Photometric Relationships Between Complementary Colors*

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Two complementary chromaticities must be additively mixed in the proper brightness ratio in order to produce a neutral mixture. This ratio, L, can be computed from any one of the following expressions.

$$L/y+1/y'=(1+L)/y_n,$$
$$L=(y'/y)\cdot(y-y_n)/(y_n-y'),$$
$$L=(y'/y)\cdot(x-x_n)/(x_n-x').$$

The symbols, x, y, and x', y', are the coordinates of the complementary chromaticities in the I.C.I. chromaticity diagram, in which the point x_n, y_n represents the chromaticity of the desired "neutral" or gray mixture. The three points (x, y), (x_n, y_n), and (x', y') must be colinear in this order. Neither of the complementaries need be spectrum colors, but the equations may be used to determine the brightness ratios of spectrum complementaries necessary for neutral mixtures. The great brightness ratios resulting from the low luminosity of the blue end of the spectrum are inconvenient for graphical representation, and can be avoided if the energy ratios of complementary spectrum lights necessary for a neutral mixture are computed. Loci can be traced in the I.C.I. chromaticity diagram to give the "moment" of one millilambert of every chromaticity with respect to any assigned neutral stimulus. The brightness ratio of two complementary chromaticities necessary for a neutral mixture is the reciprocal of their moments per millilambert with respect to the neutral. The locus of constant moment (per millilambert), m, is a conic section having the $y=0$ axis as its directrix, the neutral point as its focus and the moment, m, as its eccentricity. This diagram is of immediate use in the plane vector solution of any problem of additive color mixture. The vector solution of the problem encountered in the projection of additive photographs, the determination of the brightness ratios of the three primaries necessary for a neutral screen, is outlined. The "neutral" chromaticity desired on the screen need not be the neutral with respect to which the moment loci are drawn. The colorimetric purity of any color is the ratio of the moment per millilambert of that color to the moment per millilambert of the spectrum color colinear with the neutral and the sample color. Formulas and tables for the interconversion of colorimetric and excitation purities are given. The dominant wave-lengths and visual efficiencies, in illuminant "C," of colors having maximum visual efficiencies are plotted as functions of the wave-length limits of the absorption bands, and the complementary relations between such colors are reviewed. Formulas are given for the determination of the chromaticities in illuminant "C" of all colors for which the spectrophotometric curves are linear functions of wave-length. All such colors have the dominant wave-length 580.1 mμ, excitation purity less than 55 percent, or dominant wave-length 480.1 mμ, excitation purity less than 35 percent.

A. DEFINITION OF COMPLEMENTARY COLOR

SEVERAL alternative definitions of "complementary colors" have been proposed. Only that definition which states that the optically additive mixture of two complementary colors must match some arbitrarily assigned "neutral" stimulus is sufficiently specific to permit the solution of the general problem of the colorimetric specification of all of the possible complementaries of any given color.

Special merits are claimed for other definitions of complementary colors. For instance, it is occasionally claimed that the color of the after-image produced by fixation on a given color followed by observation of an equally bright neutral stimulus is a better aesthetic complementary than any of the additive complementaries of the given color. Since there are few data available concerning these "after-image complementaries," and since there is evidence that these after-image colors vary continually in hue as well as in saturation,[1] there is no apparent way in which the after-image or "fatigue complementary" of any given color can be identified unambiguously. Such identification seems to be a prerequisite to any serious study of the harmonious relationship which is said to exist between the after-image "complementaries." In some discussions, the after-image colors are assumed, either implicitly or explicitly, to be identical with the additive complementaries. This assumption is unsupported by any experimental data with which the present author is acquainted, and appears to have originated in an over-simplified conception of the Young-Helmholtz theory of color vision. Other older definitions of complementary colors, based on the behavior of subtractive mixture of colorants, are

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rather generally recognized to be untenable, since they originate in fortuitous phenomena, governed by the spectrophotometric characteristics of the colorants, having little or no physiological or psychological significance, and failing even more seriously than the after-image phenomenon to identify "complementary" colors in any unambiguous manner. Doubtless, a thorough survey of the literatures of art and of psychology would reveal other definitions of complementary colors, but, so far as the present author is aware, none of these definitions permits the unambiguous identification of the colors complementary to any given color.

