INFLATION TOLERANCE BANDS AND PRIVATE SECTOR BELIEFS

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Abstract. This paper estimates a New Keynesian model with a nonlinearity in the monetary policy rule to capture the practice of inflation targeting with target zones or tolerance bands. Private-sector agents form subjective expectations, update their beliefs over time using a perceived model of the economy, and are subject to shifts in sentiment. The model is estimated using data on realized macroeconomic variables and survey data on expectations for four inflation-targeting economies: Australia, Canada, New Zealand, and Sweden.

The results show that central banks do not treat target bands as zones of inaction. Their policy reactions when inflation falls within the band are comparable to or even exceed the reactions when inflation moves outside the band, and they always satisfy the Taylor principle. Private-sector expectations respond similarly to structural and sentiment shocks regardless of the inflation regime. In all cases, the data favor the simpler specification in which agents form expectations based on linear perceived laws of motion, without accounting for the nonlinearity induced by monetary policy.

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1. Introduction

The practice of Inflation Targeting, originally introduced by the Reserve Bank of New Zealand in 1990, has grown to become the dominant framework for the conduct of monetary policy around the world. It is now adopted almost universally by central banks in developed economies and by a growing number of those in emerging economies.

The specific implementation details of inflation targeting, however, vary across countries: some central banks simply identify a point target, which is communicated to the public as the objective for inflation; others adopt tolerance bands or target ranges. Targeting regimes with tolerance bands identify an interval within which fluctuations in inflation are tolerated; with target ranges, instead, the whole interval becomes the target of policy.

Policies based on bands and ranges explicitly recognize the uncertainty surrounding monetary policy outcomes, since inflation rates that cannot be perfectly controlled by policymakers.[1] Target zones, for example, may help central banks retain credibility when inflation deviates from the preferred numerical target. If inflation stays in the tolerance zone, private sector agents may interpret the deviation as due to normal, perhaps only temporary, conditions, and not to a failure of monetary policy. When the whole range serves as the target, the probability of missing the policy objective can also decrease significantly. On the other hand, target zones may become destabilizing whenever they are interpreted as zones of indifference and inaction, with central banks failing to react unless inflation moves outside the zone boundaries. They may also be taken as evidence that the central bank is too lenient toward inflation fluctuations.

The targeting of inflation ranges can be rationalized as optimal by revisiting the conventional central bank's loss function (Orphanides and Wieland, 2000). Unlike the more widespread quadratic loss function, policymakers' preferences become characterized by a quadratic loss function only when inflation falls outside of a target zone, while they remain flat at approximately zero insofar as inflation moves within the zone. Under zone-quadratic preferences, linear monetary policies are no longer desirable. With inflation near target, the policymaker is unwilling to suffer the output gap cost needed to fully stabilize inflation. But the existing uncertainty about structural shocks prevents policy from becoming entirely

[1] For example, the Sveriges Riksbank explicitly stated in a 2017 press release that: *"The Riksbank will also use a variation band of 1–3 per cent for outcomes for CPIF inflation to illustrate that monetary policy is not able to steer inflation in detail, but that inflation normally varies around the target."*

inactive: even within the band, the central bank would therefore find it optimal to react to inflation (although with a smaller coefficient) to reduce the likelihood that it will eventually move out of control.

Orphanides and Wieland provide a fully normative analysis of target zones. However, given their significance in the real world, there has been surprisingly little work expanding on the theory of inflation bands within general equilibrium models and equally limited empirical evidence. In terms of theory, the work by Le Bihan *et al.* (2023) constitutes a recent exception. The paper derives the consequences of tolerance ranges in a benchmark New Keynesian model. The analytical results reveal the dangers of turning ranges into zones of inaction: the failure to satisfy the Taylor principle within the range leads to sunspot equilibria, regardless of the threat of a higher response outside of the band. Moreover, the trade-off between reacting inside versus outside of the band is steep: for any reduction of the inflation response within the band, there has to be an increase in the reaction outside of the band that is 2.5 times as large (for example, a Taylor rule coefficient that is reduced from 1.5 to 1 would require an increase from 1.5 to 2.75 outside). And the larger reaction will come at the cost of higher interest rate and output gap variability.

The empirical evidence, instead, has so far been based on reduced-form methods, aimed at evaluating the effects of different types of inflation targeting on inflation expectations (Ehrmann, 2021, Castelnuovo et al., 2003, and Grosse-Steffen, 2021). But there aren't studies that investigate empirically the behavior of target ranges and bands within estimated, quantitative, general equilibrium models, and that assess in those contexts the interactions between such policies and the public's expectations.

This paper aims to fill this gap by estimating a New Keynesian model with a monetary policy implemented in terms of tolerance bands or target ranges, and with evolving beliefs and learning by the private sector.

The details of monetary policy are, in fact, strictly interconnected with the formation of private-sector expectations. Target zones should affect expectations by conducting them toward the central bank's objective and making them less responsive to exogenous shocks. In principle, agents should recognize that some degree of fluctuations is normal, as long as inflation remains within the band; moreover, a potential threat of a higher response outside the band should help keep expectations anchored.

The real-world experience on target bands presents additional complications. In most cases, the details of policy have changed over time, with shifts both in the midpoint and in the width of the range.

Therefore, to model the formation of expectations in such an environment, I deviate from the conventional assumption of full-information rational expectations. Economic agents form subjective expectations and learn about the economic environment, about the monetary policy rule, and about asymmetries or nonlinearities in policy due to the target range. They also learn about the dynamics of other variables, such as inflation and output, and they may also attempt to learn about whether their statistical processes differ depending on inflation falling within or outside the target zone.

I estimate the structural model using Bayesian methods and exploiting data on both realized macroeconomic variables and a set of observed expectation series for four inflation-targeting countries, which have adopted different forms of target zones: Australia, Canada, New Zealand, and Sweden. The implementation details differ across countries and typically over time within the same country.[2] Given the time variation in the type of policy, it is natural for economic agents to lack full knowledge, or understanding, about the dynamics of the economy and to attempt to learn over time based on their past experiences and the historical data that they encounter. Their expectations have two components: the first, due to learning, captures an endogenous reaction to the state of economy; the second accounts for possible exogenous deviations of their expectations from the point forecast implied by their learning model. These deviations are modeled as sentiment shocks as in Milani (2011, 2017).

Survey expectations are included in the set observables, as they help discipline the dynamics of beliefs and extract the sentiment terms.[3] They refer to expected output growth, inflation, and short-term nominal interest rates.

The estimated model allows us to tackle the following questions:

- Do central banks respond differently to inflation when it falls inside versus outside target ranges?

[2]I follow the classification in Ehrmann (2021) for the types of inflation targeting regimes in the four countries. In the model, however, tolerance bands and target ranges will be treated identically.

[3]The use of expectational data in the estimation of DSGE models is advocated and reviewed in Milani (2023).

- Do agents incorporate the policy nonlinearity in their perceived model of the economy? Do expectations about other variables also inherit the nonlinearity induced by policy?
- Do inflation and inflation expectations respond differently to shocks depending on whether they fall within or outside the band?

1.1. **Results.** The estimation results make clear that central banks don't treat inflation bands as zones of inaction. In all cases, the reaction coefficients within the zone are consistent with the Taylor principle and, if assessed in relation to contemporaneous macroeconomic data, they are often more aggressive than the corresponding reactions when inflation falls outside of the bands.

A lower reaction outside of the target band may suggest that central banks voluntarily let inflation fluctuate away from the target in those situations. One possibility, however, is that fluctuations are driven by inflationary shocks that are judged by central banks as transitory. This hypothesis finds empirical support. When the model is re-estimated by allowing for forecast-based policy rules, in which interest rates are modified in reaction to shifts in expected inflation rather than contemporaneous inflation, the reaction coefficients are found to be of comparable magnitude within and outside the band and easily lead to a satisfaction of the Taylor principle. Therefore, the results suggest that the risks of target range policies highlighted by Le Bihan *et al.* (2023) are averted by the central banks considered in the analysis.

A major rationale for the use of target bands is that they can interact with the formation of private-sector's expectations and help in keeping them anchored. In the estimation, I envision two cases: in the first, agents form the same expectations that they would form in a fully linear model. They learn continuously about the economy and about the policy rule, but they don't recognize two distinct regimes. In the second case, agents can incorporate the nonlinearity and estimate a perceived law of motion that allows for different coefficients, depending on whether inflation falls within or outside the target band. Even in this situation, however, they are still learning about the economy's coefficients in every period, regardless of the regime.

In the case of linear expectations, the responses of inflation and inflation expectations to structural and sentiment shocks remain largely similar within and outside the band.

In all countries, the data prefer the specification in which agents use a linear perceived model to form expectations rather than a nonlinear, asymmetric, perceived law of motion. The fit worsens under nonlinear expectations as the loss of parsimony is not counterbalanced by the importance of shifts in beliefs in and out of the bands.

The main disturbances that drive fluctuations also remain similar across the two regimes. The empirical results reveal a large role played by sentiment shocks related to future inflation expectations, and, particularly for Canada, New Zealand, and Sweden, sentiment related to future aggregate demand.

1.2. **Related Literature.** First and foremost, the paper aims to contribute to the theoretical and empirical literature on the use and consequences of inflation tolerance bands and ranges. Despite being adopted by several central banks worldwide, there is to this date only a limited body of work that analyzes their properties.

On the theoretical side, Le Bihan *et al.* (2023) studies equilibrium determinacy when monetary policy adopts tolerance bands. They show that tolerance ranges should not become zones of inaction: monetary policy must be active (by satisfying the Taylor principle) even within the band to prevent the economy from becoming subject to sunspot-driven fluctuations. Nonetheless, a trade-off still exists between the degree of policy reaction within and outside the band, with the volatility of policy rates that increases with the differential between the two.

Orphanides and Wieland (2000) offer a theoretical rationale for the use of inflation zone targeting by central banks. They do so by resorting to zone-quadratic (or, as an alternative, zone-linear) preferences: the policymaker's loss function implies quadratic losses for inflation deviations outside the range, and a zero or near-zero loss when inflation falls within the indicated zone. They also highlight the role of uncertainty about structural shocks: uncertainty makes it optimal to respond to shocks even within the target band. This is, in fact, what I find for the central banks in my sample.

Tetlow (2008) also provides theoretical insights that are relevant for the use of bands in monetary policy, without modeling them explicitly. He models, instead, a time-varying inflation target as a random walk with upper and lower bounds. The target boundaries can introduce a nonlinearity in expectations, which helps directing expectations toward the objectives of policy. In fact, public expectations ease the work of the monetary authority by introducing a "honeymoon effect", akin to the one that exists with exchange rate bands and which constrains inflation variability beyond the degree that would be implied by policy choices. His paper abstracts from any learning, since the target is fully known. Another related paper is Erceg (2002), which regards the target band as a confidence interval for inflation: under this interpretation, central bankers will select a point on the policy frontier, with a desired level of inflation volatility that allows them to achieve the target zone with x-percent confidence.

Finally, Davig and Foerster (2023) focus on the benefits of inflation tolerance ranges for central bank communication. They show that strategies that define tolerance ranges, and pair them with horizons within which inflation is expected to return to target, enhance monetary policy credibility.

These papers provide either positive or normative theoretical analyses. My paper builds on their findings and provides quantitative evidence, in the context of an estimated structural model, on the use of target zones by actual central banks.

On the empirical side, the existing papers that have analyzed target bands so far have used reduced-form regressions to study their impact on inflation expectations.

Castelnuovo *et al.* (2003) estimate the effects of inflation targeting on long-term private-sector forecasts finding no substantial differences, whereas Ehrmann (2021) focuses on short-term expectations and concludes that ranges and bands typically outperform point targets. My paper provides complementary empirical evidence through the lens of a general equilibrium model. It investigates differences in policy reactions within and outside the bands as well as the responses of inflation and interest rate expectations.

The paper is also more broadly related to work that incorporates a nonlinearity in the monetary policy rule. Bianchi *et al.* (2021), for example, consider an asymmetric policy rule, but with a different focus, as they concentrate on asymmetric reactions in which policy responds less aggressively to inflation when it's above target, as a way to correct the deflationary bias of conventional, symmetric, monetary policy strategies. Although it is not their main focus, in the later part of the paper, they also investigate target ranges and show that they can be effective in removing deflationary biases.

