Spatial Hedonics and the Willingness to Pay for Residential Amenities

Kenneth A. Small
Department of Economics
University of California at Irvine
Irvine, CA 92697-5100
ksmall@uci.edu

Seiji S.C. Steimetz
Department of Economics
California State University at Long Beach
1250 Bellflower Boulevard
Long Beach, CA 90840
ssteimet@csulb.edu

revised Dec. 5, 2011

Forthcoming, Journal of Regional Science

Abstract

We investigate the role of spatial multipliers when using spatial-lag hedonic models to measure the benefits of residential amenity improvements. Such benefits are commonly measured using the product of the coefficient of the amenity in question and a spatial multiplier. We show this is correct only when the spatial spillovers transmitted through the multiplier are strictly technological. If they are instead strictly pecuniary, then benefits are fully measured using the coefficient with no spatial multiplier. In either case, or in intermediate cases where both types of spillovers exist, a spatial-lag specification is required to obtain valid benefits measures.

JEL codes: Q51, R21

Keywords: spatial hedonics, spatial autoregression, spatial lag, willingness to pay, welfare

1. Introduction

It is well known that, because housing prices depend on residential amenities and local public goods, hedonic price techniques may provide a way to measure households' willingness to pay for them.[1] The basic idea is that in competitive equilibrium, households equate their marginal willingness to pay to the marginal price of the amenity that they face in the market. It is more recently established that housing prices can be influenced by the characteristics of surrounding houses.[2] A common structural specification, sometimes called "the spatial lag model", contains the price variable itself as the one that influences the price of neighboring properties. This model is especially appropriate if the influence is posited to be through some direct causal mechanism; a frequently cited example is the "sales comparison approach", or "grid adjustment technique", where the prices of "comparable" properties are considered when devising offering prices for real-estate transactions.[3]

Spatial lags complicate benefit measurement because an amenity improvement at a given property now affects the values of neighboring properties, which in turn exert an additional effect on the first property. In practice this has given rise to a "spatial-multiplier" approach to measuring benefits, where the direct effect of an improvement is magnified by the spillover effects among neighboring properties. (The effect could be diminished rather than magnified, but

[1] Rosen (1974). Several pioneering empirical studies focus on air pollution, for example Ridker and Henning (1967) and Freeman (1974). See Freeman (1993), Smith and Huang (1995), and Chay and Greenstone (2005) for summaries of more recent evidence.

[2] See Case (1991), Can (1992), Anselin (2002), and Anselin (2003) for discussions of various spatial econometric models and how to distinguish them empirically.

[3] See, for example, Anas and Eum (1984), Can and Megbolugbe (1997), Pace and Gilley (1998), and Kim et al. (2003). See Isakson (2002) for an overview of the "sales comparison approach" practiced by real-estate agents and appraisers. Other models have also been developed to account for spatially varying factors, for example geographically weighted regression (Brundson *et al.* 1996, 1999; Mei *et al.* 2004), which allows the model's coefficients to vary spatially. But the spatial lag model remains popular empirically, as illustrated in Section 4, and it allows for a direct causal interpretation which is useful for welfare analysis.

that is rare.) This multiplier is determined by the coefficient of a weighted transformation of spatially lagged prices.

We examine the economic interpretations of these direct and spillover effects. We find that the spatial-multiplier approach to benefit measurement is valid only under fairly strong assumptions about the economic nature of the spatial spillovers. In particular, we draw a distinction between two types of spillovers that are each consistent with a spatial lag specification: *pecuniary* spillovers, where spatial dependence is purely monetary, and *technological* spillovers, where the enjoyment of living at one location is directly influenced by the values of neighboring locations.[4] We show that only in the second case does the spatial-multiplier approach give a correct benefit measure.

To illustrate, assume that property values are generated by a process consistent with a spatial-lag structural specification. Then consider a neighborhood that enjoys a uniform improvement in air quality. The improvement raises property values through two mechanisms. First, cleaner air at a given location increases the utility from living there: this is the "direct effect" of the improvement, measured by the coefficient on air quality in the structural model. Second, that location's value is further raised by the mutually reinforcing price increases described above: this is the "indirect effect" caused by spillovers, related to the coefficient of the model's spatially lagged dependent variable. Together, the direct and indirect effects on *prices* are unambiguously captured by multiplying the air-quality coefficient by a spatial multiplier.

