Monopsonistic Labor Markets and International Trade

Appendix — For Online Publication

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A Decomposing Expected Maximized Utility

This section shows how to obtain equation (20) for the expected maximized utility of workers. Using the expression for $w(\varphi)$ in (5) and $g(\varphi|\varphi \geq \hat{\varphi}) = \frac{k\hat{\varphi}^k}{\varphi k+1}$, we obtain that

$$\int_{\hat{\varphi}}^{\infty} w(\varphi)^\theta g(\varphi|\varphi \geq \hat{\varphi}) d\varphi = \left(1 + \frac{\theta}{\gamma}\right) \hat{w}^\theta = \left(1 + \frac{\theta}{\gamma}\right)\left(1 - \frac{1}{\gamma}\right)^\theta \bar{w}^\theta, \tag{A-1}$$

where the second equality uses that $\bar{w} = \frac{\gamma \hat{w}}{\gamma - 1}$ (see section 5.1). Plugging in (A-1) into (4), we rewrite $\mathbb{U}$ as

$$\mathbb{U} \equiv \ln \bar{w} + \frac{1}{\theta} \ln M + \mathcal{D},$$

where $\mathcal{D} \equiv \frac{1}{\theta} \ln \left[\left(1 + \frac{\theta}{\gamma}\right)\left(1 - \frac{1}{\gamma}\right)^\theta\right]$.

The term $\mathcal{D}$ is negative, taking the value of $\frac{\sigma-1}{k\sigma} + \ln\left(1 - \frac{\sigma-1}{k\sigma}\right) < 0$ for $\theta = 0$, and approaching 0 as $\theta \to \infty$. We know that $\frac{\sigma-1}{k\sigma} + \ln\left(1 - \frac{\sigma-1}{k\sigma}\right) < 0$ because of the natural log inequality that says that $x < -\ln(1-x)$ for $x < 1$ and $x \neq 0$. To show that $\mathcal{D}$ is negative when $\theta > 0$, it is sufficient to show that $\left(1 + \frac{\theta}{\gamma}\right)\left(1 - \frac{1}{\gamma}\right)^\theta < 1$, which is true if $\frac{1}{\theta} \ln\left(1 + \frac{\theta}{\gamma}\right) < -\ln\left(1 - \frac{1}{\gamma}\right)$. Using natural log inequalities we know that $\frac{1}{\theta} \ln\left(1 + \frac{\theta}{\gamma}\right) < \frac{1}{\gamma}$ (because $\ln(1+y) < y$ for $y > -1$ and $y \neq 0$) and that $\frac{1}{\gamma} < -\ln\left(1 - \frac{1}{\gamma}\right)$ (because $x < -\ln(1-x)$ for $x < 1$ and $x \neq 0$), which then shows that $\frac{1}{\theta} \ln\left(1 + \frac{\theta}{\sigma}\right) < -\ln\left(1 - \frac{1}{\gamma}\right)$. We also know from above that $\frac{d\gamma}{dk} > 0$ and $\frac{d\gamma}{d\sigma} < 0$, and thus $\mathcal{D}$ is increasing in k and decreasing in σ.

We can also write $\mathbb{U}$ in terms of the labor supply shifter, $\mathbb{B}$. From section 4.2, $\mathbb{B}$ is defined as $\mathbb{B} \equiv \left(M \int_{\hat{\varphi}}^{\infty} w(\varphi)^\theta g(\varphi|\varphi \geq \hat{\varphi}) d\varphi\right)^{-1} \mathbb{L}$, and thus, we can rewrite (4) as

$$\mathbb{U} \equiv \frac{1}{\theta} \ln \left(\frac{\mathbb{L}}{\mathbb{B}}\right).$$

Therefore, $\mathbb{U}$ is positively related with $\mathbb{L}$ and negatively related with the labor-supply shifter, $\mathbb{B}$.

B Derivation of Equilibrium in the Open Economy

As in the closed-economy model, the average productivity in each country is given by

$$\bar{\varphi} = \left[\int_{\hat{\varphi}_N}^{\infty} \varphi^{\beta} g(\varphi | \varphi \geq \hat{\varphi}_N) d\varphi \right]^{\frac{1}{\beta}} = \left(\frac{k}{k-\beta} \right)^{\frac{1}{\beta}} \hat{\varphi}_N,$$

which is increasing in θ. We can also obtain the average productivity for each type of firm as

$$\bar{\varphi}_N = \left[\frac{k}{k-\beta} \left(\frac{\lambda^k - \lambda^{\beta}}{\lambda^k - 1} \right) \right]^{\frac{1}{\beta}} \hat{\varphi}_N \qquad \text{and} \qquad \bar{\varphi}_T = \left(\frac{k}{k-\beta} \right)^{\frac{1}{\beta}} \hat{\varphi}_T,$$

with the overall average productivity calculated as $\bar{\varphi} = \left[\frac{M_N}{M_P} \bar{\varphi}_N^{\beta} + \frac{M_T}{M_P} \bar{\varphi}_T^{\beta} \right]^{\frac{1}{\beta}}$, where M_s denotes the mass of firms with status $s \in \{N, T\}$ in each country, and $M_P \equiv M_N + M_T$ is the total mass of producers in each country.

The aggregate price P can be conveniently written as

$$\begin{aligned} P &= \left[M_N p_N (\bar{\varphi}_N)^{1-\sigma} + M_T p_D (\bar{\varphi}_T)^{1-\sigma} + M_T p_X (\bar{\varphi}_T)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\ &= \left[M_N p_N (\bar{\varphi}_N)^{1-\sigma} + M_T \left(1 + \tau^{1-\sigma} \right)^{\frac{1+\theta}{\sigma+\theta}} p_N (\bar{\varphi}_T)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \end{aligned}$$

where the second equality uses the relationships between $p_N(\varphi)$, $p_D(\varphi)$, and $p_X(\varphi)$ described in section 6.1. Using the equation for $p_N(\varphi)$ in (7), the expressions for M_N and M_T in terms of M_E, and given our assumption that $P = 1$ (the final good is the numéraire), it follows that the equation for price index above yields

$$\frac{M_E}{\delta} = \left\{ \left[\frac{\sigma (1+\theta)}{(\sigma - 1)\theta} \right]^{\theta} \frac{\mathbb{A}}{\mathbb{B}} \right\}^{\frac{\sigma-1}{\sigma+\theta}} \left\{ \frac{1}{[G(\hat{\varphi}_T) - G(\hat{\varphi}_N)] \bar{\varphi}_N^{\beta} + [1 - G(\hat{\varphi}_T)](1 + \tau^{1-\sigma})^{\frac{1+\theta}{\sigma+\theta}} \bar{\varphi}_T^{\beta}} \right\}, \tag{A-2}$$

with average productivities $\bar{\varphi}_N$ and $\bar{\varphi}_T$ defined above.