In the interests of clarity, the additive mixtures discussed in this paper will be considered to be the superposition of two images on a perfectly reflecting and perfectly diffusing screen. These images exhibit separately the brightnesses and chromaticities described for each case. When spectrophotometric characteristics are specified, these can be thought of as the spectral transmittance of light filters placed in the beams forming the images. The light source is assumed to have the spectral energy distribution of the I.C.I. illuminant "C."[2,3] A spinning disk (Maxwell disk, or color top) having equal (180°) segments of the two specified colors, consisting of opaque surfaces having the specified colorimetric or spectrophotometric characteristics, and illuminated with the I.C.I. illuminant "C," would be colorimetrically equivalent to the projection screen method. In the case of the composite image on the screen, the tristimulus values (and brightness) of the mixture are the sums of the corresponding tristimulus values of the two superimposed images.[3] In the case of the spinning disk, these tristimulus values are the averages of the corresponding tristimulus values of the two equal segments. The color of the screen with any intensity of "C" illumination will be taken arbitrarily as the neutral stimulus throughout this discussion.

It is generally known that the additive mixture of any color with a suitable brightness of any other color will produce a neutral mixture, provided that the point representing a neutral stimulus in the chromaticity diagram lies on the straight line drawn between the points representing the two colors. Such pairs of colors are complementary by the definition adopted in the first paragraph. It follows that there are an infinite number of colors complementary to any given color. Of these complementary colors, there is at most only one chromaticity having any assigned brightness. Conversely, any one of these complementary chromaticities is additively complementary to the assigned color in only one brightness. Two colors will be considered complementary colors only if they have the correct relative brightnesses as well as the correct chromaticities necessary to produce a neutral additive mixture in the projection screen experiment. Colors which have suitable chromaticities but incorrect brightness ratios will be said to have complementary chromaticities, but will not be called complementary colors.

B. Photometric Relationships Between Complementary Colors

Figure 1 permits the identification of the complementary color having its brightness in any given ratio to the brightness of the original color. Conversely, it can be used to determine the brightness ratio of any pair of complementary chromaticities necessary for a neutral additive mixture. The ratio of brightnesses necessary for a neutral additive mixture of any two chromaticities colinear with the neutral point ($x=0.3101$, $y=0.3163$) is the inverse of the ratio of the moments of one millilambert of the respective chromaticities, indicated by the loci in Fig. 1. For instance, one millilambert of a color having dominant wave-length 570 mμ and excitation purity 72 percent ($x=0.4067$, $y=0.4882$) has a moment of 0.40 with respect to the I.C.I. illuminant "C." The complementary color having the same brightness must have the same moment per millilambert with respect to illuminant "C." Consequently, it must be represented at the intersection of the extended 570 mμ dominant wave-length line with the locus of moment 0.40. This intersection is at $x=0.2880$, $y=0.2775$, and the corresponding color has the dominant wave-length 457.9 mμ and 13.4 percent excitation purity. The color having dominant wave-length 457.9 mμ and 45 percent excitation purity ($x=0.2370$, $y=0.1862$) has a moment of 0.80 per millilambert. Consequently, this color should

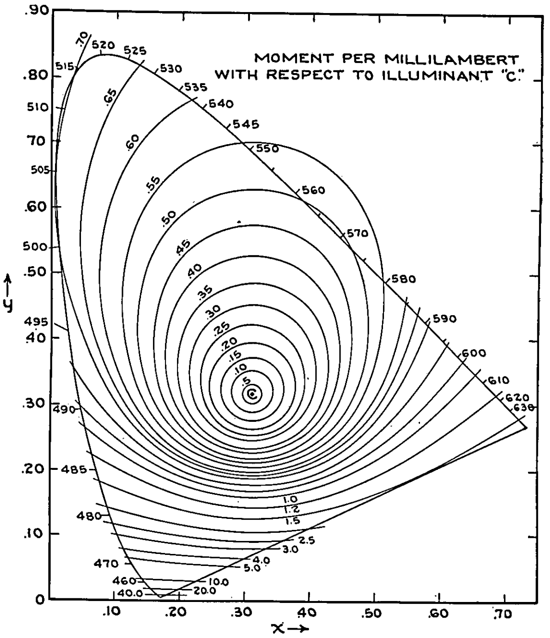

FIG. 1. Moment diagram, showing moment of one millilambert of any color, with respect to the I.C.I. illuminant "C" as neutral. Ratio of brightnesses of two complementary colors is the inverse of the ratio of the indicated moments per millilambert.

be only one-half as bright as the complementary yellow having 72 percent excitation purity, in order to result in a neutral additive mixture. The colorimetric purity of any color is the ratio of the moment per millilambert of that color to the moment per millilambert of the spectrum color colinear with the neutral and the sample color.

