Social Networks 7 (1985) 77–103
North-Holland

DOES *STRUCTURE* FIND STRUCTURE?: A CRITIQUE OF BURT'S USE OF DISTANCE AS A MEASURE OF STRUCTURAL EQUIVALENCE *

Katherine FAUST and A. Kimball ROMNEY

University of California, Irvine **

This paper examines some of the assumptions and consequences of the use of distance as a measure of structural equivalence, as implemented in Burt's STRUCTURE program. We take the general perspective that for a measure to be useful it should not confound separate types of information which are theoretically and mathematically independent. The mathematical relationship between distance and the Pearson product moment correlation coefficient is presented. We show that use of distance as a measure of similarity without proper attention to appropriate standardization procedures confounds information on differences between means and differences between variances with information on the similarity of the patterns between pairs of individuals, e.g. correlation. A detailed examination of Burt and Bittner's analysis of Bernard, Killworth and Sailer's Ham radio operator group is presented, and it is demonstrated that use of distance as a measure of structural equivalence led to nonsensical results.

1. Introduction

In a series of articles Burt has presented a method for detecting structurally equivalent actors in a network (Burt 1976; 1978; 1980; 1982; Burt and Minor 1983). This method, implemented in the computer program STRUCTURE (Burt n.d.), uses a combination of distance as a measure of structural equivalence and connectedness cluster-

* The authors would like to thank Linton Freeman and Sue Freeman for discussion and ideas freely given at all stages of the research. Portions of the work were funded by NSF Grant Number SES-8320173 to A.K. Romney and W.H. Batchelder. We are indebted to Steve Borgatti for some of the derivations appearing in the section on "Distance as a measure of similarity."

** School of Social Sciences, University of California, Irvine, CA 92717, U.S.A.

ing for identifying jointly occupied positions. The STRUCTURE approach to structural equivalence has been used in a number of empirical studies: on Laumann and Pappi's German elites (Burt 1976), on elites in sociological methodology (Burt 1983), and in an analysis of Bernard, Killworth and Sailer's (hereafter BKS) Ham radio operators (Burt and Bittner 1981). However, we have encountered instances where the method gives what we believe are nonsensical results. In this paper we examine in detail why a measure of structural equivalence based on distance, as used in STRUCTURE, runs into difficulties when applied to empirical data.

Structural equivalence is essentially a concept of similarity, and as such falls within the general discussion of measures of similarity across arrays of observations. In a network context, we are interested in the similarity of two actors' relations with other actors in the network. Given a sociomatrix (or matrices) in which the entries are the presence, absence or intensity of a given relation (or set of relations) among actors, the question is: how is similarity of relations of actors to be measured? This question requires both a model or set of precise definitions of the concept, and a measure which is the formalization of that definition.

Burt has provided the following definition. "The extent to which specific relational patterns occur repeatedly across multiple actors is captured by the concept of structural equivalence." (1980: 101) In an earlier paper he states "Two or more actors jointly occupy the same network position when they have similar relations to and from each actor in the network." (Burt 1976: 93) We interpret this to mean that the identification of positions within a network requires grouping actors with similar "relational patterns".

While there seems to be a fair amount of agreement about the intuitive notion of structural equivalence, there is wide debate about the proper computation of such a measure. Lorrain and White (1971) offered one formalization, based on algebraic models. They argued that two actors are structurally equivalent if they have identical relations to and from all other actors in a network. That is, "... *a* is structurally equivalent to *b* if *a* relates to every object *x* of *C* in exactly the same ways as *b* does." (1971: 63) However, this definition is generally recognized to be too strict. Recently several researchers have offered approaches of regular equivalence attempting to employ a more general notion of structural equivalence (Boyd 1983; Mandel and Winship

1979; Sailer 1978; White and Reitz 1983). These approaches share the common objective that actors be structurally equivalent if they are related in the same ways to *equivalent* others, not to *identical* others (Sailer 1978).

Other alternatives to the algebraic approach deriving from Lorrain and White are embodied in CONCOR, which employs iterative correlations as an algorithm for clustering actors based on the similarity of their relations (Breiger *et al.* 1975), and Burt's STRUCTURE, which uses a distance measure instead of a correlation. Drawing on the definition of structural equivalence proposed by Lorrain and White (1971) Burt notes that "Two actors I and J are structurally equivalent under this strong criterion when the distance between their respective network positions is zero" (Burt 1980: 102).

One would expect that since both CONCOR and STRUCTURE were designed to study structural equivalence by computing a measure of similarity between actors' across relations, that their results would be somewhat similar. However, the two programs give very different results for some data sets. We shall indicate why below.

One application of STRUCTURE which led us to question its usefulness is Burt and Bittner's re-analysis of data on a group of Ham radio users (Hams) collected by BKS. In that paper Burt and Bittner report on a most anomalous kind of social network structure. "Instead of the usual elitist center–periphery structure in which actors at the center of the system occupy its core status and jointly define the role of being a leading member of the system, the MWA (Hams) has a pluralistic center–periphery structure. The one clear status in the Association is occupied by Hams isolated from members of the Association. It is a status for Hams who are members more in name than in radio contact activity" (1981: 84–85). They conclude that "there is one status occupied at its core by Hams clustered together under a strong criterion of equivalence (e.g. Hams 23, 36 and 41)." (1981: 80) What seems unusual about all this is that Hams 23 and 36 are two of three Hams who were never observed to have communicated on the radio, and Ham 41 was observed only once.

Figure 1 shows the structure of the Hams group as envisioned by Burt and Bittner. We examine this figure in detail below.