Similar to the closed economy case, the aggregate expenditure on final goods in each country is the sum of workers' expenditure on final-good consumption and firms' final-good requirements to cover the fixed and entry costs. In an open economy, consumption expenditure on each country's final good comes from both countries' workers. This difference, however, is inconsequential because exports of a country exactly cancel out with its imports, so that consumption expenditure in each country's final good continues to be equal to the wage bill, $\mathbb{W}$. Therefore, the total expenditure on each country's final good is $\mathbb{A} = \mathbb{W} + M_N f + M_T (f + f_X) + M_E f_E$, which considers that trading firms must also pay the exporting fixed cost, f_X. Using the expressions for M_N and M_T above, we rewrite total expenditure as

$$\mathbb{A} = \frac{M_E}{\delta} \left[\int_{\hat{\varphi}_N}^{\infty} w(\varphi) L(\varphi) g(\varphi) d\varphi + [1 - G(\hat{\varphi}_N)] f + [1 - G(\hat{\varphi}_T)] f_X + \delta f_E \right]. \tag{A-3}$$

where

$$L(\varphi)=\begin{cases}L_N(\varphi) \text{ if } \varphi\in[\hat\varphi_N,\hat\varphi_T)\\ L_T(\varphi)\equiv L_D(\varphi)+L_X(\varphi) \text{ if } \varphi\geq\hat\varphi_T\end{cases} \quad \text{and} \quad w(\varphi)=\begin{cases}w_N(\varphi) \text{ if } \varphi\in[\hat\varphi_N,\hat\varphi_T)\\ w_T(\varphi) \text{ if } \varphi\geq\hat\varphi_T,\end{cases} \tag{A-4}$$

with $L_s(\varphi)$, $L_r(\varphi)$, and $w_s(\varphi)$ for $s\in\{N,T\}$ and $r\in\{D,X\}$ described as in section 6.1.

Finally, the labor-supply shifter, $\mathbb{B}\equiv\left[\int_{\nu\in\Omega}w(\nu)^\theta d\nu\right]^{-1}\mathbb{L}$, of the firm-level labor supply in (3) can also be written in terms of M_E as

$$\mathbb{B}=\left[\frac{M_E}{\delta}\int_{\hat\varphi_N}^{\infty}w(\varphi)^\theta g(\varphi)d\varphi\right]^{-1}\mathbb{L}. \tag{A-5}$$

We can now define the equilibrium in the open-economy model.

Definition. *An open-economy equilibrium solves for $\hat\varphi_N$ and $\hat\varphi_T$ from (21) and (23), and then solves for M_E, $\mathbb{A}$, and $\mathbb{B}$ from (A-2), (A-3), and (A-5), with $L(\varphi)$ and $w(\varphi)$ defined as in (A-4).*

The equilibrium expressions for M_E, $\mathbb{A}$, and $\mathbb{B}$ in the open-economy case are reported in section 6.3.

C Proofs

Proof of Lemma 2. From the definition of Ψ in (26), we obtain that the elasticity of Ψ with respect to τ is given by $\zeta_{\Psi,\tau}=\zeta_{\Psi,\lambda}\times\zeta_{\lambda,\tau}$. From Lemma 1 we know that $\zeta_{\lambda,\tau}>0$ and thus, the sign of $\zeta_{\Psi,\tau}$ is given by the sign of $\zeta_{\Psi,\lambda}$. After using (24) to rewrite (26) as

$$\Psi\equiv\frac{(\mathcal{F}+\lambda^k)^{\frac{k+1}{k}}}{\lambda\left[\lambda^k+(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}}-\lambda^{\frac{\beta\theta}{1+\theta}}\right]},$$

we obtain

$$\zeta_{\Psi,\lambda}=-\left[\frac{1}{\lambda^k+(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}}-\lambda^{\frac{\beta\theta}{1+\theta}}}\right]\Bigg\{\underbrace{\left(\frac{k\lambda^k-\mathcal{F}}{\mathcal{F}+\lambda^k}\right)}_{>0\ \text{(Term 1)}}\underbrace{\left[\mathcal{F}-(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}}+\lambda^{\frac{\beta\theta}{1+\theta}}\right]}_{>0\ \text{(Term 2)}}$$
$$+\mathcal{F}\left(1-\underbrace{\frac{1}{\lambda^{\frac{\beta}{1+\theta}}}\left[\frac{(\sigma-1)\theta}{(\sigma+\theta)(1+\theta)}\right]}_{<1\ \text{(Term 3)}}\underbrace{\frac{(1+\theta)\lambda^\beta}{\mathcal{F}}\left[1-\left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)^{\frac{1}{1+\theta}}\right]}_{<1\ \text{(Term 4)}}\right)\Bigg\}<0. \tag{A-6}$$

For term 1, note first from (21) that in $\lambda^\beta=\frac{\mathcal{F}}{(1+\tau^{1-\sigma})^{(1+\theta)/(\sigma+\theta)}-1}$, the denominator in the right-hand side is less than one because $1+\tau^{1-\sigma}<2$ and $\frac{1+\theta}{\sigma+\theta}<1$. It follows that $\lambda^\beta>\mathcal{F}$, and thus $k\lambda^k>\mathcal{F}$ because $\lambda\geq1$, $k>\beta$, and $k>2$. Hence, term 1 is positive. Term 2 is positive iff $\frac{\mathcal{F}}{\lambda^{\beta\theta/(1+\theta)}}+1>\left(\frac{\mathcal{F}}{\lambda^\beta}+1\right)^{\theta/(1+\theta)}$, which is always true because $\lambda^\beta\geq1$ and $\frac{\theta}{1+\theta}<1$, so that $\lambda^{\beta\theta/(1+\theta)}\leq\lambda^\beta$ and $\frac{\mathcal{F}}{\lambda^\beta}+1>\left(\frac{\mathcal{F}}{\lambda^\beta}+1\right)^{\theta/(1+\theta)}$. Term 3 is less that one because $\lambda^{\beta/(1+\theta)}>1$, $\frac{\sigma-1}{\sigma+\theta}<1$ and $\frac{\theta}{1+\theta}<1$.

For term 4, let us rewrite it as $(1+\theta)z\left[1-\left(\frac{z}{1+z}\right)^{1/(1+\theta)}\right]$, where $z \equiv \frac{\lambda^\beta}{\mathcal{F}}$ is positive, strictly increasing in τ, and going to infinity as $\tau \to \infty$. Applying L'Hôpital's rule we obtain that for every σ and θ, $\lim_{\tau\to\infty}(1+\theta)z\left[1-\left(\frac{z}{1+z}\right)^{1/(1+\theta)}\right] = 1$, and thus, a sufficient condition for term 4 to be less than one is that $(1+\theta)z\left[1-\left(\frac{z}{1+z}\right)^{1/(1+\theta)}\right]$ is increasing in τ.[1] Taking the derivative, we obtain that it is greater than zero iff $\frac{1}{1+z} < (1+\theta)\left[\left(\frac{1}{z}+1\right)^{1/(1+\theta)}-1\right]$. Using the natural log inequality that says that $\ln x \leq n(x^{1/n}-1)$ for $n>0$ and $x>0$, it follows that $\ln\left(\frac{1}{z}+1\right) \leq (1+\theta)\left[\left(\frac{1}{z}+1\right)^{1/(1+\theta)}-1\right]$. Lastly, from the natural log inequality that says that $\frac{1}{x+1} < \ln(x+1)$ for $x>-1$ and $x \neq 0$, it follows that $\frac{1}{1+z} < \ln\left(\frac{1}{z}+1\right)$, and therefore, the derivative is positive and term 4 is less than one.