But price changes need not equal welfare changes. For welfare analysis, the salient question is: how exactly are residents benefiting from the spillover effect? If the answer is simply through higher rents (or sales values), as in the "sales comparison approach" example,

[4] See Baumol and Oates (1988, pp. 31-32) for the general distinction between technological and pecuniary externalities, focusing on why the former but not the latter produces welfare effects in a competitive market.

then the spillover is a transfer from renters to landlords (or from future homeowners to current homeowners) and thus is a *pecuniary* externality which is welfare neutral. If instead the answer is that residents derive pleasure from their neighbors' higher property values, perhaps because those neighbors use some of their capital gains to beautify their houses in ways unobservable to the outside analyst, then the spillover effect represents a *technological* externality because it directly increases each resident's utility.

In practice it may be difficult to know whether a spatial spillover is pecuniary or technological. Indeed, many if not all relationships described empirically as spatial interactions are best viewed as resulting from a reduced-form version of an unknown structural model. Thus in the examples just given, my neighbors' house prices affect my utility because of unobserved aspects of the real estate market or of my neighbors' behavior. If the mechanism is "comparable sales," the appropriate structural model would involve real estate transactions under limited information, explaining how such a mechanism can persist in equilibrium when, on the face of it, it appears inconsistent with rational behavior. If the mechanism is prestige, one needs a theory of status effects that predicts to what extent my status is raised at someone else's expense. For other mechanisms, one may need to consider altruism, maintenance externalities (as in our verbal example), misperceptions, or other phenomena.[5] The problem is that empirical data typically cannot identify the full structural model behind the observed empirical relationships. But if we at least understand how different structural explanations determine the welfare implications of empirically observed relationships, we will be able to make better welfare judgments.

[5] Can and Megbolugbe (1997) discuss how either the use of "comparable sales" in real-estate appraisal or the existence of "snob value" related to higher neighboring prices can imply a spatial lag specification. Wendner and Goulder (2008) and Bergstrom (2006) discuss how welfare evaluation is affected by "status effects" and altruism, respectively. Portney (1992) and Viscusi (2000, pp. 867-8) discuss perception.

Here we examine two economic theories that are consistent with the same empirical specification. In one, a household's utility depends only on its own residential characteristics, even though in equilibrium its housing price depends on neighboring prices. In the other, the household's utility (perhaps a partially indirect utility incorporating certain neighborhood market mechanisms) can be written as depending on neighboring prices. Both theories are best viewed as partially reduced models arising from some unknown but more complex structural model of behavior or of housing markets. We show that the direct benefits of an amenity change are magnified by a spatial multiplier in the second case but not in the first. We also show through some published examples that benefits estimates can differ greatly depending on which of these economic theories of spatial dependence is assumed to be operating.

We proceed by first reviewing the conventional hedonic model, clarifying the cancellation that makes its welfare analysis especially simple (Section 2). We then consider pecuniary and technological externalities as alternative theoretical sources for spatial lags, showing that the same cancellation occurs in the former but not the latter (Section 3). We review empirical studies that have used spatial multipliers for welfare analysis (Section 4), and draw conclusions for applied research (Section 5).

2. Conventional Hedonic Welfare Analysis

We begin by reviewing hedonic welfare analysis when there is no assumed spatial interdependence among housing prices, following Freeman (1974) and Small (1975). While this material is not new, we point out certain cancellations in aggregation that help clarify just where the analysis is changed by the presence of spatially-lagged prices.

Hedonic welfare analysis is based implicitly or explicitly on an assumed long-run equilibrium in which potential occupants of properties trade off amenities against rents (Rosen 1974). For simplicity we call these potential occupants "households" and assume that owner-occupants can distinguish between their roles as owner and renter of a capital asset.

For concreteness, we identify one amenity, assumed to be valued positively by all households, for special attention: call it air quality q. Let $q_j(\theta)$ measure the air quality associated with location j, which depends on some scalar index θ of pollution-abatement measures, such that $dq_j/d\theta>0$. The rental price of the property at j is $r_j=r(x_j,q_j)$, where x_j is a vector of characteristics of that property and $r(\cdot)$ is a hedonic rent function.

Households derive utility from a numeraire good z, housing characteristics, and air quality. Thus when deciding where to live, household i with income y_i chooses a location that maximizes utility $u_i(z,x,q)$ subject to budget constraint $z+r(x,q)=y_i$. Households need not have identical utility functions. Following Rosen (1974) and the applied hedonic price literature, we assume that a continuum of values of x and q are available in the market.[6] We can then represent this decision process as:

$$\text{(1)} \qquad \underset{z,x,q,\lambda}{Max}\ \Lambda_i \equiv u_i(z,x,q) + \lambda_i \cdot [y_i - z - r(x,q)].$$

Assuming an interior solution, properties of Lagrangian multipliers imply that λ_i is the marginal utility obtained by increasing income. Thus, the household's willingness to pay for marginal

