# Urban Growth Boundaries: An Effective Second-Best Remedy for Unpriced Traffic Congestion?

by

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#### Abstract

This paper evaluates the efficacy of the urban growth boundary (UGB) as a second-best substitute for a first-best toll regime in a congested city. Numerical results show that, while a UGB is welfare improving, validating previous theoretical results, the utility gain it generates is a very small fraction of that achieved under a toll regime. Thus, the paper suggests that a UGB may not be a useful instrument for attacking the distortions caused by unpriced traffic congestion.

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#### 1. Introduction

In response to a growing focus on the phenomenon of urban sprawl by the press, policy-makers, and the general public, economists have begun to apply the tools of urban economics to analysis of the sprawl issue. The resulting small literature has identified several proximate causes for the rapid spatial expansion of cities, such as investment in an automobile-oriented transportation system (see Glaeser and Kahn (2004) and Nechyba and Walsh (2004)). In addition, the analysis has drawn a distinction between spatial expansion that is warranted on efficiency grounds and expansion that is excessive, being a result of market failures and other distortions that impart an upward bias to urban growth (see Brueckner (2000, 2001)). One such market failure is the failure by developers to account for the potential amenity value of open space around cities, which can lead to excessive development at the urban fringe. Similarly, a failure to account for the externality involved in traffic congestion, which makes the social cost of commuting higher than the private cost, leads to commute trips that are inefficiently long and cities that are excessively spread out.

A favored policy instrument for dealing with urban sprawl is the "urban growth boundary," or UGB, which specifies a city boundary beyond which development may not take place.<sup>1</sup> As argued by Brueckner (2001), a UGB works perfectly as a restraint on inefficient growth in some circumstances. For example, if the market failure leading to excessive expansion is an overlooked open-space amenity, then the social optimum can be achieved either by a development tax equal to vacant land's amenity value per acre, or by a UGB set at the appropriate distance from the urban center. However, a UGB cannot fully correct some other sprawl-inducing market failures, with the congestion externality being a case in point. To fully correct this distortion, policymakers must instead levy a congestion toll on urban commuters. By raising

the cost of intracity travel, this toll shrinks the spatial size of the city while greatly increasing central population densities. This densification, which is illustrated in the numerical results of Wheaton (1998), is socially desirable because it limits severe traffic congestion to a relatively small area around the CBD. By contrast, a UGB does not promote central densification to the same extent as a toll regime, limiting its efficacy, although it does address one symptom of market failure by reducing the city's spatial size.

Despite these limitations, a properly chosen UGB is nevertheless welfare-improving in a congested monocentric city, as demonstrated in an earlier theoretical literature. This conclusion can be inferred from the results of Kanemoto (1977) and Arnott (1979), who show that the shadow value of land is less than the market value at the city's edge in a laissez-faire equilibrium. The benefit of a UGB as a second-best instrument is more clearly highlighted in the analysis of Pines and Sadka (1985), who extend and synthesize the work of Kanemoto and Arnott.

The purpose of the present paper is to provide quantitative, as opposed to qualitative, evaluation of the UGB as a second-best instrument in a congested city. Using a numerical urban model, the paper addresses the following question: how large is the welfare gain from imposition of an optimal UGB compared to the welfare gain from a first-best congestion-toll regime? Thus, while the earlier theoretical literature proved that a properly chosen UGB raises welfare, the present paper evaluates the magnitude of the resulting gain.

The research reported in this paper was partly inspired by the earlier study of Anas and Rhee (2004), who provide a numerical appraisal of tolls and UGBs in a city that is congested but differs substantially from the standard monocentric model, which was used in the above analyses. Their city has dispersed, instead of centralized, employment, and intracity travel consists of both commuting and shopping trips. In addition, consumer location choices are influenced by random idiosyncratic preferences. The authors' numerical results show that, in such a framework, a congestion-toll regime raises welfare while imposition of a UGB is welfare-reducing. The UGB's harmful impact differs, of course, from the positive impact that arises in the standard model, and this difference may reflect Anas and Rhee's criterion for setting the UGB or perhaps the atypical nature of their model (see Pines (2006)). In carrying out the present research, the goal was to provide a counterpoint to this negative finding by highlighting

and quantifying the positive impact of UGBs in the standard model. However, as seen below, the present results convey a message that, in the end, is not too different from that of Anas and Rhee.

