Modal Symmetry and Modal Structure*

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Abstract

When does adding a tensor field to a spacetime increase its structure? One influential answer: whenever the tensor field is not preserved by some "symmetry" of the spacetime. This answer is inadequate if symmetries are interpreted as structure preserving maps from the spacetime to itself; so-called "Heraclitus" spacetimes have no nontrivial symmetries of this kind and therefore preserve any tensor field whatsoever. Here, we propose a more adequate "modal" approach to symmetries as structure preserving maps between spacetimes. Modal symmetries can be used to show that Heraclitus spacetimes are, in fact, not necessarily "maxed out" structurally.

1 Introduction

Spacetime symmetry and spacetime structure seem to be closely connected (Earman 1989; North 2021). Consider a (relativistic) spacetime, i.e. a pair (M, g) where M is a connected, Hausdorff manifold and g is a smooth Lorentzian metric on M. Let X be a smooth tensor field on M. One wonders: When does the triple (M, g, X) have more structure than the pair (M, g)? An influential answer has the following general form (Swanson and Halvorson 2012; Barrett 2015).

(Sym) The triple (M, g, X) has more structure than the pair (M, g) iff there is a "symmetry" of (M, g) that fails to preserve X.

In what follows, we will consider several ways of making the **(Sym)** condition precise. It is well known that a "global" variant breaks down when the

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spacetime (M, g) under consideration fails to have non-trivial global symmetries (Wilhelm 2021; Barrett et al. 2023). Moving to a "local" variant does not seem to help given the existence of "Heraclitus" spacetimes that fail to have non-trivial local symmetries (Manchak and Barrett 2026). Here, we introduce a "modal" variant of **(Sym)** which seems to more adequately handle the problem cases. We then consider an influential "categorical" approach to understanding symmetry and structure due to Baez et al. (2006). We use this approach to formulate a categorical variant of **(Sym)** and show how it is related to the modal variant. Along the way, we illustrate the ideas with a number of simple examples and articulate some open questions of philosophical interest.

2 Global Symmetry

The original approach to **(Sym)** concerned global symmetries (Swanson and Halvorson 2012; Barrett 2015). A **global symmetry** of a spacetime (M, g) is an isometry from the spacetime to itself, i.e. a diffeomorphism $f : M \to M$ such that $f^*(g) = g$ (Wald 1984). We say a smooth tensor field X on M is **preserved** by a global symmetry $f : M \to M$ of (M, g) if $f^*(X) = X$. Now consider the following condition.

(Glob Sym) The triple (M, g, X) has more structure than the pair (M, g) iff X is not preserved by some global symmetry of (M, g).

The condition is perfectly adequate when (M, g) is Minkowski spacetime. For example, the only metrics h on M that are preserved by all of the global symmetries of Minkowski spacetime (M, g) are of the form $h = cg$ for some positive real number c. If h does not have this form, then (M, g, h) has more structure than (M, g) according to **(Glob Sym)** as one would expect. But now consider a spacetime (M, g) whose only global symmetry is the identity map on M. Perhaps the simplest example is the following.

Example. Let $(\mathbb{R}^2, g)$ be two-dimensional Minkowski spacetime in standard (t, x) coordinates. Now let $M = \mathbb{R}^2 - \{(0, 0), (1, 0), (0, 1)\}$. One can verify that the only global symmetry of the spacetime (M, g) is the identity map on M. The three "missing" points destroy all of the non-trivial symmetries of two-dimensional Minkowski spacetime: translations, reflections, and boosts.

The example poses problems for **(Glob Sym)** since any tensor field X

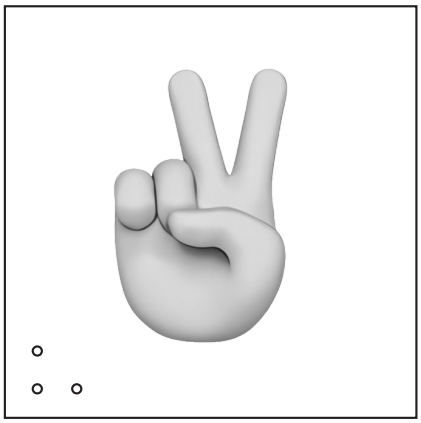

Figure 1: A scalar field bearing a striking resemblance to a peace sign emoji is preserved by the global symmetries of the mutilated Minkowski spacetime.

on M is preserved by the global symmetries of the spacetime (Wilhelm 2021; Barrett et al. 2023). This means that the triple (M, g, X) does not have more structure than the pair (M, g) according to **(Glob Sym)** no matter the choice of X. Suppose, for example, that X is a scalar field on M with contours that bear a striking resemblance to a peace sign emoji (see Figure 1). Surely (M, g, X) has more structure than (M, g) and yet, according to **(Glob Sym)**, it does not. Making matters worse, it turns out that problem cases like this are not rare. Indeed, the property of having only trivial global symmetries seems to be "generic" among all spacetimes (Mounound 2015).

