ASTRONOMICAL AND BIOCHEMICAL ORIGINS AND THE SEARCH FOR LIFE IN THE UNIVERSE
Eds. C.B. Cosmovici, S. Bowyer and D. Werthimer

SURMISING COGNITIVE UNIVERSALS FOR EXTRATERRESTRIAL INTELLIGENCES

Louis Narens
Department of Cognitive Sciences, University of California, Irvine, CA 92697, USA

ABSTRACT

Cognitive universals are concepts that our civilization and technologically advanced extraterrestrial civilizations can easily interpret. The universality of certain mathematically and perceptually based concepts are discussed. It is argued that continuously based concepts are more fertile ground for surmising cognitive universals than discretely based ones, and in particular, one should be suspicious of the use of inductively based numerical concepts, including the totality of natural numbers. Ideas about intuitive evolutionary theory, physical and perceptual invariance, and the efficient processing of information are linked to provide a framework for searching for cognitive universals.

Describing «concepts» that Extraterrestrial Intelligences (ETIs) are likely to process appears at first sight to be impossible: there are no known products of ETI activity to analyze, and any theoretical investigation of ETI cognitions appears to be doomed by the lack of adequate theories of «intelligence». However, if it could be established with a reasonable degree of confidence that there were a large number of ETI «civilizations» with advanced technological capabilities, then perhaps theories relating cognition to technological development could be utilized for making inferences about concepts that are universally needed for developing advanced technologies.

Based upon theories of star and planet formation and biochemical, biological, and evolutionary considerations, many informed astronomers and biologists have reached the conclusion that it is likely that there are numerous extraterrestrial «civilizations» populating our galaxy that have the capability of sending and receiving interstellar messages, including the kinds of messages that our civilization is capable of sending and receiving. This has prompted a respected part of the scientific community to conduct a systematic Search for Extra-Terrestrial Intelligence (SETI). Of course, part of the rationale for the method of searching is grounded in psychological assumptions about the transmitting ETIs[1].

COGNITIVE UNIVERSALS AND SETI

In this article, *cognitive universals* will refer to concepts that are easily interpretable by us and ETI civilizations that (i) are at a technological level to have the capability of sending and receiving communications with other extraterrestrial civilizations, including our own, and (ii) are able to place objects off-planet. (i) is a standard assumption of SETI, and (ii) is assumed to guarantee a pragmatic utilization of our civilizations concept of «local spatial geometry». Civilizations that satisfy (i) and (ii) are called *target civilizations*.

In the literature, cognitive universals have been surmised by various means, including *pragmatic considerations*, e.g., what the ETIs must know to build sending and receiving equipment; *considerations about intelligence*, e.g., consciousness is a highly probable consequence of a high level of intelligence; and *theoretical extrapolation based on geological, biological, and evolutionary facts and theories*. Obviously, in surmising the universality of a concept, care should be taken to account for bias due to our culture (*ethnocentrism*) as well as bias due to our human nature (*homocentrism*). In general, a complete avoidance of these biases may be impossible, since it is easily arguable (by me, at least) that the methods of inference, means, and theories of science are heavily ethno – and homocentric.

The exemplars of cognitive universals put forth in the literature have been constructed out of mathematical, physical, and chemical concepts. It is generally held that elementary arithmetic is necessary for technological development and thus that various cognitive universals can be constructed by arithmetic means. Because there is currently no deep and formal understanding of technological development, reasoned arguments for the necessity of arithmetic for technological development are inherently difficult. The ones I am familiar with appear to be based on some variant of «I can't imagine how it could be otherwise» kind of reasoning.

One way to argue for the *non-universality* of a target concept is to show that its intended function can be accomplished by concepts of a cognitive-like system *about which it can be proven* that the target concept is not derivable within the system. This amounts to producing an «It could be otherwise» example. Such an example will now be presented for arithmetic.

