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## Structural Factor-Augmented VARs (SFAVARs) and the Effects of Monetary Policy

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# Structural Factor-Augmented VARs (SFAVARs) and the Effects of Monetary Policy\*

Francesco Belviso and Fabio Milani

## Abstract

Factor-augmented VARs (FAVARs) have combined standard VARs with factor analysis to exploit large data sets in the study of monetary policy. FAVARs enjoy a number of advantages over VARs: they allow a better identification of the monetary policy shock; they avoid the use of a single variable to proxy theoretical constructs; they allow researchers to compute impulse responses for hundreds of variables. Their shortcoming, however, is that the factors are not identified and lack an economic interpretation.

This paper seeks to provide an interpretation to the factors. We propose a novel Structural Factor-Augmented VAR (SFAVAR) model, where the factors have a clear meaning: Real Activity factor, Inflation factor, Financial Market factor, Credit factor, Expectations factor, and so forth. The paper employs a Bayesian approach to jointly estimate the factors and the dynamic model. This framework is then used to study the effects of monetary policy on a wide range of macroeconomic variables.

**KEYWORDS:** VAR, dynamic factors, monetary policy, structural FAVAR

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# 1 Introduction

Vector autoregressions (VARs) are a standard framework to study the effects of monetary policy shocks on macroeconomic variables. With few exceptions, the VARs employed in the literature are fairly small in order to save degrees of freedom. Typical monetary VARs in fact include a measure of output or the output gap, a measure of inflation, the federal funds rate, and few other variables.<sup>1</sup> The small number of variables, however, is at odds with the information set actually available to central banks. Indeed, central banks in the real world monitor a huge amount of economic data and indicators.

Since a failure to account for the true information set available to the policymaker leads to an incorrect measurement of monetary policy innovations, recent studies have attempted to incorporate larger information sets in VARs. Bernanke and Boivin (2003) and Bernanke, Boivin, and Elias (2005) have combined VAR models with factor analysis to measure the effects of monetary policy in a “data-rich” environment. They develop Factor-Augmented VARs (FAVARs), in which they add common factors to a standard VAR specification.

With the FAVAR approach, however, it is impossible to assign any sort of economic meaning to the estimated factors. In this paper, we follow the recent literature, but we try to propose a solution, which can make the economic interpretation of the factors possible.

We analyze monetary policy and the dynamics of the economy exploiting more information than a typical VAR analysis. We start from the FAVAR approach and individuate plausible restrictions that allow us to give a structural interpretation to the factors. That is, we seek to identify each factor as a basic force that governs the economy. Therefore, we propose a vector autoregression augmented with what we think are more economically interpretable (and in this sense more ‘structural’) factors: we label this novel approach Structural Factor-Augmented VAR (SFAVAR).

In the analysis we include: a real activity factor, which we regard as more suitable than a single observable variable to capture the theoretical and unobservable macroeconomic concept of ‘output gap’, an inflation factor, a long-term interest rates factor, a financial market factor, and money and credit factors. We also include an expectations factor that may shed light on the interactions between expectations and the real economy.

Our proposal shares FAVARs’ advantages over conventional VARs. First, as Bernanke, Boivin, and Elias (2005) emphasize, FAVARs allow a better

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<sup>1</sup>Christiano, Eichenbaum, and Evans (2000) is a standard reference. Leeper, Sims, and Zha (1996), using Bayesian methods, manage to estimate larger VARs, but still with fewer than 20 variables.

identification of the monetary policy shock, since they condition on a more realistic information set. Moreover, in low-dimensional VARs, impulse responses can be derived only for the few included variables. By contrast, FAVARs allow us to construct the impulse responses for all the numerous economic series used in the construction of the factors.

Other recent papers apply restrictions to the FAVAR model to allow a clearer interpretation of the factors. Forni et al. (2004), for example, exploit factors in a structural VAR to identify the response of macroeconomic variables to a long-run (productivity) shock. Justiniano (2004) adopts Bayesian methods to derive factors that can be interpreted as country-specific shocks. Similarly, Sala (2003) analyzes the transmission of common monetary shocks across European countries. A dynamic factor model is used to extract the common European monetary shock.

We use likelihood-based Bayesian methods and Gibbs sampling to jointly estimate factors and VAR parameters on U.S. data. In this way, we can exploit VAR dynamics when extracting the factors. A similar methodological approach has been followed by Bernanke, Boivin, and Elias (2005) and by Kose, Otrok, and Whiteman (2000). On the contrary, standard approaches in the factor analysis literature consist of deriving factors either through principal components, as in the approximate factor model of Stock and Watson (2002), or through spectral analysis, as in Forni et al. (2000). In those cases, the estimation works in two steps. First the factors are extracted and then they are taken as given for the estimation of the model parameters. An advantage of our Bayesian approach is that it facilitates the introduction of restrictions on the loadings, thus facilitating also the economic interpretation of the factors. Moreover, the joint estimation allows us to associate the factors with an accurate indication of the uncertainty surrounding their estimation.

There are two potential disadvantages to our approach. First, to extract the factors using our Bayesian approach, we need to impose assumptions about the errors that may not be met empirically, even though they are typical in this literature. Second, the economic interpretation of our restrictions may be questioned. The assignment of variables to different groups, which allows the interpretation of the factors, carries elements of arbitrariness. SFAVARs' advantage over conventional FAVARs hinges on the plausibility of those restrictions.

We use our SFAVAR model to evaluate the responses of a wide range of macroeconomic variables to monetary policy shocks. Moreover, the estimation allows us to evaluate which variables are important to study the dynamics of the economy as a whole.

## 2 The Model

Let  $Y_t$  and  $X_t$  be two vectors of economic variables, with dimensions  $M \times 1$  and  $N \times 1$  respectively and where  $t = 1, 2, \dots, T$  is a time index.  $Y_t$  denotes the policy instrument controlled by the central bank, such as the Federal Funds rate in the U.S., and  $X_t$  is a large data set of economic variables. Assume that there exist some unobservable fundamental forces that affect the dynamics of  $X_t$ , which can be summarized by a  $K \times 1$  vector of factors  $F_t$ , so that

$$X_t = \Lambda F_t + e_t \tag{1}$$

where  $e_t$  are errors, such that  $E(e_t | F_t) = 0$  and  $E(e_{m,t}e_{n,t}) = 0$  for all  $m, n = 1, \dots, N$  and  $m \neq n$ . Take a partition of  $X_t$ , say  $X_t^1, X_t^2, \dots, X_t^I$ , where  $X_t^i$  is a  $N_i \times 1$  vector and  $\sum_i N_i = N$ . Assume that each of the vectors  $X_t^i$  is now explained by only some of the elements of the vector  $F_t$ . That is, there is a partition of  $F_t$  given by  $F_t^1, F_t^2, \dots, F_t^I$  where  $F_t^i$  is a  $K_i \times 1$  vector,  $\sum_i K_i = K$  and  $K_i < N_i$  for all  $i$ . Also, assume that  $X_t^i$  is solely explained by  $F_t^i$ . Hence we have

$$\begin{bmatrix} X_t^1 \\ X_t^2 \\ \dots \\ X_t^I \end{bmatrix} = \begin{bmatrix} \Lambda_1^f & 0 & \dots & 0 \\ 0 & \Lambda_2^f & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \Lambda_I^f \end{bmatrix} \cdot \begin{bmatrix} F_t^1 \\ F_t^2 \\ \dots \\ F_t^I \end{bmatrix} + e_t \tag{2}$$

where  $E(e_t | F_t^1, \dots, F_t^I) = 0$  and  $E(e_{m,t}e_{n,t}) = 0$  for all  $m, n = 1, \dots, N$  and  $m \neq n$ . The restriction we impose on the model is that each element of  $X_t$  is influenced by the state of the economy only through the corresponding factor. For the rest of the paper, we assume that each segment of  $X_t$  is explained by exactly one factor, that is  $K_i = 1$  for all  $i$ . This is in principle a testable assumption. It is possible to derive the number of factors for each subgroup, in fact, that leads to the highest posterior model probabilities. When two or more factors are needed, a possible solution, besides allowing for the additional factor, would be to split the original category and check if one factor becomes sufficient for each of the new categories. Due to the computational intensity of the model comparison exercise, we focus in this paper on the simpler case of one factor for each category and leave the model comparison for future research.<sup>2</sup>

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<sup>2</sup>Some quick evidence on the plausibility of the single factors may be gauged in a principal component framework by looking at the percentage of variation explained by the first factor. In our case for 7 of the 9 cases, the first component explains 54 to 94 percent of the total variability; in two cases it explains slightly less than 50 percent.

Also assume that the dynamics of  $(Y_t, F_t^1, F_t^2, \dots, F_t^I)$  is given by a factor-augmented vector autoregression:

$$\begin{bmatrix} F_t^1 \\ F_t^2 \\ \dots \\ F_t^I \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1}^1 \\ F_{t-1}^2 \\ \dots \\ F_{t-1}^I \\ Y_{t-1} \end{bmatrix} + \nu_t \quad (3)$$

where  $\Phi(L)$  is a conformable lag polynomial of finite order  $d$  and  $\nu_t$  is an error term such that  $E(\nu_t | \Omega_{t-1}) = 0$  for  $\Omega_{t-1} = \{F_{t-j}^1, \dots, F_{t-j}^I, Y_{t-j}\}_{j=1,2,\dots}$ . Clearly, the difference between this model and a standard VAR is the presence of unobservable factors.

Therefore, our main identifying assumption requires that the errors are uncorrelated both within factor subgroups and across different subgroups. This assumption may not be met empirically and represents an important drawback of our approach. For instance, according to the model, we should have  $E[X_{m,t}^1 X_{n,t}^2 | F_t] = 0$ . This means that, conditional on the factors, the contemporaneous covariances among variables should all be zero. One implication is that stock returns should have zero contemporaneous correlation with interest rates, inflation, and output, which may be unrealistic. In other words, we assumed that the relation (and so correlation) of the variables is completely explained by the value of the factors. If this assumption is not met empirically, not only our model may be statistically misspecified, but the economic interpretation of the factors may be questionable. Our assumption on the residuals is testable and we will provide some evidence later, when discussing the results of our estimation.

Our contribution is given by the set of restrictions illustrated in equation (2). The vector of economic variables  $X_t$  can be divided in subsets of similar variables. For example, a subset of variables related to the real activity, a subset of variables related to inflation, and so on. The common force that moves these variables, the dynamic factor, is now economically interpretable. For instance, these forces represent wide concepts such as economic activity, basic movements in prices, and so forth.

Bernanke, Boivin, and Elias (2005) provide a motivation for a standard FAVAR model in the context of a simple macroeconomic model, to explain why researchers need to condition their models on a richer information set. As a model of central bank's behavior, our framework similarly assumes that the central bank observes only the policy instrument  $Y_t$  (the federal funds rate) and a large set of noisy indicator variables  $X_t$ . Alternative information assumptions, however, are easily introduced in the framework.

Our SFAVAR approach enjoy some advantages over the estimation of simple VARs. First, using factors may reduce measurement problems. Some factors are in fact extracted from similar variables, such as disaggregate or regional versions of the main variable. For instance, a ‘Real Activity’ factor can be extracted, among other series, from ‘New Orders in durable good industries’ as well as ‘New Orders in non-defense capital goods’.

But what is the nature of the structural factors? Factors are more than simple re-aggregation of variables. Indeed, in our model the loadings are also unknown and need to be estimated. Hence, what criteria does the model use when fixing the loadings?

The Bayesian joint estimation of equations (2) and (3) helps answering this question. Factors are the unobserved variables that determine at the same time the value of all the other variables in the economy and the dynamics of the whole economy. Indeed each factor, through equation (2), is solely responsible for today’s value of the variables related to it, with the exception of an idiosyncratic error. This error may correspond to measurement errors as well as true idiosyncratic (i.e. relative to a single sector or region) shocks to the single variable.

Factors, together with the policy instrument, also enter the VAR equation (3). That is, given the state of the economy today, the future depends only on the level of current and past values of the factors and policy instruments. All the idiosyncratic shocks will be ‘reabsorbed’. That is, we expect that an idiosyncratic shock to a single variable will not affect the path of the economy.

Continuing the example of the ‘Real Activity’ factor, it may be that for a few months ‘New Orders in durable good industries’ may be well above average. But this does not necessarily mean that the whole economy will be affected by such sectorial shocks. In our framework this is equivalent to say that we do not expect the general level of production, inflation, or of the other fundamental forces of the economy, to be affected. Hence, with our estimation we try to ‘clean’ the dynamics of the observed variables to find the main interactions between the different parts of the economy.

### 3 The Estimation

In this section we describe the Bayesian estimation of the model. Before going into details about the Bayesian method, we discuss the alternative procedure of Principal Components estimation. Both approaches have advantages and disadvantages.

Principal Components estimation of the factors is computationally simple

and requires much less computer time. Moreover it employs a semi-parametric, rather than fully parametric, approach. The reason we prefer the Bayesian joint estimation to principal components is that the Principal Components approach constructs the estimated factors using only (2), and thus it ignores the restrictions on the dynamics of the factors given by (3). Indeed, as discussed by Eliasz (2002), the factors estimated by Principal Components have unknown dynamic properties. Loosely speaking, the factors estimated by PC are an unknown moving average of some more fundamental factors, where the fundamental factors are identified through the VAR dynamics. As we have already discussed, considering the dynamics of the factors is important for their estimation and interpretation.

In addition, the higher complexity of the Bayesian estimation is repaid by an easier assessment of the level of uncertainty: the error bands are simple to construct and to interpret. Moreover, the number of variables  $N_i$  in each subsegment  $X_t^i$  can be rather small. Therefore, the standard asymptotic results may no longer hold, since Principal Components give consistent estimates only for  $T$  and  $N_i$  both limiting to infinity. This complication does not arise with the Bayesian approach.

In this paper we will use Principal Components estimation to generate alternative starting values for the Bayesian Procedure. We describe the Principal Components estimation procedure in Appendix D.

