cost of voting for the two signal types, denoted by c_r^*, c_b^*, for which each type is made indifferent between voting and abstaining.

Table 2 summarizes the equilibrium predictions for our two voluntary voting treatments.

A final issue concerns the accuracy of group decisions. Let us denote by $W(\rho)$ and $W(\beta)$ the

Voluntary Voting Treatment & Eq. Type	v_r^*	v_b^*	p_r^*	p_b^*	c_r^*	c_b^*
VN (costless) sincere	1.000	1.000	0.5397	1.000	n/a	n/a
VN (costless) insincere	0.844	1.000	1.0000	1.000	n/a	n/a
VC (costly) sincere	1.000	1.000	0.2700	0.5497	2.70	5.50

Table 2: Equilibrium Predictions for the Voluntary Voting Treatments

equilibrium probabilities of making a correct decision by the group assigned to the red and the blue jar, respectively (recall that the red jar corresponds to state ρ while the blue jar, corresponds to state β). For the VN mechanism, we shall assume that the sincere voting equilibrium is selected; otherwise the accuracy of group decisions under the VN mechanism is no different than that reported for the C mechanism.

The theory predicts that $W(\rho)$ is greater than $W(\beta)$ under all three mechanisms (compulsory, voluntary and costless, and voluntary and costly) although the difference is negligible under the voluntary and costly mechanism. $W(\rho)$ and $W(\beta)$ are measures of the accuracy of group decisions, hence the group assigned to the red jar (which entails more precise correct signals) is predicted to be more accurate in its decisions. Table 3 shows the predicted values for $W(\rho)$ and $W(\beta)$.

Voting Mechanism	$W(\rho)$	$W(\beta)$	$\frac{1}{2}W(\rho) + \frac{1}{2}W(\beta)$
C	0.9582	0.8485	0.9033
VN*	0.9513	0.9106	0.9309
VC	0.8572	0.8501	0.8536

* Assumes play of the sincere voting equilibrium.

Table 3: Group Decision Accuracy Comparisons Across Voting Mechanisms

As Table 3 further reveals, if we take the average of $W(\rho)$ and $W(\beta)$ as an overall group decision accuracy measure for each voting mechanism (recall the equal prior over the two states), then the theory also gives us a ranking of the mechanisms in terms of the accuracy of group decisions; namely, the voluntary and costless mechanism (VN) is the best, the compulsory mechanism (C) is second best and the voluntary and costly mechanism (VC) is the worst. If we were to consider the aggregate cost spent by those who participate in voting under the VC mechanism, then VC is even worse. We note again that the difference in accuracy between the VN and C mechanisms would disappear if under the VN mechanism voters coordinated on the insincere voting equilibrium that is the unique symmetric equilibrium under the C mechanism.

Based on the equilibrium predictions, we now formally state our research hypotheses:

H1. The fraction of those who vote *against* their signals (insincerely) is significantly greater than zero (15.6% of subjects with signal r) when voting is compulsory while it is zero when voting is voluntary.

H2. Under the voluntary voting mechanisms, subjects with b signals (type-b) participate at a higher rate than subjects with r signals (type-r); $p_r^* < p_b^*$. Furthermore, the participation rate is higher under the voluntary and costless mechanism than under the voluntary and costly mechanism for each signal type (Table 2).

H3. Under all three voting mechanisms, the probability of making a correct decision is strictly higher for the group assigned to the red jar than for the group assigned to the blue jar; $W(\rho) > W(\beta)$. Moreover, assuming selection of the sincere voting equilibrium in the VN mechanism, the three voting mechanisms can be ranked according to their ex-ante, equal-weighted group decision accuracy ($\frac{1}{2}W(\rho) + \frac{1}{2}W(\beta)$); $VN > C > VC$.

6 Experimental Results

We report results from twelve experimental sessions (four sessions for each of the compulsory, voluntary and costless, and voluntary and costly treatments) with 18 subjects playing 20 rounds in each session. Overall, we find strong support for all three of our main research hypotheses. The next three sections discuss the support for each hypothesis in detail.

6.1 Sincerity/Insincerity of Voting Decisions

Finding 1 *Consistent with theoretical predictions, there is strong evidence of insincere voting by red-signal types under the compulsory voting mechanism. By contrast, nearly all voters of both signal types vote sincerely under both voluntary mechanisms (costless and costly).*

Figure 1 shows the observed frequency of *in*sincere voting under the three treatments. Under the compulsory voting mechanism (C), the proportion of type-r voters (those who drew a red ball) who voted insincerely was greater than 10% (recall that red (r) signal types are the only type who are predicted to vote insincerely with positive probability). By contrast the frequency of insincere voting by signal type-b voters (those who drew a blue ball) under the compulsory voting mechanism (C) as well as both signal types under the two voluntary voting mechanisms (VN and VC) was always less than 5%. Thus Figure 1 suggests that there is a large difference in the sincerity of voting decisions between type-r voters in treatment C and all other signal types in the three treatments of our experiment.

Table 4 shows disaggregated, session-level averages of the frequency of *sincere* voting in all 12 sessions by signal type, denoted by v_s where $s = r$ (red) or $s = b$ (blue). The table reveals that Nash equilibrium performs rather well in predicting the qualitative (if not the quantitative) results for our voting games of compulsory or voluntary participation. With a couple of exceptions, the frequency of sincere voting is close to 100% under the voluntary voting mechanisms. The decomposition of sincere voting behavior by signal types indicates that, consistent with theoretical predictions, subjects who participated in voting voted sincerely regardless of the signals drawn under both voluntary voting mechanisms. On the other hand, we do find evidence for insincere (or strategic) voting under the compulsory mechanism among subjects drawing a red ball; slightly more than 10% of type-r voters voted insincerely which is close to, though slightly lower than the equilibrium prediction of 15.6%. It is also interesting to note that the behavior of subjects under the compulsory mechanism was remarkably consistent across sessions in terms of the average

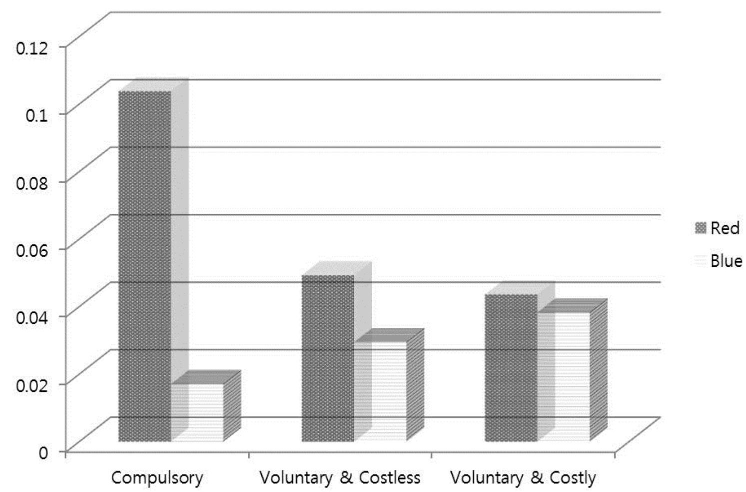

Figure 1: Overall Frequency of Insincere Voting. Pooled Data from All Rounds of All Sessions of Each of the Three Treatments

frequencies of sincere voting between signal types. The data seem to confirm the prediction that the voting mechanism in place (compulsory vs. voluntary) affects the incentives for subjects to vote sincerely or insincerely.

Are the differences in voting behavior between the three mechanisms statistically significant? To answer this question, we conducted a nonparametric Wilcoxon-Mann-Whitney (WMW) test using the four session-level averages for each treatment as reported in Table 4. The null hypothesis is that the frequencies of sincere voting between the two mechanisms under consideration come from the same distribution. Table 5 reports p-values from pairwise WMW tests of this null of no difference using session-level averages from all 20 rounds ("All rounds") or from the first or the last 10 rounds.[18]

Consider first the sincerity of voting by type-r subjects (those receiving a red signal). Over all 20 rounds, the difference between the compulsory (C) and the voluntary but costly (VC) treatment yields a statistically significant difference ($p = .011$).[19] Given the high frequency of sincere voting under the VC mechanism, we can say that subjects indeed behaved strategically under the C mechanism. We obtain the same result in the comparison between type-r subjects in the compulsory (C) treatment and type-r subjects in the combined voluntary treatments (V=VN+VC) as a group.

[18] We report asymptotic p-values for the non-parametric tests reported in Tables 5, 6, 8, 9, 12, and 14.

[19] We report p-values from one-sided tests of the null of no difference in all pairwise comparisons (in Table 5) between treatment C and the 'V' treatments, VN, VC or V=VN+VC that involves voting behavior by type-r subjects. That is because for those comparisons we have a clear directional hypothesis that type-r subjects should have voted "less sincerely" in the C treatment than in the 'V' treatments. The same reasoning applies to all subsequent comparisons (in Table 6, Table 8, Table 9, and Table 12) for which one-sided tests and p-values are reported.

Treatment/ Session	v_r (# obs.)	v_b (# obs.)
C1	0.8956 (249)	0.9910 (111)
C2	0.8730 (244)	0.9914 (116)
C3	0.8970 (233)	0.9921 (127)
C4	0.9190 (247)	0.9558 (113)
C Overall	0.8962 (973)	0.9829 (467)
C Predicted	0.8440	1.0000
VN1	0.8871 (186)	0.9914 (116)
VN2	1.0000 (154)	0.9848 (132)
VN3	0.9752 (161)	0.9048 (105)
VN4	0.9524 (168)	0.9917 (121)
VN Overall	0.9507 (669)	0.9705 (474)
VN Predicted	1.0000	1.0000
VC1	0.9794 (97)	0.9600 (75)
VC2	0.9706 (102)	1.0000 (86)
VC3	0.9444 (108)	0.9574 (94)
VC4	0.9277 (83)	0.9286 (84)
VC Overall	0.9564 (390)	0.9617 (339)
VC Predicted	1.0000	1.0000

Table 4: Observed Frequency of Sincere Voting by Signal Type, v_s

Red Signal	C vs. VN	C vs. VC	VN vs. VC	C vs. V=VN & VC
All rounds	0.075*	0.011*	0.773	0.014*
First 10 rounds	0.075*	0.075*	0.773	0.045*
Last 10 rounds	0.042*	0.011*	0.885	0.009*
Blue Signal	C vs. VN	C vs. VC	VN vs. VC	C vs. V=VN & VC
All 20 rounds	0.663	0.564	0.773	0.552
First 10 rounds	0.554	0.309	0.237	0.795
Last 10 rounds	0.561	0.758	0.561	0.604

* One-sided p-values.

Table 5: p-values from Wilcoxon-Mann-Whitney Test of Differences in the Sincerity of Voting Between Treatments by Signal Type

Furthermore, we cannot reject the null hypothesis that the frequency of sincere voting by type-r subjects in both voluntary mechanisms (VN versus VC) is the same ($p > .10$).

The evidence for a significant difference in sincere voting behavior by type-r subjects in the C and VN treatments using data from all 20 rounds is weaker ($p = .0745$), suggesting that subjects under the voluntary but costless (VN) treatment have voted "less sincerely" as compared with the voluntary and costly (VC) treatment. Nevertheless, we further observe in Table 5 that if we restrict attention to the *last* 10 rounds of play, the difference in the sincerity of voting between the C and VN treatments becomes statistically more significant ($p = .0415$). According to the theory, the existence (or absence) of voting costs affects only participation decisions and *not* the sincerity of voting decisions; hence, if subjects were playing in accordance with the sincere voting equilibrium they should have voted sincerely regardless of their voting cost under both voluntary mechanisms. The weakly significant difference between the VN and C treatments has two possible explanations. First, recall that under the VN treatment, the symmetric *in*sincere voting equilibrium of the C treatment coexists with the symmetric *sincere* voting equilibrium; the coexistence of these two symmetric equilibria may have resulted in a coordination problem for subjects. As a second explanation, subjects in the VN treatment might not have thought very seriously about their participation/abstention decisions, at least initially, because in the VN treatment participation is "free," and given that participation rates by type-r subjects are higher than the predicted rates (as we will show below), these type-r subjects might have been better off voting insincerely to raise the probability of reaching a correct decision in the event that their group is assigned to the blue jar. We will come back to the latter explanation later in the paper when we attempt to rationalize the departures we observe from sincere voting using behavioral models.

As for the voting behavior of type-b subjects, we cannot reject the null hypothesis that there is no difference in the sincerity of voting for any of the four pairwise comparisons (C vs. VN, C vs. VC, VN vs. VC and C vs. V, where V again stands for the combined data from the costly and costless voluntary mechanisms) over all rounds or over the first or last 10 rounds. Thus, consistent with equilibrium predictions, the high sincerity of type-b subjects' voting decisions is constant across all treatments of our experiment. The test statistics also suggest that type-b subjects voted slightly "more sincerely" under the C treatment though that difference is not statistically significant at conventional levels.

	C	VN	VC	V (VN & VC)
All Rounds	0.034*	0.715	0.465	0.575
First 10 rounds	0.072*	0.465	1.000	0.484
Last 10 rounds	0.034*	0.715	0.593	0.612

* One-sided p-values.

Table 6: p-values from Wilcoxon Signed Ranks Test of Differences in the Sincerity of Voting Between Signal Types Within a Treatment

As a further test of the equilibrium predictions, we ask whether red and blue signal types behaved the same (in terms of sincere voting) *within* a single voting mechanism/treatment. Table 6 shows the results of a Wilcoxon signed-ranks test for matched pairs with the null hypothesis being that the frequencies of sincere voting are the same between signal types under a fixed voting mechanism. For the purpose of this test, we paired both types' observed frequencies of sincere voting in each session and generated 4 signed differences for each of the 3 treatments and 8 signed differences for the voluntary treatment as a group. In addition, we performed the same test on the first and second half of data from the four sessions of each treatment. Clearly, the only mechanism under which both types' behavior exhibits a significant difference was the compulsory (C) voting mechanism. This finding again confirms our hypothesis regarding equilibrium voting behavior, which postulates that only the signal type r under the C treatment will vote insincerely. Under the two voluntary mechanisms individually or taken together, we never find a significant difference in the sincerity of voting decisions between signal types, which is consistent with equilibrium predictions.

6.2 Participation Decisions

Finding 2 *Under voluntary voting, the difference in participation rates by signal types are in accordance with the symmetric, sincere voting equilibrium predictions. However, subjects in both voluntary voting treatments and of both signal types over-participate relative to these equilibrium predictions.*

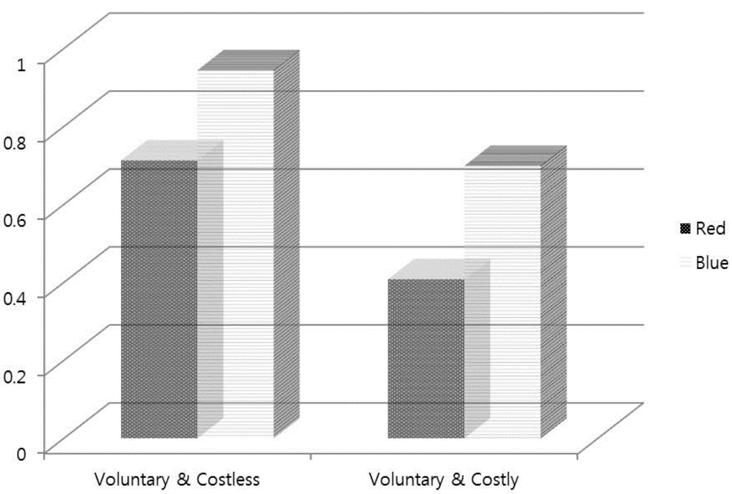

Figure 2: Overall Participation Rates, Pooled Data from All Rounds of All Sessions of Each of the Two Voluntary Voting Treatments

Support for Finding 2 comes from Figure 2 and Table 7 which reports participation rates p_s by signal type, $s = r$ or $s = b$. Consistent with theoretical predictions, we see that the participation rate of type-b voters was substantially greater than that of type-r voters in all sessions of the voluntary voting treatments. Since blue balls are rarer relative to red balls, type-b voters have more of an incentive to participate in voting decisions (and of course to vote sincerely). As reported in Table 8, Wilcoxon signed-rank tests (on the session level data shown in Table 7) lead us to reject the null hypothesis of no difference in participation rates at the lowest possible significance level given four observations for each of the two voluntary treatments (or eight observations for the voluntary treatments as a group) over all rounds or over the first and last 10 rounds of sessions. This finding is a natural consequence of the fact that the observed difference between participation rates $(\hat{p}_b - \hat{p}_r)$ in each session was always positive without exception in both voluntary voting treatments.