3 Local Symmetry

The problems associated with **(Glob Sym)** suggest a natural fix: liberalize the notion of symmetries so as to include all "local" types as well. A **local symmetry** of a spacetime (M, g) is an isometry between a pair of open sets $U, V \subseteq M$, i.e. a diffeomorphism $f : U \to V$ such that $f^*(g) = g$. A smooth tensor field X on M is **preserved** by a local symmetry $f : U \to V$ of (M, g) if $f^*(X) = X$. Now consider the following condition (Barrett and Manchak 2025).

(Loc Sym) The triple (M, g, X) has more structure than the pair (M, g) iff X is not preserved by some local symmetry of (M, g).

To appreciate the difference between **(Glob Sym)** and **(Loc Sym)**, let us return to the mutilated Minkowski spacetime example from the previous section. Because this spacetime (M, g) is flat, any pair of points in M will have neighborhoods U and V that are isometric. So the peace sign emoji field X will not be preserved by some local symmetry (see Figure 2). It follows that the triple (M, g, X) has more structure than the pair (M, g) according to **(Loc Sym)** as one would hope.

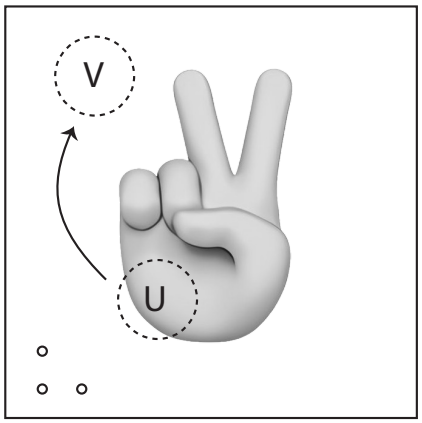

Figure 2: The peace sign emoji field is not preserved by the local symmetry taking the U into V.

But now there is a new problem: there are so called "Heraclitus" spacetimes with only trivial local symmetries (Manchak and Barrett 2026). A spacetime (M, g) with the Heraclitus property is such that, for all open connected open sets $U, V \subseteq M$ and any isometry $f : U \to V$, we have $U = V$ and f is the identity map. As with spacetimes with only trivial global symmetries, it would seem that Heraclitus spacetimes are generic in some sense (cf. Sunada 1985). Let's take a look at an example.

Example. Let (M, g) be two-dimensional Minkowski spacetime in standard (t, x) coordinates. Let N be the region of M such that $t, x > 0$ and $t^2 > x^2$. Let $\Omega = 1/(t^2 + x^2)$ and let $h = \Omega^2 g$. One can verify that the spacetime (N, h) is Heraclitus. To see this, consider the invariant scalar curvature functions $Q, R : N \to \mathbb{R}$ where R is the Ricci scalar and Q is the magnitude of dR according to the inverse of the metric h. One can show that, for any distinct points $p, q \in N$, either $Q(p) \neq Q(q)$ or $R(p) \neq R(q)$. Since Q and R are invariant under any isometry $f : U \to V$ between open connected open

sets $U, V \subseteq N$, we know that f must take any point into itself, i.e. it must be the identity map on $U = V$.

The Heraclitus example poses problems for **(Loc Sym)** since any tensor field X on N is preserved by the local symmetries of the spacetime. This means that the triple (N, h, X) does not have more structure than the pair (N, h) according to **(Loc Sym)** no matter what one chooses for X, e.g. a peace sign emoji scalar field. So (N, h, X) does not have more structure than (N, h) according to **(Loc Sym)**.

What are we to make of this state of affairs? Intuitively, adding a peace sign emoji scalar field to a Heraclitus spacetime seems to increase the structure. But perhaps this wrong. Here, we draw attention to the fact that some (all?) Heraclitus spacetimes can, in some sense, explicitly define any scalar function via some choice of invariant scalar curvature functions.