To perform Bayesian estimation, we impose normality for the errors. Hence, the model can be written as:

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Lambda & 0 \\ 0 & I_M \end{bmatrix} \begin{bmatrix} F_t \\ Y_t \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix} \quad (4)$$

where  $\Lambda$  has all the restrictions we have imposed in (2), and

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \nu_t. \quad (5)$$

where  $e_t \sim i.i.d.N(0, R)$  with  $R$  diagonal,  $\nu_t \sim i.i.d.N(0, Q)$ ,  $E(e_t|F_t) = 0$ ,  $E(\nu_t | \Omega_{t-1}) = 0$  for  $\Omega_{t-1} = \{F_{t-j}^1, \dots, F_{t-j}^I, Y_{t-j}\}_{j=1,2,\dots}$ , and  $\nu_t$  and  $e_t$  are independent. These are the main identifying assumptions. As discussed in the previous section, we are aware that some of those assumptions may not hold empirically, even though they are common in the Bayesian factor analysis literature. To identify the factors, we impose the restriction that the first element of  $\Lambda_i^f$  is one for all  $i$ . The estimation procedure is discussed in Appendix A.



## 4 Empirical Framework

### 4.1 Structural Factors

We partition the vector of economic variables  $X_t$  so that each variable is explained by one of the following structural factors:

- **REAL ACTIVITY factor.** This factor can be re-conducted to the theoretical macroeconomic concept of ‘output gap’, providing a summary of the state of real activity. It determines variables such as industrial production, capacity utilization, employment indicators, inventories, new and unfilled orders, housing starts, and consumer expenditures.
- **INFLATION factor.** This factor indicates a broader concept of inflation, incorporating data regarding a variety of consumer and producer prices, wages, and oil prices.
- **INTEREST RATES factor.** This factor explains public and private bonds yields at different maturities.
- **FINANCIAL MARKET factor.** The introduction of this factor is motivated by the recent interest in evaluating whether monetary policy responds to movements in asset prices (see Bernanke and Gertler 2001, among others); moreover, this factor allows us to verify the relevance of a financial market channel of monetary policy transmission. It includes stock price and dividend indexes.
- **MONEY factor.** This factor explains a number of money stock variables.
- **CREDIT factor.** This factor allows us to verify the empirical importance of the credit channel of monetary transmission, which is potentially important but usually disregarded in standard VARs. It includes many private credit and loans variables.
- **EXPECTATIONS factor.** Expectations regarding production, employment, inventories, new orders, and future inflation are taken from NAPM and other surveys. The dynamics of expectations with respect to the other variables of the system is an interesting issue to examine.

The complete list of variables and their associated factors are reported in Appendix B. Finally, we assume that  $Y_t$ , the policy variable, is exogenously

set by the central bank. In our estimation the policy variable is the Federal Funds rate.

The choice of the variables and the assignment of variables to categories has important elements of arbitrariness. It is admittedly hard to argue that the expectations variables coming from NAPM surveys about production or prices are driven only by the Expectations factor, and they do not contemporaneously respond to the Real Activity or Inflation factors. Or that long term rates are only driven by the Interest Rates factor and not by the Inflation factor as well. Some of these problems may be partially solved by a different regrouping of the variables or by extracting more than one factor from some of the categories (for example, extracting three factors, “level”, “slope”, and “curvature”, for the interest rates group).

## 4.2 SFAVAR Estimation

The data set builds upon the balanced panel employed by Stock and Watson (2002). Their data set consists of 120 monthly time series, covering a sample from January 1960 to December 1998. We extend this panel of data by adding several other variables, mainly for the money and credit sectors.<sup>3</sup> We also eliminate some variables whenever an aggregate variable and its components are both present in the data set (see below). We end up with a balanced data set consisting of 145 variables for estimation, spanning the period 1960:01-1998:12. All the series have been transformed to reach stationarity, if necessary. The series have also been demeaned and standardized.<sup>4</sup>

In the VAR we consider 13 lags for all the variables to allow sufficient dynamics. We jointly estimate the system (2)-(3) by Gibbs sampling as illustrated in section 3. The total number of parameters and factors to be estimated is 4,453, so that we have approximately 15 data points for each parameter. The estimates are based on 5,000 draws, with the first 2,000 omitted to reduce the influence of the initial guess on final results.

In Appendix C we evaluate the convergence of the Gibbs sampler. We perform a series of diagnostics, both through graphical methods and tests, to explore convergence properties of our estimation. Moreover, we consider multiple, distinct starting points to check the robustness of our estimates. The results suggest that convergence has been achieved and that our results are robust to different starting values.

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<sup>3</sup>The additional data are taken from FRED, the database of the Federal Reserve Bank of Saint Louis or from Datastream.

<sup>4</sup>Appendix B reports our data set with the complete list of variables grouped by their factor, their source and the transformations applied.

In our model, each variable within a factor group is the underlying factor plus an idiosyncratic error. An important assumption is that these errors are uncorrelated across variables. As extensively discussed by Eliaz (2002), this assumption is standard in the FAVAR literature, independently from the estimation method. While we can expect the factor to explain common fluctuations in a group, it is likely that some clusters of variables may retain their stronger correlation even once we control for the factor. An approach to this problem is to use the data to test if one factor or more than one are needed to explain fluctuations of the variables in a group. This could be done together with a careful choice of which variables to exclude *a priori* from the sample, which is equivalent to impose additional restrictions on the loadings. We have eliminated the most obvious ones: for instance, we eliminated “Industrial Production: consumer goods” given that we have “Industrial Production: durable consumer goods” and “Industrial Production: nondurable consumer goods”.<sup>5</sup>

As discussed when we presented the model, we assumed that the errors are uncorrelated not only within, but also across different factor groups. An implication of this assumption is that, conditional on the estimated factors, variables in a group should have zero contemporaneous correlations with variables in the other groups. We have calculated some correlations of estimated errors for variables that are not in the same factor group: some of these correlations are large, contradicting our assumption.

## 5 Results

Figure 1 shows the factors estimated with the Bayesian procedure and their 95% probability bands. The error bands are almost indistinguishable from the estimated series, signalling that factors are tightly estimated.

In Figure 2, we plot the estimated loadings for each factor. Given that the original time series have been standardized, we can directly compare the loadings to assess the importance of a given variable in the determination of the respective factor and in the aggregate dynamics of the economy. The loadings

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<sup>5</sup>While we do not perform tests, we can analyze some representative correlations of estimated errors. We calculate mean correlations over the last 1,000 draws. The result is that in general correlations are low, but some are remarkably high. For instance, we can consider the variables 101 and 102 (refer to Appendix B for series’ ID), two S&P’s stock price indexes: their errors have correlation 1.00. The correlation for deviations of 36 and 37, consumption expenditure for durables vs nondurables, is 0.49. However, the correlation is only 0.14 for deviations of 2 and 3: production of durable consumer goods vs nondurable. These results suggest that attention is needed when extracting a limited number of factors from numerous variables.

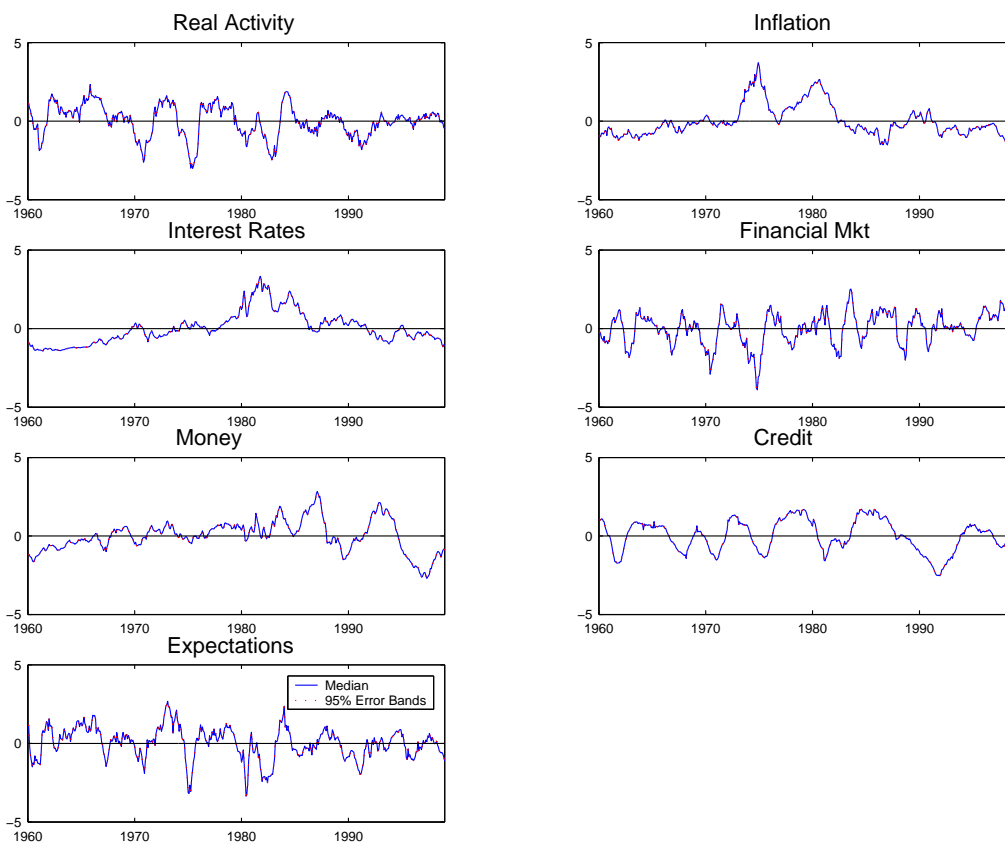


Figure 1: Estimated structural factors with error bands.

generally have comparable absolute values. This implies that the factors do not just closely follow a single variable: almost all of the included variables are selected by the estimation to determine the factors.

For a given normalization, each loading has sign consistent with our structural interpretations of the factors. For instance, unemployment rate and unemployed workers disaggregated by duration (series 14-18) all have negative loadings in the Real Activity factor. It is also possible to use the estimates to evaluate which are the most relevant series for the aggregate dynamics. For instance we observe that, in the Money factor, the aggregate M1 (series 108) has loading 1, M2 (109) has loading 0.27 and M3 (110) has loading 0.11. Hence the model selects markedly in favor of the less inclusive description of the money stock.

Given that our factors are economically interpretable, we can examine their responses to a monetary policy shock. We identify the system using a Cholesky

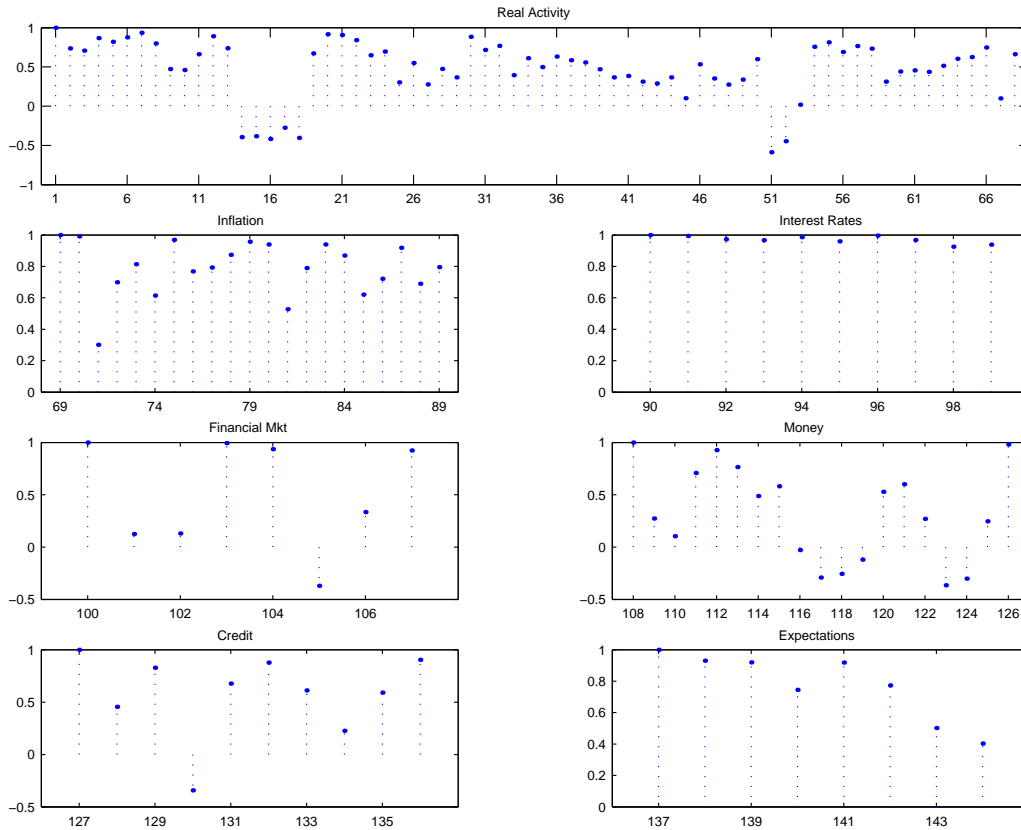


Figure 2: Estimated loadings. The labels on the x-axis refers to the variable’s ID (see Appendix B).

decomposition.<sup>6</sup> Therefore, we recursively order the variables. One issue arising in our model is the presence of the Interest Rates factor, which includes data on several long-term interest rates. Allowing the Federal Funds rate to respond to several market rates would potentially lead to indeterminacy. We could face an identification problem, with the risk of confusing an arbitrage condition with the policy rule. This issue is discussed in greater detail in Leeper, Sims, and Zha (1996). In a similar context, they assume that the policy maker can observe and react to the state of the economy. Therefore, variables dependent on expectations about the economy, such as long-term interest rates, do not contain additional information besides what is directly

<sup>6</sup>Other identification schemes are possible and can be easily accommodated in our framework, for instance exploiting long-run restrictions. We use the relatively simple Cholesky decomposition to keep the computational costs at a minimum.

observed. For this reason, we similarly assume that the monetary authority does not react to the Interest Rates factor. Following the same line of reasoning, assuming that it is possible for the monetary authority to observe the current state of the economy, we exclude a contemporaneous response of the policy rate to the Expectations factor. For the Cholesky ordering, the Interest Rates factor and Expectations are therefore ordered after the Federal Funds rate while the Federal Funds rate can respond immediately to the other factors. Symmetrically, the other factors can react to policy only with one or more lags. We consider the following ordering of factors and the policy rate: Inflation, Real Activity, Credit, Money, Financial Market, Federal Funds rate, Interest Rates, and Expectations. Note that even if monetary and financial variables are likely to react faster than one or two months to policy innovations, Federal Funds rate changes happen after Federal Open Market Committee (FOMC) meetings, which take place approximately every six weeks. Since the variables in our data set are monthly averages, a response within the same month would be incorrect if the meeting is not held in the first days of the month.<sup>7</sup>

Figure 3 shows the impulse responses for all the factors and the Federal Funds rate to a one standard deviation monetary policy shock. The error bands represent 68% probability bands, derived as the 16<sup>th</sup> and 84<sup>th</sup> percentile of the obtained response functions from Gibbs sampling. This procedure should give a more accurate indication of the total uncertainty, since it takes also into account the uncertainty surrounding the estimation of the factors as well as of the VAR parameters. The estimated error bands are tight compared to other VAR studies as Sims and Zha (1999) and Waggoner and Zha (2003). We can rule out the possibility that this is due to a series of draws close to the initial guess of the parameters. Indeed, tight error bands are obtained for various initial guesses, as shown in Appendix C. Moreover, in the same Appendix we show that the draws for many parameters have non-negligible variances. Hence it is likely that disperse draws of the parameters, while exploring the distribution, deliver similar estimated impulse response functions.