Figure 1 is easily constructed by making use of the fact that for the purposes of color computation with the aid of the center of gravity principle, the "mass" of any color in the I.C.I. chromaticity diagram is the brightness divided by the y coordinate.[3, 4] The moment of any color with respect to illuminant "C" is the product of this mass times the distance in the I.C.I. diagram from the point representing the color to the point representing illuminant "C." For one millilambert brightness of any color, this moment is therefore defined as the ratio, A/B, of the distances: A, from the sample point to the neutral point; and B, from the sample point to the horizontal (x) axis of the I.C.I. diagram. Therefore, each locus of any assigned value of the moment per millilambert, m, shown in Fig. 1 is a conic section having the neutral point as its focus, the x axis as its directrix, and the assigned moment per millilambert, m, as its eccentricity.

Since, in order to produce a neutral mixture, two colors must be colinear with the neutral point, and must have equal moments with respect to illuminant "C," their brightnesses must be in inverse proportion to their moments per millilambert, as indicated in the figure. If x, y, are the coordinates of the point representing one color, if x', y' are the coordinates of some color complementary to the first color, and if L represents the ratio of the brightness of the first color as compared with that of the second, then it can be shown that:

$$L=(y'/y)\cdot(y-0.3163)/(0.3163-y') \quad \text{(1a)}$$

or

$$L=(y'/y)\cdot(x-0.3101)/(0.3101-x') \quad \text{(1b)}$$

or

$$L/y+1/y'=(L+1)/0.3163. \quad \text{(2)}$$

These equations can be used in place of Fig. 1 for the solution of the problems discussed above. Eqs. (1a) and (2) should not be used when the dominant wave-length of either of the complementaries is between 485 mμ and 495 mμ, because the differences of the y coordinates of such colors are subject to relatively great rejection errors. Eq. (1b) is designed to handle these cases. It is essential to remember that the points (x, y) and (x', y') must represent complementary chromaticities, that is, they must be colinear with the neutral point ($x=0.3101$, $y=0.3163$).

Figure 2, showing the brightness of the spectrum complementary necessary for a neutral mixture with unit brightness of each spectrum color[5–8] can be determined immediately from Fig. 1. The values shown by the curves in Fig. 2 can be computed more accurately from Eqs. (1a) and (1b) by substituting the values of the coordinates of spectrum colors for every pair of complementary wave-lengths, given in Tables XXIV and VIII, respectively, in the *Handbook of Colorimetry*.[3]

Curve (a) in Fig. 3 shows the moment per millilambert of each spectrum color. This curve can be determined directly from Fig. 1. This curve resembles the curve given by Sinden in his Fig. 3[6] and named by Sinden, in accordance

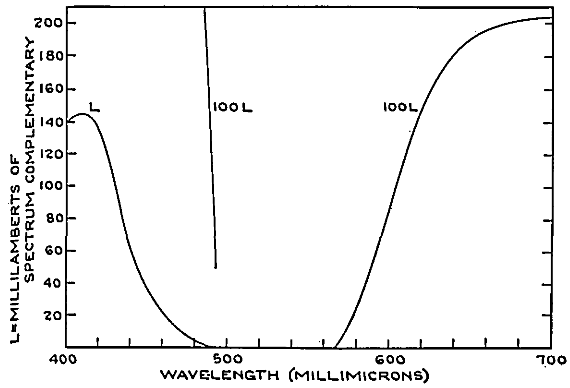

FIG. 2. Brightness of complementary wave-length necessary to "neutralize" one millilambert of indicated wave-length.

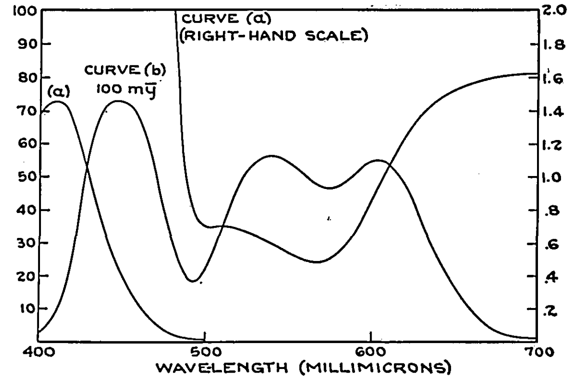

FIG. 3. (a) Moment of one millilambert of indicated wave-length. (b) Moment of one watt of indicated wave-length.

with the suggestions of Helmholtz[5] and others,[8] "relative subjective saturation of the spectral hues." This curve is shown by the present derivation to be a consequence of the objectively verifiable laws of additive color mixture, and no support for the claim of subjective significance can be found in this derivation. Curve (b) shows the relative moment per watt of each spectrum color, determined by multiplying each ordinate of curve (a) by the corresponding value of the luminosity[9] function, $\bar{y}$. The inconveniently high values shown by curve (a) for the short wave-lengths are thus eliminated, by placing the data on an energy basis.