Example. Let (N, h) be the Heraclitus spacetime constructed above and let Q and R be the invariant curvature functions defined there. One can verify that $R(t, x) = 8(t^2 - x^2)$ and $Q(t, x) = -32R(t^2 + x^2)^2$ (Manchak and Barrett 2026). For convenience, let $A = R/8$ and $B = (-Q/32R)^{1/2}$. Now, consider the invariant curvature functions $S = ((B - A)/2)^{1/2}$ and $T = ((B+A)/2)^{1/2}$. A bit of algebra shows that S and T perfectly mimic the coordinates: $S(t, x) = x$ and $T(t, x) = t$. This means that $X(t, x) = X(T, S)$ for any scalar field $X : N \to \mathbb{R}$. In other words, any scalar function X can be represented as a function only of invariant structures explicitly definable from the spacetime (M, g).

Given this example, perhaps it is not too troubling that (N, h, X) does not have more structure than the pair (N, h) according to **(Loc Sym)** no matter the choice of X. Maybe the radical asymmetry present in Heraclitus spacetimes means that they are structurally "maxed out" in some sense? In the following section, we consider this question.

4 Modal Symmetry

So far, we have taken spacetime "symmetries" to mean structure preserving maps from (a portion of) a spacetime to (a portion of) itself. But focusing on symmetries of this kind suggests that Heraclitus spacetimes might be "maxed out" structurally: adding an arbitrary tensor field to a Heraclitus spacetime does not increase its structure according to the variants

of **(Sym)** considered so far. In order clarify the situation, we now transition to a more general understanding of symmetries as structure preserving maps from (a portion of) one spacetime to (a portion of) another spacetime. These "modal" symmetries are closely related to spacetime "extensions" (see Manchak 2021) and can used to show that some Heraclitus spacetimes are, in fact, not structurally "maxed out" after all. To do this, we introduce a modal variant of the **(Sym)** condition that more adequately captures the relationship between spacetime symmetry and spacetime structure. This makes precise the informal suggestion of Manchak and Barrett (2026, p. 22) along these lines:

> "In order to judge amounts of structure, one is now not just looking at maps from the spacetime to itself. Rather, one has to take a much more holistic approach. Only by looking at maps from our spacetime to and from other spacetimes of the same kind (and maps between these other spacetimes too) can one hope to use symmetries as a complete guide to amounts of structure."

Let us say that a (local) **modal symmetry** of a spacetime (M, g) is an isometry from an open set $U \subseteq M$ to an open set $V \subseteq N$ of some spacetime (N, h), i.e. a diffeomorphism $f : U \to V$ such that $f^*(h) = g$. A tensor field X on (M, g) is **preserved** by a modal symmetry $f : U \to V$ of (M, g) from an open set $U \subseteq M$ to an open set $V \subseteq N$ of some spacetime (N, h) if one can define a smooth tensor field Y on N such that $f^*(Y) = X$. Now consider the following condition.

(Mod Sym) The triple (M, g, X) has more structure than the pair (M, g) iff X is not preserved by some modal symmetry of (M, g).

To appreciate the difference between **(Mod Sym)** and the previous **(Sym)** variants, consider the following example.

Example. Let (N, h) be the Heraclitus spacetime constructed above. Consider the Heraclitus spacetime (O, h) where O is the $t < 1$ portion of N. Let $X : O \to \mathbb{R}$ be the scalar field on the spacetime (O, h) given by $X(t, x) = 1/(1 - t)$. Of course, the inclusion map $i : O \to N$ defined by $i(t, x) = (t, x)$ is an isometry onto its image O and therefore a modal symmetry of (O, h). Now let Y be any tensor on N. If it were the case that $i^*(Y) = X$, it would follow that $Y = i_*(X) = X$ on the $O \subset N$ portion of

the spacetime (N, h). But this cannot be since X is the function $1/(1-t)$ and thus cannot be smoothly extended across $t = 1$ (see Figure 3). So $i^*(Y) \neq X$ and therefore X is not preserved by a modal symmetry of (O, h).