For the elasticity of Ψ with respect to θ, $\zeta_{\Psi,\theta}$, we obtain

$$\zeta_{\Psi,\theta} = \frac{\theta}{(1+\theta)(\sigma+\theta)}\Bigg\{\underbrace{\frac{\sigma(1+\theta)(\lambda^\beta-\lambda^{\frac{\beta\theta}{1+\theta}})\ln(1+\tau^{1-\sigma})}{(\sigma+\theta)(\mathcal{F}+\lambda^k)}}_{\geq 0\ \text{(Term 1)}} + \underbrace{\left[\frac{1}{\lambda^k+(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}}-\lambda^{\frac{\beta\theta}{1+\theta}}} - \frac{1}{\mathcal{F}+\lambda^k}\right]}_{>0\ \text{(Term 2)}}\times$$

$$\left[\ln(1+\tau^{1-\sigma})\Big[\underbrace{(k+1)\lambda^k - \frac{\sigma(1+\theta)\lambda^{\frac{\beta\theta}{1+\theta}}}{\sigma+\theta}}_{>0\ \text{(Term 3)}} + \underbrace{\left(k-\frac{\beta\theta}{1+\theta}\right)\frac{\lambda^{k+\beta}}{\mathcal{F}}}_{>0\ \text{(Term 4)}}\Big] + (\sigma-1)\ln\lambda(k+1)\lambda^k\right]\Bigg\} > 0.$$

Term 1 is greater or equal than 0 because $\lambda^\beta \geq 1$ and $\frac{\theta}{1+\theta} < 1$, and then, $\lambda^\beta \geq \lambda^{\frac{\beta\theta}{1+\theta}}$. Term 2 is positive iff $\mathcal{F} - (\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}} + \lambda^{\frac{\beta\theta}{1+\theta}} > 0$, which we know it is true from term 2 in the expression for $\zeta_{\Psi,\lambda}$ above. Term 3 is positive because $k > \sigma-1$, $\frac{1+\theta}{\sigma+\theta} < 1$, and $\lambda^k \geq \lambda^{\frac{\beta\theta}{1+\theta}}$—the latter follows because $\lambda \geq 1$ and $k > \frac{\beta\theta}{1+\theta}$ (recall that $k > \beta$). Lastly, term 4 is positive because $k > \frac{\beta\theta}{1+\theta}$.

We now have to show that if $k > \frac{\sigma-1}{\sigma-2}$, for every τ there is a unique level of θ, $\hat{\theta}$, such that $\zeta_{M_P,\tau} < 0$ if $\theta < \hat{\theta}$ and $\zeta_{M_P,\tau} > 0$ if $\theta > \hat{\theta}$; otherwise, $\zeta_{M_P,\tau} < 0$ for every τ and θ. We know that $\zeta_{M_P,\tau} = \left(\frac{\sigma-1}{\sigma-2}\right)\zeta_{\Psi,\tau} - k\zeta_{\hat{\varphi}_N,\tau}$, and thus, using $\zeta_{\hat{\varphi}_N,\tau} = -\left(\frac{\mathcal{F}}{\mathcal{F}+\lambda^k}\right)\zeta_{\lambda,\tau}$ and (A-6), and rearranging terms we obtain

$$\zeta_{M_P,\tau} = \underbrace{\zeta_{\lambda,\tau}\left[\frac{k(\sigma-1)\mathcal{F}\lambda^k}{(\sigma-2)(\mathcal{F}+\lambda^k)(\lambda^k+(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}}-\lambda^{\frac{\beta\theta}{1+\theta}})}\right]}_{>0\ \text{(Term 1)}}\Bigg\{\underbrace{\left[k-\frac{\sigma-1}{\sigma-2}\right]\left[\frac{\sigma-2}{k(\sigma-1)}\right]\left[1+\frac{(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}}-\lambda^{\frac{\beta\theta}{1+\theta}}}{\lambda^k}\right]}_{>0\ \text{if}\ k>\frac{\sigma-1}{\sigma-2};\ \leq 0\ \text{otherwise (Term 2)}}$$

$$\underbrace{-\frac{\lambda^{\frac{\beta\theta}{1+\theta}}}{\mathcal{F}}\left[\left(\frac{\mathcal{F}}{\lambda^{\frac{\beta\theta}{1+\theta}}}+1-\frac{(\sigma-1)\theta}{k(\sigma+\theta)}\left(\frac{\mathcal{F}}{\lambda^k}+1\right)\right)-\left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)^{\frac{1}{1+\theta}}\left(\frac{\mathcal{F}}{\lambda^\beta}+1-\frac{(\sigma-1)\theta}{k(\sigma+\theta)}\left(\frac{\mathcal{F}}{\lambda^k}+1\right)\right)\right]}_{<0\ \text{(Term 3)}}\Bigg\}.$$

Term 1 is positive because $\sigma > 2$, $(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}} > \lambda^{\frac{\beta\theta}{1+\theta}}$, and $\zeta_{\lambda,\tau} > 0$ (see Lemma 1). Thus, the sign of $\zeta_{M_P,\tau}$ is determined by the sum of Terms 2 and 3. Note that Term 2 is positive if and only $k > \frac{\sigma-1}{\sigma-2}$; otherwise, it less or equal than zero. For the sign of Term 3, note first that $\lambda^k > \lambda^\beta > \lambda^{\frac{\beta\theta}{1+\theta}}$, and thus, $\frac{\mathcal{F}}{\lambda^{\beta\theta/(1+\theta)}}+1 > \frac{\mathcal{F}}{\lambda^\beta}+1 > \frac{\mathcal{F}}{\lambda^k}+1$, and given that $\frac{(\sigma-1)\theta}{k(\sigma+\theta)} < 1$ and $\left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)^{\frac{1}{1+\theta}} < 1$, it follows that the

[1] Applying L'Hôpital's rule, the limit of term 4 when $\theta \to \infty$ is $\frac{\ln(1+\tau^{1-\sigma})}{\tau^{1-\sigma}}$, which is less or equal than one—this follows from the natural log inequality that says that $\ln x \leq x-1$ for $x>0$. Of course, it is the case that $\lim_{\tau\to\infty}\frac{\ln(1+\tau^{1-\sigma})}{\tau^{1-\sigma}} = 1$.

term between brackets is positive, which then implies that Term 3 is negative. Therefore, if $k \leq \frac{\sigma-1}{\sigma-2}$, the sum of Terms 2 and 3 is always negative, and hence $\zeta_{M_P,\tau} < 0$.

