# Centrality in affiliation networks

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### Abstract

This paper discusses the conceptualization, measurement, and interpretation of centrality in affiliation networks. Although centrality is a well-studied topic in social network analysis, and is one of the most widely used properties for studying affiliation networks, virtually all discussions of centrality and centralization have concerned themselves with one-mode networks. Bonacich's work on simultaneous group and individual centralities is a notable exception (*Social Networks*, 1991, 13, 155–168). I begin by outlining the distinctive features of affiliation networks and describe four motivations for centrality indices in affiliation networks. I then consider properties of some existing centrality indices for affiliation networks, including the relationship between centralities for actors and events in these networks, and present a new conceptualization of centrality that builds on the formal properties of affiliation networks and captures important theoretical insights about the positions of actors and events in these networks. These centralities are then illustrated on Galaskiewicz's data on club and board memberships of a sample of corporate executive officers (*Social Organization of an Urban Grants Economy*. New York: Academic Press, 1985). The conclusion to this paper discusses strengths and weaknesses of centrality indices when applied to affiliation networks. © 1997 Elsevier Science B.V.

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## 1. Affiliation networks

Many social network relations consist of the linkages among actors through their joint participation in social activities or membership in collectivities. This common activity creates a network of ties among actors. Similarly, collectivities, communities, or social occasions are linked to each other through the multiple memberships of actors. Such networks of actors tied to each other through their participation in collectivities, and collectivities linked through multiple memberships of actors, are referred to as *affiliation networks*, *membership networks*, *dual networks*, or *hypernetworks* (Breiger, 1974, 1990; Seidman, 1981, 1985; McPherson, 1982; Wasserman and Faust, 1994).

Formally, an affiliation network consists of two key elements: a set of actors and a collection of subsets of actors (called *events*). Thus, an affiliation network is a two-mode, non-dyadic network. The two modes are the set of actors and the set of

events. An affiliation network is non-dyadic because the affiliation relation relates each actor to a subset of events, and relates each event to a subset of actors. Affiliation networks have been referred to as dual networks because of the complementary perspectives through which actors are linked to each other as members of collectivities, and collectivities are linked to each other through shared members (Breiger, 1974, 1990).

Substantive examples of affiliation networks are widespread, and include, for example, interlocking boards of directors (Levine, 1972; Mariolis, 1975; Sonquist and Koenig, 1975; Mintz and Schwartz, 1981a,b; Allen, 1982; Mizruchi, 1982; Bearden and Mintz, 1987); memberships in voluntary organizations (McPherson, 1982); club memberships (Bonacich, 1978); social gatherings (Davis et al., 1941; Homans, 1950; Breiger, 1974); ceremonial events attended by community members (Foster and Seidman, 1984; Schweizer et al., 1993); observations of informal social interactions (Bernard et al., 1980, 1982; Freeman and Romney, 1987; Freeman et al., 1987; Freeman et al., 1989) and so on. Regardless of their substance, these applications share a formal similarity in that each consists of a set of actors and a collection of subsets of actors.

In this paper I will use two examples of affiliation networks as illustrations. The first is a hypothetical affiliation network of six actors and three events from Wasserman and Faust (1994). The second example is a subset of the data collected by Galaskiewicz (1985) for his study of the urban grants economy in the Minneapolis-St. Paul area. This affiliation network consists of 26 corporate executive officers and their membership in 15 clubs, boards of cultural organizations, and corporate boards. (See Galaskiewicz, 1985; Wasserman and Faust, 1994, for further discussion of these data.)

To illustrate an affiliation network, consider the hypothetical example of six actors and three events. The set of actors is denoted by $\mathcal{N} = \{n_1, n_2, \ldots, n_g\}$, and the set of events is denoted by $\mathcal{M} = \{m_1, m_2, \ldots, m_h\}$. There are $g$ actors and $h$ events. The affiliation network matrix for this example is presented in Table 1. This matrix, denoted by $A = \{a_{ik}\}$, shows the affiliation of the actors with the events. A '1' in row $i$, column $k$ of $A$ indicates that actor $n_i$ is affiliated with event $m_k$. Table 2 gives the one-mode matrix of actor co-memberships. This matrix, denoted by $X^{\mathcal{N}}$, indicates the number of memberships shared by each pair of actors. Table 3 gives the one-mode matrix of event overlaps, denoted by $X^{\mathcal{M}}$, which gives the number of actors shared by each pair of events.

Table 1
Affiliation network matrix

|       | $m_1$ | $m_2$ | $m_3$ |
|-------|-------|-------|-------|
| $n_1$ | 1     | 0     | 1     |
| $n_2$ | 0     | 1     | 0     |
| $n_3$ | 0     | 1     | 1     |
| $n_4$ | 0     | 0     | 1     |
| $n_5$ | 1     | 1     | 1     |
| $n_6$ | 1     | 1     | 0     |

Table 2
Actor co-membership matrix

|       | $n_1$ | $n_2$ | $n_3$ | $n_4$ | $n_5$ | $n_6$ |
|-------|-------|-------|-------|-------|-------|-------|
| $n_1$ | 2     | 0     | 1     | 1     | 2     | 1     |
| $n_2$ | 0     | 1     | 1     | 0     | 1     | 1     |
| $n_3$ | 1     | 1     | 2     | 1     | 2     | 1     |
| $n_4$ | 1     | 0     | 1     | 1     | 1     | 0     |
| $n_5$ | 2     | 1     | 2     | 1     | 3     | 2     |
| $n_6$ | 1     | 1     | 1     | 0     | 2     | 2     |

As Breiger (1974) has noted, the affiliation matrix is related to the actor co-membership matrix and to the event overlap matrix through the following equations:

$$X^{\mathcal{N}} = AA'$$                                                           (1)

and

$$X^{\mathcal{M}} = A'A.$$                                                          (2)

The affiliation network matrix uniquely determines both $X^{\mathcal{N}}$ and $X^{\mathcal{M}}$, but the reverse is not true. A given actor co-membership matrix or event overlap matrix may be generated by a number of different affiliation network matrices (Breiger, 1990).

An affiliation network can also be represented as a bipartite graph (Wilson, 1982). In a bipartite graph nodes can be partitioned into two subsets and all lines are between nodes from different subsets. In the bipartite graph for an affiliation network the two sets of nodes are the set of actors, $\mathcal{N}$, and the set of events, $\mathcal{M}$, so there are $g + h$ nodes. Since the lines indicate ties of affiliation they are always between actors and events. Fig. 1 shows the hypothetical affiliation network as a bipartite graph.

The bipartite graph can also be presented in a sociomatrix, denoted by $X^{\mathcal{NM}}$. This matrix has $g + h$ rows and columns indexing actors and events, and has the form:

$$X^{\mathcal{NM}} = \begin{bmatrix} 0 & A \\ A' & 0 \end{bmatrix}$$                 (3)

where actors occupy the first $g$ rows and columns and events occupy the last $h$ rows and columns.

In studying an affiliation network one can look at properties of actors in the one-mode relation of actor co-memberships, properties of events in one-mode relation of event overlaps, or properties of both actors and events in the two-mode affiliation relation.

Table 3
Event overlap matrix

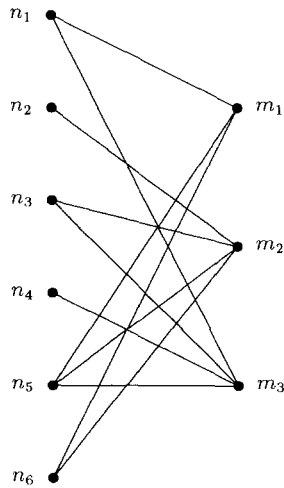|       | $m_1$ | $m_2$ | $m_3$ |
|-------|-------|-------|-------|
| $m_1$ | 3     | 2     | 2     |
| $m_2$ | 2     | 4     | 2     |
| $m_3$ | 2     | 2     | 4     |

Fig. 1. Bipartite graph of affiliation network.

Centrality is one network property that frequently has been used to study actors or events in affiliation networks (Bonacich, 1972, 1991; Mariolis, 1975; Stokman, 1977; Bonacich and Domhoff, 1981; Mintz and Schwartz, 1981a,b; Mizruchi and Bunting, 1981; Mariolis and Jones, 1982; Mizruchi, 1982, 1992; Rosenthal et al., 1985; Mizruchi et al., 1986; Fernandez and McAdam, 1988). However, virtually all analyses of centrality in affiliation networks neglect the inherent properties of these networks: the presence of two modes, the duality of actors and events, and the non-dyadic affiliation relation. In the following sections I address the issue of conceptualization and measurement of centrality in affiliation networks, with special attention to the interpretation of existing centrality measures as applied to affiliation networks and to possible future directions for developing centrality approaches that are especially appropriate for affiliation networks.

## 2. Centrality

The general notion of centrality encompasses a number of different aspects of the 'importance' or 'visibility' of actors within a network. Discussions of centrality can be found in Freeman (1979), Knoke and Burt (1983), Friedkin (1991), Faust and Wasserman (1992) and Wasserman and Faust (1994). In general, there are four common motivations for centrality in one-mode dyadic networks:
- actors are central if they are active in the network (motivating degree centrality);
- actors are central if they can contact others through efficient (short) paths (motivating closeness centrality);
- actors are central if they have the potential to mediate flows of resources or information between other actors (motivating betweenness centrality); and
- actors are central if they have ties to other actors that are themselves central (motivating eigenvector centrality).

Freeman (1979) presents three sets of indices that formalize the ideas of degree (activity), closeness (efficiency), and betweenness (control) actor centrality and graph centralization. From a slightly different perspective, Friedkin (1991) reviews the theoretical basis for centrality measures within the framework of models of interpersonal influence. Within this framework he distinguishes between 'total', 'immediate', and 'mediative' effects centrality, roughly analogous to the total influence of an actor, an actor's closeness to others, and the betweenness of the actor. At the conclusion of his paper Friedkin cites Coombs's position on the theoretical basis for measurement and scaling models in general, commenting:

> Coombs reminds us that 'a measurement or scaling model is actually a theory about behavior, admittedly on a miniature level, but nevertheless theory' [Coombs, 1964, p. 5]. By this criterion, every new proposal of a centrality measure presents new theoretical material. (Friedkin, 1991, p. 1498.)

It is in this spirit that I intend to review centrality measures for affiliation networks. The theoretical foundations for affiliation networks as well as their formal properties differ in important ways from the more usual one-mode dyadic networks. Thus, centrality indices that have been developed for one-mode dyadic social networks may or may not be appropriate for studying affiliation networks. In addition, there may be theoretical insights gained from affiliation networks that will suggest new centrality approaches.

In the next section I discuss four motivations for centrality in affiliation networks, with special attention to those features of affiliation networks that are distinctive and lead to centrality motivations that are different from centrality motivations for one-mode networks.

## 3. Centrality motivations for affiliation networks

This section presents four ideas that can motivate centralities in affiliation networks. Two of these ideas arise from the formal properties of affiliation networks (two-mode, non-dyadic networks), and two arise from empirical and theoretical insights suggested by researchers who have studied affiliation networks. The four ideas that I present below are: centralities for both actors and events, centralities for subsets of actors and/or subsets of events, the importance of linkages created by actors and events, and the importance of subset–superset relationships in actors' affiliations and events' memberships.

### 3.1. Centralities for actors and events

Since affiliation networks are two-mode networks, a complete analysis should give centrality indices for both actors and events. As we shall see below, calculating indices for both actors and events is straightforward. The important question is: what, if anything, is the relationship between the centralities of events and the centralities of the actors that belong to the events? Different authors have taken different perspectives on this question. In their discussion of the relationship between the centrality of corporate

boards and the centrality of individuals who sit on the boards, Perrucci and Lewis (1989, p. 216) argue that

> These two networks can reveal considerable overlap in that the organizations that occupy central positions in their network are also those organizations that are the primary affiliations of the leaders that hold central positions in their network. On the other hand, the two networks can be independent in that the central organizations and the central leaders are drawn from two different subgroups.