[6] Alternatively, one could recognize explicitly that there are a finite number of possible choices, and formulate a discrete choice model for household choice. Such a model measures ratios of marginal utilities but not the rent function $r(\cdot)$.

pollution abatement is the resulting utility change divided by λ, less any monetary cost arising through rent increases:

(2) $$M_{i\theta} = \frac{u_{qi}}{\lambda_i}\frac{dq_i}{d\theta} - \frac{dr_i}{d\theta}.$$

The first-order condition with respect to q is:

(3) $$\frac{u_{qi}}{\lambda_i} = r_{qi}$$

where $r_{qi} \equiv \partial r(x_i, q)/\partial q$ is the implicit price of air quality at the chosen location. The derivatives in (2) and (3) are evaluated at the solution values, which we denote $(z_i, x_i, q_i, \lambda_i)$. (For notational simplicity, we henceforth use the same index i for the household and for its location in equilibrium.)

Aggregating (2) over all households and applying (3) shows that aggregate household willingness to pay for abatement is:

(4) $$M_\theta \equiv \sum_i M_{i\theta} = \sum_i r_{qi}\frac{dq_i}{d\theta} - \sum_i \frac{dr_i}{d\theta} \;.$$

The first term on the right-hand side of equation (4) shows the increased amenity value of the improvement, a "technological effect" because it reflects that utility depends directly on air quality. The second term shows the rent increase that is induced, a pecuniary effect.

Aggregate rents received by landlords increase with θ by the marginal amount:

(5) $$R_\theta \equiv \sum_i \frac{dr_i}{d\theta}.$$

Therefore the rent changes, $\Sigma_i dr_i/d\theta$, cancel when aggregating the benefits to all residents and landlords:

(6) $$B_\theta \equiv M_\theta + R_\theta = \sum_i r_{qi} \frac{dq_i}{d\theta}.$$

Equation (6) shows that the social benefit of the improvement depends only on the technological effects. Part or all of these benefits might be captured by landlords, but the total level of the benefit is unaffected by such pecuniary transfers.[7] A special case of (6) is when abatement is uniform across all locations and measured in the same units as air quality: then $dq_i/d\theta$=1 and

(7) $$B_\theta = \sum_i r_{qi} .$$

[7] We abstract from any cost savings that the landlord may enjoy, such as less frequent painting or air-filter replacement. Because we assume a fixed supply of housing, (6) applies even if landlords exercise market power; such power would affect the division of benefits between renters and landlords (via the terms $dr_i/d\theta$), but not the total. Equation (6) applies to a *marginal* air-quality improvement, for which the effects of any induced relocations cancel out because of the envelope theorem applied to each household; thus it corresponds to the "Stage 1" and "Stage 2" effects described in Bartik (1988, pp. 176-177).

To illustrate concretely, a common empirical hedonic specification is linear in an unknown parameter vector β:

$$\mathbf{r} = X\beta + \varepsilon \tag{8}$$

where $\mathbf{r}$ is an Nx1 vector of equilibrium property prices, distinguished from the scalar function $r(\cdot)$ by bold font; X is an Nxk data matrix including observations on both x and q (possibly including transformations and interactions); β is a kx1 parameter vector; ε is an Nx1 vector of mean-zero disturbances (possibly mutually correlated); and N is the number of households. In the special case where air quality appears linearly in $X\beta$ with a single coefficient β_q, then $r_{qi}=\beta_q$ and (7) shows that the marginal aggregate willingness to pay for a uniform improvement in air quality is simply

$$B_\theta = N\beta_q\,. \tag{9}$$

3. Welfare Analysis with Spatial Lags

Spatial Multipliers

We now consider a hedonic specification in which the rent at each location depends empirically on rents at other locations. We focus on the spatial lag model to demonstrate how spatial-multiplier effects on rents can arise in hedonic analysis. A common form of this model is

$$\mathbf{r} = \rho W\mathbf{r} + X\beta + \varepsilon \tag{10}$$

where W is an NxN spatial weighting matrix whose diagonal elements are zero, ρ is a scalar spatial coefficient to be estimated, and ε is a mean-zero error vector.[8] As is usual, we assume W to be normalized to sum to one by row: $W\boldsymbol{\iota}=\boldsymbol{\iota}$ where $\boldsymbol{\iota}$ is the Nx1 unit vector; and we assume $|\rho|<1$; this guarantees that $[I-\rho W]$ can be inverted. We also assume that in any empirical application, there is sufficient independent variation among the vectors $W\mathbf{r}$ and the columns of X to identify their coefficients. Other than these properties, W is quite general and can, for example, relate near neighbors more closely than far neighbors in some prespecified manner.

