

# Prices and Exchange Rates: A Theory of Disconnect

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## Appendix B (Online)

### B.1 The Model with CES Preferences

The purpose of this section is to show the similarities and differences between the model with translog preferences and the standard model with CES preferences introduced by Melitz (2003) (and later extended by Ghironi and Melitz (2005) in a DSGE structure). Most of the notation from the main text carries out to this Appendix.

#### B.1.1 Preferences, Pricing, and Production

The representative Home household defines its preferences over the continuum of available goods from Home and Foreign. With CES preferences, the utility function is given by

$$U = \left[ \int_{i \in \Delta'} q_i^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}, \quad (\text{B-1})$$

where  $\nu > 1$  denotes the elasticity of substitution between varieties. As before, the set of available goods at Home  $\Delta'$  has measure  $N$ . It follows that the demand for good  $i$  is given by  $q_i = \left[ \frac{p_i}{P} \right]^{-\nu} Q$ , where  $p_i$  is the price of good  $i$ , and  $P = \left[ \int_{i \in \Delta'} p_i^{1-\nu} di \right]^{\frac{1}{1-\nu}}$  is the price of the aggregate consumption good  $Q$ . The total consumption expenditure of the representative Home household is given by  $I = PQ$ . Recall that households are located in the unit interval, so that household demand and market demand are equal.

Assuming that the marginal cost of producer  $i$  is constant and given by  $mc_i$ , its profit maximizing price is given by  $p_i = (1 + \mu)mc_i$ , where the markup over marginal cost is constant and equal to

$$\mu = \frac{1}{\nu - 1}. \quad (\text{B-2})$$

Production is defined as in section 3.1.3, with the extra assumption of fixed costs from selling in each market. In the absence of fixed costs in the CES model, firms always find it optimal to produce a positive amount for each of the markets. In terms of units of effective labor, I assume that the fixed cost from selling in market  $r$  is  $f_r$  for Home firms and  $f_r^*$  for Foreign firms, for  $r = \{D, X\}$ .

The pricing equations for a Home firm with productivity  $\varphi$  are given by  $p_D(\varphi) = (1 + \mu) \frac{W}{Z\varphi}$  and  $p_X(\varphi) = (1 + \mu) \frac{\tau W}{\varepsilon Z \varphi}$ . It follows that this firm's profit functions (*before* fixed costs) from

selling in each market—in the destination country’s currency—are

$$\pi_D(\varphi) = \frac{1}{\nu} \left( \frac{p_D(\varphi)}{P} \right)^{1-\nu} I \quad \text{and} \quad \pi_X(\varphi) = \frac{1}{\nu} \left( \frac{p_X(\varphi)}{P^*} \right)^{1-\nu} I^*.$$

Analogously, the pricing equations for a Foreign firm with productivity  $\varphi$  are  $p_D^*(\varphi) = (1+\mu)\frac{W^*}{Z^*\varphi}$  and  $p_X^*(\varphi) = (1+\mu)\frac{\tau^*\mathcal{E}W^*}{Z^*\varphi}$ , and its profit functions are given by

$$\pi_D^*(\varphi) = \frac{1}{\nu} \left( \frac{p_D^*(\varphi)}{P^*} \right)^{1-\nu} I^* \quad \text{and} \quad \pi_X^*(\varphi) = \frac{1}{\nu} \left( \frac{p_X^*(\varphi)}{P} \right)^{1-\nu} I.$$

I can now define the cutoff productivity levels as

$$\varphi_r = \inf \left\{ \varphi : \pi_r(\varphi) > \frac{f_r W}{\mathcal{E}\mathbb{1}\{r=X\}Z} \right\} \quad \text{and} \quad \varphi_r^* = \inf \left\{ \varphi : \pi_r^*(\varphi) > \frac{\mathcal{E}\mathbb{1}\{r=X\}f_r^*W^*}{Z^*} \right\}$$

for  $r \in \{D, X\}$ , where  $\mathbb{1}\{r = X\}$  is an indicator function that takes the value of 1 if  $r = X$ . Hence, we get the following zero-cutoff-profit conditions:

$$\begin{aligned} \frac{1}{\nu} \left( \frac{p_D(\varphi_D)}{P} \right)^{1-\nu} I &= \frac{f_D W}{Z} \\ \frac{1}{\nu} \left( \frac{p_X(\varphi_X)}{P^*} \right)^{1-\nu} I^* &= \frac{f_X W}{\mathcal{E}Z} \\ \frac{1}{\nu} \left( \frac{p_D^*(\varphi_D^*)}{P^*} \right)^{1-\nu} I^* &= \frac{f_D^* W^*}{Z^*} \\ \frac{1}{\nu} \left( \frac{p_X^*(\varphi_X^*)}{P} \right)^{1-\nu} I &= \frac{\mathcal{E}f_X^* W^*}{Z^*}. \end{aligned}$$

Similar to the derivation of equations (8) and (9) in the translog case, we obtain from the previous conditions the following relationships between the cutoff productivity levels:

$$\varphi_X^* = \tau^* \left[ \left( \frac{f_X^*}{f_D} \right)^{\frac{1}{\nu}} \frac{\mathcal{E}W^*}{Z^*} \right]^{\frac{\nu}{\nu-1}} \varphi_D \tag{B-3}$$

$$\varphi_X = \tau \left[ \left( \frac{f_X}{f_D^*} \right)^{\frac{1}{\nu}} \frac{W}{Z} \right]^{\frac{\nu}{\nu-1}} \varphi_D^*. \tag{B-4}$$

We can also use the zero-cutoff profit conditions to substitute for  $P$  and  $P^*$  in the profit functions, so that we can rewrite them as

$$\begin{aligned} \pi_D(\varphi) &= \left( \frac{\varphi_D}{\varphi} \right)^{1-\nu} \frac{f_D W}{Z} & \pi_X(\varphi) &= \left( \frac{\varphi_X}{\varphi} \right)^{1-\nu} \frac{f_X W}{\mathcal{E}Z} \\ \pi_D^*(\varphi) &= \left( \frac{\varphi_D^*}{\varphi} \right)^{1-\nu} \frac{f_D^* W^*}{Z^*} & \pi_X^*(\varphi) &= \left( \frac{\varphi_X^*}{\varphi} \right)^{1-\nu} \frac{\mathcal{E}f_X^* W^*}{Z^*}. \end{aligned}$$

## B.1.2 Prices and the Composition of Firms

As before, I assume a Pareto distribution of productivity so that equation (12) and Lemma 1 hold. For a solution to exist, the productivity dispersion parameter must satisfy the condition  $k > \nu - 1$ . The aggregate price at Home can be written as  $P = [N_D \bar{p}_D^{1-\nu} + N_X^* \bar{p}_X^{*1-\nu}]^{\frac{1}{1-\nu}}$ , where

$$\bar{p}_D = \left[ \int_{\varphi_D}^{\infty} p_D(\varphi)^{1-\nu} g(\varphi | \varphi \geq \varphi_D) d\varphi \right]^{\frac{1}{1-\nu}}$$

$$\bar{p}_X^* = \left[ \int_{\varphi_X^*}^{\infty} p_X^*(\varphi)^{1-\nu} g(\varphi | \varphi \geq \varphi_X^*) d\varphi \right]^{\frac{1}{1-\nu}}$$

denote respectively the average prices of Home and Foreign goods sold at Home, and  $N_D$  and  $N_X^*$  denote the number of sellers (at Home) from each country. We can then write the following Proposition.

### Proposition B.1 (*Domestic and imported goods average prices in the CES case*)

1.  $\bar{p}_D = \vartheta(\nu, k) \frac{W}{Z \varphi_D}$ ,  $\bar{p}_X^* = \vartheta(\nu, k) \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi_X^*}$ ,  $\bar{p}_D^* = \vartheta(\nu, k) \frac{W^*}{Z^* \varphi_D^*}$ , and  $\bar{p}_X = \vartheta(\nu, k) \frac{\tau W}{\mathcal{E} Z \varphi_X}$ , where  $\vartheta(\nu, k) = \left( \frac{k}{k - (\nu - 1)} \right)^{\frac{1}{1-\nu}} (1 + \mu) = \left( \frac{k}{k - (\nu - 1)} \right)^{\frac{1}{1-\nu}} \frac{\nu}{\nu - 1}$ .
2.  $\bar{p}_X^* = \left[ \frac{f_D W}{\mathcal{E} f_X^* W^*} \right]^{\frac{1}{\nu-1}} \bar{p}_D$  and  $\bar{p}_X = \left[ \frac{\mathcal{E} f_D^* W^*}{f_X W} \right]^{\frac{1}{\nu-1}} \bar{p}_D^*$ .

**Proof.** I obtain the results in part 1 by using Lemma 1. For the results in part 2, I use the average in prices in part 1 and equations (B-3) and (B-4). ■

Hence, the difference in the average prices of domestic and imported goods depends on the ratio of fixed costs of production—in terms of a common currency—and the elasticity of substitution between goods. In particular, they are identical if fixed costs are equal or if the elasticity of substitution tends to infinity. Recall that the translog model does not need to impose fixed costs to pin down cutoff productivity levels. Assuming fixed costs in the translog model also generates a difference between average domestic and imported goods, but only complicates the analysis without altering the model's results on pass-through and expenditure-switching effects of exchange rates.

With respect to the number of firms selling in each market, equations (14) and (15) hold for the CES case. To solve for the pools of firms  $N_P$  and  $N_P^*$ , we write the aggregate price equations  $P = [N_D \bar{p}_D^{1-\nu} + N_X^* \bar{p}_X^{*1-\nu}]^{\frac{1}{1-\nu}}$  and  $P^* = [N_D^* \bar{p}_D^{*1-\nu} + N_X \bar{p}_X^{1-\nu}]^{\frac{1}{1-\nu}}$  in terms of the cutoff rules and  $N_P$  and  $N_P^*$  by using the expressions in Proposition B.1, equations (14) and (15), and the zero-cutoff-profit conditions to substitute for  $P$  and  $P^*$ . We obtain

$$N_P = \frac{k - (\nu - 1)}{k} \frac{Z}{\nu W \varphi_{\min}^k} \left[ \frac{(\tau \tau^*)^k F^{\frac{k - (\nu - 1)}{\nu - 1}} \frac{\varphi_D^k I}{f_D} - \frac{\varphi_X^k \mathcal{E} I^*}{f_X}}{(\tau \tau^*)^k F^{\frac{k - (\nu - 1)}{\nu - 1}} - 1} \right] \quad (\text{B-5})$$

$$N_P^* = \frac{k - (\nu - 1)}{k} \frac{Z^*}{\nu W^* \varphi_{\min}^k} \left[ \frac{(\tau \tau^*)^k F^{\frac{k - (\nu - 1)}{\nu - 1}} \frac{\varphi_D^* I^*}{f_D^*} - \frac{\varphi_X^* I}{f_X^* \mathcal{E}}}{(\tau \tau^*)^k F^{\frac{k - (\nu - 1)}{\nu - 1}} - 1} \right], \quad (\text{B-6})$$

where  $F = \frac{f_X f_X^*}{f_D f_D^*}$ . In this case  $N = N_D + N_X^*$  and  $N^* = N_D^* + N_X$  can change with the exchange rate.