ARITHMETIC

Elementary arithmetic consists of (i) means of adding, multiplying, and comparing in terms of magnitude two individual natural numbers, and (ii) the set of natural numbers. I will allow that (i) is important for technological development; (ii) I find problematic for this purpose. The following is a cognitive-like system that is of fundamental importance to mathematics and science in which (i) holds but (ii) fails.

The system has the following logical symbols: $\forall$, $\exists$, $\neg$, $\wedge$, $\vee$, and $=$, (to be interpreted respectively as «for all», «for some», «not», «and», «or», and «equals»); the following set, relational, and individual symbols: $\Re$, $\oplus$, $\otimes$, $<$, $\mathbf{0}$, and $\mathbf{1}$ (to be interpreted respectively as the set of real numbers, addition, multiplication, less than, the number 0, and the number 1); and variable symbols, x, y, z,..., and separation symbols, (,). Using these symbols, various expressions formulated through the syntax of first-order logic, including sentences like $\forall x \forall y (x \oplus y = y \oplus x)$, which under the above interpretations is a true statement about the *ordered field of real numbers*, that is, a true statement about the structure that has the set of real numbers as its domain and addition, multiplication, less than, 0, and 1 as its operations, relation, and constant symbols.

It is well-known that a particular set Γ of simple sentences formulated in terms of the above symbol system completely characterizes all *algebraic* properties and truths about the ordered field of real numbers. (Γ describes an algebraic structure known as a «real closed ordered field»). The above system of symbols and Γ can express for all pairs of natural numbers, all additions, multiplications, and comparisons in terms of magnitudes. For example, letting **2** stand for the expression (**1**⊕**1**) and **3** stand for ((**1**⊕**1**)⊕**1**), which by convention may be written as (**2**⊕**1**), and **6** stand for (((((**1**⊕**1**)⊕**1**)⊕**1**)⊕**1**)⊕**1**), it then follows from the set of axioms Γ above that **2**⊗**3** = **6**. However, the above symbol system and Γ cannot express the *set* of natural numbers[2].

For the sake of argument, I will allow that analogues of various kinds of calculations of elementary arithmetic are universally important for the development of advanced technologies. However, for this purpose, I see no reason why the analogues need be anything more than combinations of additions and multiplications of *particular* natural numbers. In particular, inductively generated concepts in terms of **1**, ⊕, and ⊗ are not needed.

The above cognitive-like system is an example where simple arithmetic calculations involving natural numbers can be performed without having the general concept of «natural number». Similar examples exist for other parts of mathematics that could be very useful in the development of advanced technologies. For example it can be shown that elementary Euclidean geometry, while having natural and easily formulable concepts of addition and multiplication of lengths of line segments, cannot have a formulable concept corresponding to the set of natural numbers[3].

Of course for human minds it is an easy matter to form a concept of natural number given the above symbol system: ignoring parentheses,

1 corresponds to **1**,
2 corresponds to **1**⊕**1**
3 corresponds to **1**⊕**1**⊕**1**
and in general,
n corresponds to the expression that is n ⊕-additions of **1**; i.e., human intelligence can form the concept of natural number through induction on expressions. I consider this form of induction to be derivative of the human metacognitive ability to talk about and judge with awareness grammatical aspects of natural languages. It is worthwhile to note that humans do not have similar metacognitive abilities for the perceptual system, although perceptions are processed through grammar-like rules. A possible evolutionary explanation for the difference is that the abovementioned metalinguistic abilities are useful for humans learning foreign languages as adults – a skill that allowed for better individual and societal survival from the beginnings of mankind until present; in the perceptual world there was no equivalent need, and as a consequence, humans are unaware of grammatical-like rules they use in perceptual processing.

The above considerations demonstrate that more than the calculative utility of natural numbers for technological development is needed for establishing the universality of inductively generated numerical concepts. For SETI this means that one should be wary of the use of concepts such as «prime number» or a «binary form of the decimal expansion of π» as a common basis for communication with ETIs.

Epistemological considerations

Ontologically, continuous structures are much more complex than discrete ones. However, for the purposes of inference and modelling in science, they appear to be much simpler epistemologically. This is in part due to the fact that many concepts that are of crucial importance in our science have simple and exact definitions in continuous structures, while their counterparts in the discrete case tend to be complex, approximate, and artificial.