Equation (10) is not the most general possible linear specification that includes spatially-lagged prices. For example, one could also allow for spatial lags in X.[9] And of course, there are any number of nonlinear, semi-parametric, and non-parametric specifications that might be appropriate. But (10) is quite commonly used in practice and our objective is to show that even within this relatively simple model, welfare analysis depends critically on theoretical assumptions about why the empirical model takes a spatially lagged form. The same theoretical point will apply, perhaps less transparently, to more sophisticated models.

Model (10) can be written in reduced form by solving for the dependent-variable vector:

$$\mathbf{r} = [I - \rho W]^{-1} X\beta + [I - \rho W]^{-1} \varepsilon \tag{11}$$

[8] This parametric modeling of spatial dependence may be viewed as somewhat restrictive given recent developments in non-parametric methods (McMillen and Redfearn 2010) and semi-parametric methods, such as locally weighted regression and geographically geighted regression (Fotheringham et al. 2002; Carruthers and Clark 2010). The objective here, however, is not to advocate parametric specifications, but rather to analyze the welfare implications of the commonly-used spatial lag model.

[9] In that case we could write the structural form as $\mathbf{r} = \rho W\mathbf{r} + X\beta + WX\gamma + \varepsilon$ with reduced form $\mathbf{r} = [I - \rho W]^{-1} X\beta + [I - \rho W]^{-1} WX\gamma + [I - \rho W]^{-1} \varepsilon$. One of its special cases is the "Spatial Durbin" or "Common Factors" model in which $\rho\beta = -\gamma$.

where I is the NxN identity matrix. The matrix $[I-\rho W]^{-1}$ is known as a "spatial-multiplier matrix" (Anselin 2003).[10] We will make use of the fact that the row normalization of W implies the following property of the spatial-multiplier matrix (Kim et al. 2003):

(12) $$[I-\rho W]^{-1}\boldsymbol{\iota} = (1-\rho)^{-1}\boldsymbol{\iota} \ .$$

Thus, post-multiplication by $\boldsymbol{\iota}$ causes the spatial-multiplier matrix to simplify to the scalar "spatial multiplier" $(1\text{-}\rho)^{-1}$.Equations (11) and (12) show that the price feedback inherent in (10) tends to magnify any equilibrium price changes caused by changes in the X variables or by shocks in the error term, provided $\rho>0$. (The same equations also show that such fluctuations are bounded so long as $\rho<1$, which in turn is implied by the row-normalization condition on W.)

We now consider how the spatial multiplier affects price changes resulting from a change in air quality. As before, for simplicity we will treat the special case of (10) where data matrix X contains air quality only in the form of a single column vector, $\mathbf{q}$, with coefficient β_q. The total derivative matrix of rent vector $\mathbf{r}$ is then

(13a) $$\frac{d\mathbf{r}}{d\mathbf{q}'} = [I-\rho W]^{-1}\beta_q \, ,$$

[10] Anselin (2003:261) calls (10) and (11) a "mixed regressive, spatial autoregressive" specification, derived from a model with "spatial externalities in both modeled and unmodeled effects". As Anselin also points out, this reduced-form model could be generalized to postulate different multiplier matrices for the two terms: $\mathbf{r} = [I-\rho W_1]^{-1}X\beta + [I-\sigma W_2]^{-1}\varepsilon$. Our welfare arguments would apply unchanged to this more general model.

expressing how rent at each location is affected by changes in air quality at its own and all other locations.[11] We can express this rent gradient as the sum of direct and indirect (spatial-spillover) effects by pre-multiplying (11) by ρW and substituting into (10), so that the non-stochastic part of r is given by $X\beta + [I-\rho W]^{-1} X\beta$. Therefore:

$$\text{(13b)} \qquad \frac{d\mathbf{r}}{d\mathbf{q}'} = \overbrace{I\beta_q}^{direct} + \overbrace{\rho W[I-\rho W]^{-1}\beta_q}^{indirect} .$$

If a policy changes air quality uniformly at every location, then the rent vector changes according to:

$$\text{(13c)} \qquad \frac{d\mathbf{r}}{d\mathbf{q}'}\boldsymbol{\iota} = (1-\rho)^{-1}\beta_q \boldsymbol{\iota} = \overbrace{\beta_q \boldsymbol{\iota}}^{direct} + \overbrace{\rho(1-\rho)^{-1}\beta_q \boldsymbol{\iota}}^{indirect} .$$

The marginal change in rent at every location is now just the air-quality coefficient multiplied by the (scalar) spatial multiplier, and the aggregate rent change is $N(1-\rho)^{-1}\beta_q$. Comparing to (9), we see that $(1-\rho)^{-1}$ is indeed a multiplier.

