Labour Market Monopsony Power and the Dynamic Gains to Openness Reforms

Appendix — For Online Publication

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A Theoretical Derivations and Proofs

A.1 Utility Function, Labour Supply, and Welfare

In what follows, we drop time t subscripts and derive the expression for the household's period utility (5) in the text. In the process, we also derive the expression for the firm level labour supply (3). Assume that the utility function of a representative worker for working at firm ω which pays a wage per efficiency unit of labour of $w(\omega)$ is given by

$$U(\omega, h(\omega)) = \frac{\varepsilon(\omega)h(\omega)w(\omega) + (\Pi + T)/\bar{L}}{P} - \frac{\phi}{1+\phi}\left(Ch(\omega)\right)^{\frac{1+\phi}{\phi}} \tag{A-1}$$

where $\varepsilon(\omega)$ is the productivity (or efficiency per unit of labour) for the worker from working at firm ω, $h(\omega)$ is the hours worked and P is the price index. C is a normalisation constant which will be set equal to $\left(\Omega^P\right)^{\frac{1}{1+\theta}}$ to neutralise the love of variety for employers. $\varepsilon(\omega)$ is drawn for each firm ω from a Frechet Distribution with distribution function

$$H(\varepsilon(\omega)) = e^{-\varepsilon(\omega)^{-(1+\theta)}}; \;\; \theta > 0. \tag{A-2}$$

Given the utility function in (A-1), if a worker decides to work for firm ω, then the hours worked, $h(\omega)$ is simply

$$h(\omega) = C^{-(1+\phi)}\left(\frac{\varepsilon(\omega)w(\omega)}{P}\right)^{\phi}. \tag{A-3}$$

Therefore, the utility from working at firm ω is

$$U(\omega) = \frac{1}{1+\phi}\left(\frac{\varepsilon(\omega)w(\omega)}{CP}\right)^{1+\phi} + \frac{(\Pi + T)/\bar{L}}{P}. \tag{A-4}$$

Now, in order for a worker to work at firm ω rather than any other firm k, the following must be true: $U(\omega) > \max_k U(k)$. Given the Frechet distribution and iid draws of $\varepsilon(k)$, the probability of a worker working at firm ω is

$$P(\omega) \equiv prob(U(\omega) > \max_k U(k)) = \int_0^\infty \cap prob\left(\varepsilon(k) < \varepsilon(\omega)\left(\frac{w(\omega)}{w(k)}\right)\right) dH(\varepsilon(\omega)). \quad \text{(A-5)}$$

Next, note that

$$\cap prob\left(\varepsilon(k) < \varepsilon(\omega)\left(\frac{w(\omega)}{w(k)}\right)\right) = e^{-\varepsilon(\omega)^{-(1+\theta)}\sum_{k\neq\omega}\left(\frac{w(k)}{w(\omega)}\right)^{(1+\theta)}}, \quad \text{(A-6)}$$

and hence

$$P(\omega) = (1+\theta)\int_0^\infty e^{-\varepsilon(\omega)^{-(1+\theta)}\sum_{k\neq\omega}\left(\frac{w(k)}{w(\omega)}\right)^{(1+\theta)}} e^{-\varepsilon(\omega)^{-(1+\theta)}}\varepsilon(\omega)^{-(1+\theta)-1}d\varepsilon(\omega). \quad \text{(A-7)}$$

Re-write by combining the sum in the exponent to get

$$P(\omega) = (1+\theta)\int_0^\infty e^{-\varepsilon(\omega)^{-(1+\theta)}\sum_k\left(\frac{w(k)}{w(\omega)}\right)^{1+\theta}} \varepsilon(\omega)^{-(1+\theta)-1}d\varepsilon(\omega). \quad \text{(A-8)}$$

Define $\Upsilon = \sum_k \left(\frac{w(k)}{w(\omega)}\right)^{1+\theta}$. Now the above is simply

$$P(\omega) = \frac{1}{\Upsilon}\, e^{-\varepsilon(\omega)^{-(1+\theta)}W}\Big|_0^\infty = \frac{1}{\Upsilon} = \frac{w(\omega)^{(1+\theta)}}{\sum_k w(k)^{(1+\theta)}}. \quad \text{(A-9)}$$

Also, note that in deriving the expressions (A-9) (and later (A-16)) we have used a discrete number of firms, however, using the equivalence to the continuous case established in Ben-Akiva, Litinas and Tsunokawa (1985) we obtain $P(\omega) = \frac{w(\omega)^{(1+\theta)}}{\int w(k)^{(1+\theta)}dk}$.

Now the total labour supplied by a worker to firm ω in efficiency units is $h'(\omega) = \varepsilon(\omega)h$. From (A-3) this is equal to $\left(\frac{w(\omega)}{P}\right)^\phi \frac{\varepsilon(\omega)^{1+\phi}}{C^{1+\phi}}$. Therefore, $Eh' = \left(\frac{w(\omega)}{P}\right)^\phi \frac{1}{C^{1+\phi}} E(\varepsilon(\omega)^{1+\phi}/U(\omega) > \max_k U(k))$. Next, we calculate $E(\varepsilon(\omega)^{1+\phi}/U(\omega) > \max_k U(k))$ as follows:

$$E(\varepsilon(\omega)^{1+\phi}/U(\omega) > \max_k U(k)) = \frac{(1+\theta)}{P(\omega)}\int_0^\infty \varepsilon(\omega)^{1+\phi}e^{-\varepsilon(\omega)^{-(1+\theta)}\Upsilon}\varepsilon(\omega)^{-(1+\theta)-1}d\varepsilon(\omega). \quad \text{(A-10)}$$

Next, use $t = \varepsilon(\omega)^{-(1+\theta)}\Upsilon$. Hence $dt = -(1+\theta)\varepsilon(\omega)^{-(1+\theta)-1}\Upsilon d\varepsilon(\omega)$, so that

$$E[\varepsilon(\omega)^{1+\phi}/U(\omega) > \max_k U(k)] = \frac{(1+\theta)}{P(\omega)}\int_0^\infty e^{-t}\frac{\varepsilon(\omega)^{-(1+\theta)+\phi}}{(1+\theta)\varepsilon(\omega)^{-(1+\theta)-1}\Upsilon}dt. \quad \text{(A-11)}$$

The equality above uses the fact that $t = \varepsilon(\omega)^{-(1+\theta)}\Upsilon$ implies that when $\varepsilon(\omega) \to 0$, $t \to \infty$ and when $\varepsilon(\omega) \to \infty$, $t \to 0$, hence the integration limits flip. Next note that $t = \varepsilon(\omega)^{-(1+\theta)}\Upsilon$ implies that $\varepsilon(\omega) = \left(\frac{\Upsilon}{t}\right)^{\frac{1}{(1+\theta)}}$ and $P(\omega) = \frac{1}{\Upsilon}$. Therefore,

$$E[\varepsilon(\omega)^{1+\phi}/U(\omega) > \max_k U(k)] = \Upsilon^{\frac{1+\phi}{(1+\theta)}}\int_0^\infty e^{-t}t^{-\frac{1+\phi}{1+\theta}}dt = \Upsilon^{\frac{1+\phi}{1+\theta}}\Gamma\left(\frac{\theta-\phi}{1+\theta}\right), \quad \text{(A-12)}$$

where Γ is the gamma function. In the remainder of the appendix as well as in the text we suppress the argument of Γ to reduce notational clutter.

If there are $\overline{L}$ workers, then the labour supply function facing firm ω offering a wage $w(\omega)$ is

$$n(\omega) = \overline{L}P(\omega)E[h'(\omega)/U(\omega) > \max U(k)] = \frac{\left(\frac{w(\omega)}{P}\right)^{\phi} \frac{1}{C^{1+\phi}} \Upsilon^{\frac{1+\phi}{1+\theta}}}{\Upsilon} \Gamma \overline{L}. \tag{A-13}$$

Simplify above to obtain

$$n(\omega) = \frac{w(\omega)^{\theta}}{P^{\phi} C^{1+\phi} \left(\sum_k w(k)^{(1+\theta)}\right)^{\frac{\theta-\phi}{1+\theta}}} \Gamma \overline{L}. \tag{A-14}$$

Next, the welfare of an agent who has received a productivity shock $\varepsilon(\omega)$ and works for firm ω is given in (A-4). However, a worker works for firm ω iff $U(\omega) > \max_k U(k)$. Therefore, the expected maximised utility from working for firm ω upon using (A-12) is

$$\begin{aligned} EU(\omega) &= \frac{1}{1+\phi} \left(\frac{w(\omega)}{CP}\right)^{1+\phi} E[\varepsilon(\omega)^{1+\phi}/U(\omega) > \max_k U(k)] + \frac{(\Pi + T)/\bar{L}}{P} \\ &= \frac{1}{1+\phi} \left(\frac{w(\omega)}{CP}\right)^{1+\phi} \Upsilon^{\frac{1+\phi}{1+\theta}} \Gamma + \frac{(\Pi + T)/\bar{L}}{P}. \end{aligned} \tag{A-15}$$

Upon using the expression for Υ re-write the above as

$$EU(\omega) = \frac{1}{1+\phi} \left(\frac{1}{CP}\right)^{1+\phi} \left(\sum_k w(k)^{1+\theta}\right)^{\frac{1+\phi}{1+\theta}} \Gamma + \frac{(\Pi + T)/\bar{L}}{P}. \tag{A-16}$$

Using the equivalence to the continous case established in Ben-Akiva, Litinas and Tsunokawa (1985), $\left(\sum_k w(k)^{1+\theta}\right)^{\frac{\theta-\phi}{1+\theta}} \approx \left(\int w(k)^{1+\theta} dk\right)^{\frac{\theta-\phi}{1+\theta}}$, re-write (A-14) as

$$n(\omega) = \frac{w(\omega)^{\theta}}{P^{\phi} C^{1+\phi} \left(\int_{k \in \Omega^P} w(k)^{1+\theta} dk\right)^{\frac{\theta-\phi}{1+\theta}}} \Gamma \overline{L} \tag{A-17}$$

and rewrite (A-16) as

$$EU = \frac{1}{1+\phi} \left(\frac{1}{CP}\right)^{1+\phi} \left(\int_{k \in \Omega^P} w(k)^{1+\theta} dk\right)^{\frac{1+\phi}{1+\theta}} \Gamma + \frac{(\Pi + T)/\bar{L}}{P}, \tag{A-18}$$

where notice that the dependence on ω is dropped since varieties are now on a continuum. As mentioned earlier, to neutralise the love of variety for employers, we use $C = {\Omega^P}^{\frac{1}{1+\theta}}$.

We also define a wage index as

$$W = \left(\Omega^P\right)^{-\frac{1}{1+\theta}} \left(\int_{k \in \Omega^P} w(k)^{1+\theta} dk\right)^{\frac{1}{1+\theta}}. \tag{A-19}$$

Therefore, if firms are homogeneous and offer identical wages w, then $W = w$. Otherwise, W is an average of firm wages. With this definition of W, the labour supply to a firm is

$$n(\omega) = \frac{w(\omega)^{\theta}}{P^{\phi}\Omega^{P}W^{\theta-\phi}}\Gamma\overline{L}, \tag{A-20}$$

which matches (3) in the text, where we use the notation $B \equiv \Gamma\overline{L}/(P^{\phi}\Omega^{P}W^{\theta-\phi})$ where Γ is the gamma function, $\overline{L}$ is the number of workers or households and W is the wage index.

And the expected utility in (A-18) can be written as

$$EU = \frac{1}{1+\phi}\left(\frac{W}{P}\right)^{1+\phi}\Gamma + \frac{\left(\Pi + T\right)/\bar{L}}{P}, \tag{A-21}$$

which matches the period utility function (5) in the text.

A.2 Proof of Proposition 1

A.2.1 Planner's Problem

Below we discuss the planner's problem when the fixed costs are in terms of labour and when they are in units of the final consumption good.

We assume that the planner can choose the mass of firms, Ω^P, employment per firm, n, and real wage per firm, r (in units of the final consumption good), to maximise output net of the fixed cost of operations as well as the disutility cost of workers. The planner is choosing these in the first stage while workers facing r and getting draws of idiosyncratic productivity make their choices of hours worked and the firm to work for in the same fashion as described in the previous section. Therefore, the planner's choice of Ω^P and r results in workers choosing n.

In the homogeneous firm case, (A-3) derived earlier implies (using $C = \left(\Omega^P\right)^{\frac{1}{1+\theta}}$)

$$h = \left(\Omega^P\right)^{-\frac{1+\phi}{1+\theta}}\left(\varepsilon r\right)^{\phi}. \tag{A-22}$$

Therefore,

$$\left(\Omega^P\right)^{\frac{1}{1+\theta}} h = \left(\Omega^P\right)^{-\frac{\phi}{1+\theta}}\left(\varepsilon r\right)^{\phi}, \tag{A-23}$$

and hence

$$E\left(\left(\Omega^P\right)^{\frac{1}{1+\theta}} h\right)^{\frac{1+\phi}{\phi}} = \left(\Omega^P\right)^{-\frac{1+\phi}{1+\theta}} r^{1+\phi} E\varepsilon^{1+\phi} \tag{A-24}$$

from the worker's optimisation problem. Next, note from (A-12) that $E\left(\varepsilon^{1+\phi}\right) = \Upsilon^{\frac{1+\phi}{1+\theta}}\Gamma = \left(\Omega^P\right)^{\frac{1+\phi}{1+\theta}}\Gamma$ in the symmetric case and hence[38]

$$E\left(\left(\Omega^P\right)^{\frac{1}{1+\theta}} h\right)^{\frac{1+\phi}{\phi}} = r^{1+\phi}\Gamma, \tag{A-25}$$

[38] Since $\Upsilon = \sum_k w^{1+\theta}/w^{1+\theta} = \Omega^P w^{1+\theta}/w^{1+\theta} = \Omega^P$.

