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VERY LOCAL STRUCTURE IN SOCIAL NETWORKS

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Triadic configurations are fundamental to many social structural processes and provide the basis for a variety of social network theories and methodologies. This paper addresses the question of how much of the patterning of triads is accounted for by lower-order properties pertaining to nodes and dyads. The empirical base is a collection of 82 social networks representing a number of different species (humans, baboons, macaques, bison, cattle, goats, sparrows, caribou, and more) and an assortment of social relations (friendship, negative sentiments, choice of work partners, advice seeking, reported social interactions, victories in agonistic encounters, dominance, and co-observation). Methodology uses low dimensional representations of triad censuses for these social networks, as compared to censuses expected given four lower-order social network properties. Results show that triadic structure is largely accounted for by properties more local than triads: network density, nodal indegree and outdegree distributions, and the dyad census. These findings reinforce the observation that structural configurations that can be realized in empirical social networks are severely constrained by very local network properties, making some configurations extremely improbable.

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1. BACKGROUND

Network approaches to social structure view human and nonhuman animal populations as systems of relations among interacting units—with examples including communications, exchange of material resources, expressions of affection, informal social interactions, kinship relations, victories in agonistic encounters, exercise of authority, and provision of social support. Relations among triads—triples of units—are fundamental to structural patterns in many of these relations and considerable theoretical, empirical, and methodological work in the social sciences concerns triads.

Simmel (1950) is often credited with the early insight that social processes are fundamentally different when three rather than two people are involved, thus focusing attention on the triad. The third person in an interaction affords possibility for mediation between the other two, the formation of coalitions, and advantages accruing through tertius strategies such as brokerage. Appreciation of the importance of triadic processes has been carried forward in a number of ways, and several influential social theories rest, at least in part, on triadic patterns. Theories on cognitive balance (Heider 1946) and its generalization to structural balance (Cartwright and Harary 1956) focus on the strain involved in triads when sentiments are not consonant—for example when two close friends are in strong disagreement about their evaluation of another person or object. Granovetter's (1973) classic strength of weak ties argument proceeds from the strong (admittedly overstated) assumption that when an individual, A, has strong ties with two others, B and C, then the tie between B and C should not be absent. A "forbidden triad" is one that violates this assumption. Granovetter then follows the social structural implications of the absence of forbidden triads for the occurrence of bridging weak ties, social integration, and information diffusion in social networks. Related ideas are found in Burt's (1992) structural holes argument and its extensions. The argument is that efficiency and effectiveness result from nonredundant ties. A person who has ties to others who are not themselves tied bridges a structural hole. This pattern, which often forms an open triangle, is an important locus for strategic action. Triadic processes are also implicated in third-party effects on trust and reputation (Burt and Knez 1995). Third parties can affect dyadic transactions, in which they are not directly involved, by conveying (or being in a position to convey) information about the actions of others, thus influencing their reputations. Third-party effects are also important in interactions among nonhuman animals, and they have been most extensively investigated in dominance encounters (Chase 1982; Chase, Tovey, and Murch 2003). Empirical evidence demonstrates that the presence of third parties affects the instability of dominance orderings and the persistence of disadvantage from prior losses. Moreover, a triadic "bystander" effect might be necessary for development of dominance hierarchies (Skvoretz, Faust, and Fararo 1996). Though in very different contexts from Simmel's (1950) original theoretical reasoning, these empirical results similarly challenge an assumption that valid conclusions about dyadic interactions can be reached by studying them in isolation.

These lines of research point to triadic configurations and interactions among triples as important for larger social processes. Since triads concern relations among social actors, most often they are studied using a social network formalization. Indeed, triads have been at the heart of theoretical and methodological advances in social network research for nearly half a century. Triads are implicated in many social network theories, as noted above. Triads and properties of triples also provide the basis for important social network methods—for example, the triad census (Holland and Leinhardt 1970), random and biased nets (Skvoretz, Fararo, and Agneessens 2004), the formal linkage between local network processes and global social structures (Johnsen 1998), transitivity indices (Frank and Harary 1982), triad based role equivalence (Burt 1990), and structural effects in many exponential random graph models (Kalish and Robins 2006; Robins and Pattison 2005; Robins et al. forthcoming; Snijders et al. 2006).

The importance of triads in social structural investigation can hardly be overstated, as Holland and Leinhardt argue,

> The essential issue of any notion of structure is how the components are combined, not the components themselves. . .this issue amounts to the proposition that the lowest interesting level of structure. . .is the level of triples of nodes—the triadic level (1979:66).

Holland and Leinhardt then recognize that nodal and dyadic properties constrain possible triadic structures that might be realized empirically, and they ask whether social network data contain any information beyond that expected from lower-level properties: constraints imposed by data collection methods (e.g., question format), differences in popularity among actors, and a tendency for sociometric choices to be reciprocated (Holland and Leinhardt 1979). The extended investigations of triads by Holland and Leinhardt as well as others following their lead tackled these related questions, but at no point have they answered the question: *How much* of the triadic structure is accounted for by nodal and dyadic properties? This paper provides an answer to that question and shows that more than 90% of the triadic structure in a collection of diverse social networks is accounted for by lower-order properties. The implications of this result are far-reaching. If a substantial portion of triadic structure is explained by nodal and dyadic features, then the theoretical and methodological importance of *triadic* properties and processes are brought into question.

The analyses that follow extend the work of Skvoretz and Faust on social network comparisons (Faust and Skvoretz 2002; Skvoretz and Faust 2002) and especially the results of Faust (2006), demonstrating that triad censuses for a collection of 51 social networks are largely explained by linear and quadratic functions of network density and dyad distributions. This paper uses a different sample of 82 social networks and a different methodological approach, comparing triad censuses for these observed networks with censuses expected given four different lower-order network properties: (1) network density, (2) the outdegree distribution, (3) the indegree distribution, and (4) the dyad census. The analysis employs low-dimensional representations of observed and expected triad censuses followed by canonical redundancy analysis to quantify the exact percentage of variance in the observed triad censuses that is explained by triad censuses expected given the lower-order network properties.

2. LOCAL STRUCTURE IN SOCIAL NETWORKS

A social network, represented as a graph or directed graph, consists, minimally, of a set of nodes (also referred to as points or vertices) representing social actors and a set of arcs (edges or ties) between pairs of nodes, representing social relations between actors. A graph with node set V and arc set E is denoted $G(V, E)$. A social network with g actors can be displayed in a $g \times g$ sociomatrix, X, in which rows and columns index actors, in identical order, and entries, x_{ij}, code the state of the arc from actor i to actor j. This minimal social network may be elaborated by allowing valued arcs, multiple relations, arcs that are nondyadic (i.e., that include more than two actors), have more than two sets of actors, or have attributes for actors or arcs. In the following analyses, all networks are directional and dichotomous ($x_{ij} = 0$ or 1), and self ties (reflexive arcs, x_{ii}) are undefined.

Properties of social networks can be defined at different levels of aggregation, from local measures for individual nodes or small subsets to global measures requiring simultaneous information about the entire graph. Local structural properties refer to network measures defined for nodes, pairs of nodes, or triples of nodes. Nodal properties include, for example, nodal outdegree—the number of nodes adjacent from the node $x_{i+} = \sum_{j=1}^{g} x_{ij}$—and nodal indegree—the number of nodes adjacent to the node $x_{+j} = \sum_{i=1}^{g} x_{ij}$. Network density, Δ, the proportion of possible ties that are present in a network, can be expressed as a network-level summary of nodal degrees:

$$\Delta = \frac{\sum_{i=1}^{g} x_{i+}}{g(g-1)} = \frac{\sum_{j=1}^{g} x_{+j}}{g(g-1)} = \frac{\sum_{i=1}^{g} \sum_{\substack{j=1 \\ j \neq i}}^{g} x_{ij}}{g(g-1)}. \tag{1}$$

2.1. *Subgraphs, Dyads, and Triads*

Dyadic and triadic social network properties are defined on subgraphs of two or three nodes, respectively. A subgraph, $G_S(V_S, E_S)$, of a graph, $G(V, E)$, consists of a subset of nodes from graph $V_S \subset V$ along with the arcs involving nodes within subset $E_S \subset E$. A dyad is a subgraph of two nodes and the states of the arc(s) between them. For a directed graph with g nodes, there are $\binom{g}{2} = \frac{g(g-1)}{2}$ dyads. A triad is a subgraph of three nodes and the states of the arc(s) between them. There are $\binom{g}{3} = \frac{g(g-1)(g-2)}{6}$ triads in a directed graph.

2.2. *Isomorphism Classes and Subgraph Censuses*

Summary measures of local structural properties are often based on a census of subgraphs of a given size from a graph and rely on isomorphism classes of these subgraphs. Two graphs (or subgraphs) are isomorphic if there is a one-to-one mapping between the nodes in the two graphs that preserves adjacency. An isomorphism class is a set of isomorphic graphs or subgraphs. Dyads in a directed graph must be in one of three isomorphism classes: (1) mutual (M), (2) asymmetric (A) ignoring the direction of the arc, or (3) null (N). The dyad census of a network is a count of the number of dyads in each of the three isomorphism classes, and it is often labeled MAN. Triads in a directed graph must be in one of 16 isomorphism classes, as presented in Figure 1. This figure uses the standard labeling indicating the number of mutual, asymmetric, and null dyads in the triad, along with an additional letter for direction (U, D, C, or T) when necessary (Holland and Leinhardt 1970). The triad census for a network is summarized in a 16 element vector, t:

$$t = (c_{003}, c_{012}, c_{102}, c_{021D}, c_{021U}, c_{021C}, c_{111D}, c_{111U}, c_{030T}, c_{030C}, c_{201}, c_{120D}, c_{120U}, c_{120C}, c_{210}, c_{300})$$

where $c_\bullet$ is the count of the number of triads of type $\bullet$ in the network. For small subgraphs (dyads or triads), a subgraph census provides substantial simplification of a network since there are relatively few isomorphism classes (Wasserman and Faust 1994). Moreover, the dyad and triad censuses retain important information about local structural properties, including graph density and the prevalence of mutuality in the dyad census, and, additionally, about transitivity, intransitivity, and three-cycles in the triad census.

Several important local social network properties are triadic, including transitivity, intransitivity, and three-cycles. A triple of nodes i, j, k is transitive if $i \rightarrow j$ and $j \rightarrow k$ implies $i \rightarrow k$. A triple of nodes i, j, k is intransitive if $i \rightarrow j$ and $j \rightarrow k$ but $i \nrightarrow k$ (where $\nrightarrow$ indicates the absence of a tie from i to k). A triple is a cycle if $i \rightarrow j, j \rightarrow k$, and $k \rightarrow i$. A descriptive measure of the tendency toward transitivity is the number of transitive triples in a graph divided by the number of potentially transitive triples—that is, the number of triples of nodes i, j, k, where the condition $i \rightarrow j$ and $j \rightarrow k$ holds:

$$\frac{\sum_{k=1}^{g}\sum_{j=1}^{g}\sum_{i=1}^{g} x_{ij}x_{jk}x_{ik}}{\sum_{k=1}^{g}\sum_{j=1}^{g}\sum_{i=1}^{g} x_{ij}x_{jk}}. \tag{2}$$

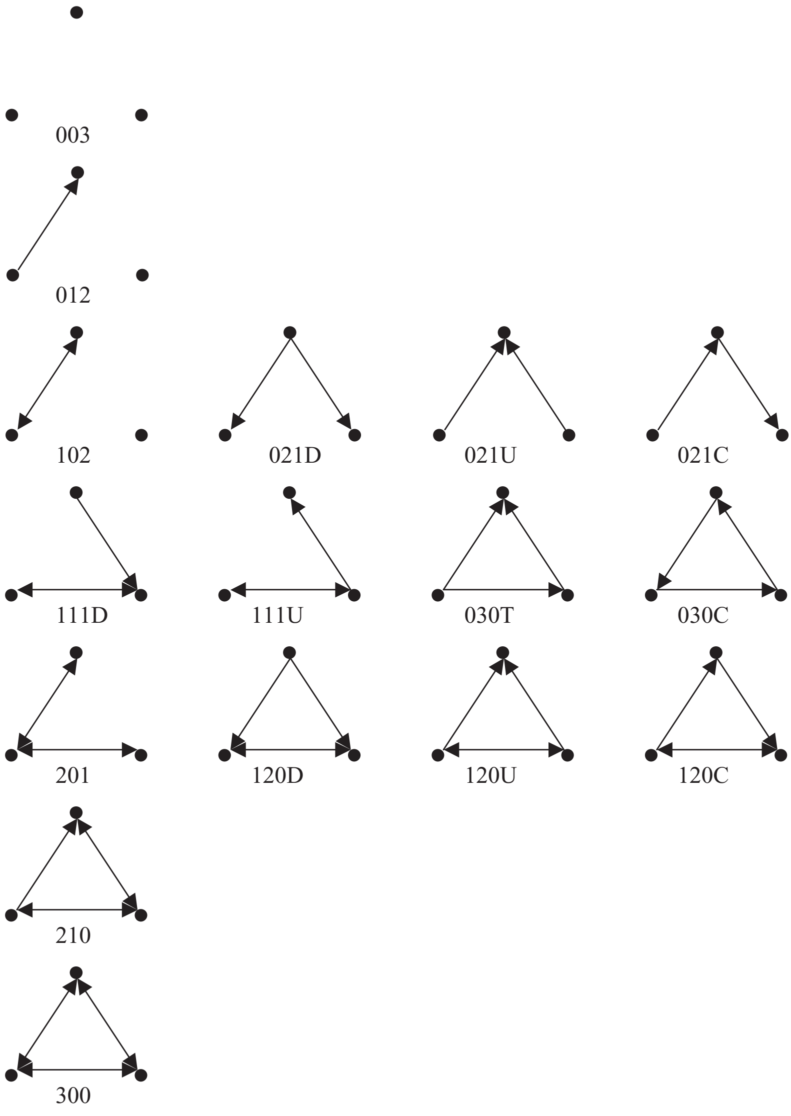

FIGURE 1. Triad isomorphism classes with *MAN* labeling.

