Work-from-Home and Cities: An Elementary Spatial Model

by

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Abstract

This paper analyzes the urban impacts of hybrid WFH in the simplest possible model, relying on Leontief utility and production functions and other simplifying assumptions. The analysis shows that introduction of WFH raises both the wage and land consumption of households while shrinking the size of the business district and reducing business land rent. When WFH requires home work-space, the city's overall spatial size increases, with residential rents rising in the suburbs while falling near the center. The decline in business rent and the rotation of the residential rent contour match empirical evidence showing that WFH reduces office-building values and flattens the residential rent gradient.

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1. Introduction

After the shock of the pandemic demonstrated its feasibility, work-from-home (WFH) has surged across the world's cities. WFH can be hybrid, with work time split between home and office, or (less commonly) fully remote, with the office seldom or never visited by the worker. The emergence of WFH has upended both labor and real-estate markets. Hybrid WFH options make labor contracts much more appealing to workers, inducing many firms to offer them despite concerns about lower productivity. With fewer workers present, employer demands for office space have plummeted, reducing the values of downtown buildings in an "office apocalypse" (Gupta, Mittal and Van Nieuwerburgh, 2022). Less-frequent hybrid commutes have made the suburbs more attractive, flattening residential price gradients, and the relocation of fully remote workers has appeared to put downward pressure on housing prices in some cities (Brueckner, Kahn and Lin, 2023; Gupta, Mittal, Peeters and Van Nieuwerburgh, 2022: Bloom and Ramani, 2022). Overall price trends have nevertheless been upward since the pandemic, and some researchers partly attribute this pattern to higher housing demand spurred by the need for work space at home (Mondragon and Wieland, 2022; Stanton and Tiwari, 2021; Gamber, Graham and Yadav, 2023).

WFH has also spurred a theoretical literature designed to portray its vast effects on the spatial structures and other features of cities, and the present paper offers a contribution to this body of work. While the models in this literature (described below) are richly detailed and highly complex, sometimes being tailored to computational exercises, the model in this paper is instead characterized by its extreme simplicity and transparency. The goal is to develop a

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model containing only the essential elements of WFH, dispensing with many details that may be realistic but stand in the way of an elementary analysis that is easily understood.

The framework is a variant of the standard urban model (Brueckner, 1987; Duranton and Puga, 2015), modified to include a central business district where firms combine land and labor to produce a tradable good. With the office apocalypse being a major empirical consequence of WFH, a model structured to capture its impact on business land rent is desirable. Another of the model's distinguishing features is the use of Leontief production and utility functions. While use of production inputs in a fixed proportion is familiar, the same feature in consumption is less so. However, the Leontief assumption on preferences is preferable to the more-extreme assumption of fixed land consumption, which is used in many urban models (including two of the WFH papers discussed below). Even though proportions are fixed, land consumption then remains endogenous and potentially impacted by WFH. In addition, the Leontief assumptions yield analytical tractability, which disappears, for example, if preferences take the familiar Cobb-Douglas form (making numerical analysis necessary).

Under a hybrid WFH arrangement, the frequency of commute trips in the model falls by the exogenous factor $\alpha < 1$, which lowers commuting cost per period. Business land usage falls in step, since fewer workers need to be accommodated. In a second version of the model, WFH also creates a need for work space at home. The analysis derives the impact of the α parameter on the endogenous variables of the model: household land consumption, the wage, and the distance to the edge of the city, with the wage solution determining business rent.

Two of the equilibrium conditions are standard, requiring that the city fit its fixed population and that residential land rent equals land's opportunity cost at the city's edge. The third condition requires equality between residential and business rents at the edge of the business district. Given the simplicity of the model, closed-form solutions emerge. They show that introduction of WFH raises both the wage and land consumption of households (raising utility) while shrinking the size of the business district and reducing business land rent. When WFH requires home work-space, the city's overall spatial size increases, with residential rents rising in the suburbs while falling near the center. While the shrinkage of the business district is a long-run effect that has yet to fully play out in US central cities, the decline in business rent and the rotation of the residential rent contour match empirical evidence showing that WFH reduces office-building values and flattens the residential rent gradient.

Previous theoretical papers analyzing the impacts of hybrid WFH include Delventhal, Kwon and Parkhomenko (DKP, 2022), Kyriakopoulou and Picard (KP, 2023), Behrens, Kichko and Thisse (BKT, 2024), Davis, Ghent and Gregory (DGG, 2024), Delventhal and Parkhomenko (DP, 2022), and Gokan, Kichko and Thisse (GKT, 2021), with the last two as yet unpublished.[1] DKP, DGG, and DP present computational models, while the remaining papers are analytical. All these papers develop models that are much richer and more detailed than the present framework. For example, all of the models except for KP have two types of workers with different skills, and some (GKT, DGG) have two types of jobs. For example, nonremote unskilled workers in GKT provide services (e.g., restaurants) in the central city, with the model capturing negative impacts in this sector when office workers stay home, a concern frequently discussed in the media. Unlike in the basic version of the current model, the productivity of work onsite and at home differ in all of the models (constituting distinct inputs in some papers), and the share of work time spent at home is chosen endogenously by workers in DKP and DP, while otherwise being exogenous (as in the present model). In some models (DKP, DP, BKT), workers choose the amount of home work-space, which is instead set exogenously in the present model. In DGG's model, commuters even experience traffic congestion.