Treatment/ Session	p_r (# obs.)	p_b (# obs.)
VN1	0.7815 (238)	0.9508 (122)
VN2	0.6906 (223)	0.9635 (137)
VN3	0.6545 (246)	0.9211 (114)
VN4	0.7273 (231)	0.9380 (129)
VN Overall	0.7132 (938)	0.9442 (502)
VN Predicted	0.5397	1.0000
VC1	0.4128 (235)	0.6000 (125)
VC2	0.4250 (240)	0.7167 (120)
VC3	0.4519 (239)	0.7769 (121)
VC4	0.3444 (241)	0.7059 (119)
VC Overall	0.4084 (955)	0.6990 (485)
VC Predicted	0.2700	0.5497

Table 7: Observed Participation Rates by Signal Type, p_s, in the Voluntary Treatments

	VN	VC	V (VN & VC)
All rounds	0.034*	0.034	0.006
First 10 rounds	0.034	0.034	0.006
Last 10 rounds	0.034	0.034	0.006

* All p-values are one-sided

Table 8: p-values from Wilcoxon Signed Ranks Test of Differences in Participation Rates Between Signal Types Within a Treatment

We further observe that each signal type participated at a higher rate under the VN treatment than under the VC treatment, which is also consistent with the theoretical prediction that the introduction of voting costs will reduce participation incentives for all types. As Table 9 reveals, a Wilcoxon-Mann-Whitney test applied to the session–level data reported in Table 7 allows us to reject the null hypothesis of no difference in participation rates by signal type between the two voluntary treatments ($p < .05$ in all pairwise tests) since all four participation observations in the VN treatment always rank higher than those in the VC treatment for both signal types over all 20 rounds or over the first or last 10 rounds of the sessions. Therefore, the participation behavior observed in our data strongly supports the qualitative predictions of the Nash equilibrium.

However, as stated in Finding 2, we also observe that subjects participated in voting at a higher rate than the equilibrium prediction with the sole exception of type-b subjects under the VN treatment (the predicted participation rate is 100% for type-b subjects in the VN treatment). This tendency for over-participation was also observed by Levine and Palfrey (2007) (when the electorate was sufficiently large, as in our case) with the rate of over-participation increasing with the group size. They explain such systematic tendency to over-participation using the notion of

VN vs. VC			
Signal Color	All rounds	First 10 rounds	Last 10 Rounds
Red	0.0105*	0.0105	0.0105
Blue	0.0105	0.0105	0.0105

* All p-values are one-sided.

Table 9: Wilcoxon-Mann-Whitney Test of Differences in Participation Rates Between Treatments

Quantal Response Equilibrium (QRE), an equilibrium concept that formalizes noisy best response. In Appendix D of the Supplementary Material, we explore whether QRE estimates of both voting behavior and participation rates can help us to better explain the data from our experiment. In particular, the participation by type-r voters was high under the VN mechanism to the point of changing their incentives with regard to voting decisions. Given such high participation rates, type-r voters should have voted insincerely with a positive (but small) probability. We speculate that, despite our neutral framing of the problem (i.e., our avoidance of all references to voting), subjects may nevertheless have had a negative feeling about selecting the "No Choice" option and thus avoided choosing it when they should have. Offering a proper incentive to select No Choice, as in our costly voting treatment with its NC bonus, provides a better test of the importance of the voluntary voting mechanism in our opinion and it appears to have worked to reduce any stigma that might have been attached to choosing "No choice".

We further note that while the participation rate of type-r subjects in the VN treatment is high, it is still well below 100 percent (the average participation rate across all sessions of this treatment is 71.3 percent). Recall that the unique symmetric *in*sincere voting equilibrium under the compulsory voting mechanism is an alternative symmetric equilibrium possibility under the VN mechanism. However, that insincere voting equilibrium would require 100 percent participation and more insincere voting by type-r subjects than we observe in the data from our VN treatment. Thus on the question of equilibrium selection, the data from our VN treatment seem closer to and more in accordance with the symmetric *sincere* voting equilibrium which, as noted earlier, payoff dominates the *insincere* voting equilibrium. We address this equilibrium selection issue in further detail in Appendix D.

Recall that for the voluntary and costly (VC) treatment only, participation decisions could also be characterized by cut-off cost thresholds c_s^* such that if a voter's cost was less than the threshold given the signal s, then they participated in (sincere) voting and otherwise they abstained (see, e.g., Table 2). Figure 3 shows participation rates disaggregated by realized cost for the two signal types using all data from all rounds of the VC treatment. The figure reveals that players did not strictly employ the equilibrium cut-off cost thresholds of the theory, namely $c_r^* = 2.7$ and $c_b^* = 5.5$. However, consistent with the theory, participation rates do decline with voting costs and are lower for red signal types than for blue signal types. Using the experimental data from our VC treatment sessions, we estimated the level of cutoff cost for each signal type. Specifically we fit the individual binary decision data on whether to vote or to abstain in each period to a logit function of the

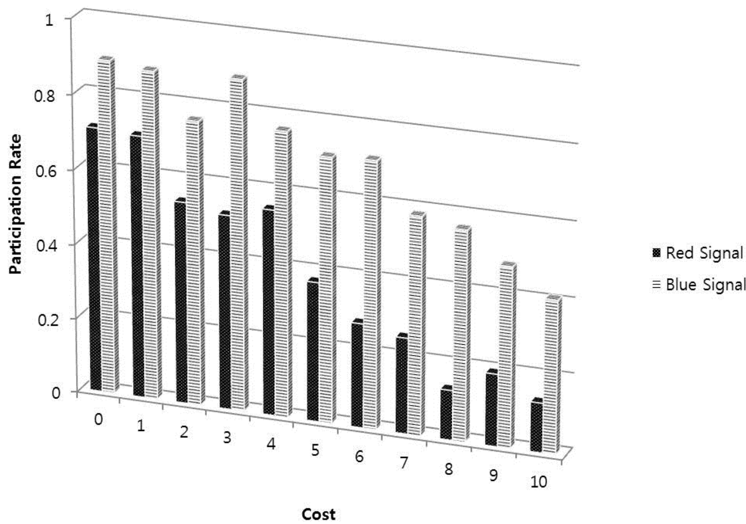

Figure 3: Participation Rates Disaggregated by Voting Costs and Signal Type, Pooled Data from All Rounds of All Sessions of the Voluntary and Costly (VC) Voting Treatment

individual's cost realization, c, their signal, s, and a constant term using all individual-level data from all rounds of the four voluntary and costly (VC) sessions. Specifically we used maximum likelihood estimation to find coefficient estimates $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ that are a best fit to the logit response function:

$$\Pr(\text{Participation}|c,s) = [1 + \exp(-\alpha - \beta c - \gamma s)]^{-1}.$$

The critical cut-off value c_s^* at which a voter is indifferent between participating and not participating given signal s obtains when $\Pr(\text{Participation}|c,s) = 0.5$. Since the signal s takes values either 0 (red signal) or 1 (blue signal), using the above logit specification and the maximum likelihood estimates of α, β and γ, we obtain the estimated cutoff costs $\hat{c}_r^* = -\hat{\alpha}/\hat{\beta}$ for the red signal type and $\hat{c}_b^* = -(\hat{\alpha}+\hat{\gamma})/\hat{\beta}$ for the blue signal type. Table 10 reports the results from our estimation of the logit response function. This table reveals that the estimated cutoff costs for both signal types

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$-\hat{\alpha}/\hat{\beta}$	$-(\hat{\alpha}+\hat{\gamma})/\hat{\beta}$
Estimate	-0.914	0.279	-1.440	3.276	8.437
Std. Error	0.114	0.020	0.132		
p-value	0.000	0.000	0.000		

Table 10: Logit Estimation of Cutoff Costs, VC Treatment

are much higher than equilibrium predictions: $-\hat{\alpha}/\hat{\beta} = 3.276 > 2.7 = c_r^*$ for red signal types and

$-(\hat{\alpha}+\hat{\gamma})/\hat{\beta} = 8.437 > 5.5 = c_b^*$ for blue signal types. This finding reflects the over-participation in voting phenomenon described earlier using participation rates. We note that consistent with the equilibrium prediction the estimated cost threshold for the blue signal type is much higher than that for the red signal type. Further, the highly significant coefficient on the signal variable, s, indicates that, consistent with the theory, blue signal types participated at a significantly greater rate than red signal types holding costs constant.

6.3 Accuracy of Group Decisions

Finding 3 *Consistent with theoretical predictions, the probability of making a correct decision is strictly higher for the group assigned to the red jar than for the group assigned to the blue jar, i.e., $W(\rho) > W(\beta)$. Further, the ranking of the voting mechanisms with respect to the ex-ante group decision accuracy measure, $\frac{1}{2}W(\rho)+\frac{1}{2}W(\beta)$, is as predicted, with $VN > C > VC$. However, these accuracy differences are not statistically significant from one another in our experimental data.*

Treatment/ Session	$W(\rho)$	$W(\beta)$	$\frac{1}{2}W(\rho)+\frac{1}{2}W(\beta)$
C1	0.9500	0.6000	0.7750
C2	1.0000	0.8500	0.9250
C3	1.0000	0.7500	0.8750
C4	1.0000	0.7000	0.8500
C Overall	0.9875	0.7250	0.8563
C Predicted	0.9582	0.8485	0.9033
VN1	1.0000	0.8000	0.9000
VN2	1.0000	0.9250	0.9625
VN3	1.0000	0.6000	0.8000
VN4	0.9750	0.8750	0.9250
VN Overall	0.9938	0.8000	0.8969
VN Predicted	0.9513	0.9106	0.9309
VC1	0.8750	0.7250	0.8000
VC2	0.9000	0.7750	0.8375
VC3	0.9250	0.9000	0.9125
VC4	0.8250	0.8250	0.8250
VC Overall	0.8813	0.8063	0.8438
VC Predicted	0.8572	0.8501	0.8536

Table 11: Observed Group Decision Accuracy, $W(\omega)$, and Aggregate Accuracy $\frac{1}{2}W(\rho)+\frac{1}{2}W(\beta)$

Recall that our measure of group decision accuracy is the equal weighted probability, $W(\omega)$, of making the correct decision in each state $\omega \in \{\rho, \beta\}$. For notational convenience, let us denote the group that is assigned to the red jar as the ρ group and the group that is assigned to the blue jar as the β group. Consistent with theoretical predictions, Table 11 reveals that the ρ group made correct decisions significantly more frequently than did the β group across all treatments.

We further observe that the frequencies of correct decisions by the ρ group tended to be higher than equilibrium predictions, while the frequency of correct decisions by the β group were generally lower than equilibrium predictions, with some exceptions in several sessions.

These success frequencies are, of course, closely tied to participation decisions and voting behavior. The observed discrepancy follows from the higher than predicted rates of voter participation under the voluntary mechanisms and from the lower than predicted rates of insincere voting under the compulsory mechanism by type-r voters who drove up the success rates when they were in the ρ group, but drove up the error rate when they were in the β group, which explains the low success rates of the β group. This same finding continues to obtain in the voluntary voting treatments where a much smaller fraction of type-r voters voted insincerely.

Finally, recall our prediction concerning the ranking of voting mechanisms in terms of group decision accuracy as given in Table 3. Groups were predicted to make correct decisions with the highest frequency under the voluntary and costless mechanism (VN), followed by the compulsory mechanism (C) and then by the voluntary and costly mechanism (VC). Our data produce this same ranking; the probability of correct decisions in the three regimes is, VN: 0.8969; C: 0.8563; and VC: 0.8438. These observed group decision accuracy measures are a little lower than equilibrium predictions under all mechanisms/treatments.

Table 12 shows the results of a WMW test of whether the observed differences in group decision accuracy are statistically significant between pairs of treatments over all 20 rounds or over the first and last 10 rounds of sessions. This test is performed for both the equal-weighted, aggregate group accuracy measure as well as conditional on the group being assigned to the red or to the blue state. Using the equal-weighted aggregate measure, we find that over all rounds we cannot reject the null hypothesis of no difference in group accuracy for any of the three pairwise treatment comparisons ($p > .10$ for all three tests). On the other hand, Table 12 reveals that if attention is restricted to just the last 10 rounds of sessions, we do find some evidence in support of the equilibrium prediction that groups make correct decisions more frequently in the VN treatment as compared with either the C treatment ($p \leq .10$) or with the VC treatment ($p < .05$). However, we continue to find that over the last 10 rounds, groups in the C treatment make correct decisions no more frequently than groups in the VC treatment ($p > .10$). Disaggregating by group type, we find, consistent with theoretical predictions that red group members are significantly more accurate in their group decisions under the C and VN mechanisms than under the VC mechanisms ($p < .05$ both tests). However in contrast to theoretical predictions there is no difference in red group decision accuracy between the VN and C treatments or in blue group decision accuracy in any of the three pairwise treatment comparisons ($p \geq .10$ for all tests).

These mixed results might be due to our small number of observations (just four independent observations for each treatment) but they could also arise from the fact that the theoretically predicted group decision accuracy differences are themselves very small for the groups of size 9 that we have considered in our experiment. Since in the limit, information aggregation holds (i.e., the probability of making a correct group decision goes to one along all the informative equilibria as the size of the electorate goes to infinity) under all three mechanisms (see, e.g., Feddersen and Pesendorfer (1998), Krishna and Morgan (2012)), we would expect that the observed differences in group decision accuracy would decrease as the size of the electorate was made even larger than in our experimental design.

Accuracy Measure	Rounds	VN vs. C	C vs. VC	VN vs. VC
Aggregate $\frac{1}{2}W(\rho) + \frac{1}{2}W(\beta)$	All rounds	0.1547*	0.2819	0.1547
	First 10 rounds	0.5000	0.3305	0.4425
	Last 10 rounds	0.0955	0.3315	0.0415
Red Group Only $W(\rho)$	All rounds	0.4250	0.0090	0.0090
	First 10 rounds	0.1585	0.0065	0.0125
	Last 10 rounds	0.1585	0.0200	0.0055
Blue Group Only $W(\beta)$	All rounds	0.1545	0.1240	0.3865
	First 10 rounds	0.5000	0.1225	0.2340
	Last 10 rounds	0.0945	0.3295	0.2325

* All p-values are one-sided.

Table 12: p-values from Wilcoxon-Mann-Whitney Test of Differences in Group Decision Accuracy Between Treatments

6.4 Individual Behavior

Thus far we have only considered behavior at the aggregate group and signal type level. In this section we delve deeper and explore the behavior of individual subjects under the three voting mechanisms. Figure 4 provides pairwise comparisons of the cumulative distributions of the frequency of sincere voting by all subjects between different voting mechanisms for each signal type or between two different signal types for a given voting mechanism. Figure 5 provides similar pairwise comparisons of the cumulative distributions of voting participation rates for the two voluntary treatments.

One implication of the theory is that the frequency of sincere voting by type-r players should be stochastically greater under the voluntary (VN or VC) mechanisms than under the compulsory (C) mechanism and that the same frequency for type-r players should be stochastically lower than that for type-b players under the compulsory (C) mechanism. This is the usual first-order stochastic dominance relationship, hence the cumulative distribution of a stochastically larger variable should lie everywhere below that of a stochastically smaller one. However, for all the other comparisons between mechanisms/types, the distributions are predicted to coincide. As Figure 4, reveals, we can indeed find this relationship in our data; in particular, the main difference between the two distributions occurs in the neighborhood of the mixed equilibrium frequency, .844, of sincere voting by type-r voters in the C treatment (which is indicated by the dashed line labeled "Nash" in graphs depicting the cumulative frequency of sincere voting by type-r subjects in the C treatments). Consider the first two graphs in the first row of Figure 4 which compare the behavior of type-r subjects in the C vs. VN and C vs. VC treatments, respectively. Consider also the comparison between the two signal types (r and b) under the C mechanism alone (the first graph in the third row of Figure 4). In these three cases alone, there is a predicted stochastic-order relationship.