Figure 4 shows the number of watts of the spectrum complementary necessary for a neutral mixture with one watt of each wave-length of the spectrum. This curve was derived by multiplying each ordinate of the curve in Fig. 2 by the ratio of the value of the luminosity function at the indicated wave-length to the value of the luminosity function at the complementary wave-length. This eliminates the excessively high values shown by the curve in Fig. 2 in the blue end of the spectrum. By thus placing the data on an energy basis, the engineering implications of the curve are more clearly shown. For instance, it is seen that, aside from the wave-lengths between 492 mμ and 567 mμ, which have either extreme red or extreme violet complementaries or no spectrum complementaries whatever, the orange at 605 mμ is the most efficient spectrum color in mixtures with its complementary. This orange spectrum color is almost twice as effective in neutralizing one watt of its blue-green spectrum complementary (490 mμ) as is the most effective blue (448 mμ) in neutralizing one watt of its yellow-green complementary (569 mμ).

C. Vector Solution of Problems of Additive Color Mixture

Figure 1 can be used to advantage in any problem in which a neutral additive mixture is to be established. If, for instance, it is necessary to secure a white image on the screen in connection with an additive method of projection of colored photographs, suitable relative brightnesses of the three projection primaries can be readily determined. Three vectors should be drawn, through the illuminant point and through the points representing the chromaticities of the three projection primaries. One of these vectors should be assigned a length equal to the moment of one millilambert of the corresponding chromaticity, as given by Fig. 1. A second of the vectors must then be given such a length that the resultant of these first two vectors will be parallel but oppositely directed to the third vector. The length of the second vector necessary for this adjustment can then be divided by the moment per millilambert of the second projection primary (determined from Fig. 1), to give as a result the proper brightness ratio of the second to the first projection primary. The chromaticity of the resultant mixture will be represented by the intersection of this resultant vector and the line joining the points representing the chromaticities of the first two projection primaries. The moment of this mixture, per millilambert of the first component only, is represented by the length of the resultant vector. Therefore, the moment of the third projection primary necessary to neutralize this additive mixture of the first two primaries is represented by the equally long but oppositely directed vector through the illuminant point and the point representing the chromaticity of the third primary. The length of this resultant vector can therefore be divided by the moment per millilambert of the third primary (determined as previously from Fig. 1) to give the required brightness ratio of the third to the first projection primaries.

The construction mentioned above, in which the chromaticity and moment of the additive mixture of any two colors can be determined is of use in all problems of additive mixture, and the moments with respect to illuminant "C" shown in Fig. 1 can be used for this construction even when illuminant "C" has no relation to the problem. The vectors must, however, be drawn through the point representing illuminant "C," if the moments from Fig. 1 are used. Any desired neutral point can be introduced as a new color having a chromaticity and moment with respect to illuminant "C."

Figure 5 illustrates the general method. The problem is to determine, using Fig. 1, the brightness ratios of the three projection primaries, *R*, *G*, and *B*, necessary to produce an image on the screen having the chromaticity of the I.C.I. illuminant "A," represented by the point, *A*, in Fig. 5. The chromaticity of the illuminant "C," the reference point for all vectors and moments, is represented by the point, *C*, in Fig. 5. Construct the vector, *g*, from the point, *C*, through the point, *G*, and having a length equal to the moment specified for the point, *G*, by the loci in Fig. 1. Any convenient scale of vector length will be satisfactory, if used consistently. The point, *F*, at the intersection of the lines, *GB* and *RA*, represents the chromaticity of the mixture of the primaries, *B* and *G*, necessary for a mixture with the primary, *R*, in order to match illuminant "A." Construct the vector, *f*, from the point, *C*, through the point, *F*, and the vector, *b*, from *C* through *B*. The vectors *f* and *b* are of such a length that *f* is the resultant of *b* and *g* as shown by the dotted parallelogram construction. The length of *b* can be divided by the moment shown for the point, *B*, in Fig. 1, to give the desired ratio of the brightness of the primary, *B*, to the brightness of the primary, *G*. The vectors, *r* and