which serves in the labour disutility term of the planner's problem. Next, (A-20) derived earlier implies $n = r^{\phi}\frac{\Gamma\overline{L}}{\Omega^P}$. We now write the per capita social planner's problem in a nested fashion (so both cases of fixed costs in terms of labour and goods can be studied). Recall that $I = 1$ captures the fixed cost in terms of labour case and $I = 0$ captures the fixed cost in terms of output case. The planner's objective maximises per capita production net of fixed costs less the cost of labour supply

$$\max_{\{\Omega^P,r\}} \frac{1}{\overline{L}}\left(\left(\Omega^P\right)^{\frac{1}{\sigma-1}} zr^{\phi}\Gamma\overline{L} - \left(\left(\Omega^P\right)^{\frac{1}{\sigma-1}} zI + (1-I)\right)\Omega^P f^C\right) - \frac{\phi}{1+\phi}r^{1+\phi}\Gamma,$$

where notice the first term $\left(\Omega^P\right)^{\frac{1}{\sigma-1}} zr^{\phi}\Gamma\overline{L}$ gives aggregate output gross of fixed costs using final goods aggregator (6).[39] The first order conditions are

$$\frac{1}{\sigma-1}\left(\Omega^P\right)^{\frac{1}{\sigma-1}-1} zr^{\phi}\Gamma\overline{L} = \left(\frac{\sigma}{\sigma-1}\left(\Omega^P\right)^{\frac{1}{\sigma-1}} zI + (1-I)\right) f^C \qquad \text{(A-26)}$$

$$\left(\Omega^P\right)^{\frac{1}{\sigma-1}} z = r. \qquad \text{(A-27)}$$

From the above two solve for Ω^P and r in each case. In the $I = 1$ case (when fixed costs are in terms of labour) obtain

$$n_{pl} = \sigma f^C;\;\; \Omega^P_{pl} = \left(\frac{z^{\phi}\Gamma\overline{L}}{\sigma f^C}\right)^{\frac{\sigma-1}{\sigma-1-\phi}}, \qquad \text{(A-28)}$$

where subscript pl denotes the value in the planner's solution when fixed costs are in terms of labour. In the $I = 0$ case (fixed cost in terms of output) we get

$$n_{po} = \frac{(\sigma-1) f^C}{r_{po}};\;\; \Omega^P_{po} = \left(\frac{z^{1+\phi}\Gamma\overline{L}}{(\sigma-1) f^C}\right)^{\frac{\sigma-1}{\sigma-2-\phi}}. \qquad \text{(A-29)}$$

A.2.2 Market Outcome with Wage-Taking Firms

Firms maximise the following objective function

$$\max_{\{n\}} A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}}\left(n - If^C\right)^{\frac{\sigma-1}{\sigma}} - wn - (1-I)f^C.$$

The first order condition with respect to n is

$$\frac{\sigma-1}{\sigma}A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}}\left(n - If^C\right)^{\frac{-1}{\sigma}} = w, \qquad \text{(A-30)}$$

where notice that the markdown $\theta/(1+\theta)$ does not appear on the right-side of the equation, in contrast with (14). From the above obtain

$$A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}}\left(n - If^C\right)^{\frac{\sigma-1}{\sigma}} = \frac{\sigma}{\sigma-1}w\left(n - If^C\right). \qquad \text{(A-31)}$$

[39] In the case of fixed costs in terms of labour ($I = 1$), see that output net of fixed costs from (6) is $A = \left(\Omega^P\right)^{\frac{\sigma}{\sigma-1}} z\left[r^{\phi}\frac{\Gamma\overline{L}}{\Omega^P} - f^C\right]$ giving the above. The above is immediate in the case where $I = 0$.

And the zero profit condition is

$$\frac{\sigma}{\sigma-1}w\left(n-If^{C}\right)=wn+(1-I)f^{C}. \tag{A-32}$$

The above implies that the amount of labour hired by firms is

$$n=\frac{1}{w}(1-I)\left(\sigma-1\right)f^{C}+I\sigma f^{C}. \tag{A-33}$$

Since firm labour supply in the symmetric case is given by $\left(\Omega^{P}\right)^{-1}w^{\phi}\Gamma\overline{L}$, labour market clearing implies

$$\left(\Omega^{P}\right)^{-1}w^{\phi}\Gamma\overline{L}=\frac{1}{w}(1-I)\left(\sigma-1\right)f^{C}+I\sigma f^{C} \tag{A-34}$$

and the CPI definition (8) becomes

$$\frac{\sigma}{\sigma-1}\frac{w}{z}\left(\Omega^{P}\right)^{\frac{1}{1-\sigma}}=1 \tag{A-35}$$

in all cases. Therefore, the 3 equations, (A-33)-(A-35) above determine n, w and Ω^{P}. As is apparent from (A-35), w, which is also a measure of welfare, is monotonically increasing in Ω^{P}. Therefore, we focus on Ω^{P} below. In the $I=1$ case (fixed cost in units of labour), we get

$$n_{l}=\sigma f^{C};\ \Omega_{l}^{P}=\left(\left(\frac{\left(\sigma-1\right)z}{\sigma}\right)^{\phi}\frac{\Gamma\overline{L}}{\sigma f^{C}}\right)^{\frac{\sigma-1}{\sigma-1-\phi}}, \tag{A-36}$$

where the subscript l above is used for the equilibrium values of endogenous variables when the fixed cost is in terms of labour. In the $I=0$ case (fixed cost in terms of output) we get

$$n_{o}=\frac{\left(\sigma-1\right)f^{C}}{w_{o}};\ \Omega_{o}^{P}=\left(\left(\frac{\left(\sigma-1\right)z}{\sigma}\right)^{\phi}\frac{z\Gamma\overline{L}}{\sigma f^{C}}\right)^{\frac{\sigma-1}{\sigma-2-\phi}}, \tag{A-37}$$

where the subscript o above is used for the equilibrium values of endogenous variables when the fixed cost is in terms of output of the final good.

A.2.3 Planner vs Wage-Taking Firms

Comparing (A-36) and (A-28) verify that $n_{pl}=n_{l}$, that is the planner chooses the same firm size as the decentralised outcome without the exercise of labour market power. As well, when $\phi\rightarrow 0$, $\Omega_{pl}^{P}=\Omega_{l}^{P}$, that is the mass of firms chosen by the planner is same as in the decentralised equilibrium with wage taking firms. When $\phi>0$, we obtain $\frac{\Omega_{pl}^{P}}{\Omega_{l}^{P}}=\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\phi(\sigma-1)}{\sigma-1-\phi}}>1$. This is due to the markup distortion in the product market which distorts labour supply because products are marked up relative to leisure as a result more leisure is consumed. This distortion is not due to the imperfection in the labour market.

Comparing (A-37) and (A-29) note that these expressions are different even when $\phi\rightarrow 0$ for the mass of firms where $\frac{\Omega_{po}^{P}}{\Omega_{o}^{P}}=\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\sigma-2}}>1$. The planner wants a larger number of firms because more firms lower the fixed cost of production, an externality, not recognised by the market. The employment size is also distorted because $n_{po} < n_o$, meaning that the planner wants more firms, each individually of smaller size.

Having established the efficiency of the market solution when fixed costs are in units of labour, firms are wage takers and $\phi \to 0$, below we verify the inefficiency introduced by firms internalising labour market power.

A.2.4 Planner vs Wage-Setting Firms

The equilibrium of the nested model with wage setting firms case is as follows.

Using the first-order condition in Equation (14) and the profit function in Equation (13), the free entry condition can be written as

$$\frac{\sigma(1+\theta)}{(\sigma-1)\theta} w\left(n - I f^C\right) = wn + (1-I)[Jw + (1-J)]f^C, \tag{A-38}$$

which yields the the optimal hiring choice as

$$n = \frac{\sigma(1+\theta)}{\sigma+\theta} I f^C + \frac{\theta(\sigma-1)}{\sigma+\theta}(1-I)\left(J + \frac{1-J}{w}\right) f^C. \tag{A-39}$$

Since $W = w$ in the homogeneous firm case, the labour market clearing condition implies

$$\left(\Omega^P\right)^{-1} w^{\phi} \Gamma \bar{L} = n. \tag{A-40}$$

The consumer price index in Equation (8) simplifies, given the marginal cost $w(1+\theta)/(\theta z)$, to

$$\left(\Omega^P\right)^{\frac{1}{1-\sigma}} \cdot \frac{\sigma(1+\theta)}{(\sigma-1)\theta} \cdot \frac{w}{z} = 1. \tag{A-41}$$

Equations (A-39), (A-40), and (A-41) jointly determine the equilibrium values of n, Ω^P, and w.

Now the fixed cost in terms of labour case is given by $I = 1$. In this case, (A-39) implies that the amount of labour hired by firms is

$$n_s = \frac{\sigma\left(1+\theta\right)}{\sigma+\theta} f^C \tag{A-42}$$

where subscript s denotes the wage setting case. Now, with this value of firm labour, equations (A-40) and (A-41) above determine w_s and Ω^P_s. The expression for Ω^P is

$$\Omega^P_s = \left(\frac{\left(\sigma+\theta\right)\left(\left(\sigma-1\right)\theta\right)^{\phi}}{\left(\sigma\left(1+\theta\right)\right)^{1+\phi}}\right)^{\frac{\sigma-1}{\sigma-1-\phi}} \left(\frac{\Gamma \overline{L} z^{\phi}}{f^C}\right)^{\frac{\sigma-1}{\sigma-1-\phi}}. \tag{A-43}$$

Comparing the firm size, n_s, in (A-42) with the firm size in the planner's solution in (A-28) verify that $n_s < n_{pl}$ for any $\theta < \infty$. So, the firm size is distorted downwards due to the exercise of monopsony power. As well, comparing the expression for Ω_s^P in (A-43) with Ω_{pl}^P in (A-28) observe that

$$\frac{\Omega_s^P}{\Omega_{pl}^P} = \left(\frac{(\sigma+\theta)}{(1+\theta)} \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)} \right)^{\phi} \right)^{\frac{\sigma-1}{\sigma-1-\phi}}. \tag{A-44}$$

Clearly, $\Omega_s^P > \Omega_{pl}^P$ in the $\phi \to 0$ case which completes the proof of proposition 1.

A.3 Proof of Proposition 2

For autarky for the case of $I = 0, J = 1$ which is the relevant case (fixed cost in units of output but indexed to wage), using (A-39), (A-40) and (A-41) obtain

$$w_D = \left(\frac{\sigma+\theta}{\theta\,(\sigma-1)} \left(\frac{\theta\,(\sigma-1)\,z}{\sigma(1+\theta)} \right)^{\sigma-1} \frac{\Gamma\bar{L}}{f^C} \right)^{\frac{1}{\sigma-1-\phi}}. \tag{A-45}$$

Now, obtain the wage in the exporting equilibrium as follows.

Given the objective function and the constraints in the text, the firm's first-order conditions with respect to n_D and n_X in the exporting case are

$$\frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} n_D^{\frac{-1}{\sigma}} = w\left(\frac{1+\theta}{\theta}\right) \tag{A-46}$$

$$\frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} \left(\frac{z}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}} n_X^{\frac{-1}{\sigma}} = w\left(\frac{1+\theta}{\theta}\right) \tag{A-47}$$

where we have invoked country symmetry (i.e., $A = A^*$). Equations (A-46) and (A-47) jointly imply

$$n_X = n_D \left(\tau^X\right)^{1-\sigma} < n_D, \tag{A-48}$$

so that the presence of the iceberg cost introduces an asymmetry in labour allocation, tilting hiring towards the domestic market.

The three equilibrium conditions described in Definition 1 continue to hold. The free entry condition becomes:

$$A^{\frac{1}{\sigma}} (z n_D)^{\frac{\sigma-1}{\sigma}} + A^{\frac{1}{\sigma}} \left(\frac{z n_X}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}} - w(n_D + n_X) = w f^X. \tag{A-49}$$

Combining Equations (A-46), (A-47), and (A-49) yields total labour demand:

$$n_T \equiv n_D + n_X = \frac{\theta(\sigma-1)}{\sigma+\theta} f^X. \tag{A-50}$$

The labour market clearing condition becomes:

$$\frac{w^{\phi}\Gamma\bar{L}}{\Omega^P} = \frac{\theta(\sigma-1)}{\sigma+\theta} f^X. \tag{A-51}$$

Using the first-order conditions along with the demand curves and production functions, the CPI condition in Definition 1 becomes:

$$1 = \Omega^P \left[1 + (\tau^X)^{1-\sigma}\right] \left[\frac{\sigma(1+\theta)}{(\sigma-1)\theta}\frac{w}{z}\right]^{1-\sigma}. \tag{A-52}$$

Equations (A-51) and (A-52) jointly determine the equilibrium values of w and Ω^P. We denote the equilibrium wage under trade by w_X, given by:

$$w_X = \left\{ \frac{\sigma+\theta}{\theta(\sigma-1)} \left[\frac{\theta(\sigma-1)z}{\sigma(1+\theta)}\right]^{\sigma-1} \left[1 + (\tau^X)^{1-\sigma}\right] \frac{\Gamma \bar{L}}{f^X} \right\}^{\frac{1}{\sigma-1-\phi}}. \tag{A-53}$$

Therefore, using (A-45) and (A-53) the expression for the gains from trade is

$$\frac{w_X}{w_D} = \left(\frac{f^C \left(1 + \left(\tau^X\right)^{1-\sigma}\right)}{f^X} \right)^{\frac{1}{\sigma-1-\phi}}. \tag{A-54}$$

Next, we verify that for exporting to be viable in the model, the following condition must be satisfied: $\frac{f^C}{f^X} > \frac{1}{(1+(\tau^X)^{1-\sigma})^{\frac{1+\theta}{\sigma+\theta}}} > \frac{1}{1+(\tau^X)^{1-\sigma}}$. For an exporting equilibrium to be viable no firm should have an incentive to stay domestic.[40] Note that the profit maximizing conditions (A-46) and (A-47) of exporting firms implies the following expression for their revenue.

$$A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} (1 + \left(\tau^X\right)^{1-\sigma})^{\frac{1}{\sigma}} n_T^{\frac{\sigma-1}{\sigma}} = w_X n_T \left(\frac{(1+\theta)\,\sigma}{\theta\,(\sigma-1)} \right). \tag{A-55}$$

Therefore, their gross profit is $\left(\frac{\sigma+\theta}{\theta(\sigma-1)}\right) w_X n_T$. Now, starting from an exporting equilibrium with wage w_X and employment n_T, for a firm to not deviate and be domestic, the following must be true

$$\left(\frac{\sigma+\theta}{\theta\,(\sigma-1)} \right) (w_{dev} n_{dev} - w_X n_T) < w_X \left(f^C - f^X\right), \tag{A-56}$$

where w_{dev} and n_{dev} are the wage and employment of the deviating firm that wants to be domestic. Using the labour supply equation, $w = (n/B)^{\frac{1}{\theta}}$ re-write the above as

$$\left(\frac{\sigma+\theta}{\theta\,(\sigma-1)} \right) \left(\frac{n_{dev}^{\frac{1+\theta}{\theta}} - n_T^{\frac{1+\theta}{\theta}}}{n_T^{\frac{1}{\theta}}} \right) < (f^C - f^X). \tag{A-57}$$

Now the first order condition of a deviant firm to stay domestic is

$$A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} n_{dev}^{\frac{\sigma-1}{\sigma}} = w_{dev} n_{dev} \left(\frac{(1+\theta)\,\sigma}{\theta\,(\sigma-1)} \right). \tag{A-58}$$

[40] This follows since revenues for a domestic firm using (14) can be written as $\left(\frac{\sigma(1+\theta)}{(\sigma-1)\theta}\right) wn$ as $I = 0$, meaning gross profits are given by $\left(\frac{\sigma+\theta}{\theta(\sigma-1)}\right) wn$. Similar expressions hold for exporting firms.