3. TRIAD CENSUS, THEORY, AND METHODOLOGY

The triad census has been the workhorse of fruitful investigations of local structure in social networks for many decades (Davis 1970, 1977, 1979; Davis and Leinhardt 1972; Faust 2006; Frank 1988; Friedkin 1998; Hallinan 1974a, 1974b; Holland and Leinhardt 1970, 1971, 1972, 1973, 1976, 1979; Johnsen 1985, 1986, 1989a, 1989b, 1998; Skvoretz et al. 2004; Wasserman 1977). Early work employing the triad census investigated the presence of theoretically important triadic properties: structural balance, clusterability, ranked clusters, and transitivity. The triad census can also be used to investigate hierarchy or linear orders, as described below. The usefulness of the triad census for these investigations arises from the formal linkage between posited global structures and permitted or forbidden local triadic processes and patterns, reflected in the triad census. As introduced in papers by Davis (1970, 1977, 1979) and Holland and Leinhardt (1970, 1971, 1972, 1973, 1976, 1979), and elaborated in the work of Johnsen (1985, 1986, 1989a, 1989b, 1998), specific global structural patterns imply the presence and absence of specific triadic configurations, just as the occurrence and nonoccurrence of specific triadic patterns imply specific global structural configurations. Thus, the triad census is useful for evaluating theories about linkage between local processes and global structures because some theoretical global structures are contradicted by specific configurations of triads. Such a global theory is evaluated by examining empirical networks for substantial occurrence of triads inconsistent with the theory.

The earliest work in this vein built on social psychological theories of cognitive balance (Heider 1946), interpersonal similarity and friendship formation (Homans 1950), and their implications for patterns of positive sentiments among individuals. Structural balance (Cartwright and Harary 1956) generalizes Heider's cognitive balance to a social structural property of a network, using a signed graph in which edges have a positive or negative valence. As a global structure, a balanced signed graph is one in which nodes can be partitioned into two subgraphs, where all ties within each subgraph have positive signs and all ties between the two subgraphs have negative signs. In a directed graph (Johnsen 1985, 1986, 1998), mutual ties are treated as positive and null ties as negative. A balanced directed graph has only mutual ties within subgraphs and only null ties between subgraphs. A balanced directed graph permits only two kinds of triads: {300 and 102}. Other triads contradict the theory.

Davis (1967) extended structural balance to the more sociologically reasonable notion of clusterability, allowing more than two subgroups. In a clusterable signed graph, nodes can be partitioned into more than two subgraphs, with positive ties only within subgraphs and negative ties only between subgraphs. In a clusterable directed graph, with mutual ties treated as positive and null ties as negative, three triads are permitted: {300, 102, 003}. All other triads violate the theory. A clusterable directed graph for a positive sentiment relation could represent patterns of friendship in a population with multiple friendship cliques, where no friendships occur between members of different cliques.

If there is hierarchical ranking between subgroups, then the ranked clusters model holds (Davis 1970; Davis and Leinhardt 1972). The ranked clusters model extends clustering to allow directed (asymmetric) ties between subgraphs, with orientation of the directed ties consistent with hierarchical ordering of subgraphs in which ties are directed from "lower" to "higher" subgraphs. The permitted triads for this model are {300, 102, 003, 120D, 120U, 030T, 021D, 021U}. The ranked clusters global model could represent a population with multiple friendship cliques ranked in prestige or popularity, where friendships between cliques are directed from lower to higher status clique members.

Transitivity is the most general global model and subsumes the other models as special cases (Holland and Leinhardt 1971). If this model holds, then, for all triples of nodes i, j, k, whenever the $i \to j$ and $j \to k$ ties are present, the $i \to k$ tie must also be present. All triads that contain triples for which transitivity is violated are forbidden by this model. For example, the 021C triad ($i \to j$, $j \to k$, and $k \to i$) lacks the required $i \to k$ tie and so violates the transitivity model, as do other triads that contain such intransitive triples. The transitivity model permits the following triads: {300, 102, 003, 120D, 120U, 030T, 021D, 021U, 012}. The global structure for transitivity allows multiple, disconnected systems of ranked clusters in a population. It could characterize friendships in a population where there is gender or ethnic segregation, so different sets of people have distinct systems of ranked friendship clusters.

Global structures associated with local processes in typically asymmetric relations, such as dominance or victories in agonistic encounters, can also be expressed as permitted and forbidden triads. A complete tournament recording wins and losses, with no ties, has only asymmetric dyads (either $i \to j$ or $j \to i$ for all i and j) and so only permits two triads: {030C, 030T}. If the tournament is incomplete (that is, some dyads may be null) and also forms a linear order, then the permitted triads are {003, 012, 021C}.

Recent work on triads has extended the census to graphs with more than one set of actors (Snijders and Stokman 1987), used local triad censuses to measure role equivalence (Burt 1990), demonstrated use of triads for studying biased networks (Skvoretz et al. 2004), developed efficient estimation procedures for large networks (Batagelj and Mrvar 2001; Karlberg 1998; Moody 1998), and used triads to investigate influence structures in scientific networks (Friedkin 1998).

Given the different local structural processes associated with different triadic global structures, it is reasonable to expect detectable triadic differences among social networks, especially when networks are formed from substantively different social relations. Thus, a heterogeneous collection of social networks of different kinds of relations is suitable for investigating the question posed at the beginning of this paper concerning how much of the triadic structure in social networks is accounted for by nodal and dyadic properties.

4. ANALYSIS OVERVIEW

An important consideration in investigation of social network structure is whether patterns observed at a given level of aggregation can be accounted for by lower-order structural features. In the current case, the question is whether the distribution of triads for a collection of social networks can be accounted for by features of nodes and dyads. If so, then a follow-up question concerns how much of the variability in the triad censuses can be explained by these lower-order properties. These questions are addressed in a multistage analysis.

First, triad censuses are found for a heterogeneous collection of social networks. The unit of analysis is a social network, and the collection of triad censuses for the networks is the object of further study. Second, triad censuses that would be expected given lower-order properties of the networks are calculated, using four lower-order properties: (1) network density, (2) the outdegree distribution, (3) the indegree distribution, and the (4) dyad census (*MAN*). Evaluating the percentage of variance in the observed triad censuses that is explained by the four lower-order structural features proceeds by comparing the observed censuses with the expected censuses. Comparison is based on low-dimensional projections from singular value decomposition (SVD), and it uses canonical redundancy analysis (described below) to assess the percentage of the SVD of observed triad censuses explained by the SVD of each of the expected censuses. Parallel analyses proceed for the spaces of networks and triads. Third, adjustment is made for the fact that low-dimensional SVD accounts for less than 100% of the observed triad censuses, to arrive at a summary of the percentage of the observed triad censuses that can be accounted for by each of four expected censuses.

5. DATA

The empirical base for the analysis that follows is a collection of 82 social networks representing a number of different species (humans, baboons, macaques, bison, cattle, goats, sparrows, caribou, and others) and a variety of social relations (friendship, negative sentiments, choice of work partners, advice seeking, reported social interactions, victories in agonistic encounters, dominance displays, and co-observation). These social networks were compiled from published sources in the social sciences and in animal behavior, or as accompaniments to standard social network software such as UCINET (Borgatti, Everett, and Freeman 2002). Descriptions of the networks are presented in Appendix A and references to their sources are in Appendix B. All networks are between individuals (rather than collectivities or aggregate units) and are treated as directed, dichotomous relations. As can be seen from the descriptive statistics in Table 1, the networks vary in size from 7 to 97 individuals and differ in their density, proportions of mutual, asymmetric, and null dyads, and tendency toward transitivity.

The triad census for each network was found (using a version of the SAS program described in Moody 1998) and arrayed in a matrix, denoted $\mathbf{T}_0$. This matrix has 82 rows indexing networks and 16 columns indexing the triad isomorphism classes. Table 2 presents this matrix, expressed in row percentages. These triad censuses show considerable variability among the networks. Notably, in 13% of the networks there are no 003 (all null) triads; however in networks #1 (rejection nominations by school children) and #28 (work choice partners) over 90% of the triads are 003. In 34% of the networks there are no 030T (transitive) triads, in contrast to networks #45 (dominance between finches) and #49 (dominance between hens) in which over 90% of triads are 030T. In 71% of the networks the 300 (all mutual) triad is absent, though in network #51 (co-observation of kangaroos) it exceeds 40%.

TABLE 1
Descriptive Statistics for Social Networks, $N = 82$

Statistic	Size	Density	Proportion Mutual	Proportion Asymmetric	Proportion Null	Proportion Transitive
Mean	24.049	0.270	0.108	0.325	0.567	0.474
Standard deviation	14.715	0.181	0.135	0.258	0.284	0.266
Minimum	7	0.012	0.000	0.000	0.000	0.000
Maximum	97	0.717	0.669	1.000	0.978	1.000
25th	16	0.125	0.022	0.121	0.375	0.272
50th	20	0.246	0.055	0.239	0.590	0.456
75th	28.250	0.376	0.162	0.521	0.810	0.665

Analyses now address the question of how much of the array of observed triad censuses is explained by lower-order properties: graph density, indegree and outdegree distributions, and the dyad census.

6. EXPECTED TRIAD CENSUSES, CONDITIONAL ON LOWER-ORDER GRAPH PROPERTIES

The question of how much of the observed triad censuses can be explained by lower-order graph properties compares the 82 observed triad censuses with those that are expected, given four lower-order graph properties: (1) density of the network Δ; (2) the dyad census MAN; (3) the outdegree distribution $\{x_{i+}\}$; and (4) the indegree distribution $\{x_{+j}\}$. In each case, expected triad frequencies are calculated directly (Wasserman 1977; Holland and Leinhardt 1976; Skvoretz et al. 2004). Formulas for finding these expected triad censuses are presented in Table 3.

Expected triad censuses, given network density, are found using a Bernoulli directed graph model. In this case triad probabilities are calculated as functions of the number of ties in a triad and the density of the network (Skvoretz et al. 2004). For example, the 300 triad has six ties, so its probability is Δ^6. Similarly, the 012 triad has one asymmetric dyad (with one tie), which may be in one of six location/orientation combinations, and two null dyads, which means that its probability is $6\Delta\,(1 - \Delta)^5$.

TABLE 2
Triad Census Array in Row Percentages

	Triad															
Net:	003	012	102	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
1	93	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	53	25	14	1	2	1	2	0	0	0	1	0	0	0	0	0
3	48	32	8	1	3	2	4	0	0	0	1	0	0	0	0	0
4	56	15	22	0	0	1	2	1	0	0	0	0	0	0	1	0
5	13	23	17	3	3	8	15	4	3	0	2	1	2	3	3	1
6	17	26	26	1	2	3	9	8	1	0	5	0	0	1	3	0
7	15	21	12	13	2	5	4	8	6	0	2	3	3	2	2	1
8	50	20	17	1	4	0	3	2	0	0	2	0	0	0	0	0
9	74	22	1	1	1	1	0	0	0	0	0	0	0	0	0	0
10	4	15	9	5	5	8	9	12	6	0	4	4	6	3	7	3
11	36	36	4	5	5	5	2	1	2	0	0	1	0	0	0	0
12	50	30	9	1	2	2	2	1	0	0	0	0	0	0	0	0
13	26	26	2	1	21	5	7	0	3	0	1	6	0	1	0	1
14	24	30	23	0	3	5	6	1	1	0	0	1	2	2	1	1
15	73	16	8	0	1	0	1	0	0	0	0	0	0	0	0	0
16	77	16	5	0	0	0	0	0	0	0	0	0	0	0	0	0
17	74	15	8	0	1	0	1	0	0	0	0	0	0	0	0	0
18	79	18	1	0	1	1	0	0	0	0	0	0	0	0	0	0
19	88	10	2	0	0	0	0	0	0	0	0	0	0	0	0	0
20	10	22	7	6	6	9	8	7	5	1	4	2	3	4	4	1
21	46	34	3	4	4	3	1	1	2	0	0	0	0	0	0	0

(Continued)

TABLE 2
(Continued)

	Triad															
Net:	003	012	102	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
22	4	12	8	4	6	4	12	7	6	0	5	7	4	4	12	5
23	62	29	3	1	2	2	1	0	0	0	0	0	0	0	0	0
24	23	36	14	1	3	6	7	2	1	1	1	1	1	1	1	0
25	65	27	1	0	4	1	2	0	0	0	0	0	0	0	0	0
26	76	16	6	0	0	0	0	0	0	0	0	0	0	0	0	0
27	64	23	8	0	1	1	1	0	0	0	0	0	0	0	0	0
28	92	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0
29	61	25	7	1	1	2	1	1	0	0	0	0	0	0	0	0
30	28	22	15	2	3	3	2	2	2	0	9	2	1	1	6	1
31	20	16	17	4	1	1	4	10	2	0	9	4	0	0	8	4
32	63	18	15	1	0	1	0	0	0	0	1	0	0	0	0	0
33	64	19	12	1	1	0	0	1	0	0	0	0	0	0	0	0
34	76	16	6	0	0	0	0	0	0	0	0	0	0	0	0	0
35	75	12	11	0	0	0	0	0	0	0	0	0	0	0	0	0
36	41	36	0	13	0	3	0	0	8	0	0	0	0	0	0	0
37	27	31	0	15	1	14	0	0	12	0	0	0	0	0	0	0
38	1	5	0	5	9	15	3	9	33	2	0	6	5	5	2	1
39	0	2	2	6	3	4	7	6	22	1	2	15	5	8	12	5
40	16	33	0	14	10	8	0	1	17	0	0	0	0	0	0	0
41	7	27	1	15	10	10	1	1	26	0	0	2	0	0	0	0
42	6	23	1	13	9	9	1	1	23	0	5	1	0	0	7	0
43	12	31	2	13	7	10	2	1	17	0	0	2	1	0	0	0