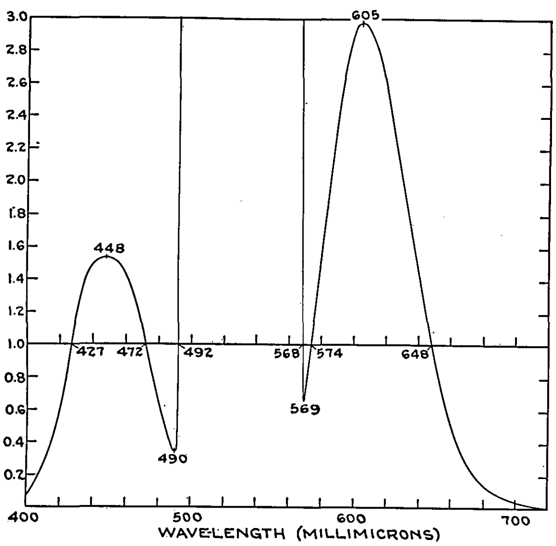

FIG. 4. Watts of complementary spectrum light necessary to "neutralize" one watt of indicated wave-length.

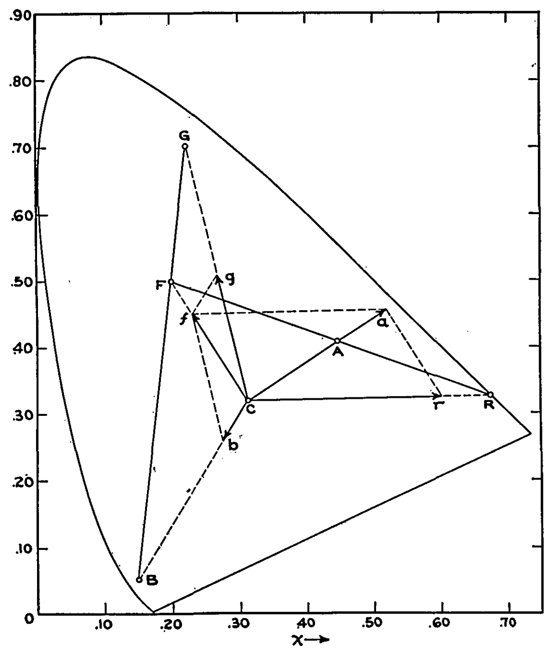

FIG. 5. Typical vector solution of a problem of additive synthesis of a desired color from three known primary chromaticities.

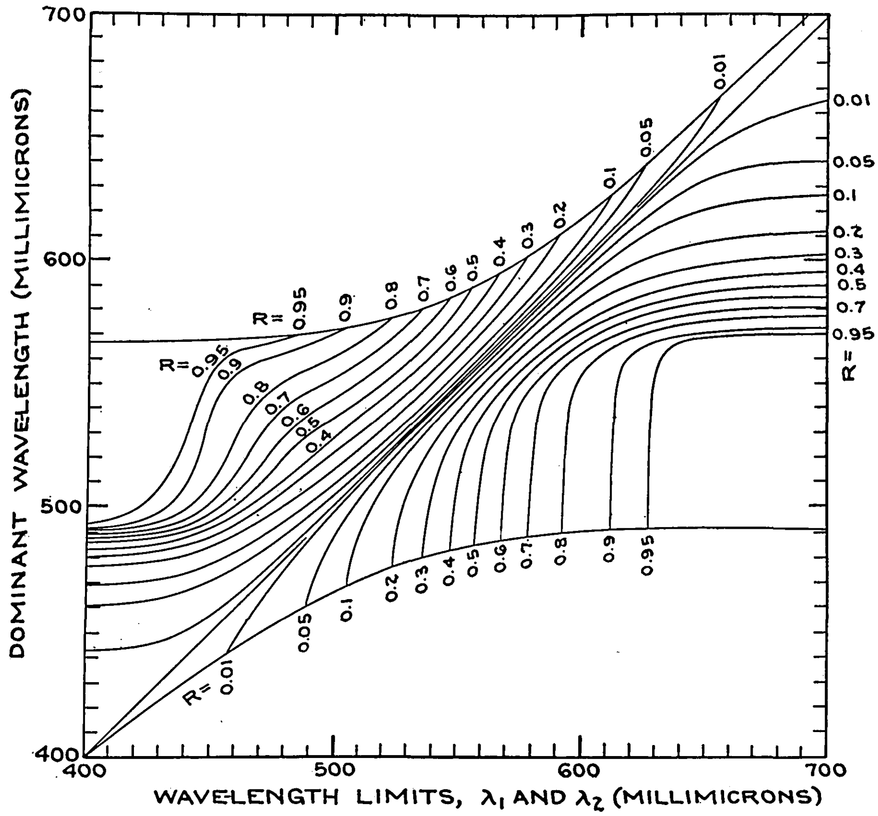

FIG. 6. Dominant wave-lengths of colors of maximum visual efficiency as functions of visual efficiency and of short or long wave-length limits of transmission or reflection band.