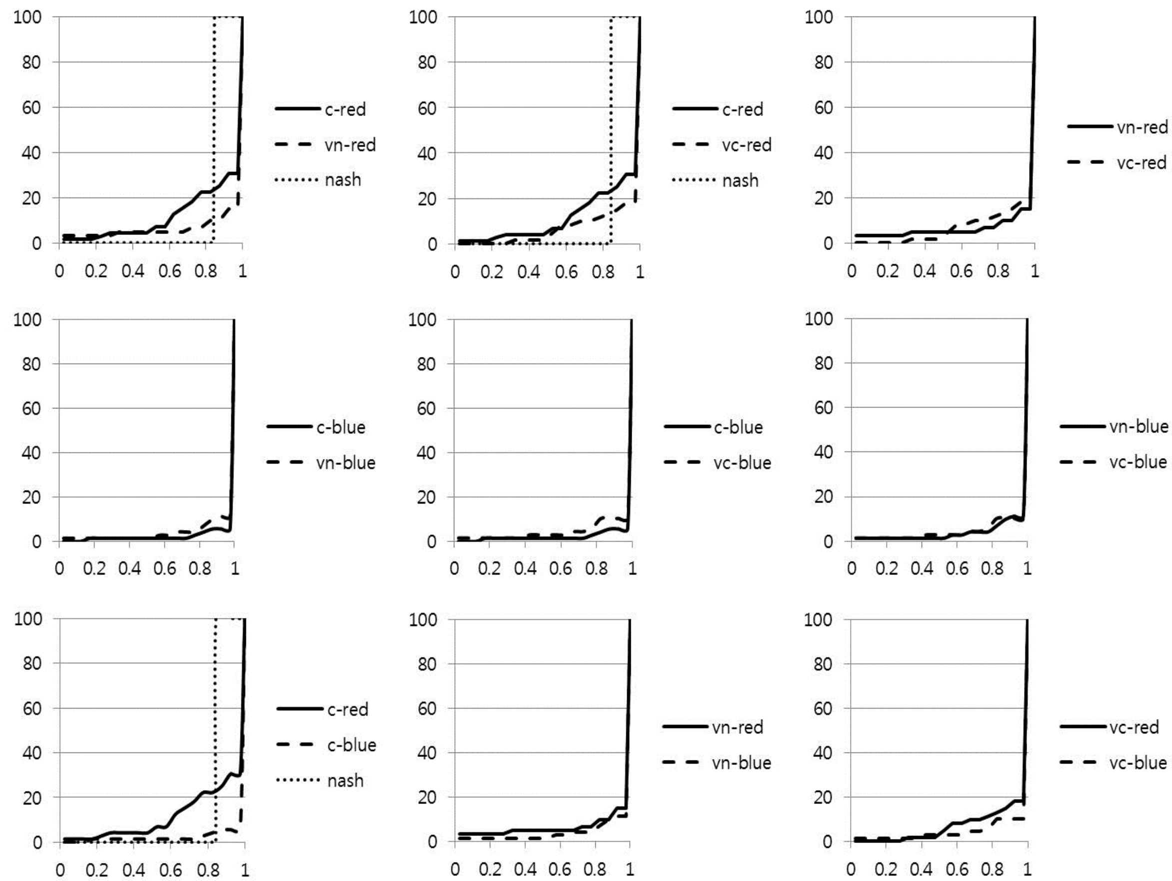

Figure 4: Distribution of the Frequency of Sincere Voting by Mechanism / Signal Type

In particular, the cumulative distribution of the frequency of sincere voting by type-r players in the C treatment should lie to the left of (or above) the cumulative distribution of the comparison group in these three graphs; more precisely the cumulative distribution of the frequency of sincere voting by type-r players in the C treatment should shift from 0% to 100% at the mixed equilibrium probability of .844. In all other pairwise comparisons the frequency of sincere voting is predicted to be 100% and so the cumulative distributions should coincide in those cases. Figure 4 reveals that, consistent with theoretical predictions, the cumulative frequency distribution of sincere voting by type-r players under the C voting mechanism is quite different from the cumulative frequency distribution of sincere voting by the comparison group. In particular, there is always a larger mass of type-r subjects voting insincerely under the C voting mechanism. Alternatively put, at 100% sincere voting, there is a large gap between the two cumulative frequencies, equal to 25% in the C/type-r vs. C/type-b comparison, 15.8% in the C/type-r vs. VN/type-r comparison or 12.6% in the C/type-r vs. VC/type-r comparison while the difference is relatively small in all other cases (precisely, it ranges from 1 to 7.7%).[20]

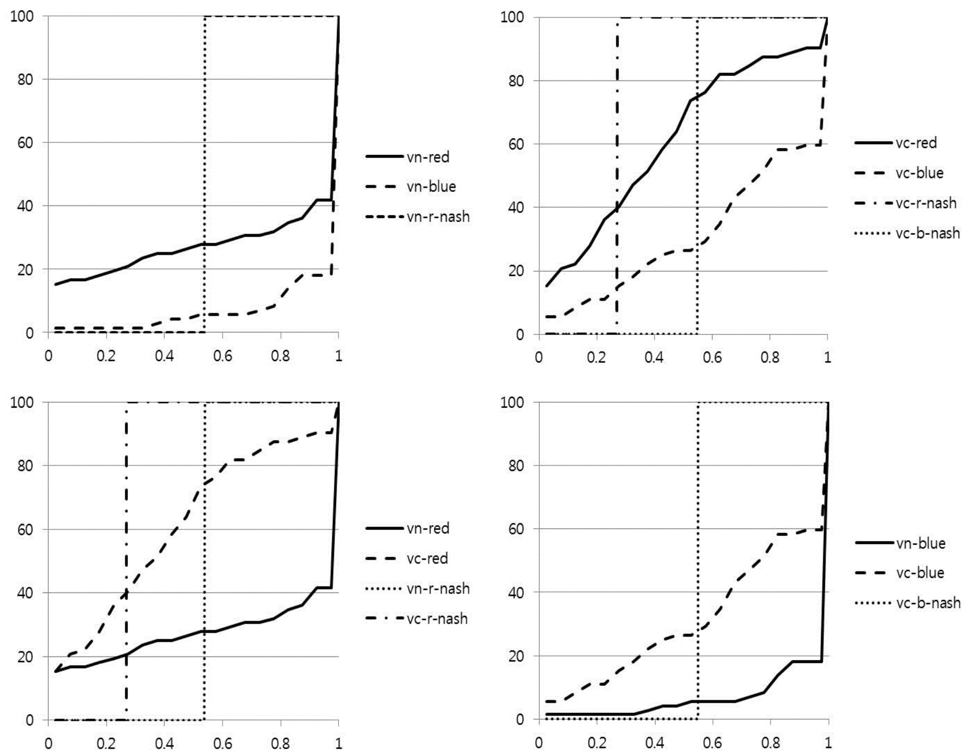

Figure 5: Distribution of Participation Rates by Mechanism / Signal Type

The theory also predicts stochastic-order relationships between the distributions of participation rates. Namely, the distribution of participation rates for type-r players should lie above the distribution of participation rates for type-b players under both voluntary voting mechanisms, and the distribution of participation rates for the VC mechanism should lie above the distribution of participation rates for the VN mechanism for both signal types. Pairwise comparisons of the cumulative frequency distributions of participation rates (and Nash equilibrium predictions) are shown in Figure 5. As that figure makes clear, the observed differences in the distributions of participation decisions are all in the right direction providing strong support for the comparative statics hypotheses about participation rates even at the individual level of our experimental data.[21]

[20] Nevertheless, a Kolmogorov-Smirnov test of differences between the cumulative frequency distributions of sincere voting fails to detect a significant difference between C/type-r and VN/type-r or C/type-r and VC/type-r (the p-values are 0.191 and 0.339, respectively). However, the difference in cumulative frequency distributions of sincere voting between C/type-r and C/type-b is significant at 1% level (p-value=0.011) according to the same test.

[21] Indeed, Kolmogorov-Smirnov tests indicate significant differences in the cumulative frequency distributions of

Summarizing our findings, we have presented strong evidence in support of the comparative statics predictions of the theory with respect to the impact of the various voting mechanisms on the sincerity of voting, participation decisions and the accuracy of group decisions. Nevertheless, we have also found some differences between the equilibrium point predictions and the experimental data, for example, over-participation relative to equilibrium predictions under the voluntary voting mechanisms. In Appendix D (Supplementary Material) we examine the performance of two models of boundedly rational behavior, "equilibrium plus noise" and quantal response equilibrium in accounting for these anomalous findings. The main message of that analysis is that these two models of bounded rationality do no improve much upon Nash equilibrium in terms of characterizing the behavior of subjects in our experiment.

7 Learning

Finally, it is of interest to consider whether there is any evidence of learning over the 20 repetitions of our voting games. In looking for evidence of learning, we compare observations in the first 10 rounds with those in the last 10 rounds. Table 13 reports the decomposition of both the voting and participation choice data into the two halves ("1st 10 rounds" and "2nd 10 rounds") and also restates the Nash equilibrium predictions. Our data on voting and participation decisions both indicate movement toward equilibrium predictions as subjects gained experience; with one exception, voting and participation decisions are always closer to Nash equilibrium predictions in the last 10 rounds as compared with the first 10 rounds. The sole exception is for the sincerity of voting decisions under the VN treatment. In that treatment, the frequencies of sincere voting by type-r voters remained essentially unchanged between the two blocks of 10 rounds, but there was a small deviation away from the Nash equilibrium prediction of 100 percent sincere voting by experienced type-b voters is the VN treatment.

Table 14 reports p-values from a Wilcoxon signed ranks test examining whether there were any significant differences in the sincerity of voting decisions or in participation rates for each signal type between the first and the last 10 rounds using session level averages. The results for changes in the sincerity of voting decisions indicate that the differences from the first to the second half are largely insignificant. However, the results for participation decisions indicate that there was significant movement toward Nash equilibrium predictions for this dimension of voting behavior in both treatments and for both signal types. The size of the learning effect is especially large under the VC mechanism. Since the game induced by the VC mechanism is the most complicated of three we have studied, the evidence for learning in the VC treatment suggests that equilibration may take longer than the time frame allowed (20 repetitions) by our experiment.

participation rates in all four pairwise comparisons, - either at the 1% level (VN/type-r vs. VN/type-b) or at the 0.1% level (the other 3 comparisons).

Sincere Voting	v_r (# obs.)	v_b (# obs.)
C - 1st 10 rounds	0.9022 (491)	0.9782 (229)
C - 2nd 10 rounds	0.8900 (482)	0.9874 (238)
C Predicted	0.8440	1.0000
VN - 1st 10 rounds	0.9507 (345)	0.9825 (228)
VN - 2nd 10 rounds	0.9506 (324)	0.9593 (246)
VN Predicted	1.0000	1.0000
VC - 1st 10 rounds	0.9461 (204)	0.9514 (185)
VC - 2nd 10 rounds	0.9677 (186)	0.9740 (154)
VC Predicted	1.0000	1.0000

Participation Rates	p_r (# obs.)	p_b (# obs.)
VN - 1st 10 rounds	0.7263 (475)	0.9306 (245)
VN - 2nd 10 rounds	0.6998 (463)	0.9572 (257)
VN Predicted	0.5397	1.0000
VC - 1st 10 rounds	0.4397 (464)	0.7227 (256)
VC - 2nd 10 rounds	0.3788 (491)	0.6725 (229)
VC Predicted	0.2700	0.5497

Table 13: Evidence of Learning Over Time

Sincere Voting	v_r	v_b
C	0.3575*	0.1367
VN	0.4264†	0.0721†
VC	0.0721	0.1766

Participation Rates	p_r	p_b
VN	0.0721	0.0340
VC	0.0340	0.0721

* All p-values are one-sided.
† Movement away from equilibrium predictions.

Table 14: Wilcoxon Signed Ranks Test: Learning

8 Conclusion

In settings where voters share a common interest but receive private, noisy (but informative) signals about the true state of the world, rational choice theory predicts that voters will adopt mixed strategies that manifest themselves in different ways depending on whether voting is compulsory or voluntary (i.e., whether abstention is allowed or not). Under the compulsory majority rule voting environment that we study, voters should play a mixed strategy with respect to whether they vote sincerely (according to their signal) when they receive an r signal, though they should always vote sincerely conditional on receiving the other b signal. Under the voluntary majority rule voting environment we study, voters should always vote sincerely, according to the signal they receive but they should play a mixed strategy with respect to their participation/abstention decision. We have designed an experiment aimed at comparing these two different voting mechanisms and testing this important qualitative difference in the type of mixed strategy that rational players should adopt and we have found compelling evidence that voters do indeed adapt their behavior to the institutional voting mechanism that is in place in the manner predicted by rational choice theory. In particular, we find that signal type-r voters vote significantly more insincerely than signal type-b voters under the compulsory voting mechanism as well as by comparison with either signal type voters under both voluntary voting mechanisms. As for the voluntary voting mechanism, we find significant variations in voter participation rates, but sincere voting among those choosing to vote, all as predicted by the theory. We also observe that the differences in the group decision accuracy of the three voting mechanisms in terms of generating the correct outcome are theoretically small. Under our parameterization of the voting model we predict and find that group decision accuracy is highest on average under the voluntary and costless voting mechanism, followed by the compulsory voting model and that group decision accuracy is lowest on average under the voluntary and costly mechanism. However, we do not find that these group decision accuracy differences are statistically significant over all 20 periods of our sessions.

Our findings may help us to understand why both compulsory and voluntary voting mechanisms are observed to co-exist in nature. Abstracting from voting costs as we do here, the two institutions may coexist because voters can and do adapt their behavior to the voting rule in place in a way that minimizes the informational efficiency differences between the two mechanisms. On the other hand, if voting costs are taken into account and if such costs are substantial, then compelling everyone to vote may be welfare reducing in that voting costs would be borne by more voters under a compulsory mechanism than under a voluntary mechanism and the slightly better accuracy of decision-making under the compulsory mechanism might not compensate for these additional voting costs. We leave such a welfare analysis to future research.

In ongoing and related experimental research, Bhattacharya, Duffy and Kim (2013), we use the same Condorcet jury model set-up of this paper but relax the assumption that information (noisy signals) about the true state of the world are provided to voters without cost. Instead, we explore the impact of variations in information purchase costs and in group size on information aggregation under the compulsory, majority rule voting mechanism. Unlike the free information environment studied in this paper, if information is costly to acquire, then as the group size becomes larger, individuals have reduced incentives to acquire information (free-riding concerns come to dominate) as the likelihood that a voter is pivotal decreases. Under majority rule and for given information acquisition costs, group accuracy first rises and then falls with increasing group size so that there is an optimal group size that maximizes information aggregation (Persico (2004)). As in this paper, in Bhattacharya et al. (2013) we consider whether and how subjects adapt their behavior to variations in group size and information costs relative to rational voter model predictions.

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Supplementary Material: Appendices A-D (for online publication only)

Appendix A: Equilibrium calculations

In deriving equilibrium predictions, we adopt the parameterization of our experimental design where the number of voters $N = 9$, $x_\rho = \Pr[r|\rho] = 0.9$ and $x_\beta = \Pr[b|\beta] = 0.6$. These choices imply that $q(\rho|r) = \frac{9}{13}$ and $q(\beta|b) = \frac{6}{7}$.

Consider first the compulsory voting mechanism (C). Let v_s denote the probability of voting sincerely given signal $s \in \{r, b\}$. A symmetric Bayesian Nash equilibrium is described by a strategy profile (v_r, v_b).

We begin by calculating the probability of pivotal events $\Pr[Piv|\omega]$. Suppose the probability of a randomly chosen voter voting for alternative A in state ω is denoted by $A(\omega)$. Then,

$$\begin{aligned} R(\rho) &= 0.9v_r + 0.1(1 - v_b), \\ B(\beta) &= 0.6v_b + 0.4(1 - v_r). \end{aligned}$$

Since only signal type-r mixes ($v_r \in (0, 1)$) while type-b plays a pure strategy of voting sincerely in our equilibrium ($v_b = 1$), these expressions can be further simplified to $R(\rho) = 0.9v_r$ and $B(\beta) = 0.6 + 0.4(1 - v_r)$, i.e., the compulsory voting equilibrium is identified with a single number, v_r.

Let (j, k) denote the event that there are j votes for R and k votes for B. Under compulsory voting, the only pivotal event is $(4, 4)$, where a vote for either R or B is pivotal. The pivot probability in each state is given by

$$\begin{aligned} \Pr[Piv|\rho] &= \Pr[(4, 4)|\rho] &= \tbinom{8}{4}[R(\rho)]^4[1 - R(\rho)]^4, \\ \Pr[Piv|\beta] &= \Pr[(4, 4)|\beta] &= \tbinom{8}{4}[B(\beta)]^4[1 - B(\beta)]^4. \end{aligned}$$

Using these expressions for the pivot probabilities, we can calculate type-r's choice probability $v_r \in (0, 1)$ by solving the following equation:

$$U(R|r) - U(B|r) = 0 \Rightarrow \frac{9}{13}\Pr[Piv|\rho] - \frac{4}{13}\Pr[Piv|\beta] = 0.$$

The equilibrium choice probability for type-r is $v_r = 0.8440$ which results in

$$U(B|b) - U(R|b) = M\left[\frac{6}{7}\Pr[Piv|\beta] - \frac{1}{7}\Pr[Piv|\rho]\right] = M \cdot 0.1389 > 0,$$

and this justifies type-b's choice of sincere voting, i.e., $v_b = 1$.