Equations (13a)–(13c) make clear how spatial multipliers apply to *price* changes. We now consider *welfare* changes by considering two distinct economic models of spatial spillovers that can be used to interpret specification (10).

[11] If **q** enters **r** non-linearly, (13a) is replaced by the more general expression $\frac{d\mathbf{r}}{d\mathbf{q}'} = [I-\rho W]^{-1}\frac{d(X\beta)}{d\mathbf{q}'}$.

Pecuniary Spatial Effects

To provide a concrete example of pecuniary spatial effects, suppose the sole motivation for specifying the spatial lag model in (10) is the "sales comparison" approach of real estate agents. In that case we would interpret the spatial dependence as pecuniary because neighboring prices do not affect the amenity value of a given location. And although the spatial spillovers may be properly modeled using spatially-lagged prices, those spillovers have no welfare implications.

For example, suppose my neighbors live in a different school district that suddenly improves, causing their property values to rise. If real-estate agents or appraisers use those property values as "comparables" to mine, then my property value could rise as well. But this price effect is pecuniary because my children cannot attend schools in the improved district and so I gain no utility from the improvement (ignoring peer-group effects across school districts).

To show this formally, and continuing with our air quality example, let $c_j(r_{-j})$ represent a scalar "comparable sales" adjustment to the rent at location j, which naturally depends on a vector of rents, r_{-j}, at locations other than j. The market rent at j then follows a hedonic process that we can write as $r_j = \tilde{r}(x_j, q_j, c_j)$. But c is not directly observed by the outside analyst; instead, the spatially-lagged rents in (10) serve as proxies for comparable-sales adjustments, the elements of W indicating which neighboring rents are relevant and how strongly they contribute to these adjustments. As before, we assume there is enough independent variation among variables (in this case, x, q, and the proxies for c) to identify their coefficients empirically as the empirical hedonic function $r(x_j, q_j, r_{-i}) \equiv \tilde{r}[x_j, q_j, c_j(r_{-j})]$. Of course, there can be specification bias from using proxies instead of the actual comparable sales adjustment (Wickens 1972).[12]

[12] One may still ask why the "sales comparison approach" can systematically affect equilibrium prices in a structural sense. One reason may be that potential homeowners have imperfect information about the housing market, as is

In this case, the location decision process is represented by

$$\text{(14)} \qquad \underset{z,x,q,\lambda}{Max}\ \Lambda_i \equiv u_i(z,x,q) + \lambda_i \cdot [y_i - z - \widetilde{r}(x,q,c)]$$

which is the same Lagrangian problem as in (1), except for the inclusion of *c* in the rent function. First-order condition (3) applies unchanged and the rest of equations (2)–(7), describing welfare calculations, still hold without modification. However, their economic interpretation is somewhat different. In (2) and (4), the total derivative $dr_i/d\theta$, reflecting the entire process of adjustment of rents to pollution abatement, now includes indirect, spatial-spillover effects via pollution's influence on values of nearby locations. (This is why we write it as a total derivative.) Those indirect effects are captured empirically by the spatial multipliers in (13a)–(13c), but that does not really matter because they cancel in (6) just as before; then (7) applies as before, with no spatial multiplier.

It is essential, however, in applying (7) that the rent function in (10) be estimated in full, including the spatial lags, so that its partial derivative can be correctly measured. Otherwise one is likely to underestimate welfare effects. For instance, if the true rent function is (10) but (8) is estimated instead, then the measured gradient r_{qi} will be overestimated to the extent that it subsumes the effects of the omitted spatial lags. In such cases the usual benefit measure given by (7) would typically result in overstated benefits. See Greenbaum (2002) and Case, Rosen and Hines (1993) for examples in labor economics and public finance of how least-squares coefficients tend to overstate marginal effects when spatial lags are omitted.

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consistent with findings that housing prices do not incorporate all known information (Case and Shiller 1989) and that different appraisers often arrive at different property valuations (Isakson 2002).

The validity of (7) shows that in computing aggregate benefits from the fully specified hedonic model with spatial lags, only the partial effect of air quality, r_{qi}, is relevant for welfare analysis. This is because the indirect spatial effects enter M_θ and R_θ with equal and opposite magnitudes, and thus cancel — just as they did in the conventional analysis of Section 2. Thus, for example, if the correct hedonic specification is (10), and the spatial price dependence it models is pecuniary, then the aggregate benefit of a uniform improvement is again (9); if we were to include the indirect effects when calculating aggregate benefits, we would erroneously inflate these benefits by a factor $(1-\rho)^{-1}$.