Despite many differences, the model of Kyriakopoulou and Picard (2023) is perhaps closest to the current one. Although it includes the complication of spatial interaction among firms as well as other differences, the analysis includes a focus on business land usage and land rent in addition to analyzing the spatial pattern of residential rents. But KP's model (along with that of GKT) is more restrictive than the current framework in one respect: it assumes fixed rather than flexible land consumption.

The plan of the paper is as follows. To orient readers, section 2 develops the model in the absence of WFH, when $\alpha = 1$. Section 3 adds WFH to the basic model without including a home work-space requirement, while section 4 includes this requirement. Section 5 analyzes

[1] By contrast, instead of focusing on hybrid WFH, Brueckner, Kahn and Lin (2023) and Brueckner and Sayantani (2022) analyze the effect of fully remote WFH, in which workers can relocate to other cities while keeping their original jobs. Multiple cities also play a role in the WFH model of Lee (2024).

WFH impacts on the urban public sector, which can be added to the model under particular assumptions. Section 6 presents a robustness analysis, asking whether a worker-productivity difference between home and office or unequal Leontief coefficients affect the results of the analysis. Section 7 offers conclusions.

2. The model without WFH

As explained above, the model is based on Leontief utility and production functions.[2] Utility depends on consumption of land, denoted q, which is a proxy for housing consumption, and nonland consumption, denoted e. The good e is a tradable commodity produced in all cities, and its price is normalized to unity. As in the standard model, all urban residents have the same utility function, and with Leontief preferences, that common function is

$$U(e,q) \quad = \quad \min\{e,\, q\}. \tag{1}$$

Although, with unitary coefficients on both goods, the corners of the right-angled Leontief indifference curves lie on the 45 degree line, the case of unequal coefficients is considered in section 6 below.

Letting r denote residential rent per unit of land, the consumer budget constraint is given by

$$e \,+\, rq \quad = \quad w \,-\, td, \tag{2}$$

where w is the wage per period, d is commuting distance, and t is round-trip money cost of commuting per mile traveled. Utility maximization yields $e = q$, with consumption occurring at an indifference-curve corner. Eliminating e from (2) using this equality and then solving for land rent yields

$$r \quad = \quad \frac{w - td}{q} \,-\, 1. \tag{3}$$

Eq. (3) gives $\partial r/\partial d = -t/q$, as in the case of general preferences, but the difference with Leontief preferences is that q is spatially constant across the city rather than increasing in d.

[2] For another use of Leontief preferences in an urban model, see Brueckner (2005).

The city is closed to migration, and for simplicity, it is assumed to be linear with unitary width. The center lies at one end, with distance to the center denoted x, and the other half of the city is suppressed without loss of generality. The business district, where production of the nonland good occurs, occupies the central portion of the city, ranging from $x = 0$ to $x = \widehat{x}$, with $\widehat{x}$ determined below.

Travel costs within the business district are assumed to be zero, so that the land is spatially undifferentiated, with business rent equal to a constant R. While some of district's output of the tradable nonland good will be consumed locally, any amount exported can then be shipped to the city center (where the export node is located) at zero cost, making the location of production within the district immaterial. Since travel cost within the business district is zero, workers commute to its outer edge, then reaching their nearby worksites at zero cost.[3]

The inputs into production of the tradable nonland good are labor, denoted H, and land, denoted L. Since constant returns to scale prevails with Leontief production, output can be viewed as coming from a single large firm that behaves competitively. Letting E denote output, the production function can be written as

$$E \quad = \quad \min\{H,\, L\}, \tag{4}$$

assuming equal labor and land coefficients. The case of unequal coefficients is considered below once the basic results are derived.

In the absence of WFH, cost minimization by the central firm leads to choice of a corner on one of the right-angled isoquants, where $H = L$. The firm's zero-profit condition is then given by

$$1 \times E \;-\; wH \;-\; RL \;\;=\;\; H \;-\; wH \;-\; RH \;\;=\;\; 0, \tag{5}$$

recalling the unitary output price and noting $E = H$ from (4). Cancelling H and using (5) to solve for the business R yields

$$R \;\;=\;\; 1 \;-\; w. \tag{6}$$

[3] Workers can be viewed as acquiring e, the amount consumed of the nonland good, during the work trip.