Using (A-55) and (A-58) obtain

$$\frac{n_{dev}^{\frac{\sigma-1}{\sigma}}}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)^{\frac{1}{\sigma}} n_{T}^{\frac{\sigma-1}{\sigma}}}=\frac{w_{dev} n_{dev}}{w_{X} n_{T}}, \tag{A-59}$$

and using $w=(n/B)^{\frac{1}{\theta}}$ write (A-59) as

$$n_{dev}=\frac{n_{T}}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)^{\frac{\theta}{\sigma+\theta}}}. \tag{A-60}$$

Using (A-60) above write the no deviation condition (A-57) as

$$\left(\frac{1}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}}-1\right) n_{T}<\frac{\theta(\sigma-1)}{\sigma+\theta}\left(f^{C}-f^{X}\right). \tag{A-61}$$

Next, use (A-50) in above and reorganise to get

$$\left(\frac{1}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}}-1\right) \frac{\theta(\sigma-1)}{\sigma+\theta} f^{X}<\frac{\theta(\sigma-1)}{\sigma+\theta}\left(f^{C}-f^{X}\right). \tag{A-62}$$

Simplify (A-62) to obtain

$$\frac{f^{C}}{f^{X}}>\frac{1}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}}. \tag{A-63}$$

Since $\frac{1}{1+\left(\tau^{X}\right)^{1-\sigma}}<\frac{1}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}}$, whenever $\frac{f^{C}}{f^{X}}>\frac{1}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}}$ is satisfied, $\frac{f^{C}}{f^{X}}>\frac{1}{\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)}$ is satisfied as well. This proves that there are gains from trade: $\frac{w_{X}}{w_{D}}>1$ in (A-54) for $\phi<\sigma-1$.[41]

Next we show that the expression for the gains from trade when fixed costs are in units of labour is exactly the same as in (A-54). The autarky wage in this case is obtained from equations (A-41) and (A-43) earlier which is given by

$$w_{l}=\left(\left(\frac{(\sigma+\theta)}{\sigma(1+\theta)}\right)\left(\frac{(\sigma-1) \theta z}{\sigma(1+\theta)}\right)^{\sigma-1}\left(\frac{\Gamma \bar{L}}{f^{C}}\right)\right)^{\frac{1}{\sigma-1-\phi}}. \tag{A-64}$$

And following the same steps as in the case when fixed costs are in units of output but indexed to wage, obtain the wage in the trading equilibrium as

$$w_{lX}=\left(\left(\frac{(\sigma+\theta)}{\sigma(1+\theta)}\right)\left(\frac{(\sigma-1) \theta z}{\sigma(1+\theta)}\right)^{\sigma-1}\left(\frac{\Gamma \bar{L}}{f^{X}}\right)\left(1+\tau^{1-\sigma}\right)\right)^{\frac{1}{\sigma-1-\phi}}. \tag{A-65}$$

Therefore, the expression for gains from trade is

$$\frac{w_{lX}}{w_{l}}=\left(\frac{f^{C}\left(1+\left(\tau^{X}\right)^{1-\sigma}\right)}{f^{X}}\right)^{\frac{1}{\sigma-1-\phi}}, \tag{A-66}$$

which is same as (A-54). This completes the proof of Proposition 2.

[41] This is a regularity/stability condition that must be obeyed throughout the derivations.

A.4 Trade Elasticity and Gains from Trade in the Homogeneous-Firm Case

We first express the own trade share, λ, as

$$\lambda = \frac{pq}{pq + p^* q^*} = \frac{q^{\frac{\sigma-1}{\sigma}}}{q^{\frac{\sigma-1}{\sigma}} + q^{*\frac{\sigma-1}{\sigma}}} = \frac{(zn_D)^{\frac{\sigma-1}{\sigma}}}{(zn_D)^{\frac{\sigma-1}{\sigma}} + \left(\frac{zn_X}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}}},$$

where the second equality follows from the demand function in Equation (7), and the third from the production function in Equation (10). We also use the fact that, due to country symmetry, the mass of producers — and hence the mass of exporters — is given by Ω_X^P in each country.

Substituting equation (A-48) into the expression above yields

$$\lambda = \frac{1}{1 + (\tau^X)^{1-\sigma}}. \tag{A-67}$$

And using (A-67) write the gains from trade in (A-54) and (A-66) as

$$\frac{w_X}{w_D} = \frac{w_{lX}}{w_l} = \left(\frac{f^C}{f^X \lambda}\right)^{\frac{1}{\sigma-1-\phi}}, \tag{A-68}$$

which is the expression for gains from trade in the text.

A.5 Heterogeneous-Firm Case

For the autarky equilibrium in the heterogeneous firm case, firms maximise

$$\max_{\{n\}} \; py - wn - Wf^C,$$

where demand is $y = Ap^{-\sigma}$, labour supply is $n = Bw^\theta$ and the fixed cost is indexed to the labour cost through the wage index W. The production function of firms is $y = zn$. Therefore, the above can be written as

$$\max_{\{n\}} \; A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} n^{\frac{\sigma-1}{\sigma}} - wn - Wf^C.$$

The first order condition is

$$\frac{\sigma-1}{\sigma} A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} n^{\frac{-1}{\sigma}} = \frac{1+\theta}{\theta} w. \tag{A-69}$$

The above can also be written as the standard markup pricing condition:

$$p = \frac{\sigma(1+\theta)}{(\sigma-1)\theta} \frac{w}{z}. \tag{A-70}$$

Next, from (A-69) obtain

$$n(z) = \left(\frac{(\sigma-1)\theta}{(1+\theta)\sigma}\right)^{\frac{\sigma\theta}{\sigma+\theta}} A^{\frac{\theta}{\sigma+\theta}} B^{\frac{\sigma}{\sigma+\theta}} z^{\frac{(\sigma-1)\theta}{\sigma+\theta}} \tag{A-71}$$

and write the profit gross of fixed cost as

$$\pi(z) = \left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\right)\left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma(1+\theta)}{\sigma+\theta}} A^{\frac{1+\theta}{\sigma+\theta}} B^{\frac{(\sigma-1)}{\sigma+\theta}} z^{\chi}, \tag{A-72}$$

where we use $\chi \equiv \frac{(\sigma-1)(1+\theta)}{\sigma+\theta}$ to reduce notational clutter. Using $\widehat{z}$ to denote the cutoff productivity, write the zero cutoff profit condition as

$$\pi(\widehat{z}) = \left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\right)\left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma(1+\theta)}{\sigma+\theta}} A^{\frac{1+\theta}{\sigma+\theta}} B^{\frac{(\sigma-1)}{\sigma+\theta}} \widehat{z}^{\chi} = W f^{C}. \tag{A-73}$$

Therefore,

$$\pi(z) = \left(\frac{z}{\widehat{z}}\right)^{\chi} W f^{C}. \tag{A-74}$$

The free-entry condition is $\int_{\widehat{z}}^{\infty}\left[\pi(z) - W f^{C}\right] g(z)dz = W f^{T}$, which using (A-74) becomes

$$\int_{\widehat{z}}^{\infty}\left[\left(\frac{z}{\widehat{z}}\right)^{\chi} - 1\right] f^{C} g(z)dz = f^{T}. \tag{A-75}$$

The above equation solves for $\widehat{z}$.

The wage index is is given by

$$W = \left(\int_{\widehat{z}}^{\infty} w(z)^{1+\theta} g(z \geq \widehat{z})dz\right)^{\frac{1}{1+\theta}}. \tag{A-76}$$

Since labour supply is $n = Bw^{\theta}$, using (A-71) we obtain the following expression for firm wage.

$$w(z) = \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma}{\sigma+\theta}}\left(\frac{A}{B}\right)^{\frac{1}{\sigma+\theta}} z^{\frac{\sigma-1}{\sigma+\theta}}. \tag{A-77}$$

Using (A-77) in (A-76) obtain

$$W = \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma}{\sigma+\theta}}\left(\frac{A}{B}\right)^{\frac{1}{\sigma+\theta}} \Phi_A^{\frac{\chi}{1+\theta}}, \tag{A-78}$$

where $\Phi_A \equiv \left(\int_{\widehat{z}}^{\infty} z^{\chi}\frac{g(z)}{1-G(\widehat{z})}dz\right)^{\frac{1}{\chi}}$ is a hyperparameter used to reduce notational clutter. Using (A-78) write the zero cutoff profit condition in (A-73) as

$$\left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\right)\left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma\theta}{\sigma+\theta}} A^{\frac{\theta}{\sigma+\theta}} B^{\frac{\sigma}{\sigma+\theta}} \widehat{z}^{\chi} \Phi_A^{-\frac{\chi}{1+\theta}} = f^{C}. \tag{A-79}$$

Using (A-70) obtain the following expression for the price index.

$$1 = \left(\Omega^{P}\right)^{\frac{1}{1-\sigma}}\left(\frac{A}{B}\right)^{\frac{1}{\sigma+\theta}}\left(\frac{\sigma\,(1+\theta)}{(\sigma-1)\,\theta}\right)^{\frac{\theta}{\sigma+\theta}} \Phi_A^{\frac{\chi}{1-\sigma}}. \tag{A-80}$$

Since $B \equiv \left(\Omega^P\right)^{-1} W^{\phi-\theta}\Gamma\overline{L}$, we can write the equation for B as

$$B = \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\frac{\sigma(\phi-\theta)}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{\phi-\theta}{\sigma+\theta}} \Phi_A^{\frac{\chi(\phi-\theta)}{1+\theta}} \left(\Omega^P\right)^{-1}\Gamma\overline{L}. \tag{A-81}$$

The 3 equations , (A-79), (A-80), and (A-81) determine Ω^P, A, and B in autarky and yield the following expression for Ω^P where we use subscript A for autarky:

$$\left(\Omega_A^P\right)^{\frac{\sigma-1-\phi}{\sigma-1}} = \left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\frac{\Gamma\overline{L}}{f^C}\right)\left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\phi}\Phi_A^{\phi-\chi}\widehat{z}^{\chi}. \tag{A-82}$$

It can be verified that for $\Phi_A = \widehat{z}$ the above yields the expression obtained in the homogeneous firm case.

Since welfare is a function of W, from (A-78) and (A-79) obtain the following useful expression for W

$$W_A = \frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\left(\Omega_A^P\right)^{\frac{1}{\sigma-1}}\Phi_A. \tag{A-83}$$

And hence

$$W_A = \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma-1}{\sigma-1-\phi}}\left(\left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\frac{\Gamma\overline{L}}{f^C}\right)\widehat{z}^{\chi}\right)^{\frac{1}{\sigma-1-\phi}}\Phi_A^{\frac{\sigma-1-\chi}{\sigma-1-\phi}}. \tag{A-84}$$

A.5.1 Trading Equilibrium in the Heterogeneous-Firm Case

The problem of non-exporting firms remains same as in autarky. The problem of exporting firms is

$$\max_{\{n_D,n_X\}} A^{\frac{1}{\sigma}}(zn_D)^{\frac{\sigma-1}{\sigma}} + A^{\frac{1}{\sigma}}\left(\frac{zn_X}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}} - w\left(n_D+n_X\right) - Wf^C - Wf^{X,C}.$$

From the solution of the exporting firms problem in the homogeneous firm case earlier, we already have $n_X = n_D\left(\tau^X\right)^{1-\sigma} < n_D$. And hence, $n_T = n_D + n_X = n_D(1+\left(\tau^X\right)^{1-\sigma})$.

We can verify that

$$w_X(z) = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}} w_N(z) \quad \text{and} \quad p_D(z) = \left[\frac{\sigma\,(1+\theta)}{(\sigma-1)\,\theta}\right]\frac{w_X(z)}{z},$$

where $w_N(z)$ is the wage offered by a non-trading firm, $w_X(z)$ is the wage offered by a trading firm, and $p_D(z)$ is the price charged by a trading firm in its home market. It follows that

$$p_D(z) = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}}\left[\frac{\sigma\,(1+\theta)}{(\sigma-1)\,\theta}\right]\frac{w_N(z)}{z} = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}} p_N(z)$$

where the expressions for $w_N(z)$ and $p_N(z)$ for a non trading firm are same as in autarky, so that it is always the case that $w_X(z) > w_N(z)$ and $p_D(z) > p_N(z)$. The price set by the trading firm in the export market is then $p_X(z) = \tau^X p_D(z)$. Note that the exporting wage premium for a firm with productivity z is given by $\frac{w_X(z)}{w_N(z)} = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}} \in \left(1, (1+\left(\tau^X\right)^{1-\sigma})^{\frac{1}{\sigma}}\right]$, which is strictly decreasing in θ; that is, a lower θ (more monopsony power) increases the exporting wage premium.

The open-economy model solves for two cutoff productivity levels, $\widehat{z}_D$ and $\widehat{z}_X$, that determine firm status: given $\widehat{z}_D < \widehat{z}_X$, a firm with productivity z does not produce if $z < \widehat{z}_D$, produces only for the domestic market if $z \in [\widehat{z}_D, \widehat{z}_X)$, and produces for the domestic and export markets if $z \geq \widehat{z}_X$. These cutoff levels satisfy the indifference conditions

$$\pi_N(\widehat{z}_D) = Wf^C, \tag{A-85}$$

$$\pi_T(\widehat{z}_X) - Wf^C - Wf^{X,C} = \pi_N(\widehat{z}_X) - Wf^C. \tag{A-86}$$

Since $\pi_T(\widehat{z}_X) - Wf^{X,C} = \pi_N(\widehat{z}_X)$, it follows that $\left[\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} - 1\right]\pi_N(\widehat{z}_X) = Wf^{X,C}$.

Using (A-46) and (A-47) for exporting firms verify that the gross profit, $A^{\frac{1}{\sigma}}(zn_D)^{\frac{\sigma-1}{\sigma}} + A^{\frac{1}{\sigma}}\left(\frac{zn_X}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}} - w_X n_T$ simply equals

$$\pi_T(z) = \left(\frac{\sigma+\theta}{(\sigma-1)\,\theta}\right) w_X(z) n_T(z) = \left(\frac{\sigma+\theta}{(\sigma-1)\,\theta}\right) Bw_X(z)^{1+\theta}.$$

Therefore,

$$\frac{\pi_T(z)}{\pi_N(z)} = \frac{w_X(z)^{1+\theta}}{w_N(z)^{1+\theta}} = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}.$$

It follows that (A-86) yields $\left(\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} - 1\right)\pi_N(\widehat{z}_X) = Wf^{X,C}$. This along with (A-85) implies $\left(\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} - 1\right)\frac{\pi_N(\widehat{z}_X)}{\pi_N(\widehat{z}_D)} = \frac{f^{X,C}}{f^C}$. Using the expression for $\pi_N(z)$ in (A-72) obtain

$$\widehat{z}_X = \mu\widehat{z}_D, \qquad \text{where} \qquad \mu \equiv \left[\frac{\mathcal{F}}{(1+\left(\tau^X\right)^{1-\sigma})^{\frac{\chi}{\sigma-1}} - 1}\right]^{\frac{1}{\chi}} \geq 1 \tag{A-87}$$

and $\mathcal{F} \equiv \frac{f^{X,C}}{f^C}$. This is one of the two equations we need to solve for $\widehat{z}_D$ and $\widehat{z}_X$. The second equation comes from the free entry condition derived below.