### B.1.3 Free-Entry Conditions and Solution

The (per period) pre-entry expected profits for Home and Foreign firms are respectively given by

$$\begin{aligned}\bar{\pi} &= \left[ \bar{\pi}_D - \frac{f_D W}{Z} \right] + \mathcal{E} \left[ \bar{\pi}_X - \frac{f_X W}{\mathcal{E} Z} \right] \\ \bar{\pi}^* &= \left[ \bar{\pi}_D^* - \frac{f_D^* W^*}{Z^*} \right] + \frac{1}{\mathcal{E}} \left[ \bar{\pi}_X^* - \frac{\mathcal{E} f_X^* W^*}{Z^*} \right],\end{aligned}$$

where  $\bar{\pi}_r = \int_{\varphi_r}^{\infty} \pi_r(\varphi) g(\varphi) d\varphi$  for  $r \in \{D, X\}$ , with a similar expression holding for  $\bar{\pi}_r^*$ . Using Corollary 2 from section 3.3 and the (before fixed costs) profit functions at the end of section B.1.1, we get

$$\begin{aligned}\bar{\pi}_D &= \frac{k}{k - (\nu - 1)} \left( \frac{\varphi_{\min}}{\varphi_D} \right)^k \frac{f_D W}{Z} & \bar{\pi}_X &= \frac{k}{k - (\nu - 1)} \left( \frac{\varphi_{\min}}{\varphi_X} \right)^k \frac{f_X W}{\mathcal{E} Z} \\ \bar{\pi}_D^* &= \frac{k}{k - (\nu - 1)} \left( \frac{\varphi_{\min}}{\varphi_D^*} \right)^k \frac{f_D^* W^*}{Z^*} & \bar{\pi}_X^* &= \frac{k}{k - (\nu - 1)} \left( \frac{\varphi_{\min}}{\varphi_X^*} \right)^k \frac{\mathcal{E} f_X^* W^*}{Z^*}.\end{aligned}$$

Hence, the free-entry conditions  $\frac{\bar{\pi}}{\delta} = \frac{f_E W}{Z}$  and  $\frac{\bar{\pi}^*}{\delta} = \frac{f_E^* W^*}{Z^*}$  can be written as

$$\frac{f_D}{\varphi_D^k} + \frac{f_X}{\varphi_X^k} = \mathfrak{F} \tag{B-7}$$

$$\frac{f_D^*}{\varphi_D^{*k}} + \frac{f_X^*}{\varphi_X^{*k}} = \mathfrak{F}^*, \tag{B-8}$$

where  $\mathfrak{F} = \frac{k - (\nu - 1)}{k \varphi_{\min}^k} (\delta f_E + f_D + f_X)$  and  $\mathfrak{F}^* = \frac{k - (\nu - 1)}{k \varphi_{\min}^k} (\delta f_E^* + f_D^* + f_X^*)$ .

We can now solve for the equilibrium cutoff productivity levels from equations (B-3), (B-4), (B-7) and (B-8). Note from these equations that, contrary to the translog case, the solution to the CES model will not depend on the income levels  $I$  and  $I^*$ . To put it differently, in the CES case the size of an economy—as captured by  $I$  and  $I^*$ —does not matter for the cutoff productivity levels. If Home and Foreign differ only in  $I$  and  $I^*$  and the exchange rate is 1, the largest country has more firms (see equations (B-5) and (B-6)) and enjoys a higher variety of goods, but the average productivities of Home and Foreign producers are identical. On the other hand, as first showed by Melitz and Ottaviano (2008) with quadratic preferences, with endogenous markups the largest country faces a tougher competitive environment and thus have larger cutoff levels. The equilibrium cutoff productivity levels for firms selling domestically are given by

$$\varphi_D = \frac{f_D^{\frac{1}{k}}}{\tau^* \mathfrak{F}^{\frac{1}{k}}} \left[ \frac{(\tau \tau^*)^k F^{\frac{k - (\nu - 1)}{\nu - 1}} - 1}{\tau^k F^{\frac{k - (\nu - 1)}{\nu - 1}} - (\rho \mathcal{E})^{\frac{k \nu}{\nu - 1}} \left( \frac{f_X^*}{f_D} \right)^{\frac{k - (\nu - 1)}{\nu - 1}}} \right]^{\frac{1}{k}} \tag{B-9}$$

$$\varphi_D^* = \frac{f_D^{*\frac{1}{k}}}{\tau \mathfrak{F}^{*\frac{1}{k}}} \left[ \frac{(\tau \tau^*)^k F^{\frac{k-(\nu-1)}{\nu-1}} - 1}{\tau^k F^{\frac{k-(\nu-1)}{\nu-1}} - \frac{1}{(\rho \mathcal{E})^{\frac{k\nu}{\nu-1}}} \left( \frac{f_X}{f_D^*} \right)^{\frac{k-(\nu-1)}{\nu-1}}} \right]^{\frac{1}{k}}, \quad (\text{B-10})$$

where  $\rho = \left( \frac{\mathfrak{F}^*}{\mathfrak{F}} \right)^{\frac{\nu-1}{k\nu}} \frac{W^*/Z^*}{W/Z}$ . For the equilibrium exporting cutoff levels  $\varphi_X^*$  and  $\varphi_X$  we substitute, respectively, equation (B-9) into equation (B-3) and equation (B-10) into equation (B-4).

### B.1.4 Exchange Rate Pass-Through in the CES Case

I begin by looking at the responses of the cutoff levels to exchange rate changes and then analyze these responses' implications for exchange rate pass-through to import prices. The following Proposition is very similar to Proposition 2 in the main text.

#### Proposition B.2 (*The cutoff level $\varphi_D$ and the exchange rate in the CES case*)

Let  $\varphi_D$  be given as in equation (B-9) and let  $\mathcal{E}$  be in a range so that an interior solution exists. Then:

1.  $\zeta_{\varphi_D, \mathcal{E}} = \frac{\nu}{\nu-1} \left[ \frac{(\rho \mathcal{E})^{\frac{k\nu}{\nu-1}} \left( \frac{f_X}{f_D^*} \right)^{\frac{k-(\nu-1)}{\nu-1}}}{\tau^k F^{\frac{k-(\nu-1)}{\nu-1}} - (\rho \mathcal{E})^{\frac{k\nu}{\nu-1}} \left( \frac{f_X}{f_D^*} \right)^{\frac{k-(\nu-1)}{\nu-1}}} \right] > 0$ , where  $\zeta_{\varphi_D, \mathcal{E}}$  is the elasticity of  $\varphi_D$  with respect to the exchange rate; and
2.  $\frac{\partial^2 \varphi_D}{\partial \mathcal{E}^2} = \frac{\varphi_D \zeta_{\varphi_D, \mathcal{E}}}{\mathcal{E}^2} \left[ (k+1) \zeta_{\varphi_D, \mathcal{E}} + \frac{\nu(k-1)+1}{\nu-1} \right] > 0$ .

That is, as in the translog case,  $\varphi_D$  is strictly increasing and strictly convex in the exchange rate.

**Proof.** For part 1 I take the logarithm of equation (B-9) and derive it with respect to  $\ln \mathcal{E}$ . For part 2 I follow the same steps as in the proof of Proposition 2. ■

It also holds that  $\varphi_D^*$  is decreasing and strictly convex in the exchange rate. Given equations (B-3) and (B-4), it follows that  $\varphi_X^*$  and  $\varphi_X$  are also convex in the exchange rate with elasticities

$$\zeta_{\varphi_X^*, \mathcal{E}} = \zeta_{\varphi_D, \mathcal{E}} + \frac{\nu}{\nu-1} > 1 + \mu \quad (\text{B-11})$$

$$\zeta_{\varphi_X, \mathcal{E}} = \zeta_{\varphi_D^*, \mathcal{E}} - \frac{\nu}{\nu-1} < -(1 + \mu), \quad (\text{B-12})$$

because  $\zeta_{\varphi_D, \mathcal{E}} > 0$ ,  $\zeta_{\varphi_D^*, \mathcal{E}} < 0$ , and  $\frac{\nu}{\nu-1} = 1 + \mu$ .

With respect to firm-level import prices and trade flows, let us focus on a Foreign firm with productivity  $\varphi \geq \varphi_X^*$ . Recall that the price this firm sets at Home (in terms of Home currency) is given by  $p_X^*(\varphi) = (1 + \mu) \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi}$ . We also have that the quantity it sells at Home is  $y_X^*(\varphi) = \frac{p_X^*(\varphi)^{-\nu}}{P^{1-\nu}} I$  and the value of its exports is  $p_X^*(\varphi) y_X^*(\varphi) = \left( \frac{p_X^*(\varphi)}{P} \right)^{1-\nu} I$ . We can then write the following Proposition:

#### Proposition B.3 (*Exchange rate pass-through to firm-level import prices, quantities, and trade flows in the CES case*)

For a Foreign firm with productivity  $\varphi \geq \varphi_X^*$ :

1. The rate of exchange rate pass-through to  $p_X^*(\varphi)$  is 1.
2. The export quantity,  $y_X^*(\varphi)$ , is decreasing and elastic with respect to the exchange rate. In particular, it declines more than  $\nu\%$  after a 1% Home-currency depreciation.
3. The value of its exports in terms of the Home currency,  $p_X^*(\varphi)y_X^*(\varphi)$ , is decreasing in the exchange rate. In particular, it declines more than  $\nu - 1\%$  after a 1% Home-currency depreciation.

**Proof.** Given that the markup is exogenous, the first result follows easily from the derivative of  $\ln p_X^*(\varphi)$  with respect to  $\ln \mathcal{E}$ . For part 2 we obtain first that  $\frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}} = -\nu + (\nu - 1)\frac{\partial \ln P}{\partial \ln \mathcal{E}}$ . From the first zero-cutoff-profit condition in section B.1.1 we obtain  $\frac{\partial \ln P}{\partial \ln \mathcal{E}} = \frac{\partial \ln p_D(\varphi_D)}{\partial \ln \mathcal{E}} = -\zeta_{\varphi_D, \mathcal{E}}$ . Hence  $\frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}} = -\nu - (\nu - 1)\zeta_{\varphi_D, \mathcal{E}} < -\nu$ . For part 3,  $\frac{\partial \ln [p_X^*(\varphi)y_X^*(\varphi)]}{\partial \ln \mathcal{E}} = 1 + \frac{\partial \ln y_X^*(\varphi)}{\partial \ln \mathcal{E}} = -(\nu - 1)(\zeta_{\varphi_D, \mathcal{E}} + 1) < -(\nu - 1)$ . ■

Therefore, the model with CES preferences implies full exchange rate pass-through to firm-level import prices and large changes in firm-level trade flows. Obviously, complete exchange rate pass-through to firm-level import prices implies an average pass-through rate of 1. But what about the pass-through rate to the aggregate import price  $\bar{p}_X^*$ ?