Consider next the voluntary and costless voting mechanism (VN). We focus here on the symmetric sincere voting equilibrium under this voting mechanism. Since we allow abstention, the event that a vote for R is pivotal may no longer coincide with the event that a vote for B is pivotal. Let us denote the former event by Piv_R and the latter event by Piv_B. We again need to calculate the pivot probabilities $\Pr[Piv_j|\omega]$, $j = R,B$.

As mentioned in footnote 14, if we denote by T, T_{-1}, and T_{+1} the events that the number of votes for R is the same as, one less than, and one more than the number of votes for B, respectively, then for each $\omega \in \{\rho, \beta\}$,

$$\begin{aligned} \Pr[Piv_R|\omega] &= \Pr[T|\omega] + \Pr[T_{-1}|\omega], \\ \Pr[Piv_B|\omega] &= \Pr[T|\omega] + \Pr[T_{+1}|\omega], \end{aligned}$$

where

$$\begin{aligned} T &\equiv \{(k,k) : 0 \le k \le 4\}, \\ T_{-1} &\equiv \{(k-1,k) : 1 \le k \le 4\}, \\ T_{+1} &\equiv \{(k,k-1) : 1 \le k \le 4\}. \end{aligned}$$

Next, let p_r and p_b denote the participation rates of type-r and type-b voters respectively. Since we have a sincere voting equilibrium under the two voluntary mechanisms, a symmetric Bayesian Nash equilibrium is described by a pair of participation rates (p_r, p_b). $A(\omega)$ is analogously defined as the probability of a randomly chosen voter choosing alternative $A \in \{R, B, \phi\}$ in state $\omega \in \{\rho, \beta\}$. Assuming sincere voting, we have:

$$\begin{aligned} &R(\rho) = 0.9p_r, \quad B(\rho) = 0.1p_b, \quad \phi(\rho) = 1 - R(\rho) - B(\rho), \\ &R(\beta) = 0.4p_r, \quad B(\beta) = 0.6p_b, \quad \phi(\beta) = 1 - R(\beta) - B(\beta). \end{aligned}$$

Under voluntary and costless voting (VN), type-r mixes between (sincere) voting and abstaining ($p_r \in (0,1)$) while type-b votes for certain ($p_b = 1$), hence $R(\rho) = 0.9p_r$, $B(\rho) = 0.1$, $R(\beta) = 0.4p_r$ and $B(\beta) = 0.6$ (i.e., the voluntary and costless voting equilibrium is again identified with a single number, p_r). Using the expressions for $A(\omega)$, we can write

$$\begin{aligned} \Pr[T|\omega] &= \sum_{k=0}^{4} \binom{n}{2k}\binom{2k}{k} R(\omega)^k B(\omega)^k (1 - R(\omega) - B(\omega))^{n-2k}, \\ \Pr[T_{-1}|\omega] &= \sum_{k=1}^{4} \binom{n}{2k-1}\binom{2k-1}{k-1} R(\omega)^{k-1} B(\omega)^k (1 - R(\omega) - B(\omega))^{n-2k+1}, \\ \Pr[T_{+1}|\omega] &= \sum_{k=1}^{4} \binom{n}{2k-1}\binom{2k-1}{k} R(\omega)^k B(\omega)^{k-1} (1 - R(\omega) - B(\omega))^{n-2k+1}. \end{aligned}$$

We now know how to express $\Pr[Piv_j|\omega]$ as a function of p_r. Type-r's equilibrium participation rate can then be obtained from

$$U(R|r) - U(\phi|r) = 0 \Rightarrow \frac{9}{13}\Pr[Piv_R|\rho] - \frac{4}{13}\Pr[Piv_R|\beta] = 0,$$

which yields $p_r = 0.5387$ and results in

$$U(B|b) - U(\phi|b) = M\left[\frac{6}{7}\Pr[Piv_B|\beta] - \frac{1}{7}\Pr[Piv_B|\rho]\right] = M \cdot (0.0342) > 0.$$

The latter condition again justifies type-b's full participation in voting ($p_b = 1$). Using the above solution for p_r, we can check that sincere voting is in fact incentive compatible. Specifically, we have:

$$\begin{aligned}
U(R|r) - U(\phi|r) &= 0, \\
U(B|r) - U(\phi|r) &= M\left[\frac{4}{13}\Pr[Piv_B|\beta] - \frac{9}{13}\Pr[Piv_B|\rho]\right] = M \cdot (-0.0402) < 0 \\
&\Rightarrow U(R|r) > U(B|r). \\
U(B|b) - U(\phi|b) &= M \cdot (0.0342) > 0. \\
U(R|b) - U(\phi|b) &= M\left[\frac{1}{7}\Pr[Piv_R|\rho] - \frac{6}{7}\Pr[Piv_R|\beta]\right] = M \cdot (-0.0693) < 0 \\
&\Rightarrow U(B|b) > U(R|b).
\end{aligned}$$

The final case of voluntary and costly voting (VC) is similar. We again have a sincere voting equilibrium and the expressions for $A(\omega)$ and the pivot probabilities $\Pr[Piv_j|\omega]$ are the same as those for the voluntary and costless voting case (VN) except that both participation rates for type-r and type-b voters are now less than 1; i.e., $p_r, p_b \in (0, 1)$ (this means that the pivot probabilities $\Pr[Piv_j|\omega]$ are functions of both p_r and p_b). In the case of voluntary and costly voting, we have a cutoff-cost equilibrium with the cutoffs given by $F^{-1}(p_r)$, $F^{-1}(p_b)$, where F is the distribution of voting costs. In other words, a type-s voter participates in voting if and only if her realized voting cost is below $F^{-1}(p_s)$, $s = r,b$. A Bayesian Nash equilibrium is defined as a pair (p_r, p_b) that solves

$$\begin{aligned}
U(R|r) - U(\phi|r) &\equiv M\left[\frac{9}{13}\Pr[Piv_R|\rho] - \frac{4}{13}\Pr[Piv_R|\beta]\right] = F^{-1}(p_r), \\
U(B|b) - U(\phi|b) &\equiv M\left[\frac{6}{7}\Pr[Piv_B|\beta] - \frac{1}{7}\Pr[Piv_B|\rho]\right] = F^{-1}(p_b).
\end{aligned}$$

If F is the uniform distribution with the support $[0, \frac{M}{10}]$ as in our laboratory voting games, the resulting solutions are $p_r = 0.2700$, $p_b = 0.5497$ as reported in Table 2. These values again insure that sincere voting is incentive compatible. Specifically, we have:

$$\begin{aligned}
U(R|r) - U(\phi|r) = M \cdot (0.0270) &> M \cdot (-0.1188) = U(B|r) - U(\phi|r) \\
U(B|b) - U(\phi|b) = M \cdot (0.0550) &> M \cdot (-0.1277) = U(R|b) - U(\phi|b)
\end{aligned}$$

Appendix B: Characterization of the set of symmetric equilibria

In this appendix, we fully characterize the set of symmetric equilibria under the three majority rule voting mechanisms that are studied in the paper: (1) compulsory voting (2) voluntary and costly voting; and (3) voluntary and costless voting.

Compulsory voting

When voting is compulsory, the action set is $\{R, B\}$. We denote by v_s the probability of sincere voting on obtaining signal $s \in \{r, b\}$. In a symmetric Bayesian Nash equilibrium, the strategy set is described by (v_r, v_b).

Under compulsory voting, there are exactly three Nash equilibria: two where a tie is impossible under any circumstances (and hence no individual's vote is ever pivotal) and one where a tie occurs with positive probability (and each individual voter conditions his vote on the event that he is pivotal).

There are two type-symmetric strategy profiles that rule out the possibility of a tie. In these two strategy profiles, all voters vote for the *same* alternative irrespective of the signal they receive. Thus, there is one profile where $v_r = 0$ and $v_b = 1$ (i.e., all voters vote for B irrespective of signals) and another where $v_r = 1$ and $v_b = 0$ (i.e., all voters vote for R irrespective of signals) where ties do not occur. Since no voter can change the outcome by changing his vote, both of these profiles are Nash equilibria. We call these two equilibria *uninformative* as the equilibrium strategies are not responsive to signals.

Next, we consider the case where strategies are such that ties occur with positive probability. In such equilibria, voters condition their decision on the event of a tie. Since a vote is pivotal in deciding the outcome only in the event of a tie, we denote the event of a tie as the pivotal event ("piv") hereafter. We show that, when the pivot probability is well-defined and positive, given any group size N and signal precisions, x_ρ and x_β, there exists a unique equilibrium. For the derivation of this equilibrium, we assume that $M = 1$ and $x_s \in \left(\frac{1}{2}, 1\right)$ and N is odd. For the parameterization of the model used in the experiment this unique equilibrium was identified in Appendix A where we showed that $v_r = 0.84$ and $v_b = 1$.

Suppose that the probability of voting for alternative R on getting signal r is v_r and the probability of voting for alternative B on getting signal b is v_b. In a symmetric Bayesian Nash equilibrium, the strategy set is described by (v_r, v_b). In what follows, we denote $\frac{N-1}{2} = n$, and thus, when there is a tie, each alternative gets n votes.

Note first that in this setting, for $s \in \{r, b\}$,

$$\begin{aligned} U(R|s) &= U(R|\rho)\Pr(\rho|piv, s) + U(R|\beta)\Pr(\beta|piv, s) = \Pr(\rho|piv, s) \\ U(B|s) &= U(B|\rho)\Pr(\rho|piv, s) + U(B|\beta)\Pr(\beta|piv, s) = \Pr(\beta|piv, s) \end{aligned}$$

Now (v_r^*, v_b^*) is an equilibrium iff

1. $\Pr(\rho|piv, r) - \Pr(\beta|piv, r) \geq 0 \Leftrightarrow v_r^* \geq 0$ and $\Pr(\rho|piv, r) - \Pr(\beta|piv, r) > 0 \Rightarrow v_r^* = 1$

2. $\Pr(\beta|piv,b) - \Pr(\rho|piv,b) \geq 0 \Leftrightarrow v_b^* \geq 0$ and $\Pr(\beta|piv,b) - \Pr(\rho|piv,b) > 0 \Rightarrow v_b^* = 1$

Suppose the probability of a randomly chosen voter voting for $A \in \{R, B\}$ in state ω is $A(\omega)$. Then,

$$R(\rho) = x_\rho v_r + (1 - x_\rho)(1 - v_b) \tag{1}$$
$$B(\beta) = x_\beta v_b + (1 - x_\beta)(1 - v_r) \tag{2}$$

Further, the pivot probabilities in each state are given by the following equations, where $R(\rho)$ is given by (1) and $B(\beta)$ by (2),

$$\Pr(piv|\rho) = \binom{2n}{n} [R(\rho)]^n [1 - R(\rho)]^n \tag{3}$$
$$\Pr(piv|\beta) = \binom{2n}{n} [B(\beta)]^n [1 - B(\beta)]^n \tag{4}$$

It is easy to see from Bayes' Rule that

$$\frac{\Pr(\rho|piv)}{\Pr(\beta|piv)} = \frac{\Pr(piv|\rho)}{\Pr(piv|\beta)} \tag{5}$$

Writing $\Pr(\rho|piv)$ as θ (it is an endogenous quantity), we can say from (3), (4) and (5) that

$$\frac{\theta}{1-\theta} = \left(\frac{R(\rho)\,[1 - R(\rho)]}{B(\beta)[1 - B(\beta)]} \right)^n \tag{6}$$

By further application of Bayes' Rule we have

$$\Pr(\rho|piv, r) = \frac{x_\rho \theta}{x_\rho \theta + (1 - x_\beta)(1 - \theta)}$$
$$\Pr(\beta|piv, r) = \frac{(1 - x_\beta)(1 - \theta)}{x_\rho \theta + (1 - x_\beta)(1 - \theta)}$$
$$\Pr(\beta|piv, b) = \frac{x_\beta (1 - \theta)}{x_\beta (1 - \theta) + (1 - x_\rho)\theta}$$
$$\Pr(\rho|piv, b) = \frac{(1 - x_\rho)\theta}{x_\beta (1 - \theta) + (1 - x_\rho)\theta}$$

Therefore, in equilibrium v_r^*, v_b^* should satisfy

$$\frac{x_\rho}{1 - x_\beta} \geq \frac{1-\theta}{\theta} \Leftrightarrow v_r^* \geq 0, \text{ and } \frac{x_\rho}{1 - x_\beta} > \frac{1-\theta}{\theta} \Rightarrow v_r^* = 1 \tag{7}$$
$$\frac{x_\beta}{1 - x_\rho} \geq \frac{\theta}{1-\theta} \Leftrightarrow v_b^* \geq 0, \text{ and } \frac{x_\beta}{1 - x_\rho} > \frac{\theta}{1-\theta} \Rightarrow v_b^* = 1 \tag{8}$$

To have the voter indifferent between actions A and B under both signals, we need $\frac{x_\rho}{1-x_\beta} = \frac{1-x_\rho}{x_\beta}$, which implies either $x_\rho + x_\beta = 1$, which is not possible, given the assumptions on x_ρ and x_β.

Therefore, we can never have both v_r and v_b lying in the interval $(0,1)$. Therefore, there are exactly three possibilities, (i) $v_r = v_b = 1$, or (ii) $v_r \in (0,1)$ and $v_b = 1$, or (iii) $v_r = 1$ and $v_b \in (0,1)$. We next find the conditions for each of these possibilities and show that they are mutually exclusive and exhaustive.

Sincere voting equilibrium

In order to have a sincere voting equilibrium, i.e., $v_r^* = v_b^* = 1$, we need $\frac{x_\rho}{1-x_\beta} \geq \frac{1-\theta}{\theta} \geq \frac{1-x_\rho}{x_\beta}$. In that case, from (1) and (2), $R(b) = x_\rho$ and $B(\beta) = x_\beta$. Then, from (3) and (4), $\frac{1-\theta}{\theta} = \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n$. The condition under which the sincere voting equilibrium obtains is

$$\frac{x_\rho}{1-x_\beta} \geq \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n \geq \frac{1-x_\rho}{x_\beta} \tag{9}$$

Equilibrium where only type-b mixes

Next, we check whether there is any equilibrium with $v_b^* \in (0,1)$. From condition (8), we have $\frac{x_\beta}{1-x_\rho} = \frac{\theta}{1-\theta}$. Also, $\frac{\theta}{1-\theta} = \frac{x_\beta}{1-x_\rho} > \frac{1-x_\beta}{x_\rho} \Rightarrow v_r^* = 1$ from condition (7).

From (1) and (2), we have

$$\begin{aligned} R(\rho) &= x_\rho + (1-x_\rho)(1-v_b^*) = 1 - v_b^*(1-x_\rho) \\ B(\beta) &= x_\beta v_b^* \end{aligned}$$

Then

$$\frac{x_\beta}{1-x_\rho} = \frac{\theta}{1-\theta} = \left[\frac{[1-v_b^*(1-x_\rho)](1-x_\rho)}{x_\beta(1-x_\beta v_b^*)}\right]^n.$$

Solving,

$$v_b^* = \frac{d-1}{dx_\beta - 1 + x_\rho}, \text{ where } d = \left(\frac{x_\beta}{1-x_\rho}\right)^{1+\frac{1}{n}}. \tag{10}$$

It is easy to verify that $v_b^* = \frac{d-1}{dx_\beta - 1 + x_\rho} > 0$. For $v_b^* < 1$, we need $dx_\beta + x_\rho - d > 0$, or $\frac{x_\rho}{1-x_\beta} > d = \left(\frac{x_\beta}{1-x_\rho}\right)^{1+\frac{1}{n}} \Rightarrow \frac{1-x_\rho}{x_\beta} > \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n$.

Therefore, we have an equilibrium with $v_b^* \in (0,1)$ and $v_r^* = 1$ whenever $\frac{1-x_\rho}{x_\beta} > \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n$, and the exact value of v_b^* is given by (10).

Equilibrium where only type-r mixes

Next, we check whether there is any equilibrium with $v_r^* \in (0,1)$. From condition (7), we have $\frac{x_\rho}{1-x_\beta} = \frac{1-\theta}{\theta}$. Also, $\frac{\theta}{1-\theta} = \frac{1-x_\beta}{x_\rho} < \frac{x_\beta}{1-x_\rho} \Rightarrow v_b^* = 1$ from condition (8).