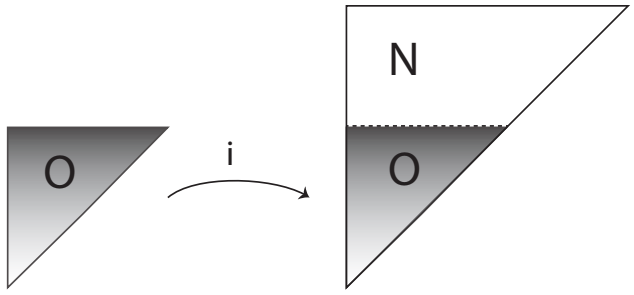

Figure 3: The scalar field X on (O, h) cannot be smoothly extended to a scalar field Y on (N, h) in such a way that $i^*(Y) = X$

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The example shows that the triple (O, h, X) has more structure than (O, h) according to **(Mod Sym)** despite the fact (O, h, X) does not have more structure than (O, h) according to both **(Glob Sym)** and **(Loc Sym)**. In other words, by considering modal symmetries, we find that a Heraclitus spacetime can fail to be "maxed out" structurally. This makes sense. The scalar field X cannot be uniformly reproduced on all spacetimes that share the same structure as (O, h), e.g. the $O \subset N$ portion of the spacetime (N, h). So $X(t, x) = 1/(1-t)$ is importantly different from other tensor fields on (O, h) such that the Ricci scalar field $R(t, x) = 8(t^2 - x^2)$ which are guaranteed be uniformly reproducible on all spacetimes that share the same structure as (O, h). For this reason, it would seem that X is not really part of the "structure" of (O, h).

Stepping back, one wonders: is there a spacetime (M, g) such that every tensor field X on M is preserved by the modal symmetries of (M, g)? We know such a spacetime would have to be Heraclitus. It would also have to be "maximal" so as to avoid the issue arising in the example just given, i.e. it could not be isometric to a proper portion of another spacetime (Manchak 2017). It is currently unknown if there exist maximal Heraclitus spacetimes (Manchak and Barrett 2024). But even if one could find an example, one must still show that that any tensor field on such a spacetime is preserved by all modal symmetries.

5 Categorical Symmetry

In the previous section, we found that the **(Mod Sym)** condition was a promising candidate for adequately capturing the relationship between symmetry and structure. We now show how this modal approach is connected to an influential "categorical" one due to Baez et al. (2006).

Essentially, a category is a collection of mathematical objects with arrows between the objects that satisfy some basic properties. The category of general relativity is sometimes defined as follows (Weatherall 2021, Halvorson and Manchak 2025): each object is a spacetimes (M, g) and each arrow is an isometry $f : M \to N$ from one object (M, g) to another object (N, h). This choice of arrows results in a meagre structure which is not very useful is capturing modal symmetries. Here is one way to see this. We say a category is **connected** if any pair of objects can be connected via a "zigzag", i.e. a finite series of arrows not necessarily pointing in the same direction. It is not difficult to verify that the category of general relativity as just defined is not connected since there is no zigzag between any pair of non-isometric spacetimes.

In order to better capture modal symmetries, let us now introduce more arrows into the category of general relativity (cf. Rosenstock et al. 2015, Barrett and Manchak 2026). An **isometric embedding** from a spacetime (M, g) to another spacetime (N, h) is a map $f : M \to N$ which is an isometry from (M, g) to (V, h) where $V = f[M]$ is some open connected subset of N. Let GR be the category where each object is a spacetimes (M, g) and each arrow is an isometric embedding $f : M \to N$ from one object (M, g) to another object (N, h). One can now show the following.

Proposition. The category GR is connected. Any pair of objects can be connected by a zigzag of four arrows or less.

Proof. Let (M, g) and (M', g') be any spacetimes and let p and p' be any points in M and M' respectively. Let (N, η) be Minkowski spacetime and let $O, O' \subset N$ be any disjoint open sets. There are neighborhoods U and U' of p and p' respectively and a spacetime (N, h) such that $h = \eta$ outside of $O \cup O'$ but (U, g) and (U', g') can be isometrically embedded into the regions O and O' respectively (see Lemma 3.2.1, Barrett and Manchak 2024). So (U, g) can be isometrically embedded into both (M, g) and (N, h) and (U', g') can be isometrically embedded into both (M', g') and (N, h). It follows that (M, g) is connected to (M', g') by a series of four arrows in the category GR (see Figure 4). □

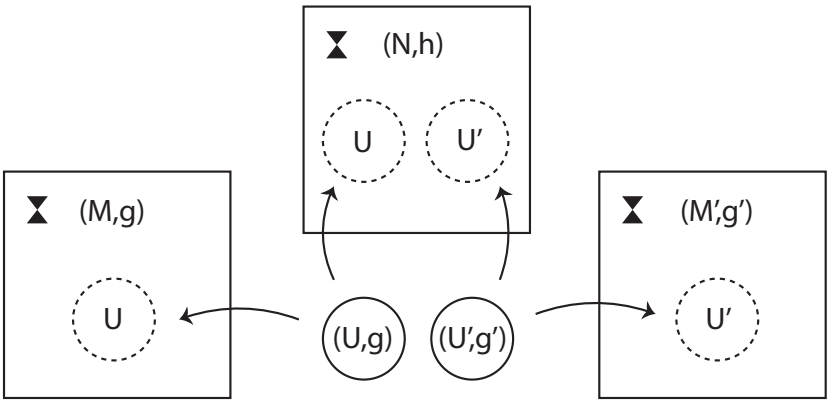

Figure 4: (U, g) can be isometrically embedded into both (M, g) and (N, h) and (U', g') can be isometrically embedded into both (M', g') and (N, h).