a, are now constructed from the point, C, through the points, R and A, respectively, and of such lengths that a is the resultant of f and r, as shown by the parallelogram construction. The length of r can be divided by the moment shown for the point, R, in Fig. 1, to give the ratio of the brightness of the primary, R, to the brightness of the primary, G. The length of the vector, a, can be divided by the moment shown for the point, A, in Fig. 1 to give the ratio of the brightness of the mixture of the three primaries to the brightness of the green component. This ratio furnishes a convenient check on the calculations.

D. Complementary Relations Between Colors Having Maximum Visual Efficiencies

The discussion thus far has been quite general in applicability, the properties of additive mixtures being independent of the spectroradiometric characteristics of the components of the mixtures. There is some interest in the complementary relationships between colors having maximum visual efficiencies.[10] From the definition of such colors it follows immediately that the brightnesses of two physically complementary[3] colors having maximum visual efficiencies must add up to the brightness of a perfect reflector in the same illumination. Very little else can be said about the complementary properties of these colors of maximum visual efficiency which has not been discussed above as general properties of all complementary colors. There is some interest, however, in the following data which have been plotted from Table II of the author's earlier paper on "Maximum Visual Efficiencies of Colored Materials."[10–15] Additional data, not contained in this table, were necessary for the completion of the curves in Fig. 6, and these data were computed according to form A of the above-mentioned article.

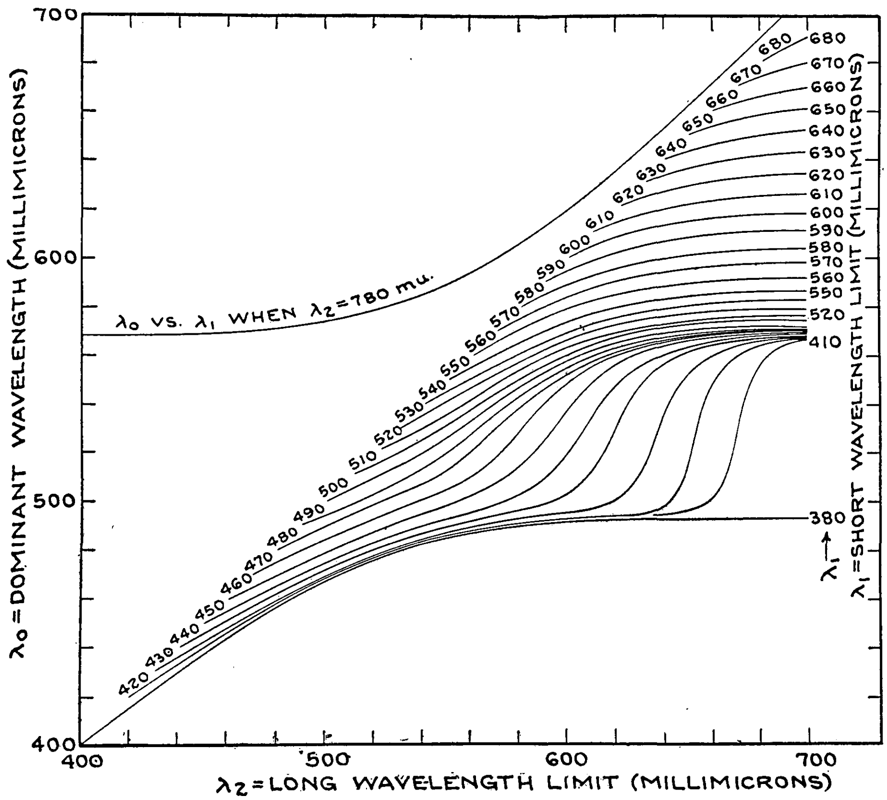

FIG. 7. Dominant wave-lengths of colors of maximum visual efficiency as functions of short and long wave-length limits of transmission or reflection band.