Labor supply by the city's workers is assumed to be perfectly inelastic. Therefore, the entire city population N is employed, with $H = N$. The land area L used in production is then equal to N, so that the distance to the edge of the business district is $\widehat{x} = N$ (recall the city's unit width). Since workers commute to the outer edge of the business district, their commuting cost from distance x is $t(x - \widehat{x}) = t(x - N)$.

Note that, while the wage is determined partly by the firm's zero-profit condition (written as (6)), other factors also enter. This dependence can be seen by writing the three equilibrium conditions for the city, imposing the standard assumption of absentee land ownership. These conditions require (i) equality between residential rent r and business rent R from (3) and (6) at the edge $\widehat{x} = N$ of the business district; (ii) equality between the opportunity cost of land (set at zero) and residential rent at $\overline{x}$, the distance to the edge of the city; (iii) equality between total residential land consumption and the city's residential area, with the latter equal to $\overline{x} - \widehat{x} = \overline{x} - N$. These conditions are written, respectively, as

$$\frac{w}{q} - 1 = 1 - w \tag{7}$$

$$w - t(\overline{x} - N) = q \tag{8}$$

$$Nq = \overline{x} - N. \tag{9}$$

Note that residential rent at the edge $\widehat{x} = N$ of the business district on the LHS of (7) reflects zero commuting cost. In (8), the $r = 0$ condition based on (3) reflects a commuting distance of $\overline{x} - N$ at the edge of the city, and the condition has been rearranged to put q on the RHS. Also, Nq in (9) gives total residential land consumption. Note that the competition for land between the central firm and residences helps determine the wage via (7).

The unknowns in (7)–(9) are w, q, and $\overline{x}$. To solve the system, $\overline{x}$ is eliminated using (8) and (9) to yield $w = (1 + tN)q$. After substituting for w on the LHS of (7), q cancels, yielding

$$w = 1 - tN. \tag{10}$$

Substituting w into the equation prior to (10) and solving for q then yields

$$q \;=\; \frac{1 \;-\; tN}{1 \;+\; tN}. \tag{11}$$

Substituting (11) into (9) to find $\overline{x}$ yields

$$\overline{x} \;=\; \frac{2N}{1+tN}. \tag{12}$$

Business rent R is not an explicit unknown in (7)–(9), but it can be recovered from (6) and (10), yielding $R = tN$, so that w and R sum to the unitary output price, as required.

Simple comparative statics can be checked against those of the standard urban model. As in that model, inspection of (11) shows that land consumption q falls as population N increases. Differentiating (12) shows that $\overline{x}$ rises with N in standard fashion. The wage is usually treated as fixed in the standard model, but (10) shows that w in the present model falls as N increases. Finally, business rent R rises with N.[4]

3. Adding WFH

Under a hybrid WFH arrangement, workers commute to the office for part of the period and work from home for the remaining portion, with the split determined exogenously by firms. Importantly, workers are assumed to have the same productivity regardless of where the work occurs, so the labor need not be decomposed into office and home components. This assumption is relaxed in a robustness check in section 6 below. Let the fraction of days that workers are present at their central worksite be denoted $\alpha < 1$. Then, commuting cost from a distance x is reduced to αtx from tx, lowering worker outlays on this important expense. The firm sets individual WFH schedules so that the number of workers present in the office is the same on each day, with that number then equal to αN.

Since fewer workers are present at the worksite, the central firm requires less land for its operations than before. Land usage falls from $L = N$ to $L = \alpha N$, declining in step with the

[4] Other standard comparative-static effects come from an increase in the commuting-cost parameter t. From (12), $\overline{x}$ falls when t rises, as in the standard model, while the spatially uniform q also declines from (11) (changes in q depend on location in the standard model). A higher t also reduces the wage from (10) while raising R.

proportional reduction in workers present. In other words, while fixed proportions ($N/L = 1$) originally applied to all workers, it applies only to those present in the office under WFH ($\alpha N/L = 1$).[5] The remaining workers use a portion of their living space (instead of business land) in generating output from home, with no loss in consumption utility from the space. The next section alters this setup by assuming that WFH requires home work space, with the amount increasing as α declines. Note finally that the reduction in business land usage under WFH reflects the long-run nature of the model, where land usage (more generally, space in office buildings and factories) is fully adjustable.

The firm's zero profit condition is now $N - wN - R(\alpha N) = 0$, indicating that while it continues to pay all its workers the wage w, land cost falls from RN to αRN. Solving for R yields

$$R = \frac{1 - w}{\alpha}. \tag{13}$$

The city's equilibrium conditions under WFH are

$$\frac{w}{q} - 1 = \frac{1 - w}{\alpha} \tag{14}$$

$$w - \alpha t(\overline{x} - \alpha N) = q \tag{15}$$

$$Nq = \overline{x} - \alpha N. \tag{16}$$

In (14), the new R from (13) appears on the RHS. Commuting cost at $\overline{x}$ in the boundary-rent condition (15) is now $\alpha t(\overline{x} - \alpha N)$, and the residential land area in (16) is now $\overline{x} - \alpha N$.