Section 2 of the paper presents the analytical framework used in the numerical calculations, section 3 presents the results of those calculations, and section 4 offers conclusions while discussing the broader applicability of the findings.

# 2. Analytical Framework

2a. The setup

The analytical framework relies on the standard model of a congested monocentric city, as developed in many previous papers. It also incorporates several auxiliary assumptions used in the model of Pines and Sadka (1985), as explained below.

Consumers are assumed to have Cobb-Douglas preferences over consumption of housing, denoted q and measured in square feet of floor space, and the nonhousing good c, with the utility function given by  $v(c,q) = c^{1-\alpha}q^{\alpha}$ , where  $0 < \alpha < 1$ . Utility is maximized with respect to the budget constraint c + pq = y - t(x), where p is the price per square foot of housing, p is income, and p is commuting cost at distance p from the CBD. Substituting the resulting demand functions back into the utility function, equating the result to a parametric utility level p0, and solving for p1 yields  $p = p(y-t(x))^{\frac{1}{\alpha}} p^{-\frac{1}{\alpha}}$ , where p1 is a constant. Substituting this housing price function into the demand function for p2 yields  $p = p(y-t(x))^{\frac{(\alpha-1)}{\alpha}} p^{\frac{1}{\alpha}}$ , where p2 is a constant.

Housing output, measured in square feet of floor space per unit of land, is given by  $\theta S^{\beta}$ , where S represents housing capital per unit of land and  $0 < \beta < 1$ . Housing developers maximize profit per unit of land, given by  $p\theta S^{\beta} - S - r$ , where r is rent per unit of land and the price of capital is normalized at unity. Solving the relevant first-order condition for S and substituting p yields  $S = \Lambda(y - t(x))^{\kappa}u^{-\kappa}$ , where  $\kappa = 1/\alpha(1-\beta)$  and  $\Lambda$  is a constant. Substituting this solution into the profit function, equating the result to zero, and solving for land rent yields

$$r = \Omega(y - t(x))^{\kappa} u^{-\kappa} \equiv r(y - t(x), u), \tag{1}$$

where  $\Omega$  is a constant. Finally, noting that population density D equals housing square feet per unit of land divided by square feet per dwelling, it follows that  $D = \theta S^{\beta}/q$ . Substituting the previous solutions,

$$D = \Phi(y - t(x))^{\kappa - 1} u^{-\kappa} \equiv D(y - t(x), u), \tag{2}$$

where  $\Phi$  is a constant.

The city is assumed to be circular, and a constant fraction  $1-\rho$  of the land at each distance is available for housing, with the fraction  $\rho$  used for a radial road network. Since the fraction of land devoted to roads appears to decline with x in real cities, the constancy of  $\rho$  is unrealistic. However, this assumption follows Pines and Sadka (1985), Wheaton (1998) and other papers in the literature. With the residential land available at distance x given by  $2\pi x(1-\rho)$ , the number of residents living beyond a distance x from the CBD is given by

$$n(x) = \int_{x}^{\overline{x}} 2\pi s (1 - \rho) D(y - t(s), u) ds, \qquad (3)$$

where  $\overline{x}$  is the distance to the urban boundary.

With congested travel, the cost per mile of commuting at distance x, denoted T(x), depends on the traffic flow across the ring at x (given by n(x)) relative to the road width at x, equal to  $2\pi x\rho$ . Adopting the functional form used in much of the prior literature,

$$T(x) = \eta + \delta \left[ \frac{n(x)}{2\pi x \rho} \right]^{\gamma}, \tag{4}$$

where all parameters are positive.