Perrucci and Lewis view the relationship between the centrality of an actor and the centralities of the events to which the actor belongs as an empirical question. In their view, a given network may or may not show any relationship between actor and event centralities. Bonacich (1991, p. 256) takes exactly the opposite position. He argues that

> Centrality involves ... the 'duality' of groups and individuals. A central firm gets its central position from the board membership patterns of its members.... Dually a central individual should be one who belongs to a variety of important firms. One kind of centrality cannot be defined without reference to the other.

Bonacich's argument builds on the theoretically important duality in the relationship between actors and events. In keeping with this perspective, Bonacich (1991) presents a centrality measure for which the centrality of an event is proportional to the sum of the centralities of the actors that are members of the event, and the centrality of an actor is proportional to the sum of the centralities of the events to which it belongs. I discuss this measure, eigenvector centrality, in detail below.

Thus, the first important feature of centralities for affiliation networks is that there should be centrality scores for both actors and events. Furthermore, there should be a clearly specifiable relationship between these quantities.

## 3.2. Centralities for subsets

Another important feature of an affiliation network is that the affiliation relation is *non-dyadic* (Wilson, 1982; Seidman, 1985). Unlike a dyadic relation that links pairs of actors (and thus is binary), a non-dyadic relation links members of subsets of arbitrary size (and is thus *n*-ary). The affiliation relation relates each event to a *subset* of actors and relates each actor to a *subset* of events.

What does this imply for centralities in these networks? Existing centrality measures apply to one of two levels: centrality indices for actors or centralization indices for whole networks (Freeman, 1979). But if we take seriously the idea that actors in affiliation networks are defined by the collection of the events to which they belong and events are defined by their collection of members (Simmel, 1955; Breiger, 1974), then the centrality of an actor should be a function of the collection of events to which it belongs and the centrality of an event should be a function of the centrality of its collection of members. In contrast to a centrality index for a node in a graph, a centrality index for a collection of nodes should quantify the importance of that collection within the entire graph. For example, a centrality index for an event in an affiliation network

should quantify the importance of the collection of actors belonging to that event. The idea is to extend centrality indices for individual nodes to centrality indices for collections of nodes in a graph (or actors or events in an affiliation network).

These proposals of centralities for two modes and for subsets of entities build on formal properties of affiliation networks. Next I discuss theoretical ideas and empirical insights that have been used to describe the 'importance' of actors and events in affiliation networks, and which thus suggest theoretically and substantively grounded motivations for centralities in affiliation networks.

### 3.3. Linkages between actors and events

A theoretically important property of an affiliation network is that actors create linkages between events and events create linkages between actors. Multiple memberships of actors provide conduits for the flow of information between events or for the coordination of activities between events. Events as collections of actors provide the opportunity for contacts between actors, for sharing of information or other resources among actors, and facilitate formation of pair-wise ties between actors (McPherson, 1982; McPherson and Smith-Lovin, 1982). Bonacich and Domhoff (1981) observe that ''no groups are directly related to each other; they are only indirectly related to each other through individuals'' (p. 178), and similarly ''central individuals link groups which tend to be disjoint'' (p. 179).

The linking function of actors and events means that an actor is central in an affiliation network if it creates ties between events and an event is central if it creates ties between actors. An actor that belongs only to one event creates no ties between events. Thus, an actor's multiple memberships contribute to its centrality because that actor creates linkages between events. It is not simply the number of memberships that an actor has that is important. An actor that belongs to few events may nevertheless create a critical tie between two or more events that otherwise would not be linked. Similarly, an event is important because it creates linkages between its members. Although events with large membership lists create more ties between pairs of actors than do events with small membership lists, it is not simply the size of an event that is important. Even a small event may bring together actors that would not meet otherwise.

Thus, the linkages between events created by actors' multiple affiliations, and the linkages between actors created by events' collections of members are important considerations in determining the centrality of both actors and events in an affiliation network. Actors are always between events and events are always between actors. This suggests that some form of betweenness centrality will be appropriate for studying affiliation networks.

### 3.4. Subset–superset inclusions of actors and events

A fourth distinctive motivation for centrality in affiliation networks arises from observations about the structure of memberships in these networks. Memberships in affiliation networks can be viewed as patterns of inclusions among events' membership

lists, as patterns of inclusions among actors' affiliations, or both (Bonacich, 1978). Here, we shall want to look not at the number of overlaps for events or co-memberships for actors, but at the patterns of inclusions among memberships. Specifically, a number of authors have observed that in terms of their participation in events, some actors' affiliations are included within the affiliations of other actors. This leads to the distinction between 'primary' and 'secondary' actors (Davis et al., 1941; Doreian, 1979a,b; Freeman and White, 1993). Many of these insights have come from analyses of the data of Davis et al. (1941) on social activities of a community of Southern women. In their original analysis of these data, Davis et al. arrange the affiliation network matrix not only to highlight two 'cliques' of women, but also to indicate which women were primary and which were secondary within each 'clique'. In his reanalysis of these data using $q$-connectivity to locate cohesive subgroups, Doreian (1979a) finds essentially the same two subgroups that Davis et al. had found, and, moreover, he describes the internal structure of these subgroups. Doreian (1979a, pp. 224–225) comments that

> each of the subgroups can be viewed as having a core and a periphery, with higher connectivity within the core, lower connectivity with the periphery.

Furthermore, Doreian identifies one of the women as a 'central core member' of one of the cliques based on her high level of connectivity to others. In their reanalysis of these data using Galois lattices (which I discuss below), Freeman and White (1993) specifically describe the nature of the internal structure of the subgroups. They observe that for secondary actors

> the events they attended are all subsets of the events attended by one or more primary actors. (p. 138.)

Freeman and White also note that events' memberships can be characterized by similar subset–superset relationships.

These observations suggest that centrality indices for affiliation networks should capture the subset–superset relationships in the affiliations of actors and events. The distinction between primary and secondary actors means that secondary actors are more likely to participate in events when primary actors are also present, and are unlikely to participate on their own. Primary actors, on the other hand, participate in events even when secondary actors are not present. The participation of secondary actors is conditional on the participation of primary actors, implying that the events to which secondary actors belong are a subset of the events to which primary actors belong (Freeman and White, 1993). In terms of centrality in affiliation networks, less central actors participate in events only in the presence of more central actors. Importantly, it is not simply the greater number of events to which more or less central actors belong, but the patterning of these events such that the events to which less central actors belong are a subset of the events to which more central actors belong (Bonacich, 1978). Similarly, one can consider the relative centrality of events using these ideas. Less central events have membership lists that are contained within the memberships of more central events. Again, it is not simply the size of the events that determines their centrality, but rather it is the patterning of memberships in these events that is important.

*3.5. Summary*

In summary, the properties of affiliation networks suggest that centralities for these networks should do four things:
1. give centrality indices for both actors and events;
2. be extendable to subsets of actors and events;
3. focus on the linkages between actors and events through overlapping memberships;
4. capture subset–superset inclusion relations between actors and events.

Analyses of affiliation networks have seldom used these ideas to study the centrality of actors and events in affiliation networks. Rather, more conventional centrality motivations for one-mode networks have been widely employed to study affiliation networks.

In addition, virtually all analyses of affiliation networks study one-mode networks of actors or events derived from the original affiliation network. For example, many researchers have studied the affiliation of corporate executives with the corporate boards on which they sit. With the exception of Bonacich (1991) and Bonacich and Domhoff (1981), centrality analyses of these data always study either the board member ties of co-membership or the corporation ties of interlocking directorates, but do not study both simultaneously.

Studying only a single mode from the two-mode network ignores the fundamental duality inherent in the affiliation relation. The relationship between the centrality of actors and the centrality of the events to which they belong, or the relationship between the centrality of events and the centrality of their members, cannot be studied directly by looking at the one-mode networks separately.

In the next sections I consider five centrality indices: degree, eigenvector, closeness, betweenness, and flow betweenness. For each I describe the application of the index to affiliation network data and discuss the results with respect to the centrality motivations introduced above.

## 4. Centralities for affiliation networks

A review of analyses of centrality in affiliation networks reveals that virtually all applications use one of two approaches to centrality: (1) some form of degree centrality or (2) some form of eigenvector centrality. I begin by discussing degree and eigenvector centrality, since they have been widely used to study centrality in affiliation networks. I then discuss closeness, betweenness, and flow betweenness centralities. As we shall see, calculating basic centrality measures for both actors and events in an affiliation network is straightforward. To study both actors and events one can analyze the bipartite graph using standard centrality approaches. An important issue for each measure is to specify, if possible, the relationship between actor and event centralities.

Since centrality indices will be defined for several different kinds of graphs and networks (a one-mode dyadic network, a bipartite graph, a one-mode actor co-membership relation, or a one-mode event overlap relation) and indices will be presented for nodes in a graph, and for actors and events in affiliation networks, we shall need

notation that reflects these distinctions. $C_A(\bullet)$ will refer to centrality definition 'A' for entity '$\bullet$'. For example, $C_C(\bullet)$ will refer to closeness centrality. The entity for which the centrality is defined can be an actor, denoted by $n_i$, an event, denoted by $m_k$, or a node in a generic graph, denoted $p_i$. In addition, when the centrality definition applies to a specific kind of relation or matrix (for example the bipartite graph, or the actor co-membership relation) I will use the following superscripts: $\mathcal{NM}$ for the bipartite graph, $\mathcal{N}$ for the one-mode actor co-membership relation, and $\mathcal{M}$ for the one-mode event overlap relation. Thus, $C_C\mathcal{NM}(n_i)$ will refer to the closeness centrality of an actor calculated on the bipartite graph. In the absence of a superscript, the definition refers to a one-mode dyadic graph or network.

## 4.1. Degree centralities

Degree centrality, denoted by $C_D(p_i)$, is one of the most straightforward centrality indices. In an affiliation network motivations for degree centrality are that actors are important because of their level of activity or the number of contacts that they have, and events are important because of the size of their memberships (Stokman, 1977; Mizruchi and Bunting, 1981; Mariolis and Jones, 1982; Fernandez and McAdam, 1988).

In a graph for a single dichotomous relation, the degree of a node is the number of nodes adjacent to it. For a nondirectional relation the degree of a node is equal to the sum of the values in the row (or column) of the sociomatrix; $C_D(p_i) = \sum_{j=1}^{g} x_{ij}$. In an affiliation network there are several ways to calculate degree centrality depending on whether one focuses on the bipartite graph (including both actors and events) or on the one-mode networks of actor co-memberships or event overlaps. Interpretations of the results will differ.

First, focus on the bipartite graph in which both actors and events are represented as nodes and lines represent affiliation ties between actors and events. In the bipartite graph the degree centrality of an actor is the number of events with which it is affiliated and the degree centrality of an event is the number of actors affiliated with it. These quantities are also equal to the entries on the main diagonal of the actor co-membership matrix $X^{\mathcal{N}}$ (for actors) or the event overlap matrix $X^{\mathcal{M}}$ (for events), or to the row total (for actors) or the column total (for events) of the affiliation matrix $A$. Thus, actor and event degree centralities in the bipartite graph are:

$$C_D^{\mathcal{NM}}(n_i) = \sum_{k=1}^{g+h} x_{ik}^{\mathcal{NM}} = x_{ii}^{\mathcal{N}} = a_{i+} \tag{4}$$

$$C_D^{\mathcal{NM}}(m_k) = \sum_{i=1}^{g+h} x_{ik}^{\mathcal{NM}} = x_{kk}^{\mathcal{M}} = a_{+k}. \tag{5}$$

Although degree centrality is usually defined for dichotomous relations, there are also a number of ways to define degree-like centralities for the one-mode valued relations of actor co-memberships and event overlaps. Both of these are valued, nondirectional relations where the values indicate the number of co-memberships for pairs of actors or the number of overlaps for pairs of events.