Following the same steps as before, the solutions to (14)-(16) are

$$w = 1 - \alpha^2 tN \tag{17}$$

$$q = \frac{1 - \alpha^2 tN}{1 + \alpha tN} \tag{18}$$

[5] If workers are all present on the same days at the office, then the firm's space needs would not decline under WFH, with the office space empty on the remaining days. As noted above, the firm schedules WFH to avoid this outcome.

$$\overline{x} \;=\; \frac{(1\;+\;\alpha)N}{1\;+\;\alpha tN}. \tag{19}$$

Using (13) and (17), business land rent is given by

$$R \;=\; \alpha tN. \tag{20}$$

The qualitative comparative-static effects of N and t on the endogenous variables remain the same as in the absence of WFH. More important are the effects of α, which is an inverse measure of the extent of WFH. Introduction of WFH corresponds to a decrease in α from a value of 1 to some smaller value, and its effect can be gauged by evaluating the α derivatives in (17)–(20). Inspection of (17), (18) and (20) shows that the wage w and land consumption q are decreasing in α and that business rent R is increasing in α. In addition, since the α-derivative of (19) has the sign of $N(1-tN)$, which is positive from (10), the boundary distance $\overline{x}$ is increasing in α. Since introduction of WFH corresponds to a decrease in α starting from 1, the following conclusions emerge:

Proposition 1. *With no home work-space requirement, introduction of WFH raises the wage w and land consumption q while reducing business rent R and the boundary distance $\overline{x}$. Given Leontief preferences, the increase in q implies higher worker utility under WFH.*

Note that the decline in $\overline{x}$ shows that the reduction under WFH in the business district's land area (which falls from N to αN) dominates WFH's positive effect on individual land consumption, leading to a smaller overall urban land area. Note also that the decline in business rent matches the predicted drop in office-building values from Gupta, Mittal and Van Nieuwerburgh (2022).

The higher q under WFH arises partly through a higher wage, although changes in residential rent also have an effect. Since business rent declines, residential rent falls at the edge of the business district ($\widehat{x} = \alpha N$), where two rents are equal. After inserting an α before $td = t(x-\alpha N)$ in (3), the slope of the residential rent function ($\partial r/\partial x$) equals $-\alpha t/q$, which can be written using (18) as

$$\frac{\partial r}{\partial x} \;=\; -\frac{\alpha t(1+\alpha tN)}{1-\alpha^2 tN}. \tag{21}$$

Since (21) becomes less negative as α decreases, it follows that residential rent falls more slowly over distance when WFH is introduced, while starting from a lower value at $\widehat{x}$. Empirical evidence in Gupta, Mittal, Peeters and Van Nieuwerburgh (2022) and Brueckner, Kahn and Lin (2023) confirms this predicted decline in the residential rent gradient.[6]

The change in rent levels farther from the city center can be deduced from the decline in $\overline{x}$ under WFH. With $\overline{x}$ falling, it follows that, despite the flatter rent curve, residential rents never rise above their initial levels anywhere in the city, falling everywhere with the introduction of WFH. This conclusion can be seen in Figure 1, where the pre- and post-WFH cases are denoted by the subscripts 0 and WFH on r, R and $\widehat{x}$. The asterisks refer to outcomes analyzed in the next section. Summarizing the land rent impacts of WFH yields

Corollary 1. *With no home work-space requirement, introduction of WFH reduces land rents throughout the city, both in its business and residential areas.*

Proposition 1 is is partly overturned once a realistic home work-space requirement is introduced, as seen in the next section.

It is important to more fully identify the ultimate sources of lower business rent and the higher wage under WFH. It appears that the root of these changes lies in the flattening of the residential rent gradient. Although this flattening is partly endogenous (being affected by q), the drop in commuting cost per mile from t to αt tends to depress the gradient in an exogenous fashion. But a lower gradient, which reflects the greater attractiveness of the suburbs relative to more-central locations under WFH, in turn tends to put downward pressure on residential rents nearer the city center. That downward pressure then lessens competition for central locations from residences, allowing firms to pay less for land in the business district. With lower business land rent, firms can then compete more vigorously for workers while still earning zero profit, pushing up the wage.

[6] The model of Kyriakopoulou and Picard (2023) also generates a decline in business rent and a smaller residential rent gradient.

