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# HOW WE MAY HAVE BEEN MISLED INTO BELIEVING IN THE INTERPERSONAL COMPARABILITY OF UTILITY[1]

## INTRODUCTION

The problem of intercomparability of utilities appears naturally in the development of welfare economics (Robbins, 1935, 1938; Samuelson, 1963; Plott, 1976). It also arose in the theory of games when von Neumann and Morgenstern (1947) provided an expected utility interpretation to the payoffs resulting from mixed strategies and, at the same time, incorporated transferability of utility in their coalition theory of $n$-person games. A recent summary of the literature is provided by Sen (1979). It appears to us that there has been relatively modest progress toward a resolution of this problem. A recent attack on it is given in Nozick (1981), a draft of which stimulated the present work.

Many economic theorists have argued that interpersonal comparisons of utilities are impossible. Their arguments are usually based on principles similar to the following by Jevons in his influential *The Theory of Political Economy*:

> The reader will find, again, that there is never, in any single instance, an attempt made to compare the amount of feeling in one mind with that in another. I see no means by which such comparison can be accomplished. The susceptibility of one mind may, for what we know, be a thousand times greater than that of another. But, provided that the susceptibility was different in a like ratio in all directions, we should never be able to discover the difference. Every mind is thus inscrutable to every other mind, and no common denominator of feeling seems to be possible. But even if we could compare the feelings of different minds, we should not need to do so; for one mind only affects another indirectly. Every event in the outward world is represented in the mind by a corresponding motive, and it is by the balance of these that the will is swayed. But the motive in one mind is weighed only against the motives in other minds. Each person is to other persons a portion of the outward world – the *non-ego* as the meta-physicians call it. Thus motives in the mind of $A$ may give rise to phenomena which may be represented by motives in the mind of $B$; but between $A$ and $B$ there is a gulf. Hence the weighing of motives must always be confined to the bosom of the individual.
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> Jevons, 1957, p. 14; the first edition of *Theory of Political Economy* appeared in 1871.

Other economic theorists have argued against this view. I. M. D. Little writes,

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No one could 'deny' interpersonal comparisons in the sense that they deny that people make them. Therefore those economists who 'deny' them must think that when a person says '$A$ is happier than $B$' he is deluding himself in thinking that he is making a statement of fact. But why should he be deluding himself? Why should it not be a statement of fact? It is probable that what is behind the idea that it is not a statement of fact, that one is not describing something one experiences, when one says '$A$ is happier than $B$', is some vague metaphysical doubt about the existence of minds other than one's own. I say about the *existence* of other minds, because nothing short of denying their existence can entitle one to say that other minds cannot be compared. If one admits that another man's behaviour, including his speech, is evidence for his having a mind, then one must admit that one can use such behavior as a good basis for saying what sort of mind he has, or what sort of a mental state he is in; that is, for saying that he is stupid or intelligent, happy or miserable, angry or pleased, and so on. But if one can say that $A$ is happy and pleased, and $B$ is miserable and angry, then one has compared their mental states. Happy and angry are relative words, but they are not relative merely to some other state of the same man. We can say of a man that he is habitually miserable, or that he has a disposition to be miserable. Obviously we cannot be meaning that he has a disposition to be more miserable than he usually is. We mean that he has a disposition to be more miserable than men usually are. We have some vague standards of happiness and misery. In other words, we use different men's behavior, in a wide sense of the word, to compare their mental states; and if we say of a man that he is always miserable, basing our judgment on how he looks and behaves, and how we know we would feel if we looked and behaved like that, and on a wide knowledge of his character gather by observing his behavior and words in a variety of situations, and on the opinions of all his friends who similarly know him well, then we would think it was just nonsense to say that he might really be deceiving everyone all the time and be the happiest of men. It is a mistake to suppose that another man's mind consists solely of feelings or images which one cannot ever experience (that is, that one's mind is a logical construction of personal feelings and images which are, by definition, not open to inspection by anyone else).
Little, 1957, pp. 54–55.