(Continued)

TABLE 2
(Continued)

	Triad															
Net:	003	012	102	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
44	17	0	63	0	0	0	0	0	0	0	0	0	0	0	0	20
45	0	0	0	0	0	0	0	0	100	0	0	0	0	0	0	0
46	0	8	0	0	17	6	2	0	40	5	0	12	7	1	1	0
47	22	38	2	10	8	10	2	1	5	0	0	1	0	0	0	0
48	19	38	2	9	8	11	2	1	8	0	0	1	1	0	0	0
49	0	0	0	0	0	0	0	0	97	3	0	0	0	0	0	0
50	4	0	27	0	0	0	0	0	0	0	46	0	0	0	0	23
51	6	0	29	0	0	0	0	0	0	0	24	0	0	0	0	42
52	2	12	0	15	7	10	1	1	47	0	0	3	1	1	0	0
53	7	27	0	15	9	14	0	0	28	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	19	0	0	19	26	0	21	14
55	7	27	0	13	9	16	0	1	24	0	0	0	1	0	0	0
56	9	14	6	37	6	0	3	3	20	0	0	3	0	0	0	0
57	0	1	0	2	4	5	2	1	55	0	0	17	10	1	3	0
58	0	29	0	0	14	0	0	0	57	0	0	0	0	0	0	0
59	0	0	0	1	0	0	1	1	29	0	0	10	30	3	16	10
60	21	21	18	4	3	3	6	8	2	0	5	2	2	1	3	2
61	81	13	3	1	1	0	0	0	0	0	0	0	0	0	0	0
62	16	20	28	1	3	3	8	5	1	0	5	1	1	1	3	2

(Continued)

TABLE 2
(Continued)

	Triad															
Net:	003	012	102	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
63	15	24	19	2	5	5	10	5	2	0	4	2	1	1	4	1
64	13	18	30	1	2	3	10	7	0	0	5	1	1	1	3	3
65	6	12	7	12	6	4	4	8	14	0	5	5	6	1	8	2
66	28	28	11	9	3	3	3	8	2	0	2	1	2	1	2	0
67	54	28	7	1	3	2	2	1	0	0	0	1	0	0	0	0
68	16	27	14	2	7	3	14	3	3	0	3	3	1	0	2	1
69	13	17	3	3	16	4	16	1	6	0	3	6	2	4	4	1
70	34	32	7	4	6	5	5	1	2	0	1	0	0	1	1	0
71	45	30	8	2	4	3	5	1	0	0	1	0	0	1	0	0
72	34	36	12	1	3	7	2	1	2	0	0	0	1	0	0	0
73	39	30	14	1	5	2	4	1	1	0	0	1	0	0	1	0
74	36	31	19	1	2	2	3	2	0	0	1	1	0	0	1	0
75	38	34	6	3	5	5	4	2	1	0	1	0	0	0	0	0
76	48	29	7	3	6	1	2	1	1	0	1	0	0	0	0	0
77	48	33	10	1	1	2	1	1	1	0	0	0	0	0	0	0
78	6	19	0	22	12	4	1	1	29	0	0	0	0	5	0	0
79	0	1	0	5	3	3	0	0	68	0	0	10	6	3	1	0
80	7	27	0	26	7	9	0	0	23	0	0	0	0	0	0	0
81	0	2	0	8	1	4	0	1	58	1	0	6	13	2	2	0
82	0	0	0	0	0	0	0	1	21	1	0	10	19	11	29	8

TABLE 3
Formulas for Expected Triad Censuses

	Condition		
Triad	Density[a]	Dyad Census[b]	Outdegrees[c]
003	$(1-\Delta)^6$	$N^{(3)}$	[000]
012	$6\Delta\,(1-\Delta)^5$	$3AN^{(2)}$	2[100] + 2[010] + 2[001]
102	$3\Delta^2\,(1-\Delta)^4$	$3MN^{(2)}$	[110] + [101] + [011]
021D	$3\Delta^2\,(1-\Delta)^4$	$\frac{3}{4}NA^{(2)}$	[200] + [020] + [002]
021U	$3\Delta^2\,(1-\Delta)^4$	$\frac{3}{4}NA^{(2)}$	[110] + [101] + [011]
021C	$6\Delta^2\,(1-\Delta)^4$	$\frac{3}{2}NA^{(2)}$	2[110] + 2[101] + 2[011]
111D	$6\Delta^3\,(1-\Delta)^3$	$3MAN$	6[111]
111U	$6\Delta^3\,(1-\Delta)^3$	$3MAN$	[210] + [201] + [120] + [102] + [021] + [012]
030T	$6\Delta^3\,(1-\Delta)^3$	$\frac{3}{4}A^{(3)}$	[210] + [201] + [120] + [102] + [021] + [012]
030C	$2\Delta^3\,(1-\Delta)^3$	$\frac{1}{4}A^{(3)}$	2[111]
201	$3\Delta^4\,(1-\Delta)^2$	$3NM^{(2)}$	[211] + [121] + [112]
120D	$3\Delta^4\,(1-\Delta)^2$	$\frac{3}{4}MA^{(2)}$	[211] + [121] + [112]
120U	$3\Delta^4\,(1-\Delta)^2$	$\frac{3}{4}MA^{(2)}$	[220] + [202] + [022]
120C	$6\Delta^4\,(1-\Delta)^2$	$\frac{3}{4}MA^{(2)}$	2[211] + 2[121] + 2[112]
210	$6\Delta^5\,(1-\Delta)$	$3AM^{(2)}$	2[221] + 2[212] + 2[122]
300	Δ^6	$M^{(3)}$	[222]

[a]Probability in Bernoulli digraph (Skvoretz et al. 2004).

[b]Numerators for probability, uniform given dyad census (MAN). The denominator is $\binom{g}{2}^{(3)}$, using descending factorial notation where $z^{(k)} = z(z-1)\cdots(z-k+1)$ (Holland and Leinhardt 1970, 1976).

[c]Where $[d_i, d_j, d_k] = \dfrac{\binom{2}{d_i}\binom{g-3}{x_{i+}-d_i}}{\binom{g-1}{x_{i+}}}\dfrac{\binom{2}{d_j}\binom{g-3}{x_{j+}-d_j}}{\binom{g-1}{x_{j+}}}\dfrac{\binom{2}{d_k}\binom{g-3}{x_{k+}-d_k}}{\binom{g-1}{x_{k+}}}$ (Wasserman 1977).

Expected triads based on the other three lower-order properties are found using conditional uniform graph distributions. Such distribution for a focal network has as its sample space all possible graphs with the same number of nodes as the focal graph and the same value(s) of the graph properties on which the distribution is conditioned (Holland and Leinhardt 1976; Wasserman and Faust 1994).

Triad probabilities conditional on the dyad census (MAN) are calculated from the numbers of M, A, and N dyads in the network (Holland and Leinhardt 1976). To illustrate, the 300 triad has three mutual dyads and no asymmetric or null dyads, so the numerator for its probability is $M^{(3)} = M(M-1)(M-2)$ and the denominator is

$$\binom{g}{2}^{(3)} = \left[\binom{g}{2} - 1\right]\left[\binom{g}{2} - 2\right],$$

the number of ways of arranging the dyads. The descending factorial notation is $z^{(k)} = z(z-1)\cdots(z-k+1)$. The 012 triad has one asymmetric dyad, which may be in one of three locations, and two null dyads. Thus, the numerator for probability of this triad is $3AN^{(2)}$ and its denominator is $\binom{g}{2}^{(3)}$.

Expected triad censuses conditional on the outdegree distribution are found by considering the outdegrees of the three nodes in a given triad (denoted d_i, d_j, d_k) and then calculating the probability of three nodes with the specific outdegrees from all possible combinations of three nodes from the graph, given their outdegrees x_{i+}, x_{j+}, x_{k+} (Wasserman 1977). Expected triad frequencies are then found by summing these probabilities across all combinations of three nodes. For an ordered triple of nodes, the probability that they form a triad with outdegrees d_i, d_j, d_k is

$$\Pr[d_i, d_j, d_k] = \frac{\binom{2}{d_i}\binom{g-3}{x_{i+}-d_i}}{\binom{g-1}{x_{i+}}}\frac{\binom{2}{d_j}\binom{g-3}{x_{j+}-d_j}}{\binom{g-1}{x_{j+}}}\frac{\binom{2}{d_k}\binom{g-3}{x_{k+}-d_k}}{\binom{g-1}{x_{k+}}}. \tag{3}$$

To illustrate, the 012 triad has one node with outdegree equal to 1 and two nodes with outdegree equal to 0. The probability that the ordered triple of nodes i, j, and k, with outdegrees x_{i+}, x_{j+}, x_{k+}, form a 012 triad is

$$\Pr[d_i, d_j, d_k] = \frac{\binom{2}{0}\binom{g-3}{x_{i+}-0}}{\binom{g-1}{x_{i+}}}\frac{\binom{2}{0}\binom{g-3}{x_{j+}-0}}{\binom{g-1}{x_{j+}}}\frac{\binom{2}{1}\binom{g-3}{x_{k+}\ \ 1}}{\binom{g-1}{x_{k+}}}. \tag{4}$$

Since the one arc in the 012 triad may be between any of three pairs of nodes, and in either orientation, three quantities, [1,0,0], [0,1,0], and [0,0,1], are summed across all combinations of three nodes in the graph to find the expected frequency of the 012 triad in the graph.

Expected triad frequencies, given the distribution of indegrees, are found according to the same logic as that related to the outdegree distribution but using the nodal indegrees and exchanging three pairs of triads: 021D and 021U; 111D and 111U; 120D and 120U. As described in Wasserman (1977) and elsewhere, density provides the least restrictive condition. The other three conditional distributions include density plus other constraints.

Expected triad censuses for the 82 networks, expressed as row proportions, were found and arranged in four network-by-triad arrays, denoted $\mathbf{T}_{E|\bullet}$, where $\bullet$ refers to the lower-order property: (1) density $\mathbf{T}_{E|\Delta}$; (2) the outdegree distribution $\mathbf{T}_{E|x_{i+}}$; (3) the indegree distribution $\mathbf{T}_{E|x_{+j}}$; and (4) dyad census MAN $\mathbf{T}_{E|MAN}$.

7. SINGULAR VALUE DECOMPOSITION

Information in a triad census array for a collection of networks is potentially 16-dimensional (the number of triads). A low-dimensional representation facilitates comparison and visual presentation. To accomplish this, singular value decomposition (SVD) is used to produce a reduced rank approximation of the characteristic structure of the matrix (Ben-Israel and Greville 1974; Digby and Kempton 1987; Weller and Romney 1990). Parallel decompositions are done for the observed and expected triad census arrays. For the observed triads censuses in $\mathbf{T}_O$, the SVD is defined as

$$\underset{82\times 16}{\mathbf{T}_O} = \underset{82\times 16}{\mathbf{U}_O}\ \underset{16\times 16}{\mathbf{D}_O}\ \underset{16\times 16}{\mathbf{V}'_O}, \tag{5}$$

where $\mathbf{U}_O$ and $\mathbf{V}_O$ are the left and right singular vectors (respectively) and $\mathbf{D}_O$ is a diagonal matrix of singular values, λ_l. Columns of $\mathbf{U}_O$ and $\mathbf{V}_O$ are orthogonal

$$\begin{aligned} \mathbf{U}'_O\mathbf{U}_O &= \mathbf{I} \\ \mathbf{V}'_O\mathbf{V}_O &= \mathbf{I}. \end{aligned} \tag{6}$$

SVD also is used to find low-dimensional representations for each of the four expected triad census arrays:

$$\underset{82\times 16}{\mathbf{T}_{E|\bullet}} = \underset{82\times 16}{\mathbf{U}_{E|\bullet}} \underset{16\times 16}{\mathbf{D}_{E|\bullet}} \underset{16\times 16}{\mathbf{V}'_{E|\bullet}}, \tag{7}$$

where $\bullet$ denotes the particular condition. A full rank solution, $W = 16$, exactly reproduces the original data. Low-dimensional representations of $\mathbf{T}_O$ or $\mathbf{T}_{E|\bullet}$ use fewer than the full set of 16 singular value and singular vector sets and approximate the original matrices. Squared singular values express the quality of the reduced rank approximation in the following way. The sum of the squared singular values is equal to the matrix norm $|| \; ||$ of $\mathbf{T}_O$ (Ben-Israel and Greville 1974; Digby and Kempton 1987):

$$\sum_{l=1}^{W} \lambda_l^2 = ||\mathbf{T}_O|| = trace(\mathbf{T}_O \mathbf{T}'_O). \tag{8}$$

Thus, the goodness-of-fit of a reduced rank solution, $w \leq W$, is given by

$$\frac{\sum_{l=1}^{w} \lambda_l^2}{\sum_{l=1}^{W} \lambda_l^2} \tag{9}$$

and is interpreted as the proportion sum-of-squares accounted for by the rank w approximation (Geenacre 1984). Since the left and right singular vectors reproduce the original data, they can be employed in further analyses that explain or account for the data. In the analyses that follow, the observed triad census array, SVD of $\mathbf{T}_O$, will be compared in turn to SVD of the triad census arrays expected given the four lower-order graph properties, $\mathbf{T}_{E|\bullet}$.