Finally, it is interesting to consider the limiting case of $\alpha = 0$, which corresponds to fully remote work, with a zero fraction of work time spent at the office. Referring to (17)-(20), $\alpha = 0$ yields $w = 1$, $q = 1$ (implying $e = 1$), $\overline{x} = N$, and $R = 0$. Residential land rent is also zero, as can be seen by replacing td in (3) with $\alpha td = 0$ and using $w = q$. In addition, the business district collapses to a point at $x = 0$ since no production land is needed. Given that q is maximal when $\alpha = 0$, workers are best off with fully remote work, which makes sense since commuting cost is then absent. However, this case is unrealistic given most employers' insistence that some work time must be spent in the office.

4. WFH with a home work-space requirement

Following the evidence of a post-pandemic surge in housing demand (cited in the introduction), suppose now that a home work-space requirement is realistically added to the model. While workers derive utility from land consumption of q, they must now purchase βq worth of land, where $\beta > 1$, to have extra space for working at home. As a result, the previous budget constraint (2) becomes

$$e + r\beta q = w - td. \tag{22}$$

But since utility is still given by $\min\{e, q\}$, q is again substituted in place of e in (21), so that the land-rent expression in (3) changes to

$$r = \frac{1}{\beta}\left(\frac{w - td}{q} - 1\right). \tag{23}$$

Again substituting $x - \alpha N$ for d, the urban equilibrium conditions now become

$$\frac{1}{\beta}\left(\frac{w}{q} - 1\right) = \frac{1 - w}{\alpha} \tag{24}$$

$$w - \alpha t(\overline{x} - \alpha N) = q \tag{25}$$

$$N\beta q = \overline{x} - \alpha N. \tag{26}$$

Note in (25) that β cancels when residential rent at the edge of the city is set equal to zero, and that residential land consumption in (26) becomes $N\beta q$.

The solutions to (24)–(26) are

$$w = 1 - \alpha^2 tN \tag{27}$$

$$q = \frac{1 - \alpha^2 tN}{1 + \alpha\beta tN} \tag{28}$$

$$\overline{x} = \frac{(\alpha + \beta)N}{1 + \alpha\beta tN}. \tag{29}$$

R again equals αtN. For a given $\alpha < 1$, the introduction of a home work-space requirement, with $\beta > 1$ instead $\beta = 1$, leaves the wage and business rent unchanged, reduces q, and (from differentiation of (29)) raises $\overline{x}$.

However, α and β will change together with the introduction of WFH, falling and rising respectively from starting values of 1. To gauge the joint impacts of these changes in α and β, their magnitudes must tied in some fashion. One way to do so is to assume that the changes are mirror images of one another, with a decrease in α requiring an equal increase in β. This assumption, along with requirement that $\beta = 1$ when $\alpha = 1$, implies $\beta = 2 - \alpha$. While (27) is unchanged, substituting this relationship into (28) and (29) reduces these solutions to

$$q = \frac{1 - \alpha^2 tN}{1 + \alpha(2 - \alpha)tN} \tag{30}$$

$$\overline{x} = \frac{2N}{1 + \alpha(2 - \alpha)tN}. \tag{31}$$

Since $\alpha(2 - \alpha)$ is increasing in α (its derivative equals $2(1 - \alpha) \geq 0$), it follows that the denominators of (30) and (31) both decrease as α declines. With the numerators of (30) and (31) rising and constant, respectively, as α declines, it follows that the introduction of WFH raises both q and $\overline{x}$, with the q increase again implying higher utility. Observe, however, that since the denominator of q in (18) is smaller than the denominator in (30) given $\alpha < \alpha(2 - \alpha)$, q is larger without a home-work space requirement than with one, making utility lower in the latter case. Summarizing yields

Proposition 2. *With a home work-space requirement, the introduction of WFH again raises the wage w and land consumption q and reduces business land rent R, but the boundary distance $\overline{x}$ now increases. Worker utility again rises under WFH, but not by as much as in the absence of a home work-space requirement.*

The increase in $\overline{x}$ makes sense given that the need for added work space means that an individual's total land consumption, given by $\beta q = (2-\alpha)q$ tends to rise more rapidly than before as α declines, now more than offsetting the drop in the area of the business district. But the home work-space requirement makes the increase in q itself smaller than in the absence of this requirement, making the utility gain from WFH smaller.[7]

Residential land rent once again declines in step with business rent at $\widehat{x}$, and the rent curve again flattens, with the numerator of (21) becoming $\alpha t(1 + \alpha(2 - \alpha))tN$, which decreases as α falls while the denominator rises. However, since $\overline{x}$ now increases, it follows that residential rents rise in the outer part of the city, as seen in Figure 1, where the r curve and $\overline{x}$ value with WFH subscripts and asterisks are revelant. The rent curve thus rotates around some central point. Summarizing yields

Corollary 2. *With a home work-space requirement, introduction of WFH again reduces business land rent, while residential rent falls near the business district but rises in the outer part of the city.*