Despite the fact that scientists have experienced considerable conceptual difficulty in uncovering a principled way to assign meaning to the statement "the utility of outcome $a$ to Person 1 is greater (or less) than the utility of outcome $b$ to Person 2," most of us agree with Little that such interpersonal comparisons are more or less successfully made in our daily lives. After all, what else could be involved when one spouse agrees to forego some desired object or activity in order to let the other have or do what he or she wishes because "It means more to my spouse than it does to me"? It is a common experience for most of us that as we get to know someone, we increasingly learn more about their preferences and how the strengths of those preferences compare with our own. One piece of evidence for this developing comparison of utilities is the fact that when confronted with choices between outcomes, two people increasingly find themselves in agreement about whose preference is the stronger. We suspect that it is the developing concensus of two people in close interaction that is the principal basis for the intuition that interpersonal comparisons of utility take place in the everyday world.

Our aim in this paper is to suggest that this conclusion is probably faulty by demonstrating the existence of mechanisms that lead to total agreement about comparative preference without forcing any true intercomparability of utility. Moreover, the sort of empirical observations needed to argue for the existence/nonexistence of a true comparability of utility are discussed.

## ASYMPTOTIC THEORY

Suppose two people, $P_1$ and $P_2$, have numerical utility functions, $u_1$ and $u_2$, respectively, that are defined over (not necessarily identical) domains of outcomes, $A_1$ and $A_2$. In order for $P_1$ to be able to answer the question,

> "Does $a_1$ from $A_1$ have more utility for me than $a_2$ from $A_2$ has for $P_2$?",

we assume that $P_1$ has a model, call it $v_{12}$, of $P_2$'s utility function. Let $\succsim_1$ be the ordering that is defined between $A_1$ and $A_2$ as follows: for each $a_1$ in $A_1$ and $a_2$ in $A_2$,

$$(1) \qquad a_1 \succsim_1 a_2 \quad \text{iff} \quad u_1(a_1) \geqslant v_{12}(a_2).$$

We refer to $\succsim_1$ as $P_1$'s *interpersonal ordering*. In like manner, $P_2$ is assumed to have a model, $v_{21}$, of $P_1$'s utility function which leads to $P_2$'s interpersonal ordering $\succsim_2$ defined by: for all $a_1$ in $A_1$ and $a_2$ in $A_2$,

$$(2) \qquad a_1 \succsim_2 a_2 \quad \text{iff} \quad v_{21}(a_1) \geqslant u_2(a_2).$$

If, in fact, these two interpersonal ordering coincide, i.e., $\succsim_1$ and $\succsim_2$ are identical, then we say that $P_1$ and $P_2$ have achieved *ordinal intercomparability of utilities*.

Several question arise naturally from this development. First, how do models of other people's utility functions and interpersonal orderings come about? Second, how is ordinal intercomparability achieved? And third, what is its relationship to what is commonly referred to as "interpersonal comparison of utility"? All of these will be discussed in the paper.

We begin by investigating the conditions that are imposed upon the models of the utility functions by the assumption that ordinal intercomparability exists.

OBSERVATION 1. *For every strictly increasing function g, if* $v_{12} = g \circ u_2$ *and* $v_{21} = g^{-1} \circ u_1$ *(where* $\circ$ *denotes the composition of functions), then* $P_1$ *and* $P_2$ *can achieve ordinal intercomparability with the interpersonal orderings* $\succsim_1$ *and* $\succsim_2$ *defined by Equations (1) and (2).*

*Proof.*
$$\begin{aligned} a_1 \succsim_1 a_2 \quad &\text{iff} \quad u_1(a_1) \geqslant v_{12}(a_2) = g \circ u_2(a_2) \\ &\text{iff} \quad v_{21}(a_2) = g^{-1} \circ u_1(a_1) \geqslant u_2(a_2) \\ &\text{iff} \quad a_1 \succsim_2 a_2. \end{aligned}$$
■

Next we show that Observation 1 essentially captures all ordinal intercomparisons:

OBSERVATION 2. *Suppose* $u_2$ *and* $v_{21}$ *are onto the same nontrivial (possibly infinite) interval and that* $P_1$ *and* $P_2$ *have achieved ordinal intercomparability. Then for some strictly increasing function g,*

$$v_{12} = g \circ u_2 \quad \textit{and} \quad v_{21} = g^{-1} \circ u_1.$$

*Proof.* Let $c_2$ and $d_2$ be any elements of $A_2$ for which $u_2(c_2) > u_2(d_2)$. Since $u_2$ and $v_{21}$ are onto the same nontrivial interval, let $b_1$ in $A_2$ be such that $u_2(c_2) > v_{21}(b_1) > u_2(d_2)$. By ordinal intercomparability, $v_{12}(c_2) > u_1(b_1) > v_{12}(d_2)$. But $u_2(c_2) > u_2(d_2)$ iff $v_{12}(c_2) > v_{12}(d_2)$ for all $c_2$, $d_2$ in $A_2$ can only happen if $v_{12}$ is a strictly increasing function of $u_2$, i.e., $v_{12} = g \circ u_2$ for some strictly increasing $g$. Then by ordinal intercomparability, $v_{21} = g^{-1} \circ u_1$. ■

We next consider the question as to how unique is the $g$ of Observation 2. The next observation formulates the fact that it is completely unique provided that there is an adequate density of equivalent comparisons.

OBSERVATION 3. *Suppose, as in Observation 2, there are two models g and h that produce exactly the same ordinal intercomparability and that for each* $a_2$ *in* $A_2$ *there is an* $a_1$ *in* $A_1$ *such that* $a_1 \sim_1 a_2$*. Then,* $g = h$.

*Proof.* Since $g$ and $h$ are each increasing functions, there is an increasing function $f$ such that $h = f \circ g$. Select any $x$ in the range of $u_2$, and let $a_2$ be such that $x = u_2(a_2)$. By hypothesis there exists an $a_1$ such that $a_1 \sim_1 a_2$. Since $g$ and $h$ each yield models,

$$\begin{aligned} a_1 \sim_1 a_2 \quad &\text{iff} \quad u_1(a_1) = g \circ u_2(a_2) = g(x) \\ &\text{iff} \quad u_1(a_1) = f \circ g \circ u_2(a_2) = f \circ g(x). \end{aligned}$$

So, $g(x) = f \circ g(x)$, whence $f$ is the identity function. ∎

Some have suggested that one person's model of another's utility is some linear function of the true utility function. In our notation this assumption takes the following form: there exist constants $\alpha_i > 0$ and $\beta_i$, $i = 1, 2$, such that

$$(3) \qquad v_{ij} = \alpha_i u_j + \beta_i, \quad i, j = 1, 2, \quad i \neq j.$$

(Throughout, we assume all linear functions to be 'positive' in the sense that the $\alpha$ parameter is positive.)

If we are willing to assume only linear models of the form of Equation (3), then the structural hypothesis of Observation 3 can be considerably weakened to the following: there exist elements $b_i$, $b_i^*$ in $A_i$, $i = 1, 2$ such that

$$b_1 \sim_i b_2, \quad b_1^* \sim_i b_2^*, \quad \text{and} \quad u_i(b_i) > u_i(b_i^*), \quad i = 1, 2.$$

The proof simply assumes two linear models, applies them to these equations, and then solves to show that the corresponding parameters must be identical.

What are these observations saying about the intercomparability of utility? First, Observations 1 and 2 say that the mere existence of ordinal intercomparability in no way forces a particular utility comparison. Second, Observation 3 raises the possibility that knowledge of a particular ordinal intercomparison might imply a unique utility comparison. Each of these points will be pursued more fully. In the next section we consider whether Observations 1 and 2 can be overcome by admitting the existence of additional ordinal information; our conclusion will be that they cannot. In the section after that it is pointed out, as is quite compatible with Observation 1, that the existence of a particular ordinal intercomparison is probably highly accidental in nature and so the apparent uniqueness of the models established in Observation 3 is equally accidental and of no real significance in establishing utility comparisons. In the final section, use of this is then made to arrive at an empirical prediction that is the negation of a consequence of the assumption of the intercomparability of utility.