8. RESULTS

Squared singular values from SVD of the observed triad census array, $\mathbf{T}_O$, are presented in column 2 of Table 4 and the first four left and right singular vectors are in Tables 5 and 6, respectively. For graphic display, singular vectors are multiplied by their singular values to emphasize the

TABLE 4
Singular Value Decomposition of Triad Arrays, Squared Singular Values, and Percentages

	Triad Census Array									
	Observed		Expected Given Density		Expected Given Outdegree Distribution		Expected Given Indegree Distribution		Expected Given Dyad Census (MAN)	
Dimension	Singular Value Squared	Percentage of Total	Singular Value Squared	Percentage of Total	Singular Value Squared	Percentage of Total	Singular Value Squared	Percentage of Total	Singular Value Squared	Percentage of Total
1	17.62	64.27	17.12	79.09	17.19	78.97	17.21	79.21	17.31	68.22
2	5.19	18.94	3.32	15.34	3.18	14.62	3.22	14.82	3.79	14.93
3	2.00	7.29	0.98	4.54	0.99	4.57	0.97	4.46	2.05	8.06
4	1.17	4.27	0.21	0.96	0.22	1.01	0.21	0.95	1.17	4.62
5	0.47	1.70	0.01	0.07	0.14	0.64	0.09	0.43	0.51	2.03
6	0.31	1.12	0.00	0.00	0.02	0.11	0.02	0.07	0.28	1.08
7	0.23	0.83			0.02	0.07	0.01	0.06	0.18	0.73
8	0.14	0.53			0.00	0.01	0.00	0.01	0.06	0.24
9	0.10	0.36			0.00	0.01	0.00	0.00	0.02	0.08
10	0.06	0.22							0.01	0.02
11	0.04	0.16								
12	0.04	0.13								
13	0.02	0.09								
14	0.02	0.08								
15	0.01	0.03								
16	0.00	0.01								

TABLE 5
Singular Value Decomposition of Observed Triad Array, Left Singular Vectors, Scores for Networks

Network	1	2	3	4	5
1	0.204	−0.081	0.225	−0.021	0.040
2	0.143	−0.020	−0.020	−0.026	−0.032
3	0.137	−0.007	−0.052	0.037	−0.009
4	0.143	−0.034	0.016	−0.119	−0.057
5	0.060	0.037	−0.155	−0.043	0.018
6	0.072	0.022	−0.172	−0.109	−0.055
7	0.063	0.049	−0.116	−0.014	0.030
8	0.133	−0.021	−0.013	−0.071	−0.040
9	0.180	−0.046	0.096	0.041	0.026
10	0.032	0.058	−0.111	−0.044	0.134
11	0.115	0.023	−0.100	0.094	0.009
12	0.140	−0.012	−0.038	0.025	−0.012
13	0.087	0.032	−0.086	0.080	0.067
14	0.092	0.019	−0.155	−0.059	−0.051
15	0.174	−0.050	0.099	−0.024	0.000
16	0.182	−0.055	0.120	−0.013	0.009
17	0.176	−0.052	0.106	−0.032	−0.002
18	0.187	−0.055	0.127	0.027	0.029
19	0.197	−0.072	0.188	−0.016	0.029
20	0.052	0.053	−0.125	0.016	0.075
21	0.134	0.005	−0.053	0.093	0.014
22	0.028	0.053	−0.100	−0.063	0.182
23	0.161	−0.025	0.021	0.058	0.013
24	0.092	0.029	−0.165	0.024	−0.019
25	0.164	−0.030	0.039	0.060	0.024
26	0.180	−0.054	0.115	−0.016	0.006
27	0.163	−0.034	0.040	0.001	−0.008
28	0.203	−0.080	0.218	−0.025	0.035
29	0.157	−0.027	0.020	0.016	−0.006
30	0.089	0.012	−0.086	−0.072	0.019
31	0.067	0.017	−0.094	−0.120	0.052
32	0.158	−0.037	0.047	−0.065	−0.038
33	0.159	−0.038	0.049	−0.040	−0.022
34	0.180	−0.054	0.114	−0.017	0.005
35	0.175	−0.057	0.116	−0.064	−0.015
36	0.125	0.038	−0.055	0.123	−0.002
37	0.093	0.071	−0.079	0.126	−0.005
38	0.017	0.160	0.018	−0.003	0.065
39	0.010	0.113	0.004	−0.072	0.234
40	0.074	0.105	−0.107	0.132	−0.015

(Continued)

TABLE 5
(Continued)

Network	1	2	3	4	5
41	0.051	0.144	−0.085	0.100	−0.028
42	0.045	0.129	−0.081	0.066	0.035
43	0.064	0.107	−0.114	0.105	−0.006
44	0.059	−0.009	−0.142	−0.494	−0.224
45	0.017	0.398	0.248	−0.094	−0.243
46	0.019	0.190	0.035	−0.006	0.070
47	0.089	0.056	−0.145	0.136	0.012
48	0.082	0.068	−0.146	0.130	0.011
49	0.017	0.386	0.241	−0.091	−0.235
50	0.021	0.004	−0.117	−0.389	−0.069
51	0.026	0.003	−0.108	−0.407	−0.032
52	0.029	0.216	0.025	0.032	−0.065
53	0.051	0.155	−0.081	0.112	−0.044
54	0.006	0.103	0.036	−0.136	0.490
55	0.050	0.141	−0.095	0.112	−0.023
56	0.044	0.118	−0.069	0.054	−0.022
57	0.013	0.237	0.117	−0.068	0.087
58	0.040	0.263	−0.006	0.060	−0.132
59	0.007	0.138	0.062	−0.119	0.398
60	0.075	0.021	−0.117	−0.078	0.008
61	0.186	−0.061	0.150	−0.010	0.025
62	0.067	0.019	−0.151	−0.149	−0.041
63	0.065	0.031	−0.152	−0.065	0.007
64	0.059	0.018	−0.162	−0.173	−0.051
65	0.033	0.087	−0.067	−0.038	0.120
66	0.094	0.021	−0.104	0.015	0.018
67	0.146	−0.016	−0.011	0.029	−0.002
68	0.070	0.035	−0.149	−0.026	0.019
69	0.052	0.050	−0.085	0.023	0.121
70	0.109	0.017	−0.097	0.060	0.006
71	0.128	−0.003	−0.056	0.034	−0.003
72	0.113	0.019	−0.119	0.037	−0.024
73	0.119	0.003	−0.085	−0.003	−0.023
74	0.115	0.005	−0.113	−0.034	−0.052
75	0.119	0.011	−0.092	0.071	0.000
76	0.135	−0.005	−0.036	0.044	−0.002
77	0.139	−0.005	−0.054	0.033	−0.017
78	0.041	0.154	−0.049	0.088	−0.027
79	0.015	0.287	0.152	−0.061	−0.037
80	0.051	0.139	−0.100	0.130	−0.031
81	0.014	0.251	0.119	−0.054	0.040
82	0.006	0.106	0.038	−0.112	0.450

TABLE 6
Singular Value Decomposition of Observed Triad Array, Right Singular Vectors, Scores for Triads

Triad	1	2	3	4	5
003	0.891	−0.212	0.384	−0.046	0.031
012	0.408	0.226	−0.657	0.373	−0.022
102	0.151	0.017	−0.392	−0.709	−0.296
021D	0.055	0.163	−0.184	0.186	0.036
021U	0.054	0.121	−0.148	0.111	0.078
021C	0.051	0.112	−0.159	0.110	0.061
111D	0.043	0.039	−0.163	−0.052	0.116
111U	0.025	0.036	−0.101	−0.054	0.100
030T	0.073	0.907	0.351	−0.102	−0.166
030C	0.002	0.014	0.002	−0.003	0.008
201	0.019	0.015	−0.124	−0.303	−0.006
120D	0.013	0.094	−0.002	−0.081	0.404
120U	0.009	0.093	0.020	−0.114	0.571
120C	0.007	0.033	−0.021	−0.025	0.150
210	0.014	0.066	−0.041	−0.124	0.561
300	0.012	0.025	−0.086	−0.392	0.149

relative importance of the dimensions. Two-dimensional plots of the left singular vectors (for the social networks) are in Figure 2 and plots of the right singular vectors (for the triads) are in Figure 3. As can be seen from the squared singular values, triad censuses for the 82 social networks are well represented using four dimensions, accounting for 94.766% of the data.

The first two left singular vectors from SVD of the observed triad census array (Figure 2 and Table 5) define a distinctly triangular space, and networks in different regions of this space have notably different local structural properties. Networks in the upper left of the plot in Figure 2 are dominance relations, characterized by asymmetric or null dyads and the absence of mutual dyads (notably, networks #45 finches, #49 hens, #79 sparrows, #58 wasps, #81 caribou, and #57 ponies). In the lower left of Figure 2 are networks of affiliation or co-observation characterized by mutual or null dyads and no asymmetric dyads (#50 howler monkeys, #51 kangaroos, and #44 dolphins). Toward the lower right are networks with the lowest density, ranging from .01 to .05 (notably #1, #18, #19, #28 and #61, all sociometric choices between humans). In addition Bernard, Killworth, and Sailer's (1980)

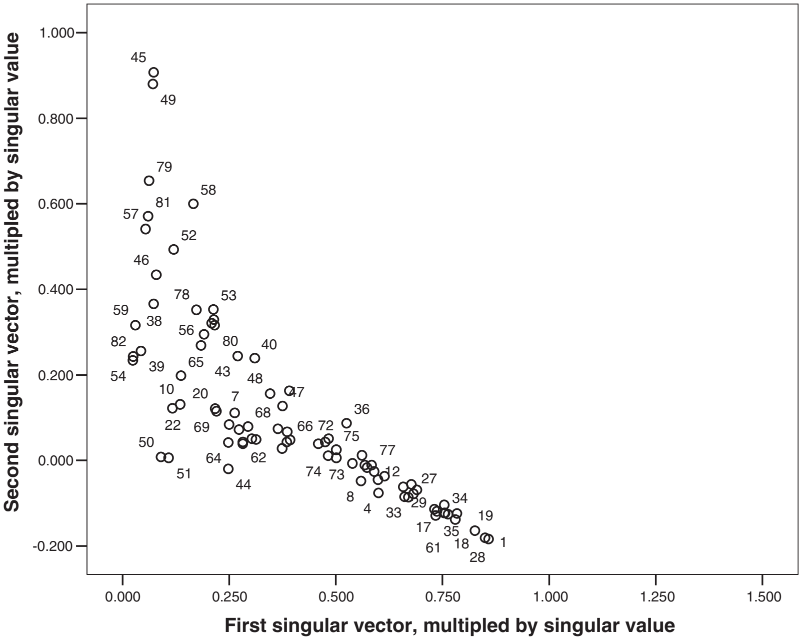

FIGURE 2. Singular value decomposition of triad census array, first two left singular vectors, multiplied by singular values, $N = 82$ networks.

reported communication between ham radio operators (network #61), which was previously shown to have a pattern almost completely described by its degree distributions (Faust and Romney 1985), is found in this corner of the space in Figure 2.

Plots of the first four left singular vectors are presented in Figure 4 and will be the basis for further comparisons. The first four left singular vectors are largely interpretable using network density and linear or quadratic functions of dyad proportions. The first left singular vector has a strong linear relationship with the proportion of null dyads in network $r^2 = 0.92$. The second left singular vector is linearly related to the proportion of asymmetric dyads, $r^2 = 0.94$. The third left singular vector is a quadratic function of the proportion of null dyads: $Y = -.17 + 1.24X - 1.28X^2$, with $r^2 = 0.87$. Finally, the fourth left singular vector has a moderate linear relationship with the proportion of mutual dyads in network $r^2 = 0.64$.

The right singular vectors for triads also reflect the local structural patterns observed for the left singular vectors (Figure 3 and

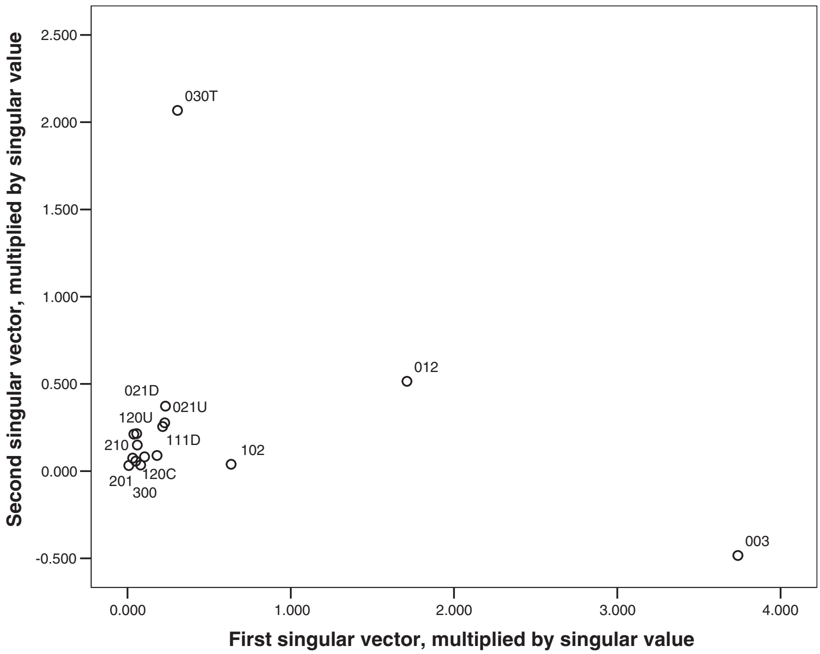

FIGURE 3. Singular value decomposition of triad census array, first two right singular vectors, multiplied by singular values, $N = 16$ triads.

Table 6). The 030T all asymmetric transitive triad is in the upper-left corner of the plot, the 300 all mutual triad is in the lower left, and the 003 all null (lowest density) triad is in the lower right.