Differentiating (4) with respect to n(x), the increase in cost per mile at x when another commuter is added to the traffic flow equals  $\gamma \delta[n(x)/2\pi x \rho]^{\gamma-1}(1/2\pi x \rho)$ . Multiplying by n(x), which gives the number of commuters affected, then yields the total damage from the congestion externality at x. The congestion toll per mile at x, which charges commuters for this damage, is thus given by

$$\tau(x) = \gamma \delta \left[ \frac{n(x)}{2\pi x \rho} \right]^{\gamma}. \tag{5}$$

Commuting cost from distance x, inclusive of the toll, equals the sum of the toll per mile and direct costs per mile across distances inside of x. Thus, the function t(x) appearing in the above equations satisfies

$$t(x) = \int_{1}^{x} [T(s) + \tau(s)]ds. \tag{6}$$

In a city where no toll is levied,  $\tau(x)$  in (5) is set to zero. Note in (6) that the CBD extends out to x = 1, with commuting cost in its interior equal to zero.<sup>2</sup>

Differentiating (3) and (6) with respect to x yields the following relationships, which play a central role in the numerical exercise:

$$n'(x) = -2\pi x (1 - \rho) D(y - t(x), u) \tag{7}$$

$$t'(x) = T(x) + \tau(x). \tag{8}$$

Eq. (7) indicates that, as x increases, the population outside of x declines at a rate equal to the population residing at x. Eq. (8) shows that commuting cost rises with x at a rate equal to the direct cost per mile at x plus the toll at x. In addition to (7), the function n(x) satisfies the constraints  $n(\overline{x}) = 0$  and n(1) = N, where N is the fixed city population.

In order to conduct a straightforward welfare analysis, the city is assumed to be fully closed, following Pines and Sadka (1985), with an equal share of differential residential land rent accruing to each urban resident as income.<sup>3</sup> In addition, congestion-toll revenue is redistributed to consumers on an equal per capita basis, possibly reflecting a reduction in other unmodeled taxes. Therefore, letting  $r_a$  denote agricultural rent, the income parameter y appearing in the equations above must satisfy

$$y = y_{exog} + \frac{1}{N} \int_{1}^{\overline{x}} 2\pi x (1 - \rho) [r(y - t(x), u) - r_a] dx + \frac{1}{N} \int_{1}^{\overline{x}} n(x) \tau(x) dx$$

$$\equiv y_{exog} + y_{rent} + y_{toll}, \tag{9}$$

where  $y_{exog}$ ,  $y_{rent}$  and  $y_{toll}$  are the components of y from exogenous sources, redistributed rents, and redistributed toll revenues, respectively. Note that although y appears explicitly in

the first integral in (9), n(x), T(x) and  $\tau(x)$  also depend implicitly on this variable. Thus, the level of y affects both  $y_{rent}$  and  $y_{toll}$ , and (9) requires that this level is self-validating in that the components on the RHS add up to y itself. Accordingly, in the numerical exercise, y is treated as an unknown to be determined, with  $y_{exog}$  set parametrically.

An additional condition that is sometimes relevant requires that rent at the edge of the city equals the agricultural rent. This condition is written

$$r(y - t(\overline{x}), u) = r_a. (10)$$

#### 2b. Finding the equilibrium

In both the laissez-faire and toll-regime cases, the variables y, u and  $\overline{x}$  must assume values such that eqs. (1)–(6) and (9)–(10) are satisfied along with the above endpoint constraints on n(x). By contrast, when a UGB is imposed,  $\overline{x}$  is set exogenously, and eq. (10) does not apply (the congestion toll  $\tau(x)$  is also set at zero).

It is important to note that, in contrast to urban models without congestion, finding the equilibrium is not simply a matter of computing the solution to a set of static simultaneous equations. To see the reason, observe that T(x), commuting cost per mile at x, depends from (4) on the distribution of the urban population across all locations in the city, which determines n(x) and thus the traffic flow at x. Since T(x) in turn helps determine t(x) and thus population density at x via (2), it follows that density at any one location in the city depends on densities at all other locations. Given this interdependence, the equilibrium must be computed using an iterative procedure that relies on the key equations (7) and (8), which involve the derivatives of the n and t functions.<sup>4</sup>

This procedure works as follows. The city is divided into narrow, discrete rings indexed by i, each with a width  $\epsilon$ , set at a small value. The relationship  $x_i = 1 + \epsilon (i - 1)$  gives the inner radius of ring i, so that ring 1 has inner radius 1, corresponding to the edge of the CBD. In computing the variables of the model, the distance measure x is replaced by the ring subscript i. Thus, r(y - t(x), u) and D(y - t(x), u) in (1) and (2) are replaced by

$$r_i = \Omega(y - t_i)^{\kappa} u^{-\kappa}; \qquad D_i = \Phi(y - t_i)^{\kappa - 1} u^{-\kappa}. \tag{11}$$