For the $k > \frac{\sigma-1}{\sigma-2}$ case, note first that if $\theta \to 0$, then $\zeta_{M_P,\tau} \to -\frac{(k+\sigma-1)\mathcal{F}\zeta_{\lambda,\tau}}{(\sigma-2)(\mathcal{F}+\lambda^k)} < 0$, whereas if $\theta \to \infty$, then $\zeta_{M_P,\tau} \to \frac{\mathcal{F}}{\mathcal{F}+\mathcal{F}^{k/(\sigma-1)}\tau^k}\left(k - \frac{\sigma-1}{\sigma-2}\right) > 0$. Therefore, for every τ there exists at least one value of θ such that $\zeta_{M_P,\tau} = 0$. To show uniqueness—so that for every τ there exists only one value of θ, $\hat{\theta}$, such that $\zeta_{M_P,\tau} < 0$ if $\theta < \hat{\theta}$ and $\zeta_{M_P,\tau} > 0$ if $\theta > \hat{\theta}$—it is sufficient to show that the sum of Terms 2 and 3 is increasing in θ. Here we show that Terms 2 and 3 are both increasing in θ, and therefore, their sum is also increasing in θ. For Term 2, we get

$$\frac{d\text{Term 2}}{d\theta} = \underbrace{\left[k - \frac{\sigma-1}{\sigma-2}\right]\left[\frac{\sigma-2}{k(\sigma-1)}\right]\frac{\Lambda}{\theta\lambda^k}}_{>0}\{\zeta_{\Lambda,\theta} \underbrace{-k\zeta_{\lambda,\theta}}_{>0}\},$$

where

$$\Lambda \equiv (\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}} - \lambda^{\frac{\beta\theta}{1+\theta}} > 0 \tag{A-7}$$

and $\zeta_{\Lambda,\theta} \equiv \frac{d\ln\Lambda}{d\ln\theta}$. The second component in the term within braces is positive because $\zeta_{\lambda,\theta} < 0$ (see Lemma 1). For $\zeta_{\Lambda,\theta}$ we obtain

$$\zeta_{\Lambda,\theta} = \frac{\theta}{(1+\theta)^2}\left\{\ln(\mathcal{F}+\lambda^\beta) + \left(\frac{1}{\sigma+\theta}\right)\ln\left(1+\frac{\mathcal{F}}{\lambda^\beta}\right)\left[\frac{\sigma(1+\theta)\lambda^{\frac{\beta\theta}{1+\theta}}}{\Lambda} - \frac{(\sigma-1)\theta\lambda^\beta}{\mathcal{F}}\right]\right\} > 0, \tag{A-8}$$

where all terms are positive because $\lambda^\beta > 1$, $\Lambda > 0$, and for the term in brackets $\sigma(1+\theta) > (\sigma-1)\theta$ and $\frac{\lambda^{\frac{\beta\theta}{1+\theta}}}{\Lambda} > \frac{\lambda^\beta}{\mathcal{F}}$ — the latter follows from $1+\frac{\mathcal{F}}{\lambda^\beta} > \left(1+\frac{\mathcal{F}}{\lambda^\beta}\right)^{\frac{\theta}{1+\theta}}$. Thus, Term 2 is increasing in θ. For Term 3, we get

$$\begin{aligned}\frac{d\text{Term 3}}{d\theta} = &\underbrace{-\frac{d\lambda^\beta}{d\theta}}_{>0}\left[\frac{1}{\beta(\mathcal{F}+\lambda^\beta)^{\frac{1}{1+\theta}}}\right]\left\{\frac{(\sigma-1)\theta}{\sigma+\theta}\left(1+\frac{\mathcal{F}}{\lambda^k}\right)\underbrace{\left(1-\frac{(\sigma-1)\theta}{k(\sigma+\theta)}\right)}_{>0}\left[\left(1+\frac{\mathcal{F}}{\lambda^\beta}\right)^{\frac{1}{1+\theta}} - 1\right]\right.\\
&\left.+\frac{\mathcal{F}}{\mathcal{F}+\lambda^\beta}\underbrace{\left[\left(1+\frac{\mathcal{F}}{\lambda^\beta}\right) - \left(\frac{(\sigma-1)\theta(1+\theta)}{k(\sigma+\theta)^2}\right)\left(1+\frac{\mathcal{F}}{\lambda^k}\right)\right]}_{>0}\right\} + \frac{(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}}\beta\ln\lambda}{(1+\theta)^2}\times\\
&\left\{1-\left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)^{\frac{\theta}{1+\theta}} + \frac{(\sigma-1)\theta}{k(\sigma+\theta)}\left[1+(k+1)\frac{\mathcal{F}}{\lambda^k}\right]\underbrace{\left[\left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)^{\frac{\theta}{1+\theta}} - \left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)\right]}_{>0}\right\}\\
&+\frac{\sigma(\sigma-1)}{k(\sigma+\theta)^2}\left[\left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)^{\frac{\theta}{1+\theta}} - \left(\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta}\right)\right]\left(1+\frac{\mathcal{F}}{\lambda^k}\right)(\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}} > 0,\end{aligned}$$

where $\frac{d\lambda^\beta}{d\theta} = -\frac{\beta\lambda^\beta}{(1+\theta)^2}\left(\frac{\mathcal{F}+\lambda^\beta}{\mathcal{F}}\right)\ln\left(\frac{\mathcal{F}+\lambda^\beta}{\lambda^\beta}\right) < 0$. All the terms in $\frac{d\text{Term 3}}{d\theta}$ were arranged so that it is straightforward to confirm their positive sign. For the three terms that have an underbrace, the first is positive because $\sigma - 1 < k$ and $\theta < \sigma + \theta$, the second is positive because $\frac{\mathcal{F}}{\lambda^\beta} > \frac{\mathcal{F}}{\lambda^k}$ and $(\sigma-1)\theta(1+\theta) < k(\sigma+\theta)^2$, and the third is positive because $\frac{\lambda^\beta}{\mathcal{F}+\lambda^\beta} < 1$ and $\frac{\theta}{1+\theta} < 1$. Therefore, Term 3 is increasing in θ. □