5. Urban doom loop?

The decline in business rent in both versions of the model as WFH reduces the need for space matches the decline in the values of US office buildings predicted by Gupta, Mittal and Van Nieuwerburgh (2022), a decline validated in various news stories (e.g., Grant, 2024). These researchers have also predicted that lower building values will generate large fiscal deficits for

[7] If a general function $\beta(\alpha)$ with $\beta' < 0$ and $\beta(1) = 1$ is used instead of $\beta = 2 - \alpha$, Proposition 2 becomes narrower. The α-derivative of the denominator of the q solution in (28) is proportional to $\beta + \alpha\beta'$, which is generally ambiguous in sign but negative or zero evaluated at $\alpha = \beta = 1$ provided $1 + \beta'(1) \leq 0$, or $-\beta'(1) \geq 1$. When this inequality holds, a marginal decrease in α starting at 1 leads to a larger or equal increase in β. Under this condition, the decrease in α reduces or leaves unchanged the denominator of (28) while increasing the numerator, thus raising q. In addition, the derivative of the $\overline{x}$ expression in (29) is proportional to $1 - \beta^2 tN + \beta'(1 - \alpha^2 tN)$. Evaluating at $\alpha = \beta = 1$ and factoring out $1 - tN$ (which is positive from (17)), the derivative is zero or negative when $1 + \beta'(1) \leq 0$ or $-\beta'(1) \geq -1$, the same condition as above. Therefore, when this condition holds, a marginal introduction of WFH (a marginal decrease in α starting at 1) raises q and raises $\overline{x}$ or leaves it unchanged. WFH again raises the wage w unconditionally.

central cities, which rely on heavily on property tax revenue from the office sector to finance public spending. They have called this secondary effect, which will depress public spending and further reduce the attractiveness of office properties, an "urban doom loop." In contrast to central cities, commentators are more optimistic about property-value changes in the suburbs, a view that is confirmed by Corollary 2.

Since the present model has no public sector, it is not ideal for depicting these budgetary impacts of WFH. However, a public sector could be added while preserving the structure of the model under particular assumptions. To do so, suppose that ad valorem taxes on rent are levied in the business district and the residential area of the city, and that the proceeds are used to provide distinct public goods, with a good valued by businesses provided in the business district and a good valued by consumers provided in the residential part of the city. Suppose in addition that the business and residential rental tax rates are equal. Equality of net-of-tax business and residential land rents would then be required at the edge $\widehat{x}$ of the business district, but this condition reduces to equality of gross-of-tax rents since the tax rates are equal, with the tax factors cancelling. The equilibrium condition (24) that embodies this requirement therefore still applies, while the boundary condition (25) also remains the same.

With the introduction of WFH, tax revenue from the business district falls, both because business rent falls and the district shrinks in size. Therefore, the post-WFH revenue is insufficient to cover the costs of providing the original level of the district's public good, which must be cut in order to balance the budget, further reducing the attractiveness of the district. Thus, a central-city doom loop is initiated.

Analysis based on the model of section 4 shows that the total residential rent in the city changes in an ambiguous direction with the introduction of WFH, a natural consequence of rent increases and decreases in different parts of the city. Therefore, while the residential area may suffer a revenue shortfall along with the central city, it may alternatively enjoy a budget surplus. As a result, the suburban public sector may be spared the experience of its central-city counterpart.

It should be noted that a central-city doom loop could be avoided if other taxes exist that could make up for the drop in property-tax revenue. For example, if the central city has a wage tax (as does New York City), it could help. Indeed, since wages in the model rise under WFH, revenue from a wage tax (if one were present) would increase, perhaps fully offsetting the decline in property-tax revenue under WFH.

6. Robustness analysis

This section investigates how relaxing two of the maintained assumptions affects the results of the analysis. First, the assumption of equal worker productivity at home and in the office is relaxed. Second, the assumption of unitary and thus equal coefficients in the Leontief utility and production functions is relaxed.

6.1. The effect of differential office and home productivities

Instead of equal productivities, suppose that worker productivity differs between the home and office, with office productivity equal to 1 and home productivity equal to δ, where δ can be greater or less than 1. Average productivity is then equal to $p(\alpha) \equiv \alpha + (1-\alpha)\delta = (1-\delta)\alpha + \delta$, with $p'(\alpha) > (<)\ 0$ as $\delta < (>)\ 1$. Thus, average productivity rises as α drops under WFH if home productivity is greater than office productivity (if $\delta > 1$), with the average falling otherwise. Evidence cited in Barrero, Bloom and Davis (2021) on this issue is mixed, with some studies showing higher home productivity and others showing the reverse.