In addition, (4) and (5) are used to write

$$T_i = \eta + \delta \left[ \frac{n_i}{2\pi x_i \rho} \right]^{\gamma}; \qquad \tau_i = \gamma \delta \left[ \frac{n_i}{2\pi x_i \rho} \right]^{\gamma}.$$
 (12)

The variable  $n_i$  is incremented using recursive relationship

$$n_{i+1} = n_i + \epsilon n'(x_i) = n_i - \epsilon 2\pi x_i (1-\rho) D_i,$$
 (13)

where the first equality is based on a first-order approximation and the second uses (7) to substitute for  $n'(x_i)$ . Similarly,  $t_i$  is incremented using

$$t_{i+1} = t_i + \epsilon t'(x_i) = t_i + \epsilon (T_i + \tau_i), \tag{14}$$

where the second equality uses (8) to substitute for  $t'(x_i)$ . The iterative process starts at i = 1, with  $t_1 = 0$  (indicating no commuting cost from the CBD edge) and  $n_1 = N$ .

The iterative process is carried out conditional on the values of y and u, but these values must be consistent with the achievement of equilibrium. To understand this point, note first that, in the laissez-faire and toll-regime cases, the iterations stop when i reaches a value  $i^*$  such that  $n_{i^*} \geq 0$  and  $n_{i^*+1} < 0$ , indicating that the population just fits inside an  $\overline{x}$  value of  $x_{i^*} = 1 + \epsilon (i^* - 1)$ . Satisfaction of two equilibrium conditions is then checked. First, the value of  $r_{i^*}$  is compared to  $r_a$ . Second, the value of  $y_{exog} + y_{rent} + y_{toll}$  from (9), which has been computed cumulatively in a discrete manner over the sequence of iterations, is compared to assumed value of y. If the comparison values diverge by more than the desired degree of accuracy in either case, the values of y and u are adjusted and the iterative process is repeated.<sup>5</sup>

In the UGB case, the iterations stop when i reaches a value  $i^{**}$  such that  $x_{i^{**}}$  equals the specified value of  $\overline{x}$ . The previous comparison involving y is then carried out, and  $n_{i^{**}}$  is compared to zero. Both y and u are adjusted until the comparison values match with a given degree of accuracy.

## 3. Numerical Results

This section presents several numerical examples, each of which involves a comparison of three equilibria: the laissez-faire equilibrium, the equilibrium under the toll regime, and the equilibrium with an optimally chosen UGB. The goal is to gauge the efficacy of a UGB by comparing the welfare gain it generates to the gain realized under the first-best toll regime.

The first four examples rely on a host of assumptions on parameter values, as follows. The housing exponent  $\alpha$  in the Cobb-Douglas utility function is set at 0.15, and the exponent  $\beta$  in the housing production function is set at 0.85 (the multiplicative factor  $\theta$  is set at 0.0001). Twenty percent of the land in each ring is devoted to roads, so that  $\rho=0.2$ . The intercept parameter  $\eta$  in the commuting-cost function (4), which is taken to represent the money cost of travel, is set at \$225, reflecting a \$0.36 cost per mile (the current Federal allowance), 250 round trips per year, and 1.25 workers per household (as in Bertaud and Brueckner (2005)). The values of the other commuting-cost parameters ( $\delta$  and  $\gamma$ ) differ across the numerical examples, as explained below. Again following Bertaud and Brueckner (2005), agricultural land rent  $r_a$  is set at \$40,000 per square mile, reflecting a land value of \$1210 per acre (average US agricultural land value in 2000) and a discount rate of 5 percent. The exogenous income value  $y_{exog}$  is set at \$40,000, a figure approximately equal to US household income in 2000, and the city population N is set at 3 million. Finally, the parameter  $\epsilon$ , which represents ring width, is set at 0.001 miles, a small value intended to achieve a high level of accuracy in identifying the equilibria.

While these parameter values generate a fairly realistic spatial size for the city, the housing exponent  $\alpha$  is arguably too small. To provide sensitivity analysis, examples 1 through 4 are supplemented with a fifth example where  $\alpha$  is raised from 0.15 to 0.35, a more realistic value. However, to keep the radius the city from expanding dramatically in response to the resulting increase in housing demand, agricultural land rent is raised by more than a factor of 6, up to a value of \$250,000 per square mile.