Figure 6 shows, for the indicated values of visual efficiency, the dominant wave-lengths of colors having complete absorption for all wave-lengths short of λ_1, complete transmission or reflection from λ_1 to λ_2, and complete absorption of all wave-lengths longer than λ_2. Conversely, Fig. 6 shows the values of λ_1 and λ_2 necessary for a color having the indicated dominant wave-length and visual efficiency. If the values of λ_1 and λ_2 necessary for a color having maximum visual efficiency and an assigned dominant wave-length and purity are to be determined, the corresponding value of the visual efficiency should be determined from Fig. 3 of the article on "Maximum Visual Efficiency of Colored Materials."[10] The values of λ_1 and λ_2 can then be determined from Fig. 6 of this report, as functions of the dominant wave-length and visual efficiency. Whenever the chromaticity to be investigated is on the purple side (toward the x axis) of the sharp ridges indicated in Figs. 3 and 4 of the article on "Maximum Visual Efficiency,"[10] the corresponding spectrophotometric curves are in class (b), typified by the solid line of Fig. 1b of the same article. For these colors, all wave-lengths of the visible spectrum are completely reflected or transmitted except those between λ_1 and λ_2, which are completely absorbed. In such cases, it is necessary to determine λ_1 and λ_2 for the physically complementary color. The dominant wave-length of this physically complementary color can be determined from the chromaticity of the desired color by the use of Table VIII and the chromaticity diagrams in the *Handbook of Colorimetry*,[3] or by computation and interpolation from D. B. Judd's[16] Table VI. Judd's table shows the slope (or the reciprocal of the slope) of the line from the point representing illuminant "C" to any sample point as a function of the dominant or complementary wave-length. The visual efficiency of the physical complementary color will be

$$R' = 1 - R, \qquad (3)$$

where R is the visual efficiency of the color transmitting or reflecting both extremes of the spectrum. The values of λ_1 and λ_2, the limits of the absorption band, can be determined from Fig. 6 as soon as the complementary wave-length and R' are known. Similarly, the converse problem of determining the chromaticity and visual efficiency of any color of maximum visual efficiency having absorption between the wave-lengths λ_1 and λ_2 can be determined as the complementary of the color which transmits only between these two limits.

Figure 7 has been constructed from Fig. 6 by holding λ_1 fixed and determining the value of λ_2 corresponding to many pairs of values of the dominant wave-length and visual efficiency. The values of the corresponding values of visual efficiency are omitted from Fig. 7 because these can be determined from Fig. 6. Fig. 7 serves primarily to determine the dominant wave-length of any color transmitting or reflecting completely all wave-lengths between λ_1 and λ_2 and absorbing all wave-lengths outside of these limits. As in the case of Fig. 6, it is necessary to determine the dominant wave-length of the complementary whenever any problem concerns maximum efficiency colors transmitting or reflecting both extremes of the visible spectrum. After the dominant wave-length corresponding to any given values of λ_1 and λ_2 has been determined from Fig. 7, the value of the visual efficiency can be determined from Fig. 6, and the purity from Fig. 3 of the article on "Maximum Visual Efficiency of Colored Materials."[10]

TABLE I. *For spectral hues.*

Dominant Wave-length (in mμ)	a	b	Dominant Wave-length (in mμ)	a	b	Dominant Wave-length (in mμ)	a	b
380	0.016	−0.984	500	1.702	0.702	590	1.338	0.338
390	0.0155	−0.9845	505	2.068	1.068	595	1.253	0.253
400	0.015	−0.985	510	2.374	1.374	600	1.178	0.178
410	0.015	−0.985	515	2.567	1.567	605	1.110	0.110
420	0.016	−0.984	520	2.654	1.654	610	1.057	0.057
430	0.022	−0.978	525	2.613	1.613	615	1.010	0.010
440	0.034	−0.966	530	2.546	1.546	620	0.974	−0.026
450	0.056	−0.944	535	2.473	1.473	625	0.946	−0.054
455	0.072	−0.928	540	2.383	1.383	630	0.924	−0.076
460	0.094	−0.906	545	2.290	1.290	635	0.904	−0.096
465	0.126	−0.874	550	2.193	1.193	640	0.889	−0.111
470	0.183	−0.817	555	2.083	1.083	645	0.876	−0.124
475	0.275	−0.725	560	1.973	0.973	650	0.867	−0.133
480	0.420	−0.580	563	1.908	0.908	660	0.855	−0.145
485	0.645	−0.355	565	1.864	0.864	670	0.848	−0.152
490	0.933	−0.067	567	1.821	0.821	680	0.844	−0.156
492	1.074	0.074	570	1.723	0.723	690	0.840	−0.160
493	1.151	0.151	575	1.645	0.645	700	0.839	−0.161
495	1.307	0.307	580	1.538	0.538	780	0.839	−0.161
497	1.461	0.461	585	1.434	0.434			