The firm's zero profit condition (in the paragraph just before (13)) is rewritten as $p(\alpha)N - wN - R\alpha N$, with output now equal to equal $p(\alpha)N$ while $L = \alpha N$ still holds.[8] Solving for R and rewriting the first WFH equilibrium condition in (23), the 1 on the RHS is replaced by $p(\alpha)$, while the other equilibrium conditions remain the same. In the solutions, the 1 in the wage solution (27) and the 1 in the numerator of the q solution in (30) are replaced by $p(\alpha)$. In the numerator of the $\overline{x}$ solution in (31), the 2 is replaced by $\alpha + \beta p(\alpha) = \alpha + (2-\alpha)p(\alpha)$, using the form of the solution in (29).

In order for the results of Proposition 2 to still apply unambiguously, $p' < 0$ must hold, indicating higher productivity at home than at the office. The productivity effect present

[8] When productivity per worker changes from 1 to some value $\omega \neq 1$ under Leontief production, the optimal input mix then satisfies $\omega N/L = 1$. In the current case, however, the αN workers that are present in the office still have productivity equal to 1 even though home productivity is now different. Therefore, the factor mix in office production remains the same as before, given by $\alpha N/L = 1$, which again yields land usage of αN.

in the solutions described above then raises both w and q, amplifying the positive α effects operating through the other terms in the solutions. In addition, the numerator of the $\overline{x}$ solution is decreasing in α when $p'(\alpha) < 0$.[9] Since the denominator is increasing in α, a reduction in α raises $\overline{x}$ as before. However, when $p' > 0$, a reduction in α has an ambiguous effect on all the solutions, unless this productivity effect is sufficiently small. Summarizing yields

Proposition 3. *When productivity is higher at home than at the office, the results in Proposition 2 continue to hold. When home productivity is lower, this conclusion is unaffected if the productivity effect is sufficient small, but otherwise the effects of WFH on the variables of the model are all ambiguous.*

6.2. The effect of unequal Leontief coefficients

Another robustness check is to relax the assumption that the Leontief coefficients are equal in the utility and production functions. This alteration is less significant than the one just investigated, but it is useful nevertheless to know how it affects the impact of WFH. To address this question, suppose that the utility and production functions are now $U(e, q) = \min\{e, \lambda q\}$ and $E = \min\{H, \theta L\}$. Then, utility and cost minimization yield $e = \lambda q$ and $H = \theta L$, making the assumptions $\lambda, \theta > 1$ natural. With these inequalities, nonland consumption exceeds land consumption and firms employ many workers per unit of land, with the latter outcome mimicking the use of tall buildings in production. Despite this view, λ and θ are unrestricted in magnitude.

Steps similar to the previous ones yield a new set of equilibrim conditions under WFH. Land usage in the business district becomes $L = (\alpha/\theta)N$, an expression that replaces αN in the equilibrium conditions (24) and (25). In addition, $-\lambda$ replaces the -1 in (24) while multiplying q on the RHS of (25). Finally, θ multiplies the RHS of (23). The new set of solutions is then

$$w \;=\; 1 \;-\; (\alpha^2/\theta)tN \tag{32}$$

$$q \;=\; \frac{1 \;-\; (\alpha^2/\theta)tN}{\lambda \;+\; \alpha(2-\alpha)tN} \tag{33}$$

[9] The derivative is $1 - p + (2-\alpha)p'$. Since $p' < 0$ implies $1 - p \leq 0$, this expression is negative.

$$\overline{x} \;=\; \frac{[2 \;+\; \alpha(\lambda/\theta - 1)]N}{1 \;+\; \alpha(2-\alpha)tN}. \tag{34}$$

From (32) and (33), the previous conclusion that w and q are decreasing in α (thus rising with the introduction of WFH) remain unchanged, while R again increases with α. However, the effect of α on the numerator of (34) is unclear, making the effect on $\overline{x}$ ambiguous in general. But if $\lambda < \theta$, then the factor multiplying α is negative and the numerator is decreasing in α. With the denominator increasing as before, $\overline{x}$ is then decreasing in α, yielding

> **Proposition 4.** *With a home work-space requirement and unequal Leontief coefficients, the introduction of WFH again raises the wage w and land consumption q and reduces business land rent R. The boundary distance $\overline{x}$ increases if $\lambda < \theta$, or if a household's nonland consumption per unit of land is less than employment per unit of land, but changes in an ambiguous direction otherwise.*

Recall that the negativity of WFH's $\overline{x}$ effect was overturned once a home work-space requirement was added to the model, in which case the expansion of the city's residential area was sufficient to offset the shrinkage of the business district. In order for this offset to persist with unequal Leontief coefficients, the land intensity of the consumption bundle (the reciprocal of λ) must be larger than the land intensity of production (the reciprocal of θ), or $\lambda < \theta$.[10]

7. Conclusion

This paper has analyzed the urban impacts of hybrid WFH in the simplest possible model, relying on Leontief utility and production functions and other simplifying assumptions. The analysis shows that introduction of WFH raises both the wage and household land consumption (raising utility) while shrinking the size of the business district and reducing business land rent. When WFH requires home work-space, the city's overall spatial size increases, with residential rents rising in the suburbs while falling near the center. The model's predicted rent impacts match empirical evidence from the WFH literature.