If it is indeed true that members of a group who never interact with the group are the central core members then we have to give up some very useful notions, both commonsense and theoretical. For example,

```
Marginal       1 1 1 1 1
total          9 3 6 4 5 6 9 1         1 2 2 1 1 3 1         4 3 1 1                               3 1
interaction    8 3 4 7 2 5 7 2 5 2 2 9 9 2 8 8 5 4 3 2 2 3 2 0 1 9 8 5 1 6 2 2 1 2 2 1 1 1 0 0 0 1 9 7
---------------------------------------------------------------------------------------------------------
               G d B R f P n T S X b e J j i N Y a m I K U k V W Z h g H M F O c A L C E Q . . . 1 D o

                                              HAMS NETWORK POSITION

 0.000         . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . XXXXX . . .
 1.255         . . . . . . . . . . . . . . . . . . . . . . . . XXX . . . . . . . . . . . . XXXXX . . .
 1.385         . . . . . . . . . . . . XXX . . . . . . . . . . XXX . . . . . . . . . . . . XXXXX . . .
 1.391         . . . . . . . . . . . . XXX . . . XXX . . . . . XXX . . . . . . . . . . . . XXXXX . . .
 1.398         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . . . . . . . . . . . . XXXXX . . .
 1.414         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . . . . XXX . XXXXXXXXXXXXXXX . . .
 1.415         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . . . . XXXXXXXXXXXXXXXXXXXXX . . .
 1.416         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . . . . XXXXXXXXXXXXXXXXXXXXXXX . .
 1.419         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . . . XXXXXXXXXXXXXXXXXXXXXXXXX . .
 1.436         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . . XXXXXXXXXXXXXXXXXXXXXXXXXXX . .
 1.484         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . . XXXXXXXXXXXXXXXXXXXXXXXXXXX XXX
 1.492         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXX . XXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXX
 1.547         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXXXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXX
 1.643         . . . . . . . . . . . . XXX . XXX XXX . . . . . XXXXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.662         . . . . . . . . XXX . . XXX . XXX XXX . . . . . XXXXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.685         . . . . . . . . XXX . . XXX . XXX XXX . . . . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.689         . . . . . . . . XXX XXX XXX . XXX XXX . . . . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.732         . . . . . . . . XXX XXX XXX . XXX XXX . . . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.733         . . . . . . . . XXX XXX XXX . XXX XXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.745         . . . . . . . . XXX XXX XXX . XXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.750         . . . . . . . . XXX XXX XXX . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.780         . . . . . XXX . XXX XXX XXX . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.782         . . . . . XXX . XXX XXX XXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.801         . . . . . XXX . XXX XXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.803         . . . . . XXX . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.910         . . . . . XXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 1.925         . . . . . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 2.328         . . . XXX XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 2.726         . . . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 3.031         . . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 3.152         . XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 3.296         XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

Figure 1. Connectedness clustering of distances between Hams standardized behavior.

we could no longer generalize that the more active, more committed, more powerful members of a group tend to occupy central roles. We would have to assume that in some "elitist" groups the active and powerful are central and in some groups like the Hams isolates are central.

In this paper we examine in detail the procedure implemented in STRUCTURE. We focus in particular on the mathematical properties of distance as a measure of similarity, its relation to the correlation coefficient, and the confounding effects of differences of means and variability among subjects. We then illustrate the contrary results arising from use of STRUCTURE by examining its application to the Hams data. We conclude that Burt and Bittner did not discover a new kind of group but instead that the use of STRUCTURE, which is based on distance, led to nonsensical results.

2. Distance as a measure of similarity

At its core, the STRUCTURE program is based on the computation of distance as a measure of structural equivalence between pairs of actors in a network. For a matrix X, with values x_{ij} between actors i and j, the distance between i and j, d_{ij}^2, is given by:

$$d_{ij}^2 = \sum_{k=1}^{N} (x_{ik} - x_{jk})^2. \tag{1}$$

We use d_{ij}^2 rather than its square root, d_{ij} (which Burt uses), because of its clear relationship to other statistics.

Distance has been widely employed as a measure of similarity in a number of fields and its mathematical properties have been explored in detail. Distance (Equation 1) is but a special case of the more general Minkowski r metrics. Distance is so widely used as a measure of similarity that no discussion of the measurement of similarity, especially in the context of clustering, scaling, or classification, is complete without a mention of it (Coxon 1982; Gordon 1981; Sneath and Sokal 1973; Sokal and Sneath 1963).

Penrose (1952) and Cronbach and Gleser (1953) provide some of the early discussions of distance as a measure of profile similarity, and both note that the magnitude of the distance increases not only with the similarity in the shape or pattern of values of two variables, but also with the differences in means of the objects measured and with the differences in variability of the values. Both propose methods for eliminating differences in mean and variance. Penrose approaches the problem by successively subtracting mean and variance components from the computed distance. Cronbach and Gleser suggest standardizations of the variables to remove mean and variance differences.