Write the gross profits of non-trading and exporting firms as follows.

$$\pi_N(z) = \left(\frac{z}{\widehat{z}_D}\right)^{\chi} Wf^C \qquad \text{and} \qquad \pi_T(z) = \left(1+\frac{\mu^\chi}{\mathcal{F}}\right)\left(\frac{z}{\widehat{z}_X}\right)^{\chi} Wf^{X,C}. \tag{A-88}$$

The free-entry condition is

$$\left\{\int_{\widehat{z}_D}^{\widehat{z}_X}\left[\pi_N(z) - Wf^C\right]g(z)dz + \int_{\widehat{z}_X}^{\infty}\left[\pi_T(z) - Wf^C - Wf^{X,C}\right]g(z)dz\right\} = Wf^T,$$

which, using (A-88), is written as

$$\int_{\widehat{z}_D}^{\widehat{z}_X} \left(\frac{z}{\widehat{z}_D}\right)^{\chi} f^C g(z)dz + \int_{\widehat{z}_X}^{\infty} \left(1+\frac{\mu^{\chi}}{\mathcal{F}}\right)\left(\frac{z}{\widehat{z}_X}\right)^{\chi} f^{X,C} g(z)dz$$
$$= f^T + [1-G(\widehat{z}_D)]\, f^C + [1-G(\widehat{z}_X)]\, f^{X,C}. \quad \text{(A-89)}$$

Note that W cancels on both sides above. Equations (A-87) and (A-89) determine $\widehat{z}_D$ and $\widehat{z}_X$.

We can also obtain the average productivity for each type of firm as

$$\overline{z}_D = \left[\frac{1-G(\widehat{z}_D)}{G(\widehat{z}_X)-G(\widehat{z}_D)} \int_{\widehat{z}_D}^{\widehat{z}_X} z^{\chi} g(z|z \geq \widehat{z}_D)dz\right]^{\frac{1}{\chi}}$$
$$\overline{z}_X = \left[\frac{1-G(\widehat{z}_D)}{1-G(\widehat{z}_X)} \int_{\widehat{z}_X}^{\infty} z^{\chi} g(z|z \geq \widehat{z}_D)dz\right]^{\frac{1}{\chi}},$$

with the overall average productivity calculated as $\overline{z} = \left[\frac{\Omega_T^D}{\Omega_T^P}\overline{z}_D^{\chi} + \frac{\Omega_T^{X*}}{\Omega_T^P}\overline{z}_X^{\chi}\right]^{\frac{1}{\chi}}$, where $\Omega_T^P \equiv \Omega_T^D + \Omega_T^{X*}$. Ω_T^P is the total mass of producers in each country in a trading equilibrium, Ω_T^D is the mass of non-exporters, and Ω_T^{X*} is the mass of exporters .

The aggregate price P can be conveniently written as

$$1 = \left[\Omega_T^D p_N(\overline{z}_D)^{1-\sigma} + \Omega_T^{X*} p_D(\overline{z}_X)^{1-\sigma} + \Omega_T^{X*} p_X(\overline{z}_X)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$
$$= \left[\Omega_T^D p_N(\overline{z}_D)^{1-\sigma} + \Omega_T^{X*}\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} p_N(\overline{z}_X)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

where the second equality uses the relationships between $p_N(z)$, $p_D(z)$, and $p_X(z)$ described earlier: $p_D(z) = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}} p_N(z); p_X(z) = \tau^X p_D(z)$.

Using the equation for $p_N(z)$ derived earlier, $w_X(z) = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}} w_N(z)$, and $w_N(z) = \left(\frac{(\sigma-1)\theta}{\sigma(1+\theta)}\right)^{\frac{\sigma}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{1}{\sigma+\theta}} z^{\frac{\sigma-1}{\sigma+\theta}}$, the equation for price index above yields

$$1 = \Omega_T^P \Phi_T^{\chi} \left(\frac{\sigma(1+\theta)}{(\sigma-1)\theta}\right)^{\frac{(1-\sigma)\theta}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{1-\sigma}{\sigma+\theta}}, \quad \text{(A-90)}$$

where Φ_T is a hyperparameter defined as

$$\Phi_T^{\chi} \equiv \frac{1}{1-G(\widehat{z}_D)}\left([G(\widehat{z}_X)-G(\widehat{z}_D)]\overline{z}_D^{\chi} + [1-G(\widehat{z}_X)](1+\left(\tau^X\right)^{1-\sigma})^{\frac{1+\theta}{\sigma+\theta}}\overline{z}_X^{\chi}\right). \quad \text{(A-91)}$$

The expression for labour market shifter B is

$$B = \left(\int_{\widehat{z}_D}^{\infty} w(z)^{1+\theta}\frac{g(z)}{1-G(\widehat{z}_D)}dz\right)^{\frac{\phi-\theta}{1+\theta}} \left(\Omega_T^P\right)^{-1}\Gamma\overline{L}. \quad \text{(A-92)}$$

Use the wage expression in above to obtain

$$B = \left(\Omega_T^P\right)^{-1} \Gamma \overline{L} \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma(\phi-\theta)}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{(\phi-\theta)}{\sigma+\theta}} \times$$
$$\left(\int_{\widehat{z}_D}^{\widehat{z}_X} z^{\chi} g(z \geq \widehat{z}_D) dz + \left(1 + \left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} \int_{\widehat{z}_X}^{\infty} z^{\chi} g(z \geq \widehat{z}_D) dz\right)^{\frac{(\phi-\theta)}{1+\theta}}.$$

Next, verify that $\left(\int_{\widehat{z}_D}^{\widehat{z}_X} z^{\chi} g(z \geq \widehat{z}_D) dz + \left(1 + \left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} \int_{\widehat{z}_X}^{\infty} z^{\chi} g(z \geq \widehat{z}_D) dz\right)$ equals Φ_T^{χ}. Therefore, write the labour market shifter B above as

$$B = \left(\Omega_T^P\right)^{-1} \Gamma \overline{L} \left(\left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma(\phi-\theta)}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{(\phi-\theta)}{\sigma+\theta}}\right) \Phi_T^{\frac{\chi(\phi-\theta)}{1+\theta}}. \tag{A-93}$$

Assuming that the least productive firm does not export, the zero cutoff profit condition is

$$\left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\right) \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma(1+\theta)}{\sigma+\theta}} A^{\frac{1+\theta}{\sigma+\theta}} B^{\frac{\sigma-1}{\sigma+\theta}} \widehat{z}_D^{\chi} = W f^C. \tag{A-94}$$

Recall that $w_N(z) = \left(\frac{(\sigma-1)\theta}{(1+\theta)\sigma}\right)^{\frac{\sigma}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{1}{\sigma+\theta}} z^{\frac{\sigma-1}{\sigma+\theta}}$ and $w_X(z) = \left(1 + \left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}} w_N(z)$. Therefore, the wage index is

$$W_T = \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{1}{\sigma+\theta}}$$
$$\left(\int_{\widehat{z}_D}^{\widehat{z}_X} z^{\chi} g(z \geq \widehat{z}_D) dz + \left(1 + \left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} \int_{\widehat{z}_X}^{\infty} z^{\chi} (z \geq \widehat{z}_D) dz\right)^{\frac{1}{1+\theta}}.$$

Using the hyperparameter Φ_T, the above can be written as

$$W_T = \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma}{\sigma+\theta}} \left(\frac{A}{B}\right)^{\frac{1}{\sigma+\theta}} \Phi_T^{\frac{\chi}{1+\theta}}. \tag{A-95}$$

Upon using (A-95) in (A-94) obtain

$$f^C = \left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\right) \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma\theta}{\sigma+\theta}} A^{\frac{\theta}{\sigma+\theta}} B^{\frac{\sigma}{\sigma+\theta}} \widehat{z}_D^{\chi} \Phi_T^{-\frac{\chi}{1+\theta}}. \tag{A-96}$$

Therefore, (A-90), (A-93), and (A-96) are the 3 key equations that solve for A, B, and Ω_T^P in the open economy. These 3 equations differ from their autarky counterpart only in the fact that Φ_A is replaced by Φ_T. Therefore, the solution to Ω_T^P in the open economy is given by

$$\left(\Omega_T^P\right)^{\frac{\sigma-1-\phi}{\sigma-1}} = \left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta} \frac{\Gamma \overline{L}}{f^C}\right) \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\phi} \Phi_T^{\phi-\chi} \widehat{z}_D^{\chi}. \tag{A-97}$$

And using (A-90) and (A-95) obtain

$$W_T = \frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)} \left(\Omega_T^P\right)^{\frac{1}{\sigma-1}} \Phi_T. \tag{A-98}$$

Using (A-97) write the above as

$$W_T = \left(\frac{(\sigma-1)\,\theta}{(1+\theta)\,\sigma}\right)^{\frac{\sigma-1}{\sigma-1-\phi}} \left(\left(\frac{(\sigma+\theta)}{(\sigma-1)\,\theta}\frac{\Gamma\overline{L}}{f^C}\right)\widehat{z}_D^{\chi}\right)^{\frac{1}{\sigma-1-\phi}} \Phi_T^{\frac{\sigma-1-\chi}{\sigma-1-\phi}}. \tag{A-99}$$

Comparing autarky with trading equilibrium, using (A-82), (A-84), (A-97) and (A-99) obtain

$$\frac{\Omega_T^P}{\Omega_A^P} = \left(\left(\frac{\Phi_T}{\Phi_A}\right)^{\phi-\chi}\left(\frac{\widehat{z}_D}{\widehat{z}}\right)^{\chi}\right)^{\frac{\sigma-1}{\sigma-1-\phi}}, \tag{A-100}$$

$$\frac{W_T}{W_A} = \left(\frac{\widehat{z}_D}{\widehat{z}}\right)^{\frac{\chi}{\sigma-1-\phi}} \left(\frac{\Phi_T}{\Phi_A}\right)^{\frac{\sigma-1-\chi}{\sigma-1-\phi}}. \tag{A-101}$$

Gains from Trade with Pareto Distribution Assume $G(z) = 1 - \left(\frac{1}{z}\right)^k$, and the probability density function is $g(z) = \frac{k}{z^{k+1}}$, where $z \in [1,\infty)$ and $k > \sigma - 1 > \chi$.

Autarky expressions are

$$\widehat{z} = \left[\left(\frac{f^C}{f^T}\right)\frac{\chi}{k-\chi}\right]^{\frac{1}{k}}, \tag{A-102}$$

$$\Phi_A \equiv \left(\int_{\widehat{z}}^{\infty} z^{\chi}\frac{g(z)}{1-G(\widehat{z})}dz\right)^{\frac{1}{\chi}} = \left(\frac{k}{k-\chi}\right)^{\frac{1}{\chi}}\widehat{z}. \tag{A-103}$$

In the trading equilibrium we get

$$\widehat{z}_D = \left[\frac{\mathcal{F}+\mu^k}{\mu^k}\right]^{\frac{1}{k}}\widehat{z};\, \widehat{z}_X = \left(\mathcal{F}+\mu^k\right)^{\frac{1}{k}}\widehat{z}. \tag{A-104}$$

As well,

$$\int_{\widehat{z}_D}^{\widehat{z}_X} z^{\chi} g(z \;\; \geq \;\; \widehat{z}_D)dz = \left[\frac{k}{k-\chi}\left(\frac{\mu^k-\mu^{\chi}}{\mu^k}\right)\right]\widehat{z}_D^{\chi} \tag{A-105}$$

$$\int_{\widehat{z}_X}^{\infty} z^{\chi} g(z \;\; \geq \;\; \widehat{z}_D)dz = \frac{k}{k-\chi}\widehat{z}_D^{k}\widehat{z}_X^{\chi-k}. \tag{A-106}$$

Therefore,

$$\Phi_T = \left(\left(1-\mu^{\chi-k}\right) + \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}\mu^{\chi-k}\right)^{\frac{1}{\chi}} \left(\frac{k}{k-\chi}\right)^{\frac{1}{\chi}} \left(\frac{\mathcal{F}+\mu^k}{\mu^k}\right)^{\frac{1}{k}}\widehat{z}, \tag{A-107}$$

and hence

$$\frac{\Phi_T}{\Phi_A} = \left(1 + \mu^{\chi-k}\left(\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} - 1\right)\right)^{\frac{1}{\chi}}\left(\frac{\mathcal{F}+\mu^k}{\mu^k}\right)^{\frac{1}{k}}. \tag{A-108}$$

Use above to write (A-101) as

$$\frac{W_T}{W_A} = \left(\frac{\mathcal{F}+\mu^k}{\mu^k}\right)^{\frac{1}{k}\frac{\sigma-1}{\sigma-1-\phi}}\left(\left(\left(1-\mu^{\chi-k}\right) + \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}\mu^{\chi-k}\right)^{\frac{1}{\chi}}\right)^{\frac{\sigma-1-\chi}{\sigma-1-\phi}}. \tag{A-109}$$

A.6 Proof of Proposition 3

The share of spending on domestic output is

$$\lambda = \frac{\int_{\widehat{z}_D}^{\widehat{z}_X}(zn_N(z))^{\frac{\sigma-1}{\sigma}}g(z\geq\widehat{z}_D)dz + \int_{\widehat{z}_X}^{\infty}(zn_D(z))^{\frac{\sigma-1}{\sigma}}g(z\geq\widehat{z}_D)dz}{\int_{\widehat{z}_D}^{\widehat{z}_X}(zn_N(z))^{\frac{\sigma-1}{\sigma}}g(z\geq\widehat{z}_D)dz + \int_{\widehat{z}_X}^{\infty}(zn_D(z))^{\frac{\sigma-1}{\sigma}}g(z\geq\widehat{z}_D)dz + \int_{\widehat{z}_X}^{\infty}\left(\frac{zn_X(z)}{\tau}\right)^{\frac{\sigma-1}{\sigma}}g(z\geq\widehat{z}_D)dz}. \tag{A-110}$$

Note that $A^{\frac{1}{\sigma}}\Omega_T^P$ is multiplied to all terms, hence it cancels out. Since $w_X(z) = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}}w_N(z)$ obtain

$$n_D(z) = n_N(z)\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{-\frac{\sigma}{\sigma+\theta}} \tag{A-111}$$

$$n_X(z) = n_N(z)\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{-\frac{\sigma}{\sigma+\theta}}\left(\tau^X\right)^{1-\sigma} \tag{A-112}$$

and substitute in the expression for λ above to obtain

$$\lambda = \frac{\int_{\widehat{z}_D}^{\widehat{z}_X} z^\chi g(z\geq\widehat{z}_D)dz + \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1-\sigma}{\sigma+\theta}}\int_{\widehat{z}_X}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz}{\int_{\widehat{z}_D}^{\widehat{z}_X} z^\chi g(z\geq\widehat{z}_D)dz + \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}\int_{\widehat{z}_X}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz}. \tag{A-113}$$