Finally, the paper adds to the expanding literature that analyzes monetary policy under deviations from rational expectations (e.g., Woodford, 2013). The behavioral elements in this paper follow the tradition of adaptive learning in macroeconomics (e.g., Sargent, 1999, Evans and Honkapohja, 2001). Several papers analyze the implications of learning for optimal monetary policy (e.g., Gaspar *et al.* 2006, Dennis and Ravenna, 2008, Eusepi and Preston, 2018, Molnár and Santoro, 2014, Gáti, 2023). Policy rules that take into account learning by the central bank or by agents are estimated in Schorfheide (2005), Milani (2008, 2020), Dennis and Ravenna (2008). A growing number of papers are exploiting survey data on private-sector forecasts to help inference in learning models (Ormeño and Molnár, 2015, Del Negro and Eusepi, 2011, Milani, 2011, 2017, Rajbhandari and Milani, 2020, Cole and Milani, 2019, Eusepi *et al.*, 2025). Finally, in this paper, I model beliefs as depending both on endogenous components, which evolve in reaction to changes in the economy, and exogenous waves of optimism/pessimism, which are orthogonal to the state economy and are defined as sentiment shocks, following previous work in Milani (2011, 2017).

2. A New Keynesian Model With Inflation Tolerance Bands

The model economy is represented by a canonical New Keynesian model (e.g., Woodford, 2003), with endogenous sources of persistence such as external habit formation in consumption and price indexation to lagged inflation, and extended to incorporate a nonlinearity in the monetary policy rule. The model's log-linearized equations are as follows:

$$y_t = \frac{\eta}{1+\eta} y_{t-1} + \frac{1}{1+\eta}\widehat{E}_t y_{t+1} - \frac{\varphi}{1+\eta}\left(i_t - \widehat{E}_t \pi_{t+1} - r_t^n\right) \tag{1}$$

$$\pi_t = \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{\beta}{1+\beta\gamma}\widehat{E}_t \pi_{t+1} + \frac{\kappa}{1+\beta\gamma}[(\omega y_t + \varphi^{-1}(y_t - \eta y_{t-1})] + u_t \tag{2}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left[\mathbf{1}_{\pi_t \in [\pi_{t,l}, \pi_{t,h}]}\chi_{\pi,in}\pi_t + (1 - \mathbf{1}_{\pi_t \in [\pi_{t,l}, \pi_{t,h}]})\chi_{\pi,out}\pi_t + \chi_y y_t\right] + \varepsilon_t, \tag{3}$$

Equation (1) is the log-linearized Euler equation, which arises from households' intertemporal optimization decisions about consumption and saving. Households obtain utility from deviations of their consumption from a stock of habits, which capture "catching up with the Joneses" effects. When written in terms of output y_t, the Euler equation states that current output depends on lagged output (through the role of habit formation), expected one-period-ahead output, and on the deviation of the *ex-ante* real interest rate (i.e., the nominal interest rate i_t minus the expected inflation rate $\widehat{E}_t \pi_{t+1}$) from the natural rate of interest r_t^n, which is typically modeled as an exogenous disturbance. The parameter η denotes the degree of habit formation, while $\varphi \equiv \sigma(1-\eta)$ governs the sensitivity of output to real interest rates and depends on η and on the elasticity of intertemporal substitution σ. Equation (2) represents a New Keynesian Phillips curve, extended to allow for firms' indexation to past inflation rates.

The contemporaneous inflation rate π_t is determined by past inflation, expectations about future inflation, by the state of real activity, and by cost-push disturbances u_t. The degree of indexation is measured by γ; β denotes the household's discount factor, ω the sensitivity of output to marginal costs, and $\kappa \equiv \frac{(1-\alpha)(1-\beta\alpha}{\alpha}$ is a composite parameter that moves inversely to the degree of price stickiness à la Calvo, α. Finally, equation (3) represents a Taylor rule, which is here modified to incorporate the use of tolerance bands or target ranges in the formulation of monetary policy.[4] The policy rate i_t is adjusted in reaction to fluctuations in inflation and output, with a gradual adjustment based on the interest rate-smoothing coefficient ρ. The term $\mathbf{1}_{\pi_t \in [\pi_{t,l}, \pi_{t,h}]}$ represents an indicator function that assumes value of one when inflation falls within the target band $[\pi_{t,l}, \pi_{t,h}]$, where $\pi_{t,l}$ and $\pi_{t,h}$ denote lower and upper bounds, and zero when it falls above or below. The monetary policy response to inflation is equal to $\chi_{\pi,in}$ when inflation falls inside the range and to $\chi_{\pi,out}$ when it's outside. The response to output is, for now, assumed unchanged at χ_y.[5] The demand and supply disturbances r_t^n and u_t follow AR(1) processes, while the monetary policy shock ε_t is conventionally modeled as *i.i.d.*

To deal with imperfect knowledge about the economy and potential shifts in monetary policy, I deviate from the assumption of full-information rational expectations. Economic agents form subjective expectations, denoted here by $\widehat{E}_t$, based on a perceived linear model of the economy, typically known as the PLM (Perceived Law of Motion).

Their PLM is given by:

$$Y_t = \widehat{a}_t + \widehat{b}_t Y_{t-1} + \epsilon_t, \tag{4}$$

where $Y_t = [y_t, \pi_t, i_t, r_t^n, u_t]'$ and the vectors and matrices of perceived coefficients are given by

$$\widehat{a}_t = \begin{bmatrix} \widehat{a}_{y,t} \\ \widehat{a}_{\pi,t} \\ \widehat{a}_{i,t} \\ 0 \\ 0 \end{bmatrix}, \quad \widehat{b}_t = \begin{bmatrix} \widehat{b}_{yy,t} & \widehat{b}_{y\pi,t} & \widehat{b}_{yi,t} & \widehat{b}_{yr,t} & \widehat{b}_{yu,t} \\ \widehat{b}_{\pi y,t} & \widehat{b}_{\pi\pi,t} & \widehat{b}_{\pi i,t} & \widehat{b}_{\pi r,t} & \widehat{b}_{\pi u,t} \\ \widehat{b}_{iy,t} & \widehat{b}_{i\pi,t} & \widehat{b}_{ii,t} & \widehat{b}_{ir,t} & \widehat{b}_{iu,t} \\ 0 & 0 & 0 & \widehat{\rho}_r & 0 \\ 0 & 0 & 0 & 0 & \widehat{\rho}_u \end{bmatrix}.$$

[4]I do not model any difference between tolerance bands and target ranges in this paper.

[5]This assumption will be relaxed in Section 6.2.

Therefore, economic agents lack full knowledge about the parameters describing the economy, both in terms of long-run means and short-term dynamics.[6] They update their beliefs over time based on the following constant-gain algorithm:

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \overline{\mathbf{g}} R_t^{-1} X_t (Y_t - \widehat{\phi}_{t-1}' X_t)', \tag{5}$$

$$R_t = R_{t-1} + \overline{\mathbf{g}} (X_t X_t' - R_{t-1}), \tag{6}$$

where $\widehat{\phi}_t = \left(\widehat{a}_t, \widehat{b}_t\right)'$ collects the beliefs about intercepts and dynamic parameters, $X_t \equiv [1, Y_{t-1}]'$ collects the PLM regressors, and R_t denotes the updated precision matrix related to the beliefs. A central parameter under learning is given by the constant gain coefficient $\overline{\mathbf{g}}$: the gain governs the responsiveness of agents' beliefs to new information, as well as the discounting of older, possibly outdated, observations. Based on the PLM and the most recently updated beliefs, private-sector agents form expectations each period as

$$\begin{aligned}\widehat{E}_{t-1} Y_{t+1} =& \left(I + \widehat{b}_{t-1}\right) \widehat{a}_{t-1} + \widehat{b}_{t-1}^2 Y_{t-1} + S_t \\ =& A(\widehat{a}_{t-1}, \widehat{b}_{t-1}) + B(\widehat{a}_{t-1}, \widehat{b}_{t-1}) Y_{t-1} + S_t, \end{aligned} \tag{7}$$

where $A(\cdot)$ and $B(\cdot)$ are matrices that depend on both structural coefficients and agents' beliefs,[7] and $S_t = [s_{y,t}, s_{\pi,t}, s_{i,t}, 0, 0]'$ is a vector of sentiment variables. Therefore, expectations are formed endogenously, with beliefs that react to the state of the economy, but they can also be subject to exogenous waves of optimism and pessimism, which are instead orthogonal to the state of the economy, and are reflected in the sentiment vector S_t. Sentiment can specifically refer to expectations about output ($s_{y,t}$), inflation ($s_{\pi,t}$), or policy rates ($s_{i,t}$). All sentiment disturbances are allowed to be persistent as they follow AR(1) processes.[8] The use of survey data as observable variables for expectations will help the inference regarding the best-fitting learning process over the sample and allow the identification of sentiment.

I will consider two cases related to expectations in the estimation. In the first, and closer to typical models with adaptive learning, agents are assumed to adopt a linear model of the

[6]Consistently with the adaptive learning literature, agents are assumed to know the autoregressive coefficients for the exogenous shocks, so $\widehat{\rho}_r = \rho_r$ and $\widehat{\rho}_u = \rho_u$.

[7]The economic system, therefore, has self-referential properties: beliefs are affected by the state of the economy, but, in turn, they also influence economic dynamics through their effects on $A(\widehat{a}_{t-1}, \widehat{b}_{t-1})$ and $B(\widehat{a}_{t-1}, \widehat{b}_{t-1})$.

[8]The modeling of sentiment follows Milani (2011, 2017). An alternative to the introduction of sentiment disturbances would be the use of measurement error related to expectations.

economy. They revise their coefficients over time, but they don't assume that zone targeting introduces a nonlinearity in their statistical model. Agents simply continuously update their coefficients each period based on new information. In the second alternative, they instead recognize the nonlinearity and incorporate it into their perceived model. The expectation formation mechanism and the estimation results for this case are discussed in Section 6.3.

On the methodological side, tolerance ranges introduce a nonlinearity that is nontrivial to manage under rational expectations. The estimation, instead, doesn't pose particular complications under learning, given that the regime is observed by agents in real time. In the benchmark case, their expectations are updated continuously based on new data and they are not subject to the discontinuity that would exist under rational expectations with the endogenous regime shift.

3. Countries and Data

3.1. **Observable Series.** The estimation aims to match observations about both realized macroeconomic variables and the available expectations data from surveys. For each country, I use quarterly data on real output growth, the inflation rate, and a short-term nominal interest rate, as realized variables for their counterparts Δy_t, π_t, and i_t in the model.[9] I obtain expectations data related to the same variables. The details of the variables being forecast sometimes differ across countries based on the specific questions that respondents need to answer in the surveys.

For the choice of countries, I choose advanced economies in which the central bank is classified in Ehrmann (2021) as adopting an inflation targeting framework with either a tolerance band or a target range, and for which I can obtain data on private-sector expectations. With these conditions, I settle on Australia, Canada, New Zealand, and Sweden, as the countries of reference.

For Canada, the realized variables are the quarterly real GDP growth rate, the CPI inflation rate (all items, seasonally adjusted), and the three-month interbank nominal interest rate.[10] Expectations data are from the Department of Finance Survey of Private Sector Economic Forecasters. I use data on expectations about real GDP growth, the CPI inflation rate, and the three-month T-bill rate. The first two series provide expectations for same-year

[9]Realized variables are obtained from FRED.

[10]For Canada only, I use the three-month, rather than an overnight, rate, for consistency, since the survey expectations that are available also refer to the three-month rate.

rates (annual growth over the calendar year), while the expectation about the interest rate refers to the average rate over the year. Expectations are available from 1994:I.

For Australia, I use data on real GDP growth, the CPI inflation rate (which I seasonally adjust), and the overnight nominal interest rate. For inflation expectations, there are different possible measures: I use Business inflation expectations, measured by the National Australia Bank Quarterly Business Survey. I choose this measure since it refers to one-quarter-ahead inflation expectations, and it's available since 1989. Consumer, union, and market economists' forecasts are also available, but for shorter time periods. Real output growth expectations (for one quarter ahead) are obtained from the Reserve Bank of Australia, building on the original forecasts' dataset assembled by Tulip and Wallace (2012). For interest rate expectations, I use the market economists' one-quarter-ahead forecasts about the cash rate. Forecasts are available at a fixed horizon of June and December of the same year. Therefore, forecasts from the surveys in February and August can be treated as one-quarter-ahead forecasts, while those from May and November surveys can be treated as two-quarter-ahead forecasts.[11]

For New Zealand, realized variables include the real GDP growth rate, the CPI all items inflation rate (which I seasonally adjust), and the overnight nominal interest rate. Expectations are from the Reserve Bank of New Zealand Survey of Expectations. I use series for annual real GDP growth and annual CPI growth expectations, both referring to one-year-out expectations. Turning to interest rates, forecasts are available for the cash rate only post-2017. Therefore, given the limited time series, in its place, I exploit data on the three-month interbank rate to extract, based on the expectations hypothesis of the term structure, implicit market expectations about future policy rates over the next quarter.[12]

Finally, for Sweden, the observable realized variables are real GDP growth, CPI inflation (already seasonally adjusted), and the repo or policy rate. All expectations series are obtained from Kantar Prospera, which carried out a survey to collect expectations from 1995

[11]In the estimation, this is equivalent to having a state-space model with changing observation equations and/or missing data. In quarters one and three, one-period-ahead interest-rate expectations are available, while in quarters two and four, two-period-ahead expectations are available.