**Proposition B.4 (Exchange rate pass-through to the aggregate import price in the CES case)**

*The rate of pass-through of exchange rate changes to the average import price is given by*

$$\Lambda_X^* = 1 - \zeta_{\varphi_X^*, \mathcal{E}} < -\mu.$$

*That is, as in the translog case, the pass-through rate is always negative, implying a decrease in the average import price from a Home-currency depreciation, and an increase from an appreciation.*

**Proof.** From Proposition B.1 we know that  $\bar{p}_X^* = \vartheta(\nu, k) \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi_X^*}$ . Then  $\Lambda_X^* = \frac{\partial \ln \bar{p}_X^*}{\partial \ln \mathcal{E}} = 1 - \zeta_{\varphi_X^*, \mathcal{E}}$ . From equation (B-11) we know that  $\zeta_{\varphi_X^*, \mathcal{E}} > 1 + \mu$ . Therefore  $\Lambda_X^* < -\mu$ . ■

Therefore, in the CES model we obtain a result similar to the example mentioned in the Introduction: full pass-through to firm-level import prices but a negative pass-through rate to the aggregate import price.<sup>1</sup>

To conclude the CES model, let us look at its implications for the impact of the exchange rate on aggregate trade flows. The value of exports of a Foreign firm with productivity  $\varphi$  can be rewritten as  $p_X^*(\varphi)y_X^*(\varphi) = s_X^*(\varphi)I$ , where  $s_X^*(\varphi) = \left(\frac{p_X^*(\varphi)}{P}\right)^{1-\nu}$  is the firm's market-share density at Home. Using the equation for  $p_X^*(\varphi)$  and the fourth zero-cutoff-profit condition from section B.1.1, we can rewrite the market-share density as  $s_X^*(\varphi) = \frac{\nu \mathcal{E} f_X^* W^*}{Z^* I} \left(\frac{\varphi_X^*}{\varphi}\right)^{1-\nu}$ . Using Lemma 1, we get that the average market share at Home of Foreign exporters is given by  $\bar{s}_X^* = \frac{k}{k-(\nu-1)} \frac{\nu \mathcal{E} f_X^* W^*}{Z^* I}$ . The total value of Foreign exports (in terms of Home currency) is then

<sup>1</sup>As mentioned in the main text, the result of negative pass-through to the aggregate import price also holds with the quasilinear-quadratic preferences of Melitz and Ottaviano (2008). See also footnote 20 in the main text.

given by  $VE^* = N_X^* \bar{s}_X^* I$ . Therefore, the elasticity of  $VE^*$  with respect to the exchange rate is  $\frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} = \frac{\partial \ln N_X^*}{\partial \ln \mathcal{E}} + 1$ . The response of the aggregate export quantity  $Y_X^*$  to the exchange rate is computed as the difference in the responses of  $VE^*$  and  $\bar{p}_X^*$ . With analogous expressions holding for the value of Home exports, I finish the CES model with the following Proposition:

**Proposition B.5** (*The impact of the exchange rate on aggregate trade flows in the CES case*)

*The quantity and value of Home exports are increasing in the exchange rate, and the quantity and value of Foreign exports are decreasing in the exchange rate. The exchange rate elasticity of the export values (in terms of the importer's currency) is in absolute value greater than  $k(1 + \mu) - 1$ , and the exchange rate elasticity of the traded quantities is in absolute value greater than  $(k - 1)(1 + \mu)$ .*

**Proof.** First I have to prove that  $\frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} = \frac{\partial \ln N_X^*}{\partial \ln \mathcal{E}} + 1 < -[k(1 + \mu) - 1]$ . Note that this result holds if  $\frac{\partial \ln N_X^*}{\partial \ln \mathcal{E}} < -k(1 + \mu)$ . Given that  $N_X^* = \left(\frac{\varphi_{\min}}{\varphi_X^*}\right)^k N_P^*$ , it follows that  $\frac{\partial \ln N_X^*}{\partial \ln \mathcal{E}} = -k\zeta_{\varphi_X^*, \mathcal{E}} + \frac{\partial \ln N_P^*}{\partial \ln \mathcal{E}}$ . As  $\zeta_{\varphi_X^*, \mathcal{E}} > 1 + \mu$ , it is enough to prove that  $\frac{\partial \ln N_P^*}{\partial \ln \mathcal{E}} < 0$ . From equation (B-6), we obtain that  $\frac{\partial \ln N_P^*}{\partial \ln \mathcal{E}} = \frac{1}{\Xi} \left[ k(\tau\tau^*)^k F^{\frac{k-(\nu-1)}{\nu-1}} \frac{\varphi_D^{*k} I^*}{f_D^*} \zeta_{\varphi_D^*, \mathcal{E}} - \frac{\varphi_X^* I}{f_X^* \mathcal{E}} (k\zeta_{\varphi_X^*, \mathcal{E}} - 1) \right]$ , where  $\Xi > 0$  is the numerator in the term in brackets in equation (B-6). Given that  $\zeta_{\varphi_D^*, \mathcal{E}} < 0$  and  $k\zeta_{\varphi_X^*, \mathcal{E}} - 1 > k(1 + \mu) - 1 > 0$ , it follows that  $\frac{\partial \ln N_P^*}{\partial \ln \mathcal{E}} < 0$ . With respect to the aggregate export quantity  $Y_X^*$ , we have  $\frac{\partial \ln Y_X^*}{\partial \ln \mathcal{E}} = \frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} - \Lambda_X^* = \frac{\partial \ln VE^*}{\partial \ln \mathcal{E}} - (1 - \zeta_{\varphi_X^*, \mathcal{E}})$ . From the first part of the proof it follows that  $\frac{\partial \ln Y_X^*}{\partial \ln \mathcal{E}} < -(k - 1)\zeta_{\varphi_X^*, \mathcal{E}} < -(k - 1)(1 + \mu)$ . We follow similar steps to prove that  $\frac{\partial \ln VE}{\partial \ln \mathcal{E}} > k(1 + \mu) - 1$  and  $\frac{\partial \ln Y_X}{\partial \ln \mathcal{E}} > (k - 1)(1 + \mu)$ . ■

Thus, as in the translog case, expenditure-switching effects of exchange rates can be substantial and this is not reflected in the rate of pass-through to aggregate import prices. Note that the effect on trade flows is larger for high values of  $k$  (lower firm heterogeneity) and high values of  $\mu$  (low elasticity of substitution  $\nu$ ). As an example, the values  $\nu = 3.8$  ( $\mu = \frac{1}{\nu-1}$  about 0.35) and  $k = 3.4$  used by Ghironi and Melitz (2005) imply elasticities higher than 3.6 for export values and higher than 3.4 for export quantities.

## B.2 The Convexity of the Cutoff Productivity Levels

Propositions 2 and B.2 state that the cutoff productivity level  $\varphi_D$  is convex in the exchange rate for both the translog and the CES cases. It also follows that the other cutoff rules  $\varphi_X$ ,  $\varphi_D^*$  and  $\varphi_X^*$  are also convex in the exchange rate. Moreover, I also show in a previous version of this paper that this result also holds if we use the quasilinear-quadratic preferences of Melitz and Ottaviano (2008).

The convexity result implies that the direction of the exchange rate change matters for the size of the responses of prices and trade flows. For firm-level import prices in the translog case, the asymmetric pass-through result in part 2 of Proposition 3 is due in part to the economy-wide effect, which depends on the responses of the cutoff levels. In this section I look further into the sources of the convexity by presenting the model in a structure that fits the translog model

(in the main text), the CES model (from section B.1), and the quasilinear-quadratic model (in a previous version of the paper).

As before, let  $\pi_r(\varphi)$  and  $\pi_r^*(\varphi)$  be the before-fixed-costs profit functions for a Home and Foreign firm with productivity  $\varphi$  selling in marker  $r$ , for  $r \in \{D, X\}$ . We define the cutoff productivity levels as  $\varphi_r = \inf \{\varphi : \pi_r(\varphi) > \text{fc}_r / \mathcal{E}^{\mathbb{1}\{r=X\}}\}$  and  $\varphi_r^* = \inf \{\varphi : \pi_r^*(\varphi) > \mathcal{E}^{\mathbb{1}\{r=X\}} \text{fc}_r^*\}$ , where  $\text{fc}_r$  and  $\text{fc}_r^*$  denote the fixed costs (in the producer's currency) from selling in market  $r$ . (In the translog and quasilinear-quadratic case it is equivalent to define the cutoff productivity levels in terms of the profit functions or the markup functions.<sup>2</sup>) We obtain then four zero-cutoff-profit conditions from which we can derive two equations for the relationship between the cutoff levels:

$$\varphi_X^* = m_1(\mathcal{E})\varphi_D \tag{B-13}$$

$$\varphi_X = m_2(\mathcal{E})\varphi_D^*, \tag{B-14}$$

where  $m_1'(\mathcal{E}) > 0$ ,  $m_2'(\mathcal{E}) < 0$ .

We also use the zero-cutoff-profit conditions to rewrite each profit function in terms of the firm's productivity level and the cutoff rule. In the three cases, the profit function  $\pi_r(\varphi)$  is homogeneous in  $(\varphi, \varphi_r)$  for  $\varphi \geq \varphi_r$  (and zero otherwise), with an analogous description holding for  $\pi_r^*(\varphi)$ .

The Home and Foreign free-entry conditions are respectively given by  $\frac{\bar{\pi}}{\delta} = \text{fe}$  and  $\frac{\bar{\pi}^*}{\delta} = \text{fe}^*$ , where  $\text{fe}$  and  $\text{fe}^*$  denote the entry costs in each country's currency and

$$\begin{aligned} \bar{\pi} &= [\bar{\pi}_D - \text{fc}_D] + \mathcal{E}[\bar{\pi}_X - \text{fc}_X / \mathcal{E}] \\ \bar{\pi}^* &= [\bar{\pi}_D^* - \text{fc}_D^*] + \frac{1}{\mathcal{E}}[\bar{\pi}_X^* - \mathcal{E}\text{fc}_X^*]. \end{aligned}$$

We can then write the free-entry conditions as

$$\begin{aligned} \bar{\pi}_D + \mathcal{E}\bar{\pi}_X &= F \\ \bar{\pi}_D^* + \frac{1}{\mathcal{E}}\bar{\pi}_X^* &= F^*, \end{aligned}$$

where  $F = \delta\text{fe} + \text{fc}_D + \text{fc}_X$  and  $F^* = \delta\text{fe}^* + \text{fc}_D^* + \text{fc}_X^*$ .