Our current science views numbers as platonic objects detached from material reality. Should we expect a similar view from ETIs? I believe not, because such a metaphysical view – independent of its correctness/incorrectness – appears to me to be at best homocentric and is arguably ethnocentric. Should we expect ETIs to have a coherent concept of number? If they have a good understanding of classical physical phenomena, then I think the answer is «Yes», because it can be shown that classical physics has qualitative algebraic systems based on empirical observations that are very natural physically and are isomorphic to the platonic ordered field of real numbers[4].

EUCLIDEAN GEOMETRY

Traditionally, elementary Euclidean geometry is formulated in terms of primitive concepts like point, line, plane, incidence, circle, sphere, angle, and congruence, with other Euclidean geometrical constructs being defined in terms of these. However, there are many ways of formulating Euclidean geometry in terms of other primitive concepts. For example, Pieri (1908) axiomatized 3-dimensional Euclidean space in terms of a single relation, R(x,y,z). In terms of the traditional Euclidean formulation, this relation may be interpreted as follows: R(x,y,z) holds iff x, y, and z are vertices of an isosceles triangle. And Tarski (1929) axiomatized 3-dimensional Euclidean space in terms of a domain whose elements intuitively correspond to solids and two primitives, a predicate B(x) corresponding to «x is a ball» and a relation C(y,z) corresponding to «y is contained in z».

Because of the importance of Euclidean geometry in the evolution of our science and technology, it is natural to investigate if the target ETI civilizations are likely to have developed versions of Euclidean 3-space, and if so, to try to ascertain concepts of their version that are likely to be interpretable by us, and vice-versa.

The erlanger program

In a famous address at Erlangen University, Klein (1872) provided a criterion for deciding whether two geometric systems captured the «same geometry». Applied to the case where we and a Target ETI both have versions of Euclidean Geometry, Kleins method produces the following: let $< A, R_1, ..., R_n >$ be our version of Euclidean geometry, say in terms of point, line, circle, sphere, etc., where we interpret A as the set of points and $R_1, ..., R_n$ as primitives. Then it is well-known that the group of Euclidean motions on A leave the primitives $R_1, ..., R_n$ invariant. (A rotation about a point is an example of such a motion: it transforms each line into a line, each circle into a circle, each intersection of two lines into the intersection of the transformed lines, etc.). Suppose an ETI version of a form of geometrical space is represented by the structure $< B, S_1, ..., S_m >$, where $S_1, ..., S_m$ are relations on B. (Note, it is not required that $A = B$). Then by Kleins criterion, the ETI version captures the same geometry as our version if and only if the group transformations on B that leave the relations S_1, ..., S_m invariant is isomorphic to the Euclidean group of motions on A.

Suppose our version and ETI version capture the same geometry. What kinds of geometrical concepts are we and the ETI likely to have in common? It can be shown that the fact that the two versions capture the same geometry is not sufficient to show that an isomorphic counterpart of the ETI version is formulable in our version through higher order logic, or vice-versa[5]. Thus to answer this question, one has to go beyond the structure of primitives. Because, by assumption, our and the ETI versions have isomorphic groups of transformations, the groups and concepts generated by them are natural places to look for common concepts. For example, the concept of «sphere» has the following formulation: for each pair of distinct points f and p consider the set X of points that are the images of p under Euclidean motions that leaves f fixed. Then it is not difficult to show that X is a sphere about f.

Note that the geometric intuition inherent in understanding the just-given, transformational concept of «sphere» is very different than the vision-based geometric intuition we normally use: it is based on easily formulable concepts in terms of the transformation group of the primitives; it is not necessarily a primitive concept nor one that is formulable in terms of primitive concepts by elementary means (i.e., through first-order logic). For the purposes of SETI, the transformational approach should be considered as a more universal form of «intuition», because it is likely to have analogs among a wider range of ETIs than the one based on (human) visual intuition.