[12]According to the expectations hypothesis, any longer term interest rate can be expressed as equal to sum of the expected short-term rates within the same horizon. Therefore, the three-month rate would be equal to the expectation of future overnight rates over the next three months, as $i_t^{3M} = \sum_{i=0}^{90} i_{t+i}^{1d} + \bar{\tau}$, where $\bar{\tau}$ accounts for a small, constant, term premium, which I remove in the estimation.

to 2024 under commission from the Swedish Riksbank.[13] Expectations cover the forecasts by labor market parties (both employees' and employers' organizations), purchasing managers, and money market players; I use the data that refer to the entirety of those respondents. From the survey, I use observations about one-year-ahead real GDP expectations, one-year-ahead inflation expectations, and one-quarter-ahead policy rate expectations.

3.2. **Inflation Targeting Bands.** Table 1 shows the information about the inflation targeting regimes in the four countries and how they changed over time. I follow the classification from Ehrmann (2021). Australia adopted an inflation targeting regime in 1993; the Reserve Bank doesn't have an explicit point target for inflation, but it aims to maintain inflation within a range between 2% and 3%. Therefore, the inflation target can be interpreted as a "thick point" (Stevens and Debelle, 1995). The Bank of Canada has both a point target and a tolerance band: the targets declined over the first two years, but they are stable at 2% for the point target with a band between 1% and 3% since the end of 1993. Monetary policy in New Zealand is subject to more variation: it starts with a [3%,5%] range in 1990, and the range is progressively adjusted downward to [0%,2%] in 1992, [0%,3%] in 1996, and then [1%,3%] in 2002. Finally, the Reserve Bank of New Zealand moved to a point and tolerance range combination in 2012, maintaining the same [1%,3%] range, and adding a 2% point target. Sweden had both a point target at 2% and a band between 1% and 3% between 1995 and 2009 and between 2017 and today. The Riksbank moved away from this framework and adopted a simple point inflation target in the middle of the sample between 2010 and 2017.

The frequent changes in policies complicate the argument that agents' beliefs can be modeled as having already converged to the rational expectations equilibrium. It is, instead, natural to assume that agents are attempting to learn in an environment characterized by shifts in policy and potential structural breaks at uncertain dates.

Target bands are violated more often in some countries than others, but in all four cases, there is enough variation between inside and outside of the band for a meaningful estimation.

[13]However, as discussed also in Appendix A, data on inflation and interest rate forecasts are publicly available from 2000 and data on output forecasts are added from 2006. The observation equations are adjusted accordingly.

4. Bayesian Estimation

I estimate the structural models on samples beginning around the implementation of inflation targeting for each country: the samples start from 1993 for Australia, 1991 for Canada, 1990 for New Zealand, and 1995 for Sweden, and end in 2024.

The sets of observation equations that link the observables to endogenous variables in the state-space model are shown in Appendix A. The observation equations vary across countries because of different survey questions related to expectations.[14] The available observations may also differ depending on the quarter: for example, in the case of Australia, given the format of the survey questions, one-quarter-ahead forecasts for the policy rate are available in odd quarters and two-quarter-ahead forecasts are available in even quarters. For same-year calendar forecasts (which are available for Canada for example and represent fixed-event forecasts), the survey data can reflect same-quarter expectations (in the fourth quarter of the year) up to three-quarter-ahead expectations (in the first quarter).

In the estimation, I use a Metropolis-Hastings algorithm to generate draws and characterize the posterior distribution of the estimated parameter vector. I only fix two parameters that wouldn't be easily identified in the estimation: the discount factor β is fixed at 0.99; the elasticity of marginal costs to income ω is fixed at 0.2. The likelihood of the system at each draw is obtained using the Kalman filter. The nonlinearity in the policy rule would make the estimation challenging under rational expectations, since, for each draw, the rational expectation solution would need to be obtained for a nonlinear DSGE model. The nonlinearity, instead, is more easily manageable under learning. The inflation rate is observable and, therefore, economic agents, when forming expectations, know whether it falls within or outside the central bank's target range.[15] They form expectations using their most recently updated beliefs, formed based on historically observed data, and they do not need to form probability distributions about future switches in policy.

Table 2 shows the prior choices, which are consistent with the previous literature. I assume a Gamma distribution with mean 1.5 and a standard deviation of 0.5 for the elasticity of intertemporal substitution σ. The Calvo parameter α follows a Beta prior distribution that

[14]For example, in some countries, survey questions ask participants to provide one-quarter-ahead expectations, while in others they may ask for growth rates over the next four quarters.

[15]As mentioned, the estimation will cover two cases: a baseline case, in which expectations are formed from a linear perceived model, and a second case, in which agents incorporate a possible nonlinearity in their PLM.

implies rigid prices for three quarters on average, with a standard deviation equal to 0.1. The monetary policy reaction coefficient to inflation follows a Normal prior with mean equal to 1.5 and standard deviation 0.25. I don't impose any *a priori* knowledge about policy asymmetries: the prior remains the same whether inflation is within or outside the target band. The prior for the reaction coefficient to output similarly follows a Normal distribution, but with lower mean 0.125 and standard deviation 0.06. A Beta distribution with mean 0.7 is assumed for the interest rate inertia parameter ρ. Non-informative Beta priors are chosen for the habit formation and indexation coefficients and for the autoregressive coefficients related to exogenous disturbances and sentiment terms (they all follow Beta(0.5,0.2) priors). Finally, as in Milani (2017), I select a Beta(0.05,0.01) prior for the constant gain coefficient $\bar{g}$.

5. Empirical Results

5.1. **Posterior Estimates.** Table 2 shows the posterior estimates across the four countries in the baseline case of a nonlinear policy rule expressed as in (3), and expectations formed from a linear, symmetric, PLM following (7).

The main coefficients of interest in the estimation are those reflecting a possible nonlinearity in the monetary policy response to inflation. The theoretical literature has highlighted the risks for macroeconomic stability that would arise if central banks didn't react aggressively enough to inflation within the target zone. As found in Le Bihan *et al.* (2023), equilibrium stability is entirely determined by what happens within the target zone, irrespectively of the potential threat of a larger reaction outside of the band.

The empirical results, however, show that the risks due to zones of inaction are avoided by our set of central banks. The estimated responses to inflation within the tolerance bands are in typical ranges and well above the values needed to satisfy the Taylor principle: the posterior means range from 1.492 for Australia to 1.801 for Canada. Perhaps surprisingly, the coefficients are consistently estimated at lower levels when inflation falls outside of the official bands. The discrepancy between the two reaction coefficients is almost non-existent for New Zealand ($\chi_{\pi,in} = 1.573$, $\chi_{\pi,out} = 1.517$), but quite sizable for the remaining economies: $\chi_{\pi,in} = 1.492$ versus $\chi_{\pi,out} = 1.284$ for Australia, $\chi_{\pi,in} = 1.801$ versus $\chi_{\pi,out} = 0.985$ for Canada, $\chi_{\pi,in} = 1.702$ versus $\chi_{\pi,out} = 1.258$ for Sweden. In the case of Sweden, the reaction to inflation falls to an intermediate level ($\chi_{\pi,point} = 1.526$) in the middle years (2010-2017:II) in which the central bank reverts to point inflation targeting. The remaining policy coefficients indicate significant inertia in policy decisions (with estimated ρ coefficients above 0.9) and a limited reaction to measures of real activity (with χ_y coefficients falling between a low of 0.039 in Canada and a high of 0.111 in Sweden).[16]

Turning to the other structural parameters, there is variation in the degree of intertemporal substitution, reflecting the wide range of estimates that typically exist in the literature. The price stickiness parameters remain between 0.825 and 0.888. There is also variation in the estimated sources of endogenous persistence: both habit formation and indexation are modest in Australia (0.089 and 0.171), but substantial in Sweden (0.59 and 0.517). For Canada and New Zealand, habits are important, while price indexation is modest.

Sentiment disturbances are, in most cases, quite persistent. Inflation sentiment is more persistent than structural cost-push shocks in all countries.[17] Sentiment about output fluctuations is quite volatile, and particularly so for New Zealand and Sweden. Finally, estimates for the constant-gain coefficients, which measure the reaction of beliefs to recent information, are not far from the values estimated in similar models on U.S. data (e.g., Milani, 2007, 2011, 2017) and range from 0.0107 for Sweden to 0.025 for Canada.

So far the estimation revealed larger policy responses within the target zone than outside of it. Do private-sector agents incorporate the knowledge that the policymaker responds asymmetrically to inflation when they form their beliefs?

To answer this question, I can estimate the following regression, which treats the evolving beliefs about the policy reaction to inflation $\widehat{b}_{i\pi,t}$ as the dependent variable and relates them to the inside/outside target zone dummy indicator $\mathbf{1}_{\pi_t \in [\pi_{t,l}, \pi_{t,h}]}$:

$$\widehat{b}_{i\pi,t} = \beta_0 + \beta_1 \mathbf{1}_{\pi_t \in [\pi_{t,l}, \pi_{t,h}]} + \nu_t \tag{8}$$

[16]Unlike under rational expectations, the estimation of the model when monetary policy fails to satisfy the Taylor principle doesn't require any extra assumption or change in procedure to find the model solution and obtain the likelihood function (see, for example, Bianchi and Nicolò, 2021, Ilabaca *et al.*, 2020, Ilabaca and Milani, 2021, for recent estimations of rational-expectations DSGE models that allow parameters to fall both in the determinacy and indeterminacy regions). Under learning, the same Metropolis-Hastings algorithm can evaluate the posterior distribution of the parameters regardless of whether they are consistent or not with the Taylor principle (and, therefore, regardless of whether they are conducive to E-stability or E-instability). The same issues are discussed in Milani (2008).

[17]Sentiment was also found to be more persistent that the corresponding structural disturbances in estimations on U.S. data (Milani, 2011, 2017).

The OLS estimates $\hat{\beta}_1$ are shown in Table 3. In all cases, the estimates are not statistically significant, indicating that the public doesn't recognize distinct regimes in the inflation reaction coefficients.[18]

5.2. **Response of Inflation and Expectations to shocks.** The existence of tolerance bands and target zones may introduce a broader nonlinearity in the economy and, potentially, in the formation of expectations. Here, I compare the sensitivity of both inflation and inflation expectations to shocks when inflation falls within the target zone or outside of the zone.

Figure 1 shows the impulse response functions of realized inflation to inflationary sentiment shocks for the four economies. The responses are almost indistinguishable regardless of whether the band is respected or breached.[19]

In Figure 2, I show the effects of a contractionary monetary policy shock on inflation: the effects are slightly smaller in New Zealand when inflation falls outside of the band. The responses are more persistent in Canada, and slightly larger in Sweden. But overall, the differences between the two regimes are not substantial. The response of inflation in Australia displays an initial increase consistent with a prize puzzle behavior, which, as we'll see below, is due to a positive response of expectations.

Figures 3 and 4 turn to the responses of inflation expectations. Figure 3 overlaps the impulse response of one-quarter-ahead inflation expectations and those of one-year-ahead expectations (except for Australia, for which the model tracks only expectations up to two quarters ahead) to sentiment shocks within and outside the zones. Again, there is no asymmetry in the responses: expectations adjust almost identically in both inflation regimes. Expectations at the one-year horizon display only a mild reaction to shocks. Figure 4 compares the responses of the same expectations to monetary policy shocks. The effects of monetary policy on expectations are smaller in the outside-band regime in Canada and New Zealand, and larger in Sweden. In Australia (and to a lesser extent in Canada), expectations increase in the short run in response to a monetary policy tightening (it's beyond the scope of the paper to investigate whether signaling effects are responsible for the increase). For Australia, the initial increase is responsible for the price puzzle observed in Figure 2.

[18]The dependent variable in the regression is a 'generated' regressor. Therefore, it is well known that the OLS standard errors likely understate the underlying degree of uncertainty. In this case, however, a correction is unnecessary, since the coefficients are already not statistically different from zero.

[19]The same is true if I show the impulse responses to cost-push shocks instead.

Finally, Figure 5 focuses on the responses of interest-rate expectations (as before one-quarter-ahead and longer) to a positive inflation sentiment shock. The impulse responses make clear that economic agents expect policy rates to increase in reaction to the shocks. The responses are identical in Australia, New Zealand, and Sweden irrespective of whether current inflation is within or outside the band objective. The responses are somewhat different for Canada, with attenuated perceived monetary policy responses outside the band. However, the differences are modest.