Consider the entries in a given row (or column) of the actor co-membership matrix, $X^{\mathcal{N}}$. The number of non-zero off-diagonal entries in the row gives the number of distinct actors with which the actor shares any event in common. However, we can also define other possible degree-like measures for this relation. Suppose we take the sum of the entries in a row. For actors in the co-membership relation this degree centrality is:

$$C_D^{\mathcal{N}}(n_i) = \sum_{j=1}^{g} x_{ij}^{\mathcal{N}} = x_{i+}^{\mathcal{N}}. \tag{6}$$

Interestingly, this quantity is equal to the sum of the sizes of the events to which the actor belongs:

$$C_D^{\mathcal{N}}(n_i) = \sum_{n_i \in m_k} x_{kk}^{\mathcal{M}} = \sum_{n_i \in m_k} a_{+k}. \tag{7}$$

We can think of $C_D^{\mathcal{N}}(n_i)$ as the actor's total 'volume' of activity. It is the number of contacts that an actor has with other actors, counting other actors each time they are encountered.

For events, the similar degree centrality is equal to

$$C_D^{\mathcal{M}}(m_k) = \sum_{l=1}^{h} x_{kl}^{\mathcal{M}} = x_{k+}^{\mathcal{M}}. \tag{8}$$

This quantity can be thought of as the total participation of the actors that belong to the event, and is equal to the sum of the number of memberships for all actors in the event:

$$C_D^{\mathcal{M}}(m_k) = \sum_{n_i \in m_k} x_{ii}^{\mathcal{N}} = \sum_{n_i \in m_k} a_{i+}. \tag{9}$$

If these centralities are calculated including the diagonal elements of $X^{\mathcal{N}}$ or $X^{\mathcal{M}}$, then they are also equal to the number of two-step walks from the actor or event node in the bipartite graph.

Taken together, Eqs. (7) and (9) show that for the one-mode networks of actor co-memberships and event overlaps there is a specifiable relationship between an actor's centrality and the centrality of the events to which it belongs, and between an event's centrality and the centrality of its members:

$$C_D^{\mathcal{N}}(n_i) = \sum_{n_i \in m_k} C_D^{\mathcal{N}\mathcal{M}}(m_k) \tag{10}$$

$$C_D^{\mathcal{M}}(m_k) = \sum_{n_i \in m_k} C_D^{\mathcal{N}\mathcal{M}}(n_i). \tag{11}$$

The degree centrality of an actor is equal to the sum of the sizes of its events (the degrees of these events in the bipartite graph) and the degree centrality of an event is equal to the sum of the number of memberships of its actors (the degrees of these actors in the bipartite graph).

Degree centrality indices for actors and events in the bipartite graph for the hypothetical example are presented in Table 4. Table 5 presents the degree centralities

Table 4
Centrality of actors and events in the bipartite graph

| | Measure of centrality | | | | | |
|---|---|---|---|---|---|---|
| | $C_D^{\mathcal{NM}}$ | $\Sigma d(i,k)$ | $C_C^{\mathcal{NM}}$ | $C_B^{\mathcal{NM}}$ | $C_F^{\mathcal{NM}}$ | $C_E^{\mathcal{NM}}$ |
| $n_1$ | 2 | 17 | 0.4706 | 1.75 | 5 | 0.286 |
| $n_2$ | 1 | 21 | 0.3810 | 0.00 | 0 | 0.154 |
| $n_3$ | 2 | 15 | 0.5333 | 3.00 | 5 | 0.309 |
| $n_4$ | 1 | 21 | 0.3810 | 0.00 | 0 | 0.154 |
| $n_5$ | 3 | 13 | 0.6154 | 6.50 | 3 | 0.440 |
| $n_6$ | 2 | 17 | 0.4706 | 1.75 | 5 | 0.286 |
| $m_1$ | 3 | 16 | 0.5000 | 4.00 | 12 | 0.365 |
| $m_2$ | 4 | 14 | 0.5714 | 10.50 | 19 | 0.428 |
| $m_3$ | 4 | 14 | 0.5714 | 10.50 | 19 | 0.428 |

Table 5
Centrality measures for actors and events in one-mode networks

| | Measure of centrality | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $C_D^{\mathcal{N}a}$ | $C_D^{\mathcal{N}b}$ | $C_B^{\mathcal{N}01}$ | $C_F^{\mathcal{N}}$ | $C_E^{\mathcal{N}}$ | $C_D^{\mathcal{M}a}$ | $C_D^{\mathcal{M}b}$ | $C_B^{\mathcal{M}01}$ | $C_F^{\mathcal{M}}$ | $C_E^{\mathcal{M}}$ |
| $n_1$ | 5 | 7 | 0.33 | 7 | 0.404 | | | | | |
| $n_2$ | 3 | 4 | 0.00 | 4 | 0.218 | | | | | |
| $n_3$ | 6 | 8 | 1.17 | 10 | 0.437 | | | | | |
| $n_4$ | 3 | 4 | 0.00 | 4 | 0.218 | | | | | |
| $n_5$ | 8 | 11 | 1.17 | 13 | 0.622 | | | | | |
| $n_6$ | 5 | 7 | 0.33 | 7 | 0.404 | | | | | |
| $m_1$ | | | | | | 4 | 7 | 0.00 | 2 | 0.515 |
| $m_2$ | | | | | | 4 | 8 | 0.00 | 2 | 0.606 |
| $m_3$ | | | | | | 4 | 8 | 0.00 | 2 | 0.606 |

[a]Excludes diagonal. [b]Includes diagonal.

for the one-mode networks of actor co-memberships and event overlaps, calculated both with diagonal entries included and with diagonal entries excluded. [1]

A number of authors have used versions of degree centrality to study centrality in affiliation networks. Romney and Weller (1984) use this index to measure the overall level of people's observed behavioral interaction in data from three groups studied by Bernard et al. (1980, Bernard et al., 1982). In their study of corporate interlock networks, Mariolis and Jones (1982) use this index as one measure of a firm's centrality, referring to it as 'the number of interlocks'. Similarly, in his study of delegations' co-sponsorship of bills within the United Nations, Stokman (1977) uses the number of co-sponsorship ties that a delegation has as an index of the delegation's centrality in the co-sponsorship network.

Some authors have criticized degree centrality because it does not consider the centrality of the actors (or events) to which an actor (or event) is adjacent. According to

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[1] Centrality indices were calculated using *UCINET IV* (Borgatti et al., 1992).

this argument, two actors may be adjacent to the same number of others, but an actor is more central if it has ties to actors that themselves are quite central. One way to deal with this issue is to incorporate the centrality of the actors to which a given actor is adjacent into the centrality index. This is what eigenvector centrality does.

## 4.2. Eigenvector centrality

The rationale for eigenvector centrality in a one-mode network is that the centrality of an actor should be proportional to the strength of the actor's ties to other network members and the centrality of these other actors (Bonacich, 1972; Mizruchi, 1982; Mizruchi et al., 1986). Originally, Bonacich (1972) motivated this centrality index as a measure of popularity, related to the measures of relative standing or status proposed by Katz (1953) and Hubbell (1965). Recently, people have defended the use of this index as a measure of the extent to which actors are in a position to influence others in the network (Mizruchi and Bunting, 1981; Fernandez and McAdam, 1988; Friedkin, 1991). Friedkin (1991) describes this index as 'total effects centrality' in the context of network influence models.

Eigenvector centrality is widely used in studies of interlocking corporate boards of directors (Mariolis, 1975; Mintz and Schwartz, 1981a,b; Mizruchi and Bunting, 1981; Mariolis and Jones, 1982; Mizruchi, 1982; Roy, 1983; Rosenthal et al., 1985). For many of these researchers centrality *is* eigenvector centrality (Mariolis, 1975; Mintz and Schwartz, 1981a,b; Roy, 1983; Rosenthal et al., 1985).

In this section I review eigenvector centrality in general and then show how the centrality indices for actors are related to the centrality indices for events in an affiliation network.

Denoting the eigenvector centrality of node $p_i$ in a one-mode network by $C_E(p_i)$, eigenvector centrality is expressed as

$$C_E(p_i) \approx C_E(p_j) x_{ij}. \tag{12}$$

The centrality of a node is proportional to the centrality of the nodes to which it is adjacent, weighted by the value of the tie between the nodes.

Finding centrality values, $C_E(p_i)$, that satisfy this equation for all nodes in a graph involves solving a system of simultaneous linear equations. This standard eigenvector–eigenvalue problem is expressed by the equation

$$Xc = \lambda c \tag{13}$$

where $X$ is a $g \times g$ sociomatrix, $\lambda$ is its largest eigenvalue, and $c$ is a vector of centrality scores (the eigenvector corresponding to the largest eigenvalue).

I will denote the vector of eigenvector centrality scores for actors by $c^{\mathcal{N}}$ and the vector of eigenvector centrality scores for events by $c^{\mathcal{M}}$. One can find these scores by analyzing the one-mode actor co-membership matrix and the one-mode event overlap matrix:

$$X^{\mathcal{N}}c^{\mathcal{N}} = AA'c^{\mathcal{N}} = \lambda^2 c^{\mathcal{N}} \tag{14}$$

$$X^{\mathcal{M}}c^{\mathcal{M}} = A'Ac^{\mathcal{M}} = \lambda^2 c^{\mathcal{M}}. \tag{15}$$

Bonacich suggests that the scores be scaled so that $\Sigma C_E^{\mathcal{N}}(n_i)^2 = \Sigma C_E^{\mathcal{M}}(m_k)^2 = 1$, though other scalings are possible. The actor and event eigenvector centralities are related to each other through the set of equations:

$$Ac^{\mathcal{N}} = \lambda c^{\mathcal{M}} \tag{16}$$

$$A'c^{\mathcal{M}} = \lambda c^{\mathcal{N}}. \tag{17}$$

Eqs. (16) and (17) show the duality of actor and event centralities for the affiliation network. The centrality of an actor is proportional to the centralities of the events with which it is affiliated, and the centrality of an event is proportional to the centralities of its members.

Bonacich (1991) shows that for an affiliation network one can find the actor and event centrality scores by analyzing a number of different matrices. Specifically, one can analyze the affiliation network matrix, $A$, the pair of one-mode matrices, $X^{\mathcal{N}}$ and $X^{\mathcal{M}}$ (Eqs. (14) and (15)), or the matrix for the bipartite graph, $X^{\mathcal{N}\mathcal{M}}$. Analysis of the bipartite graph is expressed in the equation:

$$\lambda \begin{bmatrix} c^{\mathcal{N}} \\ c^{\mathcal{M}} \end{bmatrix} = \begin{bmatrix} 0 & A \\ A' & 0 \end{bmatrix} \begin{bmatrix} c^{\mathcal{N}} \\ c^{\mathcal{M}} \end{bmatrix} \tag{18}$$

(Bonacich, 1991, p. 158). The generality of this approach is apparent in the fact that the eigenvector centrality scores from any of these analyses are equivalent, once they are identically scaled.