## AGGREGATION THEORY

As was just noted, the main significance of Observation 1 is that the existence of ordinal intercomparability, by itself, is not sufficient to determine a unique intercomparability of utility. The question must next be raised whether some additional ordinal information is sufficient to pin down the comparisons of utility.

Suppose $P_1$ were asked,

> "Is it true that $a_1$ given to you and $a_2$ given to $P_2$ is of more benefit to you and $P_2$ jointly than $b_1$ given to you and $b_2$ and $P_2$?"

Let us assume that people can answer such questions, which we formalize as follows: $P_1$ has an ordering $\succsim_1'$ on $A_1 \times A_2$ and a numerical function $H_1$ on Re $\times$ Re such that, for all $(a_1, a_2), (b_1, b_2)$ in $A_1 \times A_2$,

$$(4) \qquad (a_1, a_2) \succsim_1' (b_1, b_2) \quad \text{iff}$$

$$H_1[u_1(a_1), v_{12}(a_2)] \geqslant H_1[u_1(b_1), v_{12}(b_2)].$$

We call $H_1$ the *joint utility function* of $P_1$. It is said to be *additive* iff $H_1(x, y) = x + y$. We assume that $P_2$ also has an ordering $\succsim_2'$ on $A_1 \times A_2$ and the corresponding joint utility function $H_2$. If $\succsim_1'$ and $\succsim_2'$ are identical, then $P_1$ and $P_2$ agree on the joint utility comparisons, and we say that they have achieved *conjoint intercomparability of utilities*.

OBSERVATION 4. *Suppose $P_1$ and $P_2$ have achieved ordinal intercomparability, conjoint intercomparability, and have additive joint utility functions. Suppose further that the utility functions $u_i$ and $v_{ij}$ are all onto the real numbers. Then,*

(i) *$v_{ij}$ is a linear function of $u_j$, $i, j = 1, 2, i \neq j$; and*
(ii) *each linear function $g$ of $u_2$ gives rise to another ordinal intercomparability with new modelling functions*

$$v_{12}^* = g \circ u_2 \quad \textit{and} \quad v_{21}^* = g^{-1} \circ u_1,$$

*and to conjoint intercomparability with these modeling functions and with additive joint utility functions.*

*Proof.* (i) Let $w$, $x$, $y$, and $z$ be arbitrary real numbers. By ordinal intercomparability and Observation 2, let $g$ be a strictly increasing function such that

$$(5) \qquad v_{12} = g \circ u_2 \quad \text{and} \quad v_{21} = g^{-1} \circ u_1.$$

Since by hypothesis, $u_1$ and $u_2$ are onto the reals, $u_1$ takes on the values $w$ and $y$ and $u_2$ the values $x$ and $z$. Then, by conjoint intercomparability with additive joint utility functions and Equation (5),

$$w + g(x) \geqslant y + g(z) \quad \text{iff} \quad g^{-1}(w) + x \geqslant g^{-1}(y) + z.$$

Since by hypothesis, $v_{12}$ and $v_{21}$ are onto the reals, choose $u$ and $v$ so that $g(u) = w$ and $g(v) = y$. Then

$$g(u) + g(x) \geqslant g(v) + g(z) \quad \text{iff} \quad u + x \geqslant v + z.$$

Since this holds for all real $x$, $u$, $v$, $z$, then by the uniqueness theorem for additive conjoint structures (Theorem 6.2 of Krantz *et al.*, 1971), it follows that $g$ must be linear.