Under our Pareto distribution of productivity with dispersion parameter  $k$ , we apply Corollary 2 from section 3.3 to obtain that  $\bar{\pi}_D$ ,  $\bar{\pi}_X$ ,  $\bar{\pi}_D^*$ , and  $\bar{\pi}_X^*$  are given by

$$\begin{aligned} \bar{\pi}_D &= \bar{h}_D \frac{\varphi_{\min}^k}{\varphi_D^{k-l}} & \bar{\pi}_X &= \bar{h}_X(\mathcal{E}) \frac{\varphi_{\min}^k}{\varphi_X^{k-l}} \\ \bar{\pi}_D^* &= \bar{h}_D^* \frac{\varphi_{\min}^k}{\varphi_D^{*k-l}} & \bar{\pi}_X^* &= \bar{h}_X^*(\mathcal{E}) \frac{\varphi_{\min}^k}{\varphi_X^{*k-l}}, \end{aligned}$$

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<sup>2</sup>Given that we assume that fixed costs are zero in the translog case, note from the profit functions in section 3.1.3 that  $\varphi_r = \inf \{\varphi : \pi_r(\varphi) > 0\}$  is equivalent to  $\varphi_r = \inf \{\varphi : \mu_r(\varphi) > 0\}$ . Therefore, in the translog case without fixed costs we can use indistinctly the terms zero-cutoff-markup conditions and zero-cutoff-profit conditions. The same is true in the quasilinear-quadratic case.



where  $l$  denotes the degree of homogeneity of the profit functions.<sup>3</sup> Combining these equations we get  $\bar{\pi}_X^* = n_1(\mathcal{E}) \left(\frac{\varphi_D}{\varphi_X^*}\right)^{k-l} \bar{\pi}_D$  and  $\bar{\pi}_X = n_2(\mathcal{E}) \left(\frac{\varphi_D}{\varphi_X^*}\right)^{k-l} \bar{\pi}_D^*$ , where  $n_1(\mathcal{E}) = \frac{\bar{h}_X^*(\mathcal{E})}{h_D}$ ,  $n_2(\mathcal{E}) = \frac{\bar{h}_X(\mathcal{E})}{h_D^*}$ ,  $n_1'(\mathcal{E}) \geq 0$ , and  $n_2'(\mathcal{E}) \leq 0$ . Using equations (B-13) and (B-14), we can rewrite the free-entry conditions as

$$\bar{\pi}_D + o_2(\mathcal{E})\bar{\pi}_D^* = F \quad (\text{B-15})$$

$$\bar{\pi}_D^* + o_1(\mathcal{E})\bar{\pi}_D = F^*, \quad (\text{B-16})$$

where  $o_1(\mathcal{E}) = \frac{n_1(\mathcal{E})}{\mathcal{E}m_1(\mathcal{E})^{k-l}}$  and  $o_2(\mathcal{E}) = \frac{\mathcal{E}n_2(\mathcal{E})}{m_2(\mathcal{E})^{k-l}}$ . In the three cases of interest we have that  $o_1'(\mathcal{E}) < 0$ ,  $o_2'(\mathcal{E}) > 0$ ,  $o_1''(\mathcal{E}) > 0$ , and  $o_2''(\mathcal{E}) > 0$ . Moreover, we also have that  $o_1(\mathcal{E})o_2(\mathcal{E}) = \bar{o}$ , where  $\bar{o}$  is a constant less than one. Solving for  $\bar{\pi}_D$  and  $\bar{\pi}_D^*$  we get

$$\bar{\pi}_D = \frac{F - o_2(\mathcal{E})F^*}{1 - \bar{o}} \quad \text{and} \quad \bar{\pi}_D^* = \frac{F^* - o_1(\mathcal{E})F}{1 - \bar{o}}.$$

Therefore, we obtain that  $\frac{\partial \bar{\pi}_D}{\partial \mathcal{E}} = -\frac{o_2'(\mathcal{E})F^*}{1-\bar{o}} < 0$ ,  $\frac{\partial \bar{\pi}_D^*}{\partial \mathcal{E}} = -\frac{o_1'(\mathcal{E})F}{1-\bar{o}} > 0$ ,  $\frac{\partial^2 \bar{\pi}_D}{\partial \mathcal{E}^2} = -\frac{o_2''(\mathcal{E})F^*}{1-\bar{o}} < 0$ ,  $\frac{\partial^2 \bar{\pi}_D^*}{\partial \mathcal{E}^2} = -\frac{o_1''(\mathcal{E})F}{1-\bar{o}} < 0$ . That is, in both countries, the per-period expect profit (before entry) from selling in the domestic market is concave in the exchange rate.

Let us focus on  $\frac{\partial^2 \bar{\pi}_D}{\partial \mathcal{E}^2} < 0$ . From above we know that  $\bar{\pi}_D = \bar{h}_D \frac{\varphi_{\min}^k}{\varphi_D}$ . Then, we can write the first derivative of  $\bar{\pi}_D$  with respect to the exchange rate as  $\frac{\partial \bar{\pi}_D}{\partial \mathcal{E}} = \frac{\partial \bar{\pi}_D}{\partial \varphi_D} \frac{\partial \varphi_D}{\partial \mathcal{E}}$  and the second derivative as

$$\frac{\partial^2 \bar{\pi}_D}{\partial \mathcal{E}^2} = \frac{\partial \bar{\pi}_D}{\partial \varphi_D} \frac{\partial \varphi_D^2}{\partial \mathcal{E}^2} + \left(\frac{\partial \varphi_D}{\partial \mathcal{E}}\right)^2 \frac{\partial^2 \bar{\pi}_D}{\partial \varphi_D^2}.$$

Note that as long as  $k > l$ ,  $\frac{\partial \bar{\pi}_D}{\partial \varphi_D} = -(k-l)\bar{h}_D \frac{\varphi_{\min}^k}{\varphi_D^{k-l+1}} < 0$  and  $\frac{\partial^2 \bar{\pi}_D}{\partial \varphi_D^2} = (k-l)(k-l+1)\bar{h}_D \frac{\varphi_{\min}^k}{\varphi_D^{k-l+2}} > 0$ . Therefore, for  $\frac{\partial^2 \bar{\pi}_D}{\partial \mathcal{E}^2}$  to be less than zero, it must be the case that

$$\frac{\partial \varphi_D^2}{\partial \mathcal{E}^2} > 0.$$

That is,  $\varphi_D$  must be convex in the exchange rate.

Although there is strong empirical support for the assumption of a Pareto distribution of productivity (see footnote 17 in the main text), I solved the model numerically to verify the robustness of the convexity result to the specification of the probability distribution. In each of the three cases of interest (translog, CES, and quasilinear-quadratic) and after trying many distributions (Normal, Gamma, Uniform, among others) I was not able to find a single case in which the convexity result was not holding.

### B.3 Firm-Level Pass-Through and Productivity

The first part of Proposition 3 states that higher productivity firms have higher exchange rate pass-through rates. In section 4.2 we mention that this result is sensitive to the choice of the util-

<sup>3</sup>The profit functions are homogeneous of degree zero for the translog and the CES case, and of degree -2 in the quasilinear-quadratic case.

ity function. We analyze three cases: the model's exact translog case (ET), the Bergin-Feenstra (BF) linear approximation to the translog case, and the Melitz-Ottaviano (MO) quasilinear-quadratic utility function case.

The Home currency import price of a good from a Foreign firm with productivity  $\varphi$  is given by

$$p_X^*(\varphi) = (1 + \mu_X^*(\varphi)) \frac{\tau^* \mathcal{E} W^*}{Z^* \varphi}.$$

According to each of the cases mentioned above, let us consider three different expressions for  $1 + \mu_X^*(\varphi)$ :

$$1 + \mu_X^*(\varphi) = \begin{cases} \Omega \left( \frac{\varphi}{\varphi_X^*} e \right) & \text{for ET} \\ \left( \frac{\varphi}{\varphi_X^*} \right)^{0.5} & \text{for BF} \\ \frac{1}{2} \left( 1 + \frac{\varphi}{\varphi_X^*} \right) & \text{for MO,} \end{cases}$$

where  $\varphi_X^*$  is the cutoff productivity rule for Foreign exporters. For each of the three cases, note that  $1 + \mu_X^*(\varphi)$  equals 1 when  $\varphi = \varphi_X^*$  (a markup of zero) and that  $1 + \mu_X^*(\varphi)$  is increasing in productivity (just as expected).

As in section 4.2, in these three cases the rate of exchange rate pass-through to  $p_X^*(\varphi)$  is given by equation (26)

$$\lambda_X^*(\varphi) = 1 - \Upsilon_X^*(\varphi) \zeta_{\varphi_X^*, \mathcal{E}},$$

where  $\zeta_{\varphi_X^*, \mathcal{E}} > 1$  and

$$\Upsilon_X^*(\varphi) = \frac{\partial \ln(1 + \mu_X^*(\varphi))}{\partial \ln u_X^*(\varphi)} = \begin{cases} \frac{1}{2 + \mu_X^*(\varphi)} & \text{for ET} \\ 0.5 & \text{for BF} \\ \frac{u_X^*(\varphi)}{1 + u_X^*(\varphi)} & \text{for MO,} \end{cases}$$

where  $u_X^*(\varphi) = \frac{\varphi}{\varphi_X^*}$ .

From the proof of Proposition 3 in Appendix A we know that  $\frac{\partial \lambda_X^*(\varphi)}{\partial \varphi} = -\zeta_{\varphi_X^*, \mathcal{E}} \frac{\partial \Upsilon_X^*(\varphi)}{\partial \varphi}$ . Given that  $\zeta_{\varphi_X^*, \mathcal{E}}$  is positive for all three cases, the positive or negative relationship between pass-through and productivity depends on the sign of  $\frac{\partial \Upsilon_X^*(\varphi)}{\partial \varphi}$ . Note that

$$\frac{\partial \Upsilon_X^*(\varphi)}{\partial \varphi} = \frac{\partial \Upsilon_X^*(\varphi)}{\partial \ln u_X^*(\varphi)} \frac{\partial \ln u_X^*(\varphi)}{\partial \varphi} = \frac{\partial \Upsilon_X^*(\varphi)}{\partial \ln u_X^*(\varphi)} \frac{1}{\varphi},$$

and

$$\frac{\partial \Upsilon_X^*(\varphi)}{\partial \ln u_X^*(\varphi)} = \frac{\partial^2 \ln(1 + \mu_X^*(\varphi))}{\partial [\ln u_X^*(\varphi)]^2} = \begin{cases} -\frac{1 + \mu_X^*(\varphi)}{(2 + \mu_X^*(\varphi))^3} < 0 & \text{for ET} \\ 0 & \text{for BF} \\ \frac{u_X^*(\varphi)}{(1 + u_X^*(\varphi))^2} > 0 & \text{for MO.} \end{cases}$$

This implies that

$$\frac{\partial \lambda_X^*(\varphi)}{\partial \varphi} \text{ is } \begin{cases} > 0 & \text{for ET} \\ 0 & \text{for BF} \\ < 0 & \text{for MO,} \end{cases}$$

so that import prices from more productive firms have higher rates of pass-through in the exact translog case, and lower rates of pass-through in the quasilinear-quadratic case.

Note that the Bergin and Feenstra's approximation assumes away the firm-specific effect. In the exact translog case, although markups do increase with  $u_X^*(\varphi) = \frac{\varphi}{\varphi_X^*}$ , the marginal change in the elasticity of  $1 + \mu_X^*(\varphi)$  with respect to  $u_X^*(\varphi)$  is decreasing (this is a stronger condition than just having that marginal markups are decreasing). The opposite happens in the quasilinear-quadratic case.