For eigenvector centralities there is a clear and specifiable relationship between the centralities for actors and the centralities for events. This relationship is more apparent if we rewrite Eqs. (16) and (17) in terms of individual actor and event centrality indices. An actor's centrality is a function of the centralities of the events to which it belongs:

$$C_E^{\mathcal{N}}(n_i) = \frac{1}{\lambda} \sum_{k=1}^{h} C_E^{\mathcal{M}}(m_k) a_{ik}. \tag{19}$$

Similarly, the centrality of an event is a function of the centralities of its members:

$$C_E^{\mathcal{M}}(m_k) = \frac{1}{\lambda} \sum_{i=1}^{g} C_E^{\mathcal{N}}(n_i) a_{ik}. \tag{20}$$

Eigenvector centrality explicitly incorporates the duality between actor and event centralities (Bonacich, 1991). This duality is expressed in the pairs of Eqs. (16) and (17) or Eqs. (19) and (20) in which actors' centralities are proportional to the centralities of the events to which they belong, and events' centralities are proportional to the centralities of their members.

A similar relationship between actor and event scores results from a correspondence analysis of the affiliation matrix, $A$ (Bonacich, 1991; Wasserman and Faust, 1994). One of the goals of correspondence analysis is to assign scores to the rows and columns of a data array so that a row's score is proportional to the weighted average of the column scores and a column's score is proportional to the weighted average of the row scores; the weights are the relative frequencies in the columns or rows, respectively. For an affiliation network, an actor's score is proportional to the average of the scores for the events with which it is affiliated and an event's score is proportional to the average of

the scores for the actors that belong to the event. Correspondence analysis of an affiliation matrix, $A$, results in three sets of information:

- a set of $g$ actor scores on each of $W$ dimensions $\{u_{iw}\}$ for $i = 1,2,\ldots,g$ and $w = 1,2,\ldots,W$;
- a set of $h$ event scores on each of $W$ dimensions $\{v_{kw}\}$ for $k = 1,2,\ldots,h$ and $w = 1,2,\ldots,W$, and
- a set of $W$ principal inertias $\{\eta_w^2\}$ for $w = 1,2,\ldots,W$.

Correspondence analysis scores for rows (actors) and columns (events) are related to each other through the following equations:

$$\eta_w u_{iw} = \sum_{k=1}^{h} \frac{a_{ik}}{a_{i+}} v_{kw}$$

$$\eta_w v_{kw} = \sum_{i=1}^{g} \frac{a_{ik}}{a_{+k}} u_{iw}.$$

(21)

These row and column scores are found through a singular value decomposition of an appropriately scaled affiliation matrix: $\tilde{A} = \{a_{ik}/\sqrt{a_{i+}a_{+k}}\}$. Although the relationship between actor and event correspondence analysis scores, Eq. (21), appears similar to the relationship between actor and event eigenvector centrality scores, Eqs. (19) and (20), the resulting scores can be quite different. This difference is due to the fact that eigenvector centralities result from a decomposition of $A$ whereas correspondence analysis scores result from a decomposition of $\tilde{A}$.

The eigenvector centralities for actors and events in the hypothetical example are presented in Table 4 (for the bipartite graph) and Table 5 (for the one-mode relations of actor co-memberships and event overlaps). To illustrate the duality of actor and event eigenvector centralities, consider the eigenvector centrality of $n_1$. For the one-mode actor co-membership relation, $\lambda = 2.775$ and $C_E^{\mathcal{N}}(n_1) = 0.404$. Actor $n_1$ is affiliated with events $m_1$ and $m_3$, with centralities in the event overlap relation of $C_E^{\mathcal{M}}(m_1) = 0.515$ and $C_E^{\mathcal{M}}(m_3) = 0.606$. Thus, $n_1$'s centrality is equal to

$$0.404 = \frac{(0.515)(1) + (0.606)(0) + (0.606)(1)}{2.775}.$$

(22)

A number of people have argued that the magnitudes of eigenvector centralities for events in the event overlap relation are undesirably affected by differences in the sizes of events (Mariolis, 1975; Mizruchi and Bunting, 1981; Mariolis and Jones, 1982; Mizruchi, 1982; Bonacich, 1991). Thus, they argue, steps should be taken to remove this effect. Two approaches have been proposed: (1) standardize the event overlap measure $x_{kl}^{\mathcal{M}}$ prior to analysis, or (2) remove from the centrality index that component which is due to the degree (i.e. the size) of the event.

The first way to accommodate different event sizes is to standardize the entries in the event overlap matrix, $X^{\mathcal{M}}$, prior to finding eigenvector centralities. Mariolis (1975) suggests that standardized values $X^{*\mathcal{M}} = \{x_{kl}^{*\mathcal{M}}\}$ be defined as:

$$x_{kl}^{*\mathcal{M}} = \frac{x_{kl}^{\mathcal{M}}}{\sqrt{x_{kk}^{\mathcal{M}} x_{ll}^{\mathcal{M}}}}.$$

(23)

Since $x_{kk}^{\mathscr{M}}$ and $x_{ll}^{\mathscr{M}}$ are the sizes of events $m_k$ and $m_l$ respectively, this standardization takes event sizes into account by dividing the measure of overlap by the square root of the product of the event sizes. However, Bonacich (1991) demonstrates that this standardization fails to remove size differences. Moreover, the event eigenvector centralities from this standardized overlap measure will not be identical to event eigenvector centralities for the bipartite graph, unless entries in $X^{\mathscr{N\!M}}$ are also appropriately standardized. The appropriate standardization defines a new affiliation network matrix $A^* = \{a_{ik}^*\}$ where $a_{ik}^* = a_{ik}/\sqrt{a_{+k}}$. The new sociomatrix for the bipartite graph, $X^{*\,\mathscr{N\!M}}$, then has the form

$$\begin{bmatrix} 0 & A^\star \\ A^{\star\prime} & 0 \end{bmatrix}. \tag{24}$$

The event eigenvector centralities for $X^{*\,\mathscr{N\!M}}$ will be identical to the event eigenvector centralities for $X^{*\,\mathscr{M}}$, once they are scaled as described above. However, the actor eigenvector centralities from $X^{*\,\mathscr{N\!M}}$ will *not* be identical to actor eigenvector centralities from $X^{\mathscr{N}}$. Thus, by standardizing $X^{\mathscr{M}}$ to 'remove' differences in event sizes, one loses the important duality between actor and event centralities.

A second way to deal with differences in event sizes exploits information about the components of the eigenvector centrality index. Bonacich (1991) shows that this index is the sum of a weighted series of components, each of which quantifies the contribution to the overall centrality of the node due to paths of length $1,2,\ldots,\infty$ from the node. One can remove the effect of event sizes by 'removing' the component of centrality due to paths of length 1 (the size of the event). The advantage of this approach is that one can analyze the bipartite graph and obtain both actor and event centrality scores, thus maintaining the duality between actor and event centralities.

One potentially undesirable feature of eigenvector centralities for studying affiliation networks is that actors that belong to a single event (and thus create no ties between events) and events that have only a single member (and thus create no ties between actors) can have non-zero eigenvector centrality. For example, in the hypothetical affiliation network, $n_2$, which belongs to only one event, has eigenvector centrality of 0.154. This is due to the centrality that actors and events derive from their ties to others that are central. Non-zero centrality is inconsistent with the theoretical argument that actors in affiliation networks are central to the extent that they link events, and events are central to the extent that they link actors. However, it is consistent with the 'status' or 'popularity' motivation for this index that central actors derive their centrality through their ties to central actors.

Both degree centrality and eigenvector centrality have been widely used to study affiliation networks. However, only Bonacich and Domhoff (1981) and Bonacich (1991) consider both actor and event centralities and the relationship between them. In the next sections I discuss centralities that were not designed to study affiliation networks and have not been used to analyze these networks.

## 4.3. Closeness centrality

Closeness centrality is based on the geodesic distances between nodes in a graph, and is not applicable to valued relations. Thus, I will restrict my attention to closeness

centralities for the bipartite graph. The basis for the closeness centrality index is the average geodesic distance that a node is from all other nodes in the graph. Thus it begins with the 'farness' of a node from other nodes in the graph. Distances between nodes are summarized in the matrix $D = \{d(i,j)\}$, where the entry in cell $(i,j)$ of this matrix is the geodesic distance from node $i$ to node $j$. In other words, it is the length of any shortest path between the nodes. Since the bipartite graph is nondirectional, the distance from $i$ to $j$ is the same as the distance from $j$ to $i$, and $D$ is symmetric.

In general, closeness centrality, $C_C(p_i)$, is the inverse of the average geodesic distance between the node and all other nodes in the graph, and is calculated as

$$C_c(p_i) = \left[ \frac{\Sigma_{j=1}^{g} d(i,j)}{g-1} \right]^{-1}. \tag{25}$$

If the graph is disconnected, then closeness centrality is undefined, since some pairs of nodes are unreachable and the distance between them is infinite.

The numerator of Eq. (25) is the sum of the distances from node $i$ to all other nodes in the graph. For an actor in an affiliation network this is the distance from the actor to other actors plus the distance from the actor to all events. However, since the affiliation network is a bipartite graph, actors are only adjacent to events, and all paths emanating from an actor must first pass through the events to which the actor belongs. Similarly, since events are only adjacent to actors, all paths from events must pass through the actors that are their members.

Consider the relationship between the closeness centrality of an actor and the closeness centralities of the events to which the actor belongs, and the relationship between the closeness centrality of an event and the closeness centralities of its actors. First focus on the distances from an actor in the bipartite graph as a function of the distances from the events to which it belongs. The distance from node $i$ representing an actor to any node $j$ (either actor or event) is $d(i,j) = 1 + \min_k d(k,j)$, for event nodes $k$ adjacent to $i$. Given this property, the numerator of Eq. (25) for the closeness centrality of an actor in the bipartite graph can be expressed as a function of the distances from the actor's events, $k$:

$$\sum_{j=1}^{g+h} d(i,j) = \sum_{j=1}^{g+h} \left[ 1 + \min_k d(k,j) \right] = g + h - 1 + \sum_{j=1}^{g+h} \min_k d(k,j) \tag{26}$$

for $k$ adjacent to $i$, and $j$ not equal to $i$. Combining Eqs. (25) and (26) gives the following expression for the closeness centrality of an actor as a function of the geodesic distances from its events to other actors and events:

$$C_C^{\mathscr{N}\mathscr{A}}(n_i) = \left[ 1 + \frac{\Sigma_{j=1}^{g+h} \min_k d(k,j)}{g+h-1} \right]^{-1} \tag{27}$$

for events $k$ adjacent to actor $i$. Thus, the closeness centrality of an actor is a function of the minimum distances from any of its events to other actors and events in the bipartite graph.

Similarly, the closeness centrality of an event is a function of the minimum distances from its actors to other actors and to other events. Eq. (27) can be written for events as:

$$C_C^{\mathcal{NM}}(m_k) = \left[ 1 + \frac{\Sigma_{i=1}^{g+h} \min_j d(i,j)}{g+h-1} \right]^{-1} \tag{28}$$

for actors $i$ adjacent to event $k$.

The combination of Eqs. (27) and (28) shows that there is a clear relationship between the closeness centralities of actors and the closeness centralities of events in an affiliation network. The closeness centrality of an actor is a function of the minimum distances to its events, and the closeness centrality of an event is a function of the minimum distances to its actors.

Table 4 gives the closeness centralities for actors and events in the bipartite graph for the hypothetical example.