Verify that if there is no selection into exporting ($\widehat{z}_X = \widehat{z}_D$) then the above is simply $\lambda = \frac{1}{1+(\tau^X)^{1-\sigma}}$ as in the homogeneous firm case. Therefore, when the selection effect is present, own trade share depends on θ. Let us re-write λ as

$$\lambda = \frac{\int_{\widehat{z}_D}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz + \left(\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1-\sigma}{\sigma+\theta}} - 1\right)\int_{\widehat{z}_X}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz}{\int_{\widehat{z}_D}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz + \left(\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} - 1\right)\int_{\widehat{z}_X}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz}. \tag{A-114}$$

Here we can define Λ as

$$\Lambda = \frac{\int_{\widehat{z}_X}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz}{\int_{\widehat{z}_D}^{\infty} z^\chi g(z\geq\widehat{z}_D)dz} < 1. \tag{A-115}$$

And then we have

$$\lambda = \frac{1 + \left(\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1-\sigma}{\sigma+\theta}} - 1\right)\Lambda}{1 + \left(\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} - 1\right)\Lambda}. \tag{A-116}$$

What if $\theta \to \infty$? In this case, we get $\lambda = \frac{1}{1+(\tau^X)^{1-\sigma}\Lambda}$ from above and $\chi = \sigma - 1$. This is exactly the expression in the Melitz model as shown in Melitz and Redding (2015). So, Λ captures the selection into exporting. Next, compute $-\frac{d\log\left(\frac{1-\lambda}{\lambda}\right)}{d\log\tau^X}$ which is the trade elasticity term. Using (A-116) obtain

$$\frac{1-\lambda}{\lambda} = \frac{H\left(\tau^X\right)^{1-\sigma}\Lambda}{1+(H-1)\Lambda} = H\left(\tau^X\right)^{1-\sigma}\Lambda^M, \tag{A-117}$$

where $H \equiv \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1-\sigma}{\sigma+\theta}}$ and $\Lambda^M = \frac{\Lambda}{1+(H-1)\Lambda}$ where Λ^M is the adjusted selection term in the monopsony case. Again verify that the above becomes $\left(\tau^X\right)^{1-\sigma}\Lambda$ as $\theta \to \infty$ which is the expression in Melitz and Redding (2015). Next,

$$-\frac{d\log\left(\frac{1-\lambda}{\lambda}\right)}{d\log\tau^X} = -\frac{d\log\left(H\left(\tau^X\right)^{1-\sigma}\right)}{d\log\tau^X} - \frac{d\log\Lambda^M}{d\log\tau^X} \tag{A-118}$$

where the first term is the intensive margin elasticity, ε^I, as verified below and the second term is the extensive margin elasticity, ε^E.

A.6.1 Intensive Margin Elasticity

Verify that

$$\varepsilon^I(\theta) = -\frac{d\log\left(H\left(\tau^X\right)^{1-\sigma}\right)}{d\log\tau^X} = (\sigma-1)\Xi; 1 \geq \Xi \equiv 1 - \frac{\sigma-1}{\sigma+\theta}\frac{\left(\tau^X\right)^{1-\sigma}}{1+\left(\tau^X\right)^{1-\sigma}} > 0.$$

From the maximization problem of the exporting firm discussed earlier, note that the revenue from exporting is $R_X = A^{\frac{1}{\sigma}}\left(\frac{zn_X}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}}$ and profit maximization implies $n_D(z) = \left(\tau^X\right)^{1-\sigma}n_X(z)$ and $n_T(z) = n_D(z) + n_X(z) = \left(1+\left(\tau^X\right)^{1-\sigma}\right)n_X(z)$. Therefore, $n_X(z) = \frac{\left(\tau^X\right)^{1-\sigma}}{1+\left(\tau^X\right)^{1-\sigma}}n_T(z)$.

We have also obtained $w_X(z) = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}}w_N(z)$. Therefore,

$$\frac{n_T(z)}{n_N(z)} = \left(\frac{w_X(z)}{w_N(z)}\right)^{\theta} = \left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{\theta}{\sigma+\theta}}. \tag{A-119}$$

Using (A-71), the expression for employment of non-trading firms, and (A-119) above obtain

$$n_T(z) = \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\frac{\sigma\theta}{\sigma+\theta}}A^{\frac{\theta}{\sigma+\theta}}B^{\frac{\sigma}{\sigma+\theta}}z^{\frac{(\sigma-1)\theta}{\sigma+\theta}}\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{\theta}{\sigma+\theta}}. \tag{A-120}$$

Therefore,

$$n_X(z) = \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\frac{\sigma\theta}{\sigma+\theta}}A^{\frac{\theta}{\sigma+\theta}}B^{\frac{\sigma}{\sigma+\theta}}z^{\frac{(\sigma-1)\theta}{\sigma+\theta}}\left(\tau^X\right)^{1-\sigma}\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{-\sigma}{\sigma+\theta}}. \tag{A-121}$$

And hence

$$R_X = \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\frac{(\sigma-1)\theta}{\sigma+\theta}} A^{\frac{1+\theta}{\sigma+\theta}} B^{\frac{\sigma-1}{\sigma+\theta}} z^{\frac{1+\theta}{\sigma+\theta}} H \left(\tau^X\right)^{1-\sigma}. \tag{A-122}$$

That is $R_X \propto H \left(\tau^X\right)^{1-\sigma}$. Hence,

$$\varepsilon^I(\theta) \equiv \left|\frac{d\log R_X}{d\log \tau^X}\right| = (\sigma-1)\,\Xi < \sigma - 1. \tag{A-123}$$

Thus, the intensive margin elasticity is less than $\sigma - 1$. In the standard Melitz model $R_X \propto \left(\tau^X\right)^{(1-\sigma)}$ and hence $\varepsilon^I = \sigma - 1$. Also verify that $\varepsilon^I(\theta) \to 1$ as $\theta \to \infty$ because $\Xi \to 1$. As well, $\frac{d\Xi}{d\theta} > 0$, so the larger the θ the larger the $\varepsilon^I(\theta)$.

A.6.2 Extensive Margin Elasticity

Denote the extensive margin elasticity by $\varepsilon^E(\theta, k)$. Therefore,

$$\varepsilon^E(\theta, k) \equiv -\frac{d\log \Lambda^M}{d\log \tau^X}. \tag{A-124}$$

Taking logs,

$$\log \Lambda^M = \log \Lambda - \log\big(1 + (H-1)\Lambda\big), \tag{A-125}$$

and differentiating with respect to $\log \tau^X$ yields

$$\frac{d\log \Lambda^M}{d\log \tau^X} = \frac{d\log \Lambda}{d\log \tau^X} - \frac{(H-1)\Lambda \frac{d\log\Lambda}{d\log\tau^X} + \Lambda \frac{dH}{d\log\tau^X}}{1 + (H-1)\Lambda}. \tag{A-126}$$

Rearranging,

$$\varepsilon^E(\theta, k) \equiv -\frac{d\log \Lambda^M}{d\log \tau^X} = -\frac{1}{1+(H-1)\Lambda}\frac{d\log \Lambda}{d\log \tau^X} + \frac{\Lambda}{1+(H-1)\Lambda}\frac{dH}{d\log \tau^X}. \tag{A-127}$$

Under Pareto with shape parameter k obtain the following expression for Λ defined in (A-115).

$$\Lambda = \left(\frac{\widehat{z}_X}{\widehat{z}_D}\right)^{\chi - k}. \tag{A-128}$$

Combining (A-128) and (A-87) yields

$$\Lambda = \mu^{\chi - k}. \tag{A-129}$$

Taking logs and differentiating with respect to $\log \tau^X$,

$$\frac{d\log \Lambda}{d\log \tau^X} = (\chi - k)\,\frac{d\log \mu}{d\log \tau^X}. \tag{A-130}$$

Next, we verifyt that $\varepsilon^E(\theta, k) > k - \sigma + 1$, where $k - \sigma + 1$ is the extensive margin elasticity in the perfect labour market case.

First, using the expression for μ in (A-87) verify that

$$\frac{d \log \mu}{d \log \tau^X} = \frac{\left(\tau^X\right)^{1-\sigma} \left(1 + \left(\tau^X\right)^{1-\sigma}\right)^{\frac{1-\sigma}{\sigma+\theta}}}{\left(1 + \left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}} - 1} > 0. \tag{A-131}$$

Easily verify from above that $d \log \mu / d \log \tau$ is strictly decreasing in θ and satisfies

$$\frac{d \log \mu}{d \log \tau^X} \to 1 \quad \text{as } \theta \to \infty, \quad \text{and} \quad \frac{d \log \mu}{d \log \tau^X} > 1 \quad \text{for any finite } \theta > 0.$$

At the same time, since

$$\chi(\theta) = \frac{(\sigma - 1)(1 + \theta)}{\sigma + \theta} < \sigma - 1 \quad \text{for all } \theta > 0,$$

and we are in the empirically relevant case $k > \sigma - 1$, it follows that

$$\chi(\theta) - k < \sigma - 1 - k < 0,$$

and $|\chi(\theta) - k| > |\sigma - 1 - k| = k - \sigma + 1$. As a result,

$$-\frac{d \log \Lambda}{d \log \tau^X} = -(\chi - k) \frac{d \log \mu}{d \log \tau^X} = (k - \chi) \frac{d \log \mu}{d \log \tau^X} > (k - (\sigma - 1)) \cdot 1 = k - \sigma + 1.$$

Thus, even if we focused on the *raw* selection term Λ (as in Melitz), the absolute selection elasticity would already exceed $k - \sigma + 1$ in our model.

Second, the transformation from Λ to Λ^M further amplifies this selection elasticity. Since $\sigma > 1$, $\theta > 0$ and $\tau^X > 1$ imply $\left(\tau^X\right)^{1-\sigma} \in (0, 1)$ and $(1 - \sigma)/(\sigma + \theta) < 0$, we have $0 < H(\tau; \sigma, \theta) < 1$. With $\Lambda \in (0, 1)$ this yields

$$0 < 1 + (H - 1)\Lambda < 1 \quad \Rightarrow \quad \frac{1}{1 + (H - 1)\Lambda} > 1.$$

From (A-127),

$$-\frac{d \log \Lambda^M}{d \log \tau^X} = \frac{1}{1 + (H - 1)\Lambda} \left(-\frac{d \log \Lambda}{d \log \tau^X}\right) + \frac{\Lambda}{1 + (H - 1)\Lambda} \frac{dH}{d \log \tau^X}, \tag{A-132}$$

and

$$\frac{dH}{d \log \tau^X} = H \frac{(1 - \sigma)^2}{\sigma + \theta} \frac{\left(\tau^X\right)^{1-\sigma}}{1 + \left(\tau^X\right)^{1-\sigma}} > 0. \tag{A-133}$$

Therefore,

$$\varepsilon^E(\theta, k) = -\frac{d \log \Lambda^M}{d \log \tau^X} > -\frac{d \log \Lambda}{d \log \tau^X} > k - \sigma + 1 = \varepsilon^E(\infty, k), \tag{A-134}$$

for any finite $\theta > 0$ and $k > \sigma - 1$.

A.6.3 Intensive Margin Elasticity with Respect to Tariffs

The spending on imports is going to be gross of tariffs. Therefore, if the pre-tariff price of imports in the importing country is $p_X(z)$ and the amount of imports is $q_X(z)$ then the spending on imports is $(1+t)p_X(z)q_X(z)$ where $q_X(z) = A\left((1+t)p_X(z)\right)^{-\sigma}$. Since $\tau q_X(z) = \varphi n_X(z)$, we can write the revenue received by an exporter as $p_X(z)q_X(z) = \frac{1}{1+t}A^{\frac{1}{\sigma}}\left[\frac{\varphi n_X(z)}{\tau^X}\right]^{\frac{\sigma-1}{\sigma}}$. Therefore, an exporter maximizes the following in the case of a tariff.

$$\max\left\{A^{\frac{1}{\sigma}}(\varphi n_D(z))^{\frac{\sigma-1}{\sigma}} + \frac{1}{1+t}A^{\frac{1}{\sigma}}\left(\frac{\varphi n_X(z)}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}} - B^{-\frac{1}{\theta}}\left[n_D(z)+n_X(z)\right]^{\frac{1+\theta}{\theta}} - Wf - Wf_X\right\}.$$

The first order conditions are

$$n_D(z) \quad : \quad \frac{\sigma-1}{\sigma}A^{\frac{1}{\sigma}}z^{\frac{\sigma-1}{\sigma}}n_D(z)^{-\frac{1}{\sigma}} = \left(\frac{1+\theta}{\theta}\right)B^{-\frac{1}{\theta}}n_T(z)^{\frac{1}{\theta}}; \tag{A-135}$$

$$n_X(z) \quad : \quad \frac{\sigma-1}{\sigma}\frac{1}{1+t}A^{\frac{1}{\sigma}}\left(\frac{z}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}}n_X(z)^{-\frac{1}{\sigma}} = \left(\frac{1+\theta}{\theta}\right)B^{-\frac{1}{\theta}}n_T(z)^{\frac{1}{\theta}}, \tag{A-136}$$

where $n_T(z) = n_D(z) + n_X(z)$. Define $K \equiv (1+t)^{-\sigma}\left(\tau^X\right)^{1-\sigma}$. Now, the two first order conditions imply that

$$n_D(z) = \frac{n_X(z)}{K}; n_T(z) = (1+K)\,n_D(z); \tag{A-137}$$

$$n_X(z) = \frac{K}{1+K}n_T(z). \tag{A-138}$$

Using (A-135)–(A-138) above obtain the following expression for total employment, $n_T = n_D + n_X$, for a trading firm.

$$n_T(z) = \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\frac{\sigma\theta}{\sigma+\theta}} A^{\frac{\theta}{\sigma+\theta}}B^{\frac{\sigma}{\sigma+\theta}}z^{\frac{(\sigma-1)\theta}{\sigma+\theta}}\,(1+K)^{\frac{\theta}{\sigma+\theta}}. \tag{A-139}$$

Next, using (A-138) and (A-139) obtain

$$n_X(z) = \left(\frac{(\sigma-1)\,\theta}{\sigma\,(1+\theta)}\right)^{\frac{\sigma\theta}{\sigma+\theta}} A^{\frac{\theta}{\sigma+\theta}}B^{\frac{\sigma}{\sigma+\theta}}z^{\frac{(\sigma-1)\theta}{\sigma+\theta}}K\,(1+K)^{-\frac{\sigma}{\sigma+\theta}}. \tag{A-140}$$

The export revenue inclusive of tariffs is $R_X(z) = (1+t)\,p_X(z)q_X(z) = A^{\frac{1}{\sigma}}\left(\frac{\varphi n_X}{\tau^X}\right)^{\frac{\sigma-1}{\sigma}}$. Therefore,

$$R_X \propto (1+K)^{\frac{1-\sigma}{\sigma+\theta}}\,K\,(1+t). \tag{A-141}$$

Hence, the intensive margin elasticity with respect to tariffs is

$$\varepsilon^I_{tariff} \equiv \left|\frac{d\log R_X}{d\log(1+t)}\right| = (\sigma-1)\left(1 - \frac{\sigma}{\sigma+\theta}\frac{K}{1+K}\right). \tag{A-142}$$

A.7 FDI Equilibrium

For a firm engaging in FDI, their gross profit per market is given by $\frac{\sigma+\theta}{\theta(\sigma-1)}wn$, as can be easily derived from firm profit maximisation.