(ii) This follows from the same uniqueness result of Krantz *et al.* and the reversal of the argument given in (i). ■

Observation 4 establishes that ordinal and conjoint comparability together do not provide enough information to allow $P_1$ and $P_2$ to recover each other's utility functions when their joint utility functions are additive. However, it is worth noting that the additional conjoint information further restricts the possible models $v_{ij}$ of $u_j$ determined by the ordinal intercomparability, from $v_{ij}$ being a strictly increasing function of $u_j$ (Observations 1 and 2) to $v_{ij}$ being a linear function of $u_j$ (Observation 4). Additivity seems the most natural form for joint utility functions, and one that can be motivated by a variety of considerations. However, if this assumption were to be rejected, then for certain other forms a unique model of the other person's true utility function results. But whether this can be put to effective use to obtain true intercomparability is another matter, one that we will discuss subsequently.

OBSERVATION 5. *Suppose* $u_i$ *and* $v_{ij}$ *are onto the nonnegative reals and* $u_j(a_j) = 0$ iff $v_{ij}(a_j) = 0$. *Suppose that ordinal and conjoint intercomparability have been achieved, the latter with joint utility functions satisfying, for all positive reals* $x, y$,

$$(6) \qquad H_i(x, y) = x + y + x^2y^2, \quad i = 1, 2.$$

*Then,* $v_{12} = u_2$ *and* $v_{21} = u_1$.

*Proof.* Let $H = H_i$, $i = 1, 2$. Example 4.2 of Cohen and Narens (1979) shows that the structure $\langle \mathrm{Re}, \geqslant, H \rangle$ has the identity as its only automorphism.[2] By Observation 2, let $g$ be a strictly increasing function such that $v_{12} = g \circ u_2$ and $v_{21} = g^{-1} \circ u_1$. Then, as in the proof of Observation 4,

$$H[g(u), g(x)] \geqslant H[g(v), g(z)] \quad \text{iff} \quad H(u, x) \geqslant H(v, z)$$

for all nonnegative $u$, $x$, $v$ and $z$. In the language of Luce and Cohen (1983), $\langle g, g\rangle$ is a factorizable order automorphism of $H$. For any fixed $x_0, y_0$, they define the mapping $\pi(x)$ by $H[x_0, \pi(x)] = H(x, y_0)$ and the binary operation $*$ by $H(x * y, y_0) = H[x, \pi(y)]$. $*$ is said to be *induced* by $x_0, y_0$. Their Theorem 8 establishes that if $\langle g, g\rangle$ is a factorizable order automorphism of $H$, then $g$ is an isomorphism of the structure $\langle \mathrm{Re}, \geqslant, *\rangle$ induced by $x_0, y_0$ onto the structure $\langle \mathrm{Re}, \geqslant, *'\rangle$ induced by $x_0' = g(x_0)$, $y_0' = g(y_0)$. In this case, we see $H(x, 0) = H(0, x) = x$, and so if we set $x_0 = y_0 = 0$, we obtain

$$x = H(x, 0) = H[0, \pi(x)] = \pi(x),$$

$$x * y = H(x * y, 0) = H[x\pi(y)] = H(x, y),$$

and

$$x_0' = g(x_0) = g(0) = 0, \quad y_0' = g(y_0) = g(0) = 0.$$

Thus, the isomorphism of $\langle \mathrm{Re}, \geqslant, *\rangle$ to $\langle \mathrm{Re}, \geqslant, *'\rangle$ becomes an automorphism of $H$ as an operation. But as Cohen and Narens (1979) showed, the only automorphism of this structure is the identity, which yields the conclusion. ∎

The joint utility functions of Equation 6 are rather artificial. They were used so that a known result could be quoted in order to keep the proof of Observation 5 short. The key to the proof is that the joint utility functions are identical and admit the identity as their only strictly increasing automorphism. One could use the methods developed in Section 4 of Cohen and Narens (1979) to construct more plausible looking joint utility functions that also have the identity as their only strictly increasing automorphism.

One is tempted to try and use Observation 5 as a basis for utility intercomparison. To this end, one would get $P_1$ and $P_2$ to adopt $H_1 = H_2$ given in Equation (6) as their joint utility functions, and try and develop a procedure that would lead asymptotically to ordinal and conjoint intercomparability. For the sake of argument, let's suppose this has been accomplished. Now have $P_1$ and $P_2$ in any real sense recovered each other's utility functions? We think not. Our argument is based upon meaningfulness considerations of measurement theory.