### 4.4. Betweenness centrality

Betweenness centrality for a one-mode dyadic network focuses on the extent to which actors sit on geodesic paths between other pairs of actors (Freeman, 1979). Since calculating betweenness centrality requires considering *all* geodesics in a graph, it seems unlikely that it will be possible to express the betweenness of a node in a graph (an actor or event in an affiliation network) as a function simply of the betweenness of the nodes to which it is adjacent. Nevertheless, this section presents some informal observations about the relationship between actor and event betweenness centralities in affiliation networks. Betweenness centrality is defined for dichotomous, nondirectional relations, so I will concentrate betweenness of actors and events in the bipartite graph for the affiliation network. However, at the end of this section I consider what would happen if one were to analyze the one-mode relations of actor co-memberships or event overlaps.

An intermediate step in calculating betweenness centrality is to find the 'partial betweenness' of nodes in the network (Freeman, 1979). Node $p_i$'s partial betweenness counts the number of pairs of other nodes whose geodesic(s) contain node $p_i$. If there is more than one geodesic between a given pair of nodes, then $p_i$ receives fractional credit, where the fraction is reciprocal of the number of geodesics between the pair. Let $g_{jk}$ be the number of geodesics between $p_j$ and $p_k$, and let $g_{jk}(p_i)$ be the number of geodesics between $p_j$ and $p_k$ that contain $p_i$. If all geodesics are equally likely, then the probability that a geodesic between $p_j$ and $p_k$ contains node $p_i$ is equal to $g_{jk}(p_i)/g_{jk}$. The betweenness centrality of node $p_i$, denoted by $C_B(p_i)$, is defined as the sum of these quantities across all pairs of nodes:

$$C_B(p_i) = \sum_{j<k} \frac{g_{jk}(p_i)}{g_{jk}}. \tag{29}$$

For a graph with $g$ nodes, $C_B(p_i)$ reaches its maximum value of $(g-1)(g-2)/2$ when node $p_i$ is on geodesics between all other pairs of nodes.

Returning to an affiliation network, recall that linkages between pairs of actors are always through actors' participation in events, thus events are always on geodesics

between actors. Similarly, linkages between pairs of events are always through the joint memberships of actors, thus actors are always on geodesics between events. Consider the relationship between the betweenness centrality of an actor and the betweenness centrality of the events with which it is affiliated, and the relationship between the betweenness centrality of an event and the betweenness of its members.

In calculating the betweenness centrality of an event, $m_k$, in an affiliation network, focus on the collection of actors that belong to that event. Event $m_k$ is on a geodesic between all pairs of actors that are members of it. Since event $m_k$ has $a_{+k}$ members, there are $a_{+k}^2$ pairs of actors that have a geodesic that contains $m_k$. If a given pair of actors, $(n_i, n_j)$, only shares event $m_k$ in common (thus $x_{ij}^{\mathcal{N}} = 1$) then $m_k$ is on the only geodesic between them, and $m_k$'s betweenness is incremented by 1. Actually, $n_i$ and $n_j$ share $x_{ij}^{\mathcal{N}}$ memberships, thus $m_k$'s betweenness is incremented by $1/x_{ij}^{\mathcal{N}}$ for each pair of actors $(n_i, n_j)$ in $m_k$. Thus, a portion of the betweenness centrality of event $m_k$ can be expressed in terms of the number of co-memberships of pairs of its members as:

$$\frac{1}{2} \sum_{n_i, n_j \in m_k} \frac{1}{x_{ij}^{\mathcal{N}}}. \tag{30}$$

From this we see that an event gains betweenness centrality to the extent that pairs of its members meet only in that event.

In addition, an event gains betweenness centrality if an individual actor belongs only to that event. In that case, all geodesics from such an actor must contain the event. Since there are $g + h$ nodes (actors and events) in the affiliation network, an event gains $g + h - 2$ betweenness 'points' for each of its members that belongs to no other event. This quantity is not independent of the count in Eq. (30).

These observations suggest that the betweenness centrality of an event increases to the extent that its members belong to no other events. Since such single-membership actors are not between any pairs of events, an event gains betweenness centrality if it contains 'non-central' actors. Also, the betweenness centrality of an event increases to the extent that pairs of actors share only that event in common.

Similar properties of betweenness centrality hold for actors. An actor gains $g + h - 2$ betweenness centrality 'points' if it is the only member of an event. An actor gains $1/x_{kl}^{\mathcal{M}}$ betweenness 'points' for all pairs of events to which it belongs.

Betweenness centralities for the actors and events in the bipartite graph for the hypothetical example are presented in Table 4. Notice that $C_B(n_2) = C_B(n_4) = 0.00$ since actors $n_2$ and $n_4$ each belong to only one event, and thus create no linkages between either actors or events.

As mentioned above, betweenness centrality is defined for networks in which relations are dichotomous and nondirectional. However, sometimes it is tempting to analyze valued nondirectional relations by dichotomizing the original values and analyzing the new dichotomous relation. For example, one could define a dichotomous actor co-membership relation, $x_{ij}^{\mathcal{N}01}$, where

$$x_{ij}^{\mathcal{N}01} = \begin{cases} 0 & \text{if} \quad x_{ij}^{\mathcal{N}} = 0 \\ 1 & \text{if} \quad x_{ij}^{\mathcal{N}} > 0. \end{cases} \tag{31}$$
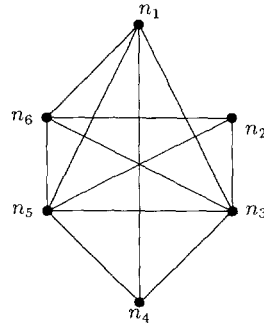
a. Dichotomous actor co-membership relation
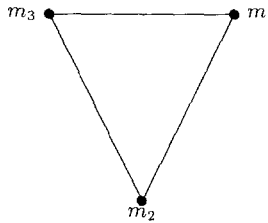


b. Dichotomous event overlap relation

Fig. 2. Graphs for dichotomous one-mode relations.

In this dichotomous relation actors are adjacent if they ever belong to the same event. This relation has been defined by Seidman (1985, p. 368) as the '$q$-overlap graph', where actors are adjacent if they share at least $q$ events, and $q = 1$. Doreian (1969) presents a similar idea for valued graphs. One could also define a dichotomous event overlap relation $x_{kl}^{\mathscr{M}01}$ where

$$x_{kl}^{\mathscr{M}01} = \begin{cases} 0 & \text{if} \quad x_{kl}^{\mathscr{M}} = 0 \\ 1 & \text{if} \quad x_{kl}^{\mathscr{M}} > 0. \end{cases} \tag{32}$$

In this dichotomous relation events are adjacent if they share any actor in common. Stokman (1977) describes either of these relations as creating a new graph by 'induction' where nodes in the new graph are adjacent if they are adjacent to the same node in the original graph.

Graphs for these two dichotomous relations for the hypothetical example are presented in Fig. 2 Notice that the graph for the dichotomous event overlap relation is complete (all events are adjacent) since each pair of events shares at least one actor. Since the graph is complete there is no differentiation among events; all are equally central in this graph. In contrast, the actor co-membership graph is not complete (some pairs of actors never belong to the same event).

What happens if we now find actor and event betweenness centralities, $C_B^{\mathscr{N}01}(n_i)$ and $C_B^{\mathscr{M}01}(m_i)$, for these relations? These centralities are presented in Table 5. Several points are important to note about these results. First, as anticipated above, all events in this

example have equal betweenness centrality for the dichotomous one-mode network of event overlaps. In contrast, in the bipartite graph, events $m_2$ and $m_3$ are clearly more central than is event $m_1$. The dichotomous event overlap relation lacks information about the number of actors shared by pairs of events, and thus lacks information about the number of geodesics between events. Second, notice that for the dichotomous actor co-membership relation, actors $n_2$ and $n_4$ have betweenness centrality equal to 0. These actors are adjacent to other actors on this relation but they are not on geodesics between them. Actors $n_2$ and $n_4$ are each affiliated with only a single event. Thus, if one of these 'single-membership' actors is adjacent to two other actors (for example, $n_2$ is adjacent to $n_5$ and $n_6$) those two actors must be adjacent to each other since all three must belong to the single event to which the 'single-membership' actor belongs. Actors that belong to only one event cannot be on geodesics between other actors.

These observations about betweenness centrality in affiliation networks are informal. Clearly, it remains for further research to express the exact relationship between actor and event betweenness centrality.

## 4.5. Flow betweenness centrality

Recently, Freeman et al. (1991) have proposed an extension of betweenness centrality that is applicable to valued relations. For a pair of actors, the value of the relation might be "their amount of interaction, the time they spend in one another's company, the range of different social settings in which they interact..." (Freeman et al., 1991, p. 145). Thus, this approach is clearly applicable to one-mode relations of actor co-memberships or event overlaps.

Flow betweenness centrality extends betweenness centrality in two ways. First it considers all paths between nodes, rather than just geodesics. Second, it is appropriate for both graphs and valued graphs in which larger values indicate stronger ties between actors. Values in the graph are considered to represent the potential for 'flow' of information or resources between nodes, with the following constraints. First, the flow between a pair of adjacent nodes, $f_{ij}$, is constrained to be less than or equal to the value of the tie between the nodes: $f_{ij} \leq x_{ij}$. Second the flow 'into' a node is equal to the flow 'out of' the node. The fact that this approach focuses on the betweenness of nodes in a graph and can be used to study valued relations suggests that it should be quite appropriate for studying centrality in affiliation networks.

To define the flow betweenness centrality of node $p_i$, one considers the extent to which the maximum flow between other pairs of nodes in the graph depends on paths that include node $p_i$. Roughly, the flow centrality index for a node in a valued graph quantifies the extent to which flow between other pairs of nodes in the graph would be reduced if that node were removed from the graph. Defining $F_{kl}(p_i)$ as the amount of flow between nodes $p_k$ and $p_l$ that passes through $p_i$, the flow betweenness centrality of $p_i$ is defined as the total flow between all pairs of nodes that depends on node $p_i$. The flow betweenness for node $p_i$, denoted by $C_F(p_i)$, is calculated as:

$$C_F(p_i) = \sum_{k<l} F_{kl}(p_i). \tag{33}$$

This index is described in detail in Freeman et al. (1991). Flow betweenness may be calculated for the one-mode networks of actor co-memberships and event overlaps, or for the bipartite graph.

Table 4 presents flow betweenness for actors and events in the bipartite graph. Table 5 presents results for the one-mode networks of actor co-memberships and event overlaps. Two results are worth noting in these tables. First, notice that actors $n_2$ and $n_4$ which each belong to only a single event have flow betweenness centralities equal to zero in the bipartite graph, but have flow centralities equal to four in the actor co-membership relation. In the one-mode network these actors cannot reside on geodesics between other actors, but they can reside on paths between other actors. Since flow betweenness considers 'flows' along all paths (not just geodesics), actors that belong to a single event can have non-zero flow betweenness in the one-mode actor co-membership relation.

A second puzzling result is seen in Table 4. Notice that for the bipartite graph actor $n_5$ (which is affiliated with all three events) has flow betweenness centrality equal to three, but actors $n_1$ and $n_3$ (which are each affiliated with two events) have centralities equal to five. However, looking at either the bipartite graph (Fig. 1) or the affiliation network matrix (Table 1), we see that the events to which actors $n_1$ and $n_3$ belong are subsets of the events to which $n_5$ belongs; $n_1$ and $n_3$ are never in an event unless $n_5$ is also a member. This anomalous result deserves further investigation.

## 5. Extensions

The centrality approaches discussed so far (degree, eigenvector, closeness, betweenness, and flow betweenness) were all designed for one-mode networks. As we have seen, all can be used to study affiliation networks by analyzing the bipartite graph. In this section I present two possible extensions of centrality using Galois lattices and graph covers. Both of these approaches are designed to study two-mode data, both focus on subsets and relations between subsets, and both allow insights into the difference between 'primary' and 'secondary' actors and events. Thus, these extensions incorporate the distinctive features of affiliation networks and might lead to centrality approaches that more naturally capture important features of these networks.