Simple re-expression of the distance formula in terms of the mean, M_i, variance, S_i^2, and product moment correlation, r_{ij}, of two variables makes the contribution of not only mean, but also variance and shape, readily apparent.

$$d_{ij}^2 = N\left[\overset{\textit{Mean}}{(M_i - M_j)^2} + \overset{\textit{Variability \& Shape}}{S_i^2 + S_j^2 - 2r_{ij}S_iS_j}\right]. \tag{2}$$

This expression makes clear that distance is to a large degree a function of the differences in mean and variance between two variables. In extreme cases, where the correlation between two variables is perfect d_{ij}^2 is purely a function of the mean, variance and number of observations.

If $r_{ij} = 1$ then

$$d_{ij}^2 = N\left[\overset{Mean}{\left(M_i - M_j\right)^2} + \overset{Variability}{\left(S_i - S_j\right)^2}\right].$$

If $r_{ij} = -1$ then

$$d_{ij}^2 = N\left[\left(M_i - M_j\right)^2 + \left(S_i + S_j\right)^2\right].$$

However, under certain conditions distance can be simply a measure of shape. For variables with equal means ($M_i = M_j$) and variances of 1, ($S_i^2 = S_j^2 = 1$) distance reduces to:

$$\overset{Shape}{d_{ij}^2{}^* = 2N\left(1 - r_{ij}\right)} \tag{3}$$

where $d_{ij}^2{}^*$ is the distance between standardized variables.

The relationship between distance and the Pearson product moment correlation for standardized variables has been presented by numerous authors (Cronbach and Gleser 1953; Coxon 1982; Fox 1982; Rohlf and Sokal 1965; Sneath and Sokal 1973; Sokal and Sneath 1963).[1] Thus distance computed on standardized variables gives a measure of similarity which is based entirely on the similarity in the pattern of observations. Furthermore, $d_{ij}^2{}^*$ has specifiable upper and lower bounds which depend on the number of observations.

If $r_{ij} = 1$ then $d_{ij}^2{}^* = 0$.

If $r_{ij} = -1$ then $d_{ij}^2{}^* = 4N$.

[1] Equation 3 expressing the relationship between distance computed on standardized variables, $d_{ij}^2{}^*$, and the Pearson product moment correlation coefficient differs from the equations in both Cronbach and Gleser (1953) and Rohlf and Sokal (1965). Cronbach and Gleser use $\sqrt{\Sigma(x_i - M)^2}$ as a measure of variability, giving $d_{ij}^2{}^* = 2(1 - r_{ij})$. Rohlf and Sokal compute variance using $(N-1)$ in the denominator, rather than N as we do, giving $d_{ij}^2{}^* = 2(N-1)(1 - r_{ij})$.

Not unexpectedly the relationship between distance and correlation holds also for dichotomously coded variables. Since much social network data is binary, where x_{ij} indicates the presence or absence of a link between actors i and j, it is worth noting the form of this relationship. Here the correlation between vectors for i and j is the phi coefficient, which is computationally equivalent to the Pearson product moment correlation between the $i\,j$ vectors.

For a 2×2 table expressed as proportions (p_{ij}) summing to 1:

		j = 1	j = 0	
i	1	p_{11}	p_{10}	$p_{1.}$
	0	p_{01}	p_{00}	$p_{0.}$
		$p_{.1}$	$p_{.0}$	1

$d_{ij}^2 = N(p_{10} + p_{01})$, as noted by Knoke and Kuklinski (1982). Or, in terms of phi:

$$d_{ij}^2 = N\left[(p_{1.} - p_{.1})^2 + p_{1.}p_{0.} + p_{.1}p_{.0} - 2\,\text{phi}(p_{1.}p_{0.}p_{.1}p_{.0})^{1/2}\right].$$

If the 2×2 table is normalized to equal marginal totals of 0.5 and N is taken to be 1, then distance is

$$d_{ij}^2 = 0.5\,(1 - \text{phi}).$$

Burt, in commenting on Cronbach and Gleser's evaluation of distance rejects correlation as a measure of structural equivalence, stating,

> ... these alternative models [correlation and covariance] are biased in the sense of ignoring aspects of relational pattern. This bias varies across different measures of relations, but results in an overestimation of structural equivalence to the extent that it exists. Accordingly, extreme caution is required in interpreting the role-sets generated by the above models." (1980: 106)

As we show in the following example, it is exactly the opposite bias, ignoring similar patterns in favor of differences in mean and variability, which leads to nonsensical results.

In choosing distance as a measure of similarity between profiles one is making implicit and often unstated assumptions about what information is to be included in determining the magnitude of similarity. In any empirical investigation it is critical to construct and employ measures which do not confound sets of information that are mathematically and theoretically independent. Any variable has in it (at least) three separate components of information: the mean, the variance and the pattern of the values in the variable. Use of distance as a measure of similarity between variables which differ in means or variances confounds the separate effects of differences in these two components of information with similarity in the patterns of values in the two variables. Since structural equivalence requires a measure of similarity of *relational pattern*, we feel that similarity as indexed by a correlation coefficient contains the information on positional structure among actors in a network.

To understand this clearly imagine that one has only the mean frequencies for BKS's Hams observed behavior. Suppose that you then produce a matrix of expected values (row total times column total all divided by grand total). In such a case we would not look for a pattern because we know there is no structural differentiation among actors. We might note here that STRUCTURE would give results that look very similar to Figure 1. This is because much of the distance comes from differences in means. CONCOR would not run because all the correlation coefficients would be 1.

3. Empirical example

As an example of the misleading results encountered in using distance as a measure of structural equivalence, we examine the analysis conducted by Burt and Bittner (1981) of data collected by BKS (1980) on communications among Ham radio operators of the Monongalia Wireless Association. We start by describing Burt's procedure for obtaining structural equivalence and then provide an interpretation of his results. We hope to illustrate that in his analysis the degree to which actors occupy the same social position (are structurally equivalent to each other) is due largely to the similarity of the mean level and variability of individual actors' interactions rather than to the similarity in the pattern of their communications.

Before looking at the concrete procedures used by Burt and Bitter we need to review the data upon which the analysis is based. To avoid any ambiguity we quote at length from BKS.