From (1) and (2), we have

$$\begin{aligned} R(\rho) &= x_\rho v_r^* \\ B(\beta) &= 1 - v_r^*(1-x_\beta) \end{aligned}$$

Then

$$\frac{x_\rho}{1-x_\beta} = \frac{1-\theta}{\theta} = \left[\frac{[1-v_r^*(1-x_\beta)](1-x_\beta)}{x_\rho(1-x_\rho v_r^*)}\right]^n.$$

Solving,

$$v_r^* = \frac{d-1}{dx_\rho - 1 + x_\beta}, \text{ where } d = \left(\frac{x_\rho}{1-x_\beta}\right)^{1+\frac{1}{n}}. \tag{11}$$

It is easy to verify that $v_r^* = \frac{d-1}{dx_\rho - 1 + x_\beta} > 0$. For $v_r^* < 1$, we need $dx_\beta + x_\rho - d > 0$, or $\frac{x_\beta}{1-x_\rho} > d = \left(\frac{x_\rho}{1-x_\beta}\right)^{1+\frac{1}{n}} \Rightarrow \frac{x_\rho}{1-x_\beta} < \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n$.

Therefore, we have an equilibrium with $v_r^* \in (0,1)$ and $v_b^* = 1$ whenever $\frac{x_\rho}{1-x_\beta} < \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n$, and the exact value of v_b^* is given by (11).

Summarizing, the unique, symmetric informative equilibrium in the compulsory voting case is completely characterized as follows:

1. If $\frac{x_\rho}{1-x_\beta} \geq \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n \geq \frac{1-x_\rho}{x_\beta}$, then $v_r^* = v_b^* = 1$.

2. If $\frac{1-x_\rho}{x_\beta} > \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n$, then $v_r^* = 1$ and $v_b^* = \frac{d-1}{dx_\beta - 1 + x_\rho}$, where $d = \left(\frac{x_\beta}{1-x_\rho}\right)^{1+\frac{1}{n}}$.

3. If $\frac{x_\rho}{1-x_\beta} < \left[\frac{x_\beta(1-x_\beta)}{x_\rho(1-x_\rho)}\right]^n$, then $v_b^* = 1$ and $v_r^* = \frac{d-1}{dx_\rho - 1 + x_\beta}$, where $d = \left(\frac{x_\rho}{1-x_\beta}\right)^{1+\frac{1}{n}}$.

We note that for our experimental parameterization where $M = 1$, $x_\rho = .9$, $x_\beta = .6$ and $n = 4$, the unique symmetric informative equilibrium under the compulsory majority rule voting mechanism is characterized by case 3.

Costly and voluntary voting

For the costly and voluntary voting case, we show that the sincere voting equilibrium we have identified in Appendix A is unique in the class of symmetric equilibria under our parameterization of the model. We proceed step by step, eliminating possibilities of other equilibria where individuals play symmetric strategies.

First, we define strategies and associated quantities. A strategy is a function $v : \{r, b\} \times [0, \bar{c}] \to \Delta\{R, B, \phi\}$. We simply write $v(A|s, c)$ as the probability of taking action $A \in \{R, B, \phi\}$ for a voter with signal $s \in \{r, b\}$ and cost c. By the nonatomicity of F, if voters for any countable number of values of c play a different strategy, then there is no change in the outcome of the game. Therefore, our symmetric equilibria are unique only up to a countable number of cost-types playing any other strategy.

Given the strategies $v(A|s, c)$, we denote by p_s^A the probability that a randomly chosen voter with signal s votes for A, i.e.,

$$p_s^A = \int_0^{\bar{c}} v(A|s, c)dF(c)$$

Denote $p_s = p_s^R + p_s^B$ as the "participation probability" of a random voter with signal s.

The probability that a randomly chosen voter votes for $A \in \{R, B\}$ in state $\omega \in \{\rho, \beta\}$ is given by:

$$
\begin{aligned}
R(\rho) &= x_\rho p_r^R + (1-x_\rho)p_b^R \\
B(\rho) &= x_\rho p_r^B + (1-x_\rho)p_b^B \\
B(\beta) &= x_\beta p_b^B + (1-x_\beta)p_r^B \\
R(\beta) &= x_\beta p_b^R + (1-x_\beta)p_r^R
\end{aligned}
$$

Define

$$
\begin{aligned}
q_r &= q(\rho|r) = \frac{x_\rho}{x_\rho + 1 - x_\beta} \\
q_b &= q(\beta|b) = \frac{x_\beta}{x_\beta + 1 - x_\rho}
\end{aligned}
$$

We now study different candidate configurations with symmetric strategies and show that none of them can be played in equilibrium.

Remark 1 *There is no equilibrium where only one action is played in equilibrium.*

Proof. If everyone votes for $A \in \{B, R\}$, then any voter with $c > 0$ is strictly better off by deviating from the prescribed strategy and abstaining. However, if everyone abstains, then those with cost $c < \min\{\frac{1}{2}[2q_b - 1], \frac{1}{2}[2q_r - 1]\}$ are strictly better off by voting sincerely. ■

Remark 2 *There is no equilibrium where only two actions are played in equilibrium with positive probability.*

Proof. First, suppose that no one abstains with positive probability in equilibrium. Then, we have the compulsory voting equilibrium strategy as the only possible equilibrium (as we have already eliminated everyone playing the same pure action by remark 1). In other words, conditioning on a tie, we have $U(R|r) = U(B|r)$ (for our data). Any voter with $c > 0$ and signal r is strictly better off abstaining. Hence we cannot have an equilibrium of this kind.

Next, suppose that in equilibrium we have $p_r^B = p_b^B = 0$, i.e., no one votes for B. Notice that $U(B|s)$ is independent of c. Hence, if some voter with signal s and cost c weakly prefers to abstain over voting for B, all voters with signal s and cost $c' > c$ strictly prefer to abstain. It follows that, in this putative equilibrium, for each signal s, there is a cutoff value c^s such that voters with costs below this cut-off value vote for B and those with costs above this cut-off value abstain. It is easy to see that $U(R|s) - U(\phi|s) = c^s$. Therefore, $F(c^s) = p_s$. Hence,

$$
\begin{aligned}
R(\rho) &= x_\rho p_r + (1-x_\rho)p_b \\
R(\beta) &= x_\beta p_b + (1-x_\beta)p_r
\end{aligned}
$$

Now, the only pivotal event is when everyone abstains, specifically:

$$
\begin{aligned}
\Pr(piv|\rho) &= [1-R(\rho)]^{N-1} \\
\Pr(piv|\beta) &= [1-R(\beta)]^{N-1}
\end{aligned}
$$

For type b to prefer to vote for R over B, we must have $\Pr(\rho|piv) > \Pr(\beta|piv)$, i.e., $\Pr(piv|\rho) > \Pr(piv|\beta)$, or $R(\beta) > R(\rho)$. Now,

$$\begin{aligned}
x_\beta p_b + (1-x_\beta)p_r &> x_\rho p_r + (1-x_\rho)p_b \\
p_b(x_\beta + x_\rho - 1) &> p_r(x_\beta + x_\rho - 1) \\
p_b &> p_r \Rightarrow c^b > c^r \\
U(R|b) - U(\phi|b) &> U(R|r) - U(\phi|r) \\
q(\rho|b)\Pr(piv|\rho) - q(\beta|b)\Pr(piv|\beta) &> q(\rho|r)\Pr(piv|\rho) - q(\beta|r)\Pr(piv|\beta) \\
\Pr(piv|\rho)[q(\rho|b) - q(\rho|r)] &> \Pr(piv|\beta)[q(\beta|b) - q(\beta|r)]
\end{aligned}$$

The left hand side is negative while the right hand side is positive leading to a contradiction. Therefore, there cannot be any equilibrium with $p_r^B = p_b^B = 0$. By the same reasoning, there cannot be any equilibrium with $p_r^R = p_b^R = 0$. ■

According to result 2, we must have $p_r > 0$, $p_b > 0$ and $p_r + p_b < 1$. These, in turn, imply that (1) $R(\omega) \neq 0$ (2) $B(\omega) \neq 0$ and (3) $R(\omega) + B(\omega) < 1$ for each ω. In other words, the probabilities $\Pr(T_{-1}|\omega)$, $\Pr(T|\omega)$ and $\Pr(T_{+1}|\omega)$ are well defined for $\omega \in \{\rho, \beta\}$.

Therefore, our pivot probabilities are well-defined. Now, we can write

$$U(R|r) - U(\phi|r) - c \equiv \frac{1}{2}q_r \Pr[Piv_R|\rho] - \frac{1}{2}(1-q_r)\Pr[Piv_R|\beta] - c \tag{12}$$

$$U(B|r) - U(\phi|r) - c \equiv \frac{1}{2}(1-q_r)\Pr[Piv_B|\beta] - \frac{1}{2}q_r \Pr[Piv_B|\rho] - c \tag{13}$$

$$U(R|b) - U(\phi|b) - c \equiv \frac{1}{2}(1-q_b)\Pr[Piv_R|\rho] - \frac{1}{2}q_b \Pr[Piv_R|\beta] - c \tag{14}$$

$$U(B|b) - U(\phi|b) - c \equiv \frac{1}{2}q_b \Pr[Piv_B|\beta] - \frac{1}{2}(1-q_b)\Pr[Piv_B|\rho] - c \tag{15}$$

Subtracting (13) from (12), we get

$$U(R|r) - U(B|r) = q_r\left(\Pr[Piv_R|\rho] + \Pr[Piv_B|\rho]\right) - (1-q_r)\left(\Pr[Piv_R|\beta] + \Pr[Piv_B|\beta]\right) \tag{16}$$

Similarly, subtracting (14) from (15), we get

$$U(B|b) - U(R|b) = q_b\left(\Pr[Piv_B|\beta] + \Pr[Piv_R|\beta]\right) - (1-q_b)\left(\Pr[Piv_B|\rho] + \Pr[Piv_R|\rho]\right) \tag{17}$$

Adding (16) and (17),

$$\begin{aligned}
& U(R|r) - U(B|r) + U(B|b) - U(R|b) \\
= \; & (q_b + q_r - 1)[(\Pr[Piv_B|\beta] + \Pr[Piv_R|\beta]) + (\Pr[Piv_B|\beta] + \Pr[Piv_R|\beta])] > 0 \\
U(B|b) - U(R|b) \; > \; & U(B|r) - U(R|r) \qquad (18)
\end{aligned}$$

Before proceeding further, we show that we cannot have insincere voting with both signal types.

Remark 3 *If a voter votes insincerely in equilibrium on getting signal s, all voters will vote sincerely on getting signal $s' \neq s$.*

Proof. Suppose some voter votes for B with a positive probability on obtaining signal r. Therefore, we must have $U(B|r) \geq \max\{U(\phi|r), U(R|r)\}$ This implies that $U(B|r) - U(R|r) \geq 0$. Hence, we must have $U(B|b) - U(R|b) > 0$ by condition (18). Thus, every voter must vote sincerely (conditional on voting) on obtaining signal b. Similarly, if some voter votes for R with a positive probability on obtaining signal b, everyone must vote sincerely on obtaining signal r. ■

Remark 4 *If some voters with signal s vote insincerely in equilibrium, it must be the case that those voters mix between R and B while those with signal $s' \neq s$ vote sincerely conditional on voting.*

Proof. Suppose c_A^s is the solution to equation

$$U(A|s) - U(\phi|s) = c, \; A \in \{R, B\} \text{ and } s \in \{r, b\}$$

From remark 2 we know that we cannot have c_B^s and c_R^s both weakly greater than $\bar{c}$. Now, suppose that $c_A^s > c_{A'}^s$ for $A \neq A'$. This implies that on obtaining signal s, the voters will never vote for A'. To see that, notice

$$c_A^s > c_{A'}^s \Leftrightarrow [U(A|s) - U(\phi|s)] > [U(A'|s) - U(\phi|s)] \Leftrightarrow U(A|s) > U(A'|s)$$

In particular, all type-s voters with $c < c_A^s$ will vote A and all type-s voters with $c > c_A^s$ will abstain.

Suppose now that some voters of type-r vote insincerely in equilibrium. It must be then that $c_R^r \leq c_B^r$. Moreover, on obtaining signal b, all voters must either abstain or vote B (by remark 3). If $c_R^r < c_B^r$, voters either vote B or abstain (and never vote for R), which cannot be an equilibrium by remark 2. So, if voters of some type s, vote insincerely in equilibrium, it must be the case that $c_R^s = c_B^s$, and the other type votes sincerely. A similar proof holds for the case where some voters of type-b vote insincerely in equilibrium. ■

By remarks 1 through 4, the only possible equilibrium configuration other than sincere voting can be (1) some type r voters mix between R and B if they vote while the type-b voters vote B (if they do not abstain), and (2) some type b voters mix between R and B if they vote while the type-r voters vote R (if they do not abstain). We next show that for our parameter values, there does not exist any equilibrium with either configuration.

First, consider the case with $c_R^r = c_B^r = c^r$. Since $c_R^r = c_B^r$, we must have $U(R|r) = U(B|r)$. Suppose that type-r voters with cost $c < c^r$ vote for B with probability $v > 0$.[22] It follows that $c_R^b < c_B^b = c^b$ and that $c^b > c^r$. All type-b voters with $c > c^b$ abstain while those with $c < c^b$ vote sincerely. Now, we have three equations in three unknowns p_r, p_b, v. Notice that we have $p_b > p_r$.

$$\begin{aligned} q_r \Pr[Piv_R|\rho] - (1 - q_r)\Pr[Piv_R|\beta] &= 2c^r && (19) \\ (1 - q_r)\Pr[Piv_B|\beta] - q_r \Pr[Piv_B|\rho] &= 2c^r && (20) \\ q_b \Pr[Piv_B|\beta] - (1 - q_b)\Pr[Piv_B|\rho] &= 2c^b && (21) \end{aligned}$$

[22] We could actually work with the function $v(c, r)$ that would denote the probability of voting B for voters with signal r and cost $c \leq c^r$, but in equilibrium, we identify only the quantity $\int_0^{c^r} v(c, r) dF(c)$ rather than the function $v(c, r)$. Hence, instead of any function $v(c, r)$ it is enough to consider $v = \frac{1}{F(c^*)} \int_0^{c^r} v(c, r) dF(c)$ for all $c \leq c^r$.

where, writing $F(c^s) = p_s$, we have

$$\begin{aligned} B(\beta) &= x_\beta p_b + (1 - x_\beta) p_r v \\ R(\beta) &= (1 - x_\beta) p_r (1 - v) \\ R(\rho) &= x_\rho p_r (1 - v) \\ B(\rho) &= x_\rho p_r v + (1 - x_\rho) p_b \end{aligned}$$

In searching for a solution to the system (19)-(21), we used our parameterization of the model where $M = 1$, $x_\rho = 0.9$, $x_\beta = 0.6$, so that $q_r = \frac{9}{13}$, $q_b = \frac{6}{7}$ and F is uniform over 0 to 0.1 so that $F(c^s) = 10c^s = p_s$. A numerical search for solutions to the system (19)-(21) (using Mathematica) yields no solution for which $1 \geq p_b > p_r > 0$ and $v \in (0, 1)$.

Similarly, for the case where type-b voters vote insincerely, mixing between voting for R and voting for B while type-r voters vote for R, there are three equations with three unknowns p_r, p_b and v where v is the probability with which type-b voters vote for R (for the types that do vote) and $p_r > p_b$:

$$(1 - q_b) \Pr[Piv_R|\rho] - q_b \Pr[Piv_R|\beta] = 2c^b \tag{22}$$

$$q_b \Pr[Piv_B|\beta] - (1 - q_b) \Pr[Piv_B|\rho] = 2c^b \tag{23}$$

$$q_r \Pr[Piv_R|\rho] - (1 - q_r) \Pr[Piv_R|\beta] = 2c^r \tag{24}$$

where, writing $F(c^s) = p_s$, we have

$$\begin{aligned} B(\beta) &= x_\beta p_b (1 - v) \\ R(\beta) &= x_\beta p_b v + (1 - x_\beta) p_r \\ R(\rho) &= x_\rho p_r + (1 - x_\rho) p_b v \\ B(\rho) &= (1 - x_\rho) p_b (1 - v) \end{aligned}$$

Again, using our parameterization of the model, a numerical search for solutions to (22)-(24) (using Mathematica) yields no solution for which $1 \geq p_r > p_b > 0$ and $v \in (0, 1)$. It therefore follows that the only equilibrium configuration is one with sincere voting, and assuming sincere voting, there is a unique cut-off value of cost for each signal type such that the voters with a given signal participate in voting only if their realized voting costs are below the cut-off value (we can again verify this for our parameter values using Mathematica). That is, under the costly and voluntary mechanism the sincere voting equilibrium we identified for our parameter values is *unique* in the class of symmetric equilibrium possibilities.