The price puzzle is alleviated. Inflation declines significantly around eight months after the shock. The price puzzle has usually been related to the omission of relevant information in the VAR (see Sims 1992). It is argued that by incorporating the knowledge central banks have when setting policy, the puzzle should disappear. This is usually accomplished by adding a commodity price measure in the VAR. In our case, the data rich model doesn't display

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<sup>7</sup>We calculate the correlation of the policy shock with contemporaneous shocks to the various factors. Our assumption is that these shocks are uncorrelated. We obtain that the estimated correlations are small with the only exception of the shocks to the Interest Rates Factor, whose correlation with the policy shocks is 0.3 with standard deviation 0.05.

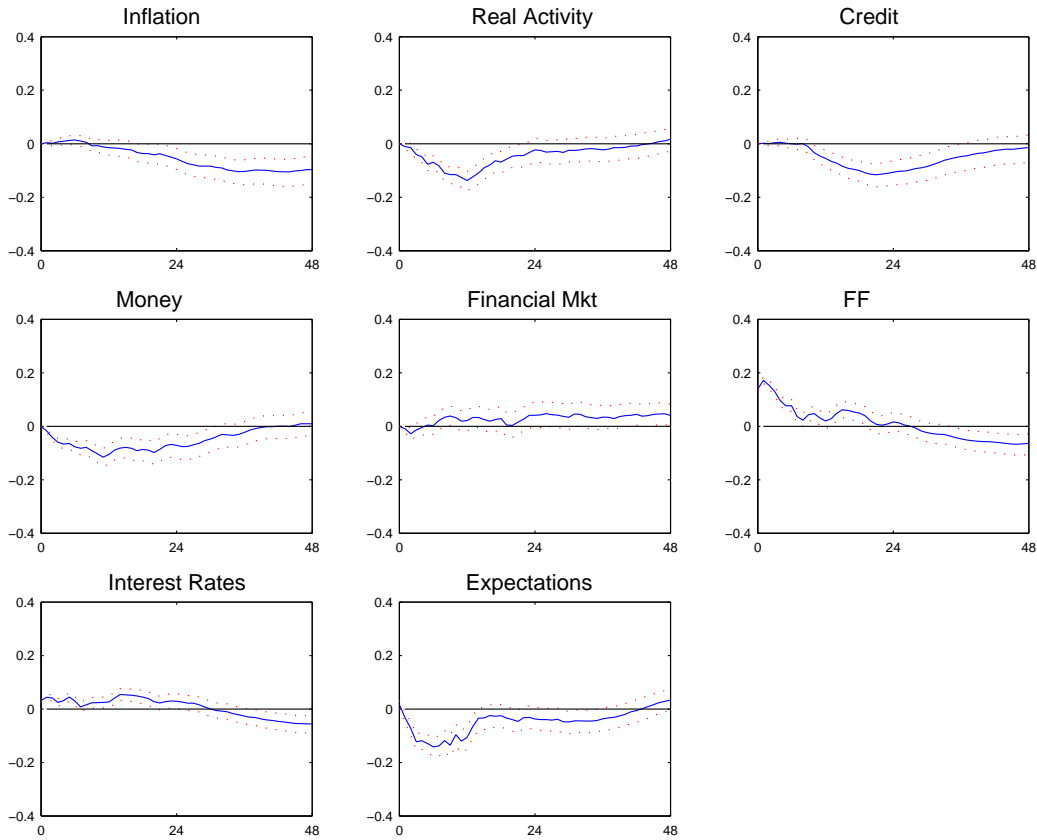


Figure 3: Impulse responses to a (one standard deviation) monetary policy shock.

the usual price-puzzle dynamics.

Real Activity drops, reaching the minimum one year after the shock, and then returns to the previous level after slightly more than two years, showing the usual hump-shaped behavior.

Credit mimics the response of the Real Activity factor, but with a delay of about 9 months and a more sluggish response. Money shows a quick and persistent downward adjustment, and it returns to the initial level only after three years. The Financial Market factor has a small downward adjustment at first, but it is mainly unaffected by the policy shock afterwards.

After a monetary contraction, we notice a hump-shaped downward adjustment of the Expectations factor, which anticipates the Real Activity factor by six to nine months for the whole time of the adjustment. The result is consistent with the values of the loadings in the Expectations factor, which mainly

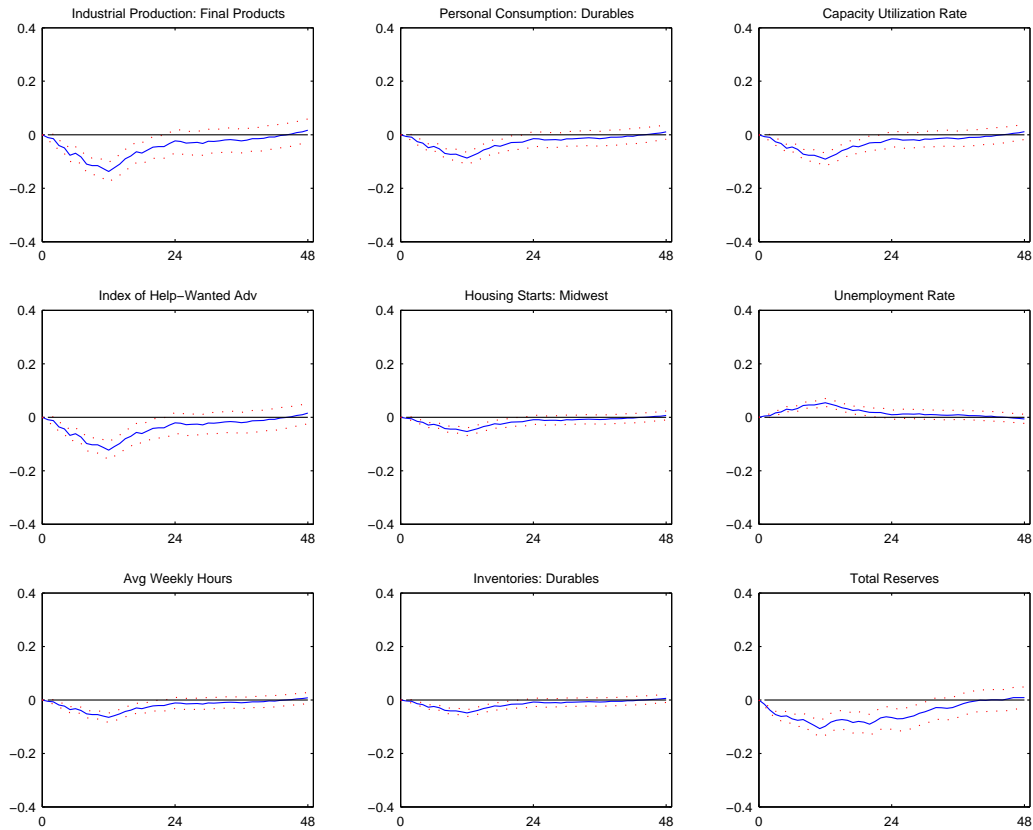


Figure 4: Impulse responses to a monetary policy shock of various variables.

denote expectations about real activity variables. Therefore, the agents of the economy appear to correctly anticipate future movements of output following an innovation in the Federal Funds rate.

A particular advantage of the factor-augmented framework is that we can derive impulse responses not only for the fundamental factors, but also for all the variables in our data set. We show the impulse responses to a monetary policy shock for some of the most interesting variables in Figures 4 and 5.

We notice that a positive shock to the federal funds rate reduces industrial production, the capacity utilization rate, and to a lesser extent, inventories. A contractionary shock increases the unemployment rate and, with a larger impact, vacancies. Spot oil price is largely unaffected.

It would be interesting to compare the fit of our restricted model as in (2) with the more conventional FAVAR approach. As already discussed, the paper aims to show that our restrictions simplify the economic interpretation of the



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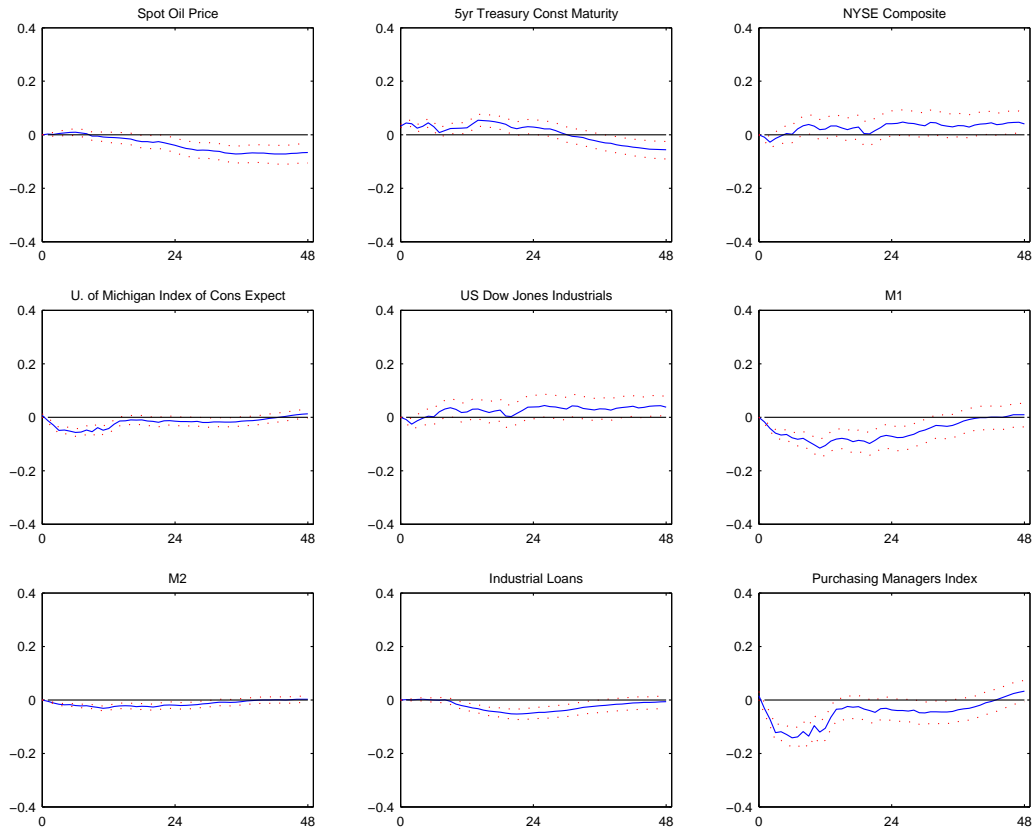


Figure 5: Impulse responses to a monetary policy shock of various variables.

factors. The fit of the restricted SFAVAR model may be superior or inferior depending on the particular application and on the choice of the subcategories. It may be possible that the estimation of fewer parameters may be beneficial for forecasting. A serious model comparison exercise, based on the log marginal likelihoods or on the forecasting performances, is beyond the scope of the current paper and is left for future research.

We argued that the Inflation factor may be preferred to a single inflation variable. A possible drawback, however, arises in the context of small monetary models. In these models, the policy reaction coefficient to inflation has important effects on the equilibrium properties of the economy. Our normalization of the Inflation factor is necessarily arbitrary and, therefore, makes the sign and size of the reaction coefficient harder to interpret.

## 6 Conclusions

Recent research has combined VAR models with factor analysis, leading to advances in the measurement of monetary policy effects. This literature has permitted researchers to incorporate larger and more realistic information sets. The main shortcoming, however, has been the inability to identify the factors, which typically lack an economic interpretation.

We suggest here a possible solution by proposing a factor-augmented VAR in which we seek to provide a structural interpretation to the factors. The factors have a more immediate economic meaning, since they explain different subcategories of the data.

We employ a Bayesian approach to estimate the factors jointly with the rest of the system, therefore exploiting the VAR dynamics to extract them. This approach allows us to study impulse responses that are obtained conditioning on a larger and more realistic amount of information.

We have pointed out in the paper, however, how particular assumptions about the errors are needed to identify the factors. Some of those assumptions are hardly met empirically. Moreover, the assignment of variables to different subcategories necessarily implies some arbitrary choices. Also, assuming that a single factor is enough to explain all the variables in each category may be overly optimistic. Through Bayesian model comparison, it is possible, however, to choose the optimal number of factors, by computing the posterior model probabilities. A more serious consideration of all these open issues is left for future research.

Also, in future research, we plan to incorporate more structure in our factor-augmented VAR. The Bayesian approach to extract the factors is rather flexible and it can be exploited to impose alternative restrictions on the loadings. An interesting extension, for example, would consist of using long-run restrictions to identify the impulse responses to technology shocks and demand shocks, in the context of our SFAVAR framework. The model may be also useful to study the effects of region-specific versus Country-specific shocks (for example, in the Euro area context) or to provide an interpretation to the factors that are helpful in explaining the term structure of interest rates (see Mönch 2005 for an analysis of the yield curve in a data-rich environment). Finally, the economically interpretable factors might be incorporated in a Dynamic Stochastic General Equilibrium (DSGE) model, following Boivin and Giannoni (2005).