Proof of Proposition 1. Note that if $\lambda = 1$ then $\mu_T = 1$, which then implies that $\tilde{\mathcal{G}} = \mathcal{G}$ in (28). If $\tau \to \infty$, we know from the definition of λ in (21) that $\lambda \to \infty$, which then makes $\frac{2(\gamma-1)(1-\mu_T)\mathcal{F}}{\mathcal{F}+\lambda^k} \to 0$ in (28), and thus $\tilde{\mathcal{G}} \to \mathcal{G}$. When $\lambda > 1$, for $\tilde{\mathcal{G}} > \mathcal{G}$ to hold it is sufficient to show that the term in braces in (28) is between 0 and 1, which follows from $\frac{\lambda^\beta}{\mathcal{F}}\left[\left(\frac{\mathcal{F}}{\lambda^\beta}+1\right)^{\theta/(1+\theta)} - 1\right] \in (0,1)$ and $\frac{\gamma(\lambda^{\beta(\gamma-1)/(1+\theta)}-1)}{(\gamma-1)(\lambda^{\beta\gamma/(1+\theta)}-1)} \in (0,1]$. To prove the last part, we show *(i)* that $\frac{\lambda^{\beta\gamma/(1+\theta)}-1}{\gamma} > \frac{\lambda^{\beta(\gamma-1)/(1+\theta)}-1}{\gamma-1}$ if $\lambda > 1$, and *(ii)* that $\frac{\gamma(\lambda^{\beta(\gamma-1)/(1+\theta)}-1)}{(\gamma-1)(\lambda^{\beta\gamma/(1+\theta)}-1)} = 1$ if $\lambda = 1$. For *(i)*, it is sufficient to show that an expression of the type $\frac{x^y-1}{y}$ is strictly increasing in y, which is proved using the log inequality $\ln x^y > \frac{x^y-1}{x^y}$ if $x^y > 0$ and $x^y \neq 1$. For *(ii)*, we use L'Hôpital's rule to show that $\lim_{\lambda\to 1} \frac{\gamma(\lambda^{\beta(\gamma-1)/(1+\theta)}-1)}{(\gamma-1)(\lambda^{\beta\gamma/(1+\theta)}-1)} = 1$. □

Proof of Proposition 2. The proofs for the first and third parts of the proposition are in the main text. For the second part of the proposition, the case of incremental trade liberalization (a reduction in τ)—which depends on the sign of $\frac{d\ln M_P}{d\ln\tau}$—appears in Lemma 2 and is proved above. For the case of moving from autarky to trade, we have to show that if $k \leq \frac{\sigma-1}{\sigma-2}$, then $M_P > M$ for every θ, whereas if $k > \frac{\sigma-1}{\sigma-2}$, for every τ there exists a unique $\hat{\varphi}$ such that $M_P > M$ is $\theta < \hat{\theta}$ and $M_P < M$ is $\theta > \hat{\theta}$. From the main text we know that $M_P = \frac{\tilde{M}_E}{\delta\hat{\varphi}_N^k}$ and $M = \frac{\tilde{M}}{\delta\hat{\varphi}^k}$, and thus, from (24), (25), and (26), we obtain

$$\frac{M_P}{M} = \Psi^{\frac{\sigma-1}{\sigma-2}}\frac{\lambda^k}{\mathcal{F}+\lambda^k} = \underbrace{\left[\frac{\mathcal{F}+\lambda^k}{\lambda^k + (\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}} - \lambda^{\frac{\beta\theta}{1+\theta}}}\right]^{\frac{\sigma-1}{\sigma-2}}}_{>1}\left(\frac{\lambda^k}{\mathcal{F}+\lambda^k}\right)^{\frac{1}{k}\left(k-\frac{\sigma-1}{\sigma-2}\right)}, \tag{A-9}$$

where the first term is larger than 1 because $\mathcal{F} > (\mathcal{F}+\lambda^\beta)^{\frac{\theta}{1+\theta}} - \lambda^{\frac{\beta\theta}{1+\theta}}$. Notice that the second term is less than 1 if $k > \frac{\sigma-1}{\sigma-2}$, and is greater or equal to 1 if $k \leq \frac{\sigma-1}{\sigma-2}$. Thus, $M_P > M$ if $k \leq \frac{\sigma-1}{\sigma-2}$. For the $k > \frac{\sigma-1}{\sigma-2}$ case, note first that $\frac{M_P}{M} = \!\!\to \left(\frac{\mathcal{F}+\lambda_0^k}{\lambda_0^k}\right)^{\frac{(k+\sigma-1)}{k(\sigma-2)}} > 1$ if $\theta \to 0$ and $\frac{M_P}{M} = \!\!\to \left(\frac{\lambda_\infty^k}{\mathcal{F}+\lambda_\infty^k}\right)^{\frac{1}{k}\left(k-\frac{\sigma-1}{\sigma-2}\right)} < 1$ if $\theta \to \infty$, where $\lambda_0 \equiv \lim_{\theta\to 0}\lambda$ and $\lambda_\infty \equiv \lim_{\theta\to\infty}\lambda$. Thus, for every τ there exists at least one level of $\theta \in (0,\infty)$, which with a slight abuse of notation we also call $\hat{\theta}$, for which $\frac{M_P}{M} = 1$. We now show uniqueness of $\hat{\theta}$. $\frac{M_P}{M}$ is not monotonically decreasing in θ, and therefore, to prove uniqueness we need to show that $\frac{dM_P/M}{d\theta} < 0$ if $\frac{M_P}{M} = 1$, which ensures that $\frac{M_P}{M}$ equals 1 only for one value of θ. We get that

$$\begin{aligned}\frac{dM_P/M}{d\theta} =& \frac{(\sigma-1)}{\theta(\sigma-2)}\left(1+\frac{\mathcal{F}}{\lambda^k}\right)^{\frac{(\sigma-1)(k+1)}{(\sigma-2)k}-1}\left[\frac{\lambda^k}{\lambda^k+\Lambda}\right]^{\frac{\sigma-1}{\sigma-2}}\Bigg\{-k\left[1-\frac{(\sigma-2)}{k(\sigma-1)}\left(k-\frac{\sigma-1}{\sigma-2}\right)\right]\left(\frac{\mathcal{F}\zeta_{\lambda,\theta}}{\mathcal{F}+\lambda^k}\right) \\ &-\left(\frac{\Lambda}{\lambda^k+\Lambda}\right)(\zeta_{\Lambda,\theta} - k\zeta_{\lambda,\theta})\Bigg\},\end{aligned}$$

where Λ and $\zeta_{\Lambda,\theta}$ are defined as in (A-7) and (A-8). From the previous equation, we obtain that $\frac{dM_P/M}{d\theta}$ is less than zero if

$$\underbrace{\left(1-\frac{\zeta_{\Lambda,\theta}}{k\zeta_{\lambda,\theta}}\right)}_{>1}\frac{\Lambda(\mathcal{F}+\lambda^k)}{(\lambda^k+\Lambda)\mathcal{F}} > 1 - \frac{(\sigma-2)}{k(\sigma-1)}\left(k-\frac{\sigma-1}{\sigma-2}\right),$$