Technological Spatial Effects

Now suppose instead that the justification for (10) is that households derive enjoyment from unobservable characteristics that are intimately associated with their neighbors' rent expenditures. For example, the perceived "quality" (or perhaps "prestige") of a location's neighborhood might be positively associated with that neighborhood's property values. In this case, spatial spillover effects among prices are indeed relevant for welfare analysis because the amenity value of a given location is influenced by rents at other locations.[13] And the "spatial multiplier" approach to welfare analysis is correct in this case.

To demonstrate this, let $n_j(r_{-j})$ denote a scalar index of location j's "neighborhood quality", which depends on a vector of rents, r_{-j}, at locations other than j, such that the hedonic rent function can be written $\hat{r}(x_j, q_j, n_j)$. Again, the newly introduced variable n is not observed by the outside analyst, so its influences on equilibrium rents are proxied by the spatially-

[13] This is similar to what Brueckner (2002) calls a "spillover model" in the context of local governments. See the discussion of Brueckner's terminology in Anselin (2002, p. 249).

weighted neighboring rents in (10). We assume for simplicity that the functional relationship $n_j(r_j)$ is not affected by the policy under consideration; if it were, another step would be necessary in translating a policy change into utility changes.

The residential location problem in this scenario is

(15) $$\underset{z,x,q,n,\lambda}{Max}\ \Lambda_i \equiv u_i(z,x,q,n) + \lambda_i \cdot [y_i - z - \hat{r}(x,q,n)]$$

noting, importantly, that n enters the utility function in (15) and is another choice variable. The condition in (3) still holds, but the choice of "neighborhood quality" among locations yields an additional first-order condition:

(16) $$\frac{u_{ni}}{\lambda_i} = \hat{r}_{ni}$$

where $u_{ni} \equiv \partial u_i / \partial n_i$ is the marginal utility of neighborhood quality and $\hat{r}_{ni} \equiv \partial \hat{r}_i / \partial n_i$ is its implicit price. Equation (16) shows that, in equilibrium, the marginal amenity value of "neighborhood quality" is balanced by the increment in rent required to obtain it through location choice.

The marginal benefit to each household from a change in abatement parameter θ is now a more complicated version of (2):

(17) $$M_{i\theta} \equiv -\left.\frac{dy_i}{d\theta}\right|_{dV_i=0} = \frac{u_{qi}}{\lambda_i}\frac{dq_i}{d\theta} + \frac{u_{ni}}{\lambda_i}\left(\frac{dn_i}{dr'_{-i}}\frac{dr_{-i}}{dq'_{-i}}\right)\frac{dq_{-i}}{d\theta} - \frac{dr_i}{d\theta}$$

where r_{-i} and q_{-i} are vectors of rents and air quality at all locations with their i-th elements replaced by zero. The first two terms on the right-hand side of (17) demonstrate two technological effects on utility: the direct amenity value of improved air quality, and the indirect amenity value of technological externalities generated by an increase in neighboring property values.[14] (The second is technological because $u_{ni} > 0$ and $dn_i/dr'_{-i} \neq 0$.) These benefits are enjoyed at the expense of a pecuniary effect: namely, the total increase in rent resulting (through all channels) from the change in policy, which is the last term in (17). We write the derivative vector dr_{-i} / dq'_{-i} as a total derivative to indicate that it includes feedback effects of r_{-i} on itself via the effect of r_{-i} on r_i; thus it is part of the matrix of derivatives on the left-hand side of (13a)–(13c).

The aggregate benefit to households (as renters) is then given by the sum over locations of (17). Applying first-order conditions (3) and (16), we can convert the marginal utilities in (17) to marginal hedonic prices and write the households' aggregate benefit in terms of rent gradients:

$$M_\theta = \sum_i \left(r_{qi} \frac{dq_i}{d\theta} + \hat{r}_{ni} \frac{dn_i}{dr'_{-i}} \frac{dr_{-i}}{dq'_{-i}} \frac{dq_{-i}}{d\theta} \right) - \sum_i \frac{dr_i}{d\theta} \ . \tag{18a}$$

The first two factors in the second term involve neighborhood quality n and so are not observed empirically; but their product is measured by including proxies r_{-i} in the hedonic rent function because:

[14] The indirect effect is added across all the neighboring properties, as indicated by the two matrix multiplication operations in the indirect-effects term — recalling that for a given i, both $\partial n_i/\partial r'_{-i}$ and $dq_{-i}/d\theta$ are vectors, the former with a zero value for component i.