Squared singular values and percentages from singular value decompositions of the four expected triad census arrays are presented in columns 4 through 11 of Table 4. Four dimensional solutions are adequate for representing the data, accounting for over 95% of the sum of squares in each case. As with the SVD of the observed triad array, left singular vectors summarize patterns for the 82 networks and right singular vectors summarize the 16 triad isomorphism classes. These vectors are used in further analyses by comparing them with the left and right singular vectors from the SVD of the observed triads, to examine how much of the observed triad censuses they explain. Focusing on the networks, plots of the first four left singular vectors are presented in Figures 5, 6, 7, and 8. Visual inspection of these Figures, in comparison with Figure 4 of the left singular vectors for the observed triad

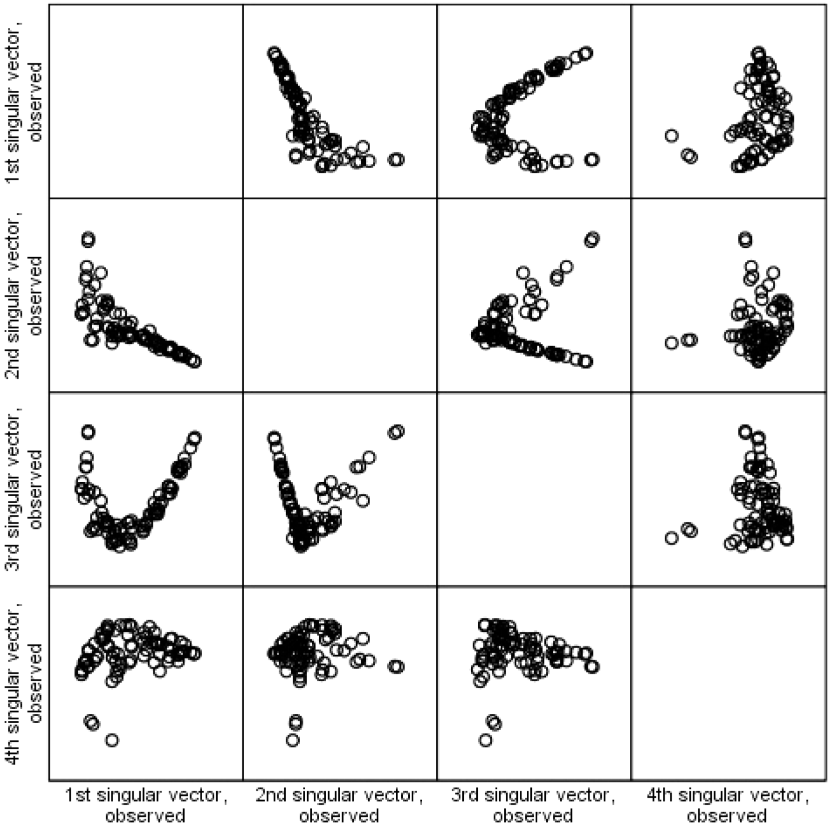

FIGURE 4. Singular value decomposition of observed triad censuses, first four left singular vectors, $N = 82$ networks.

censuses, shows considerable similarity, especially between results for the observed triad censuses and those expected given the dyad census (MAN). Systematic assessment of this similarity is accomplished using canonical redundancy analysis, as described in the next section.

9. CANONICAL REDUNDANCY

For two sets of variables **X** and **Y**, canonical redundancy $\mathbf{R}^2_{Y\bullet X}$ expresses the extent to which linear combinations of one set (**X**, the explanatory variables) explain the variability in linear combinations of the other set (**Y**, the response variables) (Lambert, Wildt, and Durand 1988; Stewart

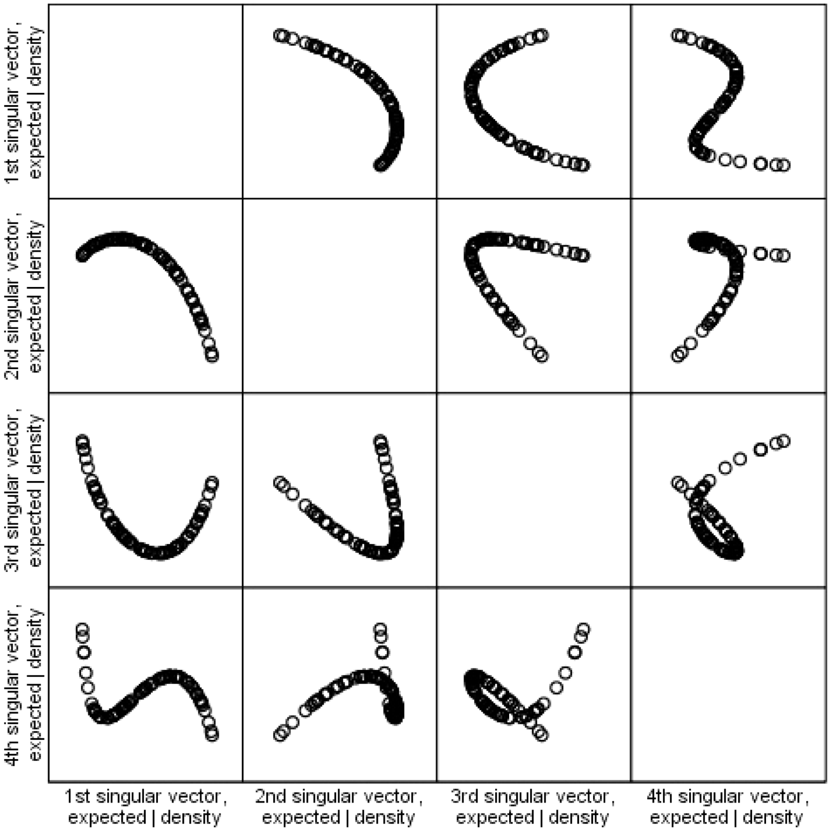

FIGURE 5. Singular value decomposition of triad censuses expected, given network density, first four left singular vectors, $N = 82$ networks.

and Love 1968). Canonical redundancy is interpreted as the proportion of variance in **Y** explained by linear combinations of **X**. Calculation of canonical redundancy can be expressed in terms of the matrices of correlations between variables from the two sets, where $\mathbf{R}_{XX}$ is the matrix of correlations between the variables in **X**, $\mathbf{R}_{XY}$ is the matrix of correlations between variables in **X** and in **Y**, and $\mathbf{R}_{YX} = \mathbf{R}'_{XY}$ is the matrix of correlations between variables in **Y** and in **X**. The proportion of variance in **Y** explained by **X** is then given by

$$\mathbf{R}^2_{Y\bullet X} = \frac{1}{p} trace\left(\mathbf{R}^{-1}_{XX}\mathbf{R}_{XY}\mathbf{R}_{YX}\right), \tag{10}$$

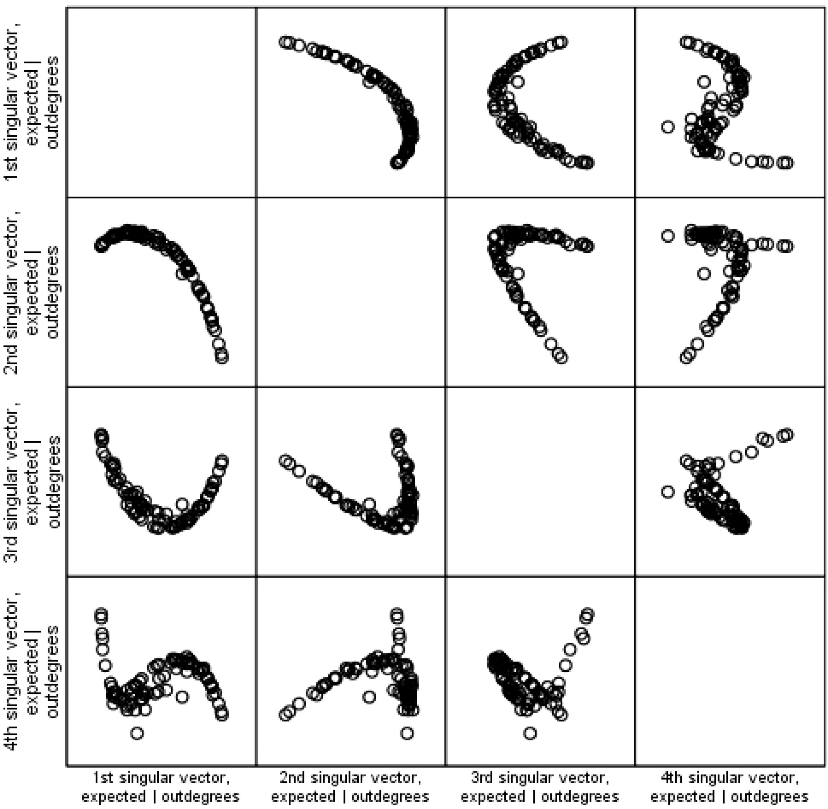

FIGURE 6. Singular value decomposition of triad censuses expected uniform given network outdegrees, first four left singular vectors, $N = 82$ networks.

where p is the number of variables in set $\mathbf{Y}$ and $\mathbf{R}_{XX}^{-1}$ is the inverse of $\mathbf{R}_{XX}$ (Lambert et al. 1988).

Parallel analyses compare the first four left singular vectors, $\mathbf{U}_O$, and right singular vectors, $\mathbf{V}_O$, from the SVD of the observed triad census (equation 5) to their counterparts from the SVD of the expected triad censuses $\mathbf{U}_{E|\bullet}$ and $\mathbf{V}_{E|\bullet}$ (equation 7). In the following canonical redundancy analyses, variable set $\mathbf{Y}$ is $\mathbf{U}_O$ or $\mathbf{V}_O$, the first four left or right singular vectors from SVD of $\mathbf{T}_O$, and variable set $\mathbf{X}$ is in turn $\mathbf{U}_{E|\bullet}$ or $\mathbf{V}_{E|\bullet}$ the first four left or right singular vectors from the SVD of $\mathbf{T}_{E|\Delta}$, $\mathbf{T}_{E|x_{i+}}$, $\mathbf{T}_{E|x_{+j}}$, or $\mathbf{T}_{E|MAN}$. Equations for the canonical redundancy calculations are

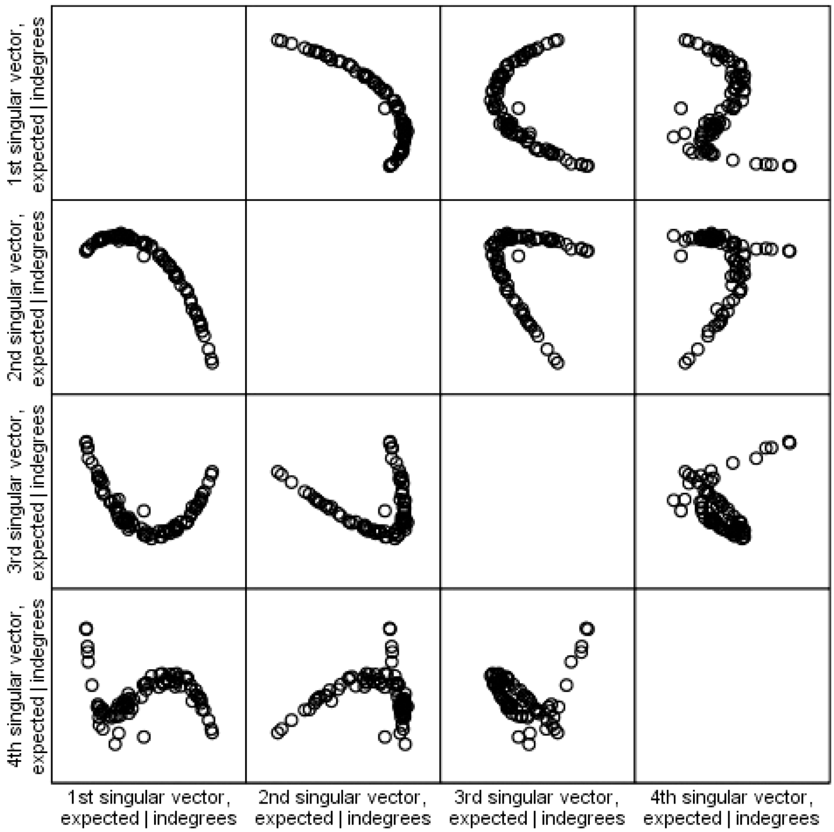

FIGURE 7. Singular value decomposition of triad censuses expected, uniform given network indegrees, first four left singular vectors, $N = 82$ networks.

$$\mathbf{R}^2_{\mathbf{U}_O\bullet\mathbf{U}_{E|\bullet}} = \frac{1}{4} trace(\mathbf{R}^{-1}_{\mathbf{U}_{E|\bullet}\mathbf{U}_{E|\bullet}} \mathbf{R}_{\mathbf{U}_{E|\bullet}\mathbf{U}_O} \mathbf{R}_{\mathbf{U}_O\mathbf{U}_{E|\bullet}}) \quad \text{and} \tag{11}$$

$$\mathbf{R}^2_{\mathbf{V}_O\bullet\mathbf{V}_{E|\bullet}} = \frac{1}{4} trace(\mathbf{R}^{-1}_{\mathbf{V}_{E|\bullet}\mathbf{V}_{E|\bullet}} \mathbf{R}_{\mathbf{V}_{E|\bullet}\mathbf{V}_O} \mathbf{R}_{\mathbf{V}_O\mathbf{V}_{E|\bullet}}). \tag{12}$$

Canonical redundancy results are presented in columns 2 and 4 of Table 7. To find the proportion of variance explained in observed triad censuses, rather than the four dimensional solutions from SVD,