Hams. Our third set of data comes from a group of amateur radio operators, commonly called "hams", living in West Virginia, western Pennsylvania, and eastern Ohio. The hams belong to the Monongalia Wireless Association (MWA), which owns and maintains WR8ABM, a 2-meter, FM repeater station....

With the cooperation of the MWA, all conversations on WR8ABM were monitored around the clock for 27 days, using a voice-operated relay between a receiver and a tape recorder. ... For current purposes, only the frequency of communication was used. ...

A "repeater" allows groups of persons to participate in conversation, so long as only one person speaks at a time. Thus, this data set includes n-tuple interactions. All dyads were listed separately for the analysis.

At the end of the 27 day monitoring period, a list of 54 users was drawn up. (Eventually, we found a total of 107 users; by the end of the monitoring period, however, we had recorded 54 users who accounted for all but a small fraction of the repeater's air time. The other 53 calls were mostly casual or transient users.) Each person was mailed a sheet with all 54 calls, and asked to scale them from 0 (no communication) to 9 (a great deal of communication). A total of 44 usable responses were obtained. (Bernard *et al.* 1980: 195)

Both BKS and Burt and Bittner were motivated by the question of the correspondence between social network data collected through observations and those collected by asking people about their communications with others. Therefore, both BKS and Burt and Bittner looked at both the observational and the reported data sets (though they arrived at different conclusions about the correspondence between the two). In the current paper we restrict our analysis to the behavioral data, though our results have implications for what correspondence might be found between behavioral and reported data.

4. STRUCTURE: Detecting the Hams postional structure

The method used by Burt and Bittner deserves careful attention, since at several points implicit assumptions are made about the nature of structural equivalence which have severe consequences for the outcome of the analysis.

One of the initial issues in analysis of any social group is delimiting the boundaries of the group under study. For the Monongalia Wireless Association BKS identified 107 members total. After recording communication for a period of 27 days and administering questionnaires they narrowed the group to 44 "core" members who also responded to their questionnaire. This is the group analyzed by BKS, and re-analyzed by Burt and Bittner. However, in scanning the marginal data on how frequently each Ham was observed in radio communication, 3 of the

Hams were never observed on the radio. An additional 15 Hams were on the radio only one or two times. The variability among individuals in the level of participation presents an important problem in the use of the distance measure. Some standardization of the observed interactions is necessary in order to make them comparable. Burt notes: "Before searching for statuses as general network subgroups, the relational pattern in which any one ham is involved must be comparable to that in which each other is involved. If the relation from ham J to ham K is in a metric different from that in which the relation from I to K is measured, then z_{jk} and z_{ik} can not be compared meaningfully..." (Burt and Bittner 1981: 78). "...relations to be compared in order to locate structurally equivalent actors in a network should be transformed into a comparable metric in order to clearly interpret distance measures of similarity between the relational patterns in which actors are involved." (Burt and Bittner 1981: 79).

The standardization proposed by Burt divides the values in each row of the data matrix by the largest value in the row. A value equal to the largest standardized value (either zero or 1) is placed on the diagonal, indicating high self–self interaction. The possible range of the total for any standardized variable is from zero to N.

In replicating the analysis of the Hams data, the 44×44 matrix of raw observations was standardized by rows by dividing each value in each row by the largest value in that row. However, this procedure did little to remove the differences in mean and variance across actors. For the 44 Hams, the correlation between an actor's mean level of communication in the raw data and their mean after standardization is $r = 0.944$. The correlation between the variance of each actor's communications and the variance after standardization is $r = 0.786$.

A critical question in the computation of similarity on social network data, as distinct from the usual profile data, is the proper treatment of diagonal elements. Arguments have been made for inclusion (Burt 1976; Guttman 1977), exclusion (Arabie *et al.* 1978; Schwartz 1977) and placement of an arbitrary value of zero on the diagonal (Breiger *et al.* 1975). An additional complication arises when a matrix is stacked atop its transpose prior to calculating similarity. Knoke and Kuklinski (1982) argue for excluding diagonal elements in the transpose of the matrix because if the corresponding diagonal elements are included for both orginal and transposed matrices, the $x_{ii}-x_{ij}$ and $x_{jj}-x_{ji}$ comparisons would be counted twice. STRUCTURE follows this rule. However, the more pressing issue is that inclusion of diagonal elements at all confounds the proximity of two individuals to each other with their similarity computed across their relations with others. An arbitrarily large value on the diagonal enhances the similarity between individuals who interact frequently with each other (are in close proximity) and detracts from the similarity of actors who never interact with each other. An arbitrarily small value on the diagonal increases the similarity of people who seldom or never interact (Schwartz 1977). In either case, inclusion of the diagonal in computation of distance (or correlation) leads to a confounding of two separate aspects of the relations between a pair of individuals: the similarity in their pattern of interactions with others in the group, and the frequency of their interaction with each other. It confounds a pairs' similarity *vis-à-vis* the rest of the group with their proximity to each other.