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We are now ready to consider the categorical approach to capturing a relationship symmetry and structure (Baez et al. 2006). Let C and D be categories. A functor $F : C \to D$ maps objects in C to objects in D and arrows in C to arrows in D in such a way that preserves some basic structure, e.g. the identity arrow from an object c in C to itself must be mapped to the identity arrow from the object Fc in D to itself. A functor $F : C \to D$ is **full** if, for all objects c and d in C and arrows $h : Fc \to Fd$ in D, there exists an arrow $f : c \to d$ in C such that $Ff = h$. We say a functor **forgets structure** if it is not full.

Example. Let Set be the category where each object is a set and each arrow is a function from one set to another. Let Top be the category where each object is a topological space and each arrow is a continuous function from one topological space to another. Let $\Pi : \text{Top} \to \text{Set}$ be the natural "projection" functor which maps any object (S, τ) in Top to the object S in Set and maps any arrow in Top to itself. The functor Π is not full and thus forgets (topological) structure. To see this, consider the set $\mathbb{R}$ and its standard topology σ. The functor Π maps the object $(\mathbb{R}, \sigma)$ in Top to the object $\mathbb{R}$ in Set. Let $h : \mathbb{R} \to \mathbb{R}$ be the function defined by $h(x) = 0$ for all $x \neq 0$ and $f(0) = 1$. This function h is an arrow in Set from the object $\mathbb{R}$ to itself. But since h is not a continuous function, there is no arrow f from

$(\mathbb{R}, \sigma)$ to itself in Top that Π maps to h in Set.

From the example, we can see how symmetry and structure are connected via the categorical approach. In the category Set, the arrow h from the object $\mathbb{R}$ to itself is a structure preserving map and thus a type of "categorical symmetry" within Set. But within the category Top, the function h is not a structure preserving map from $(\mathbb{R}, \sigma)$ to itself. So one might say that the topological structure σ is not "preserved" by the categorical symmetry h within Set. More generally, the fact that the projection functor Π is not full, i.e. forgets structure, signals that there are some topological structures that are not preserved by the categorical symmetries of Set. We now apply this general idea to the spacetime context.

In the natural way, we can take the "categorical symmetries" of GR to be its collection of arrows. For each spacetime (M, g), let $X_{(M,g)}$ be a smooth tensor field on (M, g). Let GR+$\{X_i\}$ be the category where each object is a triple $(M, g, X_{(M,g)})$ and each arrow from an object $(M, g, X_{(M,g)})$ to an object $(N, h, X_{(N,h)})$ is an isometric embedding $f : M \to N$ from (M, g) to (N, h) such that $f^*(X_{(N,h)}) = X_{(M,g)}$. The **projection functor** $\Pi : \text{GR} + \{X_i\} \to \text{GR}$ associated with $\{X_i\}$ is defined by taking each object $(M, g, X_{(M,g)})$ to the object (M, g) and each arrow $f : M \to N$ from $(M, g, X_{(M,g)})$ to $(N, h, X_{(N,h)})$ to itself. We say $\{X_i\}$ is **preserved** by the categorical symmetries of GR if the projection functor $\Pi : \text{GR}+\{X_i\} \to \text{GR}$ is full, i.e. does not forget structure.

(Cat Sym) The category GR+$\{X_i\}$ has more structure than the category GR iff $\{X_i\}$ is not preserved by the categorical symmetries of GR.

We now highlight a connection between the modal and categorical approaches to symmetry and structure: if **(Mod Sym)** detects extra structure (on a single spacetime), then **(Cat Sym)** also detects extra structure (on the collection of all spacetimes). This shows a sense in which the modal approach is well-calibrated. From previous sections, we know that **(Mod Sym)** is more sensitive to extra structure than both **(Glob Sym)** and **(Loc Sym)**. But now we find that **(Mod Sym)** is also not overly sensitive: it doesn't erroneously detect extra structure where there isn't any (according to the well-established categorical approach). To state the result, we need an intuitive definition: a collection of tensor fields $\{X_i\}$ on each spacetime is an **extension** of a tensor field X on a spacetime (M, g) if $X_{(M,g)} = X$. We have the following.