### 5.1. Galois lattices

Patterns of inclusion and overlap in events' memberships and in actors' affiliations are theoretically and empirically important components of actor and event centrality. As an early attempt to capture these relationships, Bonacich (1978) proposed using homomorphisms of Boolean algebras. The resulting homomorphisms were represented as lattices showing unions and intersections of events' members (with respect to a subset of actors), or of actors' affiliations (with respect to a subset of events). As noted by Bonacich, the height of an event or an actor in the lattice is related to its relative centrality. Bonacich's representation nicely shows the structure of inclusions among actors or among events, but it is limited in that it requires two separate representations

(one for actors and one for events). In addition, it captures only a portion of the inclusions among actors and events because the lattice for actors is constructed with respect to only a subset of events, and the lattice for events is constructed with respect to only a subset of actors. A Galois lattice provides a more appropriate approach since a single representation presents patterns of inclusions for all actors and events simultaneously.

Galois lattices have only recently been used to study affiliation networks (Schweizer, 1991; Freeman and White, 1993; Wasserman and Faust, 1994), though applications to other kinds of data are more widespread. General discussions of Galois lattices can be found in Birkhoff (1940) and Wille (1984).

A Galois lattice represents the relationship between two sets of objects in terms of inclusion mappings between subsets of objects from each set. For an affiliation network we have the two sets: $\mathcal{N} = \{n_1, n_2, \ldots, n_g\}$ and $\mathcal{M} = \{m_1, m_2, \ldots, m_h\}$. We also have the affiliation relation, denoted by $\alpha$, from $\mathcal{N}$ to $\mathcal{M}$, where $n_i \alpha m_k$ if $n_i$ 'is affiliated with' $m_k$. Thus $n_i \alpha m_k$ if actor $n_i$ belongs to event $m_k$. The inverse relation, $\alpha^{-1}$, is a mapping from $\mathcal{M}$ to $\mathcal{N}$, where $m_k \alpha^{-1} n_i$ if event $m_k$ includes actor $n_i$.

Extending the relations $\alpha$ and $\alpha^{-1}$ from individual actors and events to subsets of actors and events, we get two complementary mappings: $\uparrow$ from subsets of actors to subsets of events and $\downarrow$ from subsets of events to subsets of actors. The $\uparrow$ mapping maps a subset of entities in $\mathcal{N}$ to a subset of entities in $\mathcal{M}$. For subsets $\mathcal{N}_S \subset \mathcal{N}$ and $\mathcal{M}_S \subset \mathcal{M}$, we define $\mathcal{N}_S \uparrow \mathcal{M}_S$ if $n_i \alpha m_k$ for all $n_i \in \mathcal{N}_S$, and $m_k \in \mathcal{M}_S$. For an affiliation network, the $\uparrow$ mapping maps a subset of actors to the subset of events attended by all of the actors in that subset.

The dual mapping, $\downarrow$, goes from subsets of $\mathcal{M}$ to subsets of $\mathcal{N}$. For subsets $\mathcal{N}_S \subset \mathcal{N}$ and $\mathcal{M}_S \subset \mathcal{M}$, we define $\mathcal{M}_S \downarrow \mathcal{N}_S$ if $m_k \alpha^{-1} n_i$ for all $n_i \in \mathcal{N}_S$ and $m_k \in \mathcal{M}_S$. In an affiliation network the $\downarrow$ mapping maps a subset of events to the subset of actors all of which attend all events in the subset.

A Galois lattice represents the relation between the sets $\mathcal{N}$ and $\mathcal{M}$ in terms of the $\uparrow$ and $\downarrow$ mappings between subsets of entities from each set. A Galois lattice is presented in a diagram in which points represent entities or subsets of entities from the two sets, and lines represent subset–superset relations in the following way. There is a line or sequence of lines in the diagram ascending from object $n_i \in \mathcal{N}$ to object $m_k \in \mathcal{M}$ if $n_i \alpha m_k$, and there is a line or sequence of lines descending from object $m_k \in \mathcal{M}$ to object $n_i \in \mathcal{N}$ if $m_k \alpha^{-1} n_i$.

Fig. 3 presents the Galois lattice for the hypothetical example of six actors and three events (this figure is adapted from Wasserman and Faust, 1994, p. 333). In this figure an ascending line or sequence of lines connects each actor to the subset of events with which it is affiliated and a descending line or sequence of lines connects each event to the actors that are affiliated with it. One can also look at lines or sequences of lines between actors or between events. A line or sequence of lines ascending from actor $n_i$ to actor $n_j$ means that the events to which actor $n_j$ belongs are a subset of the events to which actor $n_i$ belongs. Similarly, a line or sequence of lines descending from event $m_k$ to event $m_l$ means that the actors that are affiliated with event $m_l$ are a subset of the actors that are affiliated with event $m_k$.

Viewing centrality in terms of subset–superset relations, actors that are relatively low
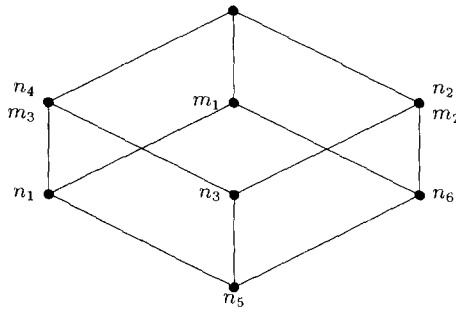
Fig. 3. Galois lattice for hypothetical example.

in the diagram are relatively more central than are actors that are relatively high in the diagram. Similarly, events that are relatively high in the diagram are relatively more central than are events that are relatively low in the diagram (Freeman and White, 1993). For the hypothetical example in Fig. 3, actor $n_5$ is the most central actor, followed by $n_1$, $n_3$, and $n_6$; $n_4$ and $n_2$ are least central. In this example the events are indistinguishable in terms of their centrality.

A Galois lattice is especially appropriate for studying affiliation networks because it explicitly represents the non-dyadic relation of inclusion between subsets of entities from the two modes. Furthermore, it represents an aspect of centrality in these networks, the subset–superset inclusions between actors and events, that is not represented by other centrality approaches. Freeman and White (1993) use Galois lattices in their reanalysis of the data of Davis et al. (1941) on Southern club women. Their results clearly showed which actors and events were primary and which were secondary in terms of the upward containment structure for actors and the downward containment structure for events.

## 5.2. Graph covers

The idea of *graph covers* was introduced by Seidman (1985) to formalize the extent to which actors potentially have access to information about different kinds of structural features of an affiliation network. An actor that is located in such a way that it can 'see' important features of other actors' membership patterns (such as pair-wise attendances or cohesive subgroups) might be well located to initiate or coordinate actions within the network. Seidman uses the idea of a graph cover to formalize exactly what features of an affiliation network an actor or a set of actors 'sees'.

We begin with a hypergraph $\mathscr{H}(\mathscr{N},\mathscr{M})$ consisting of a set of points $\mathscr{N}$ and a set of edges $\mathscr{M}$, where each edge is a subset of points from $\mathscr{N}$. For an affiliation network it seems natural to start with the set of points being the set of actors, $\mathscr{N}$, and the set of edges being the set of events, $\mathscr{M}$. (We could also reverse the roles of $\mathscr{N}$ and $\mathscr{M}$ and have the dual hypergraph $\mathscr{H}^*(\mathscr{M},\mathscr{N})$.) We next define a duality function, $\delta(n_i)$ for points. This duality function is a mapping from point $n_i$ to the set of all edges with which it is incident. In an affiliation network, this function for actors maps the actor to

the set of all events with which it is affiliated. Formally we can express this duality function as

$$\delta(n_i) = \{m_k \in \mathcal{M} \mid n_i \in m_k\}. \tag{34}$$

Using the notation from Galois lattices (above), $\delta(n_i)$ is the subset of events $\mathcal{M}_s$ for which $n_i \alpha m_k$.

Seidman (1985) then uses this duality function to define two different kinds of graph covers: a *strict cover* and a *wide cover*. A set is a wide (or strict) cover (as defined below) of some set of objects $\mathcal{A}$, where the objects are a specified set of entities. For example, the set $\mathcal{A}$ might be a subset of actors $\{n_1, n_2, n_3\}$ or it might be a subset of pairs of actors $\{(n_1, n_2), (n_1, n_3), (n_2, n_3)\}$.

In general, for some set of objects $\mathcal{A}$, a set $\mathcal{N}_s$ is a *wide cover* of $\mathcal{A}$ if the union of the duals of elements of $\mathcal{A}$ is included in the union of the duals of elements of $\mathcal{N}_s$. Formally, for object set $\mathcal{A}$ (a set of actors), $\mathcal{N}_s$ is a wide cover of $\mathcal{A}$ if

$$\bigcup_{a \in A} \delta(a) \subset \bigcup_{n_i \in \mathcal{N}_s} \delta(n_i). \tag{35}$$

Since the duality function $\delta(a)$ maps an actor to the set of events with which it is affiliated, for a subset of actors the union of these mappings is the collection of all events with which any actor in the subset is affiliated. Consider the case in which $\mathcal{A} = \mathcal{N}_t$ (a subset of actors). Another subset of actors, $\mathcal{N}_s$, is a wide cover of $\mathcal{N}_t$ if some actor in $\mathcal{N}_s$ was present at each and every event at which any actor from $\mathcal{N}_t$ was present. The events to which members of $\mathcal{N}_t$ belong are a subset of the events to which members of $\mathcal{N}_s$ belong. If $\mathcal{N}_s$ consists of a single actor, $\mathcal{N}_s = \{n_i\}$, then $n_i$ is a wide cover of the set of all actors that were never present unless $n_i$ was present.

To illustrate, in the hypothetical affiliation network of six actors and three events, $n_5$ is a wide cover of the set consisting of the remaining five actors, $\{n_1, n_2, n_3, n_4, n_6\}$; $n_3$ is a wide cover of the set $\{n_2, n_4\}$; $n_6$ is a wide cover of $\{n_2\}$; and $n_1$ is a wide cover of $\{n_4\}$. The set $\{n_1, n_6\}$ is a wide cover of the set $\{n_2, n_4\}$.

The second kind of graph cover is a *strict cover*. A strict cover is more general than a wide cover. For object set $\mathcal{A}$ (a set of actors), $\mathcal{N}_s$ is a strict cover of $\mathcal{A}$ if there exists some $n_i \in \mathcal{N}_s$ such that

$$\delta(n_i) \bigcap_{a \in \mathcal{A}} \delta(a) \neq \varnothing. \tag{36}$$

This means that for each object $a \in \mathcal{A}$ there exists some $n_i \in \mathcal{N}_s$ such that the dual of $a$ intersects the dual of $n_i$. In an affiliation network, $\mathcal{N}_s$ is a strict cover of $\mathcal{A}$ if, for every actor in $\mathcal{A}$, some actor in $\mathcal{N}_s$ belongs to some event with that actor. To illustrate, for the hypothetical example $n_6$ is a strict cover of $\{n_1, n_2, n_3, n_5\}$.

Every wide cover is a strict cover, but the reverse is not true. In an affiliation network, if $\mathcal{N}_s$ is a wide cover of $\mathcal{A}$, some actor from $\mathcal{N}_s$ is present each and every time any member of $\mathcal{A}$ is present. In other words, a member of $\mathcal{A}$ is never in an event unless a member of $\mathcal{N}_s$ is also present. In contrast, if $\mathcal{N}_s$ is a strict cover of $\mathcal{A}$, then each member of $\mathcal{A}$ must belong to at least one event with some member of $\mathcal{N}_s$, though members of $\mathcal{A}$ might belong to events where no member of $\mathcal{N}_s$ is present.