where the first term is larger than 1 because $\zeta_{\Lambda,\theta} > 0$ and $\zeta_{\lambda,\theta} < 0$ (see Lemma 1). Hence, a sufficient condition for uniqueness of $\hat{\theta}$ is that $\frac{\Lambda(\mathcal{F}+\lambda^k)}{(\lambda^k+\Lambda)\mathcal{F}} > 1 - \frac{(\sigma-2)}{k(\sigma-1)}\left(k - \frac{\sigma-1}{\sigma-2}\right)$ when $M_P = M$. From (A-9) we obtain that if $\frac{M_P}{M} = 1$, then it must be the case that

$$\frac{\Lambda}{\lambda^k} = \left(1 + \frac{\mathcal{F}}{\lambda^k}\right)^{1-\frac{(\sigma-2)}{k(\sigma-1)}\left(k-\frac{\sigma-1}{\sigma-2}\right)} - 1,$$

which then implies that

$$\frac{\Lambda(\mathcal{F}+\lambda^k)}{(\lambda^k+\Lambda)\mathcal{F}} = \frac{(1+\mathcal{F}/\lambda^k) - (1+\mathcal{F}/\lambda^k)^{\frac{(\sigma-2)}{k(\sigma-1)}\left(k-\frac{\sigma-1}{\sigma-2}\right)}}{\mathcal{F}/\lambda^k}$$

at $\frac{M_P}{M} = 1$. Let $x \equiv 1 + \frac{\mathcal{F}}{\lambda^k} > 1$ and $c \equiv 1 - \frac{(\sigma-2)}{k(\sigma-1)}\left(k - \frac{\sigma-1}{\sigma-2}\right) \in (0,1)$, where $c < 1$ follows from $k > \frac{\sigma-1}{\sigma-2}$ and $c > 0$ follows from $\frac{k(\sigma-2)}{\sigma-1} - 1 < k$. Therefore, the sufficient condition for uniqueness of $\hat{\theta}$ is $\frac{x-x^{1-c}}{x-1} > c$, which can be rewritten as

$$1 - \frac{1}{x^c} > c\left(1 - \frac{1}{x}\right).$$

The previous condition holds because both sides approach zero when $x \to 1$, but the left-hand side grows faster than the right-hand side as x increases: the derivatives with respect to x are respectively given by $\frac{c}{x^{c+1}}$ and $\frac{c}{x^2}$, with $\frac{c}{x^{c+1}} > \frac{c}{x^2}$ because $x > x^c$ ($x > 1$ and $c \in (0,1)$). Thus, if $k > \frac{\sigma-1}{\sigma-2}$, for every τ there is a unique $\hat{\theta}$ such that $M_P > M$ if $\theta < \hat{\theta}$ and $M_P < M$ if $\theta > \hat{\theta}$. □

D The Distribution of Wages in the Open Economy

Each non-trading firm with productivity φ offers wage $w_N(\varphi)$, so that the fraction of workers receiving this wage is $\ell_N(\varphi) \equiv \frac{M_N L_N(\varphi)}{\mathbb{L}}$. With $g(\varphi|\varphi \in [\hat{\varphi}_N, \hat{\varphi}_T))$ denoting the productivity distribution of non-trading active firms, it follows that the productivity-based probability density function of wages is $h_N(\varphi) \equiv \ell_N(\varphi) g(\varphi|\varphi \in [\hat{\varphi}_N, \hat{\varphi}_T))$ for $\varphi \in [\hat{\varphi}_N, \hat{\varphi}_T)$. Similarly, for trading firms the productivity-based probability density function of wages is $h_T(\varphi) \equiv \ell_T(\varphi) g(\varphi|\varphi \geq \hat{\varphi}_T)$ for $\varphi \geq \hat{\varphi}_T$, with $\ell_T(\varphi) \equiv \frac{M_T L_T(\varphi)}{\mathbb{L}}$. Therefore, the average wage across all workers in each country is $\tilde{w} = \int_{\hat{\varphi}_N}^{\infty} w(\varphi)h(\varphi)d\varphi$ where $h(\varphi) = h_N(\varphi)$ if $\varphi \in [\hat{\varphi}_N, \hat{\varphi}_T)$, $h(\varphi) = h_T(\varphi)$ if $\varphi \geq \hat{\varphi}_T$, and $w(\varphi)$ is given by (A-4). Note that we use $\tilde{w}$ to denote the average wage in the open economy, which is convenient for later when we compare it against the closed-economy average wage, $\bar{w}$.

Similar to section 5.1, we apply a change of variables to obtain the direct distribution of wages. Letting $F(w)$ denote the cumulative distribution function of wages, we obtain

$$F(w) = \begin{cases} \mu_N \left(\frac{\lambda^k}{\lambda^k - \lambda^{\beta\theta/(1+\theta)}}\right)\left[1 - \left(\frac{\hat{w}_N}{w}\right)^{\gamma}\right] & \text{if } w \in \left[\hat{w}_N, \lambda^{\frac{\beta}{1+\theta}}\hat{w}_N\right) \\ \mu_N & \text{if } w \in \left[\lambda^{\frac{\beta}{1+\theta}}\hat{w}_N, \hat{w}_T\right) \\ \mu_N + \mu_T\left[1 - \left(\frac{\hat{w}_T}{w}\right)^{\gamma}\right] & \text{if } w \geq \hat{w}_T, \end{cases} \tag{A-10}$$

where $\mu_N \equiv \frac{\lambda^k - \lambda^{\beta\theta/(1+\theta)}}{\lambda^k + (\mathcal{F}+\lambda^\beta)^{\theta/(1+\theta)} - \lambda^{\beta\theta/(1+\theta)}}$ is the fraction of workers in non-trading firms, $\mu_T \equiv 1 - \mu_N$ is the fraction of workers in trading firms, $\hat{w}_N = w_N(\hat{\varphi}_N)$, $\hat{w}_T = w_T(\hat{\varphi}_T)$, $\hat{w}_T = (\mathcal{F} + \lambda^\beta)^{\frac{1}{1+\theta}} \hat{w}_N$, and γ is defined as above. Recall that at productivity $\hat{\varphi}_T$ firms have a jump in their wages as they become exporters; equation (A-10) shows that the wage jump is from $\lambda^{\frac{\beta}{1+\theta}} \hat{w}_N$ to $\hat{w}_T$.

