SUPPLEMENT TO "UNCOVERING THE DISTRIBUTION OF MOTORISTS' PREFERENCES FOR TRAVEL TIME AND RELIABILITY"

BY KENNETH A. SMALL, CLIFFORD WINSTON, AND JIA YAN

STATED PREFERENCE SURVEY QUESTIONNAIRE

EIGHT HYPOTHETICAL COMMUTING SCENARIOS were constructed for respondents who travel on SR91. Respondents who indicated that their actual commute was less (more) than 45 minutes were given scenarios that involved trips ranging from 20–40 (50–70) minutes. An illustrative scenario follows:

SCENARIO 1

Free Lanes	Express Lanes
Usual Travel Time: 25 minutes	Usual Travel Time: 15 minutes
Toll: None	Toll: $3.75
Frequency of Unexpected Delays of 10 minutes or more: 1 day in 5	Frequency of Unexpected Delays of 10 minutes or more: 1 day in 20
Your Choice (check one):	
Free Lanes □	Toll Lanes □

DERIVATION OF LIKELIHOOD FUNCTION

Let ψ_1 represent the vector of all nonrandom parameters. Let Θ_i^k represent the value of the random parameters (except η_i^k) for individual i in subsample k; the parameters $\Theta_i \equiv (\Theta_i^{\mathrm{BR}}, \Theta_i^{\mathrm{BS}}, \Theta_i^{\mathrm{C}})$ have a joint distribution with density function $f(\Theta_i|\psi_2)$. Define the choice variable as $y_{it}^k = 1$ if the express lanes are chosen and 0 otherwise. The likelihood function of our model is then

$$L(\psi_1, \psi_2) = \prod_i \int_{\Theta_i} \prod_{k,t} P(y_{it}^k|\psi_1, \Theta_i) f(\Theta_i|\psi_2)\, d\Theta_i,$$

where i runs through all individuals; k runs through the one or two samples in which that individual appears; t runs through all responses for that individual and subsample (up to eight when $k = SP$ and just one otherwise); and $P(y_{it}^k|\psi_1, \Theta_i)$, the individual's conditional likelihood, takes the binary logit form. With integration replaced by Monte Carlo simulation, we obtain the simulated likelihood function

$$SL(\psi_1, \psi_2) = \prod_i \frac{1}{R} \sum_{r=1}^{R} \prod_{k,t} P(y_{it}^k|\psi_1, \Theta_i^r),$$

where Θ_i^r is a random draw from $f(\Theta_i|\psi_2)$.

Lee (1992) and Hajivassiliou and Ruud (1994) show that under regularity conditions, the parameter estimates obtained by maximizing the simulated likelihood function are consistent when the number of replications rises at any rate with the sample size, and are asymptotically normal and equivalent to maximum likelihood estimates when the number of replications rises faster than the square root of the sample size.

DESCRIPTIVE STATISTICS

Table A.I shows descriptive statistics, including some variables omitted from the corresponding table in the paper. The distributions of the RP sample's commuting times and route share are close to the ones in the Cal Poly data and in other survey data collected by University of California at Irvine in 1998 (Lam and Small (2001)). Our sample's median income (approximately $46,250) is higher than the average incomes in the two counties where our respondents lived ($36,189 and $39,729 in 1995, as estimated by the Population Research Unit of the California Department of Finance).

We estimate the average wage rate to be about $23 per hour, as follows. Data from the U.S. Bureau of Labor Statistics (BLS) for the year 2000 record the mean hourly wage rate by occupation for residents of Riverside and San Bernardino Counties. We combine the BLS occupational categories into six groups that match our survey question about occupation, then assign to each person in our sample the average BLS wage rate for that person's occupational group. We then add 10% to reflect the higher wages likely to be attracting these people to jobs that are relatively far away.

In the Brookings RP sample, which contains information for multiple days, choices do not vary much from day to day: 87% of respondents made the same choice every day during the survey week and nearly all the others varied on only one day. Nearly half of the Brookings RP respondents do not have a transponder and thus have chosen in advance not to use the express lanes on a given day. Among the 41 Brookings RP respondents who have a transponder, 11 made different choices on different days; this amounts to 27% of those with a transponder but only 13% from the entire Brookings RP sample. The latter statistic is relevant for judging constancy of choice because we model lane choice unconditionally on transponder (hence not getting a transponder is a natural concomitant of a persistent decision to take the free lanes). Six of those eleven respondents who varied their choice made trips on all five weekdays; four of the six chose the free lanes on all but one day, and one chose the express lanes on all but one day, leaving only one who made a 3:2 split.

The Cal Poly sample is partly choice-based, some of it being obtained from license-plate observations on SR91. However, its express-lane share is so similar to the Brookings sample, which is random, that correcting for choice-based sampling makes virtually no difference to the estimation results. Time-of-day

TABLE A.I

EXTENDED DESCRIPTIVE STATISTICS

	Value or Fraction of Sample		
	Cal Poly RP	Brookings RP	Brookings SP
Route share			
91X	0.26	0.25	
91F	0.74	0.75	
One-week trip pattern			
Never use 91X		0.68	
Sometimes use 91X		0.13	
Always use 91X		0.19	
Percent of trips by time period			
4–5 am	0.11	0.15	
5–6 am	0.22	0.13	
6–7 am	0.23	0.26	
7–8 am	0.20	0.21	
8–9 am	0.14	0.15	
9–10 am	0.10	0.10	
Age of respondents			
<30	0.11	0.12	0.10
30–50	0.62	0.62	0.64
>50	0.27	0.26	0.26
Sex of respondents			
Male	0.68	0.63	0.63
Female	0.32	0.37	0.37
Household income ($)			
$<40{,}000$	0.14	0.23	0.24
40,000–60,000	0.24	0.60	0.59
60,000–100,000	0.40	0.15	0.13
$>100{,}000$	0.22	0.02	0.04
Flexible arrival time			
Yes	0.40	0.55	0.48
No	0.60	0.45	0.52
Trip distance (mi)			
Mean	34.23	44.26	42.66
Standard deviation	14.19	26.90	27.38
Number of people in household			
Mean	3.53	2.91	3.44
Standard deviation	1.51	1.63	1.55
Number of respondents	438	84	81
Number of observations	438	377	633

patterns in the Cal Poly data are also similar to those in the Brookings RP sample, as are most other observables including age and sex.

CONSTRUCTION OF RP VARIABLES ON TRAVEL-TIME SAVINGS AND RELIABILITY

Travel times on the free lanes (91F) were collected on 11 days: first by the California Department of Transportation on October 28, 1999 (six weeks before the first wave of our survey) and then by us on July 10–14 and September 18–22, 2000 (which are the time periods covered by two later waves of our survey).

Data were collected from 4:00 to 10:00 am on each day, for a total of 210 observations y_i of the travel-time savings from using the express lanes at times of day denoted by x_i, $i = 1, \ldots, 210$. Our objective is to estimate the mean and quantiles of the distribution (across days) of travel time y conditional on time of day x. To do so, we use nonparametric methods of the class of locally weighted regressions: specifically, the form known as *local linear fit*. For each value of x on a prechosen grid, it estimates a linear function $y_i = a + b(x_i - x) + \varepsilon_i$ in the region $[x - h, x + h]$, where h is a bandwidth chosen by the investigator. It does so by minimizing a loss function $g(\cdot)$ of the deviations between observed and predicted y.