## A Likelihood-Based Gibbs Sampling

We estimate the parameters  $\theta = (\Lambda, R, \text{vec}(\Phi), Q)$  and the factors  $\{F_t\}_{t=1}^T$ . We start from the state-space model in (4) and (5), where  $\Lambda$  is restricted as described in the text,  $e_t \sim \text{i.i.d. } N(0, R)$ ,  $\nu_t \sim \text{i.i.d. } N(0, Q)$ ,  $E(e_t | F_t) = 0$ ,  $E(\nu_t | \Omega_{t-1}) = 0$  for  $\Omega_{t-1} = \{F_{t-j}^1, \dots, F_{t-j}^J, Y_{t-j}\}_{j=1,2,\dots}$ ,  $\nu_t$  and  $e_t$  are independent, and  $R$  is diagonal. We can use Gibbs sampling to estimate the model. We closely follow Elias (2002), to whom we refer for more details.

We can rewrite the model defining  $\mathbf{X}_t = (X'_t, Y'_t)'$ ,  $\mathbf{F}_t = (F'_t, Y'_t)'$ , and  $\mathbf{e}_t = (e'_t, 0, \dots, 0)'$ :

$$\mathbf{X}_t = \Lambda \mathbf{F}_t + \mathbf{e}_t \quad (6)$$

$$\mathbf{F}_t = \Phi(L) \mathbf{F}_t + \nu_t \quad (7)$$

where  $\mathbf{e}_t \sim \text{i.i.d. } N(0, \mathbf{R})$ ,  $\Lambda = \begin{bmatrix} \Lambda & 0 \\ 0 & I_M \end{bmatrix}$ ,  $\mathbf{R} = \begin{bmatrix} R & 0 \\ 0 & 0_M \end{bmatrix}$ .

Recall that  $\Phi(L)$  is of finite order  $d$ . We rewrite the VAR as a first-order Markov process. Let  $\Phi(L) = \Phi_1 L + \Phi_2 L^2 + \dots + \Phi_d L^d$ .

Define  $\bar{\mathbf{F}}_t = (\mathbf{F}'_t, \mathbf{F}'_{t-1}, \dots, \mathbf{F}'_{t-d+1})'$ ,  $\bar{\nu}_t = (\nu_t, 0, \dots, 0)'$ ,

$$\bar{\Phi} = \begin{bmatrix} \Phi_1 & \Phi_2 & \dots & \Phi_{d-1} & \Phi_d \\ I_{(K+M)} & 0 & \dots & 0 & 0 \\ 0 & I_{(K+M)} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_{(K+M)} & 0 \end{bmatrix} \quad (8)$$

to get

$$\bar{\mathbf{F}}_t = \bar{\Phi} \bar{\mathbf{F}}_t + \bar{\nu}_t, \quad (9)$$

where  $\bar{\nu}_t = (\nu'_t, 0, \dots, 0)$ ,

$$\bar{Q} = \begin{bmatrix} Q & 0 & \dots & 0 \\ 0 & 0_{(K+M)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0_{(K+M)} \end{bmatrix}. \quad (10)$$

We can also write

$$\mathbf{X}_t = \bar{\Lambda} \bar{\mathbf{F}}_t + \mathbf{e}_t \quad (11)$$

where  $\bar{\Lambda} = \begin{bmatrix} \Lambda & 0 & \dots & 0 \end{bmatrix}$ . Hence, the system to be estimated is

$$\mathbf{X}_t = \bar{\Lambda} \bar{\mathbf{F}}_t + \mathbf{e}_t \quad (12)$$

$$\bar{\mathbf{F}}_t = \bar{\Phi} \bar{\mathbf{F}}_{t-1} + \bar{\nu}_t. \quad (13)$$

For simplicity, we omit the “bar” notation. According to the Bayesian approach, we treat the model’s parameters  $\theta = (\Lambda, R, \text{vec}(\Phi'), Q)$  and the factors  $\{F_t\}_{t=1}^T$  as random variables. Let  $\tilde{X}_T = (X_1, \dots, X_T)$  and  $\tilde{F}_T = (F_1, \dots, F_T)$  be the histories of  $X$  and  $F$ , respectively. We need to derive the posterior densities of  $F$  and  $\theta$ :  $p(\tilde{F}_T) = \int_{\Omega} p(\tilde{F}_T, \theta) d\theta$  and  $p(\theta) = \int_F p(\tilde{F}_T, \theta) d\tilde{F}_T$ , where  $p(\tilde{F}_T, \theta)$  is the joint posterior distribution and  $\Omega$  and  $F$  are the supports of  $\theta$  and  $F$ .

We apply multi-move Gibbs sampling, to obtain an empirical approximation of the joint distribution. We start with an initial set of values,  $\theta^0$ . Then, conditional on  $\theta^0$  and  $\tilde{X}_T$ , we draw  $\tilde{F}_T^1$  from the conditional density  $p(\tilde{F}_T | \tilde{X}_T, \theta^0)$  and  $\theta^1$  from the conditional distribution  $p(\theta | \tilde{X}_T, \tilde{F}_T^1)$ .<sup>8</sup> These steps are repeated for  $s$  iterations, until the empirical distributions of  $\tilde{F}_T^s$  and  $\theta^s$  have converged. It can be proven that, as  $s \rightarrow \infty$ , under regularity conditions, the marginal and joint distributions of sampled parameters  $(\tilde{F}_T^s, \theta^s)$  converge to the true distributions  $(F_T, \theta)$ , at an exponential rate (see Geman and Geman 1994).

The procedure is as follow.

1. Choice of starting value  $\theta^0$ . It is advisable to start with a dispersed set of parameter values, verifying that they lead to similar empirical distributions. Unless otherwise specified, we set the loading of the first variable of each factor group to 1, and all the other loadings to 0.<sup>9</sup> The remaining variances and VAR parameters are estimated with equation-by-equation OLS. Clearly this satisfies our normalization that the first element of  $\Lambda_i^f$  is one for all  $i$ . We check the robustness of our estimates to alternative starting points.
2. How to draw from  $p(\tilde{F}_T | \tilde{X}_T, \theta)$ . This conditional distribution can be expressed as the product of conditional distributions:

$$p(\tilde{F}_T | \tilde{X}_T, \theta) = p(F_T | \tilde{X}_T, \theta) \prod_{t=1}^{T-1} p(F_t | F_{t+1}, \tilde{X}_t, \theta) \quad (14)$$

which is derived exploiting the Markov property of the state-space model.

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<sup>8</sup>Notice that to simplify the notation we leave the conditioning on the data  $Y_T$  implicit throughout this appendix.

<sup>9</sup>Refer to Appendix B for which variable is the first in each factor group.

The model is linear and Gaussian, therefore we have:

$$F_T \mid \tilde{X}_T, \theta \sim N(F_{T|T}, P_{T|T}), \quad (15)$$

$$F_t \mid F_{t+1}, \tilde{X}_t, \theta \sim N(F_{t|t+1, F_{t+1}}, P_{t|t, F_{t+1}}), \quad t = T - 1, \dots, 1, \quad (16)$$

where

$$\begin{aligned} F_{T|T} &= E(F_T \mid \tilde{X}_T, \theta), \\ P_{T|T} &= Cov(F_T \mid \tilde{X}_T, \theta), \\ F_{t|t+1, F_{t+1}} &= E(F_t \mid \tilde{X}_t, F_{t+1}, \theta) = E(F_t \mid F_{t+1}, F_{t|t}, \theta), \\ P_{t|t, F_{t+1}} &= Cov(F_t \mid F_{t+1}, \tilde{X}_t, \theta) = Cov(F_t \mid F_{t+1}, F_{t|t}, \theta). \end{aligned} \quad (17)$$

Here  $F_{t|s}$  refers to the expectation of  $F_t$  conditional on information dated  $s$  or earlier. We obtain  $F_{t|t}$  and  $P_{t|t}$ ,  $t = 1, \dots, T$ , using the Kalman Filter conditional on  $\theta$  and the data  $\tilde{X}_t$ . For instance, we can follow the formulas in Hamilton (1994) starting with the arbitrary, but computationally convenient, initial values  $F_{1|0} = 0_{Id \times 1}$  and  $P_{1|0} = I_{Id}$ . From the last iteration, we obtain  $F_{T|T}$  and  $P_{T|T}$  that we use to draw  $F_T$  from (15). Then, we can go backwards through the sample, deriving  $F_{T-1|T-1, F_t}$  and  $P_{T-1|T-1, F_t}$  by Kalman Filter, drawing  $F_{T-1}$  from (16) and so on for  $F_t$ ,  $t = T - 2, T - 3, \dots, 1$ . A modification of the Kalman filter procedure, as described in Kim and Nelson (1999), is necessary when the number of lags  $d$  in (7) is greater than 1.

3. How to draw from  $p(\theta \mid \tilde{X}_T, \tilde{F}_T)$ . Conditional on the data and on the factors generated by the previous step, we can draw values for  $\theta$ . As the factors are taken as known, (6) and (7) can be treated as two separate sets of equations, the former specifying the distribution of  $\Lambda$  and  $R$ , the latter that of  $vec(\Phi')$  and  $Q$ . Using (6), we can apply equation-by-equation OLS, to obtain  $\hat{\Lambda}$  and  $\hat{e}$ . We have  $\hat{R}_{ii} = \hat{e}'\hat{e}/(T - K_i)$ , where  $K_i$  is the number of regressors in equation  $i$ , and we set  $R_{ij} = 0$ , for  $i \neq j$ . With an uninformative prior, we have

$$R_{ii} \mid \tilde{X}_T, \tilde{F}_T = (T - K_i) \frac{\hat{R}_{ii}}{x} \text{ where } x \sim \chi^2(T - K_i). \quad (18)$$

After drawing  $R_{ii}$ , we draw  $\Lambda_i^{nn} \sim N(\hat{\Lambda}_i, R_{ii}[\tilde{F}_T^{(i)'} \tilde{F}_T^{(i)}]^{-1})$ .  $\Lambda_i$  is then obtained normalizing  $\Lambda_i^{nn}$  so that the first element of the vector is one for all  $i$ . Equation (7) is a standard VAR system, which can be estimated equation by equation to get  $vec(\hat{\Phi})$  and  $\hat{Q}$ . Then, with a flat prior on

$\log |Q|$ , we can draw  $Q$  from

$$InvWishart \left( \left[ (T - d)\widehat{Q} \right]^{-1}, T - (K + M)d \right) \quad (19)$$

and, conditional on the generated  $Q$ , we draw

$$vec(\Phi') \sim N(vec(\widehat{\Phi}'), Q \otimes (\widetilde{F}'_T \widetilde{F}_T)^{-1}), \quad (20)$$

where  $vec(\Phi')$  contains the rows of  $\Phi'$  in stacked form, forming a vector of length  $d(K + M)^2$  and  $\otimes$  refers to the Kronecker product.

Steps 2 and 3 are repeated for each iteration  $s$ . Then, inference is based on the distribution of  $(\widetilde{F}_T^s, \theta^s)$ , after convergence (that is, discarding a big enough number  $B$  of initial draws). We calculate medians and percentiles of  $(\widetilde{F}_T^s, \theta^s)$  for  $s = B + 1, \dots, S$  to form estimates of the factors and model parameters and of the associated uncertainty. Also, we evaluate the impulse response functions for each draw and calculate their medians and percentiles.

## B The Data Set

The data are taken from Stock and Watson (2002), FRED or Datastream (DS).

### 1. Real Activity Factor.

ID	Mnemonic	Description	Source	T
1	IPF	Industrial Production: final products (92=100,sa)	SW	3
2	IPCD	Industrial Production: dur consumer goods (92=100,sa)	SW	3
3	IPCN	Industrial Production: nondur consumer goods (92=100,sa)	SW	3
4	IPE	Industrial Production: business equipment (92=100,sa)	SW	3
5	IPI	Industrial Production: intermediate products (92=100,sa)	SW	3
6	IPM	Industrial Production: materials (92=100,sa)	SW	3
7	IPD	Industrial Production: dur manufacturing (92=100,sa)	SW	3
8	IPN	Industrial Production: nondur manufacturing (92=100,sa)	SW	3
9	IPMIN	Industrial Production: mining (92=100,sa)	SW	3
10	IPUT	Industrial Production: utilities (92=100,sa)	SW	3
11	IPXMCA	Capacity Util rate: mfg, total (% of capacity,sa)	SW	1
12	LHEL	Index of help-wanted advertising in newspapers (67=100,sa)	SW	3

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13	LHEM	Civilian Labor Force: employed, total (thous,sa)	SW	3
14	LHUR	Unemployment rate: all workers, 16 years & over (% ,sa)	SW	1
15	LHU5	Unemploy. by duration: unempl. 0-5 wks (thous,sa)	SW	1
16	LHU14	Unemploy. by duration: unempl. 5-14 wks (thous,sa)	SW	1
17	LHU15	Unemploy. by duration: unempl. 15+ wks (thous,sa)	SW	1
18	LHU26	Unemploy. by duration: unempl. 15-26 wks (thous,sa)	SW	1
19	LPCC	Employees on nonag. payrolls: contract construction (thous,sa)	SW	3
20	LPEM	Employees on nonag. payrolls: manufacturing (thous,sa)	SW	3
21	LPED	Employees on nonag. payrolls: durable goods (thous,sa)	SW	3
22	LPEN	Employees on nonag. payrolls: nondurable goods (thous,sa)	SW	3
23	LPSP	Employees on nonag. payrolls: service-producing (thous,sa)	SW	3
24	LPTc	Employees on nonag. payrolls: wholesale & retail (thous,sa)	SW	3
25	LPFR	Employees on nonag. payrolls: fin., ins. & real est. (thous,sa)	SW	3
26	LPS	Employees on nonag. payrolls: services (thous,sa)	SW	3
27	LPGOV	Employees on nonag. payrolls: government (thous,sa)	SW	3
28	LPHRM	Avg. weekly hrs. of prod. wkrs.: mfg (sa)	SW	1
29	LPMOSA	Avg. weekly hrs. of prod. wkrs.: mfg, overtime hrs. (sa)	SW	1
30	MSDQ	Manuf. & trade: mfg; dur goods (mil92\$,sa)	SW	3
31	MSNQ	Manuf. & trade: mfg; nondur goods (mil92\$,sa)	SW	3
32	WTDQ	Merchant wholesalers: dur goods tot (mil92\$,sa)	SW	3
33	WTNQ	Merchant wholesalers: nondur goods tot (mil92\$,sa)	SW	3
34	RTQ	Retail trade: total (mil92\$,sa)	SW	3
35	RTNQ	Retail trade: nondur goods (mil92\$,sa)	SW	3
36	GMCDQ	Personal consumption expend-total durables (bil92\$,saar)	SW	3
37	GMCNQ	Personal consumption expend-total nondurables (bil92\$,saar)	SW	3
38	GMCSQ	Personal consumption expend-services (bil92\$,saar)	SW	3
39	GMCANQ	Personal consumption expend-new cars (bil92\$,saar)	SW	3
40	HSNE	Housing starts: northeast (thous,sa)	SW	4
41	HSMW	Housing starts: midwest (thous,sa)	SW	4
42	HSSOU	Housing starts: south (thous,sa)	SW	4
43	HSWST	Housing starts: west (thous,sa)	SW	4
44	HSBR	Housing authorized: total new priv housing units (thous,saar)	SW	4
45	HMOB	Mobile homes: manufacturers' shipments (thous,saar)	SW	4
46	IVMTQ	Manufacturing & trade inventories: total (mil92\$,sa)	SW	3
47	IVMFDQ	Inventories, business dur (mil92\$,sa)	SW	3
48	IVMFNQ	Inventories, business, nondur (mil92\$,sa)	SW	3
49	IVWRQ	Manufacturing & trade inv.: merch. wholes. (mil92\$,sa)	SW	3
50	IVRRQ	Manufacturing & trade inv.: retail trade (mil92\$,sa)	SW	3