Overall, the similarity of all impulse responses inside and outside the bands are suggestive of high degrees of credibility by the respective central banks: agents know that, independently from current conditions, monetary policy will actively respond to prevent inflation from worsening.

5.3. **Role of Shocks.** I can also test whether there are significant differences in the shocks that drive inflation and inflation expectations in the two regimes. Table 4 shows the forecast error variance decomposition results, by averaging the shares of the variance explained by each shock in periods in which inflation falls within the band and periods in which it moves outside.

For all countries, there are only minor variations in the shares attributed to each shock in the two regimes. In Australia, inflation is mostly driven by three disturbances: the price-markup disturbance, the sentiment shock about future aggregate demand, and the inflation sentiment shock. The latter is predominant for inflation expectations. In the other countries, sentiment still plays a large role, and it is sentiment about demand that is singled out as the main driver of fluctuations at business cycle frequencies. The importance of sentiment confirms the findings in Milani (2011, 2017) based on U.S. data.

6. Alternative Specifications

In what follows, I investigate the empirical results under different modeling assumptions. Tables 5 to 8 report the estimates for all alternative cases, with each table collecting the coefficients for each country for ease of comparison. We also show the corresponding marginal likelihoods for each specification to assess the relative fit of each model to the data.

6.1. **Alternative Timing in the Monetary Policy Rule.** In the benchmark estimation, I assumed a conventional timing in the Taylor rule, with the central bank adjusting interest rates based on the values of contemporaneously observed variables. An alternative that has been proposed in the literature consists of using an "operational" Taylor rule,[20] i.e., a rule that responds to lagged variables, whose values have already been released and are available to policymakers when they set rates in period t.

The results under the Taylor rule with lagged variables are shown in the columns numbered (2) in Tables 5-8. The differentials between the inflation reaction coefficients $\chi_{\pi,in}$ and $\chi_{\pi,out}$ are generally smaller than under the contemporaneous rules. Only for Sweden, the response outside of the band exceeds the response inside, but the difference is trivial. The estimates confirm that central bank policies remain active and therefore consistent with the Taylor principle on either side of the band.

6.1.1. *Why is the Response to Contemporaneous Inflation Lower When Inflation is Outside the Tolerance Band?* It may seem puzzling that monetary policy is estimated to react with a higher coefficient when inflation is under control than when inflation moves outside of the band. One possible explanation is that, on those occasions, the central bank expects inflation to be driven by transient shocks and, therefore, to rapidly revert to target without an aggressive intervention.

To test this hypothesis, I re-estimate the model with a Taylor rule that responds to expected, rather than contemporaneous, inflation. The corresponding estimates are shown in the tables in columns (6).[21]

When policy is allowed to react to expectations, the coefficients toward inflation actually increase when the variable falls outside of the band: the posterior mean estimates for $\chi_{\pi,out}$ are above $\chi_{\pi,in}$ in all countries, although they remain of comparable magnitudes for Canada, New Zealand, and Sweden. Only for Australia, $\chi_{\pi,out}$ is significantly higher at 1.497 compared with $\chi_{\pi,in}$, which equals 1.068.

Therefore, the results are consistent with the hypothesis. When inflation is expected to be transitory, the central bank doesn't necessarily react to the increase. But *ex post*, when the rule is estimated based on contemporaneous inflation, it gives the appearance of an attenuated reaction.

[20]The terminology originates from McCallum (1999).

[21]I report the estimates for a policy that reacts to one-quarter-ahead expectations. I've also estimated cases with forecast-based rules at longer horizons, up to four-quarter-ahead, with similar results.

Overall, these results echo the discussion by Orphanides and Wieland (2000), who show that violations of the band do not necessarily imply that the central bank needs to react with aggressive stabilization as long as the driving shock is expected to be transitory.

The results are also consistent with the horizons within which the central banks in the sample aim to reach their targets. The Reserve Bank of New Zealand seeks to maintain inflation within the boundaries of the range over the "medium term". The Government's Remit for the Monetary Policy Committee states that the committee should "discount events that have only transitory effects on inflation, setting policy with a medium-term orientation". The Reserve Bank of Australia in the "Statement on the Conduct of Monetary Policy" clarifies that the 2-3% target is a medium-term goal and that they allow for fluctuations in inflation over shorter economic cycles. The Bank of Canada is more specific, writing that it aims to achieve the target "typically within a horizon of six to eight quarters". The Riksbank sets policy with the expectation that inflation is close to target within a two-year horizon (although there is no rule regarding the speed with which inflation deviations are closed down).[22]

The estimates of high reaction coefficients within the band conform with some policymakers' statements. For example, the Reserve Bank of Australia makes clear that inflation numbers within the thick band do not imply a zone of policy inaction (Debelle, 2018). Policymakers at the Riksbank also explained in press conferences that they still seek to achieve the same 2% point target even when inflation already falls within the [1%,3%] band.

6.2. **Alternative Nonlinearity in the Monetary Policy Rule.** Theoretical work (LeBihan *et al.*, 2023) has assumed a similar policy rule to (3), with an asymmetric reaction only for the inflation response coefficient. But it cannot be ruled out that all policy coefficients may switch depending on whether inflation falls within or outside the band. Orphanides and Wieland (2000), for example, in their normative analysis, observe that the output objective should dominate when inflation is within the zone and become secondary when inflation moves outside. The results shown under column (5) refer to the corresponding estimation, which allows all policy coefficients ρ, χ_π, and χ_y to vary between the two inflation regimes.

The empirical results regarding χ_y are generally consistent with Orphanides and Wieland's prediction. The monetary policy reactions to output are larger when inflation is within

[22]Chung et al. (2020) provide a more detailed discussion of the policy implementation details for many central banks.

the target zone in all countries, except for New Zealand, for which the coefficients are of comparable magnitudes. The estimated responses to output $\chi_{y,in}$ and $\chi_{y,out}$ shift from 0.144 to 0.073 for Australia, 0.0801 to 0.0286 for Canada, and 0.1120 to 0.0763 for Sweden, based on the regime. It should be noted, however, that this specification always achieves a lower fit than the simpler alternatives.

Additionally, I still need to test whether the nonlinearity is actually needed. The data may be better explained by a specification that ignores the existence of target bands. The results in columns (4) consider this case: the nonlinearity is removed by imposing $\chi_{\pi,in} = \chi_{\pi,out}$ in the estimation. The model fit becomes somewhat lower for Australia, Canada, and New Zealand, suggesting that accounting for target bands is preferred by the data. The fit instead improves for Sweden, where the more parsimonious model works better than one aiming to account for multiple regimes during the sample (within the band, outside of the band, and conventional point targeting).

6.3. **Nonlinearity/Asymmetry in Expectations.** In the benchmark case, I have assumed that, despite the nonlinearity in policy, agents form expectations from a linear perceived law of motion. Here, I relax this assumption. Now economic agents can recognize that the dynamics of inflation, output, and interest rates, may differ depending on whether inflation is within the tolerance zone or outside of it. They incorporate such nonlinearity into their PLM, which becomes:

$$Y_t = \mathbf{1}_{\pi_t \in [\pi_{t,l}, \pi_{t,h}]} \left[\widehat{a}_{in,t} + \widehat{b}_{in,t} Y_{t-1}\right] + (1 - \mathbf{1}_{\pi_t \in [\pi_{t,l}, \pi_{t,h}]}) \left[\widehat{a}_{out,t} + \widehat{b}_{out,t} Y_{t-1}\right] + \epsilon_t, \qquad (9)$$

where $\widehat{a}_{in,t}$, $\widehat{b}_{in,t}$, $\widehat{a}_{out,t}$, $\widehat{b}_{out,t}$ denote regime-specific beliefs. Agents update their beliefs as before according to the same constant-gain learning formulas as in (5)-(6) and form expectations based on (9) and the updated beliefs.

The posterior estimates are shown in the tables in columns (3). Overall, the estimates remain comparable to those obtained under linear PLMs. But the fit is consistently lower: the data substantially reject the specification that assumes that agents recognize the nonlinearity induced in the economy by target bands. Hence, the full-information Bayesian estimation, which allows for the asymmetry in expectations, provides results that are consistent with the simpler, reduced-form, evidence from regression (8).

6.4. **Alternative Initialization of Beliefs.** So far, the agents' learning process has been initialized using pre-sample data. As an alternative, I re-estimate the models now setting initial beliefs equal to their value under the model's rational expectations equilibrium. The estimates are shown in columns (7) in the tables. The case of RE initial beliefs wasn't used as benchmark since it leads to significantly lower fit in all cases. Aside from the lower fit, the coefficient estimates for the policy parameters would still lead to similar conclusions.

7. Conclusions

Several central banks that adopt inflation targeting as their monetary policy framework also combine it with tolerance bands or target ranges.

Despite their real-world relevance, only a limited theoretical literature has analyzed the properties of such arrangements. More studies examine the empirical evidence, but they are typically based on reduced-form regressions.

This paper uses a different approach, by analyzing a general equilibrium, New Keynesian, model that incorporates a nonlinearity in the policy rule to capture the existence of target zones. The structural model is estimated using Bayesian techniques.

In light of the nonlinearity and the many shifts in policy over the sample, it is desirable to consider deviations from the assumption of full-information rational expectations. I model expectations as subjective and exploit data on private-sector expectations from surveys related to future output, inflation, and policy rates, in the estimation. Economic agents are assumed to learn based on perceived models of the economy, either linear or nonlinear, and are subject to exogenous shifts in optimism or pessimism, defined as sentiment shocks. The expectations data help strengthen inferences about both the agents' learning process and the sentiment terms.

The estimation results indicate that central banks largely avoid the risks of inaction when inflation falls within the target zone, which were highlighted by previous research. A failure of the Taylor principle within the zone would lead to instability and multiple equilibria. In the estimation, however, the policy responses remain consistent regardless of whether inflation falls within or outside the target zone, and always at levels consistent with the Taylor principle.

The results do not support the evidence of a significant nonlinearity in the formation of expectations. The responses of inflation and policy expectations to shocks are very similar irrespective of violations of the band. This finding likely reflects the credibility gained by inflation-targeting central banks in the sample.

I have focused on four advanced-economy central banks that have been largely successful in controlling inflation. In future research, I can extend the analysis to study the effects of target zones in emerging economies. Policy coefficients may vary more across regimes in those cases and, in particular, private-sector beliefs may become more sensitive to shocks.

Appendix A. Observation Equations

The state variables in the model and the observable data series are related through a set of observation equations. Since the definitions for some of the available survey expectations differ across countries, the relevant observation equations will also be country-specific.

For Australia, the set of observation equations in the state-space model is given by:

$$\begin{bmatrix} Real\ Output\ Growth_t \\ Inflation_t \\ Nominal\ Interest\ Rate_t \\ Expected\ Real\ Output\ Growth_{t+1} \\ Expected\ Inflation_{t+1} \\ \begin{bmatrix} Expected\ Interest\ Rate_{t+1} \\ Expected\ Interest\ Rate_{t+2} \end{bmatrix}' \mathcal{Q}_t \end{bmatrix} = \begin{bmatrix} \delta \\ \pi^* \\ \pi^* + r^* \\ \delta \\ \pi^* \\ \pi^* + r^* \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ \pi_t \\ i_t \\ \widehat{E}_{t-1}\left[y_{t+1}\right] - y_t \\ \widehat{E}_{t-1}\pi_{t+1} \\ \begin{bmatrix} \widehat{E}_{t-1} i_{t+1} \\ \widehat{E}_{t-1} i_{t+2} \end{bmatrix}' \mathcal{Q}_t \end{bmatrix}$$

where, given the timing in the survey, only one between the one-quarter-ahead and the two-quarter-ahead is available each quarter. The survey data for interest rates refer to fixed-event forecasts for June and December each year. Therefore, survey responses in the February and August surveys provide one-quarter-ahead forecasts, while survey responses in May and November provide two-quarter-ahead forecasts. In the observation equation for expected interest rates, this is denoted using the quarter indicator $\mathcal{Q}_t = [q_t, 1 - q_t]'$, with $q_t = 1$ representing odd quarters.