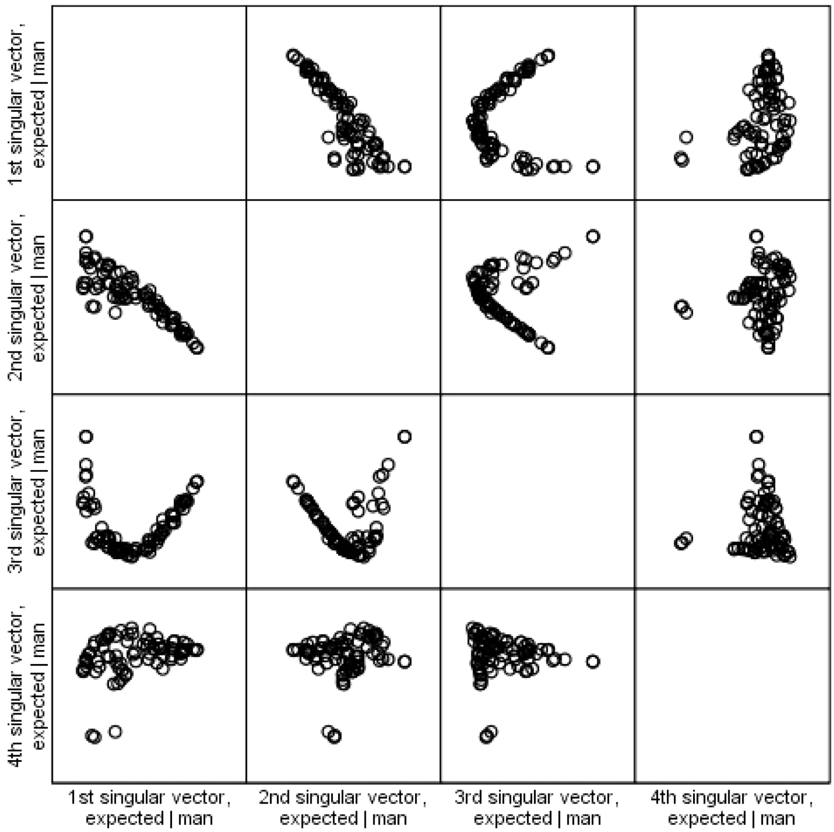

FIGURE 8. Singular value decomposition of triad censuses expected, uniform given dyad census MAN, first four left singular vectors, $N = 82$ networks.

the canonical redundancy must be discounted by the proportion of the observed triad censuses explained by the first four singular value, singular vector sets:

$$\frac{\sum_{l=1}^{4} \lambda_l^2}{\sum_{l=1}^{16} \lambda_l^2} \times \mathbf{R}^2_{\mathbf{U}_{O\bullet}\mathbf{U}_{E|\bullet}} \quad \text{or} \quad \frac{\sum_{l=1}^{4} \lambda_l^2}{\sum_{l=1}^{16} \lambda_l^2} \times \mathbf{R}^2_{\mathbf{V}_{O\bullet}\mathbf{V}_{E|\bullet}}. \tag{13}$$

TABLE 7
Canonical Redundancy and Proportion of Observed Triad Census Array Explained by Expected Triad Census Arrays, 82 Networks, and 16 Triads

	Networks		Triads	
Lower-Order Property for Expected Triad Census Array	Canonical Redundancy	Proportion of Observed Triad Censuses Array Explained	Canonical Redundancy	Proportion of Observed Triad Censuses Array Explained
Δ	0.644	0.610	0.571	0.541
$\{x_{i+}\}$	0.706	0.669	0.621	0.588
$\{x_{+j}\}$	0.686	0.650	0.610	0.578
MAN	0.983	0.932	0.924	0.876

These proportions are shown in columns 3 and 5 of Table 7, for the network and triad spaces respectively.[1]

To summarize these results, the dyad census (MAN) accounts for 93.2% of the variance among social networks and 87.6% of the variance among triad isomorphism classes for the 82 social networks, confirming the visual similarity between configurations in Figures 4 and 8. Even simpler local features account for a substantial portion of the variance. Network density accounts for 61% of the variance among networks and 54.1% of the variance among triad isomorphism classes.

[1] In addition, canonical variables for the relationship between the left singular vectors for observed triad censuses and those expected conditional on the dyad census (MAN) were calculated using the coefficients for the linear combinations of $\mathbf{X}$ and $\mathbf{Y}$. Both sets of canonical variables are interpretable as linear and quadratic functions of network density and dyad proportions. Focusing on the coefficients for the left singular vectors from the SVD of the observed triad censuses, $\mathbf{U}_O$, there is a substantial linear relationship between the first canonical variable and the proportions of null dyads ($r^2 = 0.934$) as well as asymmetric dyads ($r^2 = 0.923$). The second canonical variable is a moderate linear function of the proportion of mutual dyads ($r^2 = 0.609$); the third is a quadratic function of the proportion of asymmetric dyads ($r^2 = 0.836$); and the fourth is a quadratic function of network density ($r^2 = 0.718$). The same pattern holds for the canonical variables for set$\mathbf{U}_{E|MAN}$, the left singular vectors from the SVD of triad censuses expected conditional on the dyad census (MAN).

The indegree distribution accounts for 65% and 57.8%, and the outdegree distribution for 66.9% and 58.8% of the variance for networks and triads respectively.

10. TRIADIC STRUCTURE?

The fact that the dyad censuses essentially reproduce the triad censuses for the collection of 82 social networks raises the question of whether, once we condition on the dyad census, there is any remaining triadic structure in the networks. To examine possible triadic effects, observed triad frequencies are compared to those expected conditional on the dyad census (MAN) using the *tau* statistic, $\tau = \frac{c_{O_k} - c_{E|MAN_k}}{\sigma_{c_k}}$ (Holland and Leinhardt 1970; Wasserman and Faust 1994), where c_{O_k} is the observed frequency of triad type k for a given network, $c_{E|MAN_k}$ is the expected frequency of triad type k for the network, given its dyad census, and σ_{c_k} is the standard deviation of the expected frequency (equations for σ_{c_k} are found in Holland and Leinhardt 1970 and Wasserman and Faust 1994). Tau statistics were calculated for each of the 16 triad isomorphism classes and in each of the 82 social networks, resulting in $82 \times 16 = 1312$ comparisons. These 1312 τ values are plotted, by triad isomorphism class, as box plots in Figure 9. In these plots each box shows the distribution of τ for 82 networks; the median is indicated by a horizontal line and the edges of the box show the 25th and 75th percentiles. Extreme observations, more than 1.5 or 3.0 box widths from the upper or lower edges of a box, are indicated as open circles or asterisks and labeled with the identification number of the network.

Keeping in mind that the τ values for a given network are not independent, several points are worth noting. A handful of social networks appear to have triadic patterning that departs from what is expected given their dyad censuses. Notably, three dominance networks—#55 (macaques), #49 (hens), and #42 (bison)—have more 030T (transitive) triads than expected, and, for networks #49 and #55, fewer 030C (cyclic) triads. Networks #60 and #61 (reported interactions between humans in a fraternity and in a group of ham radio operators) have more 300 (all mutual) triads and also more 003 (all null) triads than expected.

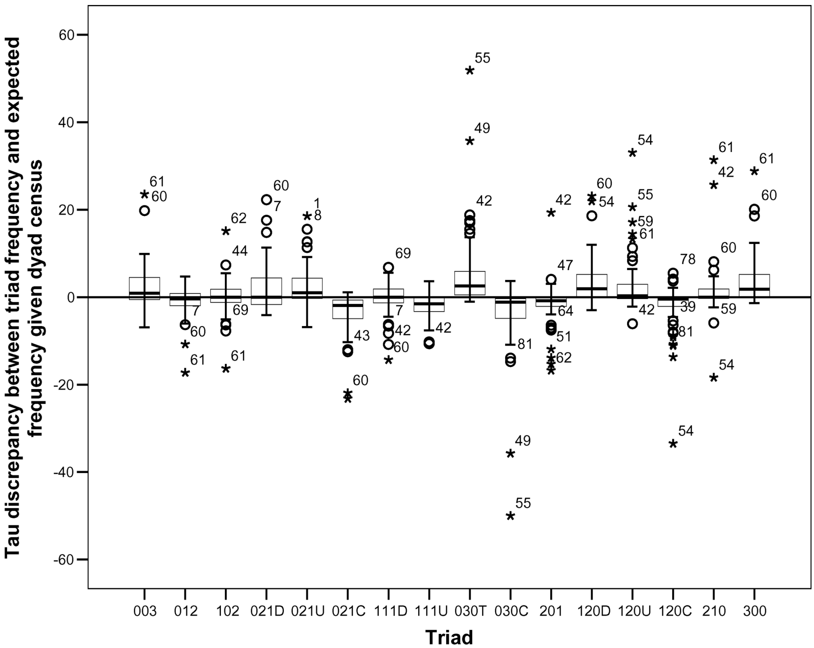

FIGURE 9. Box plots of tau statistics for discrepancy between triad frequency and expected frequency given dyad census, $N = 82$ networks.

Importantly, networks that show triadic tendencies are, with the exception of network #49, not the networks that anchor different corners of the triangular space defined by the first two left singular vectors from SVD of the observed triad censuses (see again Figure 2). In other words, triad censuses that are distinctive in comparison to censuses from other networks do not necessarily exhibit substantial triadic tendencies, beyond what is expected from their dyad censuses.

11. DISCUSSION

These results demonstrate that the vast majority of the variance in triad censuses for a diverse collection of 82 social networks is explained by properties that are more local than triadic—network density, the indegree and outdegree distributions, and the distribution of mutual, asymmetric, and null dyads. This clearly replicates and extends earlier findings (Faust 2006) demonstrating that triad censuses for a wide range of social networks are largely accounted for by linear and quadratic functions of network density and dyad censuses proportions. Three questions follow from these results. First, why is triadic structure in social networks overwhelmingly described by very local network features? Second, what are the implications for social network theory and methodology? Third, what research should be pursued to further elucidate triadic structures in social networks?

11.1. *Why Very Local Structure?*

One key to understanding these results is that triad census probabilities for a given network are largely circumscribed by the network's density, degree, and dyad distributions. This is consistent with previous work on triad censuses (Faust 2006) and local effects on other graph-level measures (Anderson, Butts, and Carley 1999; Butts 2006). For many social networks, lower-order properties severely limit the range of possible triadic outcomes. Thus, in comparative perspective, local structural features explain observed triad censuses for a collection of social networks.

To illustrate, consider four social networks that occupy different regions of the two-dimensional SVD solution (Figure 2) and have different densities, outdegree, and dyad distributions. Network #49, dominance relations between 32 hens, has only asymmetric dyads, which means that only two triad configurations (030T and 030C) are possible; all others are impossible. In this network 97% of the triads are 030T and 3% are 030C. Given its dyad census, 75% are expected to be 030T and 25% are expected to be 030C. Network #50, co-observations of 17 howler monkeys, has only mutual and null dyads, which means that only four triad configurations (003, 102, 201, and 300) are possible: These four configurations occur in 4%, 27%, 46%, and 23% of the triads in this network. These triads are expected to occur 5%, 26%, 44%, and 24% of the time, given this network's dyad census. Given the dyad censuses for networks #49 and #50, it is impossible for them to have any triadic configurations in common (though they share ten impossible configurations). Network #1, nominations of rejection between 97 schoolchildren, has the lowest density of the 82 networks in the sample

($\Delta = 0.01$). Given this density, the probability of a 300 (all mutual) triad is $(.01)^6 = .000000000001$ and the probability of a 003 (all null) triad is $(.99)^6 = .9983$. There are no 300 triads in network #1 and 93% of its triads are 300. Though 030T and 030C triads are not impossible in network #1, in comparison to network #49, their occurrence must be rare. Outdegree and indegree distributions also constrain possible triad configurations that can be realized. As an extreme case, consider network #28 in which each of 63 people nominated a single other person they would like to work with, giving $x_{i+} = 1$ for all people and $\Delta = 0.02$. In this case only seven triadic configurations are possible: 003, 012, 102, 021U, 021C, 111D, and 030C. These configurations occur in 92%, 6%, 1%, 0.08%, 0.05%, 0.06%, and 0.002% of the triads in network #28, illustrating the constraints that the combination of fixed degree and low network density have on triad probabilities.

These four social networks illustrate the severe constraints that very local graph properties place on possible triadic outcomes. Therefore, when analyzed in aggregate, variation among triad censuses for a collection of social networks is overwhelmingly explained by the networks' states on these lower-order features. Stated more abstractly, although information in the triad census array is potentially 16 dimensional (the number of triad isomorphism classes), the possible location of a particular network's triad census in this space is extremely constrained by its nodal and dyadic properties.

11.2. *Implications for Social Network Methodology and Theory*

Several lines of research on social network methodology and theory are impacted by these results. Research attempting to formulate exact local-global structural theories based on specific permitted and forbidden triads is most directly in jeopardy, especially when such efforts do not explicitly consider lower-order graph properties (Davis 1970; Friedkin 1998; Johnsen 1985, 1986, 1989a, 1989b, 1998). That these lower-order features are vexing for such theoretical formulations is seen in the persistent statement of separate theoretical models for different size networks (Davis 1970; Johnsen 1985, 1986, 1989a, 1989b, 1998) and in observations that some triads should not be forbidden in large networks (Friedkin 1998). Network size affects the triad census through its effect on network density, a point that needs to be addressed explicitly in constructing social network theories.

Statistical models for social networks, including exponential random graph models and their extensions (Pattison and Wasserman 1999; Robins and Pattison 2005; Robins et al. forthcoming; Snijders et al. 2006; Wasserman and Pattison 1996; Wasserman and Robins 2005) are also affected by these results. Constraints that nodal and dyadic properties imposed on possible triadic configurations underscore the importance of including parameters for these lower-order effects in models examining triadic or higher-order effects.