51	IVSRMQ	Ratio for mfg & trade: mfg; inventory/sales (87\$,sa)	SW	2
52	IVSRWQ	Ratio for mfg & trade: wholesaler; inventory/sales (87\$,sa)	SW	2
53	IVSRRQ	Ratio for mfg & trade: retail trade; inventory/sales (87\$,sa)	SW	2
54	MOCMQ	New orders (net)-consumer goods & materials (92\$)(bci)	SW	3
55	MDOQ	New orders, dur goods industries (92\$)(bci)	SW	3
56	MSONDQ	New orders, nondefense capital goods (92\$)(bci)	SW	3
57	MDO	mfg new orders: dur g indust (mil\$,sa)	SW	3
58	MDUWU	mfg new orders: dur g indust-unfilled orders (mil\$,sa)	SW	3
59	MNO	mfg new orders: nondur g indust (mil\$,sa)	SW	3
60	MNOU	mfg new orders: nondur g indust-unfilled orders (mil\$,sa)	SW	3
61	MU	mfg unfilled orders: all mfg industries (mil\$,sa)	SW	3
62	MDU	mfg unfilled orders: dur goods industries (mil\$,sa)	SW	3
63	MNU	mfg unfilled orders: nondur goods industries (mil\$,sa)	SW	3
64	MPCON	contracts & orders for plant & equipment (bil\$,sa)	SW	3
65	DSPIC96	Real Disposable Personal Income	FRED	3
66	EMRATIO	Civilian Employment-Population Ratio	FRED	3
67	CIVPART	Civilian Participation Rate	FRED	3
68	USSHIM...A	US Shipments - All Manufacturing Industries CURN	DS	3

## 2. Inflation Factor.

69	PWFSA	Producer price index: finished goods (82=100,sa)	SW	3
70	PWFCSA	Producer price index: finished consumer goods (82=100,sa)	SW	3
71	PSM99Q	Index of sensitive materials prices (1990=100)(bci-99a)	SW	3
72	PU83	CPI-U: apparel & upkeep (82-84=100,sa)	SW	3
73	PU84	CPI-U: transportation (82-84=100,sa)	SW	3
74	PU85	CPI-U: medical care (82-84=100,sa)	SW	3
75	PUC	CPI-U: commodities (82-84=100,sa)	SW	3
76	PUCD	CPI-U: durables (82-84=100,sa)	SW	3
77	PUS	CPI-U: services (82-84=100,sa)	SW	3
78	PUXF	CPI-U: all items less food (82-84=100,sa)	SW	3
79	PUXHS	CPI-U: all items less shelter (82-84=100,sa)	SW	3
80	PUXM	CPI-U: all items less medical care (82-84=100,sa)	SW	3
81	LEHCC	Avg. hr earnings of constr wkrs: construction (\$,sa)	SW	3
82	LEHM	Avg. hr earnings of prod wkrs: manufacturing (\$,sa)	SW	3
83	PFCGEF	Producer Price Index: Finished Consumer Goods-non Foods	FRED	3
84	PPICPE	Producer Price Index Finished Goods: Capital Equipment	FRED	3
85	PPICRM	Producer Price Index: Crude Materials	FRED	3



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86	PPIFCF	Producer Price Index: Finished Consumer Foods	FRED	3
87	PPIITM	Producer Price Index: Intermediate Materials	FRED	3
88	OILPRICE	Spot Oil Price: West Texas Intermediate	FRED	3
89	USLABCOSE	US Unit Labor Costs in Manufacturing (62=100,sa)(bci)	DS	3

### 3. Interest Rates Factor.

90	FYGT5	US Treasury const maturities 5-yr (% per annum,nsa)	SW	1
91	FYGT10	US Treasury const maturities 10-yr (% per annum,nsa)	SW	1
92	FYAAAC	Bond yield: moody's aaa corporate (% per annum)	SW	1
93	FYBAAC	Bond yield: moody's baa corporate (% per annum)	SW	1
94	FYFHA	Secondary market yields on fha mortgages (% per annum)	SW	1
95	GS1	1-Year Treasury Constant Maturity Rate	FRED	1
96	GS3	3-Year Treasury Constant Maturity Rate	FRED	1
97	LTGOVTBD	Long-Term U.S. Government Securities	FRED	1
98	TB3MS	3-Month Treasury Bill: Secondary Market Rate	FRED	1
99	TB6MS	6-Month Treasury Bill: Secondary Market Rate	FRED	1

### 4. Financial Market Factor.

100	FSNCOM	NYSE common stock price index: composite (65=50)	SW	3
101	FSPCOM	S&P's common stock price index: composite (1941-43=10)	SW	3
102	FSPIN	S&P's common stock price index: industrials (1941-43=10)	SW	3
103	FSPCAP	S&P's common stock price index: capital gds (1941-43=10)	SW	3
104	FSPUT	S&P's common stock price index: utilities (1941-43=10)	SW	3
105	FSDXP	S&P's composite stock: dividend yield (% per annum)	SW	1
106	FSPXE	S&P's composite stock: price-earnings ratio (% ,nsa)	SW	1
107	USSHRPRCF	US Dow Jones Industrials Share Price Index (EP)	DS	3

### 5. Money Factor.

108	FM1	Money stock: m1 (bil\$,sa)	SW	3
109	FM2	Money stock: m2 (bil\$,sa)	SW	3
110	FM3	Money stock: m3 (bil\$,sa)	SW	3
111	FMFBA	Monetary base, adj for reserve requirement (mil\$,sa)	SW	3
112	FMRRA	Depository inst reserves: total(mil\$,sa)	SW	3
113	FMRNBC	Depository inst reserves: nonborrowed (mil\$,sa)	SW	3

114	CURRSL	Currency Component of M1	FRED	3
115	DEMDEPSL	Demand Deposits at Commercial Banks	FRED	3
116	EXCRESNS	Excess Reserves of Depository Institutions	FRED	2
117	LGTDCBSL	Large Time Deposits at Commercial Banks	FRED	3
118	LTDSL	Large Time Deposits - Total	FRED	3
119	NFORBRES	Net Free or Borrowed Reserves of Depository Institut	FRED	2
120	REQRESNS	Required Reserves	FRED	3
121	RESBALNS	Reserve Bal with FRBs, Not Adj for reserve requir	FRED	3
122	SAVINGSL	Savings Deposits - Total	FRED	3
123	STDCBSL	Small Time Deposits at Commercial Banks	FRED	3
124	STDSL	Small Time Deposits - Total	FRED	3
125	SVGCSL	Savings Deposits at Commercial Banks	FRED	3
126	TCDSL	Total Checkable Deposits	FRED	3

#### 6. Credit Factor.

127	AUTOSL	Total Automobile Credit Outstanding	FRED	3
128	BUSLOANS	Commercial and Indust Loans at All Comm Banks	FRED	3
128	CONSUMER	Consumer (Individual) Loans at All Commercial Banks	FRED	3
130	INVEST	Total Investments at All Commercial Banks	FRED	3
131	LOANS	Total Loans and Leases at Commercial Banks	FRED	3
132	NONREVSL	Total Nonrevolving Credit Outstanding	FRED	3
133	OTHERSL	Total Other Credit Outstanding	FRED	3
134	OTHSEC	Other Securities at All Commercial Banks	FRED	3
135	REALLN	Real Estate Loans at All Commercial Banks	FRED	3
136	TOTALSL	Total Consumer Credit Outstanding	FRED	3

#### 7. Expectations Factor.

137	PMI	Purchasing managers'index (sa)	SW	1
138	PMP	NAPM production index (percent)	SW	1
139	PMEMP	NAPM employment index (percent)	SW	1
140	PMNV	NAPM inventories index (percent)	SW	1
141	PMNO	NAPM new orders index (percent)	SW	1
142	PMDEL	NAPM vendor deliveries index (percent)	SW	1
143	PMCP	NAPM commodity prices index (percent)	SW	1
144	HHSNTN	U.of Mich. index of consumer expectations (bcd-83)	SW	1

8. Federal Funds Rate.

145	FEDFUNDS	Effective Federal Funds Rate	FRED	1
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Note: T is the transformation code: 1) no transformation, 2) annual difference, 3) annual difference of logarithms, 4) logarithm.

## C Convergence of the Gibbs Sampler

In this Appendix we evaluate convergence of our Gibbs sampler. Although the simulated chains theoretically converge to the true distribution of the parameters, this has to be confirmed in practice.

There is not a single methodology to assess convergence of a chain of draws for a parameter. Hence, judgments by the statistician are required. This issue is even more relevant in our model, due to the the very big number of parameters to be estimated. Therefore, we perform a series of diagnostics, both graphical methods and tests, to explore convergence proprieties of our estimation.

### C.1 Graphical Methods

#### C.1.1 Trace Plots

As a first diagnostic for convergence, we look at trace plots for a number of variables. Figure 6 shows trace plots for selected loadings. These trace plots display good convergence properties.

We are particularly interested in estimating precisely the impulse responses to a shock to the Federal Funds rate. Hence, it is reasonable to analyze the convergence proprieties of the impulse responses, rather than the various parameters of the VAR. Figure 7 shows trace plots for the median estimates of the responses of all variables to a policy shock at selected dates. Most of the trace plots display no trend, even if some seem to display an increase in variance at the very end.

#### C.1.2 First Half vs Second Half of Draws

Another graphical diagnostic is to compare estimates calculated using the first half of the kept draws with those derived from the second half.

Figure 8 shows the estimated loadings calculated from the first half of the kept draws, together with those derived from the second half. Figure 9 does

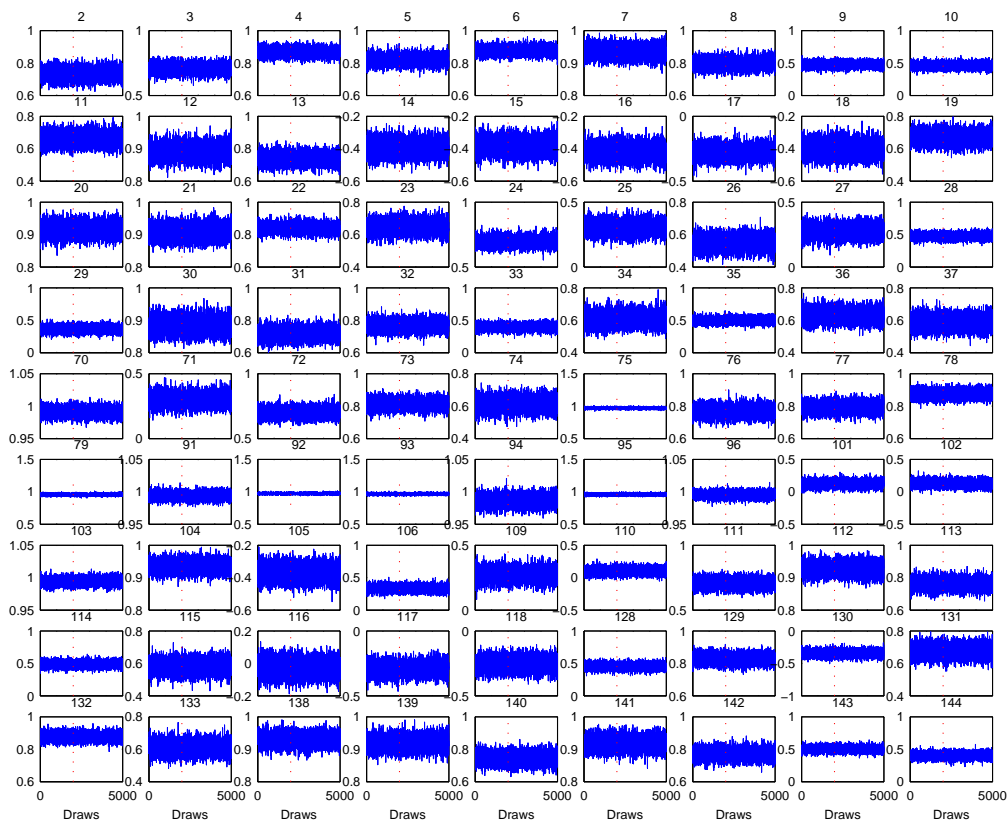


Figure 6: Trace plots: selected loadings. The title of each subplot refers to the variable’s ID (see Appendix C).

the same with the impulse responses to a policy shock. The two halves give indistinguishable estimates, suggesting good convergence proprieties.

## C.2 Tests

We perform several diagnostics and tests for a number of selected parameters, both loadings and impulse responses. The results are displayed in Tables 1, 2, 3, 4 and 5.