Proposition. Let X be a smooth tensor field on a spacetime (M, g). If (M, g, X) has more structure than (M, g) according to **(Mod Sym)**, then for any extension $\{X_i\}$ of X, the category GR+$\{X_i\}$ has more structure than the category GR according to **(Cat Sym)**.

Proof. Let X be a smooth tensor field on a spacetime (M, g). Suppose (M, g, X) has more structure than (M, g) according to **(Mod Sym)**. So X is not preserved by some modal symmetry of (M, g). It follows that there is an isometry $f : U \to V$ from some connected open set $U \subseteq M$ to some connected open set $V \subseteq N$ of a spacetime (N, h) such that there is no tensor Y on N such that $f^*(Y) = X$. Let $\{X_i\}$ be any extension of X. So $f^*(X_{(N,h)}) \neq X_{(M,g)}$. Since U is a connected open set in M, we can consider the spacetime (U, g) and its associated tensor field $X_{(U,g)}$. The inclusion map $i : U \to M$ is an isometric embedding from (U, g) to (M, g). Let $j : U \to N$ be the isometric embedding from (U, g) to (N, h) defined by $j = f \circ i$. Now assume both $i^*(X_{(M,g)}) = X_{(U,g)}$ and $j^*(X_{(N,h)}) = X_{(U,g)}$. We show a contradiction.

Because $j = f \circ i$, we find that $j^*(X_{(N,h)}) = i^*(f^*(X_{(N,h)}))$. Since $j^*(X_{(N,h)}) = X_{(U,g)}$ by assumption, we have $X_{(U,g)} = i^*(f^*(X_{(N,h)}))$. So $i_*(X_{(U,g)}) = f^*(X_{(N,h)})$. The assumption $i^*(X_{(M,g)}) = X_{(U,g)}$ just means $X_{(M,g)} = i_*(X_{(U,g)})$. So $X_{(M,g)} = f^*(X_{(N,h)})$: a contradiction. So either $i^*(X_{(M,g)}) \neq X_{(U,g)}$ or $j^*(X_{(N,h)}) \neq X_{(U,g)}$. Assume the first case obtains (the other case is handled similarly).

Consider the objects $(U, g, X_{(U,g)})$ and $(M, g, X_{(M,g)})$ in the category GR+$\{X_i\}$. Because $i : U \to M$ is an isometric embedding, it is an arrow from the object (U, g) to the object (M, g) in the category GR. But $i^*(X_{(M,g)}) \neq X_{(U,g)}$ by assumption. So i is not an arrow from the object $(U, g, X_{(U,g)})$ to the object $(M, g, X_{(M,g)})$ in GR+$\{X_i\}$. This means the projection functor $\Pi : \text{GR} + \{X_i\} \to \text{GR}$ is not full, i.e. forgets structure. So $\{X_i\}$ is not preserved by the categorical symmetries of GR and thus the category GR+$\{X_i\}$ has more structure than the category GR according to **(Cat Sym)**. □

6 Conclusion

The abstract **(Sym)** condition informally captures a relationship between spacetime symmetry and spacetime structure: the triple (M, g, X) has more structure than the spacetime (M, g) iff there is a "symmetry" of (M, g) that fails to preserve the tensor field X. We have considered several ways

of making this condition precise via global, local, modal, and categorical symmetries. The global variant **(Glob Sym)** is inadequate for spacetimes having only trivial global symmetries. In that case, the triple (M, g, X) never has more structure than (M, g) – no matter the tensor field X. A local variant **(Loc Sym)** avoids this consequence but an analogous problem arises for spacetimes having only trivial local symmetries, i.e. Heraclitus spacetimes.

Both the global and local variants of **(Sym)** take symmetries to be types of structure preserving maps from a spacetime to itself. A more general modal variant **(Mod Sym)** takes symmetries to be structure preserving maps from one spacetime to another. According to this condition, Heraclitus spacetimes are not necessarily "maxed out" structurally as **(Glob Sym)** and **(Loc Sym)** both judge them to be. Although this modal approach to symmetries is more sensitive to extra structure than both the global and local approaches, it is also not overly sensitive: if **(Mod Sym)** detects extra structure (on a single spacetime), then a natural categorical variant **(Cat Sym)** also detects extra structure (on the collection of all spacetimes).

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