An initial measure of structural equivalence was computed by calculating the distance between each pair of actors across both the rows and the columns of the standardized matrix (or, equivalently, across the columns of the initial standardized matrix stacked atop its transpose). d_{ij} rather than d_{ij}^2 was used for comparability with Burt and Bittner. Diagonals were treated as in Burt and Bittner.

The resulting 44×44 symmetric matrix of distances computed across the standardized matrix and its transpose provides the initial measure of structural equivalence. As Burt has stated two actors with zero distance in this matrix are structurally equivalent under the strong definition provided by Lorrain and White (1971).

However, many researchers including Burt have recognized the necessity for relaxing this strong definition of structural equivalence (Breiger *et al.* 1975; Burt 1976; Heil and White 1976). Burt comments "When dealing with actual networks of relations, the strong definition of equivalence has little utility since there are likely to be minor differences between structurally equivalent positions due to sampling variability, errors of observation, and/or theoretically trivial differences between actors." (1976: 96) Or, "There are good reasons for expecting two occupants of a single status not to be structurally equivalent under a strong criterion. Obviously, random errors and arbitrary decisions in measuring relations could result in the ostensible nonequivalence of status occupants. More importantly, statuses should not be defined too strictly in terms of relations among individual actors. Given two actors occupying the same status, there is no reason to expect them both to perform their role relations in an identical manner." (1980: 104).

Degrees of structural equivalence were found by employing connectedness method clustering, as advocated by Burt. The resulting structure provides a partition of the actors in the group into structurally non-equivalent positions. Results of the cluster analysis are presented in Figure 1. Actors are labeled with their total frequency of observed communications.

Figure 1 reproduces, within rounding error, the figure presented in Burt and Bittner (1981: 87) of the Hams behavioral data. They interpret this figure along with the cognitive data: "...we envision the MWA as a single status system in which the one status is occupied by isolates as the bulk of the Association. ...Hams vary in their distance from this status according to the frequency of their radio contact with others." (Burt and Bittner 1981: 84).

5. Interpreting the Hams positional structure

Our interpretation of this result is quite different. We agree that the Hams who interact frequently with others (say, are on the radio more than 50 times during the observation period) are on the periphery of the structure as shown in Figure 1, while those who never or seldom communicate with others are in the center. However, we attribute this to an artifact of the procedure for identifying the status, rather than to the positional structure of the Hams group.

Multidimensional scaling provides an alternative means for representing the distances among actors which makes this more apparent (Kruskal *et al.* 1973). Figure 2 shows the results of a two-dimensional multidimensional scaling of the same data (distances among the standardized interactions) clustered in Figure 1. Marginal totals from the raw interactional data are mapped on top of the representation. It is clear that the dense core on the right of the picture is composed of people who have very low participation in the group, and that the high interactors spread in concentric circles from this core. Although in Burt's analysis a standardization was employed to remove scale (elevation) from the raw data there remains a large component attributable to differences in marginal frequencies.

If we look at the structure of the group based on the similarity in pattern as measured by the Pearson product moment correlation coeffi-

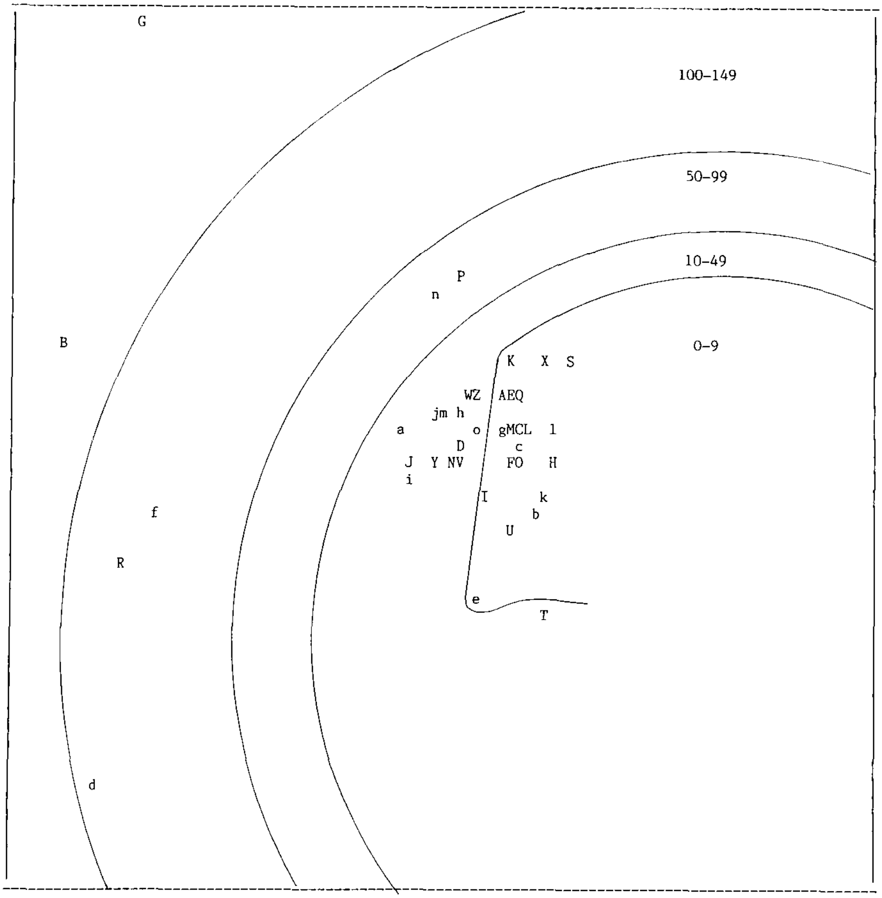

Figure 2. Multidimensional scaling of distances between Hams standardized behavior. Raw behavior marginal totals mapped on top. Stress = 0.1096.

cient, it is quite different from the structure based on the distances. Figure 3 shows the multidimensional scaling of the correlations among actors' standardized interactions (leaving out those three Hams who were never observed communicating with others). The positional assignment described by Burt and Bittner from Figure 1 is mapped on top of the multidimensional scaling result. High interactors (more than 50 observed communications) are indicated by underlines. The mapping of the structure found by Burt and Bittner in Figure 3 makes no sense.

Their analysis give a central place to those Hams who have low levels of interaction in the group, and who have patterns of communication which are dissimilar from each other and from other members of the group.

Two hints about why the Burt and Bittner analysis is inside out have been given above. First, equation 2 "partitioning" the distances shows that differences in mean and variance lead to large distances between variables even though they have similar patterns as measured by the

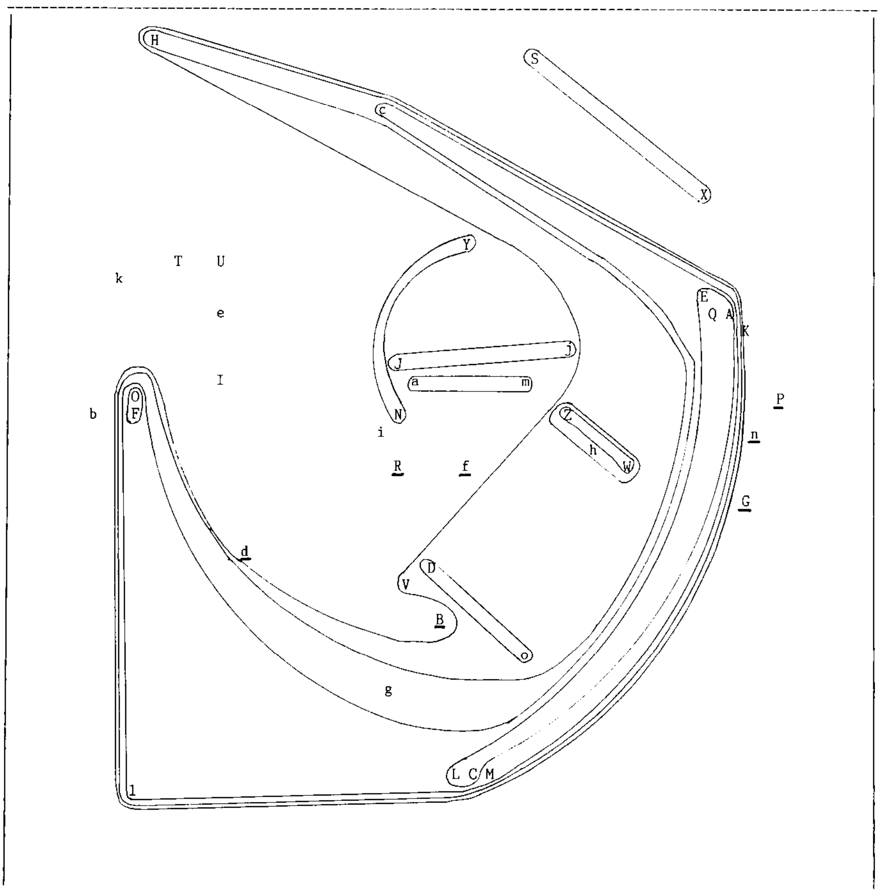

Figure 3. Multidimensional scaling of correlations between Hams standardized behavior. Positions from connectedness clustering mapped on top. Stress = 0.2134.

correlation coefficient. Second, even though a standardization procedure was employed to remove these differences, it failed to perform as advertised.

We take two approaches to demonstrating the relationship between individuals' means and variances and the magnitude of the distances. First we examine the relationship between the magnitude of the distance and the pairwise differences in mean and variance. Second, we demonstrate how transformations on the original variables to remove differences in mean and variance affect distance and correlation computed on these variables.

In the group of 41 Hams there are 820 pairwise distances. We now examine the relationship between the magnitude of these distances and the components of mean, variability and shape. If the distances are primarily composed of similarities in the shape of the variables, those pairs with low distances will be those with correspondingly high Pearson product moment correlations. Results of pairwise distances and pairwise correlations will be virtually inversely identical. If, on the other hand, the distances are composed of mean and variance effects, then pairwise distances and pairwise correlations will not be similar. Removing mean and variance effects should have a profound effect on the distances and increase their similarity to the correlations. We found a profound effect.