11.3. *Future Directions*

In their 1979 paper Holland and Leinhardt demonstrated that the extent of intransitivity detected in social networks of positive sentiments depended, in large part, on the conditional distribution used as referent against which triadic structural tendencies were assessed. Presciently, they chose the uniform distribution, conditional on the dyad census, as the distribution for their comparisons. They concluded that "employing the higher level of conditioning revealed that, at least in some cases, what was previously thought to be structure was spurious, the result of lower level constraints operating on the digraphs" (1979:77). Despite this bleak observation, work on triad distributions in social and other networks continues to this day. However, the profound constraint that lower-order properties impose on possible triadic outcomes is often underappreciated. Future research should continue to address the exact range of possible triad censuses and higher-order graph configurations that can arise, given various lower-order structural properties, and how these constraints impact other social network methodologies.

In conclusion, the results presented in this paper do not imply that there are no differences between empirically observed triad censuses and those expected given the four lower-order graph properties investigated here. What they do demonstrate is that lower-order structural properties so severely limit possible triadic outcomes that, in comparative perspective, differences among triad censuses for diverse social networks are overwhelmingly explained by the lower-order properties. Open questions remain concerning exactly how, and to what extent, triadic processes and properties vary across different kinds of social relations.

APPENDIX A:
DESCRIPTION OF SOCIAL NETWORKS

	Species	Relation	Size	Source
1	Human	Rejection	97	Smucker (1947:378)
2	Human	Eating preference	25	Jennings (1937:114)
3	Human	Lunch preference	16	Bronfenbrenner (1944)
4	Human	Sociometric choice	25	Chabot (1950:133)
5	Human	Like most	10	Katz and Powell (1960, Table 3:304)
6	Human	Who likes you most	10	Katz and Powell (1960, table 3a:304)
7	Human	Work partner choice	24	Holland and Leinhardt (1978, fig. 5:248)
8	Human	Positive choice	25	Moreno in Forsyth and Katz (1946, fig. 3:345)
9	Human	Rejection	25	Moreno in Forsyth and Katz (1946, fig. 3:345)
10	Human	Like to work with	23	Zeleny (1947:398)
11	Human	Not like to work with	23	Zeleny (1947:398)
12	Human	Seating choice	25	Taba (1955, table 4:5)
13	Human	Friendship	13	Fine (1987, fig. 6.1:141)
14	Human	Friendship	13	Fine (1987, fig. 6.2:144)
15	Human	Friendship	25	French (1963, fig. 1:148)
16	Human	Friendship	25	French (1963, fig. 2:148)
17	Human	Friendship	23	French (1963, fig. 3:149)
18	Human	Friendship	40	Weintraub and Bernstein (1966, fig. 1:513)
19	Human	Friendship	63	Weintraub and Bernstein (1966, fig. 2:514)
20	Human	Indifferent	29	McKinney (1948, table 1:358)
21	Human	Rejection	29	McKinney (1948, table 1:358)
22	Human	Acceptance	29	McKinney (1948, table 1:358)
23	Human	Wood cutting partner	34	Sanders (1939:64)
24	Human	Choice to sit with	14	Sandman (1952:413)
25	Human	Choice not to sit with	14	Sandman (1952:413)
26	Human	Friendship	41	Venable (1954:356)
27	Human	Friendship	24	Grossman and Wrighter (1948:353)

(Continued)

APPENDIX A
(Continued)

	Species	Relation	Size	Source
28	Human	Work choice	63	Faunce and Beegle (1948:211)
29	Human	Friend	14	Hayes and Conklin (1953:25)
30	Human	Friend	13	Hayes and Conklin (1953:26)
31	Human	Friend	12	Hayes and Conklin (1953:27)
32	Human	Friend	23	Hayes and Conklin (1953:29)
33	Human	Friend	24	Hayes and Conklin (1953:30)
34	Human	Best friend	44	Cook (1944:252)
35	Human	Best friend	42	Cook (1944:253)
36	Ants	Dominance	16	Cole (1981:83
37	Ants	Dominance	13	Cole (1981:84)
38	Baboons	Dominance	10	McMahan and Morris (1984:377)
39	Baboons	Dominance	16	Gauthier and Strayer (1986:124)
40	Big horn sheep	Dominance	20	Hass (1991:513)
41	Big horn sheep	Dominance	24	Hass (1991:514)
42	Bison	Dominance	25	Lott (1979)
43	Cattle	Dominance	28	Schein and Fohrman (1955:49)
44	Dolphin	Affiliation	13	Connor, Smolker, and Richards (1992:422)
45	Finches	Dominance	8	Marler (1955:111)
46	Geldings	Dominance	9	deVries, Netto, and Hanegraaf (1993:169)
47	Goats	Dominance	21	Fournier and Festabianchet (1995:1453)
48	Goats	Dominance	30	Fournier and Festabianchet (1995:1454)
49	Hens	Dominance	32	Guhl (1953)
50	Howler monkeys	Co-observation	17	Froehlich and Thorington (1981)
51	Kangaroos	Co-observation	17	Grant (1973:453)
52	Macaques	Dominance	16	Strayer and Cummins (1980)
53	Macaques	Win/loss	16	Strayer and Cummins (1980)
54	Macaques	Win access to water	30	Strayer and Cummins (1980)
55	Macaques	Dominance	62	Takahata (1991:125)
56	Orangutans	Dominance	7	Utimai et al. (1997:917)

(Continued)

APPENDIX A
(Continued)

	Species	Relation	Size	Source
57	Ponies	Threats	17	Cluton-Brock, Greenwood, and Powell (1976)
58	Wasps	Shares forage	7	Eberhard (1969:25)
59	Wolves	Deference	16	van Hooff and Wensing (1987)
60	Humans	Reported interaction	58	Bernard, Killworth, and Sailer (1980:211-212)
61	Humans	Reported interaction	44	Bernard, Killworth, and Sailer (1980:213-214)
62	Humans	Reported interaction	40	Bernard, Killworth, and Sailer (1980:215)
63	Humans	Reported interaction	34	Bernard, Killworth, and Sailer (1980:216-217)
64	Humans	Reported interaction	18	Borgatti, Everett, and Freeman (2002)
65	Humans	Advice	21	Krackhardt (1987:129-132)
66	Humans	Friendship	21	Krackhardt (1987:129-132)
67	Humans	Advice	32	Coleman, Katz, and Menzel (1966)
68	Humans	Liking	17	Newcomb (1961)
69	Humans	Not liking	17	Newcomb (1961)
70	Humans	Disesteem	18	Sampson (1968:466)
71	Humans	Dislike	18	Sampson (1968:465)
72	Humans	Esteem	18	Sampson (1968:466)
73	Humans	Influence	18	Sampson (1968:467)
74	Humans	Like	18	Sampson (1968:465)
75	Humans	Negative influence	18	Sampson (1968:467)
76	Humans	Negative praise	18	Sampson (1968:468)
77	Humans	Praise	18	Sampson (1968:468)
78	Sparrows	Dominance	13	Moller (1987:1639)
79	Sparrows	Dominance	10	Moller (1987:1639)
80	Sparrows	Dominance	14	Moller (1987:1640)
81	Caribou	Win/loss in interactions	20	Barrette and Vandal (1986:125)
82	Caribou	Win/loss in interactions	20	Barrette and Vandal (1986:126)

APPENDIX B: REFERENCES FOR SOCIAL NETWORK DATA

[The network numbers follow in brackets.]

Barrette, Cyrille, and Denis Vandal. 1986. "Social Rank, Dominance, Antler Size, and Access to Food in Snow-bound Wild Woodland Caribou." *Behaviour* 97:118–46. [81, 82]

Bernard, H. Russell, Peter Killworth, and Lee Sailer. 1980. "Informant Accuracy in Social Network Data IV: A Comparison of Clique-level Structure in Behavioral and Cognitive Data." *Social Networks* 2:191–218. [60 fraternity, 61 ham radio operators, 62 office, 63 technical office]

Borgatti, Stephen P., Martin G. Everett, and Linton C. Freeman. 2002. *UCINET 6 for Windows Software for Social Network Analysis*. Harvard, MA: Analytic Technologies. [64]

Bronfenbrenner, Urie. 1944. "The Graphic Presentation of Sociometric Data." *Sociometry* 7:283–89. [3]

Chabot, James. 1950. "A Simplified Example of the Use of Matrix Multiplication for the Analysis of Sociometric Data." *Sociometry* 13:131–40. [4]

Cluton-Brock, T. H., J. P. Greenwood, and R. P. Powell. 1976. "Ranks and Relationships in Highland Ponies and Highland Cows." *Zeitschrift Tierpsychologie* 41:202–16. [57]

Cole, B. J. 1981. "Dominance Hierarchies in Leptothorax Ants." *Science* 212:83–84. [36, 37]

Coleman, James S., E. Katz, and H. Menzel. 1966. *Medical Innovation*. New York: Bobbs-Merrill. Cited in Ronald S. Burt. 1991. *Structure*. New York: Center for the Social Sciences, Columbia University. [67]

Connor, R. C., R. A. Smolker, and A. F. Richards. 1992. "Dolphin Alliances and Coalitions." Pp. 415–44 in *Coalitions and Alliances in Humans and Other Animals*, edited by A. H. Harcourt and F. B. M. deWaal. Oxford, England: Oxford University Press. [44]

Cook, Lloyd Allen. 1944. "An Experimental Sociographic Study of a Stratified Tenth Grade Class." *American Sociological Review* 10:250–61. [34, 35]

de Vries, H., W. J. Netto, and P. L. H. Hanegraaf. 1993. "Matman: A Program for the Analysis of Sociometric Matrices and Behavioral Transition Matrices." *Behaviour* 125:157–75. [46]

Eberhard, M. J. W. 1969. "The Social Biology of Polistine Wasps." Ann Arbor, MI: Museum of Zoology, University of Michigan, Miscellaneous Publications No. 140, p. 25. [58]

Faunce, Dale, and J. Allan Beegle. 1948. "Cleavages in a Relatively Homogenous Group of Rural Youth." *Sociometry* 11:207–16. [28]

Fine, Gary A. 1987. *With the Boys: Little League Baseball and Preadolescent Culture*. Chicago: University of Chicago Press. [13, 14]

Forsyth, Elaine, and Leo Katz. 1946. "A Matrix Approach to the Analysis of Sociometric Data: Preliminary Report." *Sociometry* 9:340–47. Also Moreno, Jacob L. 1934. *Who Shall Survive: A New Approach to the Problem of Human Inter-relations*. New York: Beacon House. [8, 9]

Fournier, F., and M. Festabianchet. 1995. "Social-dominance in Adult Female Mountain Goats." *Animal Behaviour* 49:1449–59. [47, 48]

French, Cecil L. 1963. "Some Structural Aspects of a Retail Sales Group." *Human Organization* 22:146–51. [15, 16, 17]

Froehlich, J. W. and R. W. Thorington, Jr., 1981, "The Genetic Structure and Socioecology of Howler Monkeys (*Alouatta palliata*) on Barro Colorado Island." Pp. 291–306 in *Ecology of Barro Colorado Island: Seasonal Rhythms and Long Term Changes in a Tropical Forest*, edited by E. G. Leigh and A. S. Randi. Washington, DC: Smithsonian Press. Also L. D. Sailer and S. J. C. Gaulin. 1984. "Proximity, Sociality and Observation: The Definition of Social Groups." *American Anthropologist* 86:91–98. [50]

Gauthier, R., and F. F. Strayer. 1986. "Empirical Techniques for Identification of Dominance Class." Pp. 120–33 in *Current Perspectives in Primate Social Dynamics*, edited by D. M. Taub and F. A. King. New York: Van Nostrand. [39]

Grant, T. R. 1973. "Dominance and Association Among Members of a Captive and a Free-ranging Group of Grey Kangaroos (*Macropus giganthus*)." *Animal Behaviour* 21:449–56. [51]

Grossman, Beverly, and Joyce Wrighter. 1948. "The Relationship Between Selection-Rejection and Intelligence, Social Status, and Personality Amongst Sixth Grade Children." *Sociometry* 11:346–55. [27]

Guhl, A. M. 1953. *Social Behavior of the Domestic Fowl*. Manhattan, KS: Kansas State College, Agricultural Experiment Station, Technical Bulletin 73. [49]

Hass, Christine. 1991. "Social Status in Female Bighorn Sheep (*Ovis canadensis*): Expression, Development and Reproductive Correlates." *Journal of the Zoological Society of London* 225:509–23. [40, 41]

Hayes, Margaret L., and Mary Elizabeth Conklin. 1953. "Intergroup Attitudes and Experimental Change." *Journal of Experimental Education* 22:19–36. [29, 30, 31, 32, 33]
Holland, Paul, and Samuel Leinhardt. 1978. "An Omnibus Test for Social Structure Using Triads." *Sociological Methods and Research* 7:227–56. [24]
Jennings, Helen. 1937. "Structure of Leadership-Development and Sphere of Influence." *Sociometry* 1:99–143. [2]
Katz, Leo, and James H. Powel. 1960. "A Proposed Index of the Conformity of One Sociometric Measurement to Another." Pp. 298–306 in *The Sociometry Reader*, edited by Jacob L. Moreno and Helen H. Jennings. Glencoe, IL: Free Press. [5, 6]
Krackhardt, D. 1987. "Cognitive Social Structures." *Social Networks* 9:104–34. [65, 66]
Lott, D. F. 1979. "Dominance Relations and Breeding Rate in Mature Male American Bison." *Zeitschrift Tierpsychologie* 49:418–32. [42]
Marler, P. 1955. "Studies of Fighting in Chaffinches. (1) Behaviour in Relation to the Social Hierarchy." *British Journal of Animal Behaviour* 3:111–17. [45]
McKinney, John C. 1948. "An Educational Application of a Two-Dimensional Sociometric Test." *Sociometry* 11:356–67. [20, 21]
McMahan, C. A., and M. D. Morris. 1984. "Application of Maximum Likelihood Paired Comparison Ranking to Estimation of a Linear Dominance Hierarchy in Animal Societies." *Animal Behaviour* 32:374–78. [38]
Moller, Anders P. 1987. "Variation in Badge Size in Male House Sparrows *Passer domesticus:* Evidence for Status Signaling." *Animal Behaviour* 35:1637–44. [78, 79, 80]
Newcomb, T. 1961. *The Acquaintance Process*. New York: Holt, Reinhardt, and Winston. Also, Nordlie P. 1958. "A Longitudinal Study of Interpersonal Attraction in a Natural Group Setting." Ph.D. dissertation, University of Michigan. In Borgatti, S.P., Everett, M.G. and Freeman, L.C. 2002. *UCINET 6 for Windows*. Harvard, MA: Analytic Technologies. [68, 69]
Sampson, S. 1968. "A Novitiate in a Period of Change: An Experimental and Case Study of Social Relationships." Ph.D. dissertation, Cornell University. [70, 71, 72, 73, 74, 75, 76, 77]
Sanders, Irwin T. 1939. "Sociometric Work with a Bulgarian Woodcutting Group." *Sociometry* 2:58–69. [23]