For Canada, the observation equations are as follows:

$$\begin{bmatrix} Real\ Output\ Growth_t \\ Inflation_t \\ Nominal\ Interest\ Rate_t \\ \begin{bmatrix} Expected\ Real\ Output\ Growth_{t-1,t+3} \\ Expected\ Real\ Output\ Growth_{t-2,t+2} \\ Expected\ Real\ Output\ Growth_{t-3,t+1} \\ Expected\ Real\ Output\ Growth_{t-4,t} \end{bmatrix}' \mathcal{Q}_t \\ \begin{bmatrix} Expected\ Inflation_{t-1,t+3} \\ Expected\ Inflation_{t-2,t+2} \\ Expected\ Inflation_{t-3,t+1} \\ Expected\ Inflation_{t-4,t} \end{bmatrix}' \mathcal{Q}_t \\ \begin{bmatrix} Expected\ Interest\ Rate_{t-1,t+3} \\ Expected\ Interest\ Rate_{t-2,t+2} \\ Expected\ Interest\ Rate_{t-3,t+1} \\ Expected\ Interest\ Rate_{t-4,t} \end{bmatrix}' \mathcal{Q}_t \end{bmatrix} = \begin{bmatrix} \delta \\ \pi^* \\ \pi^* + r^* \\ 4\delta \\ 4\pi^* \\ 4\left(\pi^* + r^*\right) \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ \pi_t \\ i_t \\ \begin{bmatrix} \widehat{E}_{t-1}\left[y_{t+3}\right] - y_{t-1} \\ \widehat{E}_{t-1}\left[y_{t+2}\right] - y_{t-2} \\ \widehat{E}_{t-1}\left[y_{t+1}\right] - y_{t-3} \\ \widehat{E}_{t-1}\left[y_{t}\right] - y_{t-4} \end{bmatrix}' \mathcal{Q}_t \\ \begin{bmatrix} \widehat{E}_{t-1} \sum\limits_{j=0}^{4} \pi_{t+j-1} \\ \widehat{E}_{t-1} \sum\limits_{j=0}^{4} \pi_{t+j-2} \\ \widehat{E}_{t-1} \sum\limits_{j=0}^{4} \pi_{t+j-3} \\ \widehat{E}_{t-1} \sum\limits_{j=0}^{4} \pi_{t+j-4} \end{bmatrix}' \mathcal{Q}_t \\ \begin{bmatrix} \widehat{E}_{t-1} \sum\limits_{j=0}^{4} i_{t+j-1} \\ \widehat{E}_{t-1} \sum\limits_{j=0}^{4} i_{t+j-2} \\ \widehat{E}_{t-1} \sum\limits_{j=0}^{4} i_{t+j-3} \\ \widehat{E}_{t-1} \sum\limits_{j=0}^{4} i_{t+j-4} \end{bmatrix}' \mathcal{Q}_t \end{bmatrix}$$

where $\mathcal{Q}_t = [q_t^1, q_t^2, q_t^3, 1 - \sum\limits_{j=1}^{3} q_t^j]'$ and each q_t^j is an indicator that equals 1 in quarter j and 0 otherwise.

For New Zealand and Sweden, the available expectation series have similar horizons and definitions. Therefore, the observation equations for both countries can be expressed similarly as follows:

$$\begin{bmatrix} Real\ Output\ Growth_t \\ Inflation_t \\ Nominal\ Interest\ Rate_t \\ Expected\ Real\ Output\ Growth_{t,t+4} \\ Expected\ Inflation_{t,t+4} \\ Expected\ Interest\ Rate_{t+1} \end{bmatrix} = \begin{bmatrix} \delta \\ \pi^* \\ \pi^* + r^* \\ 4\delta \\ 4\pi^* \\ \pi^* + r^* \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ \pi_t \\ i_t \\ \widehat{E}_{t-1}\left[y_{t+4} - y_t\right] \\ \widehat{E}_{t-1}\sum_{j=1}^{4}\pi_{t+j} \\ \widehat{E}_{t-1}i_{t+1} \end{bmatrix}$$

For these two countries, the same expectation series are available each quarter: therefore, the observation equations don't include the quarter indicator. Only for Sweden, however, some expectation series are not available for the full sample. Swedish series about expected inflation and expected interest rate begin in 2000, whereas the survey question about expected GDP is added starting in 2006. The estimation considers them as missing data until they become available (the size of the observation vector, therefore, expands over time).

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Country	Starting Date	Point Target	Tolerance Range
Australia	1993:II	-	[2,3]
Canada	1991:I	3	[2,4]
	1992:I	2.5	[1.5,3.5]
	1993:IV	2	[1,3]
New Zealand	1990:II	-	[3,5]
	1990:IV	-	[2.5,4.5]
	1991:IV	-	[1.5,3.5]
	1992:IV	-	[0,2]
	1996:IV	-	[0,3]
	2002:IV	-	[1,3]
	2012:III	2	[1,3]
Sweden	1995:I	2	[1,3]
	2010:I	2	-
	2017:III	2	[1,3]

TABLE 1. Point Targets and Tolerance Bands/Ranges under Inflation Targeting Regimes for Australia, Canada, New Zealand, and Sweden.

Param.	**Prior Distr.**	**Posterior Distributions**			
		Australia	Canada	New Zealand	Sweden
'Deep'					
σ	Γ(1.50,0.5)	0.304 [0.16,0.49]	1.643 [0.89,2.61]	1.97 [1.11,2.99]	1.954 [1.03,2.91]
α	B(0.66,0.1)	0.888 [0.85,0.92]	0.872 [0.82,0.93]	0.853 [0.80,0.91]	0.825 [0.76,0.90]
η	B (0.5,0.2)	0.089 [0.03,0.16]	0.64 [0.44,0.82]	0.523 [0.38,0.70]	0.590 [0.41,0.83]
γ	B (0.5,0.2)	0.171 [0.03,0.41]	0.107 [0.02,0.26]	0.21 [0.04,0.49]	0.517 [0.18,0.76]
Policy					
ρ	B(0.7,0.1)	0.973 [0.95,0.99]	0.928 [0.90,0.96]	0.924 [0.89,0.95]	0.934 [0.90,0.96]
$\chi_{\pi,in}$	N(1.5,0.25)	1.492 [1.01,1.98]	1.801 [1.36,2.25]	1.573 [1.06,2.07]	1.702 [1.10,2.12]
$\chi_{\pi,out}$	N(1.5,0.25)	1.284 [0.80,1.78]	0.985 [0.52,1.52]	1.517 [1.06,1.97]	1.258 [0.75,1.82]
$\chi_{\pi,point}$	N(1.5,0.25)				1.526 [1.16,1.97]
χ_y	N(0.125,0.06)	0.072 [-.01,0.17]	0.039 [-.01,0.11]	0.072 [0.00,0.16]	0.111 [0.05,0.18]
Disturbances					
ρ_r	B (0.5,0.2)	0.528 [0.36,0.68]	0.115 [0.03,0.23]	0.655 [0.55,0.75]	0.727 [0.60,0.85]
ρ_u	B (0.5,0.2)	0.113 [0.02,0.25]	0.147 [0.04,0.31]	0.222 [0.07,0.40]	0.102 [0.03,0.19]
σ_r	Γ^{-1}(0.30,1)	0.374 [0.32,0.4]	1.804 [1.52,2.09]	1.022 [0.87,1.22]	1.046 [0.86,1.33]
σ_u	Γ^{-1}(0.30,1)	0.613 [0.50,0.77]	0.646 [0.55,0.77]	0.497 [0.40,0.64]	0.881 [0.63,1.05]
σ_ε	Γ^{-1}(0.30,1)	0.104 [0.09,0.12]	0.155 [0.14,0.18]	0.209 [0.18,0.24]	0.132 [0.12,0.15]
Sentiment					
ρ_{e^y}	B (0.5,0.2)	0.271 [0.12,0.43]	0.865 [0.82,0.92]	0.96 [0.93,0.99]	0.951 [0.90,0.98]
ρ_{e^π}	B (0.5,0.2)	0.854 [0.77,0.93]	0.303 [0.19,0.41]	0.752 [0.64,0.85]	0.84 [0.75,0.92]
ρ_{e^i}	B (0.5,0.2)	0.737 [0.63,0.84]	0.642 [0.54,0.74]	0.502 [0.35,0.64]	0.690 [0.57,0.84]
σ_{e^y}	Γ^{-1}(0.30,1)	1.142 [1.02,1.29]	1.1953 [1.69,2.29]	4.25 [3.65,4.30]	2.512 [2.17,2.98]
σ_{e^π}	Γ^{-1}(0.30,1)	0.192 [0.17,0.22]	0.503 [0.43,0.59]	0.198 [0.16,0.24]	0.115 [0.10,0.14]
σ_{e^i}	Γ^{-1}(0.30,1)	0.149 [0.12,0.18]	0.226 [0.19,0.27]	0.167 [0.15,0.19]	0.08 [0.07,0.09]
Learning					
$\bar{g}$	B(0.05,0.01)	0.012 [0.009,0.015]	0.025 [0.021,0.029]	0.014 [0.01,0.019]	0.0107 [0.008,0.013]

TABLE 2. Posterior Estimates: Baseline Estimation. *Note*: The Table reports mean posterior estimates, along with 2.5% and 97.5% percentiles of the posterior distribution (in brackets).

	Australia	Canada	New Zealand	Sweden
$\hat{\beta}_1$	-0.0005 (0.0014) [−0.342]	-0.0001 (0.0043) [−0.031]	0.0027 (0.0016) [1.69]	-0.0007 (0.0017) [−0.398]

Table 3. Regressions of Evolving Beliefs on Within-Target-Range dummy. *Note*: Dependent variable is estimated belief $\widehat{b}_{i\pi,t}$. Under each estimated coefficient, the table reports the standard error (in parentheses) and the t-statistic (in square brackets).

	r_t^n		u_t		ε_t		$s_{y,t}$		$s_{\pi,t}$	
	(in)	(out)	(in)	(out)	(in)	(out)	(in)	(out)	(in)	(out)
Australia										
π_t	0.0622	0.0594	0.3873	0.3967	0.0295	0.0274	0.2793	0.2669	0.2417	0.2496
$\widehat{E}_t\pi_{t+1}$	0.0457	0.0468	0.0160	0.0175	0.0341	0.0384	0.2072	0.2056	0.6970	0.6918
Canada										
π_t	0.0347	0.0280	0.2572	0.2783	0.0102	0.0100	0.6768	0.6619	0.0211	0.0218
$\widehat{E}_t\pi_{t+1}$	0.0373	0.0324	0.0087	0.0070	0.0153	0.0133	0.5563	0.6118	0.3825	0.3355
New Zealand										
π_t	0.0043	0.0044	0.1045	0.1011	0.0054	0.0059	0.8138	0.8185	0.0720	0.0702
$\widehat{E}_t\pi_{t+1}$	0.0067	0.0060	0.0031	0.0023	0.0074	0.0085	0.8826	0.9023	0.1002	0.0809
Sweden										
π_t	0.0265	0.0331	0.1922	0.2407	0.0072	0.0072	0.7383	0.6738	0.0357	0.0452
$\widehat{E}_t\pi_{t+1}$	0.0284	0.0471	0.0110	0.0098	0.0106	0.0110	0.7811	0.7868	0.1688	0.1454

TABLE 4. Variance Decomposition *Note*: The table shows the share of the forecast error variance of inflation and next-quarter inflation expectations that can be explained by each shock. The shares represent averages over the periods in which inflation falls within the tolerance band and for those in which it falls outside the boundaries of the band.