E Inequality and Variable Trade Costs

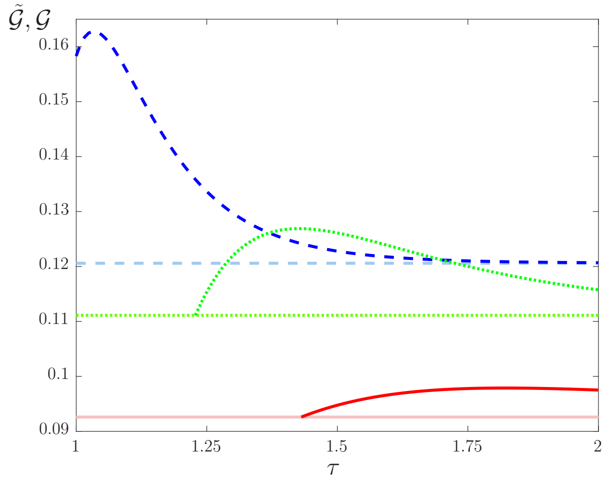
This section presents a corollary to Proposition 1, showing that an inverted-U relationship between trade liberalization and inequality is not guaranteed when monopsony power is high and fixed costs are sufficiently large.

Corollary 1. *Given that $\tau \geq 1$, λ is bounded below by $\underline{\lambda} = \max\left\{1, \left[\frac{\mathcal{F}}{2^{(1+\theta)/(\sigma+\theta)}-1}\right]^{1/\beta}\right\}$. If $\underline{\lambda} > 1$, then $\frac{d\underline{\lambda}}{d\mathcal{F}} > 0$ and $\frac{d\underline{\lambda}}{d\theta} < 0$. The main implication is that low values of θ (i.e., high monopsony power) or high values of $\mathcal{F} \equiv \frac{f_X}{f}$ yield levels of $\underline{\lambda}$ that occur at the downward-sloping region of $\tilde{\mathcal{G}}$ and λ, so that in the entire range of τ, trade liberalization always yields higher inequality.*

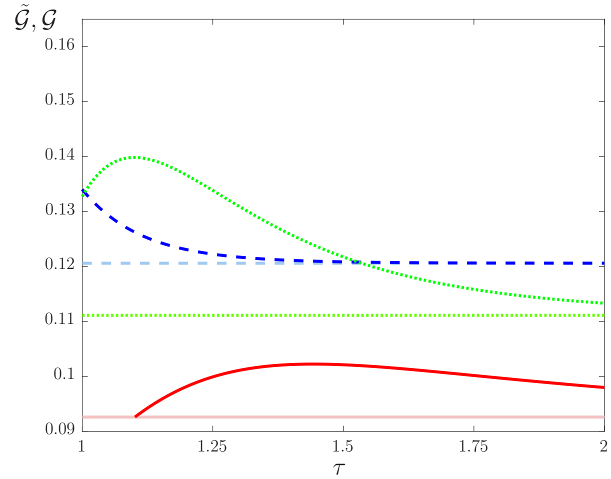
Figure A-1 illustrates this point by showing the relationship between τ and $\tilde{\mathcal{G}}$ for different levels of θ and $\mathcal{F}$. As in Figure 1, we use $\sigma = 3.8$, $k = 3.4$, $\theta^L = 0.15$, $\theta^M = 1.8$, $\theta^H = 6$, with each panel showing a different value of $\mathcal{F} \in \{0.25, 0.5, 0.75, 1\}$. The horizontal lines in each panel indicate the closed-economy Gini coefficients, $\mathcal{G}$, for the different values of θ. The dashed lines show that when monopsony power is high ($\theta^L = 0.15$), an inverted-U relationship between $\tilde{\mathcal{G}}$ and τ only emerges when fixed exporting costs are the smallest ($\mathcal{F} = 0.25$), whereas the solid lines show that if monopsony power is low ($\theta^H = 6$), an inverted-U shape appears in all cases. That is, for high degrees of monopsony power, and provided that fixed exporting costs are not too low, trade liberalization increases inequality monotonically. Intuitively, monopsony power inhibits exporting (see Figure 1), and when it is sufficiently high, the fraction of workers employed in trading firms will never be sufficient to cause a reduction in inequality after trade liberalization.[2]

As shown in section 5.1, more monopsony power (a reduction in θ) increases inequality in the closed economy. This is clearly shown in the ordering of the horizontal lines in Figure A-1. In the open economy, however, it is not necessarily the case that a lower θ increases $\tilde{\mathcal{G}}$. Note, for example, that in panels (c) and (d) of Figure A-1, the dashed line ($\tilde{\mathcal{G}}(\theta^L)$) is below the dotted line ($\tilde{\mathcal{G}}(\theta^M)$) for low values of τ, so that in those cases, a reduction in θ from θ^M to θ^L reduces inequality. This is also a consequence of the negative effect of monopsony power on exporting: inequality goes down after a reduction in θ because of the decline in the fraction of workers employed in trading firms (*i.e.*, there is less inequality because there are less high-wage exporting firms).

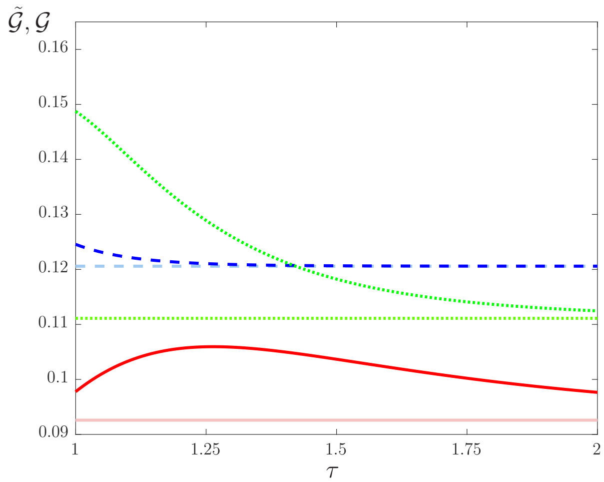
[2]The result in Corollary 1 can also be seen in the intercepts for s_T in Figure 1. The fraction of trading firms has an upper bound given by $\underline{\lambda}^{-k}$; for the cases in Figure 1 with $\underline{\lambda} > 1$, this upper bound corresponds to the values of s_T when $\tau = 1$.



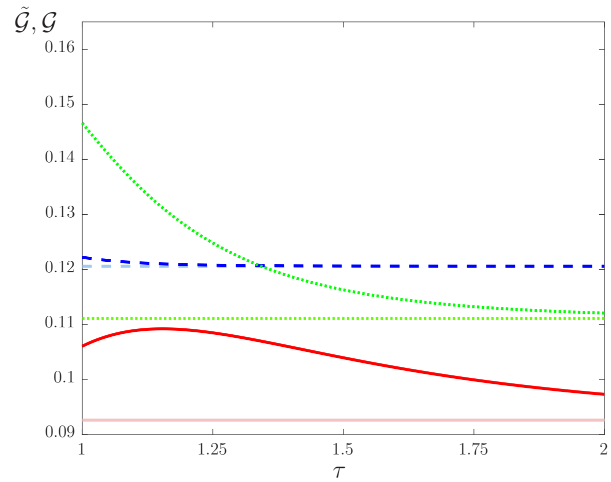
(a) $\mathcal{F} = 0.25$



(b) $\mathcal{F} = 0.5$



(c) $\mathcal{F} = 0.75$



(d) $\mathcal{F} = 1$

Figure A-1: Trade costs, inequality, and monopsony power: θ^L (dashed), θ^M (dotted), θ^H (solid)