(18b)
$$\frac{dr_i}{dr'_{-i}} = \hat{r}_{ni} \frac{dn_i}{dr'_{-i}}$$

where $r(x_i, q_i, r_{-i}) \equiv \hat{r}[x_i, q_i, n_i(r_{-i})]$ is the rent function regarded as a function of observables, i.e. the hedonic function that is estimated empirically. Substituting (18b) into (18a):

(18c)
$$\begin{aligned} M_\theta &= \sum_i \left(r_{qi} \frac{dq_i}{d\theta} + \frac{\partial r_i}{\partial r'_{-i}} \frac{dr_{-i}}{dq'_{-i}} \frac{dq_{-i}}{d\theta} \right) - \sum_i \frac{dr_i}{d\theta} \\ &= \sum_i \left(\frac{\partial r_i}{\partial q'_i} \frac{dq_i}{d\theta} + \frac{dr_i}{dq'_{-i}} \frac{dq_{-i}}{d\theta} \right) - \sum_i \frac{dr_i}{d\theta} \\ &= \sum_i \frac{dr_i}{d\mathbf{q}'} \cdot \frac{d\mathbf{q}}{d\theta} - \sum_i \frac{dr_i}{d\theta} \ . \end{aligned}$$

The aggregate benefit to landlords is still given by (5). When we add the benefits to households and landlords, the last term in (18c) is canceled by (5) just as in the conventional analysis of Section 2, leaving:

(19)
$$B_\theta = M_\theta + R_\theta = \sum_i \frac{dr_i}{d\mathbf{q}'} \cdot \frac{d\mathbf{q}}{d\theta} = \sum_i \left(\overbrace{r_{qi} \frac{dq_i}{d\theta}}^{\text{direct}} + \overbrace{\frac{dr_i}{dr'_{-i}} \frac{dr_{-i}}{dq'_{-i}} \frac{dq_{-i}}{d\theta}}^{\text{indirect}} \right).$$

Equation (19) shows the aggregate benefit as a sum of direct and indirect effects to each household. The direct effect is the same as the conventional result (6), whereas the indirect (spillover) effect captures the end result of accounting for mutual price interactions. Furthermore, the two ways of writing equation (19) correspond to the two ways of writing the rent change in the empirical model, equations (13a) and (13b).

In the case considered before of a uniform improvement in pollution levels and an empirical specification in which **r** is linear in *q*, these direct and indirect effects are given by (13c), and their sum implies an aggregate benefit of:

$$B_\theta = N(1-\rho)^{-1}\beta_q . \tag{20}$$

This is equal to (9) multiplied by $(1\text{-}\rho)^{-1}$. Thus if technological externalities motivate the spatial lag specification, the spatial-multiplier approach to benefits measurement correctly measures the welfare effect of a uniform improvement.[15]

Both Pecuniary and Technological Spatial Effects

What if *both* pecuniary and technological spatial effects motivate a spatial-lag specification such as (10)? We could then model the corresponding location choice problem as

$$\underset{z,x,q,n,\lambda}{Max}\ \Lambda_i \equiv u_i(z,x,q,n) + \lambda_i \cdot [y_i - z - r(x,q,c,n)], \tag{21}$$

[15] An analogous, but more complex result holds for the more general structural specification $\mathbf{r} = \rho W\mathbf{r} + X\beta + WX\gamma + \varepsilon$, where the aggregate marginal price effect of a uniform air quality improvement is $N(1-\rho)^{-1}(\beta_q + \gamma_q)$ for the case where *X* is linear in *q*. Now we could consider separately whether the spatial spillover represented by each of the terms $\rho W\mathbf{r}$ and $WX\gamma$ is pecuniary or technological. Depending on the answers, a spatial multiplier might be applied to neither or to one or both parts of the marginal direct benefit measure $N\beta_q$+$N\gamma_q$.

where now the technological externality n is included in both utility and the rent function, but the pecuniary externality c appears only in the rent function. Equation (18a) still applies, but it is no longer equivalent to a simple spatial multiplier because the right-hand side of (18b) will now contain an additional term, $\tilde{r}_{ci} \cdot dc_i / dr'_{-i}$. Therefore, (18a) no longer reduces to the simple form (19). Rather, in order to measure welfare change, we would need some way to postulate or estimate the relative importance of the two forces as measured by the terms $\tilde{r}_{ci} \cdot dc_i / dr'_{-i}$ and $\hat{r}_{ni} \cdot dn_i / dr'_{-i}$. Suppose we postulate that the terms are of relative sizes (1-α) and α, respectively, where α is the relative importance of technological externalities in explaining the spatial-lag term in the empirical specification. Then (19) and (20) are replaced as measures of total benefits by:

(22) $$B_\theta = \sum_i \left(\overbrace{r_{qi} \frac{dq_i}{d\theta}}^{\text{direct}} + \overbrace{\alpha \cdot \frac{dr_i}{dr'_{-i}} \frac{dr_{-i}}{dq'_{-i}} \frac{dq_{-i}}{d\theta}}^{\text{indirect}} \right)$$

and:

(23) $$B_\theta = N \cdot \left[1 + \alpha\rho(1-\rho)^{-1}\right] \cdot \beta_q .$$

As the relative importance of the indirect effect's technological externalities varies from zero to one, the benefit measure varies from one that includes only direct effects to one that fully incorporates the spatial multiplier.