Denote the pth quantile value of y, given x, by $q_p(x)$. Following Koenker and Bassett (1978), we estimate it with the *local linear quantile regression*

$$\text{(A1)} \qquad \hat{q}_p(x) = \arg\min_a \sum_{i=1}^{n} g_p[y_i - a - b(x_i - x)] \cdot K\left[\frac{x_i - x}{h}\right],$$

where n is the total number of observations and $g_p[\cdot]$ is the loss function, which is asymmetric except when $p = 0.5$,

$$\text{(A2)} \qquad g_p(t) = \big[|t| + (2p - 1)t\big]/2,$$

in which case equation (A1) defines. Yu and Jones (1998) show that the estimated percentile values converge in probability to the actual percentile values as the number of observations n grows larger, provided the bandwidth h is allowed to shrink to zero in such a way that $nh \to \infty$. In the case of the median ($p = 0.5$), this is a least-absolute-deviation loss function and, therefore, the estimator can be thought of as a nonparametric least-absolute-deviation estimator.

Similarly, denoting the mean of y given x by $m(x)$, its estimate is given by (A1), but with subscript p replaced by m and with loss function $g_m(t) = t^2$.

The choice of kernel function has no significant effect on our results. We use the *biweight* kernel function

$$\text{(A3)} \qquad K(u) = \begin{cases} 15/16(1 - u^2)^2, & |u| \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

The choice of bandwidth, however, is important. We first tried the bandwidth proposed by Silverman (1985),

$$\text{(A4)} \qquad h = 0.9n^{-0.5}\min\{\operatorname{std} x, (\operatorname{iqd} x/1.34)\},$$

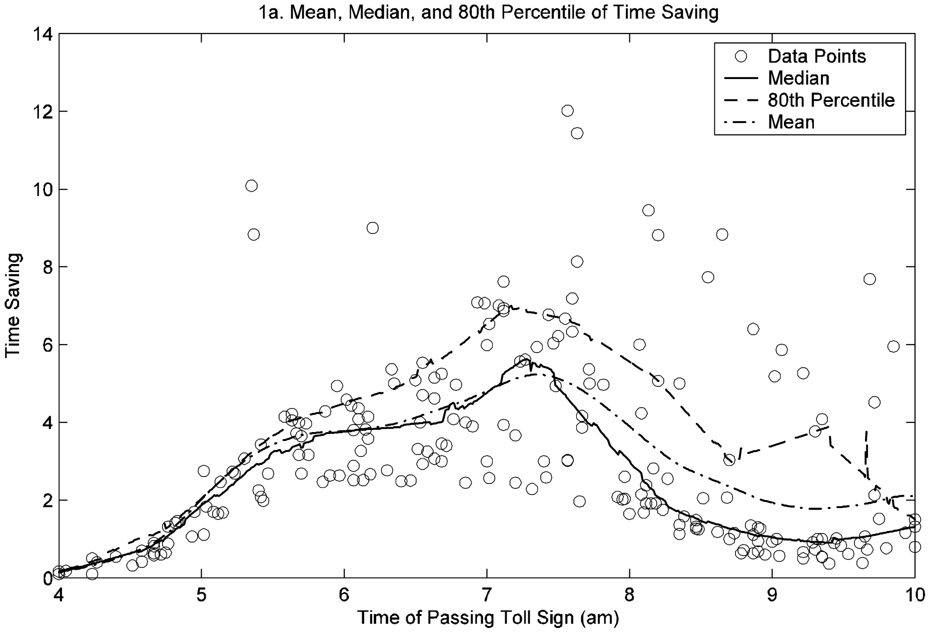
where std x and iqd x are the standard deviation and interquartile difference of the empirical distribution of x. This bandwidth turns out to be about 0.5 hour for our data. However, there is rather extreme variation in our data at particular times of day, especially around 6:00 am, due to accidents that occurred on two days around that time. While these accidents are part of the genuine history and we want to include their effects, they produce an unlikely time pattern for reliability when used with the bandwidth defined by (A4)—namely, one with a sharp but narrow peak in the higher percentiles around 5:30 am, followed by the expected broader peak centered near 7:30 am. We therefore increased the bandwidth to 0.8 hour to smooth out this first peak.

We also estimated the standard deviation as the square root of the estimated variance of time saving, obtained by a similar nonparametric regression of the squared residuals $[y_i - \hat{m}(x_i)]^2$ on time of day.

Results are shown in Figures A1 and A2. Figure A1(a) shows the raw field observations of travel-time savings with the nonparametric estimates of mean, median, and 80th percentile superimposed. Median time savings reach a peak of 5.6 minutes around 7:15 am.

The pointwise confidence intervals of the median time savings, shown in Figure A1(b), and of our preferred reliability measure, shown in Figure A2(b), are constructed using the paired bootstrap (Hardle (1990), Buchinsky (1998)). We randomly sample pairs (y_i, x_i) with replacement to form the bootstrap sample with the same size as the original data and compute the local linear quantile estimator for both the median and the 80th percentile. The procedure is repeated 100 times. The empirical distributions of the median time savings and unreliability (80th–50th percentile) are used to construct the upper and lower bounds of the 90% confidence intervals for the two estimates. The 90% confidence band is indicated by the lines labeled "CI-UP" and "CI-LO."

Figure A2(a) shows the same raw observations after subtracting our nonparametric estimate of median time savings by time of day. An interesting pattern emerges. Up to 7:30 am, the scatter of points is reasonably symmetric around zero with the exception of three data points, but after 7:30 am the scatter becomes highly asymmetric, with dispersion in the positive range (the upper half of the figure) apparently continuing to increase until well after 8:00 am while dispersion in the negative range decreases. This feature is reflected in the three measures of dispersion, or unreliability, that are also shown in the figure: the standard deviation and the 80th–50th and 90th–50th percentile differences. The standard deviation peaks at roughly 7:45 am, the other two peak considerably later. The reason for these differences is that traffic in the later part of the peak is affected by incidents that occur either then or earlier. This mostly