5.1. Distance and differences in mean and variance

In this section we explore the degree to which pairwise distances are predictable from differences in mean and variability or, alternatively, from similarity in pattern measured by the correlation coefficient. A matrix of mean differences between actors was constructed by computing for each actor the mean interactions in the standardized data, and then for each pair of actors taking the square of the difference between their means. The difference in variability was done in a similar fashion, taking the square of the difference between the standard deviations. The correlations are the Pearson product moment correlations between actors' standardized interactions. For each of these three calculations the data is the 82×41 matrix of standardized interactions (the original 41×41 standardized matrix stacked atop its transpose) excluding corresponding diagonal elements (thus for each pair, computations are across 80 observations).

Since comparisons are between matrices, the quadratic assignment program, developed by Hubert and his collegues provides the appropriate statistical test. "A permutation distribution and an associated significance test are developed for the specific hypothesis of 'no conformity' reinterpreted as a random matching of the rows and (simultaneously) the columns of one sociometric matrix to the rows and columns of a second." (Hubert and Baker, 1978: 31). The program provides an index of association, Gamma, along with the first two moments of the permutation distribution, allowing an approximate normal curve test.

Table 1 presents the results of comparison of the distances between all pairs of 41 Hams and the differences in mean (squared), the differences in standard deviations (squared) and the Pearson product moment correlations between actors. In addition to the Z scores from the quadratic assignment, Pearson product moment correlations and Goodman–Kruskal gammas between the matrices are presented. Distances among actors are strongly and significantly predicted by the

Table 1
Comparison of distances and levels of Hams structural equivalence with differences in mean and variability, and Pearson correlations of standardized behavior

	Distances		
	Comparison measure		
	QAP Z	Pearson r	gamma
Mean difference $(M_i - M_j)^2$	7.111	0.892	0.600
Variability difference $(S_i - S_j)^2$	6.999	0.828	0.605
Pearson correlation r_{ij}	−2.975	−0.239	−0.231
	Level of structural equivalence		
	Comparison measure		
	QAP Z	Pearson r	gamma
Mean difference $(M_i - M_j)^2$	6.844	0.896	0.632
Variability difference $(S_i - S_j)^2$	6.915	0.854	0.689
Pearson correlation r_{ij}	0.215	0.018	−0.007

Comparisons are for lower half of 41×41 matrices.
$N = 820$.

differences in the mean and variability of communications. However, similarity measured using distance and similarity measured using correlation are correlated −0.24. Clearly one of the measures is misleading.

As a way of relaxing the strong definition of structural equivalence as zero distance, Burt advocates using connectedness method clustering to define structurally non-equivalent positions in the group, and to assign people to these positions. Figure 1 depicts this positional structure. As with the distances we again examine the degree to which assignment to the position is a function of differences in mean and variability of actors' communications and the Pearson product moment

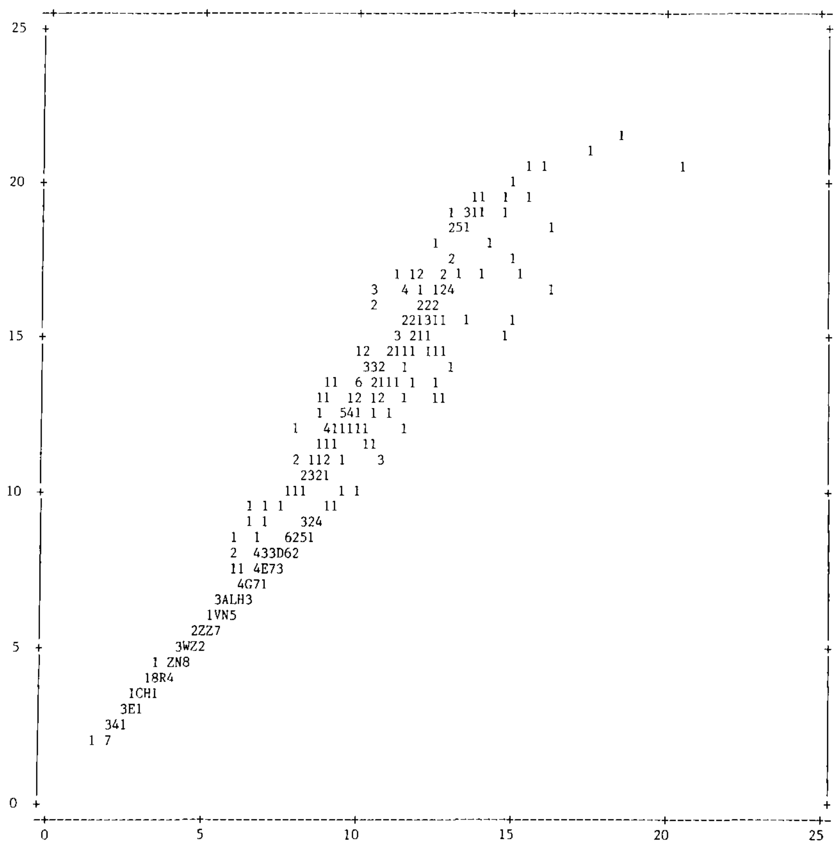

Figure 4a. Distances on Hams standardized behavior *vs*. Distances on Hams standardized behavior minus mean. $r = 0.957$.

correlation between them. Table 1 presents comparisons with the levels from the connectedness method clustering. The level is the value of the lowest point at which actors *i* and *j* are joined into a single status. As with the distances, the levels in the clustering, the degree to which two actors are jointly members of the status, is strongly predicted by the differences in mean and variability. However, the similarity in pattern of interactions among actors as measured by the correlations has no relation to how equivalent they are.

These results indicate that an approach to structural equivalence based on distance, as in STRUCTURE, and one based on correlation,

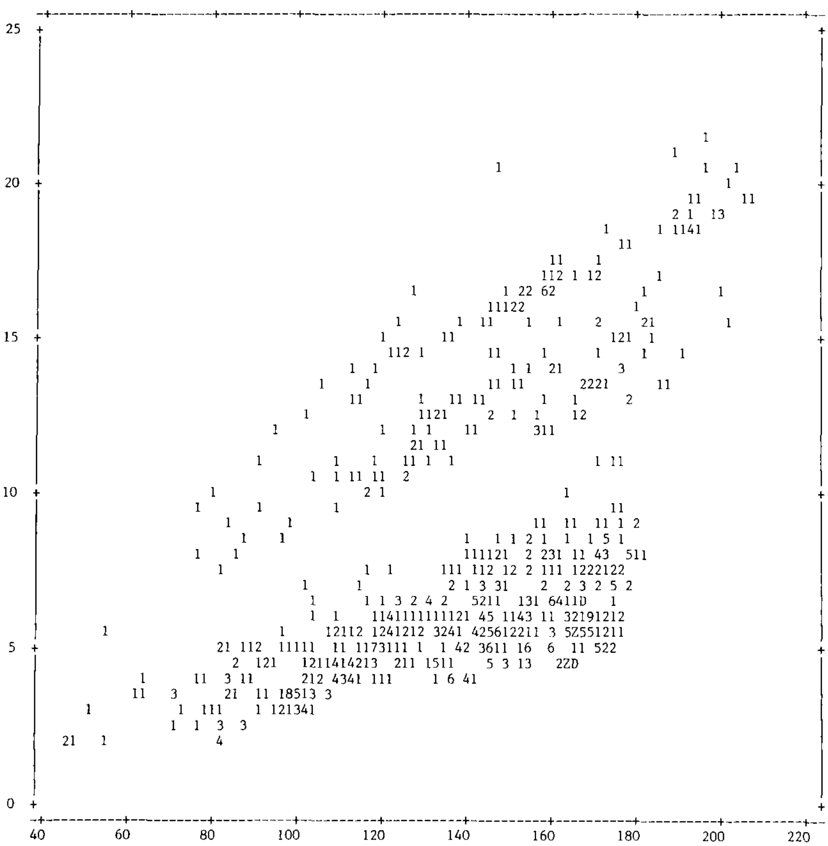

Figure 4b. Distances on Hams standardized behavior *vs*. Distances on Hams standardized behavior divided by standard deviation. $r = 0.423$

as in CONCOR, lead to quite different results for these data. In the following section we examine when distance and correlation converge on the same interpretation.

5.2. Removing mean and variance effects from the distance

Our second strategy for demonstrating the confounding of differences in mean and variance with the distances among actors in the Hams data follows the formal procedure outlined by Cronbach and Gleser for partitioning the distance into what they refer to as elevation (mean),

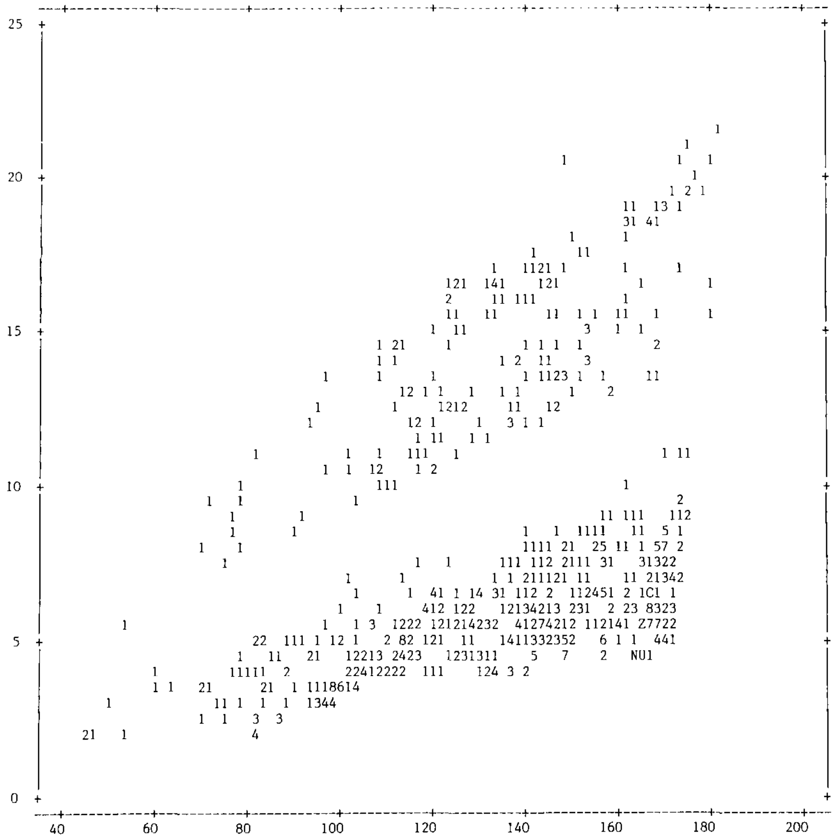

Figure 4c. Distances on Hams standardized behavior *vs*. Distances on Hams standardized behavior minus mean and divided by standard deviation. $r = 0.240$.

scatter (variability) and shape (Cronbach and Gleser 1953). Distance calculated on raw data includes effects of all three of these factors, as indicated in Equation 2. Mean and variance differences can be successively removed, leading to a distance measure which is purely due to shape. If two variables are transformed to have means of zero (by subtracting the mean from each value) then the distance computed between these variables is composed of the two effects: variance and shape. This is mathematically equivalent to subtracting $N(M_i - M_j)^2$ from the distance d_{ij}^2 (Cronbach and Gleser 1953: 460). If variables are further transformed by dividing each value by the standard deviation

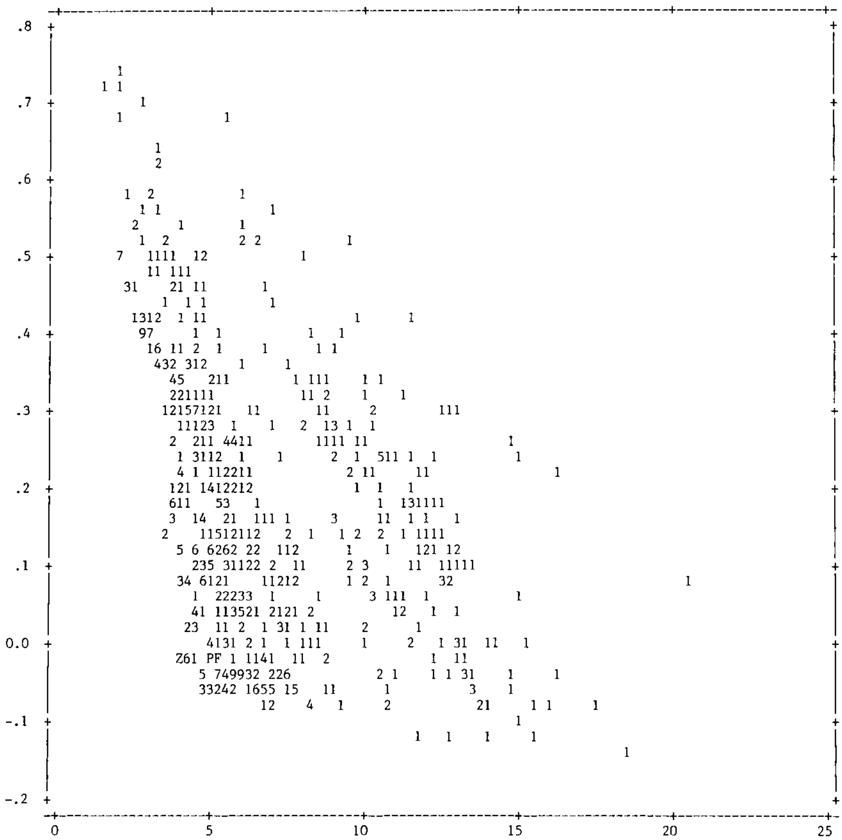

Figure 4d. Correlations on Hams standardized behavior *vs.* Distances on Hams standardized behavior minus mean. $r = -0.303$.

(i.e. converted to Z scores) then the distance is simply a function of the Pearson product moment correlation and the number of cases, as in Equation 3. These transformations have no effect on the correlations.