In a symmetric FDI equilibrium, the zero-profit condition becomes:

$$2\left[\frac{\sigma+\theta}{\theta\left(\sigma-1\right)}\right]wn=wf^{M}, \tag{A-143}$$

which yields the following expression for employment by a multinational firm, denoted with subscript M, in each market:

$$n_{M}=\frac{\theta\left(\sigma-1\right)}{2\left(\sigma+\theta\right)}f^{M}. \tag{A-144}$$

The labour market clearing condition is then given by

$$\frac{w^{\phi}}{\Omega^{P}}\Gamma\overline{L}=\frac{\theta\left(\sigma-1\right)}{2\left(\sigma+\theta\right)}f^{M}, \tag{A-145}$$

where Ω^{P} is the mass of producers (comprising both home-based and foreign-based multinationals). Finally, the equation for the price index is given by

$$\Omega^{P}\left[\frac{\sigma(1+\theta)}{(\sigma-1)\,\theta}\frac{w}{z}\right]^{1-\sigma}=1. \tag{A-146}$$

Combining Equations (A-145) and (A-146), we obtain the following expression for the equilibrium wage under FDI, denoted by w_{M}:

$$w_{M}=\left\{\left[\frac{\sigma(1+\theta)}{(\sigma-1)\,\theta z}\right]^{1-\sigma}\frac{2\left(\sigma+\theta\right)}{\theta\left(\sigma-1\right)}\frac{\Gamma\overline{L}}{f^{M}}\right\}^{\frac{1}{\sigma-1-\phi}}. \tag{A-147}$$

use Equations (A-53) and (A-147) to obtain (20) reported in the text.

A.8 Proof of Proposition 4

Let us start with a candidate exporting equilibrium and find out under what condition an individual firm will have an incentive to deviate to FDI.

As shown earlier, the gross profit of firms doing FDI can be written as $2\left(\frac{\sigma+\theta}{\theta(\sigma-1)}\right)wn$. Suppose we are in an exporting equilibrium with the wage w_{X}. Since exporting firms are making zero profits, a firm will deviate to FDI if $2\left(\frac{\sigma+\theta}{\theta(\sigma-1)}\right)w_{dev}n_{dev}>w_{X}f^{M}$ where we use w_{dev} to denote the wage and n_{dev} to denote the employment of the deviating firm. Also recall that the the zero profit condition in an exporting equilibrium is $\left(\frac{\sigma+\theta}{\theta(\sigma-1)}\right)w_{X}n_{T}=w_{X}f^{X}$ where $n_{X}=n_{D}\left(\tau^{X}\right)^{1-\sigma}; n_{T}=n_{D}+n_{X}=\frac{\theta(\sigma-1)}{\sigma+\theta}f^{X}$. Therefore, for a firm to deviate to FDI the following inequality must be satisfied:

$$\left(\frac{\sigma+\theta}{\theta\left(\sigma-1\right)}\right)\left(2w_{dev}n_{dev}-w_{X}n_{T}\right)>w_{X}\left(f^{M}-f^{X}\right). \tag{A-148}$$

Using $w = (n/B)^{\frac{1}{\theta}}$ re-write the above as

$$\left(\frac{\sigma+\theta}{\theta\left(\sigma-1\right)}\right)\left(\frac{2n_{dev}^{\frac{1+\theta}{\theta}}-n_T^{\frac{1+\theta}{\theta}}}{n_T^{\frac{1}{\theta}}}\right) > (f^M - f^X). \tag{A-149}$$

Now the first order condition of a deviant firm choosing FDI is

$$A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} n_{dev}{}^{\frac{\sigma-1}{\sigma}} = w_{dev} n_{dev}\left(\frac{\left(1+\theta\right)\sigma}{\theta\left(\sigma-1\right)}\right). \tag{A-150}$$

Using $w = (n/B)^{\frac{1}{\theta}}$ along with (A-55) and (A-150) obtain

$$n_{dev} = \frac{n_T}{\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{\theta}{\sigma+\theta}}}. \tag{A-151}$$

Using (A-151) rewrite (A-149) as

$$\left(\frac{2}{\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}}-1\right)n_T > \frac{\theta\left(\sigma-1\right)}{\sigma+\theta}\left(f^M - f^X\right). \tag{A-152}$$

Next, use $n_T = \frac{\theta(\sigma-1)}{\sigma+\theta} f^X$ derived in (A-50) earlier in (A-152) above and reorganise to get

$$\frac{f^M}{f^X} < \frac{2}{\left(1+\left(\tau^X\right)^{1-\sigma}\right)^{\frac{1+\theta}{\sigma+\theta}}} \equiv \overline{f}. \tag{A-153}$$

Then, if the inequality above is satisfied, starting from a candidate exporting equilibrium, firms will deviate to FDI. Alternatively, if $\frac{f^M}{f^X} > \overline{f}$, then we have an exporting equilibrium.

Now suppose the initial equilibrium is one where all firms engage in FDI. Under what condition will an individual firm have an incentive to become an exporter instead?

Since the initial equilibrium is one with FDI, the zero profit condition of FDI firms is $2\left(\frac{\sigma+\theta}{\theta(\sigma-1)}\right) w_M n_M = w_M f^M$. If a firm deviates and becomes an exporter, its first order conditions will be given by (A-46)and (A-47) and let us denote its wage and employment by $w_{T'}$ and $n_{T'}$, respectively. For the deviation to be profitable we need $w_{T'} n_{T'}\left(\frac{\sigma+\theta}{\theta(\sigma-1)}\right) > w_M f^X$.

Therefore, profitable deviation requires

$$\left(\frac{\sigma+\theta}{\theta\left(\sigma-1\right)}\right)\left(w_{T'} n_{T'} - 2 w_M n_M\right) > w_M (f^X - f^M). \tag{A-154}$$

Using $n = B w^{\theta}$ write the above as

$$\left(\frac{\sigma+\theta}{\theta\left(\sigma-1\right)}\right)\left(\frac{n_{T'}^{\frac{1+\theta}{\theta}}}{n_M^{\frac{1}{\theta}}} - 2n_M\right) > \left(f^X - f^M\right). \tag{A-155}$$

Since the profit maximisation condition of the deviating exporting firm remains (A-46)and (A-47), equation (A-55) remains valid which we reproduce below with subscript T being replaced by T'.

$$A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} (1 + \left(\tau^X\right)^{1-\sigma})^{\frac{1}{\sigma}} n_{T'}^{\frac{\sigma-1}{\sigma}} = w_{T'} n_{T'} \left(\frac{(1+\theta)\,\sigma}{\theta\,(\sigma-1)}\right). \tag{A-156}$$

And the profit maximizing condition of FDI firms remains

$$A^{\frac{1}{\sigma}} z^{\frac{\sigma-1}{\sigma}} n_M{}^{\frac{\sigma-1}{\sigma}} = w_M n_M \left(\frac{(1+\theta)\,\sigma}{\theta\,(\sigma-1)}\right). \tag{A-157}$$

From the above two obtain (upon using $n = Bw^{\theta}$)

$$n_{T'} = (1 + \left(\tau^X\right)^{1-\sigma})^{\frac{\theta}{\sigma+\theta}} n_M = (1 + \left(\tau^X\right)^{1-\sigma})^{\frac{\theta}{\sigma+\theta}} \frac{\theta\,(\sigma-1)}{2\,(\sigma+\theta)} f^M. \tag{A-158}$$

The last equality above follows from the fact that in an FDI equilibrium the expression for n_M is given in (A-144). Now use above in (A-155) to obtain

$$\frac{f^M}{f^X} > \overline{f}, \tag{A-159}$$

where $\overline{f}$ is defined in (A-153). Therefore, as long as $\frac{f^M}{f^X} < \overline{f}$, no firm wants to deviate to exporting. Hence, we have an FDI equilibrium whenever $\frac{f^M}{f^X} < \overline{f}$. Alternatively, if $\frac{f^M}{f^X} > \overline{f}$ then we have an exporting equilibrium. In the knife edge case of $\frac{f^M}{f^X} = \overline{f}$ either equilibrium is possible. This completes the derivation required for Proposition 4.

B Computational Algorithms

The computational algorithms here follow similar procedures to those used in Borota, Defever, Impullitti and Spencer (2019), Ding, Spencer and Wang (2025b) and Ding, Spencer and Wang (2025a).

Steady State

Calibrated Steady State

For the computation of the steady state in the calibration step (note that aggregate objects are stationary and so their time subscripts are omitted):

1. Fix the W and W^* indices to unity (we will adjust the fixed costs of entry to make this consistent with free entry later). In what follows, variables with hats will denote conjectures in the computations. Conjecture objects $\widehat{A}$, $\widehat{A}^*$, $\widehat{\Omega}^P$ and $\widehat{\Omega}^{P*}$ — the demand shifters for final output in H, F, the mass of producing firms in H and F, respectively. Recall that the final good in H is the numéraire and since the two countries are symmetric, it follows that $P = P^* = 1$.

2. Solve the incumbent firm's Bellman equation (22). This step yields their value function as well policy functions for price, employment, discrete choice and wage policy functions. All these objects are functions of the firm's state space. Solve this part of the problem using value function iteration.

3. Solve the entrant's problem (25) using the value function obtained from Step 2 and assuming the fixed entry cost $f^T = 0$ (this is a free parameter given the fixed W above). After obtaining entry value v^T, set $f^T = v^T$.

4. Find the steady-state cross-sectional distribution of H and F firms: normalising to have a unit measure of each set of firms. Do this by firstly re-writing the cross-sectional law of motion in equation (26) in matrix notation as

$$\mu_{t+1} = \zeta_t \mu_t + \Omega_t^T G^T \tag{A-160}$$

where μ_t the vector of the measure of firms across the state space and ζ_t is a Markov transition matrix that depends on the productivity process for incumbents, the stochastic process for the iceberg costs, their equilibrium discrete choices and the stochastic process for the sunk cost draws. We can then find the invariant stationary distribution $\widetilde{\mu} = \widetilde{\mu}_t = \widetilde{\mu}_{t+1}$ through inverting the steady-state version of equation (A-160) as $\widetilde{\mu} = \widetilde{\Omega}^T (I - \zeta)^{-1} G^T$. Here I is the identity matrix and $\widetilde{\Omega}^T$ is the measure of entrants that normalises the overall firm measure to unity (giving the stationary distribution).

5. Find variable averages implied by the stationary distribution found in Step 4.

6. Find the measures of firms using the linearity of the stationary measure such that the definition of the wage indices hold in each country as given in equation (4). Recall that these indices are set equal to unity as per the normalisation above. Note that this step utilises the average employment levels found using the stationary distribution, in Step 5.

7. Aggregate using the variable averages and firm measures, found in Step 6.
8. Compute the following metrics of distance

$$
\begin{aligned}
\Delta^{A} &= |\widehat{A} - A| \\
\Delta^{A*} &= |\widehat{A}^{*} - A^{*}| \\
\Delta^{MP} &= |\widehat{\Omega}^{P} - \Omega^{P}| \\
\Delta^{MP*} &= |\widehat{\Omega}^{P*} - \Omega^{P*}|.
\end{aligned} \tag{A-161}
$$

See that the first two objects in the set of equations in (A-161) represent the distance of the conjectured demand shifter from the supply of final goods implied from the previous step in H and F, respectively. The second two equations give the distance of the conjectured masses of producers from those implied in H and F, respectively. If $\max(\Delta^{A}, \Delta^{A*}, \Delta^{MP}, \Delta^{MP*})$ is sufficiently small, then stop. Otherwise update the conjectures $\widehat{A}$, $\widehat{A}^{*}$, $\widehat{\Omega}^{P}$ and $\widehat{\Omega}^{P*}$, return to Step 2 above and repeat until convergence. Once converged, compute the list of equilibrium objects in equation (28) for this calibrated steady state. Label this list of objects Υ_0.

Counterfactual Steady State

When running a counterfactual, use the following procedure:

A Conjecture objects $\widehat{A}, \widehat{A}^{*}, \widehat{P}^{*}, \widehat{W}$, $\widehat{W}^{*}$, $\widehat{\Omega}^{P}$ and $\widehat{\Omega}^{P*}$. Notice now that the two wage indices now need to be found endogenously.

B As in Step 2 in the calibration procedure, solve the incumbent firm's Bellman equation using value function iteration.

C Solve the entrant's problem (25) using the value function obtained from Step B and the fixed cost of entry implied by Step 3 of the calibration procedure.

D As in Step 4 of the calibration procedure, find the stationary cross-section of firms.

E As in Step 5 of the calibration procedure, find the averages of variables using the stationary distribution.

F Similarly to Step 6 of the calibration procedure, find the measures of firms implied by the current wage index conjectures $\widehat{W}$ and $\widehat{W}^{*}$.

G As in Step 7 of the calibration procedure, aggregate using the measures of firms found in Step F.