What do we observe empirically? To my knowledge, the only empirical paper that deals with the issue of firm-level productivity and pass-through by looking at exporters' markups is the recent paper by [Berman, Martin and Mayer \(2009\)](#). They divide French exporters in two groups, low-performance exporters and high-performance exporters, and find that the elasticity of exporters' markups to real exchange rates is not statistically different from zero for the low-performance group, while the elasticity is significant and 0.21—in their benchmark specification—for the high-performance group (if they do not divide the exporters, the elasticity is 0.17 and is statistically significant). These results imply that pass-through rates are close to 1 for low-performance exporters and 0.79 for high-performance exporters (with an average of 0.83 for the entire group). The authors suggest that this evidence supports the hypothesis that firm-level pass-through rates are lower for more productive firms. Some observations with respect to these results are: 1) the implied firm-level pass-through rates seem very high, which suggests that markup adjustment is not that relevant for exchange-rate pass-through (this contradicts empirical evidence cited in section 2); 2) instead of dividing exporters in two groups, a natural equation to estimate would be to include an interaction term between the productivity variable and the real exchange rate; and 3) the regressions are in levels, but prices and real exchange rates are not necessarily stationary (it would seem more appropriate to run first-difference regressions).

## B.4 The General Equilibrium Model

Table B.1 presents the 31 equations of NOEM model. The 31 endogenous variables are  $\varphi_{D,t}$ ,  $\varphi_{X,t}$ ,  $\varphi_{D,t}^*$ ,  $\varphi_{X,t}^*$ ,  $P_t$ ,  $P_t^*$ ,  $W_{t+1}$ ,  $W_{t+1}^*$ ,  $\mathcal{E}_t$ ,  $\bar{\pi}_t$ ,  $\bar{\pi}_t^*$ ,  $C_t$ ,  $C_t^*$ ,  $v_t$ ,  $v_t^*$ ,  $N_{E,t}$ ,  $N_{E,t}^*$ ,  $N_{P,t}$ ,  $N_{P,t}^*$ ,  $N_{D,t}$ ,  $N_{X,t}$ ,  $N_{D,t}^*$ ,  $N_{X,t}^*$ ,  $L_t$ ,  $L_t^*$ ,  $A_{t+1}$ ,  $A_{t+1}^*$ ,  $B_{t+1}$ ,  $B_{t+1}^*$ ,  $r_{t+1}$ , and  $r_{t+1}^*$ . The exogenous variables are  $M_t$ ,  $M_t^*$ ,  $Z_t$ ,  $Z_t^*$ ,  $\tau_t$ ,  $\tau_t^*$ ,  $f_{E,t}$ , and  $f_{E,t}^*$ . We can solve the model for shocks to one or more of the exogenous variables. In the main text, we focused on a 1% permanent and unexpected increase in the Home money supply (and assumed that the rest of the exogenous variables were constant). In this section, I develop a sensitivity analysis for the monetary shock in the main text, and also solve the model for transitory and permanent Home productivity shocks.

Table B.1: The New Open Economy Macroeconomics Model

Aggregate prices	$P_t = \Gamma(k) \frac{W_t}{Z_t \varphi_{D,t}}, P_t^* = \Gamma(k) \frac{W_t^*}{Z_t^* \varphi_{D,t}^*}$
Cutoff productivity rules	$\varphi_{X,t}^* = \tau_t^* \left[ \frac{\varepsilon_t W_t^*}{Z_t^*} \right] \varphi_{D,t}, \varphi_{X,t} = \tau_t \left[ \frac{W_t}{Z_t} \right] \varphi_{D,t}^*$
Average profits	$\bar{\pi}_t = \frac{\psi P_t C_t}{\varphi_{D,t}^k} + \mathcal{E}_t \frac{\psi P_t^* C_t^*}{\varphi_{X,t}^k}, \bar{\pi}_t^* = \frac{\psi P_t^* C_t^*}{\varphi_{D,t}^k} + \frac{1}{\mathcal{E}_t} \frac{\psi P_t C_t}{\varphi_{X,t}^k}, \text{ where } \psi = \frac{\gamma \bar{\mu}(k) \varphi_{\min}^k}{k+1}$
Free-entry conditions	$\bar{\pi}_t + v_t = \frac{f_{E,t} W_t}{Z_t}, \bar{\pi}_t^* + v_t^* = \frac{f_{E,t}^* W_t^*}{Z_t^*}$
Number of entrants	$N_{E,t} = N_{P,t} - (1 - \delta) N_{P,t-1}, N_{E,t}^* = N_{P,t}^* - (1 - \delta) N_{P,t-1}^*$
Pool of producers	$N_{P,t} = \frac{1}{\gamma \bar{\mu}(k) \varphi_{\min}^k} \left[ \frac{(\tau_t \tau_t^*)^k \varphi_{D,t}^k - \varphi_{X,t}^k}{(\tau_t \tau_t^*)^{k-1}} \right], N_{P,t}^* = \frac{1}{\gamma \bar{\mu}(k) \varphi_{\min}^k} \left[ \frac{(\tau_t \tau_t^*)^k \varphi_{D,t}^k - \varphi_{X,t}^k}{(\tau_t \tau_t^*)^{k-1}} \right]$
Actual producers	$N_{D,t} = \left( \frac{\varphi_{\min}}{\varphi_{D,t}} \right)^k N_{P,t}, N_{X,t} = \left( \frac{\varphi_{\min}}{\varphi_{X,t}} \right)^k N_{P,t}$ $N_{D,t}^* = \left( \frac{\varphi_{\min}}{\varphi_{D,t}^*} \right)^k N_{P,t}^*, N_{X,t}^* = \left( \frac{\varphi_{\min}}{\varphi_{X,t}^*} \right)^k N_{P,t}^*$
Labor demands	$L_t^D = N_{E,t} \frac{f_{E,t}}{Z_t} + \frac{k}{k+1} \frac{\gamma \bar{\mu}(k)}{W_t} (N_{D,t} P_t C_t + N_{X,t} \mathcal{E}_t P_t^* C_t^*)$ $L_t^{*D} = N_{E,t}^* \frac{f_{E,t}^*}{Z_t^*} + \frac{k}{k+1} \frac{\gamma \bar{\mu}(k)}{W_t^*} (N_{D,t}^* P_t^* C_t^* + N_{X,t}^* \frac{P_t C_t}{\mathcal{E}_t})$
Euler equations (bonds)	$\frac{1+\eta A_{t+1}}{1+r_{t+1}} = \beta E_t \left[ \frac{C_t}{C_{t+1}} \right], \frac{1+\eta B_{t+1}}{1+r_{t+1}^*} = \beta E_t \left[ \frac{C_t}{C_{t+1}} \frac{Q_{t+1}}{Q_t} \right], \text{ where } Q_t = \frac{\varepsilon_t P_t^*}{P_t}$ $\frac{1+\eta B_{t+1}^*}{1+r_{t+1}^*} = \beta E_t \left[ \frac{C_t^*}{C_{t+1}^*} \right], \frac{1+\eta A_{t+1}^*}{1+r_{t+1}^*} = \beta E_t \left[ \frac{C_t^*}{C_{t+1}^*} \frac{Q_{t+1}}{Q_t} \right]$
Euler equations (shares)	$v_t = \beta(1 - \delta) E_t \left[ \frac{P_t C_t}{P_{t+1} C_{t+1}} (\bar{\pi}_{t+1} + v_{t+1}) \right], v_t^* = \beta(1 - \delta) E_t \left[ \frac{P_t^* C_t^*}{P_{t+1}^* C_{t+1}^*} (\bar{\pi}_{t+1}^* + v_{t+1}^*) \right]$
Euler equations (money)	$\frac{M_t}{P_t} \left( 1 - \beta E_t \left[ \frac{P_t C_t}{P_{t+1} C_{t+1}} \right] \right) = \chi C_t, \frac{M_t^*}{P_t^*} \left( 1 - \beta E_t \left[ \frac{P_t^* C_t^*}{P_{t+1}^* C_{t+1}^*} \right] \right) = \chi C_t^*$
Euler equations (labor)	$W_{t+1} E_t \left[ \frac{L_{t+1}}{P_{t+1} C_{t+1}} \right] = \frac{\theta \kappa}{\theta - 1} E_t [L_{t+1}^2], W_{t+1}^* E_t \left[ \frac{L_{t+1}^*}{P_{t+1}^* C_{t+1}^*} \right] = \frac{\theta \kappa}{\theta - 1} E_t [L_{t+1}^{*2}]$
Bond market equilibrium	$A_{t+1} + A_{t+1}^* = 0, B_{t+1} + B_{t+1}^* = 0$
Net foreign assets	$P_t A_{t+1} + \mathcal{E}_t P_t^* B_{t+1} = P_t (1 + r_t) A_t + \mathcal{E}_t P_t^* (1 + r_t^*) B_t - \frac{1}{2} (P_t C_t - \mathcal{E}_t P_t^* C_t^*)$ $+ \frac{1}{2} (W_t L_t - \mathcal{E}_t W_t^* L_t^*) - \frac{1}{2} (N_{E,t} v_t - N_{E,t}^* \mathcal{E}_t v_t^*) + \frac{1-\delta}{2} (N_{P,t-1} \bar{\pi}_t - N_{P,t-1}^* \mathcal{E}_t \bar{\pi}_t^*)$

### B.4.1 Sensitivity Analysis

Tables B.2 and B.3 show the model's responses to a 1% permanent increase in Home money supply at different time horizons and for many different parameter values. We focus on the responses of the 20 variables in Figure 2. With the exception of the last two columns, the responses represent percent deviations from the initial steady state. The last two columns are presented as proportion of total consumption expenditure. Table B.2 presents the responses at time 1 (the time of the shock) and time 5, while Table B.3 presents the responses at time 10 and time 20. For each time period, the top row shows the model's responses in the benchmark case, with parameter values given by  $\beta = 0.96$ ,  $\delta = 0.1$ ,  $\gamma = 1$ ,  $\varphi_{\min} = 1$ ,  $k = 4$ ,  $\chi = 0.1$ ,  $\theta = 5$ ,  $\kappa = 0.75$ ,  $Z = Z^* = 1$ ,  $f_E = f_E^* = 0.2$ ,  $\tau = \tau^* = 1.4$ ,  $\eta = 0.0025$ , and  $\varsigma = 0.001$ . We analyze the model's responses to different values of  $\delta$ ,  $\gamma$ ,  $k$ ,  $\theta$ ,  $\kappa$ ,  $\tau$ ,  $\tau^*$ ,  $f_E$ , and  $f_E^*$ .