The basic difficulty is this: the transformations on the utility functions that leave invariant ordinal intercomparability do not leave invariant the interpersonal conjoint orderings determined by the $H$ of Equation (6). To be explicit, *suppose that for some real, strictly increasing* $\varphi$,

$$(7) \qquad u_1^* = \varphi \circ u_1,$$

*and that the pairs* $u_1$, $v_{12}$, *and* $u_1^*$, $v_{12}^*$ *yield [by Equations (1) and (2)] identical interpersonal orderings. Let* $H_1$ *be given by Equation (6), and suppose that* $u_1$, $v_{12}$, $H_1$ *and* $u_1^*$, $v_{12}^*$, $H_1$ *yield [by Equation (4)] the interpersonal conjoint orderings* $\succsim_1'$ *and* $\succsim_1'^*$, *respectively. Then,* $u_1 = u_1^*$ *iff* $\succsim_1' = \succsim_1'^*$.

*Proof.* From Equation (7) and the fact that for all $a_1$ in $A_1$ and $a_2$ in $A_2$,

$$(8) \qquad a_1 \succsim_1 a_2 \quad \text{iff} \quad u_1(a_1) \geqslant v_{12}(a_2) \quad \text{iff} \quad u_1^*(a_1) \geqslant v_{12}^*(a_2),$$

we see that

$$(9) \qquad v_{12}^* = \varphi \circ v_{12}.$$

So, $\succsim_1'$ is determined by $H_1(u_1, v_{12})$ and $\succsim_1'^*$ by $H_1(\varphi \circ u_1, \varphi \circ v_{12})$. Thus, $\succsim_1' = \succsim_1'^*$ iff $\varphi$ is an automorphism of $H_1$. By Example 4.2 of Cohen and Narens (1979) this means that $\varphi$ is the identity, whence $\succsim_1' = \succsim_1'^*$ iff $u_1 = u_1^*$.

Since we want $\succsim_1'$ and $\succsim_1'^*$ to be identical whenever we are dealing with functionally equivalent utility functions, we see that the freedom of representing the interpersonal order is incompatible with the lack of freedom permitted by the joint utility function of Equation (6). Put in the language of measurement theory, the interpersonal conjoint ordering induced by Equation (6) is not meaningful vis-a-vis the interpersonal orderings.

The above remarks can be summarized as follows:

OBSERVATION 6. *There are severe difficulties – perhaps insurmountable – in employing situations similar to the one in Observation 5 to achieve true interpersonal comparisons of utilities.*

It is worthwhile to note that if $P_1$ and $P_2$ have interval scale families of utility functions and achieve ordinal and conjoint intercomparability with additive joint utility functions, then assumptions of meaningfulness provide no extra restrictions or difficulties, as is readily seen by applying Observation 4.

Much more can be said about the impact of meaningfulness considerations upon joint utility functions and other kinds of joint utility relations, but to do so would cause us to stray from the main topic of the paper.

## DYNAMICAL CONSIDERATIONS

OBSERVATION 7. *Ordinal intercomparability is possible.*

*Proof.* Suppose that $P_1$ and $P_2$ know each other's utility functions up to strictly monotonic transformations. This can arise in a variety of ways: an exchange of information about these utility functions, or mutual witnessing of preference behavior, or by other processes. Let $P_1$ select some model of $P_2$'s utility that satisfies two conditions: $v_{12} = g \circ u_2$, where $g$ is strictly increasing, and $v_{12}$ has the same range as $u_1$, say the reals. The former is possible because $P_1$ knows $u_2$ up to strictly increasing function, and the latter is not particularly restrictive. Now, suppose $P_1$ informs $P_2$ fully of the interpersonal ordering $\succsim_1$ that arises from comparing $u_1$ and $v_{12}$ and that $P_2$ elects to accept $\succsim_1$ as his or her own interpersonal ordering $\succsim_2$. By Observations 1 and 2, there is one and only one model of $u_1$ that is consistent with $\succsim_2$, namely, $v_{21} = g^{-1} \circ u_1$. From his knowledge of $\succsim_2$, $P_2$ can readily construct $v_{21}$. ■