[10] Another possible robustness check would involve replacement of absentee landownwership with internal ownership, where total urban rent is equally redistributed among the residents (the city is then fully closed). To carry out such an analysis, an expression for total rent per capita, denoted I, is required. However, I itself also appears on the RHS of the equality defining I, since rent depends on income, now equal to $w+I$. But with Leontief preferences, I cancels from both sides of this equation, so that a fully closed city cannot be analyzed.

The analysis complements a set of previous papers whose models are richer and more detailed and thus more complex than the present one, perhaps being inaccessible to some readers interested in the WFH topic. It is hoped that this paper, through its simplicity and thus its pedagogical value, will provide additional illumination on how WFH affects cities.

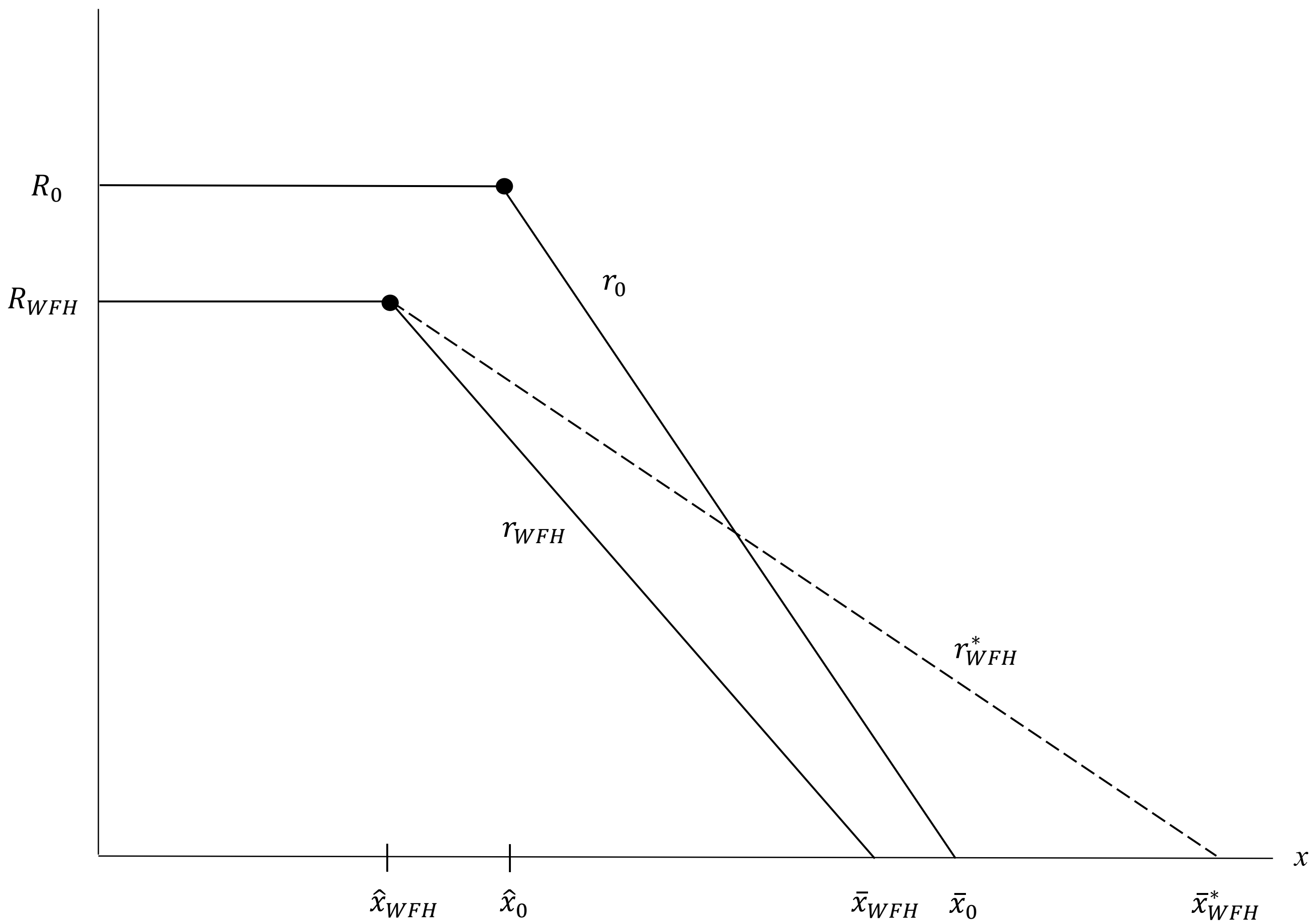

Figure 1: Spatial-structure effects of WFH

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