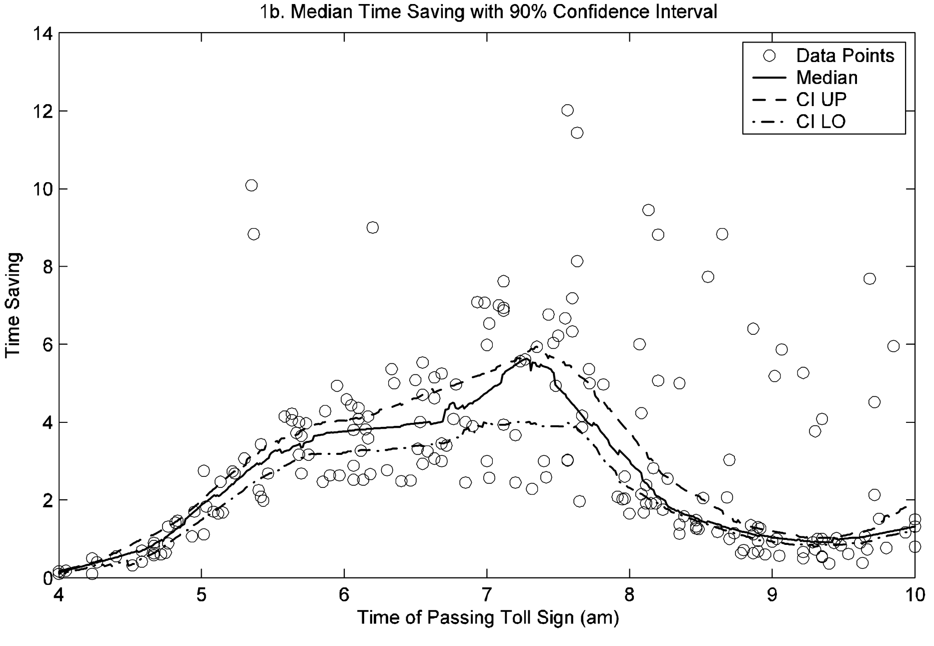
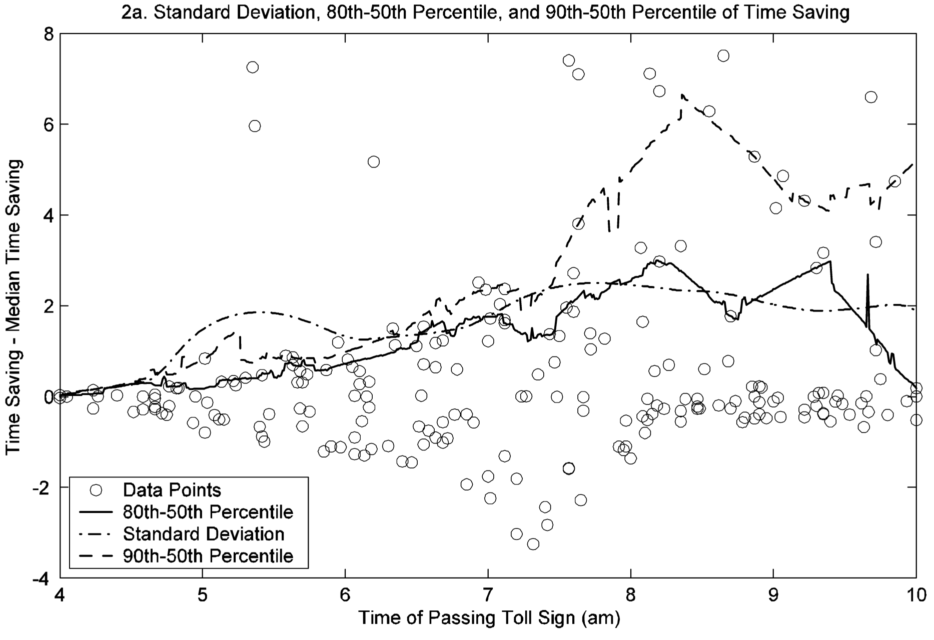


FIGURE A1.—Time saving.



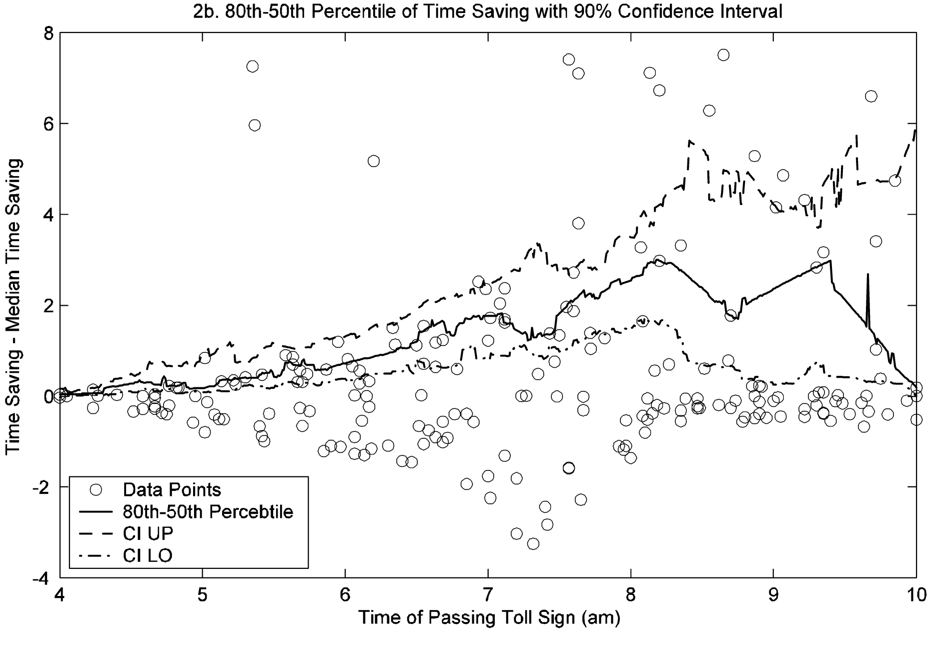


FIGURE A2.—Dispersion of time saving.

affects the upper tails of the distribution of travel-time savings and so is most apparent in the percentile differences. The standard deviation, by contrast, is higher early in the rush hour because of days with little congestion—showing up as negative points in Figure A2(a). Note that the confidence bands in Figure A2(b) suggest that unreliability as we choose to measure it most likely continues to rise until after 8:00 am, generating a pattern that is different from that of median travel time.

In our estimations, we obtained the best statistical fits (in terms of log likelihood) using the 80th–50th percentile difference. The 90th–50th percentile difference fits almost as well and resulted in similar coefficient estimates. The 75th–50th percentile difference, not shown in Figures A1 and A2, fits noticeably less well and gave statistically insignificant results for the reliability measure. The same was true for the standard deviation.

OTHER INDEPENDENT VARIABLES

The "flexible arrival time" variable was designed to be similar to a variable found in Small (1982) to be important in explaining the cost of early or late arrivals. The question, identical in the Brookings and Cal Poly RP surveys, was, "Could you arrive late at work on that day without it having an impact on your job?"

MODELS ESTIMATES ON RP-ONLY AND ON SP-ONLY DATA

Table A.II shows (in columns 2 and 3) our "best" RP-only model and SP-only model, along with the joint RP/SP model reported in the paper (column 4). Also shown (in column 1) is a joint RP/SP model estimated without randomizing the time and unreliability coefficients. The coefficients of this model may be compared to those of the RP-only model in column 2 because they have the same error structure and thus the coefficients are of comparable scale. All coefficients that are even close to statistical significance are of similar magnitudes in these two columns.

Many of the coefficients of the SP-only model of column 3 are imprecisely estimated; the others align closely with the comparable joint SP/RP model of column 4. This may not appear to be the case with the standard deviation of the constant (σ_ξ) among the SP variables, but actually they are similar because in the joint model the comparable standard deviation is that of the combined terms $\rho\nu^{\mathrm{BR}} + \xi^{\mathrm{BS}}$, which is $[(\rho \cdot 1)^2 + \sigma_\xi^2]^{1/2} = (3.2882^2 + 0.4800^2)^{1/2} = 3.3230$, which is within 2 standard deviations of the estimate of σ_ξ^2 in column 3.