Helmholtz-lie theory

Von Helmholtz (1868) gave a mathematical argument that if geometrical objects can move freely about in physical space without changing their shape, then physical space must have constant curvature. As a consequence, physical space must be a spherical geometry, or the one that results from Euclids axioms (Euclidean geometry), or one of the two geometries (hyperbolic or elliptic) that result from Euclids axioms with the Parallel Postulate replaced by its negation. Given a space of constant curvature, a variety of simple conditions can be added to obtain Euclidean geometry as the only possibility. Von Helmholtzs «proof» had a gap that was filled by Lie (1886), who reformulated Helmholtzs theory in terms of transformation groups. Various improvements and alternatives were suggested over time by other mathematicians, with Fruedenthal (1965) providing a particularly elegant and improved version of the Helmholtz-Lie theory[6].

Rigid bodies are physical objects whose inter-point distances between its parts do not change when the object is moved in space. Because fabrication of equipment calls for various rearrangements of rigid bodies within the local environment, I consider it reasonable that the ETI accomplishes these rearrangements by means based partially on equivalences of the concepts «rigid body», «motion», and «local space». (By similar reasoning, this view is also supported by the assumption that the ETI is able to place objects off-planet). The Helmholtz-Lie theory gives credence to the idea that these rearrangements lead to a Euclidean concept of space. Another consideration in favor of this conclusion involves efficacy of information processing: if one knows the Euclidean-shape of a complicated rigid body at one location, then one can compute its shape at any other location by an appropriate Euclidean motion.

The above arguments in favor of the universality of Euclidean geometrical concepts are derived from pragmatic considerations about technology and the manipulations of objects in local physical space. Additional arguments based on evolutionary and psychological considerations are presented next.

PERCEPTUAL CONSTANCIES

I believe it is reasonable to assume that the ETIs underwent considerable biological evolution before they began large-scale technological development. I will also assume that the target ETIs either utilized concepts based on information processing schemes extant in their biological antecedents during the later stages of this largely pretechnological development or have the means to access, recover, or reconstruct such concepts. These assumptions allow for linkages between evolutionary approaches to biological informational processing and cognitive universals.

The kind of linkage discussed here is between the physical environment and the perception of it. Because the environment in which ETI evolution took place may be very different from those having occurred on Earth and may include features that our scientific community has never considered, these universals need to be inferred by abstract considerations about the evolutionary pressures that produced them. Only two kinds of such universals are discussed here: the cognitive representation of physical space and the cognitive representation of physical intensity of objects from a physical dimension. The ideas and arguments presented about these universals generalize to several other kinds of universals.

Object and lightness constancies

There is believed to be an evolutionary advantage for humans and animals to be able to identify, classify, and remember objects across contexts. Experimental research has shown that humans are good at judging whether an object viewed from one perspective is the same object when viewed from another. This is an example of *object constancy*. It has been shown that humans employ a number of strategies for accomplishing this form of object constancy, including performing an informational analog of a Euclidean motion by comparing the objects perceived shape in one context with a memory of its shape in a previous context. This situation is much more complex than the geometric one of von Helmholtz that was discussed earlier: in object constancy, memory is involved, both the viewer and object may be in motion, the viewer may have no information of how the object got from one context to another, the viewer receives information about the physical environment through 2-dimensional projections of that environment, etc. However, the final result is the similar: the viewer is able to perform the equivalent of Euclidean motions on his or her mental representations. Thus by the Erlanger Program, we may view *this part* of the mental representation taking place in a 3-dimensional Euclidean Geometry.

In a normal viewing condition, a gray patch on a white wall in a lighted room will appear the same subjective lightness and color (gray) under changes in intensity and color of the rooms lighting. This is an example of *lightness and hue constancies (for gray)*. These constancies help in the identification, classification, and remembrance of gray objects across contexts that may have different illuminations. If one looked through a tube so that only the gray is visible, then its lightness and color will change with changes in intensity and color of the room lights. In the normal viewing condition, psychologists have found that it is the ratio of the physical intensity of the light reflected from the gray with the physical intensity of the light reflected from the white part of the wall that is determining the lightness appearance of the gray patch, and this ratio is invariant under changes of intensity of the illumination. The explanation of hue constancy is more complicated and will not be given here.