For each chain of kept draws we calculate autocorrelations, that are generally small. We then calculate the Raftery and Lewis’s (1995) diagnostic for each chain, calculated for the 2.5% quantile, with precision 1% and confidence level 95%. This method suggests a small number of total draws (inferior to our 3,000), a minimum additional burn-in to achieve convergence for all the

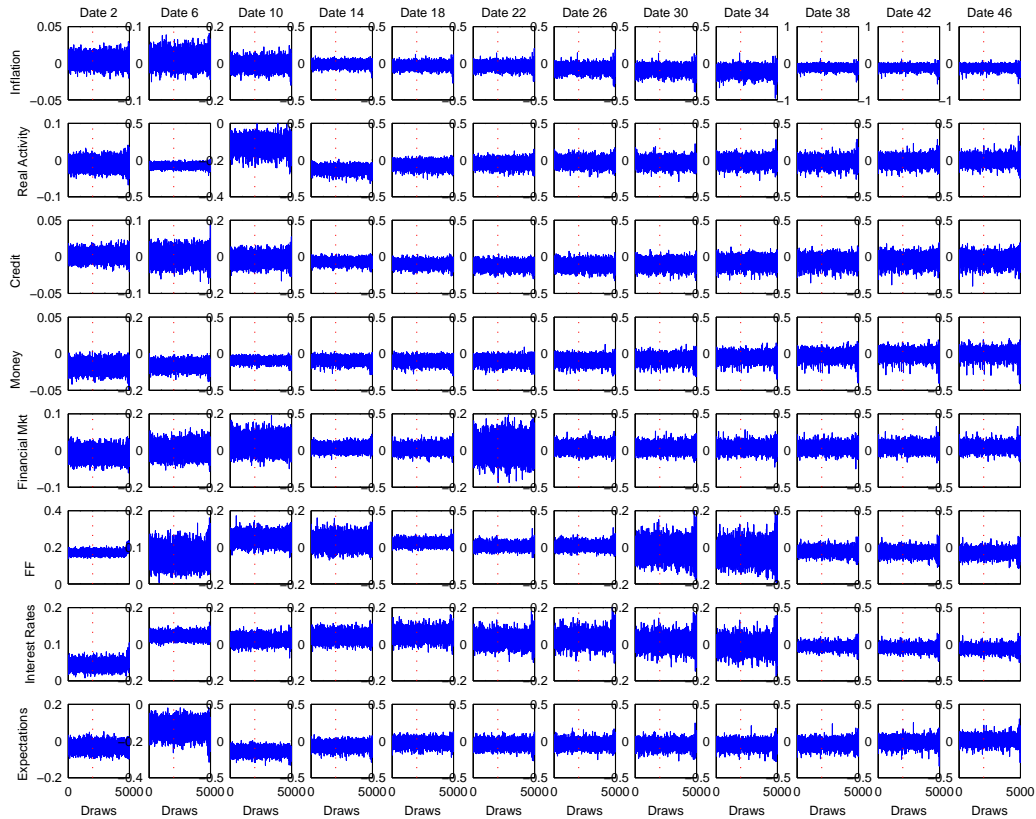


Figure 7: Trace plots: selected dates for impulse responses to a policy shock.

parameters evaluated and a thinning parameter equal to 1.

We also use the Geweke (1992) test. With this test, we compare the partial mean of each parameter considered over the first 20% of the draws with the partial mean over the last 50%. Results vary across the various parameters

Finally we calculate the effective sample size  $N_{eff}$ . Indeed, draws are typically correlated for each parameter. The effective sample size represents the number of independent draws for the estimation of the given parameter. A reasonable estimate of the effective sample size can be obtained by dividing the estimated variance by the square of the numerical standard error. This last is obtained by Geweke (1992), estimating spectral density. In our estimation, all the parameters have effective sample size equal to the number of draws.

In conclusion, diagnostic methods suggest reasonably good convergence proprieties for our estimation. A different method to assess how far we are from the true distribution is to verify the stability of our estimation with

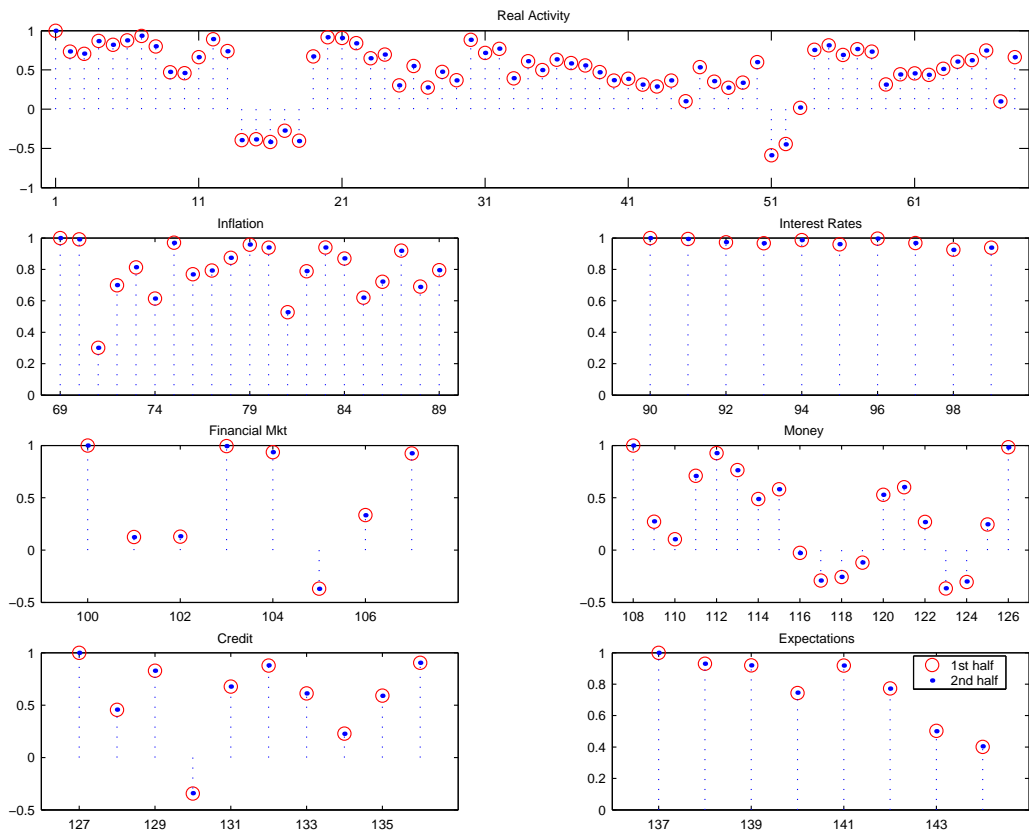


Figure 8: Convergence: loadings, first half vs. second half of the sampling.

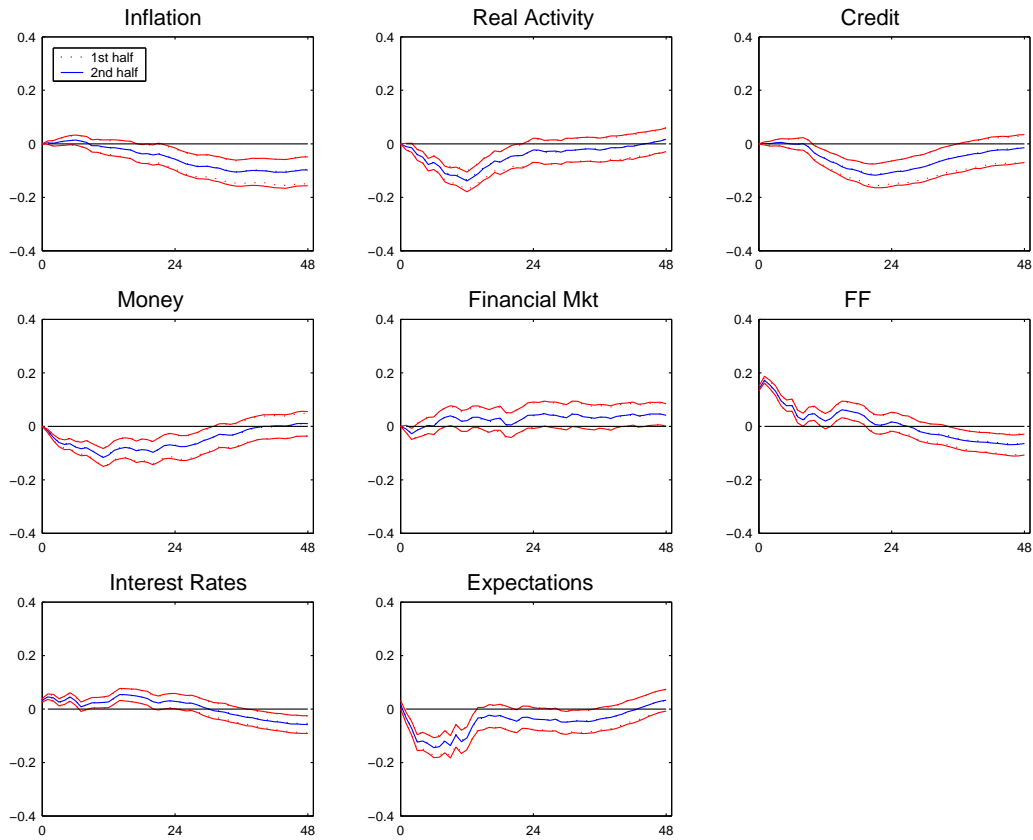


Figure 9: Convergence: impulse response functions, first half vs. second half of the sampling.

respect to different and distinct starting points.

### C.3 Multiple starting points

As discussed in Appendix A, the estimation procedure requires us to select a starting value for all the parameters. In our main estimation, we set the loading of the first variable of each factor group to 1, and all the other loadings to 0.<sup>10</sup> The remaining variances and VAR parameters are estimated with equation-by-equation OLS.

To test for the robustness of our main estimates with respect to other starting values, in this section we run shorter simulations with distinct starting

<sup>10</sup>Refer to Appendix B for which variable is the first in each factor group.

values. Then, we compare our main estimation and each alternative starting point along to two central dimensions: the estimation of the loadings and of the impulse responses to a policy shock.

For each simulation with the alternative starting values, the estimates are based on 600 draws of the base model, with the first 200 draws omitted to reduce the influence of the initial guess on the estimates. We consider the following alternative starting points:

- A. We estimate the factors with Principal Components (see Appendix D) and we use the estimated loadings as initial conditions. The loadings are then transformed to satisfy the normalization that the first element of  $\Lambda_i^f$  is one for all  $i$ . The remaining variances and VAR parameters are estimated with equation-by-equation OLS. The resulting estimates are compared to the main simulation in Figures 10 and 11. The two estimations with different starting values deliver very close estimates.
- B. We set all the loadings to 1. This is a rather extreme starting value, since at least some of the loadings are expected to be negative. The remaining variances and VAR parameters are estimated with equation-by-equation OLS. The resulting estimates are compared to the main simulation in Figures 12 and 13. The two estimations with different starting values deliver close estimates. There are significant deviations only in the impulse responses for Expectations and Federal Funds rate.
- C. We set all the loadings to 1. This is a rather extreme starting value, since at least some of the loadings are expected to be negative. Moreover, we estimate the VAR parameters not using the factors calculated with such loadings, but the most representative variable of each factor group, that is the first of each group. The resulting estimates are compared to the main simulation in Figures 14 and 15. The two estimations with different starting values deliver very close estimates.

We conclude that even starting with very different and arbitrary starting points and running short simulations, the estimates of the model fall rather close. This suggests that convergence has been achieved.



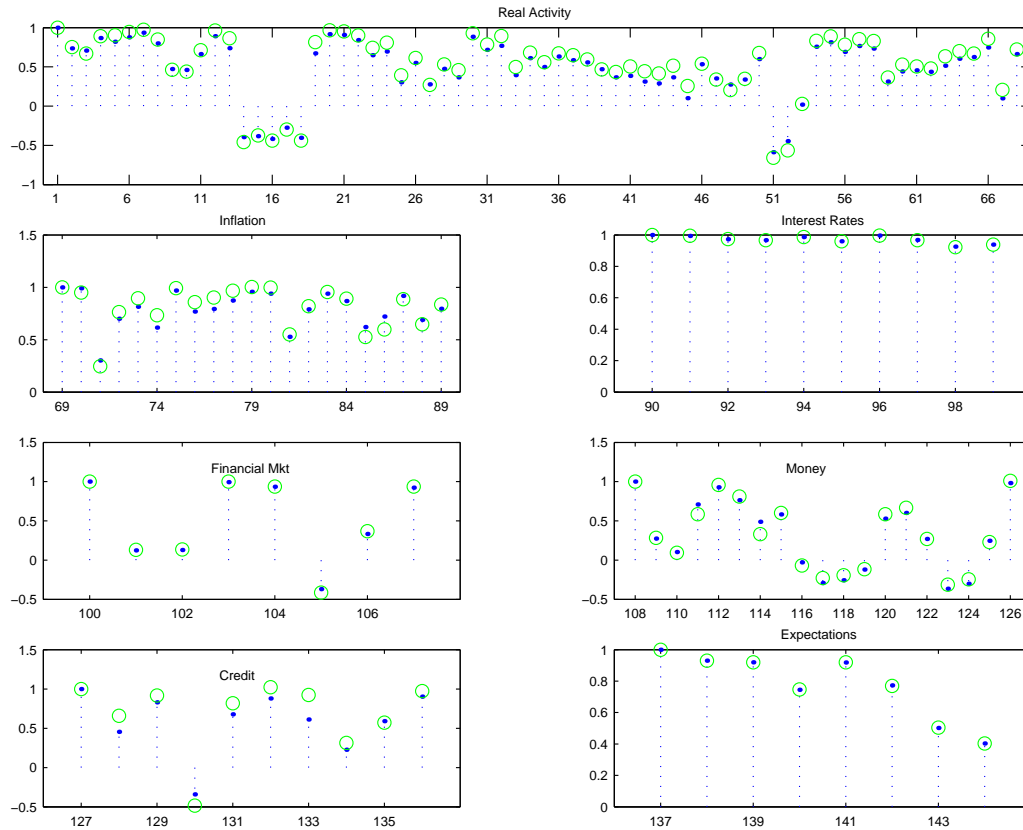


Figure 10: Loadings. Main estimates (full dot) are plotted against the estimates with the alternative starting point A (circle)