Param.	Posterior Distributions						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
'Deep'							
σ	0.304 [0.16,0.49]	0.302 [0.16,0.48]	0.331 [0.18,0.53]	0.288 [0.16,0.45]	0.292 [0.16,0.48]	0.293 [0.15,0.44]	0.236 [0.13,0.36]
α	0.888 [0.85,0.92]	0.888 [0.85,0.93]	0.880 [0.84,0.92]	0.891 [0.86,0.93]	0.890 [0.85,0.93]	0.892 [0.86,0.93]	0.908 [0.88,0.94]
η	0.089 [0.03,0.16]	0.089 [0.03,0.16]	0.086 [0.03,0.15]	0.092 [0.04,0.16]	0.088 [0.03,0.16]	0.084 [0.03,0.15]	0.051 [0.01,0.10]
γ	0.171 [0.03,0.41]	0.166 [0.03,0.42]	0.170 [0.03,0.40]	0.169 [0.04,0.37]	0.125 [0.03,0.27]	0.139 [0.03,0.35]	0.095 [0.02,0.22]
Policy							
ρ	0.973 [0.95,0.99]	0.984 [0.97,0.99]	0.973 [0.95,0.99]	0.972 [0.95,0.99]	-	0.968 [0.94,0.99]	0.971 [0.95,0.99]
$\chi_{\pi,in}$	1.492 [1.01,1.98]	1.371 [0.90,1.86]	1.504 [1.03,2.02]	1.242 [0.78,1.67]	1.406 [0.99,1.92]	1.068 [0.56,1.55]	1.501 [1.12,1.99]
$\chi_{\pi,out}$	1.284 [0.80,1.78]	1.361 [0.86,1.86]	1.277 [0.78,1.79]	-	1.267 [0.81,1.75]	1.497 [0.91,1.98]	1.259 [0.74,1.96]
χ_y	0.072 [-0.02,0.17]	0.090 [-0.02,0.20]	0.063 [-0.03,0.17]	0.069 [-0.02,0.17]	-	0.097 [0.01,0.21]	0.084 [-0.01,0.19]
ρ_{in}					0.971 [0.95,0.99]		
ρ_{out}					0.927 [0.89,0.96]		
$\chi_{y,in}$					0.144 [0.03,0.24]		
$\chi_{y,out}$					0.073 [-0.02,0.2]		
Disturbances							
ρ_r	0.528 [0.36,0.68]	0.530 [0.37,0.68]	0.538 [0.38,0.70]	0.503 [0.34,0.64]	0.522 [0.36,0.67]	0.528 [0.37,0.67]	0.449 [0.31,0.56]
ρ_u	0.113 [0.02,0.25]	0.112 [0.02,0.25]	0.102 [0.02,0.24]	0.115 [0.03,0.24]	0.125 [0.03,0.27]	0.113 [0.03,0.22]	0.041 [0.01,0.10]
σ_r	0.374 [0.32,0.44]	0.373 [0.32,0.44]	0.375 [0.32,0.44]	0.374 [0.32,0.44]	0.373 [0.32,0.44]	0.371 [0.32,0.43]	0.354 [0.31,0.41]
σ_u	0.613 [0.50,0.77]	0.608 [0.50,0.77]	0.608 [0.50,0.77]	0.612 [0.51,0.77]	0.585 [0.50,0.69]	0.593 [0.50,0.72]	0.563 [0.48,0.66]
σ_ε	0.104 [0.09,0.12]	0.104 [0.09,0.12]	0.104 [0.09,0.12]	0.103 [0.09,0.12]	0.104 [0.09,0.12]	0.103 [0.09,0.12]	0.104 [0.09,0.12]
Sentiment							
ρ_{e^y}	0.271 [0.12,0.43]	0.270 [0.10,0.43]	0.299 [0.15,0.45]	0.256 [0.10,0.47]	0.246 [0.11,0.38]	0.274 [0.10,0.43]	0.334 [0.19,0.50]
ρ_{e^π}	0.854 [0.77,0.93]	0.855 [0.76,0.93]	0.895 [0.82,0.96]	0.856 [0.77,0.94]	0.847 [0.74,0.94]	0.854 [0.77,0.93]	0.719 [0.59,0.85]
ρ_{e^i}	0.737 [0.63,0.84]	0.743 [0.64,0.84]	0.763 [0.66,0.86]	0.743 [0.64,0.85]	0.743 [0.62,0.87]	0.733 [0.62,0.84]	0.789 [0.69,0.89]
σ_{e^y}	1.142 [1.02,1.29]	1.146 [1.01,1.30]	1.166 [1.04,1.33]	1.143 [1.01,1.30]	1.144 [1.01,1.30]	1.145 [1.01,1.31]	1.431 [1.27,1.59]
σ_{e^π}	0.192 [0.17,0.22]	0.193 [0.17,0.22]	0.204 [0.18,0.23]	0.192 [0.17,0.22]	0.193 [0.17,0.22]	0.193 [0.17,0.22]	0.254 [0.22,0.29]
σ_{e^i}	0.149 [0.12,0.18]	0.148 [0.12,0.18]	0.148 [0.13,0.18]	0.148 [0.12,0.18]	0.149 [0.12,0.18]	0.150 [0.13,0.18]	0.183 [0.15,0.22]
Learning							
$\bar{g}$	0.012 [0.009,0.015]	0.012 [0.009,0.015]	0.007 [0.005,0.008]	0.012 [0.009,0.015]	0.012 [0.009,0.015]	0.012 [0.009,0.015]	0.026 [0.021,0.030]
Marginal Likelihood	-222.05	-225.21	-244.76	-222.43	-228.28	**-221.61**	-293.81

TABLE 5. Posterior Estimates: Alternative Specifications - Australia. *Note*: The Table reports mean posterior estimates, along with 2.5% and 97.5% percentiles. The columns refer to the following specifications: (1) baseline; (2) lagged variables in the monetary policy rule; (3) nonlinear PLM; (4) same policy coefficients within and outside the band; (5) all monetary policy coefficients change in and out of the band; (6) forecast-based Taylor rule; (7) REE-based initial beliefs.

Param.	**Posterior Distributions**						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
'Deep'							
σ	1.6433 [0.89,2.61]	1.6286 [0.87,2.75]	1.6298 [0.90,2.63]	1.4104 [0.81,2.13]	1.4243 [0.90,1.96]	1.4884 [0.93,2.38]	1.4226 [0.73,2.37]
α	0.8716 [0.82,0.93]	0.8771 [0.82,0.93]	0.8826 [0.83,0.94]	0.8838 [0.83,0.93]	0.8776 [0.83,0.93]	0.8817 [0.83,0.94]	0.8759 [0.83,0.93]
η	0.6400 [0.44,0.82]	0.6307 [0.44,0.82]	0.6474 [0.46,0.83]	0.5882 [0.37,0.78]	0.6370 [0.44,0.81]	0.6190 [0.45,0.81]	0.7098 [0.47,0.90]
γ	0.1066 [0.02,0.26]	0.1064 [0.02,0.24]	0.1846 [0.04,0.40]	0.1059 [0.02,0.23]	0.1075 [0.02,0.25]	0.1111 [0.02,0.26]	0.0565 [0.01,0.14]
Policy							
ρ	0.9284 [0.90,0.96]	0.9496 [0.93,0.97]	0.9386 [0.90,0.97]	0.9445 [0.91,0.97]	-	0.9380 [0.90,0.97]	0.9334 [0.90,0.96]
$\chi_{\pi,in}$	1.8006 [1.36,2.25]	1.4407 [0.99,1.89]	1.6331 [1.11,2.13]	1.4547 [0.94,1.99]	1.8276 [1.40,2.33]	1.4100 [0.93,1.90]	1.6340 [1.07,2.14]
$\chi_{\pi,out}$	0.9852 [0.52,1.52]	1.4586 [0.99,1.93]	1.2656 [0.76,1.77]	-	1.2626 [0.74,1.75]	1.4568 [1.08,1.84]	1.2407 [0.80,1.69]
χ_y	0.0392 [-0.02,0.11]	0.0680 [-0.01,0.16]	0.0586 [-0.02,0.14]	0.0673 [0.00,0.15]	-	0.0602 [-0.01,0.17]	0.0641 [0.00,0.14]
ρ_{in}					0.8639 [0.79,0.92]		
ρ_{out}					0.9443 [0.91,0.97]		
$\chi_{y,in}$					0.0801 [0.02,0.16]		
$\chi_{y,out}$					0.0286 [-0.04,0.12]		
Disturbances							
ρ_r	0.1151 [0.03,0.23]	0.1210 [0.03,0.24]	0.1145 [0.03,0.22]	0.1265 [0.04,0.24]	0.1134 [0.03,0.23]	0.1178 [0.03,0.23]	0.1274 [0.04,0.24]
ρ_u	0.1469 [0.04,0.31]	0.1584 [0.04,0.32]	0.1427 [0.03,0.30]	0.1557 [0.03,0.35]	0.1656 [0.04,0.33]	0.1484 [0.04,0.30]	0.0854 [0.02,0.18]
σ_r	1.8037 [1.52,2.09]	1.7949 [1.52,2.10]	1.8511 [1.55,2.18]	1.7676 [1.47,2.05]	1.8023 [1.53,2.13]	1.7765 [1.52,2.09]	1.9802 [1.66,2.29]
σ_u	0.6463 [0.55,0.77]	0.6326 [0.54,0.76]	0.6716 [0.56,0.84]	0.6261 [0.53,0.74]	0.6450 [0.55,0.78]	0.6345 [0.54,0.76]	0.5920 [0.52,0.68]
σ_ε	0.1547 [0.14,0.18]	0.1635 [0.15,0.19]	0.1679 [0.15,0.19]	0.1695 [0.15,0.19]	0.1605 [0.14,0.18]	0.1677 [0.15,0.19]	0.1670 [0.15,0.19]
Sentiment							
ρ_{e^y}	0.8654 [0.82,0.92]	0.8676 [0.82,0.92]	0.8870 [0.84,0.93]	0.8731 [0.83,0.92]	0.8632 [0.82,0.91]	0.8700 [0.83,0.92]	0.8074 [0.76,0.85]
ρ_{e^π}	0.3035 [0.19,0.41]	0.3007 [0.19,0.42]	0.3896 [0.28,0.51]	0.3037 [0.20,0.41]	0.2959 [0.19,0.40]	0.2832 [0.18,0.39]	0.3368 [0.24,0.44]
ρ_{e^i}	0.6425 [0.54,0.74]	0.6409 [0.54,0.74]	0.6358 [0.53,0.74]	0.6392 [0.55,0.73]	0.6450 [0.53,0.76]	0.6502 [0.54,0.76]	0.7410 [0.64,0.83]
σ_{e^y}	1.9529 [1.69,2.29]	1.9459 [1.67,2.28]	2.0221 [1.75,2.33]	1.9242 [1.65,2.24]	1.9451 [1.70,2.24]	1.9332 [1.69,2.22]	2.4218 [2.10,2.79]
σ_{e^π}	0.5027 [0.43,0.59]	0.5025 [0.43,0.58]	0.5125 [0.44,0.60]	0.4990 [0.43,0.58]	0.4990 [0.43,0.58]	0.5055 [0.44,0.58]	0.5445 [0.47,0.64]
σ_{e^i}	0.2257 [0.19,0.27]	0.2256 [0.19,0.26]	0.2457 [0.21,0.29]	0.2252 [0.20,0.26]	0.2263 [0.19,0.26]	0.2253 [0.19,0.27]	0.2225 [0.19,0.26]
Learning							
$\bar{g}$	0.0249 [0.02,0.03]	0.0249 [0.02,0.03]	0.0192 [0.02,0.02]	0.0250 [0.02,0.03]	0.0249 [0.02,0.03]	0.0248 [0.02,0.03]	0.0309 [0.03,0.03]
Marginal Likelihood	**-701.53**	-703.98	-740.58	-707.15	-707.47	-705.61	-746.66

TABLE 6. Posterior Estimates: Alternative Specifications - Canada. *Note*: The Table reports mean posterior estimates, along with 2.5% and 97.5% percentiles. The columns refer to the following specifications: (1) baseline; (2) lagged variables in the monetary policy rule; (3) nonlinear PLM; (4) same policy coefficients within and outside the band; (5) all monetary policy coefficients change in and out of the band; (6) forecast-based Taylor rule; (7) REE-based initial beliefs.