Abstractly, perceptual constancies are perceptual features that remain invariant across a naturally occurring class of contexts. The case of particular importance for this article is where the contexts are related by a group of transformations on the physical environment. In this case, the constant perceptual features are associated with physical stimuli that remain invariant under transformations from the group. Intuitively, the transformational group corresponds to a kind of redundancy across contexts of physical information. Thus successful employment of an analog of the transformational group will greatly aid in the efficacy of the processing of constant perceptual features by capitalizing on the redundancy inherent in the perceptual situation. This efficiency allows for possible evolutionary advantage.

In terms of intuitive evolutionary theory, the class of contexts and the transformational group exist, the former because it is physical and the latter because it is platonic. As organisms evolve, some develop perceptual processing. And as the latter continue to evolve, some develop perceptual processing of features that are approximate invariants of the physical transformational group. This gives the latter an evolutionary advantage. In general, there are evolutionary pressures to refine these percepts so that they are percepts of better approximations of invariants of the physical transformation group. There are also evolutionary pressures for more efficient processing. Both kinds of pressures are simultaneously met by having a good approximation of the processing analog of the physical transformation group. The feasibility of evolving such a processing analog depends a great deal on the nature of the transformation group. Analogs to physical similarity groups (which are isomorphic to the numerical group of positive, real multiplications) are easy to achieve by a number of physiological and biological-like processes. This makes them especially good candidates for perceptual evolution[7].

PSYCHOPHYSICAL LAWS

Since the beginning of experimental psychology, it has been acknowledged that there is a lawful relationship between a stimulus physical intensity and the subjective intensity of its percept. Characterizations of this relationship have been highly controversial in the psychological literature. Because one side of this relationship is observable and the other is not, the controversy was not unexpected. The non-observability of a subjective experience does not preclude a scientific analysis of its subjective content; it only requires more subtle forms of scientific analyses than those usually encountered in the physical and biological sciences.

The function that associates a usual measurement of the physical intensity of light from a star with the historical method of assigning an apparent magnitude to its perceived brightness, produces a function – called a «psychophysical function» – that behaves in a very lawful way – in this case, perceived brightness is the logarithm of physical intensity. Similar results hold for many other kinds of stimuli and for other modalities, namely, the measurements of subjective intensity are the logarithms of measurements of physical intensity. This is the *logarithm form* of the psychophysical law. Of course, different ways of measuring the intensities of the stimuli or percepts may result in different laws. In particular, measurement procedures that rely on different methods of the elicitation of subjective judgments of intensity may produce a different form of the psychophysical law, for example, a *power form*, where subjective intensity is a power of physical intensity. In my view, both of these forms and other forms of the psychophysical law are equally valid: the psychophysical law is really not about numbers but about how the transformation group on the physical dimension is related to the transformation group on the subjective dimension. The relationship is simple, they are isomorphic, and in fact similarity groups[8].

One should note that this as an instance of the Kleins Erlanger Program generalized to a non-geometric situation. Application of the Erlanger Program to this situation yields that physical intensity and subjective intensity have the same, abstract, content. It can be shown that this content has a beautiful mathematical structure to it. For the purposes of this article, this structure may be viewed as an analog of basic real algebra, which includes analogs of the real numbers, its ordering, addition, multiplication, and the operations of raising to powers and taking logarithms[9].

For SETI, these considerations lead to the following intuitive theory: it is plausible that consistent judgments of intensity of various physical dimensions of objects have positive evolutionary value. For reasons like those given for object constancy, it is plausible that the resulting evolutionary pressure leads processing to employ analogs of the similarity group. Thus, independent of which physical dimensions are being perceived, we and the ETI share the same form of qualitative psychophysical function. Because the mathematical content of our and the ETIs subjective dimensions are the same (by the Erlanger Program) the rich algebraic structure of certain invariants that are easily definable in terms of the similarity group is fertile ground to look for commonality for the purposes of communication. The algebra closely resembles the basic real algebra of our mathematics.