H Compute the metrics of distance of (A-161) in the calibration procedure, as well as

$$
\begin{aligned}
\Delta^{P*} &= |\widehat{A}^{*} - C^{*} - L^{*}| \\
\Delta^{W} &= |v^{T}| \\
\Delta^{W*} &= |v^{T*}|
\end{aligned}
$$

where the top equation is the difference between the conjectured final output in F and aggregate demand, while the second and third are the distances of the free entry conditions from holding. If $\max(\Delta^A, \Delta^{A*}, \Delta^{MP}, \Delta^{MP*}, \Delta^{P*}, \Delta^W, \Delta^{W*})$ is sufficiently small, then stop. Otherwise update the conjectured objects $\widehat{A}, \widehat{A}^*, \widehat{\Omega}^P, \widehat{\Omega}^{P*}, \widehat{P}^*, \widehat{W}, \widehat{W}^*$, return to Step B and repeat until convergence. Once stopped, compute the list of objects in (28) for this steady state, label this list $\Upsilon_{\widehat{\mathcal{T}}}$

Transition Dynamics

One uses the economies given by Υ_0 and Υ_1 as boundary conditions for the simulation, which starts at time $t = 1$. After having solved for the two steady states, follow the procedure below.

a Conjecture the length of time to convergence, label this number $\widehat{\mathcal{T}} \in \mathbb{N}$.

b Conjecture time paths for aggregate variables $\{\widehat{\Upsilon}_t\}_{t=1}^{\widehat{\mathcal{T}}-1}$ where

$$\widehat{\Upsilon}_t \equiv (\widehat{A}_t, \widehat{A}^*_t, \widehat{P}^*_t, \widehat{W}_t, \widehat{W}^*_t, \widehat{\Omega}^P_t, \widehat{\Omega}^{P*}_t, \widehat{M}^T_t, \widehat{M}^{T*}_t).$$

Notice that now we need to also conjecture the masses of entrants in each country.

c Take $v_{\widehat{\mathcal{T}}}(z, s, \tau^s)$ to be the endpoint for the H incumbent value function. This serves as the continuation value for the final period of the transition. Iterate backwards on the firm Bellman equation (22) to the initial period. This gives a sequence $\{v_t(z, s, \tau^s)\}_{t=1}^{\widehat{\mathcal{T}}-1}$. Do the same for the F firms.

d Using the sequence $\{v_t(z, s, \tau^s)\}_{t=1}^{\widehat{\mathcal{T}}-1}$, iterate backwards on the entrant Bellman equation (25). This gives a sequence of entry values $\{v^T_t\}_{t=1}^{\widehat{\mathcal{T}}-1}$. Do the same for F firms.

e Using the policy functions found in c and d, as well as μ_0 from list Υ_0 as a starting point, iterate forwards on the law of motion (A-160) to obtain a sequence of cross-sectional measures $\{\mu_t(z, s, \tau^s)\}_{t=1}^{\widehat{\mathcal{T}}-1}$. Do the same for the F firms.

f Compute the following sequence of distance metrics $\{\Delta_t\}_{t=1}^{\widehat{\mathcal{T}}-1}$

$$\Delta_t = (\Delta^A_t, \Delta^{A*}_t, \Delta^{P*}_t, \Delta^W_t, \Delta^{W*}_t, \Delta^{MP}_t, \Delta^{MP*}_t, \Delta^M_t, \Delta^{M*}_t)$$

where

$$\begin{aligned}
\Delta^A_t &= |\widehat{A}_t - A_t| \\
\Delta^{A*}_t &= |\widehat{A}^*_t - A^*_t| \\
\Delta^{P*}_t &= |\widehat{A}^*_t - C^*_t - F^*_t| \\
\Delta^W_t &= |\widehat{W}_t - W_t| \\
\Delta^{W*}_t &= |\widehat{W}^*_t - W^*_t| \\
\Delta^{MP}_t &= |\widehat{\Omega}^P_t - \Omega^P_t| \\
\Delta^{MP*}_t &= |\widehat{\Omega}^{P*}_t - \Omega^{P*}_t| \\
\Delta^M_t &= |v^T_t| \\
\Delta^{M*}_t &= |v^{T*}_t|
\end{aligned}$$

are the time-varying versions of distance metrics defined for the steady states. Note we also consider Δ_t^W and Δ_t^{W*} along the transition since we no longer normalise the cross-sectional measure. If $\max_t\{\max \Delta_t\}_{t=1}^{\widehat{\mathcal{T}}-1}$ is sufficiently small then stop. Otherwise, update the sequences of $\{\widehat{\Upsilon}_t\}_{t=1}^{\widehat{\mathcal{T}}-1}$, return to Step c and repeat.

g Check to see if the system has converged by time $\widehat{\mathcal{T}}$. If not, update your guess of $\widehat{\mathcal{T}}$, return to Step b and repeat until convergence.

C Extensions and Robustness

In this section we give some consideration to extensions to the model.

C.1 Parameter Robustness

	Baseline		**$\theta = 6.5$**	**$\phi = 0.0$**		**$\sigma = 4.5$**		**$\sigma = 4.0$**	
	Imperfect	Perfect	Imperfect	Imperfect	Perfect	Imperfect	Perfect	Imperfect	Perfect
Welfare	2.623	0.629	1.499	2.343	0.460	2.751	0.078	2.664	0.203
Consumption	2.801	0.762	1.684	2.343	0.460	2.966	0.182	2.870	0.318
Disutility	3.746	1.452	2.645	0.000	0.000	4.111	0.716	3.971	0.909
Measure D	-88.11	-74.79	-89.31	-92.24	-74.85	-90.88	-51.95	-85.59	-60.84
Measure X	143.0	119.6	144.0	165.3	119.1	174.8	89.45	151.9	82.95
Measure T	-18.82	-31.48	-32.56	-38.98	-31.65	-33.84	23.60	-14.41	-5.855
Measure HQ	-3.150	-4.636	-3.449	-3.365	-4.865	-3.147	-8.324	-3.709	-5.855
Profits	3.757	1.452	2.651	3.304	1.148	4.122	0.716	3.983	0.909
Taxes	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0	-100.0
W index	3.122	1.208	2.204	3.304	1.147	3.424	0.596	3.308	0.757

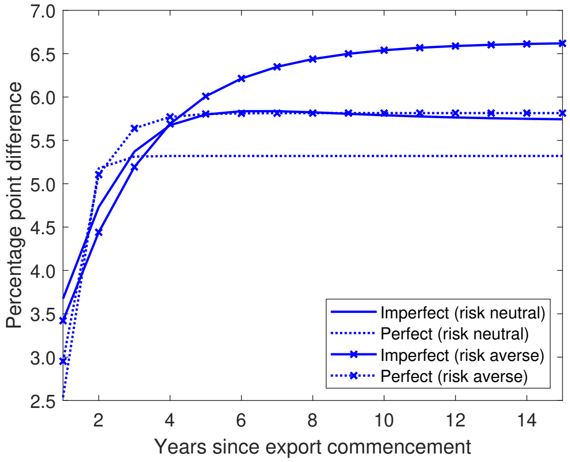
Table A-1: Parameter robustness. All parameterisations are without FDI and counterfactuals consider bilateral trade liberalisation. All numbers are after multiplication by 100.

C.2 Decomposing WMD and USLS

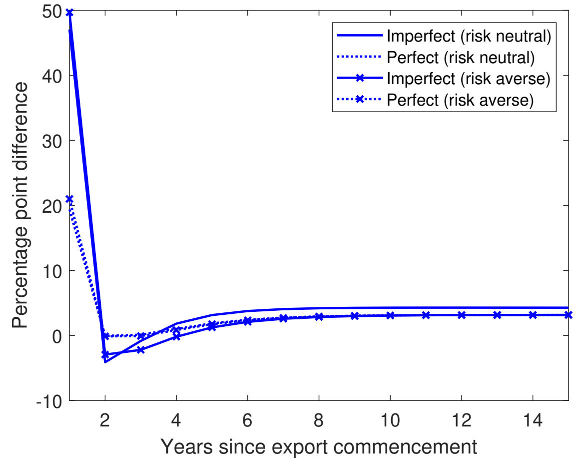
	Trade			**FDI**		
	Imperfect	No WMD	Perfect	Imperfect	No WMD	Perfect
Welfare	2.623	2.525	0.629	-0.721	-3.235	0.000
Consumption	2.801	2.676	0.762	-0.700	-3.208	0.010
Disutility	3.746	3.444	1.452	-0.589	-3.070	0.078
Measure D	-88.11	-88.41	-74.79	-0.557	-26.47	-1.471
Measure X	143.0	145.6	119.6	-13.44	-24.21	-11.09
Measure M				61.27	348.0	52.46
Measure T	-18.82	-23.77	-31.48	-0.966	-9.677	-1.332
Measure HQ	-3.150	-2.685	-4.636	-1.328	-8.239	-1.332
Measure U	0.361	0.371	0.283	-1.721	0.802	-1.262
Profits	3.757	3.463	1.452	-0.509	-2.835	0.344
Taxes	-100.0	-100.0	-100.0	-20.43	-35.16	-19.30
Wage index	3.122	2.878	1.208	-0.500	-2.579	0.065

Table A-2: Steady-state changes in aggregate variables from bilateral liberalisation episodes. All numbers are percentage deviations from the calibrated steady state (post-multiplication by 100). No WMD stands for no wage markdowns, but retaining upward-sloping labour supply (i.e. C_U for the trade case and C_U^{FDI} for the FDI case).

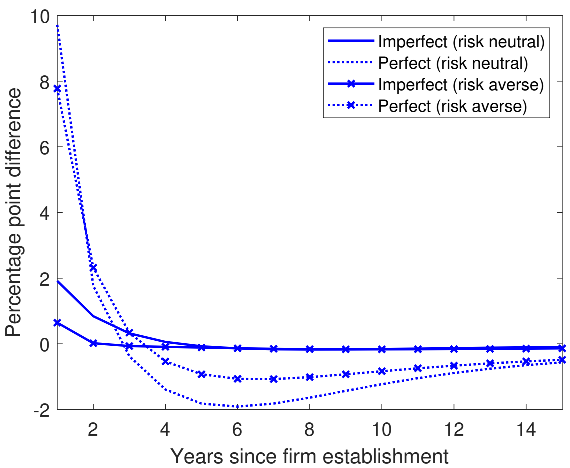
C.3 Risk Aversion



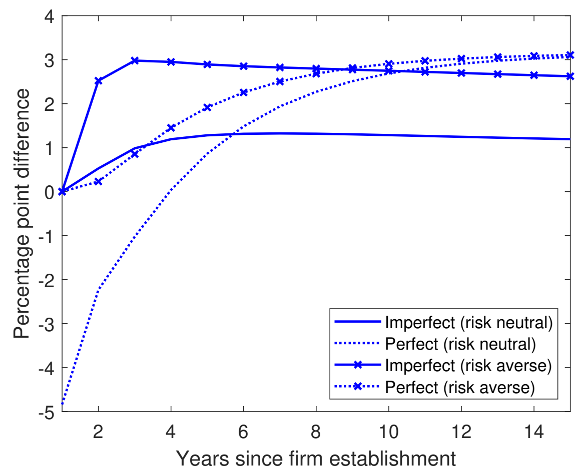
(a) New exporter intensity



(b) New exporter survival rates



(c) New entrant sales growth



(d) New entrant survival rates

Figure A-1: Firm expansion trajectories and survival rates with risk aversion. Cross-sectional comparisons across pre and post-liberalisation steady states. All variables are expressed as percentage point differences of the relevant rate in the post-liberalisation steady state from the pre-liberalisation steady state for the same given year since firm establishment/export commencement. All numbers are after multiplication by 100.

We start by noting that equation (5) in the text can be approximated with a reduced-form household problem (proof to follow)

$$\max_{C_t, N_t, \{l_t(\omega)\}_{\omega \in \Omega_t^P}} \sum_{t=0}^{\infty} \beta^t \left[C_t - \frac{N_t^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right] \tag{A-162}$$

	Risk neutrality		Risk aversion	
	Imperfect	Perfect	Imperfect	Perfect
Welfare	2.623	0.629	1.753	0.550
Consumption	2.801	0.762	1.870	0.674
Disutility	3.746	1.452	0.727	0.761
Measure D	-88.11	-74.79	-91.07	-77.89
Measure X	143.0	119.6	143.1	124.5
Measure T	-18.82	-31.48	-39.59	-40.49
Measure HQ	-3.150	-4.636	-2.540	-4.816
Measure U	36.14	28.32	37.41	29.49
Profits	3.757	1.452	2.765	1.468
Taxes	-100.0	-100.0	-100.0	-100.0
Wage index	3.122	1.208	2.623	1.340

Table A-3: Steady-state changes in aggregate variables from bilateral liberalisation episodes. All numbers are percentage deviations from the calibrated steady state (post-multiplication by 100).

subject to

$$
\begin{aligned}
N_t &= \left(\Omega_t^P\right)^{-\frac{1}{1+\theta}} \left(\int_{\omega \in \Omega_t^P} n_t(\omega)^{\frac{1+\theta}{\theta}} d\omega \right)^{\frac{\theta}{1+\theta}} \\
P_t C_t &= W_t N_t + \Pi_t + T_t,
\end{aligned}
$$

which has the interpretation of a risk neutral household choosing its consumption, aggregate labour disutility and portfolio of work choices over employers ω. Motivated by this, we extend the model to solve a problem with risk aversion of the form

$$
\max_{C_t, N_t, \{l_t(\omega)\}_{\omega \in \Omega_t^P}} \quad \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\nu}}{1-\nu} - \frac{1}{1+\frac{1}{\phi}} N_t^{1+\frac{1}{\phi}} \right] \tag{A-163}
$$

subject to

$$
\begin{aligned}
N_t &= \left(\Omega_t^P\right)^{-\frac{1}{1+\theta}} \left(\int_{\omega \in \Omega_t^P} n_t(\omega)^{\frac{1+\theta}{\theta}} d\omega \right)^{\frac{\theta}{1+\theta}} \\
P_t C_t + A_{t+1} &= W_t N_t + (1+R_t) A_t + \Pi_t + T_t,
\end{aligned}
$$

where variable A_t is a zero net supply riskless bond with interest rate R_t.[42] Note that this representation can also be approximately micro-founded with a random productivity specification as in the main text (proof shown below).[43] We then consider a parameterisation

[42] One could assume that this bond is traded internationally. This would only be material though in the case of unilateral reforms; we abstract from such discussion given our focus on bilateral liberalisation.

[43] The non-labour income terms for profits Π_t and tax redistributions T_t create complications with the equivalence. As long as these terms are small, the microfoundation approximately holds.

of the model with export engagement, akin to that studied in Section 7.1 and compare the long-run aggregate results (Table A-3) as well as firm lifecycle responsiveness (Figure A-1). We restrict attention to calibrations with both features of monopsony present (C_I^{RA}) and the perfect labour market structure with neither (C_P^{RA}). We focus our attention on counterfactual predictions across steady states, exploring the effects on aggregate variables and the firm lifecycle. The risk neutral household of the baseline analysis ($\nu = 0$) will always work more as real wages rise. However the risk averse household ($\nu > 0$) also exhibits an income effect, which pushes-back against the labour supply expansion.

We now give a proof of the equivalence between the micro-founded model with random productivity and the reduced form problems (A-162) and (A-163) for risk neutral and risk averse workers, respectively. The setup with risk aversion nests that with risk neutrality — we therefore provide derivations for the former. We start by finding an expression for the period expected utility of a worker in the micro-founded framework without non-labour income. We then undertake derivations for the problem in (A-163) and show the associated solutions are equivalent.