MODELS TESTING SENSITIVITY TO IDENTIFYING ASSUMPTIONS

In our base model, any taste differences across different times of day are assumed to be captured by interacting the alternative-specific dummy variable

TABLE A.II

COMPARISON OF MODELS ESTIMATED ON RP, SP, OR COMBINED DATA SETS

Dependent Variable: 1 if Toll Lanes Chosen, 0 Otherwise				
	Coefficient (Standard Error)			
Independent Variable	Joint RP/SP (Fixed)	RP Only (Fixed)	SP Only (Random)	Joint RP/SP (Random)
RP variables				
Constant				
BR subsample ($\bar{\theta}^{BR}$)	−0.0437	−0.6046		0.1489
	(1.3267)	(0.9854)		(0.8931)
C subsample ($\bar{\theta}^{C}$)	−1.8146	−2.0108		−1.6349
	(0.8665)	(0.8537)		(1.1040)
Cost ($)	−1.7539	−1.5533		−1.8705
	(0.6761)	(0.6245)		(0.5812)
Cost × dummy for medium income ($60,000–100,000)	0.5470	0.5375		0.5438
	(0.2420)	(0.1990)		(0.2549)
Cost × dummy for high income (>$100,000)	1.0765	1.0507		1.1992
	(0.3997)	(0.3488)		(0.3849)
Median travel time (min) × trip distance (in units of 10 mi)	−0.3695	−0.3057		−0.4088
	(0.1388)	(0.1296)		(0.1536)
Median travel time × (trip distance squared)	0.0618	0.0484		0.0695
	(0.0270)	(0.0240)		(0.0276)
Median travel time × (trip distance cubed)	−0.0026	−0.0020		−0.0029
	(0.0011)	(0.0009)		(0.0012)
Unreliability of travel time (min)	−0.7567	−0.7063		−0.5778
	(0.3027)	(0.2823)		(0.2435)
SP variables				
Constant ($\bar{\theta}^{BS}$)	−1.7845		−0.8180	−1.6107
	(1.1025)		(1.5111)	(0.8943)
Standard dev. of constant (σ_ξ)	0.1606		4.4873	0.4800
	(0.6603)		(0.8610)	(0.6305)
Cost	−0.9575		−1.3835	−1.0008
	(0.2842)		(0.3465)	(0.2849)
Cost × dummy for high income (>$100,000)	0.3934		0.4091	0.2842
	(0.7427)		(0.9382)	(0.9714)
Cost × dummy for medium income ($60,000–100,000)	−0.2827		−0.2951	−0.2317
	(0.4014)		(0.6695)	(0.5407)
Travel time (min) × long-commute dummy (>45 min)	−0.1908		−0.2450	−0.1965
	(0.0392)		(0.0638)	(0.0522)
Travel time ×(1 − long-commute dummy)	−0.2212		−0.2919	−0.2146
	(0.0477)		(0.0727)	(0.0618)
Unreliability of travel time (probability)	−5.4733		−7.5635	−5.6292
	(2.1784)		(2.0451)	(2.3819)

Continues

for express lane use with individuals' characteristics, especially characteristics already suspected to affect travelers' time-of-day choice. In this section, we test the sensitivity of our results to this assumption.

TABLE A.II—*Continued*

Dependent Variable: 1 if Toll Lanes Chosen, 0 Otherwise				
	Coefficient (Standard Error)			
Independent Variable	Joint RP/SP (Fixed)	RP Only (Fixed)	SP Only (Random)	Joint RP/SP (Random)
Variables pooled in joint RP/SP model				
Female dummy	1.2260	1.3003	0.4125	1.3267
	(0.4475)	(0.3776)	(1.4493)	(0.6292)
Age 30–50 dummy	1.1980	1.3428	−0.1976	1.2362
	(0.5467)	(0.4544)	(1.0270)	(0.5121)
Flexible arrival-time dummy	0.4687	0.2699	2.3446	0.5903
	(0.4920)	(0.3597)	(1.0611)	(0.6994)
Household size (number of people)	−0.4989	−0.4395	−0.9089	−0.5497
	(0.2123)	(0.1968)	(0.4493)	(0.2248)
Standard dev. of coeff.'s of travel time (part of Ω)			0.2188	0.1658
			(0.0643)	(0.0457)
Ratio of std. dev. to mean for coeff.'s of unreliability (σ_ω)			1.1009	1.0560
			(0.3449)	(0.2754)
Other parameters				
Scale parameter				
C subsample (μ^C)	0.4228	0.4495		0.4118
	(0.1544)	(0.1909)		(0.1688)
SP subsample (μ^{BS})	1.0634			1.3368
	(0.2185)			(0.3741)
Correlation parameter (ρ)	3.4555			3.2882
	(0.7153)			(0.8320)
Summary statistics				
Number of observations	522	522	633	1,155
Number of persons	522	522	81	548
Number of replications (R)	4,500	4,500	4,500	4,500
Log-likelihood	−510.77	−267.91	−241.19	−501.57
Pseudo R^2	0.3589	0.3507	0.4503	0.3704

First, we take advantage of the low correlation built by design into the independent variables in the SP subsample. About two-thirds of the SP respondents also answered RP questions so that we know their time of day of travel. Therefore we use this subsample to test whether adding time-of-day dummy variables, one for each hour of the day (with 7:00–8:00 am omitted as the base hour), improves the fit and changes results of interest.

Table A.III shows the resulting estimates. The model in the first column has the same specification as the "SP Only" model in Table A.II,[1] but is estimated

[1]It is parameterized slightly differently by estimating the standard deviation of the coefficient of reliability directly, rather than its ratio to the mean coefficient. Since in SP-only there is only one such coefficient, the two parameterizations contain the same information.

TABLE A.III

SP PARAMETER ESTIMATES WITH AND WITHOUT TIME-OF-DAY DUMMY VARIABLES ON SUBSET OF SP DATA[a]

	SP Model Without Time Dummies	SP Model with 5 Time Dummies
Constant ($\bar{\theta}^{BS}$)	−3.3976	−7.5484
	(1.0696)	(2.7081)
Standard deviation of constant (σ_ξ)	5.0741	4.4774
	(1.1871)	(0.9892)
Cost	−1.3736	−1.4106
	(0.3758)	(0.4176)
Cost × dummy for high household income	1.1043	1.5854
(>\$100,000)	(0.9037)	(1.3433)
Cost × dummy for medium household	−0.2547	−0.1013
income (\$60,000–100,000)	(0.5837)	(0.7527)
Travel time (min) × long-commute	−0.2548	−0.2390
dummy (>45 min)	(0.0646)	(0.0772)
Travel time ×(1 − long-commute dummy)	−0.3439	−0.3816
	(0.0959)	(0.0908)
Unreliability of travel time (probability)	−6.8380	−6.8387
	(0.7550)	(1.0872)
Female dummy	1.4710	2.1094
	(1.1129)	(1.3880)
Age 30–50 dummy	−1.1926	−0.3462
	(1.2027)	(1.4856)
Flexible arrival-time dummy	3.1764	2.7374
	(1.0885)	(1.1891)
Household size (number of people)	−0.6959	−0.7530
	(0.5252)	(0.5139)
Time dummy		
Between 4 and 5 am		3.8750
		(3.2170)
Between 5 and 6 am		3.5643
		(2.3979)
Between 6 and 7 am		5.2648
		(2.5304)
Between 7 and 8 am		
Between 8 and 9 am		7.8265
		(2.5848)
Between 9 and 10 am		4.8480
		(2.5887)
Standard deviation of coefficients of travel	0.2601	0.2939
time (part of Ω)	(0.0785)	(0.0883)
Standard deviation of coefficient of	1.5310	1.4474
unreliability (part of Ω)	(0.4824)	(0.4908)
Summary statistics		
Number of observations	433	433
Number of persons	55	55
Log-likelihood	−152.98	−149.11

[a]Numbers in parentheses are robust standard errors. Monte Carlo integration performed using 4,500 random draws.

on only the subsample of 55 people who answered both SP and RP questions. The second column shows the same model with the addition of five time-of-day dummies. The results show that after controlling for individuals' characteristics, people traveling between 7:00 and 8:00 am are different from those traveling in other periods. There seems to be little difference among the five time periods other than 7:00–8:00, except possibly 8:00–9:00. Likelihood ratio tests show that the five time dummies taken together do not significantly improve the fit, with $2\Delta L = 6.52$ well short of the chi-square critical value (at 5% significance level) for 5 degrees of freedom. (A likelihood ratio test for including just the single dummy for 8:00–9:00, compared to the model of column 1, yields a value almost exactly equal to the critical value for 1 degree of freedom, but given that we had no a priori reason to single out that period, such a test is biased toward rejecting the null.)