Returning to the Galois lattice diagram for an affiliation network, we see that a single

actor, $n_i$, is a wide cover of the set of all actors that reside on lines ascending from it. The events to which any of these other actors belong are a subset of the events to which $n_i$ belongs. A set of actors is a wide cover of that set of actors that reside on lines ascending from any of the actors in the set. A single actor is a strict cover of the set of all actors with which it is ever a co-member. In the Galois lattice this is the set of all actors from which lines intersect either ascending or descending lines from $n_i$ at any point before the topmost point in the diagram. Any subset of actors is a strict cover of the set of all actors that ever belonged to any event with any actor in the set. In the Galois lattice this is the set of all actors whose lines (ascending or descending) intersect a line from any member of the cover subset of actors, before the topmost point in the diagram.

Seidman (1985) elaborates in detail how graph covers can be used to study structural features of hypergraphs by varying the definition of the elements of $\mathscr{A}$. For example, if $\mathscr{A}$ consists of a set of pairs of actors, then a wide cover of $\mathscr{A}$ is a set of actors that are always present at any event at which any pair of actors in $\mathscr{A}$ is present.

## 5.3. Summary

Both the Galois lattice and the concept of graph covers nicely present subset–superset relationships among actors and events. This subset–superset relation is exactly the centrality idea of primary and secondary actors, as suggested by Davis et al. (1941), Doreian (1979a) and Freeman and White (1993). The Galois lattice incorporates the duality in the relation between actors and events in a single representation and makes the relative centrality of actors and events quite apparent in their vertical positions in the diagram. The distinction between primary events and actors and secondary events and actors is easily seen in the relative vertical positions of actors and events and in the patterns of inclusions among them.

The concept of graph cover formalizes the idea of the 'importance' of an actor (or a subset of actors) in terms of the 'view' that the actor has of the network. One might think of a graph cover in terms of the kind and extent of information about an affiliation network that is available to an actor or to a subset of actors. An actor that 'covers' a large number of other actors, or 'covers' a large portion of the co-memberships between pairs of actors potentially has access to information about those features of the network.

Both Galois lattices and graph covers provide a set of concepts for thinking about centrality in affiliation networks that is not provided by any of the other centrality indices. Both are naturally appropriate for two-mode non-dyadic networks and focus on subset–superset relations between actors and/or events in a way that captures theoretical and empirical insights about the relative importance of actors in these networks.

Neither the Galois lattice nor the formalism of a graph cover provides an index or quantification of actors' or events' centrality. One possibility would be to define a new relation between actors based on subset–superset relationships between their memberships. For example, one could define $x_{ij} = 1$ if $\delta(n_j) \subset \delta(n_i)$, and $x_{ij} = 0$ otherwise. The quantity $\sum_j x_{ij}$ would give the number of actors for which $n_i$ is a wide cover, while $\sum_j x_{ji}$ would give the number of actors that are wide covers of $n_i$. Exploring other possible indices is a fruitful arena for further research.

Now let us turn to another illustration.

## 6. Another example

As another example, consider the affiliation network of club and board memberships of corporate executive officers studied by Galaskiewicz (1985) as part of his extensive research on the urban grants economy in Minneapolis-St.Paul. I will focus on a subset of 26 CEOs and 15 clubs/boards from Galaskiewicz's data. The affiliation network matrix for this example is presented in Table 6. Centralities for CEOs and for the boards/clubs are in Table 7 (for the bipartite graph) and Table 8 (for the one-mode relations).

Overall there is considerable agreement among the centrality measures in their identification of which clubs and CEOs are relatively central. Tables 9–11 present correlation matrices for the five centrality indices, calculated on the bipartite graph (including all nodes, and for clubs and CEOs separately) and on the one-mode relations of club overlaps and actor co-memberships. The correlations between the measures of centrality for the clubs in the bipartite graph range from a low of 0.890 to a high of 0.992 with a median of 0.968; for CEOs in the bipartite graph the range is from 0.041 to 0.867 with a median of 0.643. For the one-mode relations, the correlation between the centrality measures for clubs ranges from 0.918 to 0.993 with a median of 0.954, and for CEOs the range is from 0.348 to 0.989 with a median of 0.711. It appears that there is

Table 6
Affiliation network matrix for CEOs and clubs

|    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| 1  | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 0  |
| 2  | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  |
| 3  | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 1  | 0  | 0  | 0  |
| 4  | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  |
| 5  | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 1  | 1  | 0  |
| 6  | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 1  | 0  |
| 7  | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1  | 1  | 0  | 0  | 0  | 0  |
| 8  | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1  | 0  | 0  | 0  | 0  | 0  |
| 9  | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1  | 0  | 0  | 0  | 0  | 0  |
| 10 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 0  |
| 11 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 0  |
| 12 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  |
| 13 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 0  |
| 14 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0  | 1  | 1  | 1  | 0  | 1  |
| 15 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 1  | 0  | 1  |
| 16 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 1  | 0  |
| 17 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0  | 1  | 1  | 0  | 0  | 1  |
| 18 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0  | 0  | 1  | 1  | 0  | 1  |
| 19 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 0  |
| 20 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0  | 1  | 0  | 0  | 0  | 1  |
| 21 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 0  |
| 22 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 1  |
| 23 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  |
| 24 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  |
| 25 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 1  | 0  | 0  |
| 26 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 1  | 0  | 0  | 0  |

Table 7
Centrality of CEOs and clubs in the bipartite graph

| | Measure of centrality | | | | | |
|---|---|---|---|---|---|---|
| | $C_D^{\mathcal{N}\mathcal{M}}$ | $\Sigma d(i,k)$ | $C_C^{\mathcal{N}\mathcal{M}}$ | $C_B^{\mathcal{N}\mathcal{M}}$ | $C_E^{\mathcal{N}\mathcal{M}}$ | $C_F^{\mathcal{N}\mathcal{M}}$ |
| $m_1$ | 3 | 112 | 35.71 | 5.53 | 0.057 | 49 |
| $m_2$ | 11 | 90 | 44.44 | 52.57 | 0.276 | 235 |
| $m_3$ | 22 | 62 | 64.52 | 374.10 | 0.476 | 527 |
| $m_4$ | 12 | 84 | 47.62 | 139.41 | 0.236 | 271 |
| $m_5$ | 3 | 112 | 35.71 | 3.91 | 0.064 | 64 |
| $m_6$ | 4 | 110 | 36.36 | 5.76 | 0.102 | 60 |
| $m_7$ | 4 | 114 | 35.09 | 12.61 | 0.049 | 125 |
| $m_8$ | 4 | 110 | 36.36 | 13.16 | 0.068 | 106 |
| $m_9$ | 6 | 104 | 38.46 | 17.13 | 0.114 | 156 |
| $m_{10}$ | 3 | 116 | 34.48 | 5.32 | 0.037 | 79 |
| $m_{11}$ | 4 | 110 | 36.36 | 5.52 | 0.118 | 45 |
| $m_{12}$ | 5 | 108 | 37.04 | 11.01 | 0.120 | 98 |
| $m_{13}$ | 5 | 106 | 37.74 | 12.56 | 0.119 | 100 |
| $m_{14}$ | 3 | 122 | 32.79 | 2.63 | 0.057 | 85 |
| $m_{15}$ | 9 | 94 | 42.55 | 37.77 | 0.237 | 146 |
| $n_1$ | 3 | 89 | 44.94 | 9.36 | 0.126 | 7 |
| $n_2$ | 3 | 93 | 43.01 | 20.88 | 0.090 | 58 |
| $n_3$ | 2 | 97 | 41.24 | 3.46 | 0.091 | 14 |
| $n_4$ | 3 | 95 | 42.11 | 3.24 | 0.151 | 5 |
| $n_5$ | 3 | 95 | 42.11 | 14.37 | 0.099 | 50 |
| $n_6$ | 3 | 97 | 41.24 | 11.75 | 0.123 | 38 |
| $n_7$ | 4 | 87 | 45.98 | 36.34 | 0.132 | 58 |
| $n_8$ | 3 | 117 | 34.19 | 9.09 | 0.049 | 59 |
| $n_9$ | 4 | 111 | 36.04 | 17.55 | 0.061 | 93 |
| $n_{10}$ | 2 | 97 | 41.24 | 3.12 | 0.090 | 6 |
| $n_{11}$ | 3 | 95 | 42.11 | 7.21 | 0.132 | 8 |
| $n_{12}$ | 2 | 119 | 33.61 | 3.02 | 0.043 | 23 |
| $n_{13}$ | 4 | 87 | 45.98 | 23.26 | 0.136 | 43 |
| $n_{14}$ | 7 | 81 | 49.38 | 49.62 | 0.241 | 52 |
| $n_{15}$ | 5 | 91 | 43.96 | 18.19 | 0.184 | 39 |
| $n_{16}$ | 5 | 91 | 43.96 | 35.86 | 0.149 | 79 |
| $n_{17}$ | 6 | 89 | 44.94 | 34.20 | 0.197 | 75 |
| $n_{18}$ | 5 | 91 | 43.96 | 24.93 | 0.126 | 35 |
| $n_{19}$ | 5 | 85 | 47.06 | 44.54 | 0.142 | 64 |
| $n_{20}$ | 5 | 85 | 47.06 | 23.79 | 0.205 | 27 |
| $n_{21}$ | 3 | 89 | 44.94 | 14.25 | 0.119 | 23 |
| $n_{22}$ | 3 | 93 | 43.01 | 11.12 | 0.119 | 25 |
| $n_{23}$ | 4 | 93 | 43.01 | 10.11 | 0.166 | 26 |
| $n_{24}$ | 5 | 85 | 47.06 | 37.87 | 0.169 | 61 |
| $n_{25}$ | 3 | 95 | 42.11 | 5.75 | 0.133 | 16 |
| $n_{26}$ | 3 | 95 | 42.11 | 6.08 | 0.133 | 16 |

greater consistency across centrality measures in the identification of central clubs than in the identification of central CEOs.