Costless and voluntary voting

In this section, we show that there are exactly four symmetric equilibria with costless voting given our parameterization of the model. One of these equilibria involves a positive probability of abstention and sincere voting. This is the equilibrium that we have identified in the main body of the paper.

There are three other symmetric equilibria when voting is voluntary but costless. These three equilibria involve no abstention (full voluntary participation) and each such equilibrium exactly replicates one of the three symmetric equilibria under the compulsory voting mechanism in terms of voter behavior. Therefore, for each equilibrium under the compulsory voting mechanism, there is an equilibrium under the voluntary and costless voting mechanism with exactly same voting behavior.

In what follows, we study different combinations of symmetric voting strategies and see which of these gives rise to an equilibrium configuration. To do so, we first define strategies. In the costless voting case, a symmetric strategy is a function $v : \{r, b\} \to \Delta\{R, B, \phi\}$. Denote by p_s^A the probability of taking action A on obtaining signal s, where $A \in \{R, B, \phi\}$ and $s \in \{r, b\}$.

We first make a remark that will be very useful for deriving our results.

Remark 5 *Suppose there is a positive probability of abstention by at least one type $s \in \{b, r\}$. If, in a symmetric equilibrium with costless voting, a voter votes insincerely with a positive probability on obtaining signal s, then voters with signal $s' \neq s$ will (i) vote sincerely and (ii) never abstain.*

Proof. The positive probability of abstention ensures that the event that no one votes for either alternative has a positive probability. Hence, pivot probabilities are well defined. First, note that in the costless case, expressions (12), (13), (14) and (15) all hold with $c = 0$. Hence, remark 3 goes through, which proves part (i), i.e., if types with signal s vote insincerely with a positive probability, they will vote sincerely conditional on voting. Now, suppose that type r votes insincerely with a positive probability. Then, subtracting (13) from (15) we get

$$\begin{aligned} & 2\left[U(B|b) - U(\phi|b)\right] - 2\left[U(B|r) - U(\phi|r)\right] \\ = \quad & (q_b + q_r - 1)(\Pr[Piv_B|\beta] + \Pr[Piv_B|\rho]) > 0 \end{aligned}$$

Therefore, we must have $U(B|b) - U(\phi|b) > U(B|r) - U(\phi|r)$. Since type r votes B with a positive probability, we must have $U(B|r) - U(\phi|r) > 0$. Therefore, we must have $U(B|b) > U(\phi|b)$, i.e., for type b, sincere voting dominates abstention. The case where type b votes insincerely can be proved exactly the same way. ■

First, consider the case where only one action is played in equilibrium. If all voters abstain then the best response for any individual voter is to vote sincerely. Hence everyone abstaining cannot be an equilibrium.

Next, consider the case where all voters vote for R with probability 1 irrespective of their signal. This is an equilibrium, but everyone votes uninformatively. Similarly, all voters voting for B irrespective of their signals is also an equilibrium. Thus, the two *uninformative* equilibria identified in the compulsory voting case are also present under the voluntary and costless voting mechanism.

Second, consider equilibrium strategies where only two actions are played.

As in the first subcase, suppose that only R and B played. Clearly, the only candidate for equilibrium is the case where $p_r^R = 0.844$, $p_r^B = 0.156$, $p_b^B = 0$ and $p_b^R = 1$. By the fact that this is an equilibrium under the compulsory voting mechanism, we know that the pivot probabilities are such that a voter with signal r is indifferent between voting for R or B and a voter with signal b strictly prefers to vote for B. In other words, we know that the posterior belief in favor of state β on obtaining signal s is given by:

$$\begin{aligned} \Pr(\beta|piv, r) &= \frac{1}{2} \\ \Pr(\beta|piv, b) &> \frac{1}{2} \end{aligned}$$

Given these beliefs, deviating and abstaining does not make either type strictly better off. Therefore, the voting probabilities dictated by the mixed strategy equilibrium under the compulsory voting mechanism continue to constitute a Nash equilibrium profile under the voluntary and costless voting mechanism.

As in the second subcase, suppose that only R and ϕ are played in equilibrium. Then, by remark 3, it must be the case that those with signal b mix, and those with signal r vote sincerely. Suppose $p_b^R = p$ and $p_b^\phi = 1 - p$. We then have that $p_r^R = 1$. For this to be an equilibrium, there is just one pivotal event, which is the case where all other voters abstain. (The case where there is just one vote for R does not qualify as a pivotal event for deciding between voting for R or abstaining)

The probability that a randomly chosen voter votes for R in state ω is given by

$$\begin{aligned} R(\rho) &= x_\rho + (1 - x_\rho)p \\ R(\beta) &= x_\beta p + 1 - x_\beta \end{aligned}$$

The pivotal probabilities in each state are

$$\begin{aligned} \Pr(piv|\rho) &= [1 - R(\rho)]^{N-1} \\ \Pr(piv|\beta) &= [1 - R(\beta)]^{N-1} \end{aligned}$$

For type b to prefer to vote for R over B, we must have $\Pr(\rho|piv) > \Pr(\beta|piv)$, i.e., $\Pr(piv|\rho) > \Pr(piv|\beta)$, i.e., $R(\beta) > R(\rho)$. Now,

$$\begin{aligned} x_\beta p + 1 - x_\beta &> x_\rho + (1 - x_\rho)p \\ &\Rightarrow p(x_\beta + x_\rho - 1) > x_\beta + x_\rho - 1, \end{aligned}$$

which is a contradiction, since $p < 1$ and $x_\beta + x_\rho > 1$. By the same logic, we will not have an equilibrium in which R is not played.

Finally, consider the case where all three actions R, B and ϕ are played in equilibrium. There are two possible subcases - one where everyone votes sincerely conditional on voting and another where some type votes insincerely. In the case of sincere voting, we have shown that there is a unique equilibrium. Thus, consider the case where some type s votes insincerely with positive probability. By remark 3, it must be the case that the other type votes sincerely and does not abstain. For all three actions to be played in equilibrium, it must be the case that the type voting insincerely plays all three actions with strictly positive probabilities.

Suppose that type b mixes between all three actions and type r votes sincerely. In that case, the indifference between R and ϕ for type b gives

$$(1-q_b)\Pr[Piv_R|\rho] = q_b \Pr[Piv_R|\beta] \tag{25}$$

The indifference between B and R for type b gives

$$q_b\left(\Pr[Piv_B|\beta] + \Pr[Piv_R|\beta]\right) = (1-q_b)\left(\Pr[Piv_B|\rho] + \Pr[Piv_R|\rho]\right)$$

Using (25), we get

$$q_b \Pr[Piv_B|\beta] = (1-q_b)\Pr[Piv_B|\rho]$$

Therefore, in this case we have two equations

$$\frac{\Pr[Piv_R|\rho]}{\Pr[Piv_R|\beta]} = \frac{\Pr[Piv_B|\rho]}{\Pr[Piv_B|\beta]} = \frac{q_b}{1-q_b} \tag{26}$$

in two unknowns $p_b^B > 0$ and $p_b^R > 0$, satisfying $p_b^B + p_b^R < 1$. Moreover, we have the voting probabilities for the two alternatives in each state given as functions of p_b^B and p_b^R as follows

$$\begin{aligned}
B(\beta) &= x_\beta p_b^B \\
R(\beta) &= x_\beta p_b^R + (1-x_\beta) \\
R(\rho) &= x_\rho + (1-x_\rho)p_b^R \\
B(\rho) &= (1-x_\rho)p_b^B
\end{aligned}$$

In searching for a solution to the equations (26) we used our parameterization of the model where $M=1$, $x_\rho = 0.9$ and $x_\beta = 0.6$. A numerical search for the solution to the equations (26) (using Mathematica) yields no solution for which $p_b^B > 0$ and $p_b^R > 0$ and $p_b^B + p_b^R < 1$.

Similarly, if type r mixes between all three actions and type b voted sincerely and with certainty, then we would have

$$\frac{\Pr[Piv_R|\rho]}{\Pr[Piv_R|\beta]} = \frac{\Pr[Piv_B|\rho]}{\Pr[Piv_B|\beta]} = \frac{q_r}{1-q_r}. \tag{27}$$

Moreover, the statewise probability that a random voter votes for a particular alternative is given, as functions of p_r^B and p_r^R, by

$$\begin{aligned}
B(\beta) &= x_\beta + (1-x_\beta)p_r^B \\
R(\beta) &= (1-x_\beta)p_r^R \\
R(\rho) &= x_\rho p_r^R \\
B(\rho) &= x_\rho p_r^B + (1-x_\rho)
\end{aligned}$$

A search for a solution to the equations (27) under our parameterization of the model (again using Mathematica) yields no solution for which $p_r^B > 0$ and $p_r^R > 0$ and $p_r^B + p_r^R < 1$. This establishes the fact that there is no symmetric equilibrium with insincere voting where all three actions are played in equilibrium.

Appendix C: Experimental Instructions

The following are the experimental instructions for the voluntary and costly voting (VC) treatment. The instructions for the other two treatments are similar, with the omission of the voting cost part for the voluntary and costless treatment and the further omission of the participation decision part for the compulsory treatment.

Overview

Welcome to this experiment in the economics of decision-making. Funding for this experiment has been provided by the University of Pittsburgh. We ask that you not talk with one another for the duration of the experiment.

For your participation in today's session you will be paid in cash, at the end of the experiment. Different participants may earn different amounts. The amount you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. Thus it is important that you listen carefully and fully understand the instructions before we begin. There will be a short comprehension quiz following the reading of these instructions which you will all need to complete before we can begin the experimental session.

The experiment will make use of the computer workstations, and all interaction among you will take place through these computers. You will interact anonymously with one another and your data records will be stored only by your ID number; your name or the names of other participants will not be revealed in the session today or in any write-up of the findings from this experiment.

Today's session will involve 18 subjects and 20 rounds of a decision-making task. In each round you will view some information and make a decision. Your decision together with the decisions of others determine the amount of points you earn each round. Your dollar earnings are determined by multiplying your total points from all 20 rounds by a conversion rate. In this experiment, each point is worth 1 cent, so 100 points = $1.00. Following completion of the 20th round, you will be paid your total dollar earnings plus a show-up fee of $5.00. Everyone will be paid in private, and you are under no obligation to tell others how much you earned.

Specific details

At the start of each and every round, you will be randomly assigned to one of two groups, the R (Red) group or the B (Blue) group. Each group will consist of 9 members. All assignments of the 18 subjects to the two groups of size 9 at the start of each round are equally likely. Neither you nor any other member of your group or the other group will be informed of whether they are assigned to the R or to the B group until the end of the round.

Imagine that there are two "jars", which we call the red jar and the blue jar. Each jar contains 10 balls; the red jar contains 9 red balls and 1 blue ball while the blue jar contains 6 blue balls and 4 red balls. The red jar is always assigned to the R (Red) group and the blue jar is always assigned to the B (Blue) group. However, recall that you do not know which group (Red or Blue) you have been assigned to; that is, you don't know the true color of your group's jar. Furthermore, your assignment to the R or B group is randomly determined at the start of every round.

To help you determine which jar is assigned to your group, each member of your group will be allowed to independently select one ball, at random, from your group's jar. You do this on the first stage screen on your computer by clicking on your choice of the ball to examine: the balls are numbered 1 to 10. Once you click on the number of a ball, you will be privately informed of the color of that ball. You will not be told the color of the balls drawn by the other members of your group, nor will they learn the color of the ball you chose, and it is possible for members of your group to draw the same ball as you do or any of the other 9 balls as well. Each member in your group selects one ball on their own, and only sees the color of their own ball. However, all members of your group (Red or Blue) will choose a ball from the *same* jar that contains the same number of red and blue balls. Recall again that if you are choosing a ball from the red jar, that jar contains 9 red balls and 1 blue ball while if you are choosing a ball from the blue jar, that jar contains 6 blue balls and 4 red balls.

After each individual has drawn a ball and observed the color of their chosen ball, each individual is asked to decide (1) whether they want to join in the group decision process and make a choice between "RED" or "BLUE" or (2) whether they do not want to join in the group decision process, corresponding to the option "NO CHOICE".

Your group's decision depends on both individual decisions.

Your 9-member group's decision will be the color chosen by the majority of those who decided to join the group decision process. Suppose for example that 6 of your group members decided to join the group decision process (i.e., 3 members selected NO CHOICE). If 4 or more of the 6 who decided to make a choice choose RED, then the group decision is RED by the majority rule. Similarly, the group's decision is BLUE if a majority of those who decided to make a choice chose BLUE. That is, your group's decision will be whichever color receives more individual choices among the members of your group who decided to make a choice. In the case of a tie, where each color receives the same number of individual choices by members of your group (for example, 3 members chose RED and the other 3 chose BLUE), the group decision is INDETERMINATE. If the number of those who decided to make a choice is odd (for example, 5 members decided to make a choice while 4 members selected NO CHOICE), then your group's decision can be either CORRECT or INCORRECT, as discussed below, but it cannot be INDETERMINATE.

If you decided not to join the group decision process, that is, you selected NO CHOICE, then you will get additional points, which we refer to as the NC BONUS. The amount of your NC BONUS is assigned randomly by the computer. In any given round, your NC bonus points for the round will be a number drawn randomly from the set $\{0, 1, 2, ..10\}$, with all numbers in that set being equally likely. Your NC BONUS in each round does not depend on your prior round NC BONUS or your decisions in any previous rounds, or on the NC BONUSes or decisions of other members. While you are told your own NC BONUS before you make any decision, you are never told the NC BONUSes of other participants. You only know that each of the other members has an NC BONUS that is some number between 0 and 10, inclusive.

The points you earn in any given round are determined as follows. Suppose you decided to join the group decision process and you then chose RED or BLUE. If your group's decision (via majority rule) is the same as the true color of the jar that is assigned to your group, then the group decision is

CORRECT, and you will earn 100 points from the group's correct decision. If your group's decision is different from the true color of your group's jar, then the group decision is INCORRECT, and you will earn 0 points from the group's incorrect decision. If the group decision is INDETERMINATE, then you will earn 50 points from the group's indeterminate decision. Suppose instead that you selected NO CHOICE. In that case, if your group's decision is the same as the true color of the jar that is assigned to your group, then the group decision is CORRECT, and you will earn 100 points plus the NC BONUS assigned to you for that round. If your group's decision is different from the true color of your group's jar, then the group decision is INCORRECT, and you will earn the NC BONUS. If your group's decision is INDETERMINATE, then you will earn 50 points plus the NC BONUS. In other words, if you decide not to join the group decision-you select NO CHOICE-then your earnings will increase by the amount of the NC BONUS that is assigned to you in each round. Notice that both decisions, your decision to make a choice or not (NO CHOICE) and, if you decide to make a choice, your decision between RED or BLUE can affect whether the overall decision of your group is CORRECT, INCORRECT or INDETERMINATE.

If the final (20th) round has not yet been played, then at the start of each new round you and all of the other participants will be randomly assigned to a new 9-person group, R or B. You will not know which group, R or B you have been assigned to but you will have the opportunity to draw a new ball from your group's jar, to decide whether to make a choice or not (NO CHOICE) and if you have decided to make a choice to choose between RED or BLUE. In other words, the group you are in will change from round to round.

Following completion of the final round, your points earned from all rounds played will be converted into cash at the rate of 1 point = 1 cent. You will be paid these total earnings together with your $5 show-up payment in cash and in private.

Questions?

Now is the time for questions. If you have a question about any aspect of these instructions, please raise your hand and an experimenter will answer your question in private.

Quiz

Before we start today's experiment we ask you to answer the following quiz questions that are intended to check your comprehension of the instructions. The numbers in these quiz questions are illustrative; the actual numbers in the experiment may be quite different. Before starting the experiment we will review each participant's answers. If there are any incorrect answers we will go over the relevant part of the instructions again.

1. I will be assigned to the same group, R or B in every round. Circle one: True False.
2. I will get a different NC Bonus in every round. Circle one: True False.
3. If I decide to make a choice I give up the NC Bonus. Circle one: True False.
4. The red jar contains ______ red balls and ______ blue balls. The blue jar contains ______ red balls and _______ blue balls.