Next, we use our full data set to reestimate our joint RP/SP base model, but adding a single time dummy for 7:00–8:00 am. This variable is chosen based on the finding from the SP subsample that this time period is the most distinct (see Table A.III, column 2). Because one-third of the SP sample (26 people) provide no information about time schedule, we divide the SP subsample into two parts, BST and BSNT, each with its own mean constant $\bar{\theta}^k$ in (3) and its own mean random coefficient vector $\bar{\beta}^k$ in (4). The time dummy is included in the variable vector W_i^k in (3) for those samples k for which time of day of travel is known, namely $k =$ C, BR, BST. In this way the BNST subsample does not bias the coefficient of the time dummy, although it also does not contribute to identifying it.

Results from this model are presented in Table A.IV. The coefficient of the peak dummy is not statistically significant but has a substantial magnitude. As shown in Table III of the paper, middle column, the estimated RP values of time and reliability are moderately higher in this model than in the base model. Through a separate experiment, not shown, we ascertained that this is due to the inclusion of the time dummy and not to allowing for separate constants and mean random parameters for the two halves of the SP subsample.

MODELS TESTING SENSITIVITY TO CONSTRAINTS ACROSS RP AND SP COEFFICIENTS

As noted in the text, we estimated models with both more and fewer constraints across the RP and SP coefficients. Table A.V shows the results. The "constrained model" forces each of the three cost coefficients to be identical between the RP and SP subsamples. These three constraints are not rejected by a likelihood ratio test ($2\Delta L = 3.90$, 3 d.f.), but as noted in the paper we nonetheless impose that constraint in our base model since the RP cost coefficient is a particularly policy-relevant parameter and we do not want to risk contaminating it with SP values if they are really different.

The two "flexible models" keep the cost coefficients unconstrained, like the base model. In addition, flexible model 1 allows two variables (namely female

TABLE A.IV

PARAMETER ESTIMATES OF JOINT RP/SP MODEL WITH DUMMY FOR TRAVEL DURING PEAK HOUR (7–8 AM)[a]

Independent Variable	Joint RP/SP (Table II of Paper)	Joint RP/SP with Time Dummy
RP variables		
Constant		
Brookings subsample ($\bar{\theta}^{BR}$)	0.1489	−0.3666
	(0.8931)	(0.9579)
Cal Poly subsample ($\bar{\theta}^{C}$)	−1.6349	−2.0826
	(1.1040)	(1.1526)
Cost ($)	−1.8705	−1.9793
	(0.5812)	(0.7088)
Cost × dummy for medium household income	0.5438	0.5819
($60,000–100,000)	(0.2549)	(0.4103)
Cost × dummy for high household income	1.1992	1.2039
(>$100,000)	(0.3849)	(0.5800)
Median travel time (min) × trip distance	−0.4088	−0.5433
(in units of 10 mi)	(0.1536)	(0.1858)
Median travel time × (trip distance squared)	0.0695	0.0908
	(0.0276)	(0.0358)
Median travel time × (trip distance cubed)	−0.0029	−0.0037
	(0.0012)	(0.0015)
Unreliability of travel time (min)	−0.5778	−0.7629
	(0.2435)	(0.4118)
SP variables		
Constant		
$\bar{\theta}^{BS}$ or $\bar{\theta}^{BST}$	−1.6107	−2.3024
	(0.8943)	(1.2070)
$\bar{\theta}^{BSNT}$		−0.6206
		(0.6780)
Standard deviation of SP constants (σ_{ξ})	0.4800	2.7149
	(0.6305)	(1.0897)
$\text{Cost}^{\text{BS or BST}}$	−1.0008	−0.9621
	(0.2849)	(0.6390)
$\text{Cost}^{\text{BSNT}}$		−1.2230
		(0.5689)
Cost × dummy for medium household income	−0.2317	−0.1171
($60,000–100,000)$^{\text{BS or BST}}$	(0.5407)	(0.7072)
Cost × dummy for medium household income		0.2069
($60,000–100,000)$^{\text{BSNT}}$		(1.0601)
Cost × dummy for high household income	0.2842	0.7294
(>$100,000)$^{\text{BS or BST}}$	(0.9714)	(0.6209)
Cost × dummy for high household income		−0.2121
(>$100,000)$^{\text{BSNT}}$		(0.7158)
Travel time (min) × long-commute dummy	−0.1965	−0.1826
(>45 min)$^{\text{BS or BST}}$	(0.0522)	(0.0681)
Travel time (min) × long-commute dummy		−0.2576
(>45 min)$^{\text{BSNT}}$		(0.1117)

Continues

TABLE A.IV—*Continued*

Independent Variable	Joint RP/SP (Table II of Paper)	Joint RP/SP with Time Dummy
Travel time $\times(1 - \text{long-commute dummy})^{\text{BS or BST}}$	−0.2146	−0.2414
	(0.0618)	(0.0793)
Travel time $\times(1 - \text{long-commute dummy})^{\text{BSNT}}$		−0.1813
		(0.1010)
Unreliability of travel time $(\text{probability})^{\text{BS or BST}}$	−5.6292	−5.5637
	(2.3819)	(2.1783)
Unreliability of travel time $(\text{probability})^{\text{BSNT}}$		−6.4284
		(2.5097)
Variables pooled in joint RP/SP model		
Female dummy	1.3267	1.3631
	(0.6292)	(1.0750)
Age 30–50 dummy	1.2362	1.3661
	(0.5121)	(0.8699)
Flexible arrival-time dummy	0.5903	0.6255
	(0.6994)	(0.9079)
Household size (number of people)	−0.5497	−0.5782
	(0.2248)	(0.1869)
Peak dummy (time of passing toll sign is between 7 and 8 am)		−1.6400
		(1.2855)
Standard deviation of coefficients of travel time	0.1658	0.1695
(part of Ω)	(0.0457)	(0.0663)
Ratio of standard deviation to the mean for	1.0560	0.9983
coefficients of unreliability (σ_ω)	(0.2754)	(0.4253)
Other parameters		
Scale parameter		
Cal Poly sample (μ^{C})	0.4118	0.4000
	(0.1688)	(0.2517)
SP sample ($\mu^{\text{BS or BST}}$)	1.3368	1.3798
	(0.3741)	(0.5438)
SP sample (μ^{BSNT})		1.1715
		(0.5026)
Correlation parameter: RP and SP (ρ)	3.2882	2.3510
	(0.8320)	(1.0682)
Summary statistics		
Number of observations	1,155	1,155
Number of persons	548	548
Log-likelihood	−501.57	−496.71

[a]Numbers in parentheses are robust standard errors. Monte Carlo integration performed using 4,500 random draws.

and household size), of the four that are pooled in the base model, to differ between RP and SP. Flexible model 2 allows all four of those variables to differ. Table A.VI shows some implications of these models for values of time and reliability.