At the time of the shock, the model's responses to different values for  $\gamma$ ,  $\theta$ ,  $\kappa$ ,  $f_E$ , and  $f_E^*$  are very similar to the benchmark case. The model is more sensitive to different values of  $\delta$ ,  $k$ , and specially  $\tau$  and  $\tau^*$ . The response of the nominal exchange rate increases with  $\delta$  (higher death rate),  $k$  (less firm heterogeneity), and the iceberg trade costs ( $\tau$  and  $\tau^*$ ). Of these, the iceberg trade costs have the higher impact on entry (as seen by the changes in the pools of firms). Overall, however, the fourteen cases present a similar picture in comparison with the benchmark case. The changes in aggregate prices and consumption are very similar for all cases. For example, the changes in the Home aggregate price and consumption are close to -0.9% and 1.9%, respectively. There are large changes in the pools of firms, particularly at Home, and there is a large improvement in the Home country's trade balance (equal to the *NFA* position at time 1).

By time 5, the cases with different values of  $\delta$  have the most important differences in real variables with respect to the benchmark case. This fact points out the relevance of the death rate for the persistence of the monetary shock. The higher the death rate, the faster the economy moves towards its new steady state. With respect to the sensitivity to trade costs and the productivity dispersion parameter, noticeable differences (in comparison to the benchmark case) remain for some variables. By time 10, there is still a difference in the deviation of the pool of Home firms,  $N_P$ , for the case of a lower death rate ( $\delta = 0.05$ ). By time 20, the deviations from the initial steady state are in all cases are very similar.

Overall, the sensitivity analysis shows that the patterns observed in Figure 2 are robust to different parameter values. Although there are not dramatic differences with respect to the benchmark case, we have found that the parameters that affect the most the magnitude and persistence of the responses are the death rate ( $\delta$ ), the productivity dispersion parameter ( $k$ ), and the iceberg trade costs ( $\tau$  and  $\tau^*$ ).

### B.4.2 Productivity Shocks

In this section I consider the effects of transitory and permanent 1 percent productivity shocks at Home. For this purpose, I assume that aggregate productivities at Home and Foreign follow

Table B.2: Responses to a 1% Increase in Home Money Supply — Sensitivity Analysis

	$\mathcal{E}$	$\varphi_D$	$\varphi_D^*$	$\varphi_X$	$\varphi_X^*$	$N_P$	$N_P^*$	$N_D$	$N_D^*$	$N_X$	$N_X^*$	$W$	$W^*$	$P$	$P^*$	$C$	$C^*$	$Q$	$\frac{TB}{PC}$	$\frac{NFA}{PC}$	
<b>Time 1</b>																					
Benchmark	<b>0.29</b>	<b>0.93</b>	<b>-0.12</b>	<b>-0.41</b>	<b>1.22</b>	<b>5.65</b>	<b>-2.38</b>	<b>1.82</b>	<b>-1.92</b>	<b>7.38</b>	<b>-7.00</b>	<b>0.00</b>	<b>0.00</b>	<b>-0.92</b>	<b>0.12</b>	<b>1.94</b>	<b>-0.12</b>	<b>1.34</b>	<b>2.81</b>	<b>2.81</b>	<b>2.81</b>
$\delta = 0.05$	0.16	0.88	-0.04	-0.19	1.04	5.10	-1.68	1.48	-1.54	5.92	-5.67	0.00	0.00	-0.87	0.04	1.89	-0.04	1.08	2.21	2.21	2.21
$\delta = 0.14$	0.36	0.93	-0.16	-0.52	1.30	5.84	-2.73	1.98	-2.10	8.08	-7.62	0.00	0.00	-0.92	0.16	1.94	-0.17	1.46	3.10	3.10	3.10
$\gamma = 2$	0.27	0.91	-0.11	-0.38	1.19	5.54	-2.28	1.77	-1.86	7.15	-6.79	0.00	0.00	-0.90	0.11	1.92	-0.11	1.30	2.72	2.72	2.72
$\gamma = 5$	0.25	0.90	-0.09	-0.34	1.15	5.40	-2.14	1.69	-1.78	6.84	-6.51	0.00	0.00	-0.89	0.09	1.91	-0.10	1.24	2.59	2.59	2.59
$k = 3$	0.13	1.06	-0.05	-0.18	1.19	5.37	-2.32	2.08	-2.16	5.93	-5.72	0.00	0.00	-1.05	0.06	2.07	-0.06	1.24	2.87	2.87	2.87
$k = 4.5$	0.36	0.86	-0.12	-0.48	1.23	5.67	-2.30	1.65	-1.76	7.99	-7.52	0.00	0.00	-0.85	0.13	1.87	-0.13	1.35	2.67	2.67	2.67
$\theta = 2$	0.32	0.95	-0.14	-0.45	1.26	5.82	-2.55	1.91	-2.02	7.75	-7.33	0.00	0.00	-0.93	0.14	1.96	-0.14	1.40	2.96	2.96	2.96
$\theta = 10$	0.28	0.92	-0.11	-0.39	1.21	5.60	-2.34	1.80	-1.90	7.28	-6.91	0.00	0.00	-0.91	0.12	1.93	-0.12	1.32	2.77	2.77	2.77
$\kappa = 0.5$	0.26	0.91	-0.10	-0.36	1.18	5.49	-2.23	1.74	-1.83	7.04	-6.69	0.00	0.00	-0.90	0.10	1.92	-0.11	1.28	2.67	2.67	2.67
$\kappa = 1$	0.31	0.94	-0.13	-0.43	1.25	5.75	-2.49	1.87	-1.98	7.61	-7.20	0.00	0.00	-0.93	0.13	1.95	-0.13	1.38	2.90	2.90	2.90
$\tau, \tau^* = 1.2$	0.06	0.82	-0.01	-0.08	0.89	6.70	-3.43	3.27	-3.39	7.02	-6.77	0.00	0.00	-0.81	0.01	1.83	-0.02	0.90	4.16	4.16	4.16
$\tau, \tau^* = 1.6$	0.46	0.92	-0.12	-0.58	1.39	4.83	-1.57	1.05	-1.11	7.29	-6.87	0.00	0.00	-0.91	0.12	1.93	-0.12	1.51	1.80	1.80	1.80
$f_E, f_E^* = 0.1$	0.27	0.91	-0.11	-0.38	1.19	5.54	-2.28	1.77	-1.86	7.15	-6.79	0.00	0.00	-0.90	0.11	1.92	-0.11	1.30	2.72	2.72	2.72
$f_E, f_E^* = 0.25$	0.29	0.93	-0.12	-0.41	1.23	5.68	-2.41	1.84	-1.94	7.45	-7.06	0.00	0.00	-0.92	0.12	1.94	-0.13	1.35	2.84	2.84	2.84
<b>Time 5</b>																					
Benchmark	<b>0.35</b>	<b>0.15</b>	<b>0.00</b>	<b>0.11</b>	<b>0.04</b>	<b>0.63</b>	<b>-0.04</b>	<b>0.05</b>	<b>-0.05</b>	<b>0.19</b>	<b>-0.19</b>	<b>0.64</b>	<b>0.18</b>	<b>0.50</b>	<b>0.18</b>	<b>0.50</b>	<b>-0.18</b>	<b>0.03</b>	<b>-0.06</b>	<b>5.92</b>	<b>5.92</b>
$\delta = 0.05$	0.22	0.29	0.02	0.13	0.17	1.37	-0.15	0.22	-0.22	0.84	-0.83	0.49	0.16	0.21	0.14	0.79	-0.15	0.15	0.18	5.82	5.82
$\delta = 0.14$	0.42	0.08	0.00	0.11	-0.03	0.29	0.03	-0.04	0.04	-0.14	0.14	0.72	0.19	0.64	0.19	0.37	-0.19	-0.03	-0.18	5.67	5.67
$\gamma = 2$	0.34	0.15	0.00	0.12	0.03	0.62	-0.03	0.04	-0.04	0.15	-0.15	0.64	0.18	0.50	0.18	0.50	-0.18	0.03	-0.07	5.65	5.65
$\gamma = 5$	0.33	0.15	0.00	0.13	0.02	0.61	-0.02	0.03	-0.03	0.10	-0.10	0.64	0.19	0.50	0.18	0.51	-0.19	0.02	-0.10	5.30	5.30
$k = 3$	0.21	0.22	0.00	0.13	0.09	0.80	-0.13	0.14	-0.14	0.39	-0.39	0.57	0.22	0.35	0.22	0.65	-0.22	0.08	0.00	6.45	6.45
$k = 4.5$	0.41	0.12	0.00	0.11	0.02	0.57	-0.01	0.02	-0.02	0.09	-0.09	0.68	0.16	0.56	0.16	0.44	-0.16	0.02	-0.07	5.46	5.46
$\theta = 2$	0.36	0.15	0.00	0.10	0.05	0.65	-0.06	0.06	-0.06	0.25	-0.25	0.65	0.18	0.50	0.18	0.50	-0.18	0.05	-0.03	6.35	6.35
$\theta = 10$	0.35	0.15	0.00	0.11	0.03	0.63	-0.04	0.04	-0.04	0.17	-0.17	0.64	0.18	0.50	0.18	0.50	-0.18	0.03	-0.06	5.81	5.81
$\kappa = 0.5$	0.34	0.15	0.00	0.12	0.03	0.62	-0.03	0.03	-0.03	0.13	-0.13	0.64	0.19	0.50	0.18	0.50	-0.18	0.02	-0.08	5.53	5.53
$\kappa = 1$	0.36	0.15	0.00	0.11	0.04	0.65	-0.06	0.06	-0.06	0.22	-0.22	0.65	0.18	0.50	0.18	0.50	-0.18	0.04	-0.04	6.18	6.18
$\tau, \tau^* = 1.2$	0.16	0.13	0.01	0.11	0.04	0.63	-0.05	0.10	-0.10	0.20	-0.20	0.55	0.29	0.41	0.28	0.59	-0.28	0.03	-0.15	8.47	8.47
$\tau, \tau^* = 1.6$	0.50	0.15	0.00	0.10	0.05	0.63	-0.04	0.03	-0.03	0.22	-0.22	0.71	0.11	0.56	0.11	0.44	-0.11	0.05	-0.01	3.93	3.93
$f_E, f_E^* = 0.1$	0.34	0.15	0.00	0.12	0.03	0.62	-0.03	0.04	-0.04	0.15	-0.15	0.64	0.18	0.50	0.18	0.50	-0.18	0.03	-0.07	5.65	5.65
$f_E, f_E^* = 0.25$	0.35	0.15	0.00	0.11	0.04	0.64	-0.05	0.05	-0.05	0.20	-0.20	0.64	0.18	0.50	0.18	0.50	-0.18	0.04	-0.05	6.00	6.00

With the exception of the last two columns, the numbers represent percent deviations from the initial steady state. The last two columns are presented as proportion of total consumption expenditure. Benchmark parameter values:  $\beta = 0.96$ ,  $\delta = 0.1$ ,  $\gamma = 1$ ,  $\varphi_{\min} = 1$ ,  $k = 4$ ,  $\chi = 0.1$ ,  $\theta = 5$ ,  $\kappa = 0.75$ ,  $Z = Z^* = 1$ ,  $f_E = f_E^* = 0.2$ ,  $\tau = \tau^* = 1.4$ ,  $\eta = 0.0025$ , and  $\varsigma = 0.001$ .