Sandman, Ellen S. 1952. "A Study in Sociometry on Kindergarten Level." *Sociometry* 25:410–22. [24, 25]
Schein, M. W., and M. H. Fohrman. 1955. "Social Dominance Relationships in a Herd of Dairy Cattle." *British Journal of Animal Behaviour* 3:45–55. [43]
Smucker, Orden. 1947. "Measurement of Group Tension Through Use of Negative Sociometric Data." *Sociometry* 10:376–83. [1]
Strayer, F. F., and Mark S. Cummins. 1980. "Aggressive and Competitive Structures in Captive Monkey Groups." Pp. 85–96 in *Dominance Relations: An Ethological View of Human Conflict and Social Interaction*, edited by Donald R. Omark, F. F. Strayer, and Daniel G. Freedman. New York: Garland STPM Press. [52, 53, 54]
Taba, Hilda. 1955. *With Perspective on Human Relations: A Study of Peer Group Dynamics in an Eighth Grade*. Washington, DC: American Council on Education. [12]
Takahata, Yukio. 1991. "Diachronic Changes in the Dominance Relations of Adult Female Japanese Monkeys of the Arashiyama B Group." Pp. 124–39 in *The Monkeys of Arashiyama*, edited by Linda Marie Fedigan and Pamela J. Asquith. Albany, NY: State University of New York Press. [55]
Utami, Sri Suci, Serge A. Winch, Elisabeth H. M. Sterck, and Jan A. R. A. M. van Hooff. 1997. "Food Competition Between Wild Orangutans in Large Fig Trees." *International Journal of Primatology* 18:909–27. [56]
van Hooff, Jan A. R. A. M., and Joep A. B. Wensing. 1987. "Dominance and its Behavioral Measures in a Captive Wolf Pack." Pp. 219–52 in *Man and Wolf*, edited by Harry Frank. Dordrecht, Netherlands: Junk. [59]
Venable, Tom C. 1954. "The Relationship of Selected Factors to the Social Structure of a Stable Group." *Sociometry* 17:355–57. [26]
Weintraub, D., and F. Bernstein. 1966. "Social Structure and Modernization: A Comparative Study of Two Villages." *American Journal of Sociology* 71:509–21. [18, 19]
Zeleny, Leslie D. 1947. "Selection of the Unprejudiced." *Sociometry* 10:396–401. [10, 11]

REFERENCES

Anderson, Brigham S., Carter Butts, and Kathleen Carley. 1999. "The Interaction of Size and Density with Graph-Level Indices." *Social Networks* 21:239–67.

Batagelj, Vladimir, and Andrej Mrvar. 2001. "A Subquadratic Triad Census Algorithm for Large Sparse Networks with Small Maximum Degree." *Social Networks* 23:237–43.

Ben-Israel, A., and T. Greville. 1974. *Generalized Inverses: Theory and Applications*. New York: Wiley.

Bernard, H. Russell, Peter Killworth, and Lee Sailer. 1980. "Informant Accuracy in Social Network Data IV: A Comparison of Clique-level Structure in Behavioral and Cognitive Data." *Social Networks* 2:191–218.

Borgatti, Stephen P., Martin G. Everett, and Linton C. Freeman. 2002. *UCINET 6 for Windows: Software for Social Network Analysis*. Harvard, MA: Analytic Technologies.

Burt, Ronald S. 1990. "Detecting Role Equivalence." *Social Networks* 12:83–97.

———.1992. *Structural Holes: The Social Structure of Competition*. Cambridge, MA: Harvard University Press.

Burt, Ronald S., and Marc Kenz. 1995. "Kinds of Third-Party Effects on Trust." *Rationality and Society* 7(3):255–92.

Butts, Carter T. 2006. "Exact Bounds for Degree Centralization." *Social Networks* 28:283–96.

Cartwright, Dorwin, and Frank Harary. 1956. "Structural Balance: A Generalization of Heider's Theory." *Psychological Review* 63:277–93.

Chase, Ivan D. 1982. "Dynamics of Hierarchy Formation: The Sequential Development of Dominance Relationships." *Behaviour* 80:218–40.

Chase, Ivan D., Craig Tovey, and Peter Murch. 2003. "Two's Company, Three's a Crowd: Differences in Dominance Relationships in Isolated Versus Socially Embedded Pairs of Fish." *Behaviour* 140:1193–217.

Coleman, James. 1988. "Social Capital in the Creation of Human Capital." *American Journal of Sociology* 94:S95–120.

Davis, James A. 1967. "Clustering and Structural Balance in Graphs." *Human Relations* 20:181–87.

———. 1970. "Clustering and Hierarchy in Interpersonal Relations: Testing Two Graph Theoretical Models on 742 Sociomatrices." *American Sociological Review* 35(5):843–51.

———. 1977. "Sociometric Triads as Multivariate Systems." *Journal of Mathematical Sociology* 5:41–59.

———. 1979. "The Davis/Holland/Leinhardt Studies: An Overview." Pp. 51–62 in *Perspectives on Social Network Research*, edited by Paul W. Holland and Samuel Leinhardt. New York: Academic Press.

Davis, James A., and Samuel Leinhardt. 1972. "The Structure of Positive Interpersonal Relations in Small Groups." Pp. 218–51 in *Sociological Theories in Progress,* Vol. 2, edited by Joseph Berger, Morris Zeldith Jr., and Bo Anderson. Boston: Houghton Mifflin.

Digby, P. G. N., and R. A. Kempton. 1987. *Multivariate Analysis of Ecological Communities*. New York: Chapman and Hall.

Faust, Katherine. 2006. "Comparing Social Networks: Size, Density, and Local Structure." *Metodoloki Zvezki (Advances in Methodology and Statistics)* 3(2):185–216.

Faust, Katherine, and A. Kimball Romney. 1985. "Does STRUCTURE Find Structure? A Critique of Burt's Use of Distance as a Measure of Structural Equivalence." *Social Networks* 7:77–103.

Faust, Katherine, and John Skvoretz. 2002. "Comparing Networks Across Space and Time, Size, and Species." Pp. 267–99 in *Sociological Methodology* 2002, vol. 32, edited by Ross Stolzenberg. Cambridge, MA: Blackwell Publishing.

Frank, Ove. 1988. "Triad Count Statistics." *Discrete Mathematics* 72:141–49.

Frank, Ove, and Frank Harary. 1982. "Cluster Inference by Using Transitivity Indices in Empirical Graphs." *Journal of the American Statistical Association* 77:835–40.

Friedkin, Noah. 1998. *A Structural Theory of Social Influence*. New York: Cambridge University Press.

Granovetter, Mark S. 1973. "The Strength of Weak Ties." *American Journal of Sociology* 78:1360–80.

Greenacre, Michael. 1984. *Theory and Applications of Correspondence Analysis*. London: Academic Press.

Hallinan, Maureen T. 1974a. *The Structure of Positive Sentiment*. New York: Elsevier.

———. 1974b. "Structural Model of Sentiment Relations." *American Journal of Sociology* 80:364–78.

Heider, Fritz. 1946. "Attitudes and Cognitive Organization." *Journal of Psychology* 21:107–12.

Holland, Paul W., and Samuel Leinhardt. 1970. "A Method for Detecting Structure in Sociometric Data." *American Journal of Sociology* 76:492–513.

———. 1971. "Transitivity in Structural Models of Small Groups." *Comparative Group Studies* 2:107–24.

———. 1972. "Some Evidence on the Transitivity of Positive Interpersonal Sentiment." *American Journal of Sociology* 77:1205–9.

———. 1973. "The Structural Implications of Measurement Error in Sociometry." *Journal of Mathematical Sociology* 3:85–111.

———. 1976. "Local Structure in Social Networks." *Sociological Methodology 1976,* 7:1–45.

———. 1979. "Structural Sociometry." Pp. 63–83 in *Perspectives on Social Network Research*, edited by Paul W. Holland and Samuel Leinhardt. New York: Academic Press.

Homans, George C. 1950. *The Human Group*. New York: Harcourt, Brace.

Johnsen, Eugene C. 1985. "Network Macrostructure Models for the Davis-Leinhardt Set of Empirical Sociomatrices." *Social Networks* 7:203–24.

———. 1986. "Structure and Process: Agreement Models for Friendship Formation." *Social Networks* 8:257–306.

———. 1989a. "Agreement-Friendship Processes Related to Empirical Social Macrostructures." Pp. 239–79 in *The Small World*, edited by Manfred Kochen. Norwood, NJ: Ablex.

———. 1989b. "The Micro-Macro Connection: Exact Structure and Process." Pp. 169–201 in *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, edited by Fred Roberts. New York: Springer-Verlag.

———. 1998. "Structures and Processes of Solidarity: An Initial Formalization." Pp. 263–302 in *The Problem of Solidarity: Theories and Models*, edited by Patrick Doreian and Thomas Fararo. Amsterdam: Gordon and Breach.

Kalish, Yuval, and Garry Robins. 2006. "Psychological Predispositions and Network Structure: The Relationship Between Individual Predispositions, Structural Holes, and Network Closure." *Social Networks* 28:56–84.

Karlberg, Martin. 1998. "Triad Count Estimation in Digraphs." *Journal of Mathematical Sociology* 23:99–126.

Lambert, Zarrel V., Albert R. Wildt, and Richard M. Durand. 1988. "Redundancy Analysis: An Alternative to Canonical Correlation and Multivariate Multiple Regression in Exploring Interset Associations." *Psychological Bulletin* 104:282–89.

Moody, James. 1998. "Matrix Methods for Calculating the Triad Census." *Social Networks* 20:291–99.

Pattison, Philippa, and Stanley Wasserman. 1999. "Logit Models and Logistic Regressions for Social Networks: II. Multivariate Relations." *British Journal of Mathematical and Statistical Psychology* 52:169–93.

Robins, Garry, and Philippa Pattison. 2005. "Interdependencies and Social Processes: Dependence Graphs and Generalized Dependence Structures." Pp. 192–214 in *Models and Methods in Social Network Analysis*, edited by Peter J. Carrington, John Scott, and Stanley Wasserman. New York: Cambridge University Press.

Robins, Garry, Tom Snijders, Peng Wang, Mark Handcock, and Philippa Pattison. Forthcoming. "Recent Developments in Exponential Random Graph (p*) Models for Social Networks." *Social Networks.*

Simmel, Georg. 1950. *The Sociology of Georg Simmel.* Translated, edited, and with an introduction by Kurt H. Wolff. New York: Free Press.

Skvoretz, John, Thomas J. Fararo, and Filip Agneessens. 2004. "Advances in Biased Net Theory: Definitions, Derivations, and Estimations." *Social Networks* 26:113–39.

Skvoretz, John, and Katherine Faust. 2002. "Relations, Species, and Network Structure." *Journal of Social Structure* 3(2) (http://zeeb.library.cmu.edu:7850/JoSS/skvoretz/index.html).

Skvoretz, John, Katherine Faust, and Thomas J. Fararo. 1996. "Social Structure, Networks, and E-State Structuralism Models." *Journal of Mathematical Sociology* 21:57–76.

Snijders, T. A. B. 1991. "Enumeration and Simulation Methods for 0-1 Matrices with Given Marginals." *Psychometrika* 56:397–417.

Snijders, T. A. B., P. Pattison, G. L. Robins, and M. Handock. 2006. "New Specifications for Exponential Random Graph Models." Pp. 99–153 in *Sociological Methodology.* vol. 36, edited by Ross M. Stolzenberg. Boston, MA: Blackwell Publishing.

Snijders, T. A. B., and Frans N. Stokman. 1987. "Extensions of Triad Counts to Networks with Different Subsets of Points and Testing Underlying Random Graph Distributions." *Social Networks* 9:249–75.

Stewart, Douglas, and William Love. 1968. "A General Canonical Correlation Index." *Psychological Bulletin* 70:160–63.
Wasserman, Stanley S. 1977. "Random Directed Graph Distributions and the Triad Census in Social Networks." *Journal of Mathematical Sociology* 5:61–86.
Wasserman, Stanley, and Katherine Faust. 1994. *Social Network Analysis: Methods and Applications*. New York: Cambridge University Press.
Wasserman, Stanley S., and Philippa Pattison. 1996. "Logit Models and Logistic Regressions for Social Networks: I. An Introduction to Markov Graphs and p*." *Psychometrika* 61:401–25.
Wasserman, Stanley S., and Garry Robins. 2005. "An Introduction to Random Graphs, Dependence Graphs, and p*." Pp. 148–61 in *Models and Methods in Social Network Analysis*, edited by Peter J. Carrington, John Scott, and Stanley Wasserman. New York: Cambridge University Press.
Weller, Susan C., and A. Kimball Romney. 1990. *Metric Scaling: Correspondence Analysis*. Newbury Park, CA: Sage.