In example 1, the congestion exponent  $\gamma$  in (4) is set equal to 1.50, with the multiplicative factor  $\delta$  set at 0.000001.<sup>6</sup> This example is shown in the topmost section of Table 1, with results for the laissez-faire equilibrium given in the first row. In this equilibrium, the city radius is

22.683 miles, and the utility level is 7771.8997. The value of  $y_{rent}$  is \$853, so that total income y equals \$40,835. Commuting cost at the edge of the city, denoted  $\overline{t}$  in the table, is \$7558, which represents 19 percent of income. Population density  $D_1$  at the edge of the CBD is 388,861 persons per square mile, and land rent  $r_1$  at this location is \$357 million per square mile.

Results for the toll regime are shown in the next line of Table 1. Under that regime, the city radius shrinks by about 1.5 miles to 21.158 miles, a decline of about 7 percent. Central density rises by almost a factor of four, to 1,432,612 persons per square mile, and central land rent rises by a similar factor. Rental income rises slightly, but income from redistributed tolls equals \$1582, leading to a notably higher y value of \$42,449. Commuting cost  $\overline{t}$  at the edge of the city, which now includes the toll, rises to \$8112. In results not shown in the Table, the toll's share in commuting cost per mile (equal to  $\tau(x)/[T(x) + \tau(x)]$ ) falls from a high of 59 percent at the CBD's edge to 40 percent at x = 10 and then to 29 percent at  $\overline{x}$ , reflecting a decline in congestion moving away from the CBD.

The utility gain under the toll regime is 66.1881 relative to the laissez-faire case. Different approaches can be used to derive the dollar equivalent of this gain, but the following approach seems most natural. The laissez-faire model is solved holding utility fixed at the first-best level achieved under the toll regime, with  $y_{exog}$  and y adjusted to achieve equilibrium. The resulting increase in  $y_{exog}$  tells how much exogenous income would have to rise in the laissez-faire case to generate the first-best utility level. For example 1, that income increase is equal to \$335, or 0.8 percent of  $y_{exog}$ . The magnitude of this dollar welfare gain is discussed further below.<sup>7</sup>

Before turning to the analysis of the UGB case, consider the laissez-faire and toll-regime equilibria for the remaining examples. Under example 2, shown in the second part of Table 1, the congestion exponent  $\gamma$  is reduced to 1.25, with  $\delta$  raised to 0.00002. Note that while the decline in  $\gamma$  indicates lower congestion, this change is partly offset by the higher  $\delta$ . Imposition of the toll regime shrinks the city radius from 23.293 (a larger value than in the first example) to 21.906 miles. Central density and land rent start out lower than before, but their proportional increases under the toll regime are similar to those in the first example. The values of  $\bar{t}$  and  $y_{toll}$  are smaller than in example 1, partly reflecting lower tolls. The toll regime raises utility

by 31.7010, a smaller increase than in example 1. This utility gain is equivalent to a \$158 increase in  $y_{exog}$ , a gain of 0.4 percent.

Congestion is reduced further in example 3, where  $\gamma=1.12$  and  $\delta=.0001$ , leading to a larger city radius and a reduction in central density and rent in the laissez-faire case. Imposition of the toll regime raises utility by 21.122, a yet-smaller amount that is equivalent to a \$107 increase in  $y_{exog}$ , while generating now-familiar changes in the remaining variables. In example 4, the last case considered,  $\gamma$  is reduced to 1.00, the value used by Wheaton (1978), while the value of  $\delta$  is unchanged. The city's laissez-faire radius rises again, while central density and rent are further reduced. The changes under the toll regime again follow previous patterns, but now the utility gain is miniscule, being equal to 1.0372. This gain is equivalent to an increase in  $y_{exog}$  of only \$5.

Consider now the UGB cases. For each example, the congestion toll is set at zero, and UGB equilibria are computed for a series of  $\overline{x}$  values ranging below the laissez-faire equilibrium value ( $\overline{x}$  is reduced in steps of 0.1 miles). The  $\overline{x}$  value in the series associated with the highest utility level is then selected. The typical pattern of utilities resulting from tightening of the UGB is shown in Figure 1, which pertains to example 3.