Figures 4a–f present scatterplots of (1) the relationship between the distances among the original data and the distances among the successively transformed data (Figures 4a–c), and (2) the relationship between the Pearson product moment correlations among the original data and the distances among the successively transformed data (Figures 4d–f). At the extreme we note a mathematical identity: distances among variables which have mean and variance effects removed (mean

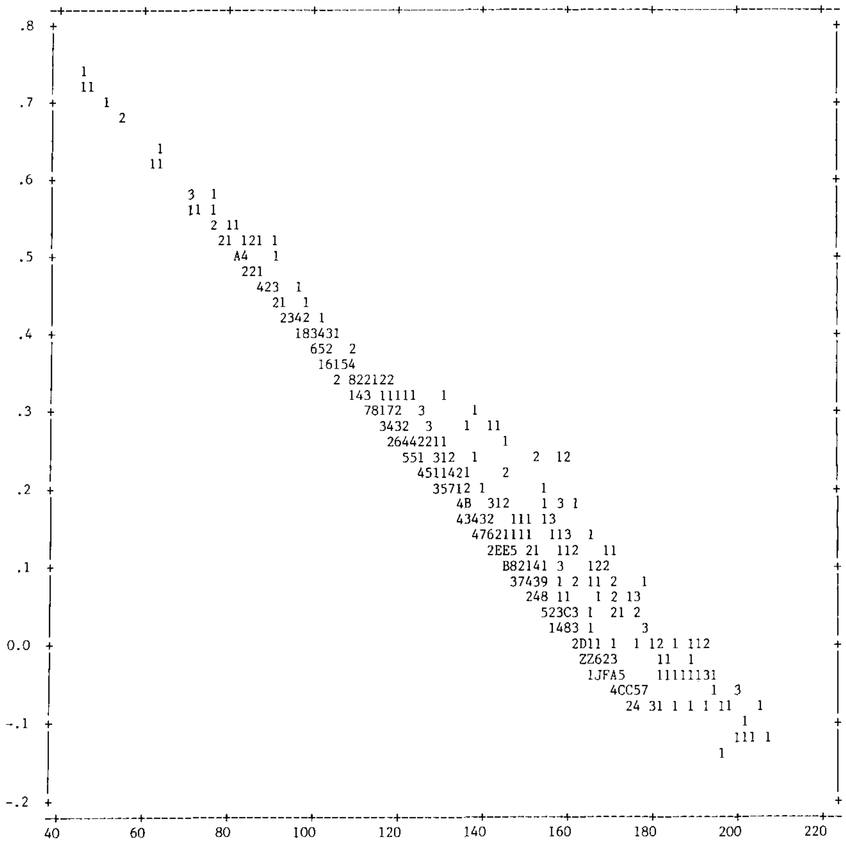

Figure 4e. Correlations on Hams standardized behavior *vs*. Distances on Hams standardized behavior divided by standard deviation. $r = -0.971$.

= 0 and variance = 1) are equivalent to correlations on the untransformed data (Figure 4f). Any departure from a straight line in Figure 4f is due to rounding error. Removal of mean effects from the distances show little departure from the original distances (Figure 4a) and only slightly closer correspondence to the correlations (Figure 4d) than the distances on the untransformed data. Removal of variance effects by dividing the values of each variable by the variable's standard deviation, has a profound effect on the distances computed among these variables as compared to distances on the untransformed variables. After removing variance effects distances look quite similar to the correlations (Figure 4e) and quite different from distances on the

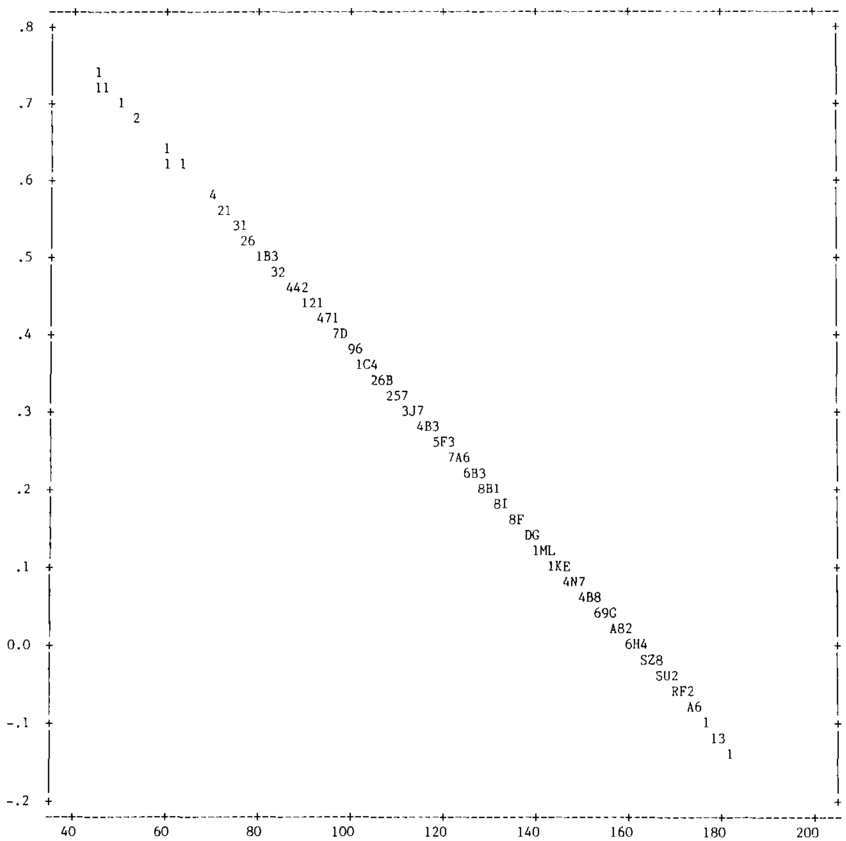

Figure 4f. Correlations on Hams standardized behavior *vs.* Distances on Hams standardized behavior minus and divided by standard deviation. $r = -1.000$.

original variables (Figure 4b). Distances computed on variables from which both mean and standard deviation effects have been removed are linearly identical to, though inverse of, correlations on the original variables (Figure 4f). Figure 4c is identical to the scatterplot of the distances on the original variables *versus* the correlations on the original variables.

The series of plots and measures of association in Figure 4 show that as differences in mean and variability are removed from the variables, distances calculated among them approach the correlation coefficient, and, with both mean and variability differences removed, distance is a linear function of the correlation.

6. Discussion

Over the past several years distance has been used as a measure of structural equivalence in a number of empirical studies (Burt 1976, 1982; Burt and Bittner 1981). In this article we have discussed the anomalous results encountered in Burt and Bittner's analysis of BKS's Hams data using STRUCTURE. However, this is not the only instance of contrary results arising from the use of STRUCTURE. In a re-analysis of Laumann and Pappi's data on German elites, Burt discusses the difference between groupings of individual distance (proximity or frequency of interaction) and social distance (the pairwise distance between individuals' relations with others). He notes that in a smallest space analysis of social distances "isolates in the economic exchange network are occupying a joint position in the network instead of being scattered around the periphery of the network as they would be in an analysis of individual distance." (1976: 119). And equally as puzzling, he notes that analysis of social distances in the information-seeking network shows " the six most influential actors in the community jointly occupy a position in the information-seeking network... because they all have the characteristic of receiving many citations from other actors in the network which they do not reciprocate. Their position in the network is placed on the periphery of the social space as represented in the smallest space analysis since they occupy a position which is very different from the positions of most actors in the network. In an analysis of individual distances these actors would be near the center of the space since so many actors claim to interact with them" (1976: 119–120). In the figure showing the results of the smallest space analysis the bulk of the network (the non-influentials) forms a dense cluster on one side of the picture while the six influentials are scattered across the other side of the picture.

The grouping of actors who seldom interact with others or who receive few citations from others in the network into a coherent position, while active or extensively cited actors are relegated to peripheral positions is the same "positional structure" resulting from a STRUCTURE analysis of the Hams data. As we have argued above, this is an artifact of confounding differences in means and variances with similarity in the patterns among individuals. This error arises when an index such as distance is computed without proper attention to standardization of the variables.

Comments on some further issues are relevant. The first is the general observation that in all branches of knowledge there is an expectation that independent methods with similar aims should give similar answers. That is, the results given by various methods should converge. Burt and Bittner found that CONCOR gave what they found to be inconclusive results so that they introduced STRUCTURE to give improved results. The striking fact that the two methods gave opposed answers did not seem to be peculiar for Burt and Bittner. We find it very satisfying that the two methods converge when one correctly removes the effect of differences in means and variances from the data before applying STRUCTURE.

The fact that CONCOR and STRUCTURE give convergent answers on the Hams observed behavior data does not necessarily mean that either method is adequate nor that there are very striking regularities in the data to be discovered. We are completely persuaded by the derivations and arguments of Schwartz (1977) that principal components analysis *always* gives more information than does CONCOR. STRUCTURE, even with proper adjustments for mean and variance effects, also seems to us to be less than ideal. For example, we might mention two points, first, stacking a matrix with its transpose, and second, using the connectedness method for clustering. Both seem ill-advised. Stacking a matrix with its transpose confounds the patterns between the "from-ness" and the "to-ness" inherent in asymmetrical matrices. The two patterns should be analyzed separately. In symmetric matrices it is simply redundant. Second, researchers such as D'Andrade (1978) and Hubert (1974) have shown that ALPAIR or the diameter method generally give more robust and interpretable answers than connectedness method. Our choice would be to map ALPAIR on a MDS representation subsequent to a principal components analysis to ensure that "significant" structure is present.