TABLE A.V

ESTIMATES OF MODELS WITH DIFFERENT CONSTRAINTS IN COMBINING RP AND SP DATA[a]

Independent Variable	Constrained Model	Flexible Model 1	Flexible Model 2
RP variables			
Constant			
Brookings subsample ($\bar{\theta}^{BR}$)	−0.2824	0.0489	−0.2490
	(1.1698)	(1.1855)	(1.1921)
Cal Poly subsample ($\bar{\theta}^{C}$)	−1.7043	−1.4813	−1.5501
	(0.9153)	(0.9314)	(0.9530)
Cost ($)		−1.6802	−1.5368
		(0.6466)	(0.6819)
Cost × dummy for medium household income ($60,000–100,000)		0.5037	0.4610
		(0.2548)	(0.2649)
Cost × dummy for high household income (>$100,000)		1.0225	0.9438
		(0.4086)	(0.4246)
Median travel time (min) × trip distance (in units of 10 mi)	−0.3670	−0.3672	−0.3429
	(0.1351)	(0.1538)	(0.1550)
Median travel time × (trip distance squared)	0.0620	0.0624	0.0575
	(0.0246)	(0.0301)	(0.0313)
Median travel time × (trip distance cubed)	−0.0026	−0.0026	−0.0024
	(0.0010)	(0.0013)	(0.0013)
Unreliability of travel time (min)	−0.5756	−0.5367	−0.4842
	(0.2403)	(0.2764)	(0.2871)
Female dummy			1.2300
			(0.5963)
Age 30–50 dummy		1.2792	1.2064
		(0.7316)	(0.6765)
Flexible arrival-time dummy		0.3179	0.3081
		(0.5484)	(0.6344)
Household size (number of people)			−0.4604
			(0.3050)
SP variables			
Constant ($\bar{\theta}^{BS}$)	−2.8392	−1.9158	−0.8929
	(1.3131)	(1.1092)	(1.0722)
Standard deviation of constant (σ_ξ)	0.7779	0.0952	0.0582
	(0.8621)	(0.6376)	(0.7978)
Cost ($)		−0.9506	−0.5974
		(0.3920)	(0.3976)
Cost × dummy for medium household income ($60,000–100,000)		−0.2366	−0.1701
		(0.5082)	(0.4395)
Cost × dummy for high household income (>$100,000)		0.2385	0.0969
		(1.3160)	(0.7285)
Travel time (min) × long-commute dummy (>45 min)	−0.2927	−0.1835	−0.1141
	(0.1145)	(0.0856)	(0.0719)
Travel time ×(1 − long-commute dummy)	−0.3311	−0.2152	−0.1350
	(0.1472)	(0.1033)	(0.1013)
Unreliability of travel time (probability)	−8.3078	−5.4302	−3.3903
	(3.7404)	(2.5374)	(2.0618)
Female dummy			0.3990
			(1.0978)

Continues

TABLE A.V—*Continued*

Independent Variable	Constrained Model	Flexible Model 1	Flexible Model 2
Age 30–50 dummy		0.8342	0.5822
		(0.9278)	(1.0957)
Flexible arrival-time dummy		1.6099	1.0396
		(1.0436)	(1.7014)
Household size (number of people)			−0.4012
			(0.6599)
	Variables pooled in joint RP/SP model		
Cost ($)	−1.6431		
	(0.6378)		
Cost × dummy for medium household income ($60,000–100,000)	0.4499		
	(0.2239)		
Cost × dummy for high household income (>$100,000)	0.9866		
	(0.4024)		
Female dummy	1.3204	1.2044	
	(0.4080)	(0.5355)	
Age 30–50 dummy	1.2589		
	(0.4978)		
Flexible arrival-time dummy	0.4281		
	(0.5046)		
Household size (number of people)	−0.4986	−0.5173	
	(0.2303)	(0.2499)	
Standard deviation of coefficients of travel time (part of Ω)	0.2248	0.1417	0.0922
	(0.0730)	(0.0656)	(0.0792)
Ratio of standard deviation to the mean for coefficients of unreliability (σ_ω)	0.9908	1.0529	1.0733
	(0.2555)	(0.3651)	(0.3291)
	Other parameters		
Scale parameter			
Cal Poly sample (μ^C)	0.4634	0.4608	0.5013
	(0.1927)	(0.2371)	(0.2645)
SP sample (μ^{BS})	0.8758	1.3915	2.2202
	(0.4061)	(0.3987)	(1.0082)
Correlation parameter: RP and SP (ρ)	5.0864	3.2013	2.0276
	(1.8389)	(0.8889)	(0.9637)
	Summary statistics		
Number of observations	1,155	1,155	1,155
Number of persons	548	548	548
Log-likelihood[b]	−503.52	−500.55	−500.23

[a]Numbers in parentheses are robust standard errors. Monte Carlo integration performed using 4,500 random draws.
[b]By comparison, the log-likelihood for the base model is −501.57 (see Table II of the paper).

MODEL WITH RELATED CHOICES TREATED SIMULTANEOUSLY

To see whether results are sensitive to the assumptions made about exogeneity or endogeneity of related choices, we estimated a model that simultane-

TABLE A.VI

VALUES OF TIME AND RELIABILITY FROM MODELS WITH DIFFERENT CONSTRAINTS IN COMBINING RP AND SP DATA[a]

	Constrained Model	Base Model	Flexible Model 1	Flexible Model 2
RP VOT				
Median	22.17	21.46	21.62	22.23
	(13.28, 40.33)	(11.47, 29.32)	(10.72, 36.60)	(10.58, 42.17)
Total heterogeneity	13.91	10.47	10.15	10.33
	(8.49, 33.62)	(5.82, 24.11)	(5.16, 24.76)	(4.15, 32.23)
RP VOR				
Median	21.89	19.56	19.86	19.35
	(10.93, 39.69)	(6.26, 42.80)	(4.51, 44.92)	(0.16, 44.71)
Total heterogeneity	28.32	26.49	27.18	28.05
	(14.05, 59.37)	(8.60, 60.40)	(6.81, 70.07)	(6.64, 76.80)
Log-likelihood	−503.52	−501.57	−500.55	−500.23

[a]Numbers in parentheses are robust standard errors. Monte Carlo integration performed using 4,500 random draws.

ously incorporated choice of route (i.e., express or free lanes), transponder, and mode (i.e., car occupancy). Ten people are omitted in this estimation because of missing mode choice. (In the base model of the paper, we assume that these ten people are solo drivers, and we tested this assumption by estimating another model in which the cost variable is interacted with a dummy that represents those ten people; this interaction term was not significant and the increase in log-likelihood value was also not significant.)

As explained in the paper, this model has nine alternatives for RP observations: namely, all the permissible combinations of two routes (regular or express lanes), two transponder choices (yes or no), and three modes (solo, two-person carpool, or three-or-more-person carpool). The three combinations that involve express lanes but no transponder are ruled out by the legal requirement to have a transponder to use the express lanes. Thus the choice set is

$$\{TF1, TF2, TF3, TX1, TX2, TX3, NF1, NF2, NF3\},$$

where each alternative is defined by T or N for having or not having a transponder; F or X for free or express lane; and 1, 2, or 3 for the number of people in the vehicle (3 means three or more).