CONSCIOUSNESS

As a psychologist, I would like to make one brief point about consciousness as a cognitive universal.

Some think consciousness is a likely consequence of intelligence and therefore be taken as a cognitive universal. *Consciousness*, as used in psychology and philosophy, is a very complicated concept with many components. Most in my view are not strongly connected with an abstract concept of intelligence. In this category I would put awareness, reflexivity, and qualia. Two key components that I view to be strongly connected with intelligence are metacognitive modeling – a system having and using an imperfect model of part of itself – and meta-metacognitive modeling – a system having and using an imperfect model of its metacognitive modeling. (Imperfect rather than perfect models are used to avoid variants of the Liar's Paradox)[10].

SUMMARY

Mathematics has always been considered an obvious place to search for cognitive universals: it is abstract, crucial in the development of our science and technology, and generally considered to express «indisputable truths». I have argued that the kinds of mathematics most likely to be universal correspond to our continuous mathematics, particularly variants of Euclidean geometry and real algebra. The arguments were based on pragmatic and evolutionary considerations and psychological theory. They involve applications of sophisticated mathematical theory.

Although various arithmetic based concepts have been proposed as universals, I am suspicious of the universality of concepts that necessarily involve the totality of natural numbers in their definition, e.g., the concept of «prime number» or concepts based on inductively generated, infinite sequences of numbers. The suspicion is based on the notion that inductively generated numerical concepts are ultimately based on metalinguistic abilities – abilities whose universality I consider to be questionable. This suspicion would diminish greatly if tougher, more rigorous arguments are given for the evolutionary likeliness of the development of inductive arithmetical concepts. It is not enough to say integers are useful in various kinds of estimations and calculations, for such estimations and calculations can take place without a concept corresponding to the totality of numbers. I have suggested that perceptual constancies are useful for efficient perceptual processing. The features exhibited by the constancies may vary considerably across ETIs. However, the formal mathematical groups associated with the constancies are of a few abstract types. I argued on evolutionary grounds that for each of these types there is much in common about the ETIs' psychological processing of the associated constancies. This conclusion provides a fertile base for surmising cognitive universals.

NOTES

[1] Similar assumptions play a major role in strategies of a related project, Communicating with Extra-Terrestrial Intelligence (CETI). For the purposes of this article, SETI and CETI may be merged, with the term «SETI» referring to both projects.

[2] The proof of this is as follows: suppose that a predicate N(x) were definable in terms of the above system of symbols and first-order logic with the following property: for each natural number n, N(n) holds if and only if n is n-additions of **1** in the sense described above. Then the interpretation of N in the ordered field of real numbers is the set of natural numbers. Then it easily follows from Γ that the system consisting of the predicate N and the restrictions of $\oplus$ and $\otimes$ to N satisfies the axioms of Robinson (1952) for a fragment of arithmetic. Then it follows by theorems of Robinson (1952) and Gödel (1931) that not every sentence formulated in the above symbol system is derivable from Γ. However, results of Tarski and McKinsey (1951) show that the opposite is true. Therefore, the predicate N cannot exist.

[3] To show this, one uses a result of McKinsey and Tarski (1951) that the truth or falsity of each sentence of elementary geometry is decidable, and then proceed in a manner analogous to the above example of the ordered field of real numbers.

[4] See Narens and Luce (1990) for a fuller discussion of the mathematics that is qualitatively and naturally inherent in physics.

[5] The relationship of the Erlanger Program to definability issues in higher-order logics is discussed in Narens (1988).

[6] This is the same Fruedenthal who created the first systematic scheme for one-way communication with ETIs in Fruedenthal (1960).

[7] I believe that currently enough is known about the processes involved in this intuitive evolutionary argument to give it a precise mathematical formulation and argument.