F Decomposing Expected Maximized Utility in the Open Economy

Analogously to equation (4), the expected maximized utility in the open economy is given by $\mathbb{U}$ is given by

$$\tilde{\mathbb{U}} \equiv \frac{1}{\theta} \ln \left(M_P \int_{\hat{\varphi}_N}^{\infty} w(\varphi)^{\theta} g(\varphi | \varphi \geq \hat{\varphi}_N) d\varphi \right). \tag{A-11}$$

Using (A-4), equation (5) for $w_N(\varphi)$, $w_T(\varphi) = \left(1+\tau^{1-\sigma}\right)^{\frac{1}{\sigma+\theta}} w_N(\varphi)$, and $g(\varphi|\varphi \geq \hat{\varphi}_N) = \frac{k\hat{\varphi}_N^k}{\varphi k+1}$, we obtain that

$$\int_{\hat{\varphi}_N}^{\infty} w(\varphi)^{\theta} g(\varphi|\varphi \geq \hat{\varphi}_N) d\varphi = \left(1+\frac{\theta}{\gamma}\right)\left[\frac{\lambda^k + (\mathcal{F}+\lambda^{\beta})^{\frac{\theta}{1+\theta}} - \lambda^{\frac{\beta\theta}{1+\theta}}}{\lambda^k}\right] \hat{w}_N^{\theta}$$

$$= \left(1+\frac{\theta}{\gamma}\right)\left(1-\frac{1}{\gamma}\right)^{\theta}\left\{\frac{[\lambda^k + (\mathcal{F}+\lambda^{\beta})^{\frac{\theta}{1+\theta}} - \lambda^{\frac{\beta\theta}{1+\theta}}]^{1+\theta}}{(\mathcal{F}+\lambda^k)^{\theta}\lambda^k}\right\} \tilde{w}^{\theta}, \qquad \text{(A-12)}$$

where the second equality uses that $\tilde{w} = \left[\frac{\mathcal{F}+\lambda^k}{\lambda^k + (\mathcal{F}+\lambda^{\beta})^{\theta/(1+\theta)} - \lambda^{\beta\theta/(1+\theta)}}\right]\frac{\gamma \hat{w}_N}{\gamma - 1}$ (see section 7.1). Plugging in (A-12) into (A-11), we rewrite $\tilde{\mathbb{U}}$ as

$$\tilde{\mathbb{U}} \equiv \ln \tilde{w} + \frac{1}{\theta}\ln M_P + \mathcal{D} + \mathcal{T},$$

where $\mathcal{D} \equiv \frac{1}{\theta}\ln\left[\left(1+\frac{\theta}{\gamma}\right)\left(1-\frac{1}{\gamma}\right)^{\theta}\right] < 0$ and $\mathcal{T} \equiv \frac{1}{\theta}\ln\left\{\frac{[\lambda^k + (\mathcal{F}+\lambda^{\beta})^{\theta/(1+\theta)} - \lambda^{\beta\theta/(1+\theta)}]^{1+\theta}}{(\mathcal{F}+\lambda^k)^{\theta}\lambda^k}\right\} \leq 0$.

We already showed in section A that $\mathcal{D}$ is negative. Here we show that the term $\mathcal{T}$ is less or equal to zero. Note first that if $\lambda = 1$, then $\mathcal{T} = 0$. We will now prove that $\mathcal{T} < 0$ if $\lambda > 1$. $\mathcal{T}$ is less than zero if the term within braces is between 0 and 1. That term is greater than zero because $(\mathcal{F}+\lambda^{\beta})^{\theta/(1+\theta)} > \lambda^{\beta\theta/(1+\theta)}$, and is below 1 if and only if

$$\lambda^k\left[\left(\frac{\mathcal{F}}{\lambda^k}+1\right)^{\frac{\theta}{1+\theta}} - 1\right] > \left(\frac{1}{\lambda^{\frac{\beta}{1+\theta}}}\right)\lambda^{\beta}\left[\left(\frac{\mathcal{F}}{\lambda^{\beta}}+1\right)^{\frac{\theta}{1+\theta}} - 1\right]. \qquad \text{(A-13)}$$

If $z \equiv y\left[\left(\frac{\mathcal{F}}{y}+1\right)^{\frac{\theta}{1+\theta}} - 1\right]$ is increasing in y, then (A-13) holds because $\lambda^k > \lambda^{\beta}$ and $1 > \frac{1}{\lambda^{\beta/(1+\theta)}}$. We obtain that $\frac{dz}{dy}$ is greater than zero if an only if $1 + \left(\frac{1}{1+\theta}\right)\frac{\mathcal{F}}{y} > \left(1+\frac{\mathcal{F}}{y}\right)^{1/(1+\theta)}$. The latter is true because both sides approach 1 if $\frac{\mathcal{F}}{y} \to 0$, so that as $\frac{\mathcal{F}}{y}$ increases in the $(0,1)$ interval (recall that $\mathcal{F} < \lambda^{\beta} < \lambda^k$ and thus $0 < \frac{\mathcal{F}}{\lambda^k} < \frac{\mathcal{F}}{\lambda^{\beta}} < 1$), the left-hand side changes at a rate $\frac{1}{1+\theta}$ that is higher than the rate of change of the right-hand side, which is given by $\frac{1}{1+\theta}\left(\frac{y}{\mathcal{F}+y}\right)^{\theta/1+\theta}$. Therefore, $\mathcal{T}$ is negative if $\lambda > 1$, and equals zero if $\lambda = 1$.

As in the closed the economy, from the definition of $\tilde{\mathbb{B}}$ we know that

$$\frac{\mathbb{L}}{\tilde{\mathbb{B}}} = M_P \int_{\hat{\varphi}_N}^{\infty} w(\varphi)^{\theta} g(\varphi|\varphi \geq \hat{\varphi}_N) d\varphi$$

and therefore, (A-11) can be rewritten as $\tilde{U} \equiv \frac{1}{\theta}\ln\left(\frac{\mathbb{L}}{\tilde{\mathbb{B}}}\right)$. Given that $\mathbb{U} = \frac{1}{\theta}\ln\left(\frac{\mathbb{L}}{\mathbb{B}}\right)$, it follows that

$$\tilde{\mathbb{U}} - \mathbb{U} = \frac{1}{\theta}\ln\left(\frac{\mathbb{B}}{\tilde{\mathbb{B}}}\right) = \frac{1+\theta}{\theta}\ln\left[\left(\frac{\hat{\varphi}_N}{\hat{\varphi}}\right)\Psi^{\frac{1}{\sigma-2}}\right],$$

where the second equality follows from the equilibrium value for $\tilde{\mathbb{B}}$ in the last paragraph of section 6.3.