Param.	Posterior Distributions						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
'Deep'							
σ	1.97 [1.11,2.99]	1.9426 [1.13,2.96]	1.8095 [0.92,2.59]	1.6849 [1.08,2.51]	2.0004 [0.93,2.68]	1.8445 [1.30,2.33]	1.2949 [0.85,1.92]
α	0.853 [0.80,0.91]	0.8546 [0.80,0.91]	0.8264 [0.76,0.89]	0.8632 [0.81,0.92]	0.8477 [0.79,0.91]	0.8600 [0.81,0.91]	0.9150 [0.87,0.95]
η	0.523 [0.38,0.70]	0.5260 [0.37,0.70]	0.5215 [0.35,0.68]	0.5082 [0.37,0.66]	0.5127 [0.39,0.65]	0.5373 [0.40,0.70]	0.5914 [0.40,0.76]
γ	0.21 [0.04,0.49]	0.2134 [0.04,0.52]	0.2175 [0.05,0.44]	0.1604 [0.03,0.43]	0.2429 [0.09,0.47]	0.1686 [0.03,0.43]	0.1138 [0.02,0.24]
Policy							
ρ	0.924 [0.89,0.95]	0.9447 [0.92,0.97]	0.9317 [0.90,0.96]	0.9263 [0.90,0.95]	-	0.9354 [0.91,0.96]	0.9101 [0.88,0.94]
$\chi_{\pi,in}$	1.573 [1.06,2.07]	1.5169 [1.03,2.00]	1.5382 [1.05,2.01]	1.6183 [1.11,2.07]	1.5947 [1.09,2.17]	1.4924 [1.07,1.93]	1.6226 [1.15,2.29]
$\chi_{\pi,out}$	1.517 [1.06,1.97]	1.3920 [0.95,1.87]	1.3539 [0.84,1.84]	-	1.5847 [1.12,2.10]	1.5101 [1.05,1.89]	1.5306 [1.06,1.89]
χ_y	0.072 [0.00,0.16]	0.0967 [0.01,0.19]	0.1219 [0.04,0.23]	0.0710 [-0.02,0.17]		0.0760 [-0.01,0.15]	-0.0410 [-0.09,0.00]
ρ_{in}					0.8875 [0.81,0.95]		
ρ_{out}					0.9236 [0.89,0.95]		
$\chi_{y,in}$					0.0752 [0.01,0.17]		
$\chi_{y,out}$					0.0722 [-0.01,0.18]		
Disturbances							
ρ_r	0.655 [0.55,0.75]	0.6564 [0.55,0.75]	0.6456 [0.53,0.75]	0.6587 [0.55,0.75]	0.6548 [0.56,0.74]	0.6490 [0.54,0.75]	0.4456 [0.26,0.64]
ρ_u	0.222 [0.07,0.40]	0.2168 [0.06,0.39]	0.1630 [0.05,0.31]	0.2376 [0.10,0.41]	0.2110 [0.08,0.36]	0.2494 [0.08,0.45]	0.0846 [0.02,0.18]
σ_r	1.022 [0.87,1.22]	1.0199 [0.86,1.22]	1.0587 [0.88,1.27]	1.0087 [0.86,1.17]	1.0064 [0.87,1.18]	1.0335 [0.86,1.23]	1.2950 [1.06,1.56]
σ_u	0.497 [0.40,0.64]	0.4976 [0.40,0.64]	0.5124 [0.41,0.63]	0.4741 [0.39,0.62]	0.5124 [0.42,0.63]	0.4782 [0.39,0.61]	0.4519 [0.39,0.54]
σ_ε	0.209 [0.18,0.24]	0.2092 [0.19,0.24]	0.2021 [0.18,0.23]	0.2076 [0.18,0.24]	0.2104 [0.19,0.24]	0.2090 [0.18,0.24]	0.1956 [0.17,0.22]
Sentiment							
ρ_{e^y}	0.96 [0.93,0.99]	0.9607 [0.93,0.99]	0.9506 [0.92,0.99]	0.9638 [0.93,0.99]	0.9631 [0.94,0.99]	0.9579 [0.93,0.99]	0.9513 [0.92,0.98]
ρ_{e^π}	0.752 [0.64,0.85]	0.7548 [0.64,0.86]	0.7725 [0.69,0.87]	0.7539 [0.65,0.85]	0.7498 [0.64,0.86]	0.7649 [0.65,0.87]	0.8099 [0.72,0.89]
ρ_{e^i}	0.502 [0.35,0.64]	0.5056 [0.36,0.64]	0.5458 [0.42,0.69]	0.5108 [0.32,0.64]	0.5209 [0.36,0.67]	0.5133 [0.39,0.64]	0.7390 [0.64,0.84]
σ_{e^y}	4.25 [3.65,4.30]	4.2576 [3.67,4.92]	4.3890 [3.73,5.02]	4.1896 [3.63,4.83]	4.1920 [3.66,4.72]	4.3564 [3.87,4.87]	4.2437 [3.59,5.18]
σ_{e^π}	0.198 [0.16,0.24]	0.1959 [0.16,0.24]	0.2652 [0.22,0.32]	0.1974 [0.16,0.24]	0.1992 [0.16,0.25]	0.1929 [0.16,0.24]	0.1566 [0.13,0.19]
σ_{e^i}	0.167 [0.15,0.19]	0.1676 [0.15,0.19]	0.1694 [0.15,0.19]	0.1677 [0.15,0.19]	0.1663 [0.15,0.19]	0.1672 [0.15,0.19]	0.2260 [0.20,0.26]
Learning							
$\bar{g}$	0.014 [0.01,0.019]	0.0140 [0.01,0.02]	0.0082 [0.01,0.01]	0.0140 [0.01,0.02]	0.0140 [0.01,0.02]	0.0138 [0.01,0.02]	0.0256 [0.02,0.03]
Marginal Likelihood	**-671.56**	-673.35	-730.93	-672.21	-677.47	-673.24 (-671.23 for 4)	-706.55

TABLE 7. Posterior Estimates: Alternative Specifications - New Zealand. *Note*: The Table reports mean posterior estimates, along with 2.5% and 97.5% percentiles. The columns refer to the following specifications: (1) baseline; (2) lagged variables in the monetary policy rule; (3) nonlinear PLM; (4) same policy coefficients within and outside the band; (5) all monetary policy coefficients change in and out of the band; (6) forecast-based Taylor rule; (7) REE-based initial beliefs.

Param.	**Posterior Distributions**						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
'Deep'							
σ	1.95 [1.03,2.91]	2.423 [1.20,4.39]	2.0192 [1.19,3.15]	1.8968 [0.96,3.29]	2.4607 [1.54,3.33]	2.71 [1.50,4.38]	1.6979 [0.93,2.90]
α	0.825 [0.76,0.90]	0.829 [0.76,0.90]	0.851 [0.76,0.92]	0.8535 [0.78,0.92]	0.8324 [0.76,0.90]	0.8523 [0.78,0.93]	0.9103 [0.86,0.95]
η	0.59 [0.41,0.83]	0.63 [0.45,0.79]	0.6421 [0.47,0.83]	0.6481 [0.43,0.83]	0.6109 [0.47,0.73]	0.6374 [0.45,0.79]	0.6444 [0.40,0.87]
γ	0.517 [0.18,0.76]	0.529 [0.11,0.92]	0.5534 [0.09,0.91]	0.3769 [0.07,0.71]	0.3102 [0.05,0.61]	0.3012 [0.12,0.50]	0.0982 [0.02,0.28]
Policy							
ρ	0.934 [0.90,0.96]	0.956 [0.94,0.97]	0.9513 [0.94,0.97]	0.9556 [0.94,0.97]	-	0.9516 [0.93,0.97]	0.9389 [0.91,0.96]
$\chi_{\pi,in}$	1.702 [1.10,2.12]	1.433 [0.97,1.88]	1.2966 [0.89,1.85]	1.3870 [0.96,1.85]	1.4267 [1.14,1.79]	1.3432 [0.87,1.84]	1.6252 [1.09,2.40]
$\chi_{\pi,out}$	1.258 [0.75,1.82]	1.521 [1.09,1.93]	1.4575 [1.11,1.84]		1.1139 [0.84,1.39]	1.4865 [1.01,1.97]	1.2706 [0.83,1.70]
$\chi_{\pi,point}$	1.526 [1.16,1.97]	1.498 [0.99,2.06]	1.4156 [1.00,1.86]		1.2648 [0.92,1.55]	1.5196 [1.02,2.09]	1.4655 [1.01,1.86]
χ_y	0.111 [0.05,0.18]	0.136 [0.06,0.22]	0.1355 [0.06,0.20]	0.1253 [0.04,0.21]	-	0.1008 [0.03,0.16]	0.0357 [-0.04,0.11]
ρ_{in}					0.9537 [0.92,0.98]		
ρ_{out}					0.914 [0.84,0.96]		
ρ_{point}					0.9413 [0.91,0.97]		
$\chi_{y,in}$					0.1120 [0.01,0.20]		
$\chi_{y,out}$					0.0763 [0.00,0.14]		
$\chi_{y,point}$					0.1214 [0.01,0.19]		
Disturbances							
ρ_r	0.727 [0.60,0.85]	0.718 [0.57,0.84]	0.6815 [0.54,0.79]	0.7052 [0.57,0.83]	0.7238 [0.58,0.86]	0.7063 [0.56,0.83]	0.3963 [0.17,0.62]
ρ_u	0.102 [0.03,0.19]	0.114 [0.02,0.24]	0.0974 [0.02,0.21]	0.1276 [0.02,0.29]	0.1236 [0.03,0.27]	0.1314 [0.03,0.28]	0.0498 [0.01,0.12]
σ_r	1.05 [0.86,1.33]	1.11 [0.89,1.36]	1.1189 [0.91,1.39]	1.1234 [0.93,1.35]	1.0502 [0.86,1.23]	1.1257 [0.90,1.36]	1.2585 [0.94,1.63]
σ_u	0.881 [0.63,1.05]	0.902 [0.60,1.18]	0.9315 [0.62,1.24]	0.800 [0.59,1.01]	0.7421 [0.56,1.01]	0.7289 [0.60,0.88]	0.6131 [0.52,0.74]
σ_ε	0.132 [0.12,0.15]	0.124 [0.11,0.14]	0.1213 [0.11,0.14]	0.1236 [0.11,0.14]	0.1281 [0.11,0.15]	0.132 [0.12,0.15]	0.1344 [0.12,0.15]
Sentiment							
ρ_{e^y}	0.951 [0.90,0.98]	0.946 [0.91,0.98]	0.9369 [0.90,0.97]	0.9432 [0.91,0.98]	0.9485 [0.92,0.98]	0.9433 [0.91,0.98]	0.9746 [0.94,0.99]
ρ_{e^π}	0.84 [0.75,0.92]	0.848 [0.76,0.93]	0.8291 [0.71,0.93]	0.8464 [0.76,0.93]	0.8548 [0.75,0.94]	0.8647 [0.78,0.95]	0.8884 [0.81,0.95]
ρ_{e^i}	0.69 [0.57,0.84]	0.692 [0.52,0.84]	0.6878 [0.54,0.83]	0.6759 [0.50,0.79]	0.7016 [0.55,0.83]	0.6774 [0.55,0.82]	0.8258 [0.67,0.93]
σ_{e^y}	2.51 [2.17,2.98]	2.48 [2.17,2.84]	2.9951 [2.56,3.43]	2.5647 [1.99,3.06]	2.5133 [2.20,2.73]	2.5337 [2.28,2.79]	2.4698 [2.12,2.88]
σ_{e^π}	0.115 [0.10,0.14]	0.114 [0.09,0.14]	0.1534 [0.12,0.20]	0.1164 [0.10,0.14]	0.1113 [0.09,0.14]	0.110 [0.09,0.14]	0.1246 [0.10,0.15]
σ_{e^i}	0.08 [0.07,0.09]	0.080 [0.07,0.09]	0.081 [0.07,0.09]	0.0800 [0.07,0.09]	0.0765 [0.07,0.09]	0.0806 [0.07,0.09]	0.1552 [0.13,0.19]
Learning							
$\bar{g}$	0.011 [0.008,0.013]	0.011 [0.0152,0.0182]	0.0093 [0.007,0.011]	0.0108 [0.008,0.0133]	0.0105 [0.0082,0.0128]	0.0108 [0.008,0.0131]	0.0199 [0.018,0.022]
Marginal Likelihood	-336.47	-333.63	-375.68	**-331.49**	-353.60	-338.32	-437.16

TABLE 8. Posterior Estimates: Alternative Specifications - Sweden. *Note*: The Table reports mean posterior estimates, along with 2.5% and 97.5% percentiles. The columns refer to the following specifications: (1) baseline; (2) lagged variables in the monetary policy rule; (3) nonlinear PLM; (4) same policy coefficients within and outside the band; (5) all monetary policy coefficients change in and out of the band; (6) forecast-based Taylor rule; (7) REE-based initial beliefs.

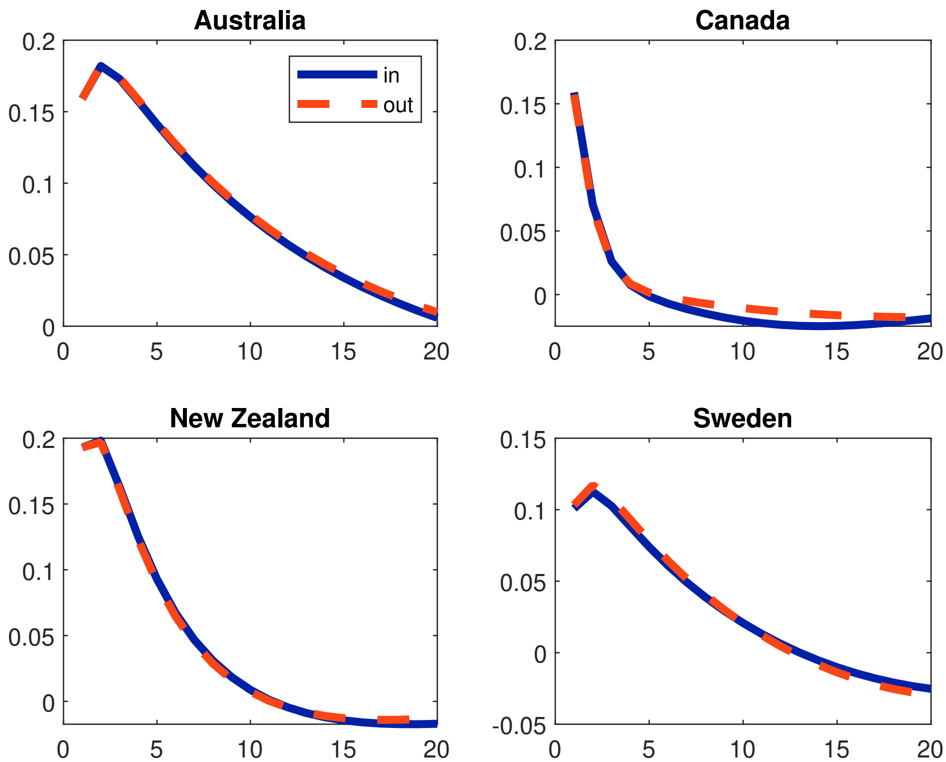

FIGURE 1. Response of Inflation to Inflationary Sentiment Shocks. *Note*: The Figure shows the impulse response functions of realized inflation to a positive inflation sentiment shock, across the four countries. Solid blue lines denote average responses when the inflation rate is within the band; dashed orange lines refer to average response when inflation falls outside the band.