Turning to the numerical results, the optimal  $\bar{x}$  value in example 1 is 18.9 miles, a radius 2.25 miles smaller than under the toll regime. Central density increases only slightly under the optimal UGB, in constrast to the dramatic increase under the toll regime, while  $\bar{t}$  falls and  $y_{rent}$  and hence y show slight increases. As noted above, the UGB's failure to foster strong central densification in a congested city limits its efficacy, a conclusion that is dramatically illustrated by the small utility gain under the UGB relative to the laissez-faire case. This increase is only 0.4409, a magnitude that represents only 0.7 percent of the utility gain under the first-best toll regime.

A similar pattern appears in examples 2–4. In each case, the optimal UGB lies inside the toll-regime's  $\overline{x}$ , as illustrated in Figure 1 for example 3. In addition, central densification with the UGB is only slight, and the utility increase relative to the laissez-fare case is small. Interestingly, for all the examples, the utility increase ranges between 0.7 and 0.8 percent of the gain under the toll regime.

To provide further sensitivity analysis, example 5 shows the effect of raising  $\alpha$ , the Cobb-Douglas housing exponent, from 0.15 to 0.35 while increasing  $r_a$  to \$250,000 ( $\gamma$  is set at 1.50). Despite a higher agricultural rent, the spatial size of the city grows in response to the larger  $\alpha$ , with the laissez-faire  $\overline{x}$  now equal to 32.058 miles. Correspondingly, central density (now 48,302 persons per square mile) is much lower than in example 1. Imposition of the toll regime shrinks the city radius to 30.1 miles, while again raising central density and land rent substantially. The toll regime generates a utility increase of 5.5875, which is equivalent to an income gain of \$327, a value close to that in example 1. Imposition of an optimal UGB (at x = 29.3) again leads to only a slight increase in central density, and it generates a utility gain of 0.1185, equal to only 2.1% of the gain under the toll regime.

Thus, example 5 reconfirms the patterns seen in examples 1–4, suggesting what seems to be a robust conclusion: a UGB is a very ineffective substitute for the first-best toll regime, yielding at best a few percentage points of the utility gain generated by that regime. For a city described by the monocentric model, the analysis therefore implies that a UGB is virtually useless as a tool for attacking the distortions induced by unpriced traffic congestion.<sup>8</sup>

While this finding constitutes the main result of the paper, another noteworthy conclusion is that the dollar-equivalent welfare gain from the toll regime can be quite small, as seen above. While the \$335 gain under example 1 nonnegligible, the \$5 gain in the less-congested case of example 4 is surprisingly slight. However, it should be noted that trip length is the only marginal of adjustment available to commuters in the present model, which may impart a downward bias to the welfare gain. In reality, consumers would adjust on other margins in response to a toll regime: switching from automobile travel to public transit, shifting travel times toward off-peak hours, carpooling, and switching to less-crowded untolled routes. The absence of these adjustments limits the gain from the toll regime, making it negligible in size in the less-congested examples in Table 1.9

## 4. Conclusion

This paper has evaluated the efficacy of urban growth boundaries as a second-best remedy for unpriced traffic congestion. The numerical results suggest that a UGB is a very poor substitute for a first-best toll regime, capturing only a tiny fraction of the welfare gain it generates. The results thus imply that a UGB is virtually useless as a second-best policy instrument in a congested city.

Since these conclusions emerge from a monocentric model, they could be undermined by criticisms that question the realism of that model, which often point to its poor representation of our increasingly polycentric cities. Despite such reservations, the conclusions of the analysis are likely to be reasonably robust to modifications that preserve the monocentric model's fundamental linkage between population density and transportation costs. As seen above, failure of the UGB to appreciably raise densities near employment centers is the main reason for its poor performance, and this failure will persist regardless of whether the city has one or many such centers.

This basic conclusion should also apply in models like that of Anas and Rhee (1994), which adds many nonstandard features beyond polycentric land use to the standard model. However, as noted above, poor UGB performance emerges in an extreme form in Anas and Rhee's analysis, which shows that a UGB set according to their criteria is actually welfare reducing. This conclusion, which contradicts monocentric theory, may arise from a number of sources, as argued by Pines (2006). However, it does coincide with the message of the present paper, which is that urban planners searching for a second-best remedy for unpriced traffic congestion should not view the UGB as a useful instrument.