Although the dynamics used to establish ordinal intercomparability in Observation 6 are quite artificial, the result does nonetheless illustrate two important points. First, ordinal intercomparability arises as a result of a particular social process, and the asymptotic intercomparability may vary with the process – in the proof, it changes as $P_1$ changes his or her choice of $g$. Second, under suitable social conditions, ordinal intercomparability may in fact be achieved.

The next question to consider is the uniqueness of the asymptotic ordinal intercomparisons which we now call *equilibrium points*. In considering this, three distinct sources of potential non-uniqueness must be distinguished. First, there is the question of where the process begins. In the example of Observation 7, this beginning point is the choice of $g$, and it is obvious that the resulting equilibrium varies with the choice of $g$. We do not know how widespread this phenomenon is, but we suspect it is true of many social dynamics. Second, there is the question of the impact of the choice of a particular social dynamic. For example, for the situation described in the proof of Observation 7, suppose the dynamic were as follows: the initial models are $g_1 \circ u_2$ and $g_2 \circ u_1$, following which each person tells the other the $g$ being used, and they then decide to use the models $g_2^{-1} \circ g_1 \circ u_2$ and $g_1^{-1} \circ g_2 \circ u_1$. This results in the equilibrium $(g_2^{-1} \circ g_1, (g_2^{-1} \circ g_1)^{-1})$ which is distinct from $(g_1, g_1^{-1})$ except if $g_2$ is the identity function. Third, and most interesting, is the question whether with a fixed dynamic process there are distinct sequences of experiences that lead to distinct equilibria. This is difficult to deal with precisely for two reasons, namely, (i) how to characterize a broad class of plausible dynamic rules and (ii) how to characterize the class of experiences that ultimately lead to equilibria. So far the authors have constructed one very specific dynamic model that for finite sets of alternatives does in fact converge to equilibria which differ according to the order of experience. We cannot at this time determine how general this result is, but we conjecture it is quite general. Note that the extent that the starting point affects the equilibrium of a noncommutative dynamic process is determined by the order of experience. For example, consider two sets of experiences that differ only in the order of the first two interpersonal comparisons. Since the dynamic process is assumed to be noncommutative, the process enters into the remaining set of common experiences at different starting points, which by hypothesis is likely to lead to different equilibria. These miscellaneous remarks can be summarized as follows:

OBSERVATION 8. *For at least one process of adjustment for each person's model of the other's utility, the process ends up at an equilibrium. Moreover, if the initial point is not an equilibrium, the end points differ as a function of the starting point. We conjecture that the asymptotic version of this statement is true for all plausible adjustment processes that do not begin too close* *to any equilibria and for which the starting points are not too similar. Further, we conjecture that from a non-equilibrium starting point, the end point equilibrium will vary as a function of the order of experience.*

## CONCLUSIONS AND EMPIRICAL IMPLICATIONS

We initially noted, as many before us have, that some pairs (or groups) of people seem to achieve interpersonal comparison of utility. Upon reformulating what such a comparison might consist of, we arrive at the concept of ordinal intercomparability. Although ordinal intercomparability leads to interpersonal agreement (Observation 3), it does not do so in a unique manner (Observations 1 and 2). Even adding more powerful conjoint comparisons does not necessarily lead to uniqueness (Observation 4). There are special cases where such conditions do lead to uniqueness (Observation 5). But because of meaningfulness considerations it is doubtful that these can be used as a basis for true intercomparability (Observation 6). There are dynamic processes that can in fact lead to ordinal intercomparability (Observation 7), but the outcome of these is likely to depend upon social and accidental matters quite external to utility considerations of the parties involved (Observation 8). If this accidental character is, in fact, correct, it should be empirically testable, and we will shortly indicate one form such a test might take.