Table B.3: Responses to a 1% Increase in Home Money Supply — Sensitivity Analysis (Continued)

	$\mathcal{E}$	$\varphi_D$	$\varphi_D^*$	$\varphi_X$	$\varphi_X^*$	$N_P$	$N_P^*$	$N_D$	$N_D^*$	$N_X$	$N_X^*$	$W$	$W^*$	$P$	$P^*$	$C$	$C^*$	$Q$	$\frac{TB}{PC}$	$\frac{NFA}{PC}$	
<b>Time 10</b>																					
Benchmark	0.44	0.02	0.00	0.14	-0.13	-0.11	0.16	-0.17	0.17	-0.67	0.67	0.79	0.20	0.78	0.21	0.22	-0.21	-0.12	-0.39	5.48	
$\delta = 0.05$	0.33	0.08	0.00	0.17	-0.09	0.17	0.15	-0.14	0.14	-0.52	0.53	0.71	0.21	0.64	0.21	0.36	-0.21	-0.10	-0.35	6.00	
$\delta = 0.14$	0.50	0.00	0.00	0.12	-0.12	-0.15	0.16	-0.17	0.17	-0.65	0.65	0.82	0.19	0.82	0.19	0.19	-0.19	-0.12	-0.37	5.07	
$\gamma = 2$	0.45	0.02	0.00	0.14	-0.13	-0.10	0.16	-0.17	0.17	-0.66	0.66	0.79	0.20	0.78	0.21	0.22	-0.21	-0.12	-0.39	5.14	
$\gamma = 5$	0.46	0.02	0.00	0.14	-0.13	-0.10	0.15	-0.17	0.17	-0.65	0.66	0.80	0.20	0.78	0.20	0.22	-0.20	-0.12	-0.38	4.68	
$k = 3$	0.35	0.03	-0.01	0.14	-0.12	-0.09	0.17	-0.19	0.19	-0.52	0.52	0.74	0.24	0.71	0.25	0.29	-0.25	-0.11	-0.45	5.98	
$k = 4.5$	0.49	0.01	0.00	0.14	-0.13	-0.11	0.15	-0.16	0.16	-0.72	0.73	0.82	0.18	0.80	0.18	0.20	-0.19	-0.13	-0.35	5.06	
$\theta = 2$	0.44	0.02	0.00	0.14	-0.13	-0.11	0.17	-0.17	0.17	-0.67	0.67	0.79	0.21	0.78	0.21	0.23	-0.21	-0.12	-0.39	6.05	
$\theta = 10$	0.45	0.02	0.00	0.14	-0.13	-0.11	0.16	-0.17	0.17	-0.66	0.67	0.79	0.20	0.78	0.21	0.22	-0.21	-0.12	-0.39	5.34	
$\kappa = 0.5$	0.45	0.02	0.00	0.14	-0.13	-0.10	0.16	-0.17	0.17	-0.66	0.66	0.80	0.20	0.78	0.20	0.22	-0.21	-0.12	-0.39	4.98	
$\kappa = 1$	0.44	0.02	0.00	0.14	-0.13	-0.11	0.17	-0.17	0.17	-0.67	0.67	0.79	0.21	0.78	0.21	0.23	-0.21	-0.12	-0.39	5.83	
$\tau, \tau^* = 1.2$	0.30	0.02	-0.01	0.09	-0.08	-0.19	0.23	-0.27	0.26	-0.55	0.55	0.70	0.31	0.68	0.32	0.32	-0.32	-0.07	-0.58	7.62	
$\tau, \tau^* = 1.6$	0.57	0.02	0.00	0.17	-0.15	-0.05	0.11	-0.11	0.11	-0.70	0.71	0.86	0.13	0.85	0.13	0.16	-0.13	-0.15	-0.24	3.75	
$f_E, f_E^* = 0.1$	0.45	0.02	0.00	0.14	-0.13	-0.10	0.16	-0.17	0.17	-0.66	0.66	0.79	0.20	0.78	0.21	0.22	-0.21	-0.12	-0.39	5.14	
$f_E, f_E^* = 0.25$	0.44	0.02	0.00	0.14	-0.13	-0.11	0.16	-0.17	0.17	-0.67	0.67	0.79	0.20	0.78	0.21	0.23	-0.21	-0.12	-0.39	5.59	
<b>Time 20</b>																					
Benchmark	0.60	0.00	-0.01	0.10	-0.10	-0.14	0.12	-0.14	0.14	-0.54	0.54	0.86	0.16	0.86	0.16	0.14	-0.16	-0.10	-0.30	3.98	
$\delta = 0.05$	0.53	0.01	-0.01	0.12	-0.12	-0.12	0.12	-0.16	0.16	-0.61	0.61	0.84	0.18	0.84	0.19	0.17	-0.19	-0.11	-0.35	4.42	
$\delta = 0.14$	0.64	0.00	0.00	0.09	-0.09	-0.13	0.12	-0.13	0.13	-0.49	0.49	0.87	0.14	0.88	0.15	0.13	-0.15	-0.09	-0.28	3.71	
$\gamma = 2$	0.62	0.00	-0.01	0.10	-0.10	-0.13	0.11	-0.13	0.13	-0.51	0.51	0.87	0.15	0.87	0.15	0.13	-0.16	-0.09	-0.29	3.58	
$\gamma = 5$	0.65	0.00	-0.01	0.09	-0.09	-0.11	0.10	-0.12	0.12	-0.47	0.48	0.88	0.14	0.88	0.14	0.12	-0.14	-0.09	-0.27	3.07	
$k = 3$	0.57	0.00	-0.01	0.09	-0.10	-0.15	0.13	-0.16	0.16	-0.43	0.43	0.85	0.17	0.85	0.19	0.15	-0.19	-0.09	-0.34	4.03	
$k = 4.5$	0.63	0.00	0.00	0.10	-0.11	-0.13	0.11	-0.13	0.13	-0.58	0.58	0.88	0.14	0.88	0.15	0.13	-0.15	-0.10	-0.28	3.75	
$\theta = 2$	0.57	0.00	0.00	0.11	-0.11	-0.15	0.13	-0.15	0.15	-0.58	0.58	0.85	0.17	0.85	0.17	0.15	-0.17	-0.11	-0.33	4.66	
$\theta = 10$	0.61	0.00	-0.01	0.10	-0.10	-0.13	0.11	-0.14	0.14	-0.53	0.53	0.87	0.15	0.87	0.16	0.14	-0.16	-0.10	-0.30	3.81	
$\kappa = 0.5$	0.63	0.00	-0.01	0.09	-0.10	-0.12	0.11	-0.13	0.13	-0.50	0.50	0.88	0.14	0.87	0.15	0.13	-0.15	-0.09	-0.28	3.40	
$\kappa = 1$	0.58	0.00	0.00	0.10	-0.11	-0.15	0.13	-0.15	0.15	-0.56	0.56	0.86	0.16	0.86	0.17	0.14	-0.17	-0.10	-0.32	4.40	
$\tau, \tau^* = 1.2$	0.52	0.00	-0.01	0.06	-0.07	-0.19	0.16	-0.20	0.20	-0.42	0.42	0.81	0.22	0.81	0.23	0.19	-0.23	-0.05	-0.43	5.26	
$\tau, \tau^* = 1.6$	0.67	0.00	0.00	0.13	-0.13	-0.09	0.08	-0.09	0.09	-0.61	0.61	0.91	0.10	0.91	0.11	0.09	-0.11	-0.13	-0.20	2.86	
$f_E, f_E^* = 0.1$	0.62	0.00	-0.01	0.10	-0.10	-0.13	0.11	-0.13	0.13	-0.51	0.51	0.87	0.15	0.87	0.15	0.13	-0.16	-0.09	-0.29	3.58	
$f_E, f_E^* = 0.25$	0.60	0.00	-0.01	0.10	-0.11	-0.14	0.12	-0.14	0.14	-0.55	0.55	0.86	0.16	0.86	0.16	0.14	-0.17	-0.10	-0.31	4.11	

With the exception of the last two columns, the numbers represent percent deviations from the initial steady state. The last two columns are presented as proportion of total consumption expenditure. Benchmark parameter values:  $\beta = 0.96$ ,  $\delta = 0.1$ ,  $\gamma = 1$ ,  $\varphi_{\min} = 1$ ,  $k = 4$ ,  $\chi = 0.1$ ,  $\theta = 5$ ,  $\kappa = 0.75$ ,  $Z = Z^* = 1$ ,  $f_E = f_E^* = 0.2$ ,  $\tau = \tau^* = 1.4$ ,  $\eta = 0.0025$ , and  $\varsigma = 0.001$ .

the processes

$$\begin{aligned}\ln Z_{t+1} &= \varrho \ln Z_t + \xi_{t+1}^z \\ \ln Z_{t+1}^* &= \varrho \ln Z_t^* + \xi_{t+1}^{z*},\end{aligned}$$

where  $\varrho$  is the persistence parameter, and  $\xi_{t+1}^z$  and  $\xi_{t+1}^{z*}$  are normal white noise processes with standard deviation  $\omega$ .

I use the same parameter values as in the benchmark case for the 1% Home monetary expansion. In this case, however,  $M_t$  and  $M_t^*$  are equal to 2.5 for every  $t$ . At time 0 we are in the initial steady state, with  $Z_0 = Z_0^* = 1$ . The standard deviation  $\omega$  is set at 0.001. We look first at the case of a transitory increase in  $Z$  at time 1.

For quarterly data, the persistence parameter for a transitory productivity shock is usually set between 0.9 and 0.95 (Ghironi and Melitz (2005) use 0.9). These values correspond to a persistence parameter between 0.65 and 0.82 for yearly data. I choose the lower bound,  $\varrho = 0.65$ , so that about 90% of the time-1 1% increase in  $Z$  has vanished by time 7. Figure B.1 presents the model's responses. As in Figure 2 in the main text, the horizontal axis represents time in years and, with the exception of the last two subfigures, the variables' responses represent percent deviations from the initial steady state.

The transitory productivity shock does not have any permanent effects—the steady state does not change. Nevertheless, there are substantial changes at the time of the shock and during the transition. Indeed, comparing Figures 2 and B.1 we observe that, with some exceptions for nominal variables, the responses to the transitory productivity shock are very similar to the responses to the permanent monetary expansion.

At the time of the positive productivity shock at Home there is an appreciation of the Home currency. The percent decline in  $\mathcal{E}$  is, however, smaller than the percent increase in  $Z$ , which implies a decline in the relative cost of Home effective labor,  $\frac{W_1/Z_1}{\varepsilon_1 W_1^*/Z_1^*}$  (recall that  $W_1$  and  $W_1^*$  were set before the shock).<sup>4</sup> This creates a response at the time of the shock that is similar to the response to the permanent Home monetary expansion analyzed in the main text (the only difference is the response of the nominal exchange rate): entry increases at Home and declines at Foreign, with higher competition at Home driving up  $\varphi_D$  and  $\varphi_X^*$  (with the opposite happening for  $\varphi_D^*$  and  $\varphi_X$ ); Foreign sellers are displaced by Home sellers in both markets, consumption increases at Home and slightly declines at Foreign, and Home runs a positive trade balance (implying a positive net foreign asset position). Note that even though there is a decline in  $\mathcal{E}$ , the real exchange rate,  $Q$ , increases because of the substantial decline in  $P$  (the decline in  $P$  is driven by the increase in both  $Z$  and  $\varphi_D$ ).