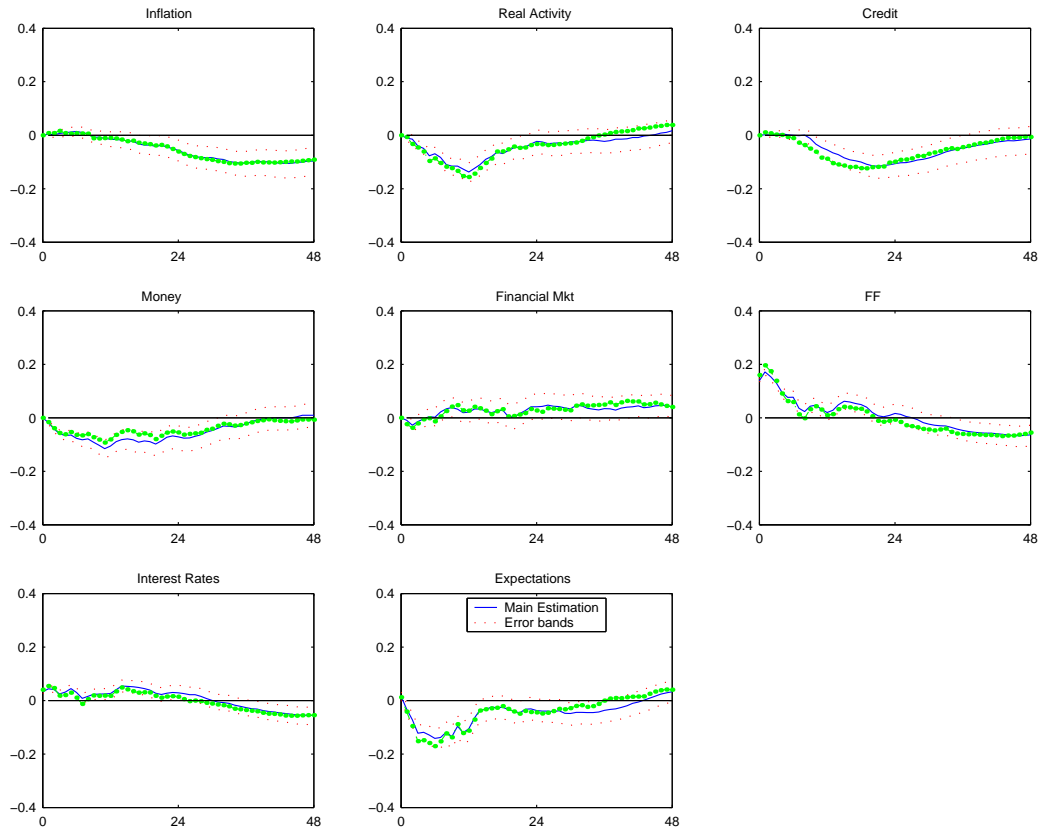


Figure 11: Impulse responses to a monetary policy shock. Main estimates and error bands are plotted against the estimates with the alternative starting point A (dotted line)

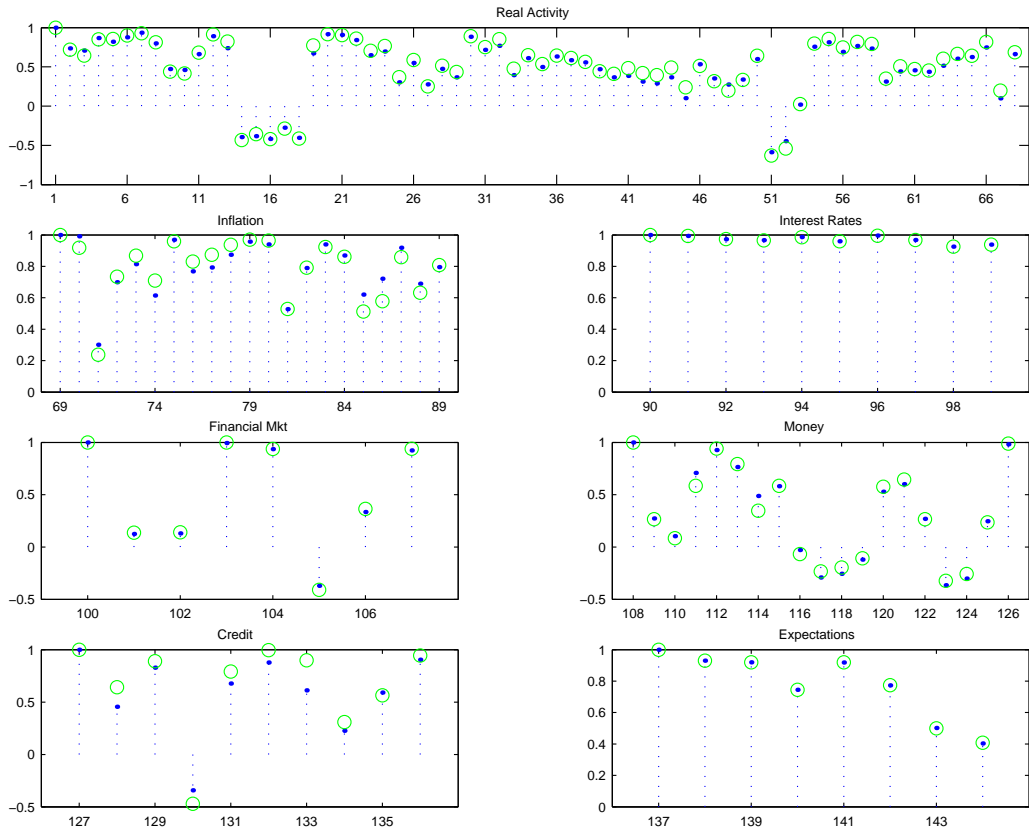


Figure 12: Loadings. Main estimates (full dot) are plotted against the estimates with the alternative starting point B (circle)

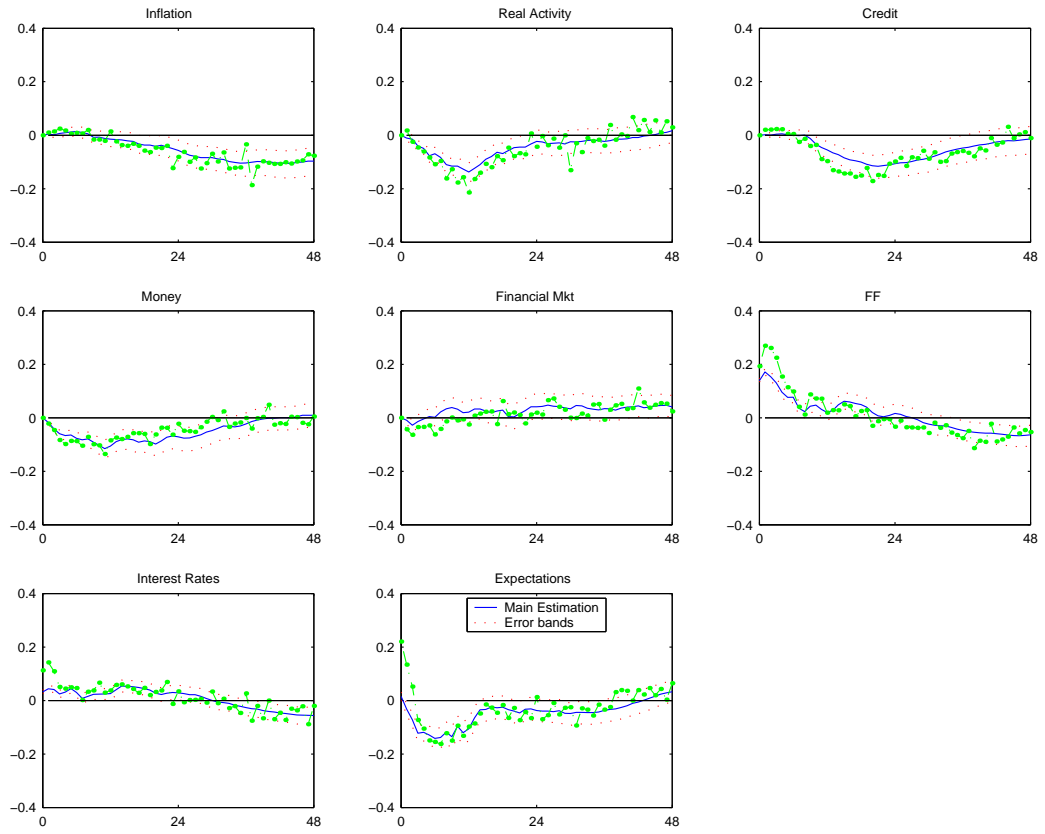


Figure 13: Impulse responses to a monetary policy shock. Main estimates and error bands are plotted against the estimates with the alternative starting point B (dotted line)

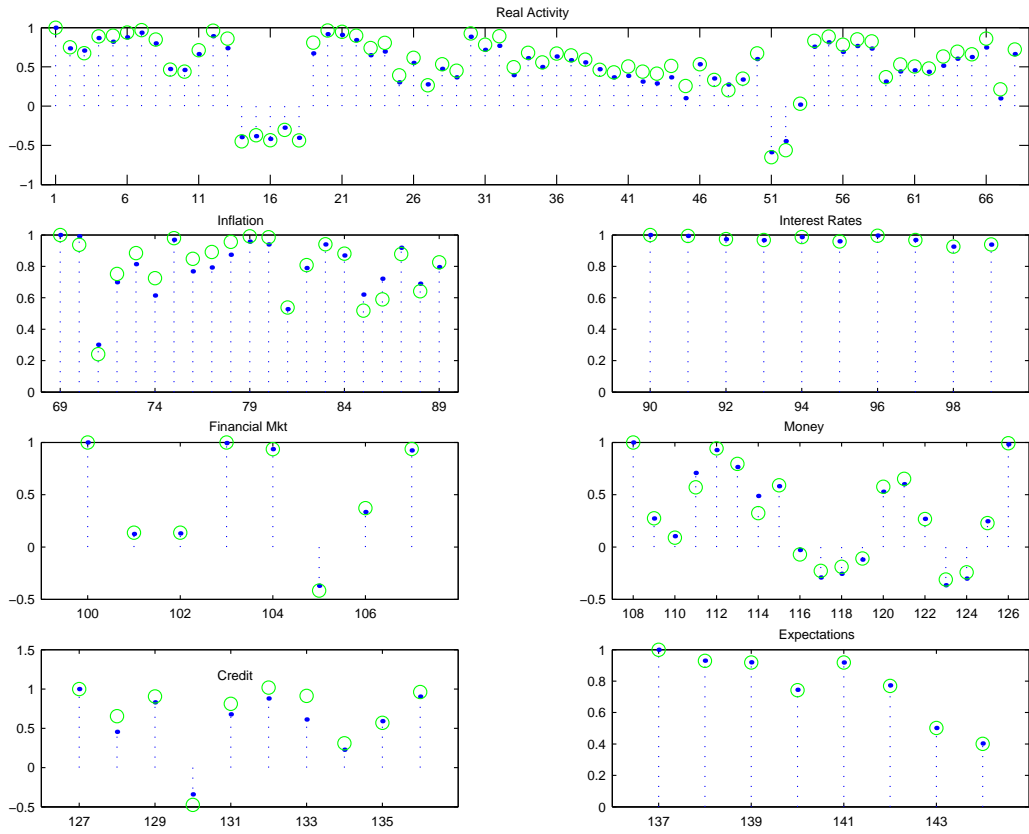


Figure 14: Loadings. Main estimates (full dot) are plotted against the estimates with the alternative starting point C (circle)

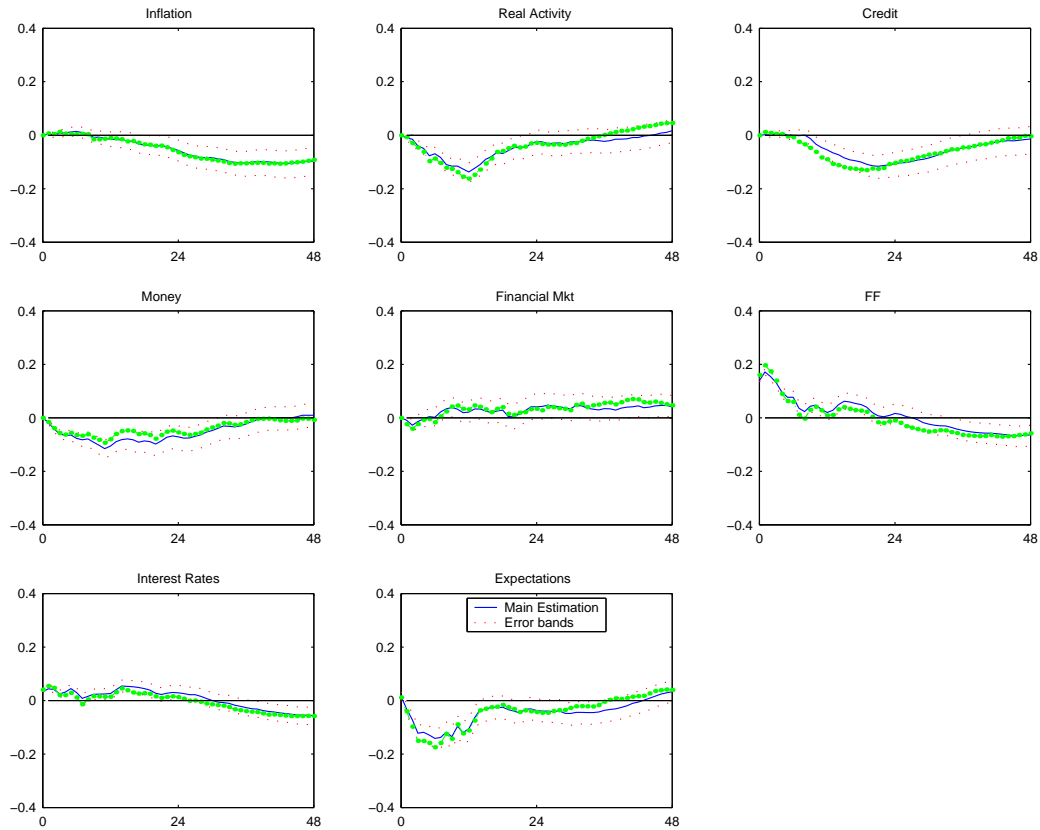


Figure 15: Impulse responses to a monetary policy shock. Main estimates and error bands are plotted against the estimates with the alternative starting point C (dotted line)

## D Estimation with Principal Components

In this section we describe how to estimate the factors using Principal Components. This estimation are used as an alternative initialization of the Gibbs Sampler in Appendix C. To estimate the factors with Principal Components we follow Bernanke, Boivin, and Elias (2005) two-step procedure. The identification of the factors is obtained by imposing  $F^{i'}F^i/T = I$ . The estimation works as follows.