Micro-founded derivations. In the absence of non-labour income, when a risk averse worker is employed at firm ω, they receive utility

$$U_t(\omega, h_t(\omega)) = \frac{1}{1-\nu}\left(\frac{\varepsilon_t(\omega)h_t(\omega)w_t(\omega)}{P_t}\right)^{1-\nu} - \frac{\phi}{1+\phi}\left(Ch_t(\omega)\right)^{\frac{1+\phi}{\phi}} \tag{A-164}$$

where similarly to equation (1), object C is a constant that neutralises the love of employer variety effect. In what follows, we drop t subscripts from variables to reduce notational clutter. Now the first order condition for $h_t(\omega)$ gives

$$h(\omega) = \frac{1}{C^{\frac{1+\phi}{1+\nu\phi}}}\left(\frac{\varepsilon(\omega)w(\omega)}{P}\right)^{\frac{(1-\nu)\phi}{1+\nu\phi}} \tag{A-165}$$

which can be substituted into (A-164) to obtain the utility from working at firm ω

$$U(\omega) = \left(\frac{1}{1-\nu} - \frac{\phi}{1+\phi}\right)\left(\frac{\varepsilon(\omega)w(\omega)}{CP}\right)^{\frac{(1-\nu)(1+\phi)}{1+\nu\phi}}. \tag{A-166}$$

Therefore, a worker will work for the firm yielding highest $\varepsilon(\omega)w(\omega)$ and hence the probability of working for firm ω is again

$$P(\omega) = \frac{1}{\Upsilon_t} = \frac{w(\omega)^{(1+\theta)}}{\int w(k)^{(1+\theta)}dk}. \tag{A-167}$$

Now the total labour supplied by a worker to firm ω in efficiency units is $h'(\omega) = \varepsilon(\omega)h(\omega)$. From (A-165) this is equal to $\left(\frac{w(\omega)}{P}\right)^{\frac{(1-\nu)\phi}{1+\nu\phi}}\left(\frac{\varepsilon(\omega)}{C}\right)^{\frac{1+\phi}{1+\nu\phi}}$. Therefore

$$Eh' = \left(\frac{w(\omega)}{P}\right)^{\frac{(1-\nu)\phi}{1+\nu\phi}}\frac{1}{C^{\frac{1+\phi}{1+\nu\phi}}}E\left(\varepsilon(\omega)^{\frac{1+\phi}{1+\nu\phi}}\middle| U(\omega) > \max_k U(k)\right). \tag{A-168}$$

Next, we calculate $E\left(\varepsilon(\omega)^{\frac{1+\phi}{1+\nu\phi}}\middle|U(\omega)>\max_k U(k)\right)$ using the same steps as in the derivation of (A-12) in the baseline risk neutral case earlier and obtain

$$E\left(\varepsilon(\omega)^{\frac{1+\phi}{1+\nu\phi}}\middle|U(\omega)>\max_k U(k)\right)=\Upsilon^{\frac{1+\phi}{(1+\theta)(1+\nu\phi)}}\Gamma\left(\frac{(\theta-\phi)+\nu\phi(1+\theta)}{(1+\theta)(1+\nu\phi)}\right) \tag{A-169}$$

where Γ is the gamma function. If there are $\overline{L}$ workers, then the labour supply function facing firm ω offering a wage $w(\omega)$ is

$$n(\omega)=\overline{L}P(\omega)E\left[h'(\omega)\middle|U(\omega)>\max U(k)\right] \tag{A-170}$$

$$=\left(\frac{w(\omega)}{P}\right)^{\frac{(1-\nu)\phi}{1+\nu\phi}}\frac{P(\omega)}{C^{\frac{1+\phi}{1+\nu\phi}}}\Upsilon^{\frac{1+\phi}{(1+\theta)(1+\nu\phi)}}\Gamma\left(\frac{(\theta-\phi)+\nu\phi(1+\theta)}{(1+\theta)(1+\nu\phi)}\right)\overline{L}. \tag{A-171}$$

Simplify the above to obtain

$$n(\omega)=\left(\frac{1}{P}\right)^{\frac{(1-\nu)\phi}{1+\nu\phi}}\frac{1}{C^{\frac{1+\phi}{1+\nu\phi}}}\left(\frac{w(\omega)^{\theta}}{\left(\int w(k)^{(1+\theta)}dk\right)^{\frac{(\theta-\phi)+\nu\phi(1+\theta)}{(1+\theta)(1+\nu\phi)}}}\right)\Gamma\left(\frac{(\theta-\phi)+\nu\phi(1+\theta)}{(1+\theta)(1+\nu\phi)}\right). \tag{A-172}$$

Therefore, the elasticity of labour supply facing a firm remains θ as before. Next, the welfare of an agent who has received a productivity shock $\varepsilon(\omega)$ and works for firm ω is given in (A-166). However, a worker works for firm ω iff $U(\omega)>\max_k U(k)$. Therefore, the expected maximised utility from working for firm ω upon using (A-166) is

$$EU(\omega)=\left(\frac{1}{1-\nu}-\frac{\phi}{1+\phi}\right)\left(\frac{w(\omega)}{CP}\right)^{\frac{(1-\nu)(1+\phi)}{1+\nu\phi}}E\left[\varepsilon(\omega)^{\frac{(1-\nu)(1+\phi)}{1+\nu\phi}}\middle|U(\omega)>\max_k U(k)\right]. \tag{A-173}$$

Following the same steps as before, obtain

$$E\left(\varepsilon(\omega)^{\frac{(1-\nu)(1+\phi)}{1+\nu\phi}}\middle|U(\omega)>\max_k U(k)\right)=\Upsilon^{\frac{(1-\nu)(1+\phi)}{(1+\theta)(1+\nu\phi)}}\Gamma\left(1-\frac{(1-\nu)(1+\phi)}{(1+\theta)(1+\nu\phi)}\right). \tag{A-174}$$

Therefore, expected utility is

$$EU(\omega)=\left(\frac{1}{1-\nu}-\frac{\phi}{1+\phi}\right)\left(\frac{w(\omega)}{CP}\right)^{\frac{(1-\nu)(1+\phi)}{1+\nu\phi}}\Upsilon^{\frac{(1-\nu)(1+\phi)}{(1+\theta)(1+\nu\phi)}}\Gamma\left(1-\frac{(1-\nu)(1+\phi)}{(1+\theta)(1+\nu\phi)}\right). \tag{A-175}$$

Upon using the expression for Υ re-write the above as

$$EU=\left(\frac{1}{1-\nu}-\frac{\phi}{1+\phi}\right)\left(\frac{1}{CP}\right)^{\frac{(1-\nu)(1+\phi)}{1+\nu\phi}}\left(\int w(k)^{(1+\theta)}dk\right)^{\frac{(1-\nu)(1+\phi)}{(1+\theta)(1+\nu\phi)}}\Gamma\left(1-\frac{(1-\nu)(1+\phi)}{(1+\theta)(1+\nu\phi)}\right). \tag{A-176}$$

Again we can use $C = \left(\Omega^P\right)^{\frac{1}{1+\theta}}$ to kill love of employer variety, and use the following expression for W

$$W = \left(\Omega^P\right)^{-\frac{1}{1+\theta}} \left(\int_{k\in\Omega^P} w(k)^{1+\theta} dk\right)^{\frac{1}{1+\theta}}. \tag{A-177}$$

Therefore, the expected utility is

$$EU = \left(\frac{1+\phi\nu}{(1-\nu)(1+\phi)}\right)\left(\frac{W}{P}\right)^{\frac{(1-\nu)(1+\phi)}{(1+\nu\phi)}} \Gamma\left(1 - \frac{(1-\nu)(1+\phi)}{(1+\theta)(1+\nu\phi)}\right). \tag{A-178}$$

Reduced-form solution. Recall from problem (A-163) that we define labour disutility as

$$N = \left(\Omega_t^P\right)^{\frac{1}{1+\theta}} \left(\int_{\omega\in\Omega} n(\omega)^{\frac{1+\theta}{\theta}} d\omega\right)^{\frac{\theta}{1+\theta}}. \tag{A-179}$$

The household problem in the absence of non-labour income is

$$\max \frac{1}{1-\nu}\left(\int w(\omega)n(\omega)\right)^{1-\nu} - \frac{N^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}$$

where recall that $P = 1$ as the numeraire. This has first order condition

$$(X)^{-\nu} w(\omega) = N^{\frac{1}{\phi}} \left(\Omega_t^P\right)^{\frac{1}{1+\theta}} \left(\int_{\omega\in\Omega} n(\omega)^{\frac{1+\theta}{\theta}} d\omega\right)^{\frac{\theta}{1+\theta}-1} n(\omega)^{\frac{1}{\theta}}, \tag{A-180}$$

where note that we've used the budget constraint with only labour income of $X = \int w(\omega)n(\omega)$ for consumption denoted by X. Re-write above as

$$(X)^{-\nu} w(\omega) = N^{\frac{1}{\phi}-\frac{1}{\theta}} \left(\Omega_t^P\right)^{\frac{1}{\theta}} n(\omega)^{\frac{1}{\theta}} \tag{A-181}$$

giving

$$n(\omega) = \frac{N^{\frac{\phi-\theta}{\phi}} (X)^{-\nu\theta}}{\left(\Omega_t^P\right)} w(\omega)^{\theta} \tag{A-182}$$

Using (A-182) in (A-179) obtain

$$N^{\frac{\theta}{\phi}} = (X)^{-\nu\theta} \left(\Omega_t^P\right)^{\frac{-\theta}{(1+\theta)}} \left(\int_{\omega\in\Omega} w(\omega)^{1+\theta} d\omega\right)^{\frac{\theta}{1+\theta}}. \tag{A-183}$$

Next, note that

$$W = \left(\Omega_t^P\right)^{-\frac{1}{1+\theta}} \left(\int w(\omega)^{1+\theta}\right)^{\frac{1}{1+\theta}}. \tag{A-184}$$

Using above re-write (A-183) as

$$N^{\frac{1}{\phi}} = (X)^{-\nu} W. \tag{A-185}$$

Next, note that

$$X = \int w(\omega)n(\omega) = \frac{N^{\frac{\phi-\theta}{\phi}} (X)^{-\nu\theta}}{(\Omega_t^P)} \left(\int w(\omega)^{1+\theta} \right) \tag{A-186}$$

implies

$$X^{1+\nu\theta} = W^{1+\theta} N^{\frac{\phi-\theta}{\phi}}. \tag{A-187}$$

Solve (A-185) and (A-187) to obtain

$$N = W^{\frac{(1-\nu)\phi}{1+\nu\phi}}. \tag{A-188}$$

Therefore,

$$N^{\frac{\phi-\theta}{\phi}} (X)^{-\nu\theta} = \frac{N}{W^{\theta}} = W^{\frac{(1-\nu)\phi-\theta(1+\nu\phi)}{1+\nu\phi}}. \tag{A-189}$$

Hence, labour supply in (A-182) is

$$n(\omega) = \frac{W^{\frac{(1-\nu)\phi-\theta(1+\nu\phi)}{1+\nu\phi}} w(\omega)^{\theta}}{\Omega_t^P}. \tag{A-190}$$

Now, welfare in the reduced form model is given by

$$EU = \frac{1}{1-\nu} (X)^{1-\nu} - \frac{N^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}. \tag{A-191}$$

Verify using (A-187) and (A-188) that $X = WN$. Using this along with (A-188) in (A-192) obtain

$$EU = \left(\frac{1+\phi\nu}{(1-\nu)(1+\phi)} \right) W^{\frac{(1-\nu)(1+\phi)}{1+\nu\phi}}. \tag{A-192}$$

Verify below that labour supply and welfare above differ from that in the microfounded model by a constant. Using the definition of W write the labour supply in (A-172) above as

$$n(\omega) = \frac{W^{\frac{(1-\nu)\phi-\theta(1+\nu\phi)}{1+\nu\phi}} w(\omega)^{\theta}}{\Omega_t^P} \Gamma \tag{A-193}$$

And welfare in (A-178) can be written as (recall that $P = 1$)

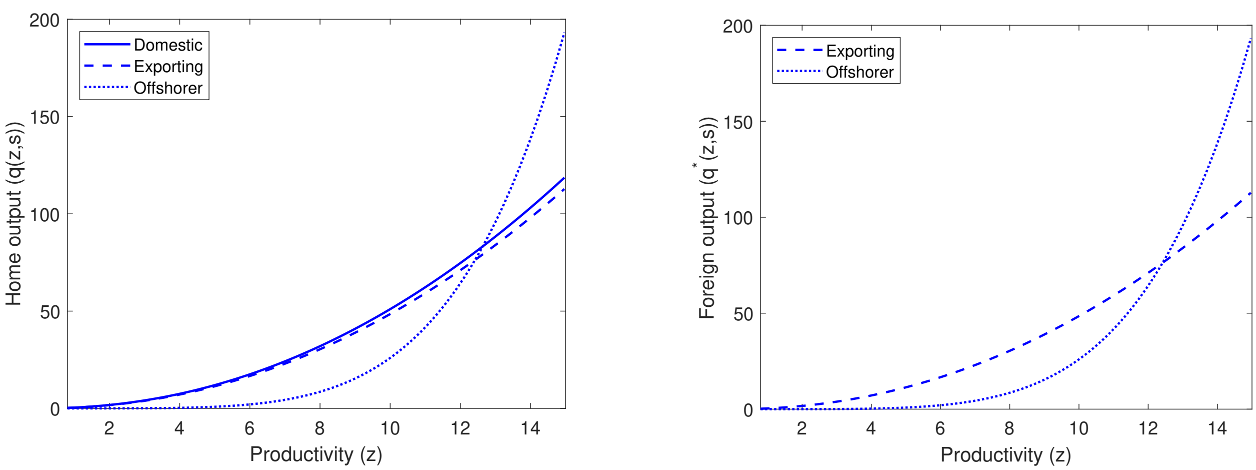
$$EU = \left(\frac{1+\phi\nu}{(1-\nu)(1+\phi)} \right) W^{\frac{(1-\nu)(1+\phi)}{(1+\nu\phi)}} \Gamma. \tag{A-194}$$

Observe that the labour supply in the reduced form case in (A-190) differs from that in the microfounded case in (A-193) by Γ, as does the welfare in the reduced form case in (A-192) from that in the microfounded case in (A-194).

C.4 Offshoring

Here we consider a model extension where firms have the option to undertake all of their production in a foreign country, then ship some of their goods back to their home market. To give the simplest possible treatment, here firms have four possible discrete choices conditional on state — exit, domestic ($s = D$), exporter ($s = X$) or offshorer ($s = O$). A firm that offshores pays an additional set of fixed costs to relocate all production to F; these firms have the interpretation of being multinationals in the vertical sense. We assume variable iceberg costs of exporting (τ^X) and an efficiency loss in the foreign subsidiary (τ^O similarly to τ^M), as in the baseline exercises. When a firm headquartered in H exports goods from F back to H, it will incur both of these variable cost inefficiencies.

To study this issue, there must be asymmetries across the two countries H and F, such that H firms find relocating all production to F favourable from a cost perspective. We consider a scenario whereby all features are held constant, except for the firm-level labour supply elasticity θ. We study the problem from the perspective of H firms and assume that the H value of θ remains at 1.88 as in the baseline quantitative exercises, but assume that in F, firms face perfect labour markets with $\theta = \infty$.



(a) Steady-state H output policy functions

(b) Steady-state F output policy functions

Figure A-2: Offshoring model output policy functions

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