As the productivity shock dissipates, the economy moves towards the original steady state following the same transition behavior as in the Home monetary expansion case. Obviously, the exceptions are the nominal variables  $\mathcal{E}$ ,  $W$  and  $P$ , which move towards higher levels after a permanent Home monetary expansion (because money is neutral in the long run) but must return to their original levels after a transitory Home productivity increase. Summarizing the

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<sup>4</sup>Under our benchmark parameter values, the relative cost of Home effective labor declines at the time of the Home productivity shock as long as the persistence parameter,  $\varrho$ , is below 0.985.



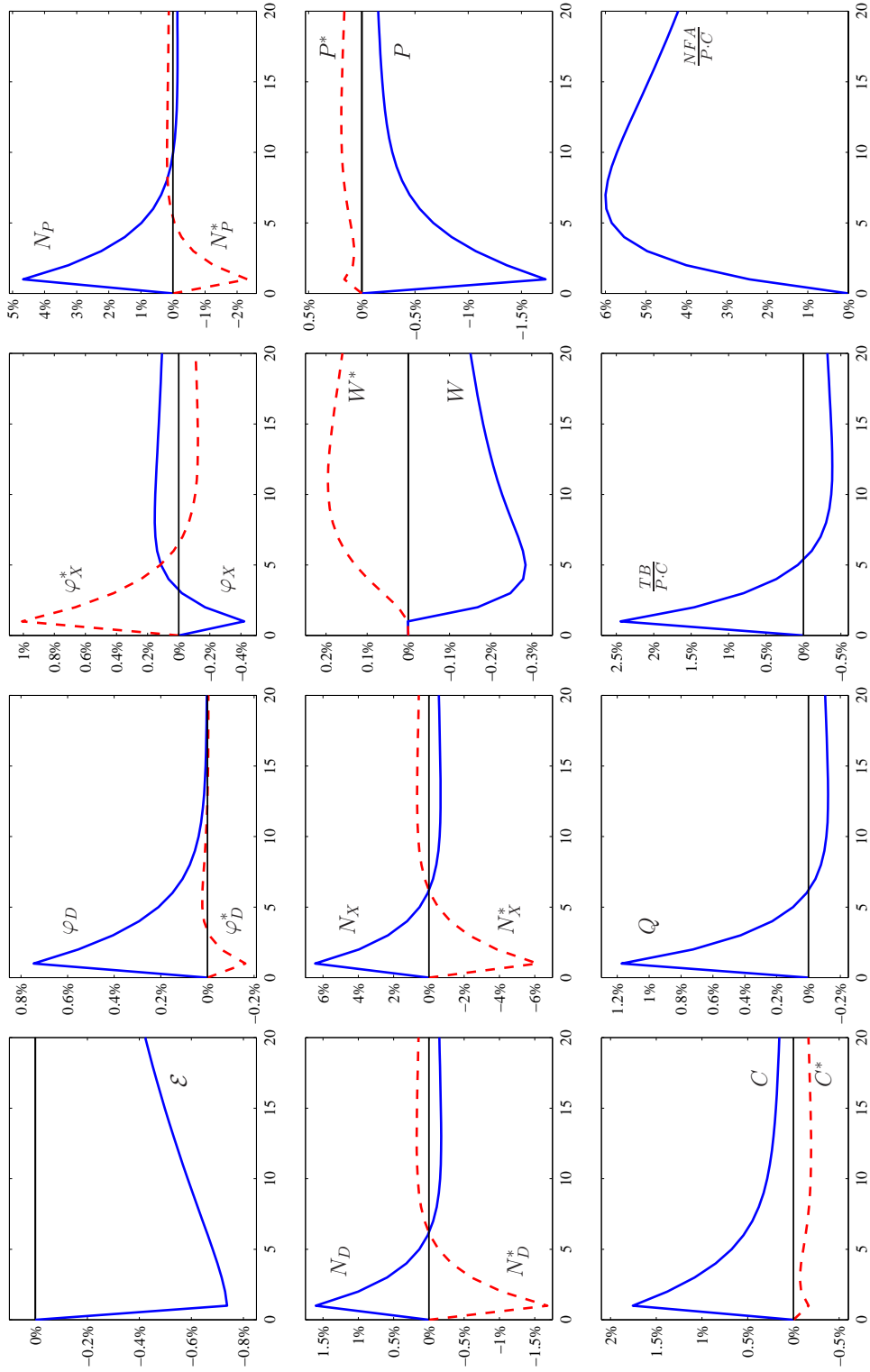


Figure B.1: A Transitory 1% Increase in Home Aggregate Productivity

transition movements, we have that as Home productivity decreases towards its original level, entry at Home declines so that the pool of Home firms moves back towards its steady state level, while the opposite happens with Foreign entry; the changes in entry patterns during the transition decrease the labor demand at Home and increase the labor demand at Foreign, generating small changes in nominal wages. After five periods of Home trade balance surpluses, the Home economy moves to trade deficits for the rest of the transition. At the end, we obtain that a 1% transitory productivity shock at Home has substantial effects in the countries' net foreign asset positions. The Home net foreign asset position reaches its peak of about 6% of total consumption expenditure by time 7 (when about 90% of the productivity shock has already vanished), and then declines slowly towards a financial autarky position.

Finally, I look into the effects of a permanent 1% productivity shock at Home. The difference with the previous experiment is that now I set  $\rho$  to 1. Figure B.2 shows the model's responses. As we can observe, all the variables in Figure B.2 jump at the time of the shock to the new steady state.<sup>5</sup> Given fixed money supplies, the nominal exchange rate jumps down immediately to its new steady state to reflect the permanent output difference between the countries. Entry increases at Home at the time of the shock to drive the pool of firms,  $N_P$ , to its higher steady state; it then moves to its new steady state of  $\delta N_P$  (higher than the original level). The increase in entry generates a jump in Home labor at the time of the shock, and then it goes back to its original level. All the cutoff rules increase, which implies that there is also an increase in average productivity at Foreign. The number of Home sellers in both markets permanently increases, and the opposite happens for Foreign sellers. Even though there is smaller mass of Foreign sellers, there are no changes in the Foreign equilibrium labor because the labor released by the Foreign firms that are no longer producing is (exactly) absorbed by the remaining Foreign exporters, who increase their output helped by the Home currency appreciation. Aggregate prices,  $P$  and  $P^*$ , decrease: at Home there is a large drop because of the increase in both  $Z$  and  $\varphi_D$ ; and at Foreign there is a slight decline because of the increase in  $\varphi_D^*$ . These price changes are mirrored by a large increase in consumption at Home and a modest increase at Foreign. As in the transitory case, although there is an appreciation of the Home currency, the real exchange rate increases (though only 0.1%) because the decrease in  $P$  dominates the declines in  $\mathcal{E}$  and  $P^*$ . The economy remains in financial autarky.

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<sup>5</sup>Ghironi and Melitz (2005) obtain dynamics for a permanent productivity shock due to their "time to build" assumption (firms start producing one period after entry).

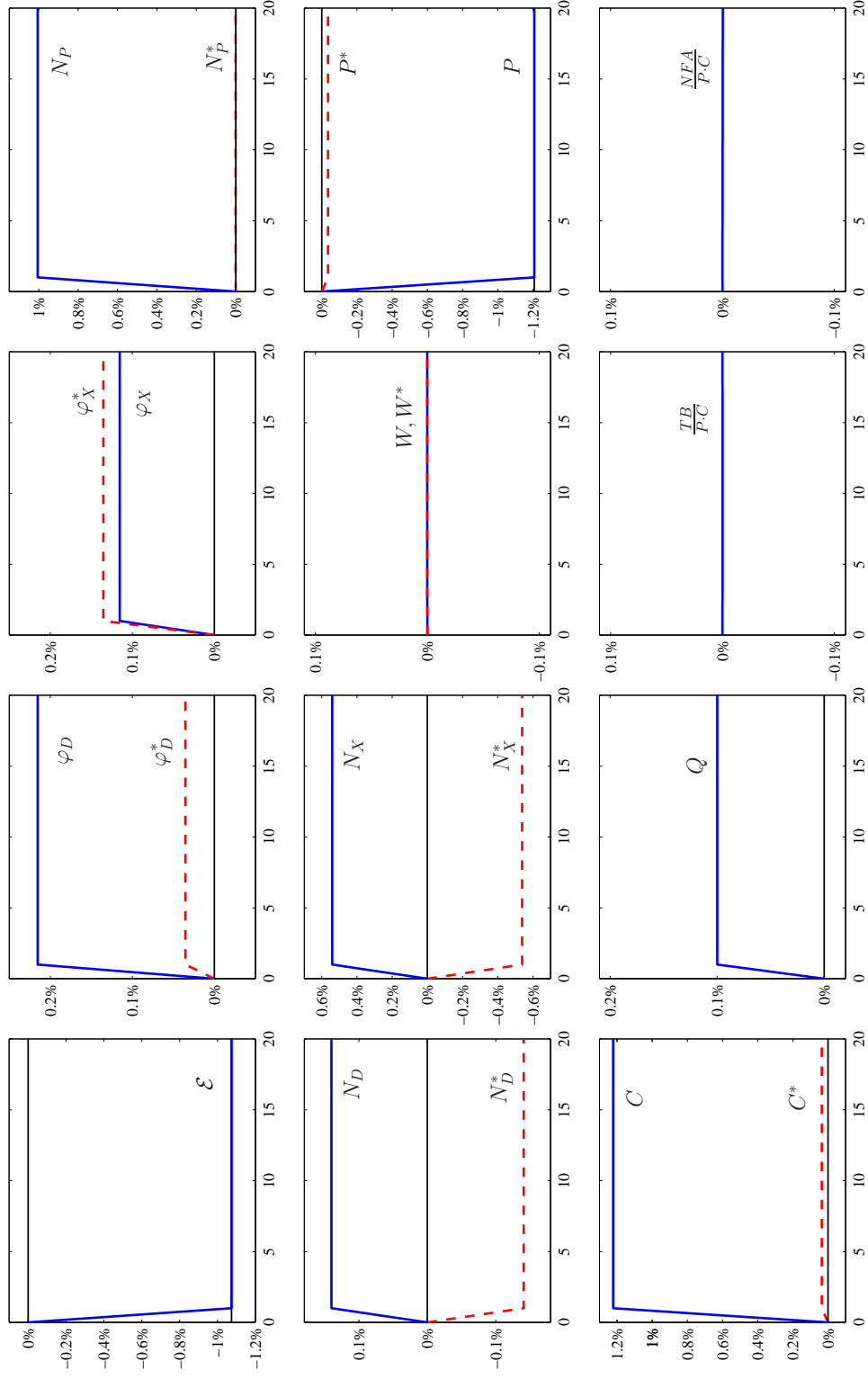


Figure B.2: A Permanent 1% Increase in Home Aggregate Productivity

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