Our impression, based on a very great deal of analysis, is that the bulk of the information in the Hams data resides in the marginals, i.e. differences in frequency of communication. The regularities remaining in interactions among Hams that might be interpreted as something like preferences are very small. These interactions appear so much like randomly distributed residuals that it is probably not profitable to seek a fancy model of explaination. The Hams seem to have logged on at more or less random times and interacted with others who also logged on at random.

Finally, we question the value of any measure which confounds phenomena which are mathematically and theoretically distinct. Distance as a measure of similarity applied to nonstandardized variables confounds information about the similarity in the patterns of values with information about the differences in the mean and variance of each variable. Of special importance to social network research is the additional problem of confounding information about the proximity of pairs of individuals with a measure of the similarity of their relations with others. Such confounding arises when diagonal elements (self–self interactions) are included in the calculation of similarity across actors. In order to construct testable models of positional, relational and individual phenomena, it is critical to employ measures which do not confound these at the outset.

References

Arabie, Phipps, Scott A. Boorman and Paul R. Levitt
1978 "Constructing blockmodels: How and why." *Journal of Mathematical Psychology 17*: 21–63.

Bernard, H. Russell, Peter D. Killworth and Lee Sailer
1980 "Informant accuracy in social network data IV: A comparison of clique-level structure in behavioral and cognitive network data." *Social Networks 2*: 191–218.

Boyce, A.J.
1969 "Mapping diveristy: A comparative study of some numerical methods." In A.J. Cole (ed), *Numerical Taxonomy*. New York: Academic Press.

Boyd, John P.
1983 "Structural similarity, semi-groups and idempotents." *Social Networks 5*: 157–172.

Breiger, Ronald L., Scott A. Boorman and Phipps Arabie
1975 "An algorithm for clustering relational data with applications to social network analysis and comparison with multidimensional scaling." *Journal of Mathematical Psychology 12*: 328–383.

Burt, Ronald S.

1976 "Positions in networks." *Social Forces 55*: 93–122.

1978 "Cohesion versus structural equivalence as a basis for network subgroups." *Sociological Methods and Research 7*: 189–212.

1980 "Models of network structure." *Annual Review of Sociology 6*: 79–141.

1983 *Toward a Structural Theory of Action: Network Models of Social Structure, Perception, and Action*. New York: Academic Press.

n.d. "STRUCTURE: a computer program providing basic data for the analysis of empirical positions in a system of actors." Survey Research Center, University of California, Berkeley.

Burt, Ronald S. and William M. Bittner

1981 "A note on inferences regarding network subgroups." *Social Networks 3*: 71–88.

Burt, Ronald S. and Michael Minor

1983 *Applied Network Analysis: A Methodological Introduction*. Beverly Hills: Sage.

Coxon, A.P.M.

1982 *The User's Guide to Multidimensional Scaling*. Exeter, NH: Heinemann.

Cronbach, Lee J. and Goldine C. Gleser

1953 "Assessing similarity between profiles." *The Psychological Bulletin 50*: 456–473.

D'Andrade, Roy G.

1978 "U-statistic hierarchical clustering." *Psychometrika 43*: 59–67.

Fox, John

1982 "Selective aspects of measuring resemblance for taxonomy." In Herschel C. Hudson (ed.), *Classifying Social Data*. San Francisco: Jossey-Bass.

Friedkin, Noah

1984 "Structural cohesion and equivalence explanations of social homogeneity." *Sociological Methods and Research 12*: 235–261.

Gordon, A.D.

1981 *Classification*. London: Chapman and Hall.

Guttman, Louis

1977 "A definition of dimensionality and distance for graphs." In J.C. Lingoes (ed.) *Geometric Representations of Relational Data*. Ann Arbor, MI: Mathesis.

Heil, Gregory H. and Harrison C. White

1976 "An algorithm for finding simultaneous homomorphic correspondences between graphs and their image graphs." *Behavioral Science 21*: 26–35.

Hubert, Lawrence

1974 "Approximate evaluation techniques for the single-link and complete-link hierarchical clustering procedures." *Journal of the Americal Statistical Association 69*: 698–704.

Hubert, Lawrence J. and Frank B. Baker

1978 "Evaluating the conformity of sociometric measurements." *Psychometrika 43*: 31–41.

Johnson, S.C.

1967 "Hierarchical clustering schemes." *Psychometrika 32*: 241–254.

Knoke, David and James H. Kuklinski

1982 *Network Analysis*. Beverly Hills: Sage.

Kruskal, Joseph B., Forrest W. Young and Judith B. Seery

1973 "How to use KYST, a very flexible program to do multidimensional scaling and unfolding." Bell Laboratories (unpublished).

Lorrain, Francois and Harrison C. White

1971 "Structural equivalence of individuals in social networks." *Journal of Mathematical Sociology 1*: 49–80.

Mandel, Michael and Christopher Winship

1979 "Roles, positions, and networks." Paper presented at the American Sociological Association Meetings, Boston, Massachusetts.

Penrose, L.S.
1952 "Distance, size and shape." *Annals of Eugenics 18*: 337–343.

Rohlf, F. James and Robert R. Sokal
1965 "Coefficients of correlation and distance in numerical taxonomy." *The University of Kansas Science Bulletin 45*: 3–27.

Sailer, Lee D.
1978 "Structural equivalence: Meaning and definition, computation and application." *Social Networks 1*: 73–90.

Schwartz, Joseph E.
1977 "An examination of CONCOR and related methods for blocking sociometric data." In David R. Heise (ed) *Sociological Methodology 1977*. San Francisco: Jossey-Bass.

Sneath, Peter H.A. and Robert R. Sokal
1973 *Numerical Taxonomy: The Principles and Practice of Numerical Classification*. San Francisco: Freeman.

Sokal, Robert R. and Peter H.A. Sneath
1963 *Principles of Numerical Taxonomy*. San Francisco: Freeman.

White, Douglas R. and Karl P. Reitz
1983 "Graph and semigroup homomorphisms on networks of relations." *Social Networks 5*: 193–234.