Table 1
Numerical Results

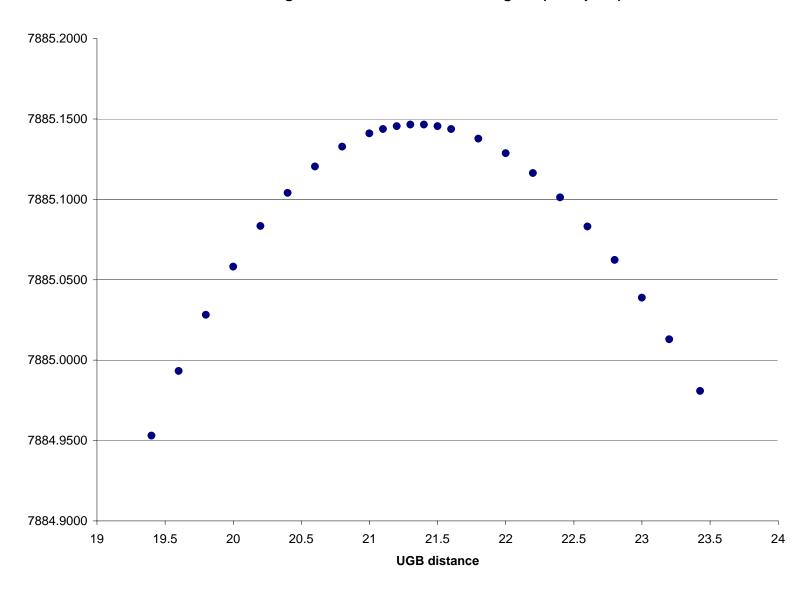
#1  $(\gamma = 1.50, \delta = .000001)$ 

			"	( )		,			
	$\overline{x}$	utility	gain	y	$y_{rent}$	$y_{toll}$	$\overline{t}$	$D_1$	$r_1$
laissez faire	22.683	7710.8997	_	40,835.28	835.28	0	7558.55	388,861	$3.573 \times 10^{8}$
toll regime	21.158	7777.0878	66.1881	42,449.13	866.47	1582.67	8886.81	1,432,612	$1.368 \times 10^9$
best UGB	18.9	7711.3406	0.4409 (0.7%)	40,841.80	841.80	0	6691.29	390,575	$3.589 \times 10^{8}$
			#	$= 2 \ (\gamma = 1.25,$	$\delta = .00002$	)			
	$\overline{x}$	utility	gain	y	$y_{rent}$	$y_{toll}$	$\overline{t}$	$D_1$	$r_1$
laissez faire	23.293	7840.0825	_	40,843.10	843.10	0	7009.01	183,377	$1.722 \times 10^{8}$
toll regime	21.906	7871.7835	31.7010	42,085.21	863.83	1221.39	8112.99	575,534	$5.450 \times 10^8$
best UGB	20.8	7840.3000	0.2175 (0.7%)	40,847.51	847.51	0	6439.94	188,026	$1.728 \times 10^8$
			#.	#3 $(\gamma = 1.12)$	$\lambda,  \delta = .0001)$	)			
	$\overline{x}$	utility	gain	y	$y_{rent}$	$y_{toll}$	$\overline{t}$	$D_1$	$r_1$
laissez faire	23.426	7884.9808	_	40,846.56	846.56	0	6818.73	145,911	$1.341 \times 10^{8}$
toll regime	22.122	7906.6760	21.6952	42,897.93	863.47	1034.45	7776.38	389,569	$3.672 \times 10^{8}$
best UGB	21.4	7885.1465	$0.1657 \ (0.8\%)$	40,850.17	850.17	0	6356.00	146,335	$1.345 \times 10^8$
			#	$\#4 \ (\gamma = 1.00)$	$\delta = .0001$				
	$\overline{x}$	utility	gain	y	<i>Yrent</i>	$y_{toll}$	$\overline{t}$	$D_1$	$r_1$
laissez faire	25.200	8107.8021		40,858.34	858.34	0	5868.49	42,820	$3.936 \times 10^{7}$
toll regime	24.826	8108.8393	1.0372	41,169.08	861.45	307.63	6174.65	59,173	$5.481 \times 10^7$
best UGB	24.8	8107.8099	$0.0078 \ (0.8\%)$	40,859.11	859.11	0	5778.20	42,853	$3.940 \times 10^7$