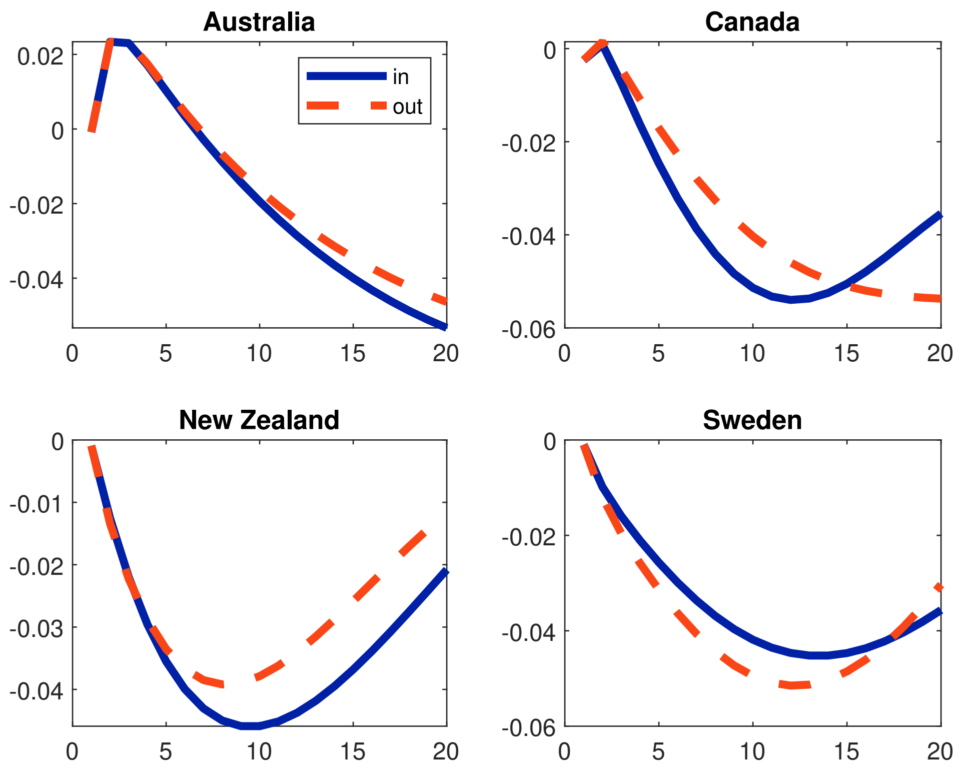

FIGURE 2. Response of Inflation to Contractionary Monetary Policy Shocks.
Note: The Figure shows the impulse response functions of realized inflation to a positive monetary policy shock, across the four countries. Solid blue lines denote average responses when the inflation rate is within the band; dashed orange lines refer to average response when inflation falls outside the band.

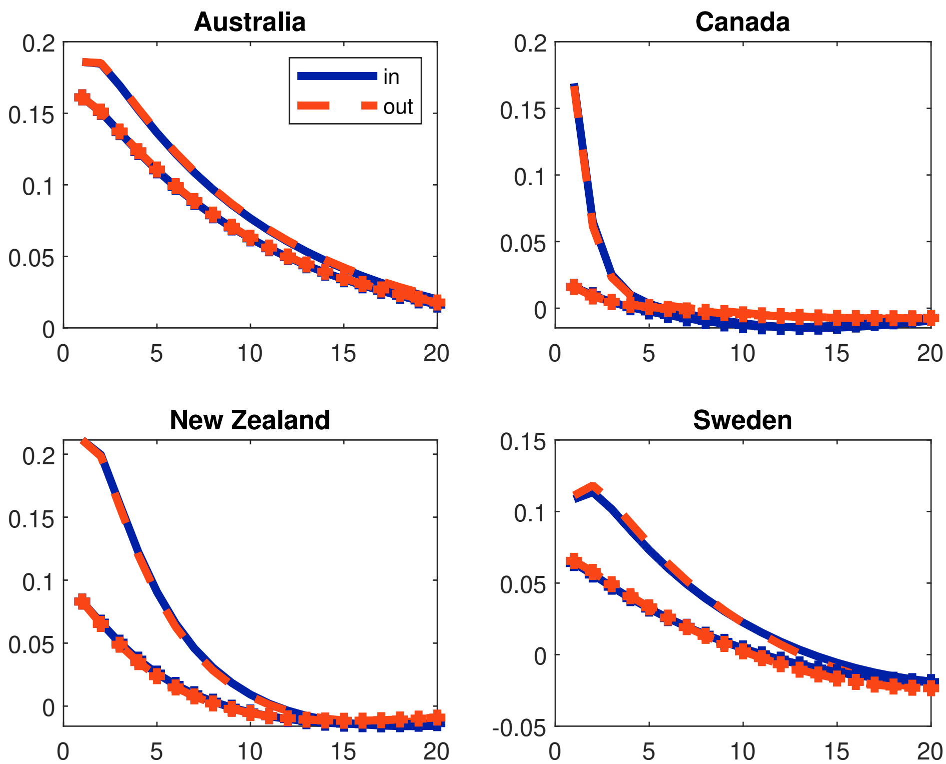

FIGURE 3. Response of Inflation Expectations to Inflationary Sentiment Shocks. *Note*: The Figure shows the impulse response functions of inflation expectations (at different horizons) to a positive inflation sentiment shock, across the four countries. Solid blue lines denote average responses when the inflation rate is within the band; dashed orange lines refer to average response when inflation falls outside the band. The lines with '+' markers (blue solid and orange dashed) display the responses of longer-term expectations (two quarters ahead for Australia, one year ahead for Canada, New Zealand, Sweden; the regular lines (blue solid and orange dashed, with no markers) show the responses of one-quarter-ahead expectations.

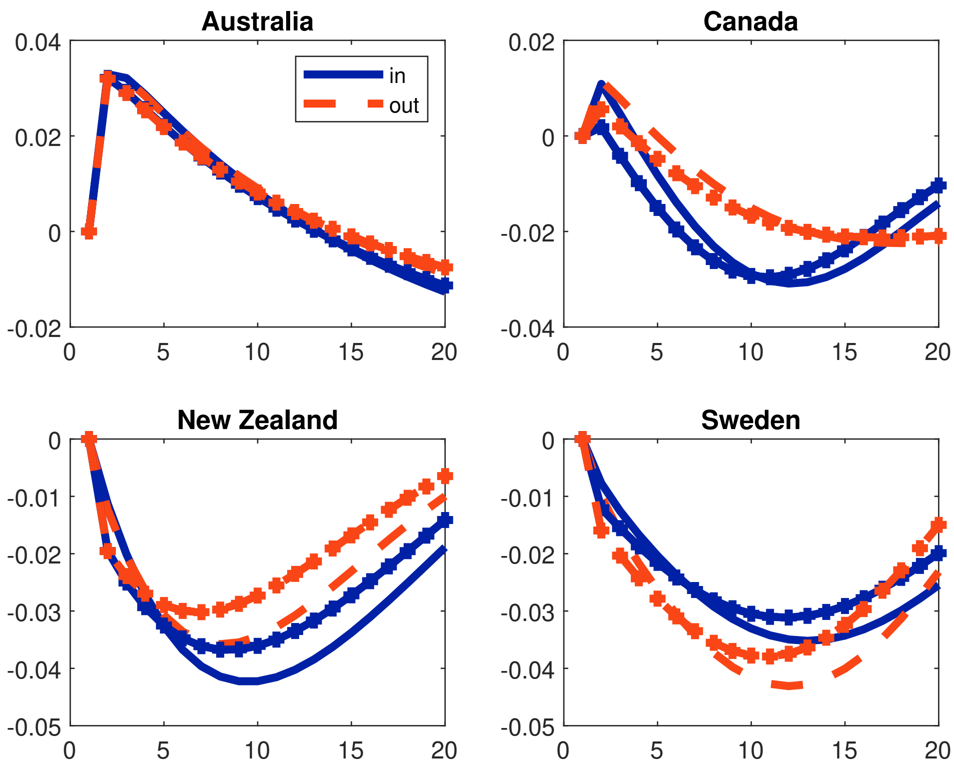

FIGURE 4. Response of Inflation Expectations to Contractionary Monetary Policy Shocks. *Note*: The Figure shows the impulse response functions of inflation expectations (at different horizons) to a positive monetary policy shock, across the four countries. Solid blue lines denote average responses when the inflation rate is within the band; dashed orange lines refer to average response when inflation falls outside the band. The lines with '+' markers (blue solid and orange dashed) display the responses of longer-term expectations (two quarters ahead for Australia, one year ahead for Canada, New Zealand, Sweden; the regular lines (blue solid and orange dashed, with no markers) show the responses of one-quarter-ahead expectations.

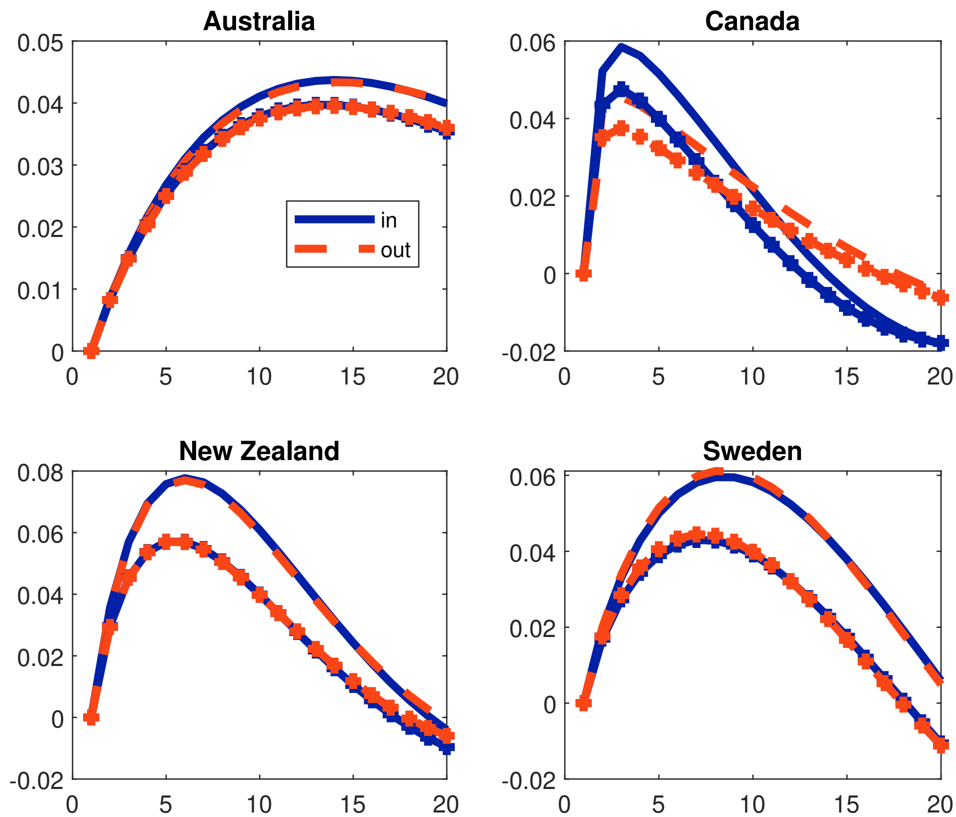

FIGURE 5. Response of Interest-Rate Expectations to Inflationary Sentiment Shocks. *Note*: The Figure shows the impulse response functions of interest-rate expectations (at different horizons) to a positive inflation sentiment shock, across the four countries. Solid blue lines denote average responses when the inflation rate is within the band; dashed orange lines refer to average response when inflation falls outside the band. The lines with '+' markers (blue solid and orange dashed) display the responses of longer-term expectations (two quarters ahead for Australia, one year ahead for Canada, New Zealand, Sweden; the regular lines (blue solid and orange dashed, with no markers) show the responses of one-quarter-ahead expectations.