1. Using principal components, we find the factors  $(F_t^1, F_t^2, \dots, F_t^I)$  from the model

$$\begin{bmatrix} X_t^1 \\ X_t^2 \\ \dots \\ X_t^I \end{bmatrix} = \begin{bmatrix} \Lambda_1^f & 0 & \dots & 0 \\ 0 & \Lambda_2^f & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \Lambda_I^f \end{bmatrix} \cdot \begin{bmatrix} F_t^1 \\ F_t^2 \\ \dots \\ F_t^I \end{bmatrix} + e_t. \quad (21)$$

We obtain  $(\hat{F}_t^1, \hat{F}_t^2, \dots, \hat{F}_t^I)$ .

2. We run a standard VAR

$$\begin{bmatrix} \hat{F}_t^1 \\ \hat{F}_t^2 \\ \dots \\ \hat{F}_t^I \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} \hat{F}_{t-1}^1 \\ \hat{F}_{t-1}^2 \\ \dots \\ \hat{F}_{t-1}^I \\ Y_{t-1} \end{bmatrix} + \nu_t \quad (22)$$

to obtain  $\hat{\Phi}(L)$ .

3. To find the loadings, we do OLS of the equation

$$\begin{bmatrix} X_t^1 \\ X_t^2 \\ \dots \\ X_t^I \end{bmatrix} = \begin{bmatrix} \Lambda_1^f & 0 & \dots & 0 \\ 0 & \Lambda_2^f & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \Lambda_I^f \end{bmatrix} \cdot \begin{bmatrix} \hat{F}_t^1 \\ \hat{F}_t^2 \\ \dots \\ \hat{F}_t^I \end{bmatrix} + e_t. \quad (23)$$

This gives us  $(\hat{\Lambda}_1^f, \hat{\Lambda}_2^f, \dots, \hat{\Lambda}_I^f)$ .

F-ID	Date	Autocorrelation				Raftery-Lewis			Geweke	$N_{eff}$
		Lag 1	Lag 5	Lag 10	Lag 50	Thin	Burn	$N_D$	Prob	
1	02	0.004	0.010	-0.036	-0.001	1	3	1081	0.45	3000
1	12	0.021	0.012	0.017	0.038	1	2	968	0.01	3000
1	22	-0.012	0.003	-0.002	-0.057	1	2	892	0.90	3000
1	32	-0.039	0.005	-0.003	-0.010	1	2	917	0.90	3000
1	42	-0.013	0.001	0.002	-0.030	1	3	1052	0.68	3000
2	02	0.012	0.010	0.009	0.001	1	2	942	0.33	3000
2	12	-0.008	0.027	0.010	-0.031	1	2	942	0.82	3000
2	22	0.015	-0.008	0.037	-0.020	1	3	1052	0.66	3000
2	32	0.048	-0.010	0.041	-0.012	1	3	1081	0.21	3000
2	42	0.031	0.028	0.030	0.024	1	3	1081	0.05	3000
3	02	0.114	0.098	0.111	0.093	1	2	968	0.00	3000
3	12	-0.030	-0.009	-0.001	0.002	1	2	942	0.92	3000
3	22	-0.040	-0.026	-0.032	-0.006	1	2	917	0.91	3000
3	32	-0.016	-0.019	-0.013	-0.000	1	2	995	0.67	3000
3	42	-0.008	-0.024	-0.000	0.002	1	2	995	0.20	3000
4	02	-0.004	0.006	0.010	0.000	1	2	942	0.14	3000
4	12	0.000	-0.007	-0.013	-0.015	1	2	942	0.13	3000
4	22	0.022	-0.021	0.021	-0.004	1	2	942	0.93	3000
4	32	0.008	-0.033	0.000	-0.014	1	2	968	0.46	3000
4	42	-0.009	0.044	-0.010	0.022	1	2	917	0.36	3000
5	02	0.041	0.001	0.009	0.000	1	3	1081	0.15	3000
5	12	0.058	0.023	0.034	0.054	1	3	1148	0.35	3000
5	22	-0.004	0.017	0.036	0.027	1	2	942	0.64	3000
5	32	-0.007	0.001	0.009	0.005	1	2	917	0.61	3000
5	42	-0.022	-0.029	0.008	0.002	1	2	968	0.57	3000

Table 1: Battery of convergence tests for selected dates of the impulse responses to a policy shock, calculated on the kept draws. F-ID refers to the Factor’s ID, as indicated in Appendix C. Raftery-Lewis is calculated with quantile 2.5%, precision 1% and confidence level 95%.  $N_D$  refers to number of draws to achieve the desired accuracy. Geweke Chi-squared diagnostic compares the first 20% of the kept draws with the last 50% and gives the probability that the two partial means are the same.  $N_{eff}$  is the effective sample size: sample size divided by the integrated autocorrelation time.



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F-ID	Date	Autocorrelation				Raftery-Lewis			Geweke	$N_{eff}$
		Lag 1	Lag 5	Lag 10	Lag 50	Thin	Burn	$N_D$	Prob	
6	02	0.020	-0.010	-0.030	-0.000	1	2	942	0.59	3000
6	12	-0.025	0.010	-0.011	0.009	1	3	1052	0.26	3000
6	22	0.007	0.026	-0.008	0.015	1	2	968	0.03	3000
6	32	0.009	0.010	-0.013	0.021	1	2	968	0.41	3000
6	42	0.015	0.015	-0.015	0.002	1	2	995	0.92	3000
7	02	0.035	-0.012	-0.025	-0.002	1	3	1024	0.06	3000
7	12	0.053	0.012	0.035	-0.009	1	3	1024	0.14	3000
7	22	0.017	-0.021	0.019	-0.015	1	2	995	0.80	3000
7	32	-0.031	0.007	0.010	0.018	1	2	968	0.44	3000
7	42	0.009	-0.003	0.006	-0.000	1	2	968	0.84	3000
8	02	0.353	0.353	0.329	0.278	1	2	968	0.00	3000
8	12	0.010	0.023	0.009	-0.006	1	2	917	0.85	3000
8	22	-0.046	-0.019	-0.006	-0.027	1	2	995	0.75	3000
8	32	-0.002	-0.013	-0.003	-0.024	1	3	1052	0.36	3000
8	42	0.001	-0.015	0.014	0.002	1	3	1052	0.17	3000

Table 2: Battery of convergence tests for selected dates of the impulse responses to a policy shock, calculated on the kept draws. F-ID refers to the Factor’s ID, as indicated in Appendix C. Raftery-Lewis is calculated with quantile 2.5%, precision 1% and confidence level 95%.  $N_D$  refers to number of draws to achieve the desired accuracy. Geweke Chi-squared diagnostic compares the first 20% of the kept draws with the last 50% and gives the probability that the two partial means are the same.  $N_{eff}$  is the effective sample size: sample size divided by the integrated autocorrelation time.

ID	Autocorrelation				Raftery-Lewis			Geweke	$N_{eff}$
	Lag 1	Lag 5	Lag 10	Lag 50	Thin	Burn	$N_D$	Prob	
2	0.026	0.003	-0.024	-0.018	1	2	995	0.00	3000
3	-0.001	-0.034	-0.023	-0.018	1	2	892	0.89	3000
4	0.002	0.016	-0.006	0.031	1	2	968	0.42	3000
5	0.010	-0.032	0.032	-0.011	1	2	942	0.24	3000
6	-0.034	0.023	-0.014	-0.010	1	2	942	0.66	3000
7	0.009	0.012	-0.016	0.003	1	2	917	0.68	3000
8	0.051	0.013	-0.018	-0.001	1	2	968	0.51	3000
9	0.014	-0.007	-0.022	-0.038	1	2	917	0.41	3000
10	-0.014	-0.027	0.024	-0.023	1	2	892	0.83	3000
11	0.013	-0.002	0.009	0.016	1	2	995	0.70	3000
12	0.016	-0.011	-0.012	0.004	1	3	1024	0.81	3000
13	-0.019	-0.014	0.004	0.024	1	2	942	0.32	3000
14	-0.003	0.033	0.015	-0.025	1	2	942	0.13	3000
15	-0.043	0.017	-0.021	0.013	1	2	968	0.58	3000
16	-0.016	-0.025	-0.024	0.002	1	2	968	0.82	3000
17	-0.007	-0.004	-0.015	0.003	1	2	917	0.91	3000
18	-0.005	0.006	0.004	0.006	1	2	917	0.30	3000
19	0.011	-0.010	-0.030	-0.007	1	2	942	0.81	3000
20	-0.011	0.009	-0.011	0.000	1	2	942	0.90	3000
21	0.003	-0.031	0.024	-0.023	1	2	917	0.20	3000
22	-0.008	-0.006	0.007	0.014	1	2	892	0.01	3000
23	0.037	-0.009	-0.024	-0.016	1	2	968	0.55	3000
24	-0.012	-0.010	-0.021	0.021	1	2	917	0.74	3000
25	-0.008	-0.006	-0.033	-0.022	1	2	942	0.33	3000
26	0.008	-0.027	-0.022	-0.006	1	2	942	0.29	3000

Table 3: Battery of convergence tests for selected loadings, calculated on the kept draws. F-ID refers to the Factor's ID, as indicated in Appendix C. Raftery-Lewis is calculated with quantile 2.5%, precision 1% and confidence level 95%.  $N_D$  refers to number of draws to achieve the desired accuracy. Geweke Chi-squared diagnostic compares the first 20% of the kept draws with the last 50% and gives the probability that the two partial means are the same.  $N_{eff}$  is the effective sample size: sample size divided by the integrated autocorrelation time.

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ID	Autocorrelation				Raftery-Lewis			Geweke	$N_{eff}$
	Lag 1	Lag 5	Lag 10	Lag 50	Thin	Burn	$N_D$	Prob	
27	-0.018	0.031	-0.009	-0.014	1	2	968	0.07	3000
28	0.021	0.016	0.009	0.032	1	2	942	0.12	3000
29	-0.032	0.011	0.009	-0.012	1	2	917	0.25	3000
30	0.023	-0.021	0.016	-0.034	1	2	968	0.92	3000
31	0.022	0.004	-0.013	-0.034	1	2	995	0.59	3000
32	0.007	-0.023	0.003	-0.028	1	2	917	0.27	3000
33	-0.025	0.014	-0.010	-0.010	1	2	942	0.34	3000
34	-0.034	0.013	0.001	0.007	1	2	917	0.85	3000
35	-0.033	0.003	-0.029	0.016	1	2	942	0.55	3000
36	0.010	0.014	-0.008	-0.011	1	2	995	0.92	3000
37	0.014	0.033	0.008	0.011	1	2	942	0.24	3000
70	-0.028	-0.003	0.000	0.002	1	2	892	0.43	3000
71	0.006	0.018	-0.042	0.012	1	2	917	0.65	3000
72	-0.018	0.049	-0.005	0.014	1	2	917	0.22	3000
73	0.014	-0.012	0.001	0.022	1	2	917	0.70	3000
74	-0.063	-0.005	-0.023	-0.003	1	2	942	0.03	3000
75	-0.007	0.037	-0.024	0.026	1	2	968	0.28	3000
76	-0.009	-0.016	0.034	-0.015	1	2	942	0.75	3000
77	0.007	-0.018	-0.004	-0.032	1	2	892	0.25	3000
78	0.026	0.007	0.022	0.032	1	2	917	0.63	3000
79	0.013	0.011	0.012	-0.012	1	2	942	0.24	3000
91	-0.019	0.002	0.003	-0.021	1	2	942	0.17	3000
92	-0.035	-0.003	-0.007	-0.018	1	2	942	0.40	3000
93	0.007	0.009	0.037	0.004	1	2	917	0.33	3000
94	-0.001	-0.011	-0.001	-0.029	1	2	917	0.09	3000

Table 4: Battery of convergence tests for selected loadings, calculated on the kept draws. F-ID refers to the Factor's ID, as indicated in Appendix C. Raftery-Lewis is calculated with quantile 2.5%, precision 1% and confidence level 95%.  $N_D$  refers to number of draws to achieve the desired accuracy. Geweke Chi-squared diagnostic compares the first 20% of the kept draws with the last 50% and gives the probability that the two partial means are the same.  $N_{eff}$  is the effective sample size: sample size divided by the integrated autocorrelation time.

ID	Autocorrelation				Raftery-Lewis			Geweke	$N_{eff}$
	Lag 1	Lag 5	Lag 10	Lag 50	Thin	Burn	$N_D$	Prob	
95	0.030	-0.029	0.020	0.003	1	2	931	0.62	3000
96	0.031	-0.001	-0.000	-0.032	1	2	892	0.77	3000
101	-0.023	0.007	-0.009	-0.003	1	2	968	0.20	3000
102	-0.009	0.025	-0.005	-0.036	1	2	968	0.98	3000
103	0.013	0.045	-0.013	-0.006	1	2	942	0.61	3000
104	0.008	0.019	-0.028	0.026	1	3	1024	0.94	3000
105	0.009	-0.004	-0.011	0.014	1	2	942	0.60	3000
106	0.019	0.034	0.002	0.004	1	2	943	0.67	3000
109	0.049	-0.021	0.044	0.007	1	3	1052	0.70	3000
110	0.021	-0.022	0.061	-0.002	1	2	917	0.61	3000
111	0.009	-0.013	-0.022	-0.009	1	2	942	0.24	3000
112	-0.011	0.001	-0.006	-0.012	1	2	917	0.18	3000

Table 5: Battery of convergence tests for selected loadings, calculated on the kept draws. F-ID refers to the Factor's ID, as indicated in Appendix C. Raftery-Lewis is calculated with quantile 2.5%, precision 1% and confidence level 95%.  $N_D$  refers to number of draws to achieve the desired accuracy. Geweke Chi-squared diagnostic compares the first 20% of the kept draws with the last 50% and gives the probability that the two partial means are the same.  $N_{eff}$  is the effective sample size: sample size divided by the integrated autocorrelation time.

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