#5  $(\gamma = 1.50, \delta = .000001, \alpha = 0.35, r_a = 250,000)$ 

	$\overline{x}$	utility	gain	y	$y_{rent}$	$y_{toll}$	$\overline{t}$	$D_1$	$r_1$
laissez faire toll regime best UGB	32.058 30.113 29.3	623.9792 626.5667 624.0977	5.5875 0.1185 (2.1%)	41,639.09 45,905.40 41,683.68	1639.09 1755.68 1683.68	$0 \\ 4149.72 \\ 0$	11324.40 15319.19 10676.69	48,302 237,006 49,067	$10.559 \times 10^{7}$ $57.119 \times 10^{7}$ $10.738 \times 10^{7}$

Figure 1: Welfare under a UGB regime (example 3)



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## **Footnotes**

- \*I thank David Pines for helpful discussions in the course of this work and am grateful to Alex Anas and Ken Small for useful comments on an early draft. I also thank Richard Arnott and two referees for helpful suggestions. These individuals are not responsible, however, for any shortcomings in the paper.
- <sup>1</sup>See Brueckner (2000, 2001) and Ding, Knaap and Hopkins (1999) for institutional discussion regarding the use of UGBs.
- <sup>2</sup>This assumption follows Wheaton (1978). While a different value for the CBD boundary would not have a qualitative effect on the results, the CBD cannot be a point at x = 0 because the adjacent road width is then zero, implying infinite commuting cost per mile near the CBD under (4).
- <sup>3</sup>It is assumed that transport land is acquired by the city at the agricultural rent, so that the differential rent it generates equals zero.
- <sup>4</sup>The equilibrium is presumed to be unique, although no proof of this assumption is given. The numerical calculations never suggested the existence of multiple equilibria.
- <sup>5</sup>The utility level u is adjusted in increments of .0001, and at the equilibrium value in the laissez-faire and toll-regime cases, changing u causes  $r_{i^*}$  to jump from one side of  $r_a$  to the other. The second requirement is that  $y_{exog} + y_{rent} + y_{toll}$  must match y to the second decimal place, a condition that is achieved by appropriate adjustment of y as u is changed. In the UGB equilibrium, changing u causes  $n_{i^{**}}$  to jump from one side of zero to the other, while y is adjusted as above.
- <sup>6</sup>An attempt was made to carry out computations for the larger  $\gamma$  value of 2.0, but the adjustment methods for u and y failed in this case (see footnote 5), so that the equilibrium could not be identified.
- <sup>7</sup>It is worth noting that Wheaton's (1998) calculations yield welfare gains from correcting the congestion externality that are much larger than the present ones. However, the approaches in the two papers differ. First, while Wheaton computes the social optimum for his city, he does not decentralize the optimum via an explicit toll regime. Redistribution of the toll revenue in the present model should apparently limit any effect from this difference, although a firm conclusion is hard to draw. Second, land rent in Wheaton's model flows to absentee owners rather than being redistributed to urban residents, and this feature

presents an obstacle to straightforward computation of an aggregate welfare gain. In fact, Wheaton's description of his procedure for aggregating the impacts on city residents and absentee landowners is not clear. Despite these differences in approach, the land-use changes in moving from the laissez-faire equilibrium to the first-best optimal city are similar in both papers.

<sup>8</sup>As discussed by Brueckner (2001), imposition of a UGB when one is not needed reduces welfare. In the present model, if congestion is eliminated by setting  $\delta$  equal to zero, and a UGB reduces the city radius by ten percent below its laissez-faire value (as in Table 1), then utility falls by 0.4088, a small effect. Brueckner (2001) shows that a much larger welfare loss can be generated by a more stringent UGB.

<sup>&</sup>lt;sup>9</sup>The presence of these additional margins could also improve the performance of the toll-regime relative to the (already poor) performance of the UGB.