5. Consider the following scenario in a round. 5 members of your group decide to make a choice and 3 of these members choose RED.

    a. How many members of your group made NO CHOICE? _________
    b. What is your group's decision? _________
    c. If the jar of balls your group was drawing from was in fact the RED jar, how many points are earned by those who made a choice? _________
    d. If the jar of balls your group was drawing from was in fact the BLUE jar, how many points are earned by those who made a choice? _________

6. Consider the following scenario in a round. 4 members of your group decide to make a choice and 2 of these members choose RED.

a. How many members of your group made NO CHOICE? _________

b. What is your group's decision? _________

c. If the jar of balls your group was drawing from was in fact the RED jar, how many points are earned by those who made a choice? _________

d. If the jar of balls your group was drawing from was in fact the BLUE jar, how many points are earned by those who made a choice? _________

Appendix D: Models of Bounded Rationality

In section 6 of the text we presented strong evidence in support of the comparative statics predictions of the theory but we also found some differences between the equilibrium point predictions and the experimental data, for example, over-participation relative to equilibrium predictions under the voluntary voting mechanisms. In this appendix we consider whether two models of boundedly rational behavior might help us to better account for these anomalous findings.

8.0.1 Equilibrium Plus Noise

Perhaps the simplest model of "noise" in the data is the so-called *equilibrium-plus-noise model.*[23] In this approach, the predicted choice probability, $p(\eta)$, (of either sincere voting or of participation in voting) is a weighted average of the equilibrium prediction, p, and a purely random choice probability of $\frac{1}{2}$:

$$p(\eta) = \eta p + (1-\eta)\frac{1}{2},$$

where $\eta \in [0,1]$ and $p \in \{v_r, v_b, p_r, p_b\}$ with v_s and p_s, respectively, representing the equilibrium probability of sincere voting (given participation, in the voluntary treatments) and the probability of participation in voting by signal type $s \in \{r, b\}$. Here, η is a simple measure of the "closeness" of the data to equilibrium predictions; $\eta = 0$ corresponds to random choices whereas $\eta = 1$ corresponds to equilibrium play. We further impose the restriction that the weight η assigned to the choice probabilities is the same for both signal types and for now we also restrict η to be the same for both voting and participation decisions in any given treatment (however, we allow η to vary from treatment to treatment).

To construct a likelihood function, let ω_s denote the total number of signal type-s subjects, let τ_s denote the total number of type-s subjects who participate in voting and let σ_s denote the total number of type-s subjects who vote sincerely (among all type-s *subjects* in the compulsory treatment and among all type-s *participants* in the voluntary treatments). The likelihood function is then proportional to

$$\mathcal{L}(\eta) = v_r(\eta)^{\sigma_r}(1-v_r(\eta))^{\omega_r-\sigma_r} v_b(\eta)^{\sigma_b}(1-v_b(\eta))^{\omega_b-\sigma_b},$$

in the case of the compulsory voting (C) treatment, and to

$$\begin{aligned}\mathcal{L}(\eta) = \quad & v_r(\eta)^{\sigma_r}(1-v_r(\eta))^{\tau_r-\sigma_r} v_b(\eta)^{\sigma_b}(1-v_b(\eta))^{\tau_b-\sigma_b} \\ & \times\, p_r(\eta)^{\tau_r}(1-p_r(\eta))^{\omega_r-\tau_r} p_b(\eta)^{\tau_b}(1-p_b(\eta))^{\omega_b-\tau_b},\end{aligned}$$

in case of the voluntary (VN or VC) treatments. Our restriction on η requires us to use pooled data from all sessions of a given treatment in maximizing the above likelihood functions.

Table 15 reports results from a maximum likelihood (ML) estimation of the *equilibrium-plus-noise model* using data from all 20 rounds or from the first or last 10 rounds of all sessions of a given treatment. The observed frequencies of sincere voting and participation (from the experimental

[23] See, e.g., Blume, Duffy and Franco (2009).

Treatment	$\hat{v_r}$	$v_r(\hat{\eta})$	$\hat{v_b}$	$v_b(\hat{\eta})$	$\hat{p_r}$	$p_r(\hat{\eta})$	$\hat{p_b}$	$p_b(\hat{\eta})$	$\hat{\eta}$ [95% CI]	LR Stat
C	0.896	0.837	0.983	0.989	n/a	n/a	n/a	n/a	0.979 [0.961, 0.990]	1236.74
First 10 rounds	0.902	0.835	0.978	0.987	n/a	n/a	n/a	n/a	0.974 [0.946, 0.991]	616.13
Last 10 rounds	0.890	0.838	0.987	0.991	n/a	n/a	n/a	n/a	0.983 [0.958, 0.996]	620.99
Nash	0.844		1.00		n/a	n/a	n/a	n/a		
VN (Sincere)	0.951	0.956	0.970	0.956	0.713	0.536	0.944	0.956	0.912 [0.891, 0.930]	1723.73
First 10 rounds	0.951	0.955	0.982	0.955	0.726	0.536	0.931	0.955	0.910 [0.879, 0.935]	855.11
Last 10 rounds	0.951	0.956	0.959	0.956	0.700	0.536	0.957	0.956	0.913 [0.832, 0.963]	868.64
Nash	1.00		1.00		0.540		1.00			
VN (Insincere)	0.951	0.753	0.970	0.867	0.713	0.867	0.944	0.867	0.734 [0.706, 0.761]	1414.56
First 10 rounds	0.951	0.755	0.982	0.871	0.726	0.871	0.931	0.871	0.741 [0.701, 0.778]	721.85
Last 10 rounds	0.951	0.750	0.959	0.864	0.700	0.864	0.957	0.864	0.727 [0.686, 0.765]	692.97
Nash	0.844		1.00		1.00		1.00			
VC	0.956	0.944	0.962	0.944	0.408	0.296	0.699	0.544	0.888 [0.848, 0.921]	764.96
First 10 rounds	0.946	0.928	0.951	0.928	0.440	0.303	0.723	0.542	0.856 [0.795, 0.906]	364.91
Last 10 rounds	0.968	0.961	0.974	0.961	0.379	0.288	0.672	0.546	0.922 [0.869, 0.959]	403.17
Nash	1.00		1.00		0.270		0.550			

Table 15: Equilibrium-Plus-Noise Model: Maximum Likelihood Estimates

data) are denoted by $\hat{v_s}$ and $\hat{p_s}$ and the corresponding estimates based on the equilibrium-plus-noise model are denoted by $v_s(\hat{\eta})$ and $p_s(\hat{\eta})$ and $\hat{\eta}$; for the latter we provide a 95% confidence interval. The table also shows the results of likelihood ratio tests that compare the likelihood function for the unrestricted equilibrium-plus-noise model (with estimates $\hat{\eta}$) with those for a restricted version where $\eta = 0$ implying purely random choices. We use the same numbers of observations (ω_s, τ_s and σ_s) when evaluating the likelihood functions of both the restricted and unrestricted models. The last column of Table 15 in particular reports the likelihood ratio (LR) test statistics (LR Stat $\equiv -2\ ln\ l$, where l is the ratio of the restricted to the unrestricted likelihood functions) that can be evaluated under the null hypothesis (H_0) of no difference between the restricted and the unrestricted models. The LR test statistic follows a χ^2 distribution with degrees of freedom equal to the number of restrictions, in this case, 1. Finally, in the case of the VN treatment only, we assess the fit of the equilibrium-plus-noise model using two different symmetric equilibrium probability vectors: one corresponding to the sincere voting equilibrium vector (labeled 'Sincere') and the other corresponding to the insincere voting equilibrium (labeled 'Insincere').

We observe that our data are very close to the Nash equilibrium point predictions for all treatments, as indicated by the high estimated values for $\hat{\eta}$. We also observe that the data from the compulsory voting treatment are significantly closer to equilibrium predictions than are the data from the two voluntary voting treatments. This difference is largely due to the over-participation we observed in the voluntary treatments as reported in the previous section. Since we measure the closeness of both the voting and participation decisions to equilibrium predictions using a single estimate, $\hat{\eta}$, for each treatment (recall our restriction on η), a consequence is that we obtain lower values for $\hat{\eta}$ for the voluntary treatments. We do find some improvement in the estimate of $\hat{\eta}$ for all voting mechanisms as we move from the first to the last 10 rounds (with the exception of $\hat{\eta}$ for the VN insincere equilibrium specification) meaning that subjects' behavior gets closer to the equilibrium predictions with experience.

Given the closeness of our data to the equilibrium predictions, it is perhaps not so surprising that we obtain the high likelihood ratio (LR) test statistics reported in Table 15. By construction, these statistics (and the corresponding p-values) measure the extent to which the equilibrium-plus-noise model outperforms a purely random choice model. Since all reported LR statistics are well above the critical value for the χ^2 statistic that corresponds to a p-value= 0.001 (which is 10.828 with d.f.=1), we can safely reject the null of random decision making in favor of the unrestricted model where subjects are close to playing the equilibrium predictions at the 0.1% level (or lower).

Regarding the issue of equilibrium selection under the VN mechanism, we can use our simple equilibrium-plus-noise model to assess which symmetric equilibrium provides a better characterization of the play of subjects in the VN treatment. As Table 15 reveals, when we use the symmetric *sincere* voting equilibrium probability vector as the benchmark, we obtain a much higher value for $\hat{\eta}$, approximately .91, than we do if we use the symmetric *insincere* voting equilibrium probability vector as the benchmark in which case the estimate of $\hat{\eta}$ is approximately .73. Moreover, the 95 percent confidence intervals for these two estimates do not overlap. We thus conclude that, on the question of equilibrium selection, behavior in the VN sessions is better characterized by the symmetric sincere voting equilibrium than by the symmetric insincere voting equilibrium.

As a robustness check, we consider a modified equilibrium-plus-noise model where we allow for two different values for the error term, η_1 and η_2, with the first being assigned to the sincerity of voting choice and the second to the participation decision. In this case, the likelihood function is proportional to

$$\begin{aligned}\mathcal{L}(\eta_1, \eta_2) = \quad & v_r(\eta_1)^{\sigma_r}(1 - v_r(\eta_1))^{\tau_r - \sigma_r} v_b(\eta_1)^{\sigma_b}(1 - v_b(\eta_1))^{\tau_b - \sigma_b} \\ & \times\, p_r(\eta_2)^{\tau_r}(1 - p_r(\eta_2))^{\omega_r - \tau_r} p_b(\eta_2)^{\tau_b}(1 - p_b(\eta_2))^{\omega_b - \tau_b}.\end{aligned}$$

Maximum likelihood estimates from this specification are given in Table 16. [24] By comparison with the one-error version of the model as reported in Table 15, the results for the two-error version of the model in Table 16 confirm that most of the departure from equilibrium predictions comes from the participation decision, as evidenced by the generally lower estimates for η_2 as compared with η_1. We note that the VN treatment estimates for η_1 do not depend on whether the benchmark equilibrium is the sincere or the insincere voting equilibrium, however estimates of η_2 are significantly closer to 1 when the benchmark equilibrium is the sincere and not the insincere equilibrium, providing further support for the selection of the sincere equilibrium as the more plausible candidate for characterizing the experimental data in the VN treatment.

8.0.2 Quantal Response Equilibrium

A main drawback of the equilibrium-plus-noise model is that it does not rationally account for the possibility that subjects may be best responding to the noise they observe in the data. An

[24] Unlike in Table 15 and 17, we don't report confidence intervals for the two–error parameter estimates in Table 16 or in the table that follows, Table 18. In the case of two-error probabilities, we would have *confidence regions* for the two-dimensional (η_1, η_2) or (λ_1, λ_2) plane, which is often represented as an ellipsoid centered around the point of the estimated errors that includes the true parameter values with, say 95% confidence. The idea behind such a confidence region is exactly the same as a confidence interval but estimating the former is computationally more demanding and for this reason we omit such an analysis here.

Treatment	$\hat{v_r}$	$v_r(\hat{\eta}_1)$	$\hat{v_b}$	$v_b(\hat{\eta}_1)$	$\hat{p_r}$	$p_r(\hat{\eta}_2)$	$\hat{p_b}$	$p_b(\hat{\eta}_2)$	$\hat{\eta}_1$	$\hat{\eta}_2$	LR Stat
VN (Sincere)	0.951	0.959	0.970	0.959	0.713	0.536	0.944	0.950	0.918	0.899	1724.52
First 10 rounds	0.951	0.964	0.982	0.964	0.726	0.535	0.931	0.938	0.927	0.875	857.83
Last 10 rounds	0.951	0.955	0.959	0.955	0.700	0.537	0.957	0.961	0.909	0.921	868.82
Nash	1.00		1.00		0.540		1.00				
VN (Insincere)	0.951	0.833	0.970	0.984	0.713	0.794	0.944	0.794	0.967	0.587	1633.86
First 10 rounds	0.951	0.837	0.982	0.991	0.726	0.796	0.931	0.796	0.981	0.592	850.00
Last 10 rounds	0.951	0.828	0.959	0.977	0.700	0.792	0.957	0.792	0.954	0.583	792.49
Nash	0.844		1.00		1.00		1.00				
VC	0.956	0.959	0.962	0.959	0.408	0.390	0.699	0.524	0.918	0.478	808.60
First 10 rounds	0.946	0.949	0.951	0.949	0.440	0.416	0.723	0.518	0.897	0.366	395.26
Last 10 rounds	0.968	0.971	0.974	0.971	0.379	0.365	0.672	0.529	0.941	0.585	418.29
Nash	1.00		1.00		0.270		0.550				

Table 16: Equilibrium-Plus-Noise Model with Two Error Probabilities: Maximum Likelihood Estimates

equilibrium concept that formalizes this idea is the *quantal response equilibrium* (QRE), see, e.g., McKelvey and Palfrey (1995) and Goeree, Holt and Palfrey (2005), which we now apply to our experimental data. In particular, we consider the *logit quantal response equilibrium model* and assume that our subjects make decisions according to a stochastic, logistic choice rule.

In the quantal response equilibrium model, we calculate the choice probabilities as (quantal response) functions of the expected payoffs. Given the slope λ of the logistic quantal response function, the voting strategy of a subject can be written as:

$$v_r(\lambda) = \frac{1}{1+\exp[-\lambda\{U(R|r)-U(B|r)\}]}, \tag{28}$$

$$v_b(\lambda) = \frac{1}{1+\exp[-\lambda\{U(B|b)-U(R|b)\}]}. \tag{29}$$

where v_s is again defined as the probability of voting sincerely, given signal $s \in \{r, b\}$. Here, λ is understood to measure the "degree of rationality"; $\lambda = 0$ corresponds to random behavior whereas $\lambda = \infty$ corresponds to equilibrium behavior (perfect rationality). We can also specify participation strategies in a similar way. Under the voluntary and costless (VN) treatment, we have:

$$p_r(\lambda) = \frac{1}{1+\exp[-\lambda\{v_r(\lambda)(U(R|r)-U(\phi|r))+(1-v_r(\lambda))(U(B|r)-U(\phi|r))\}]}, \tag{30}$$

$$p_b(\lambda) = \frac{1}{1+\exp[-\lambda\{v_b(\lambda)(U(B|b)-U(\phi|b))+(1-v_b(\lambda))(U(R|b)-U(\phi|b))\}]}, \tag{31}$$

and under the voluntary and costly (VC) treatment we have,

$$p_r(\lambda) = \frac{1}{1+\exp[\lambda\{\frac{p_r(\lambda)}{10}-v_r(\lambda)(U(R|r)-U(\phi|r))-(1-v_r(\lambda))(U(B|r)-U(\phi|r))\}]}, \tag{32}$$

$$p_b(\lambda) = \frac{1}{1+\exp[\lambda\{\frac{p_b(\lambda)}{10}-v_b(\lambda)(U(B|b)-U(\phi|b))-(1-v_b(\lambda))(U(R|b)-U(\phi|b))\}]}, \tag{33}$$

where p_s is, as before, the rate of participation in voting, given signal $s \in \{r, b\}$. We treat the model parameter λ as a constant to be estimated. For the compulsory (C) treatment, we solve for $(v_r(\lambda), v_b(\lambda))$, the system of equations (1)-(2). For the voluntary treatments, we solve for $(v_r(\lambda), v_b(\lambda), p_r(\lambda), p_b(\lambda))$, the system of equations (1)-(4) for the VN mechanism and the system of equations (1)-(2) and (5)-(6) for the VC mechanism. We initially restrict λ to be the same for both signal types and for both voting and participation strategies in any given treatment (however, we allow λ to vary from treatment to treatment).