We can also look at the relationship between centrality measures for the bipartite graph and for the one-mode networks. Table 12 presents the correlation between each

Table 8
Centrality measures for CEOs and clubs in one-mode networks

| | Measure of centrality | | | | |
|---|---|---|---|---|---|
| | $C_D^{\mathcal{N}\,a}$ | $C_D^{\mathcal{N}\,b}$ | $C_B^{\mathcal{N}\,01}$ | $C_E^{\mathcal{N}}$ | $C_F^{\mathcal{N}}$ |
| $m_1$ | 11 | 14 | 1.85 | 0.080 | 55 |
| $m_2$ | 36 | 47 | 4.07 | 0.391 | 122 |
| $m_3$ | 62 | 84 | 10.70 | 0.673 | 280 |
| $m_4$ | 38 | 50 | 6.97 | 0.333 | 200 |
| $m_5$ | 10 | 13 | 1.09 | 0.091 | 35 |
| $m_6$ | 15 | 19 | 1.27 | 0.144 | 52 |
| $m_7$ | 9 | 13 | 0.70 | 0.070 | 45 |
| $m_8$ | 11 | 15 | 2.10 | 0.096 | 49 |
| $m_9$ | 16 | 22 | 1.99 | 0.162 | 51 |
| $m_{10}$ | 8 | 11 | 0.95 | 0.052 | 41 |
| $m_{11}$ | 18 | 22 | 1.51 | 0.167 | 55 |
| $m_{12}$ | 18 | 22 | 0.29 | 0.170 | 51 |
| $m_{13}$ | 18 | 23 | 2.11 | 0.169 | 57 |
| $m_{14}$ | 8 | 11 | 0.17 | 0.080 | 28 |
| $m_{15}$ | 34 | 43 | 3.24 | 0.336 | 106 |
| $n_1$ | 37 | 40 | 3.85 | 0.178 | 407 |
| $n_2$ | 26 | 29 | 2.60 | 0.127 | 278 |
| $n_3$ | 25 | 27 | 0.15 | 0.128 | 248 |
| $n_4$ | 39 | 42 | 0.15 | 0.213 | 337 |
| $n_5$ | 27 | 30 | 0.15 | 0.140 | 274 |
| $n_6$ | 33 | 36 | 0.00 | 0.174 | 312 |
| $n_7$ | 37 | 41 | 3.85 | 0.187 | 392 |
| $n_8$ | 16 | 19 | 0.13 | 0.069 | 207 |
| $n_9$ | 18 | 22 | 0.43 | 0.086 | 206 |
| $n_{10}$ | 26 | 28 | 0.15 | 0.127 | 269 |
| $n_{11}$ | 36 | 39 | 0.15 | 0.187 | 345 |
| $n_{12}$ | 14 | 16 | 0.13 | 0.061 | 172 |
| $n_{13}$ | 39 | 43 | 3.85 | 0.192 | 429 |
| $n_{14}$ | 61 | 68 | 3.85 | 0.341 | 586 |
| $n_{15}$ | 46 | 51 | 0.15 | 0.261 | 406 |
| $n_{16}$ | 39 | 44 | 1.09 | 0.211 | 387 |
| $n_{17}$ | 48 | 54 | 0.15 | 0.278 | 431 |
| $n_{18}$ | 32 | 37 | 3.09 | 0.178 | 294 |
| $n_{19}$ | 42 | 47 | 3.85 | 0.201 | 504 |
| $n_{20}$ | 53 | 58 | 3.85 | 0.289 | 483 |
| $n_{21}$ | 35 | 38 | 3.85 | 0.168 | 388 |
| $n_{22}$ | 32 | 35 | 1.24 | 0.168 | 301 |
| $n_{23}$ | 42 | 46 | 0.15 | 0.235 | 354 |
| $n_{24}$ | 45 | 50 | 3.85 | 0.239 | 437 |
| $n_{25}$ | 35 | 38 | 0.15 | 0.188 | 323 |
| $n_{26}$ | 35 | 38 | 0.15 | 0.188 | 324 |

[a] Excludes diagonal. [b] Includes diagonal.

kind of centrality index calculated on the bipartite graph and on the one-mode relation, for CEOs and clubs separately. For example, the first line in the body of the table gives the correlation between the degree centralities for CEOs and for clubs in the bipartite

Table 9
Correlations between centrality measures: clubs and CEOs in the bipartite graph

|  | $C_D^{\mathcal{N}\mathcal{M}}$ | $C_C^{\mathcal{N}\mathcal{M}}$ | $C_B^{\mathcal{N}\mathcal{M}}$ | $C_E^{\mathcal{N}\mathcal{M}}$ |
|---|---|---|---|---|
| $C_C^{\mathcal{N}\mathcal{M}}$ |  |  |  |  |
| All nodes | 0.694 |  |  |  |
| Clubs | 0.992 |  |  |  |
| CEOs | 0.627 |  |  |  |
| $C_B^{\mathcal{N}\mathcal{M}}$ |  |  |  |  |
| All nodes | 0.928 | 0.752 |  |  |
| Clubs | 0.948 | 0.968 |  |  |
| CEOs | 0.867 | 0.659 |  |  |
| $C_E^{\mathcal{N}\mathcal{M}}$ |  |  |  |  |
| All nodes | 0.889 | 0.872 | 0.816 |  |
| Clubs | 0.980 | 0.967 | 0.890 |  |
| CEOs | 0.780 | 0.823 | 0.576 |  |
| $C_F^{\mathcal{N}\mathcal{M}}$ |  |  |  |  |
| All nodes | 0.938 | 0.485 | 0.892 | 0.696 |
| Clubs | 0.977 | 0.969 | 0.960 | 0.921 |
| CEOs | 0.572 | 0.041 | 0.696 | 0.045 |

Table 10
Correlations between centrality measures: one-mode matrix of club overlaps

|  | $C_D^{\mathcal{M}}$ | $C_C^{\mathcal{M}}$ | $C_B^{\mathcal{M}}$ | $C_E^{\mathcal{M}}$ |
|---|---|---|---|---|
| $C_C^{\mathcal{M}}$ | 0.958 |  |  |  |
| $C_B^{\mathcal{M}}$ | 0.937 | 0.954 |  |  |
| $C_E^{\mathcal{M}}$ | 0.993 | 0.930 | 0.918 |  |
| $C_F^{\mathcal{M}}$ | 0.959 | 0.954 | 0.985 | 0.936 |

Table 11
Correlations between centrality measures: one-mode matrix of CEO co-memberships

|  | $C_D^{\mathcal{N}}$ | $C_C^{\mathcal{N}}$ | $C_B^{\mathcal{N}}$ | $C_E^{\mathcal{N}}$ |
|---|---|---|---|---|
| $C_C^{\mathcal{N}}$ | 0.734 |  |  |  |
| $C_B^{\mathcal{N}}$ | 0.449 | 0.687 |  |  |
| $C_E^{\mathcal{N}}$ | 0.989 | 0.664 | 0.348 |  |
| $C_F^{\mathcal{N}}$ | 0.942 | 0.788 | 0.641 | 0.888 |

Table 12
Correlations between centralities for the bipartite graph and the one-mode relations, for CEOs and clubs

| Centrality measure | CEOs | Clubs |
|---|---|---|
| Degree | 0.767 | 0.979 |
| Betweenness[a] | 0.634 | 0.946 |
| Eigenvector | 1.000 | 1.000 |
| Flow | 0.137 | 0.944 |

[a] Betweenness centralities for dichotomized one-mode relation.

Fig. 4. Galois lattice for CEOs and clubs.

graph and in the one-mode actor co-membership relation (the $C_D^{\mathcal{N}\mathcal{M}}(n_i)$ and the $C_D^{\mathcal{N}}(n_i)$ and the $C_D^{\mathcal{N}\mathcal{M}}(m_k)$ and the $C_D^{\mathcal{M}}(m_k)$). For these data, there is greater discrepancy for actors (the CEOs) than for events (the clubs) between the centralities from the bipartite graph and the centralities from the one-mode relation.

Fig. 4 presents the Galois lattice for the CEOs and clubs. [2] First, consider the relative centrality of the clubs. Clubs 3 and 4 are the only clubs whose memberships contain the memberships of other clubs (club 4 contains clubs 1 and 10; club 3 contains clubs 2, 5, 6, 11 and 14). Clubs 3 and 4 are located toward the top of the diagram in Fig. 4 and the containments are indicated by the lines descending from these clubs. More complicated patterns of containment are present among the CEOs. CEO 14 contains 25, 26 and 20 (who in turn contains 4); 15 contains 23 and 25; 17 contains 26 (who in turn contains 3); 8 contains 12; 11 contains 10; 13 contains 1 (who in turn contains 10); 19 contains both 1 and 12; and 16 contains 6. This suggests at least three levels of centrality for CEOs. The most central CEOs are 8, 11, 13, 14, 15, 16, 17 and 19; second are 1, 6, 12, 20, 23, 25 and 26; and at the third level are 3, 4 and 23. The memberships of the remaining CEOs are not included within the memberships of others, nor do they include others, thus, their centrality relative to others cannot be determined using this approach. Further analyses show that other indices of centrality for the CEOs are consistent with these three levels in the Galois lattice. CEOs in the first level have higher centrality on other indices than do CEOs in the second level. CEOs in the second level have higher centrality values than do the CEOs in the third level. CEOs whose centrality is

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[2] This figure was produced using the program *DIAGRAM* (Vogt and Bliegener, 1990).

indeterminate in the Galois lattice have moderate to high levels of centrality on other centrality indices.

In conclusion, let us consider some of the strengths and weaknesses of centrality indices for studying affiliation networks.


## 7. Discussion

The introduction to this paper presented four ideas that might motivate centrality approaches that would be especially appropriate for affiliation networks. Let us now consider existing centrality indices and possible extensions in light of these ideas.

First, it is important to have centrality scores for both actors and events in an affiliation network. This is not a problem for any of the approaches, since all can be used to analyze the bipartite graph. However, because of the duality in the relationship between actors and events, we might hope that the centralities of actors should be related in specifiable ways to the centralities of the events with which they are affiliated, and similarly the centralities of events should be related in specifiable ways to the centralities of the actors they include. These relationships hold for degree centralities calculated on the one-mode networks of actor co-memberships and event overlaps (Eqs. (10) and (11)), for closeness centrality in the bipartite graph (Eqs. (27) and (28)), and for eigenvector centralities in the bipartite graph or in the pair of one-mode matrices (Eqs. (16) and (17) or Eqs. (19) and (20)). Relationships between actor and event centralities are not as straightforward for betweenness centrality and flow betweenness centrality.

A second feature of affiliation networks is that the affiliation relation is non-dyadic, and thus focuses on subsets. To what extent do centralities in affiliation networks allow us to quantify the centrality of a subset of actors or a subset of events? In one sense, any index for which an actor's centrality is a function of the centralities of its events, or an event's centrality is a function of the centralities of its members, does capture the centralities of subsets. An actor's centrality is a function of the centrality of the subset of events to which it belongs. An event's centrality is a function of the subset of actors belonging to it. As noted above, these properties hold for degree, closeness, and eigenvector centralities, but not for betweenness and flow betweenness.

A third important theoretical property of an affiliation network is that actors create linkages between events and events create linkages between actors. Central actors should link events and central events should link actors. Theoretically, actors that do not link events, and events that do not link actors, should not be considered central. The property of linkages among actors and/or events seems most naturally embodied in betweenness, flow betweenness, and eigenvector centrality, but is also most problematic for these approaches. Part of the problem is that misleading results arise when one analyzes the one-mode relations of actor co-memberships or event overlaps. These relations lack important information about the pattern of affiliation ties. When used to analyze the bipartite graph, betweenness centrality and flow betweenness centrality give centralities equal to zero for actors that belong only to one event and to events with only a single member, as desired. However, eigenvector centrality gives non-zero values to such actors and events, even in the bipartite graph. This property is consistent with the

intuitive motivation for eigenvector centrality that a node's centrality is a function of the centralities of the nodes to which it is adjacent, but is inconsistent with the idea that centrality in affiliation networks is a result of the linkages created by actors and events.

The fourth empirically and theoretically important aspect of centrality in an affiliation network is the subset–superset (i.e. inclusion) relationships between actors' affiliations and events' memberships. These relationships capture the distinction between 'primary' and 'secondary' actors and events observed by numerous researchers. However, this distinction is not quantified by any of the existing centrality indices reviewed here. Subset–superset relationships are nicely displayed in the Galois lattice and are formalized by the idea of graph covers. Developing a centrality index based on these inclusion relations seems like a fruitful area for future research.

A final practical note is in order. Seidman (1981) discusses the misleading conclusions that can result when one studies one-mode networks of actor co-memberships or event overlaps for an affiliation network. Specifically, he demonstrates that cliques of actors in the co-membership relation (or events in the overlap relation) are not necessarily complete subgraphs in the affiliation relation. Thus, one is not necessarily justified in drawing conclusions about collections larger than pairs from the one-mode relations. As we have seen, similar cautions are in order for centrality analyses of the one-mode networks derived from the affiliation network. In going from the affiliation relation to either the actor co-membership relation or the event overlap relation, one loses information about the patterns of affiliation between actors and events. Thus, one needs to be cautious when interpreting centralities for these one-mode relations.

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