For SP observations, there are still only two choices, express or regular lanes. Thus these observations help identify the coefficients that apply to those choice, but not those that apply only to mode or transponder choice.

The Cal Poly sample was actually formed by the Cal Poly researchers from four separate subsamples, one random and three choice-based. We could ignore this in our models of route choice only, because the route shares are nearly identical across all samples. However, with this expanded choice set,

TABLE A.VII

CHOICE SHARES AND WEIGHTS FOR CAL POLY DATA SUBSETS[a]

	Population Share	Sample Share					
		New-Plates Subsample		Repeat Subsample		UCI Subsample	
Choice Alternative	%	%	Weight	%	Weight	%	Weight
Solo_notrans_91f	42	28	1.5000	17	2.4706	11	3.8182
Solo_trans_91f	18	26	0.6923	33	0.5455	39	0.4615
Solo_trans_91x	18	15	1.2000	16	1.1250	22	0.8182
Hov2_notrans_91f	5	9	0.5556	3	1.6667	0	NA
Hov2_trans_91f	2	8	0.2500	16	0.1250	6	0.3333
Hov2_trans_91x	6	5	1.2000	7	0.8571	22	0.2727
Hov3_notrans_91f	4	3	1.3333	0	NA	0	NA
Hov3_trans_91f	2	3	0.6667	3	0.6667	0	NA
Hov3_trans_91x	3	3	1.0000	5	0.6000	0	NA
No. of observations	302	191		58		18	

[a] Solo, Hov2, and Hov3 refer to occupancy one, two, and three or more, respectively; notrans and trans refer to no transponder and transponder; 91f and 91x refers to free lanes and express lanes. Weight = (population share)/(sample share). NA denotes not applicable.

the shares of the choice-based portions of the sample differ substantially from those of the random portions. We use the weighted exogenous sample maximum likelihood estimator to account for this (Manski and Lerman (1977)). The population choice shares for each alternative are constructed based on the 302 respondents in the Brookings RP sample (84 respondents) and the Cal Poly random subsample (218 respondents). Table A.VII shows the weights and how they are computed.

We allow for a new complication that arises from the possibility of special correlation patterns among the nine alternatives. These patterns could arise from individual-specific preferences for features that are shared by several alternatives.

We assume specifically that such features are shared by four groups of alternatives: those alternatives that involve owning a transponder (T), those that make use of the express lanes (E), those that involve a high-occupancy vehicle of 2 people ($H2$), those that involve a high-occupancy vehicle of 3 people or more ($H3$). We define dummy variables D_j^T, D_j^E, D_j^{H2}, and D_j^{H3} for alternatives in each of these groups. Thus for example the dummy variable D_j^T is defined as $D_j^T = 1$ if $j \in \Omega^T$ and 0 if $j \in \Omega^N$, where

$$\Omega^T = \{TF1, TF2, TF3, TX1, TX2, TX3\},$$
$$\Omega^N = \{NF1, NF2, NF3\}.$$

We allow each of these four dummy variables D^k to have a random coefficient π^k with its own variance (except that we constrain π_j^{H2} and π_j^{H3} to have the same variance). Brownstone and Train (1999) show how such a structure mimics a generalized extreme value model analogous to a generalized nested logit, in which the error terms in certain groups of alternatives (such as Ω^T) are correlated with each other.

Table A.VIII shows the estimation results. In the RP portion of the model, the variables previously interacted with express lane dummy (in the base model

TABLE A.VIII

MODEL OF SIMULTANEOUS ROUTE, TRANSPONDER, AND MODE CHOICE[a]

Model Specification	Model 1	Model 2
RP estimates		
Generic variables		
Toll	−2.1584 (0.8076)	−2.3242 (0.7927)
Toll × high household income	1.7290 (0.6924)	1.8653 (0.6468)
Toll × medium household income	0.7949 (0.4055)	0.8611 (0.4519)
Median travel time × trip distance	−0.5724 (0.1885)	−0.7301 (0.1966)
Median travel time × trip distance squared	0.1057 (0.0412)	0.1330 (0.0418)
Median travel time × trip distance cubed	−0.0046 (0.0020)	0.0058 (0.0020)
Travel time uncertainty	−0.8432 (0.3488)	−1.0284 (0.3694)
Transponder choice		
Transponder dummy× Brookings dummy	−3.4748 (1.2356)	−3.6485 (1.1985)
Transponder dummy × Cal Poly dummy	−3.9872 (1.1263)	−4.1199 (1.1201)
Female dummy × transponder dummy	2.0008 (0.7575)	2.1197 (0.7643)
Household size × transponder dummy	−0.5278 (0.3216)	−0.5467 (0.3093)
Commute dummy × transponder dummy	1.6617 (0.9047)	1.6973 (0.9568)
Age 30–50 dummy × transponder dummy	1.4734 (0.8102)	1.5966 (0.7502)
Flexible arrival time dummy × transponder dummy	1.0900 (0.6531)	1.0799 (0.6969)
Std. dev. of transponder dummy ($\sigma^2_{\pi T}$)	0.5148 (1.1836)	0.5593 (1.2038)
Route choice		
Express lane dummy × Brookings dummy	0.9480 (1.1598)	0.2766 (1.1373)
Express lane dummy × Cal Poly dummy	−0.2209 (0.9747)	−0.6633 (0.9799)
Travel between 7–8 am × express lane		−1.7420 (1.0506)
Std. dev. of express lane dummy ($\sigma^2_{\pi E}$)	1.8667 (0.7009)	2.0697 (0.7634)
Carpool choice		
Carpool dummy × Brookings dummy	−12.0415 (3.5257)	−13.1853 (4.8817)
Carpool dummy × Cal Poly dummy	−7.6201 (1.9693)	−8.1681 (3.0250)
Female dummy × age 30–50 dummy × carpool dummy	4.2380 (1.2671)	4.5499 (1.8136)
Commute dummy × carpool dummy	−2.0038 (1.0197)	−2.1747 (1.1157)
HOV3 dummy × Brookings dummy	2.5501 (2.5805)	2.4164 (2.4015)
HOV3 dummy × Cal Poly dummy	1.4015 (1.5798)	1.3327 (1.5475)
Flexible arrival time dummy × HOV3 dummy	−3.1861 (1.4337)	−3.4114 (1.5671)
Small family size (≤4) × HOV3 dummy	−1.7968 (1.0710)	−1.9213 (1.1970)
Log of trip distance × HOV3 dummy	−2.1905 (1.1829)	−2.1981 (1.1853)
Common std. dev. of HOV2, HOV3 dummies ($\sigma^2_{\pi H}$)	6.0159 (2.9544)	6.6347 (3.1331)