To construct the likelihood function, let ω_s denote the total number of type-s subjects, let τ_s denote the total number of type-s subjects who participate in voting and let σ_s denote the total number of type-s subjects who vote sincerely (among all type-s *subjects* in the compulsory treatment and among all type-s *participants* in the voluntary treatments). The likelihood function is then proportional to

$$\mathcal{L}(\lambda) = v_r(\lambda)^{\sigma_r}(1-v_r(\lambda))^{\omega_r-\sigma_r}v_b(\lambda)^{\sigma_b}(1-v_b(\lambda))^{\omega_b-\sigma_b}$$

in case of the compulsory (C) treatment, and to

$$\begin{aligned}\mathcal{L}(\lambda) = \quad & v_r(\lambda)^{\sigma_r}(1-v_r(\lambda))^{\tau_r-\sigma_r}v_b(\lambda)^{\sigma_b}(1-v_b(\lambda))^{\tau_b-\sigma_b} \\ & \times\, p_r(\lambda)^{\tau_r}(1-p_r(\lambda))^{\omega_r-\tau_r}p_b(\lambda)^{\tau_b}(1-p_b(\lambda))^{\omega_b-\tau_b}\end{aligned}$$

in case of the voluntary (VN or VC) treatments. For robustness, as in the equilibrium-plus-noise model for the two voluntary voting treatments we also study the case where there are two different estimated values for λ, λ_1 for the sincerity of voting decision and λ_2 for the participation decision and in that case we estimate the likelihood function $\mathcal{L}(\lambda_1, \lambda_2)$ In all instances, we use pooled data from all sessions of a given treatment in maximizing the above likelihood function. The results of our maximum likelihood estimation are shown in Tables 17 for the one-lambda model and Table 18 for the two-lambda model.[25]

Treatment	$\hat{v_r}$	$v_r(\hat{\lambda})$	$\hat{v_b}$	$v_b(\hat{\lambda})$	$\hat{p_r}$	$p_r(\hat{\lambda})$	$\hat{p_b}$	$p_b(\hat{\lambda})$	$\hat{\lambda}$ [95% CI]	LR Stat
C	0.896	0.797	0.983	0.994	n/a	n/a	n/a	n/a	42.33 [36.28, 49.67]	1191.53
First 10 rounds	0.902	0.795	0.978	0.992	n/a	n/a	n/a	n/a	40.22 [32.69, 49.76]	590.69
Last 10 rounds	0.890	0.800	0.987	0.996	n/a	n/a	n/a	n/a	45.16 [35.80, 58.26]	601.35
Nash	0.844		1.00		n/a	n/a	n/a	n/a		
VN	0.951	0.944	0.970	0.997	0.713	0.531	0.944	0.877	48.59 [42.85, 54.88]	1652.40
First 10 rounds	0.951	0.950	0.982	0.998	0.726	0.531	0.931	0.888	51.92 [43.36, 61.67]	843.75
Last 10 rounds	0.951	0.939	0.959	0.996	0.700	0.531	0.957	0.868	45.69 [38.26, 54.16]	809.67
Nash	1.00		1.00		0.540		1.00			
VC	0.956	0.909	0.962	0.984	0.408	0.361	0.699	0.554	20.75 [18.18, 23.60]	798.82
First 10 rounds	0.946	0.888	0.951	0.976	0.440	0.371	0.723	0.550	18.37 [15.32, 21.84]	387.61
Last 10 rounds	0.968	0.934	0.974	0.992	0.379	0.348	0.672	0.558	24.39 [20.01, 29.55]	415.67
Nash	1.00		1.00		0.270		0.550			

Table 17: Quantal Response Equilibrium: Maximum Likelihood Estimates

As in the previous subsection, $\hat{v}_s$ and $\hat{p}_s$ denote the observed probabilities of sincere voting and participation while $v_s(\hat{\lambda})$ and $p_s(\hat{\lambda})$ denote the estimated probabilities. Table 17 reports the estimates $v_s(\hat{\lambda})$, $p_s(\hat{\lambda})$ and $\hat{\lambda}$ along with a 95% confidence interval for the latter and Table 18 reports the estimates $v_s(\hat{\lambda}_1)$, $p_s(\hat{\lambda}_2)$, $\hat{\lambda}_1$ and $\hat{\lambda}_2$ with a 95% confidence interval for the latter two estimates. Also shown for comparison purposes are the actual, observed frequencies $\hat{v}_s$ and $\hat{p}_s$ from all rounds as well as from the first and the last 10 rounds of all sessions of each treatment. Finally, the table also reports the results of likelihood ratio (LR) tests that compare the unrestricted model with the restricted one, with the former being the quantal response equilibrium model and the restriction in the latter model being that $\lambda = \lambda_1 = \lambda_2 = 0$ (purely random behavior). The details concerning the LR test statistics are exactly the same as in the previous subsection.

As Table 17 reveals, the estimated slope coefficients, $\hat{\lambda}$, of the quantal response function are quite high for all three treatments. In other words, subjects demonstrated a substantial degree of rationality in all three voting treatments. Similar evidence of rational voter behavior is also found in previous studies by Guarnaschelli, McKelvey and Palfrey (2000), Levine and Palfrey (2007) and Battaglini, Morton and Palfrey (2010); their estimated values for λ are also high. Notice further that the $\hat{\lambda}$ values are comparatively lower for the voluntary and costly (VC) mechanism, which intuitively makes sense as this mechanism entails the most complicated game that subjects in our

[25]Unlike Table 15 for the VN treatment we cannot use QRE estimates to compare between the two symmetric equilibrium possibilities that arise under the VN mechanism, as they involve different likelihood functions (one with participation choices and the other without participation choices) preventing us from making a fair comparison between the two types of equilibria. For this reason, we only report in Table 17 QRE estimates for the sincere voting equilibrium specification using the VN treatment data.

Treatment	$\hat{v_r}$	$v_r(\hat{\lambda}_1)$	$\hat{v_b}$	$v_b(\hat{\lambda}_1)$	$\hat{p_r}$	$p_r(\hat{\lambda}_2)$	$\hat{p_b}$	$p_b(\hat{\lambda}_2)$	$\hat{\lambda}_1$	$\hat{\lambda}_2$	LR Stat
VN	0.951	0.899	0.970	0.979	0.713	0.605	0.944	0.964	34.76	94.22	1773.96
First 10 rounds	0.951	0.908	0.982	0.983	0.726	0.596	0.931	0.958	36.99	88.81	887.52
Last 10 rounds	0.951	0.892	0.959	0.975	0.700	0.615	0.957	0.971	33.02	100.43	888.36
Nash	1.00		1.00		0.540		1.00				
VC	0.956	0.909	0.962	0.984	0.408	0.360	0.699	0.554	20.65	21.38	798.84
First 10 rounds	0.946	0.886	0.951	0.978	0.440	0.378	0.723	0.550	18.65	16.42	387.79
Last 10 rounds	0.968	0.935	0.974	0.991	0.379	0.342	0.672	0.558	23.76	27.51	415.84
Nash	1.00		1.00		0.270		0.550				

Table 18: Quantal Response Equilibrium with Two Error (λ) Parameters: Maximum Likelihood Estimates

experiment were asked to play.[26]

This finding is also consistent with the findings of the previous section, i.e., the data from the VC treatment were found to be the furthest from the equilibrium predictions according to the estimates, $\hat{\eta}$. Finally, as in the equilibrium-plus-noise model, we again observe an improvement in $\hat{\lambda}$ as we move from estimates based on the first 10 rounds of data to estimates based on the last 10 rounds of data under both the C and VC mechanisms. While $\hat{\lambda}$ decreases with experience under the VN mechanism, from $\hat{\lambda} = 51.92$ to 45.69, both estimates still indicate a high degree of rationality; indeed, these estimates are higher than the $\hat{\lambda}$ estimates for the other two treatments.

When we allow two different estimates for λ as in Table 18 we again find that the estimates of both λ values are high and significantly different from zero. We further find that allowing for two different estimates (for the sincerity of voting and the participation decision) does not matter in understanding the data from the VC treatment, where the two estimates $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are not statistically different from one another. However, for the VN treatment we find that $\hat{\lambda}_1$ is now significantly lower than $\hat{\lambda}_2$. Nevertheless, we also observe in Table 18 that the QRE estimates using the two-parameter approach of $v_s(\hat{\lambda}_1)$ and $p_s(\hat{\lambda}_2)$ are indeed closer to the actual data, $\hat{v}_s$ and $\hat{p}_s$, respectively, than are estimates found using the one-parameter approach (as reported in Table 17).

Let us focus further on the QRE predictions regarding the voting decisions and participation rates. Notice first that the QRE estimates for the frequency of sincere voting are lower for type-r players than for type-b players under both voluntary mechanisms and using either the one- and two-parameter estimation approaches. This stands in contrast to the Nash equilibrium prediction that both frequencies should be the same for both types. This reflects the pattern in our data that type-b players tend to vote "more sincerely" than type-r players in these treatments. Second, the QRE estimates of participation rates are consistent with the comparative statics prediction of the theory. As in our experimental data, the QRE predicts a higher participation rate for type-b players than for type-r players under each voluntary mechanism, and a higher participation rate under the VN mechanism than under the VC mechanism for each type. Finally, the QRE predicts under-participation in the VN mechanism and over-participation in the VC mechanism, relative to the Nash equilibrium predictions. However, our experimental data exhibit a strong tendency of over-participation in all cases except for type-b players under the VN mechanism.[27] This final observation suggests that QRE does not do a very good job of predicting the participation rates observed in our experimental data.

On the other hand, the QRE approach again achieves very high likelihood ratio (LR) statistics for the comparison between the unrestricted QRE model and the restricted model of random behavior ($\lambda = \lambda_1 = \lambda_2 = 0$) with the LR statistics reported in Tables 17-18 exceeding to a high degree the critical value of the χ^2 statistic at the 0.1% significance level, equal to 10.828 (with d.f.=1 Table 17) and equal to 13.816 (with d.f.=2 Table 18) enabling us to reject at the 0.1% level, the null of no difference between the restricted and unrestricted QRE models.

[26]Subjects in the VC treatment have to process additional information concerning their private voting cost and must condition their participation decision on that cost. Hence, one can argue that the cognitive burden is higher under the VC mechanism.

[27]Of course type-b players cannot over-participate in the VN treatment as the Nash prediction for their participation rate is one.

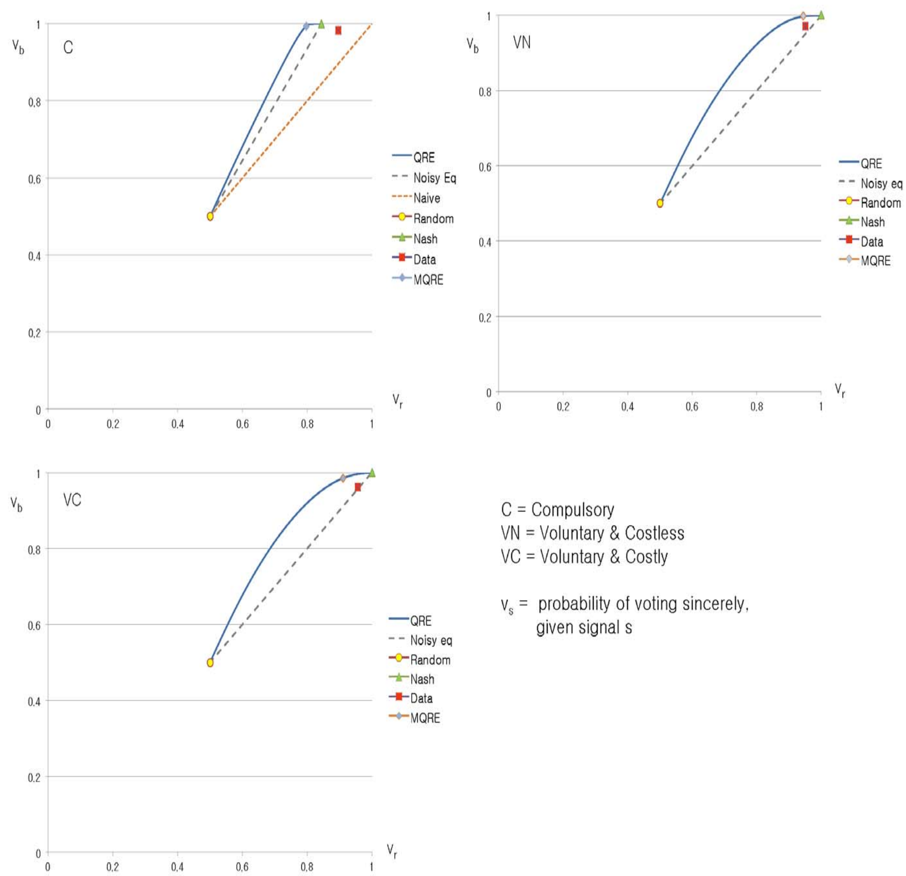

Figure 6: Data and Model Predictions Regarding the Sincere Voting Decisions, v_s, in Each Mechanism

To further investigate the relationship between our data, the equilibrium predictions and the two models of boundedly rational behavior, consider Figure 6 which illustrates the sincerity of voting decisions under all three voting mechanisms and Figure 7 which illustrates participation decisions under the two voluntary voting mechanisms. The circular dot in the middle represents random play in which the subjects mix between their two available actions (sincere/insincere voting or vote/abstain) with equal probability. The triangular dot in the upper right corner (Figure 6), or on the uppermost line or on the middle left (Figure 7) is the Nash equilibrium prediction. The straight line between these two dots corresponds to the predictions from the single error probability equilibrium-plus-noise model for various values of η. As we change the values of η from 0 to 1, we travel on the line from the point of random play toward the Nash equilibrium. Similarly, the curved line between these same two points represents the single-lambda QRE predictions for various levels of λ. As we change the value of λ from 0 to ∞, we move from random play to the Nash equilibrium point. Finally, the square dot with the cross ($\times$) represents our data and the diamond dot on the QRE curve represents the maximum likelihood estimate for the QRE prediction (labeled MQRE).

Focusing on just the sincerity of voting decisions by signal type, Figure 6 suggests that our data are pretty close to the Nash equilibrium predictions under all three voting mechanisms. This can be anticipated from the high estimated values for $\hat{\eta}$ and $\hat{\lambda}$ in Tables 15 and 17. We conclude that Nash equilibrium performs very well in making quantitative predictions of voting behavior. When we compare QRE with the equilibrium-plus-noise model, it seems that our data are somewhat closer to the predictions of the latter model.

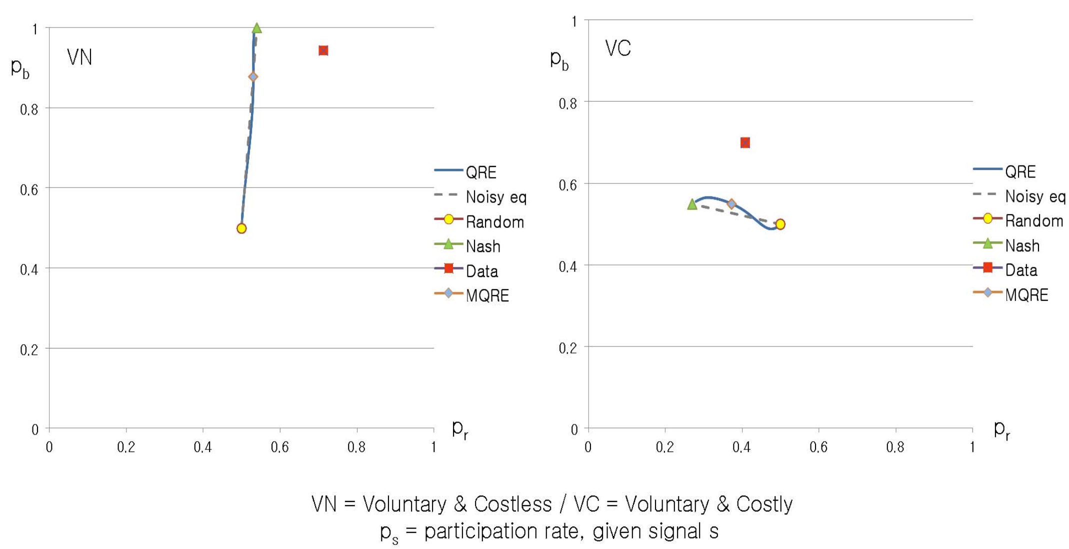

Figure 7: Data and Model Predictions Regarding Participation Decisions, p_s, in the Two Voluntary Voting Mechanisms

On the other hand, as Figure 7 reveals, our data on participation decisions exhibit deviations from Nash equilibrium point predictions. For both voluntary voting mechanisms, over-participation by one signal type was too great to be justified by using either the Nash or QRE predictions. Specifically, in the VN treatment, type-r voters participated at rates greater than possible under any QRE parameter λ while in the VC treatment it was type-b voters who over-participated relative to QRE predictions. In the VC treatment, type-r voters also participated at a rate that is much higher than the Nash equilibrium prediction. These findings suggest that neither Nash nor QRE may yield good point predictions for the participation rates observed in our voluntary voting games. Nevertheless, as emphasized earlier, we do find strong support for the comparative statics predictions of the theory both in the data and in the estimated predictions using the two models of boundedly rational behavior.