4. Review of Spatial-Multiplier Evidence

Several recent papers employ the spatial-multiplier approach to estimate the benefits of residential amenity improvements, based on structural specifications that include spatially-lagged equilibrium property values. Here we briefly review some of their findings to numerically demonstrate the importance of distinguishing between pecuniary and technological spatial effects.

From the above analysis we see that if spatial spillovers are *strictly* pecuniary and the model is correctly specified as a spatial-lag model, but the spatial-multiplier approach to benefits measurement is employed, then benefits will be incorrectly inflated by the magnitude of the spatial multiplier, $(1-\rho)^{-1}$. But if the spillovers are *strictly* technological, then ignoring the spatial multiplier will understate these benefits by a factor $(1-\rho)$.

For example, Kim et al. (2003) estimate the benefits of a uniform reduction in air pollution and report a spatial-lag parameter estimate of ρ=0.55, implying a spatial multiplier of 2.22. Thus if the spatial spillovers in their sample are strictly pecuniary, then the spatial-multiplier approach would overstate benefits by 122%. But if those spillovers are strictly technological, then omitting the spatial multiplier when calculating benefits would understate them by 55%. If the spatial-lag parameter estimate reflects a mixture of pecuniary and technological effects, then the true benefits are somewhere between 45% and 100% of those calculated in their paper.

Table 1 provides the spatial-lag parameter estimates and corresponding spatial multipliers reported by several recent studies. The table implies that benefit estimates can differ substantially according to the presumed economic nature of spatial spillovers. Of course, they might depart even further if the correct structural model is not in fact compatible with the empirical model being estimated.

Table 1: Spatial Parameter and Multiplier Estimates from Recently Published Studies

Study	Amenity Considered	Spatial Parameter ρ	Spatial Multiplier $(1-\rho)^{-1}$
Kim et al. (2003)	Air Quality	0.55	2.22
Anselin et al. (2008)[a]	Access to Potable Water	0.24	1.32
Cohen & Coughlin (2008)[b]	Airport Noise	0.54	2.17
Andersson et al. (2009)[a]	Road and Railway Noise	0.00–0.52	1.00–2.08
Anselin & Lozano-Gracia (2009)[a]	Air Quality	0.33	1.49
Kim & Goldsmith (2009)	Distance from Animal Feeding Operations	0.14–0.23	1.16–1.30

[a] These studies discuss the distinction made here between pecuniary and technological spatial externalities, citing an earlier draft of our paper.

[b] This study estimates airport noise-reduction benefits from a semi-logarithmic autoregressive specification in which noise levels are characterized by dummy variables. Applying a spatial multiplier to its relevant parameter estimates increases estimated benefits by 145% — see Steimetz (2010).

5. Conclusion

The fairly recent development of spatial-hedonic housing-price models introduces a spatial dimension to estimating the willingness to pay for residential amenity improvements. In particular, spatial-lag models have given rise to a spatial-multiplier approach, where both the direct and spatial-spillover effects of an improvement are included in benefits measurement. We demonstrate that this approach is only appropriate when spatial dependence among properties is strictly technological, as opposed to pecuniary. Moreover, we demonstrate that even when this dependence is correctly modeled from an empirical standpoint, differing assumptions about the nature of spatial dependence can drastically change the estimated benefits of an amenity improvement.

The intuition for our findings is straightforward. If, for example, reduced pollution increases my neighbors' property values, thereby increasing the value of my house, but does not further improve the amenity value of my house, then the spatial effect is pecuniary and, therefore, welfare-neutral. If, on the other hand, I derive increased utility through my neighbors' rise in property values, then the spatial effect is technological and is appropriately included in welfare analysis. In the former case, the direct coefficient on pollution in a spatial-lag specification produces the correct measure, whereas in the latter case the spatial-multiplier approach produces the correct measure.

We have not attempted to provide an empirical means to determine whether spatial dependence is generated by technological or pecuniary externalities. Doing so would presumably require data on the mechanism underlying spatial price interactions; for instance, temporal sales data might allow for identifying the nature of housing-price spillovers through sales-price comparisons, following Pace et al. (1998). Such efforts may be especially worthwhile in light of our discussion of cases when pecuniary and technological effects are empirically confounded. They might also bring to light situations where the spatial lag model is misspecified and hence neither of the welfare measures we discuss is correct.

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