Continues

TABLE A.VIII—*Continued*

Model Specification	Model 1	Model 2
SP estimates		
Express lane dummy	−0.6475 (1.4869)	−0.5533 (1.1885)
Std. dev. of express lane dummy	1.4452 (0.6802)	1.4513 (0.6397)
Toll	−1.4007 (0.3058)	−1.4181 (0.3034)
Toll × dummy for high household income	0.6561 (1.0086)	0.6854 (0.9838)
Toll × dummy for medium household income	−0.1722 (0.5870)	−0.1345 (0.5611)
Travel time (min) × long commute dummy	−0.2281 (0.0537)	−0.2283 (0.0519)
Travel time × (1 − long commute dummy)	−0.2788 (0.0653)	−0.2742 (0.0645)
Travel-time uncertainty	−7.4717 (2.2717)	−7.5477 (2.3027)
Female dummy × express lane dummy	1.3131 (1.0265)	1.2922 (0.9653)
Flexible arrival-time dummy × express lane dummy	1.5888 (1.1100)	1.5208 (1.0894)
Age 30–50 dummy × express lane dummy	−1.0218 (1.0618)	−0.9845 (0.9816)
Household size × express lane dummy	−0.5848 (0.4050)	−0.5973 (0.3724)
Correlation between RP and SP express lane choice	2.3538 (0.8024)	2.1204 (0.7096)
Combined estimates		
Std. dev. of travel time coefficient	0.2149 (0.0462)	0.2140 (0.0468)
Ratio between mean and std. dev. of travel-time uncertainty	0.9052 (0.3192)	0.8744 (0.3004)
Parameters associated with scaling		
Scale parameter		
Cal Poly sample	0.3498 (0.0598)	0.3406 (0.0539)
Brookings RP sample	0.6379 (0.1916)	0.5805 (0.1302)
Number of observations	1,124	1,124
Number of persons	538	538
Log-likelihood	−1,002.06	−1,001.25

[a]Numbers in parentheses are robust standard errors. Monte Carlo simulations performed using 4,000 randomized and shuffled Halton draws. (Results were stable for more than 2,000 draws.) The Halton sequences are first randomized by adding a uniform random number to each element and are then shuffled by reordering the elements randomly to avoid correlation across sequences in high-dimensional integration. See Train (2003, Sections 9.3.3–9.3.4) and Hess, Train, and Polak (2005). Instead of normalizing the variance of the Brookings RP remaining error term and rescaling the other two, as in our base model, here we normalize the variance of Brookings SP and rescale the two RP variances; this proved to have better numerical properties.

of the paper) are now interacted with a transponder dummy instead, which we found improves the goodness of fit. As a result, all these variables are now being estimated separately across RP and SP data sets since there is no transponder dummy in the SP model. In model 2, we include a time-of-day dummy interacted with express lane, just as in Table A.VIII.[2]

Table A.IX reports the distribution of the estimated RP VOT and VOR from the two models shown in Table A.VIII.

[2]We take advantage of our earlier finding that omitting this information from the SP sample does not affect values of time and reliability. We therefore include the time-of-day dummy only for the RP observations. This avoids dividing the SP sample into two parts, which would have increased the number of parameters to estimate.

TABLE A.IX

VALUES OF TIME AND RELIABILITY FROM MODEL WITH 9 CHOICE ALTERNATIVES

	Model 1 (Without Time-of-Day Dummies)	Model 2 (with Time-of-Day-Dummy)
RP VOT		
Median	23.58	28.50
	(12.73, 36.48)	(17.42, 42.17)
Total heterogeneity	12.50	12.55
	(8.01, 21.41)	(7.62, 22.54)
RP VOR		
	25.47	28.35
Median	(6.94, 42.18)	(11.92, 47.87)
Total heterogeneity	29.45	30.96
	(10.32, 61.44)	(8.52, 64.57)

INTERPRETATION AS A SELECTION MODEL FOR TRANSPONDER ACQUISITION

Our nine-alternative mixed-logit model for RP observations can be interpreted as a selection model, in which one first selects transponder (denoted by choice variable T_i), then conditional on that chooses among occupancies if $T_i = 0$ (with no transponder) and among occupancies and lanes if $T_i = 1$ (with transponder). We could write the observation likelihood as a conditional probability multiplied by a marginal probability, the form depending on whether or not transponder was chosen for this observation:

$$L_i = \sum_{j\in\Omega^T} y_{ij}\,\Pr(y_{ij} = 1|T_i = 1)\cdot \Pr(T_i = 1) + \sum_{j\in\Omega^N} y_{ij}\,\Pr(y_{ij} = 1|T_i = 0)\cdot (1 - \Pr(T_i = 1)), \tag{A5}$$

where y_{ij} indicates the observed choice for traveler i (equal to 1 if the commuter chooses alternative j; 0 otherwise). The choices involved in $\Pr(y_{ij} = 1|T_i = 1)$ are more extensive than those in $\Pr(y_{ij} = 1|T_i = 0)$ because they include lane choice.

If we had no random parameters, $\Pr(T_i = 1)$ would be binary logit and the conditional probabilities in (A5) would be multinomial logit; transponder selection and lane choice would then be correlated in a manner determined entirely by the logit functional form. However, our use of random parameters results in a more flexible correlation pattern that depends on the estimated variances of these random parameters, especially π_i^T (the random coefficient of the transponder dummy). As it happens, we do not estimate a statistically significant variance for π_i^T nor are the results of our nine-alternative model affected in any important way by whether or not we include π_i^T in the model.

Nevertheless, the exclusion restrictions that identify the selection model—namely, the inclusion of certain variables under "transponder choice" but not under "route choice" in Table A.VIII—are based on statistical significance rather than any theoretical exclusion requirement. Therefore, the mixed-logit functional form chosen for the nine-alternative model effectively identifies the selection model.

Dept. Economics, University of California, Irvine, CA 92697-5100, U.S.A.; ksmall@uci.edu; http://www.socsci.uci.edu/˜ksmall/,

Brookings Institution, 1775 Massachusetts Avenue, N.W., Washington, DC 20036, U.S.A.; cwinston@brook.edu,

and

Dept. of Logistics, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong; lgtjiay@polyu.edu.hk.

Manuscript received August, 2002; final revision received January, 2005.

REFERENCES

BROWNSTONE, D., AND K. TRAIN (1999): "Forecasting New Product Penetration with Flexible Substitution Patterns," *Journal of Econometrics*, 89, 109–129.

BUCHINSKY, M. (1998): "Recent Advances in Quantile Regression: A Practical Guide for Empirical Research," *Journal of Human Resources*, 33, 88–126.

HAJIVASSILIOU, V., AND P. RUUD (1994): "Classical Estimation Methods for LDV Models Using Simulation," in *Handbook in Econometrics*, Vol. IV, ed. by R. Engle and D. MacFadden. New York: Elsevier Science, 2383–2441.

HARDLE, W. (1990): *Applied Nonparametric Regression*. Cambridge, U.K.: Cambridge University Press.

HESS, S., K. TRAIN, AND J. POLAK (2005): "On the Use of a Modified Latin Hypercube Sampling (MLHS) Method in the Estimation of a Mixed Logit Model for Vehicle Choice," *Transportation Research B*, forthcoming.

LAM, T. C., AND K. A. SMALL (2001): "The Value of Time and Reliability: Measurement from a Value Pricing Experiment," *Transportation Research Part E: Logistics and Transportation Review*, 37, 231–251.

LEE, L. (1992): "On Efficiency of Methods of Simulated Moments and Maximum Simulated Likelihood Estimation of Discrete Response Models," *Econometric Theory*, 8, 518–552.

MANSKI, C. F., AND S. R. LERMAN (1977): "The Estimation of Choice Probabilities from Choice Based Samples," *Econometrica*, 45, 1977–1988.

SILVERMAN, B. W. (1985): *Density Estimation for Statistics and Data Analysis*. London: Chapman & Hall.

SMALL, K. A. (1982): "The Scheduling of Consumer Activities: Work Trips," *American Economic Review*, 72, 467–479.

YU, K., AND M. C. JONES (1998): "Local Linear Quantile Regression," *Journal of the American Statistical Association*, 93, 228–237.