E. FORMULAS AND TABLES FOR INTERCONVERSION OF COLORIMETRIC AND EXCITATION PURITIES

It is occasionally necessary in these and other problems to convert data from excitation purity[3] to colorimetric purity,[16, 17] or to make the reverse transformation. This transformation can be made graphically by the use of Fig. 1, using the rule given at the end of the first paragraph of Section B. The fundamental algebraic formulas for this transformation are given on page 60 of the *Handbook of Colorimetry*[3] but the following simplified formulas and tables of parameters for illuminant "C" are very convenient. The formula for the conversion from excitation purity, p_e, to colorimetric purity, p_c, is:

$$p_c = a\cdot p_e/(1+b\cdot p_e). \tag{4}$$

The formula for the reverse transformation is:

$$p_e = p_c/(a-b\cdot p_c). \tag{5}$$

For spectrum hues the parameter, a, is given by

$$a = y_\lambda/y_w, \tag{6}$$

where y_λ is the y coordinate of the spectrum color having the dominant wave-length of the sample and where y_w is the y coordinate of the neutral point. For spectrum hues, the parameter, b, is:

$$b = a-1.0. \tag{7}$$

The values of the parameters, a and b, for spectrum hues and for the I.C.I. illuminant "C" taken as the neutral are given by Table I.

TABLE II. *For purples.*

Complementary Wave-length (in mμ)	a	b
492	−1.808	−0.124
493	−1.601	−0.210
495	−1.402	−0.330
497	−1.295	−0.408
500	−1.169	−0.483
505	−1.069	−0.552
510	−1.030	−0.596
515	−1.027	−0.625
520	−1.040	−0.649
525	−1.082	−0.669
530	−1.127	−0.684
535	−1.178	−0.703
540	−1.248	−0.723
545	−1.329	−0.748
550	−1.427	−0.777
555	−1.571	−0.816
560	−1.758	−0.866
563	−1.908	−0.908
565	−2.033	−0.942
567	−2.183	−0.985

For purples:

$$a=(y_\lambda/y_w)\cdot(y_{\lambda'}-y_w)/(y_\lambda-y_w), \quad (8)$$

where y_λ and y_w are defined as above, and $y_{\lambda'}$ is the y coordinate of the intersection of the line drawn between the extremities of the spectrum locus with the straight line through the sample and the neutral points. The parameter, b, for the purples is given by

$$b=(y_{\lambda'}-y_w)/y_w. \quad (9)$$

Table II gives the values of the parameters, a and b, for purple hues and for the I.C.I. illuminant "C," as functions of the complementary wave-lengths.

F. Linear Spectrophotometric Curves

A brief mention of a subject different from, but not wholly unrelated to, the topic of complementary colors will probably not be too much out of place here. Many spectrophotometric curves of naturally occurring substances exhibit an almost linear relationship between reflectance (or transmittance) and wave-length. This characterizes most of the spectrophotometric curves for human hair,[18] in addition to many of those for wood, paper pulp, and untinted papers, textiles, and minerals. It has been noticed frequently that such samples have dominant wave-lengths in the neighborhood of 575 mμ to 580 mμ. If the reflectance is assumed to bear a strictly linear relationship to the wave-length, within the visible spectrum, then it can be represented by the formula:

$$R_\lambda=c+d\cdot(\lambda-400\text{ m}\mu)/(300\text{ m}\mu) \quad (10)$$

between the wave-lengths 400 mμ and 700 mμ. In this formula, the constant, c, is the value of the reflectance, R_λ, at 400 mμ and d is the difference between the values of the reflectances at 700 mμ and 400 mμ. Computation for the case of $c=0$ and $d=1$, with illuminant "C," results in the tristimulus values:

$$X=0.5400,\ Y=0.5243,\ Z=0.2172,$$

corresponding to $x=0.4220$, $y=0.4095$, dominant wave-length 580.1 mμ and excitation purity 54.9 percent.

In general, it can be shown that:

$$X=0.9804c+0.5400d,$$
$$Y=c+0.5243d,$$
$$Z=1.1812c+0.2172d,$$
$$x=\frac{0.9804c+0.5400c}{3.1616c+1.2815d},$$
$$y=\frac{c+0.5243d}{3.1616c+1.2815d}.$$

Dominant wave-length = 580.1 mμ when d is greater than zero.

Dominant wave-length = 480.1 mμ when d is less than zero. In the extreme case for which $c=1.0$ and $d=-1.0$, the dominant wave-length is of course 480.1 mμ, the excitation purity is 34.8 percent, and the brightness (Y) relative to magnesium oxide (a practically perfect diffuse reflector) in the same illumination is 47.6 percent.

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