We should perhaps restate here that the principal aim of this paper is to examine cases where individuals reach interpersonal agreement among themselves. These form only one variety of situations where interpersonal comparisons play a central role, but a variety that we believe is critical for the establishment of the 'existence' of true intercomparability, and it is the one that researchers in favor of such intercomparisons invariably cite as the basis for the belief that such intercomparisons can be carried out successfully.

We will now extend our investigation from the two individual case to situations involving three individuals.

Suppose there are three people $P_1$, $P_2$, and $P_3$ with utility functions $u_1$, $u_2$, and $u_3$. Suppose $P_1$ and $P_2$ have interacted together sufficiently that they have a common interpersonal ordering, which we denote $\succsim_{12}$. Similarly, suppose that $P_2$ and $P_3$ have, quite independently of $P_1$, interacted and arrived at $\succsim_{23}$, and that $P_1$ and $P_3$, independent of $P_2$, have arrived at $\succsim_{13}$. In our understanding of the traditional view of the interpersonal comparisons of utility, one would predict: for all $a_i$ in $A_i$, $i = 1, 2, 3$,

$$(10) \qquad \text{if} \quad a_1 \succsim_{12} a_2 \quad \text{and} \quad a_2 \succsim_{23} a_3, \quad \text{then} \quad a_1 \succsim_{13} a_3.$$

Put it in words: if $a_1$ means more to $P_1$ than $a_2$ does to $P_2$, and in turn $a_2$ means more to $P_2$ than $a_3$ means to $P_3$, then if there were a true interpersonal comparison of utility applicable to $a_1$, $a_2$, and $a_3$, we would anticipate the result that $a_1$ means more to $P_1$ than $a_3$ does to $P_3$. The following observation illustrates a type of constraint on interpersonal orderings that is imposed by Equation (10):

OBSERVATION 9. *For $i, j = 1, 2, i \neq j$ suppose $u_i$ are given, $v_{ij} = \alpha_{ij}u_j + \beta_{ij}$, for all $a_i$ in $A_i$ and $a_j$ in $A_j$,*

$$a_i \succsim_{ij} a_j \quad \textit{iff} \quad u_i(a_i) \geqslant v_{ij}(a_j),$$

*$\succsim_{ji}$ is the converse of $\succsim_{ij}$, and Equation (10) holds. Also suppose there are $b_i$ and $b_i^*$ in $A_i$ such that*

$$b_1 \sim_{12} b_2, \quad b_2 \sim_{23} b_3, \quad b_1^* \sim_{12} b_2^*, \quad b_2^* \sim_{23} b_3^*,$$

*and*

$$b_1 \succ_{12} b_2^*.$$

*Then*

$$\alpha_{13} = \alpha_{12}\alpha_{23} \quad \textit{and} \quad \beta_{13} = \alpha_{12}\beta_{23} + \beta_{12}.$$

*Proof.* Apply Equation (3) to the given equivalences and collect terms. ∎

The empirical test we suggest is based upon the assumption that when the interpersonal orderings of $P_1$, $P_2$, and $P_3$ are arrived at independently, Equation (10) will most likely not hold. The major empirical task is to devise procedures (either experimental or observational) where we are confident that pairwise ordinal intercomparability holds independently for three people. That done, we obtain the orderings for each pair of the three people and then check for transitivity, Equation (10).

Common observation suggests the outcome. Most of us are familiar with the experience of knowing well two people who have not yet met. Using our own utility function as the common scale, we feel confident in predicting compatibility based on our knowledge of each person's preferences and strengths of preference. Yet when we cause them to meet, conflicts and violations of our predictions unexpectedly occur, much to our chagrin. Apparently, in these situations, independent pairwise consensus does not a group consensus make.

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## NOTES

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[2] An *automorphism* of a function $G$ from $X \times X$ into $X$ is a one-to-one function $\alpha$ from $X$ onto $X$ such that for each $r, s$ in $X$, $\alpha[G(r, s)] = G[\alpha(r), \alpha(s)]$.

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