[8] In psychology, this characterization of the psychophysical law was first put forth in Luce (1959). That paper contained some epistemological principles that Luce later considered to be wrong, and Luce (1990) revised his epistemological theory and gave a qualitative formulation of the psychophysical law similar to the one presented here. Narens (in press) provides a theory of the psychophysical law that is consistent with the ideas presented throughout this article and explains the pattern of different laws that are observed by different methods of elicitations of subjective intensities.

[9] Detailed constructions are given in Chapter 7 of the authors forthcoming book *Theories of Meaningfulness.*

[10] A fuller presentation of this point of view for the psychological area of learning and memory is given in Narens, Graf, and Nelson (1996).

ACKNOWLEDGEMENTS

I would like to thank Dr. Dieter Heyer for encouragement and advice and the Zentrum für Interdisziplinarische Forschung for the use of its facilities in preparing this article. Research for the article was supported by the NSF grant SBR-9520107 to the University of California, Irvine.

REFERENCES

FREUDENTHAL, H. (1960): *Lincos: Design of a Language for Cosmic Intercourse*, Amsterdam, North-Holland Pub. Co.

FREUDENTHAL, H. (1965): Lie groups in the foundations of Geometry, *Advances in Mathematics*, **1**, 145-190.

GÖDEL, K. (1931): Über formal unentscheidbare sätze per principia Mathematica und verwandter Systeme I, *Monatshefte für Mathematik und Physik*, 38, 173-98.

HELMHOLTZ, H. VON (1868): Über die thatsachen, die der Geometrie zu Grunde liegen, *Göttingen Nachrichten*, 9, 193-221.

KLEIN, F. (1872): Vergleichende Betrachtungen über neuere geometrische Vorschungen, Programm zu Eintritt in die philosophische Facultät und den Senat der Universität zu Erlangen, Erlangen, Deichert.

LIE, S. (1886): Bemerkungen zu Helmholtz' Arbeit über die Thatsachen, die der Geometrie zu Grunde liegen, *Berichte über der Verhandlungen der Königlich Sächischen Gesellschaft der Wissenschaften zu Leipzig, Mathematisch-Physische Klasse*, 38, 337-342.

LUCE, R.D. (1959): On the possible psychophysical laws, *Psychological Review*, 66, 81-95.

LUCE, R.D. (1990): «On the possible psychophysical laws» revisited: remarks on cross-modal matching, *Psychological Review*, 97, 66-77.

NARENS, L. (1988): Meaningfulness and the Erlanger Program of Felix Klein, *Mathématiques Informatique et Sciences Humaines*, **101**, 61-72.

NARENS (in press): On subjective intensity and its measurement, *Choice, Decision, and Measurement: Essays in Honor of R. Duncan Luce* (A.A.J. MARLEY, Ed.), Mahwah: Lawrence Erlbaum and Associates.

NARENS L.: *Theories of Meaningfulness* (Manuscript).

NARENS, L., GRAF, A. and NELSON, T.O. (1996): Metacognitive Aspects of Implicit/Explicit Memory, in *Implicit Memory & Metacognition* (L. REDER, Ed.), Mahwah: Lawrence Erlbaum and Associates, 137-170.

NARENS, L. and LUCE, R.D. (1990): Three aspects of the effectiveness of mathematics in science, in *Mathematics and Science, World Scientific Press.* (R. MICKENS, Ed.), 122-135.

PIERI, M. (1908): La geometria elementare istituita sulle nozioni di «punto» e «sfera», *Memorie di Matematica e de Fisica della Società Italiana delle Scienze, Ser. 3*, **15**, 345-350.

ROBINSON, R.M. (1952): An essentially undecidable axiom system, Proceedings of the International Congress of Mathematicians, 1952, **1**, 729-730.

TARSKI, A. (1929): Les fondements de la géométrie des corps, *Ksiega Pamiatkowa Pierwszego Polskiego Zjasdu Matematykw Krakw*, 29-33.

TARSKI, A. and MCKINESY, J.C.C. (1951): *A Decision Method for Elementary Algebra and Geometry*, 2nd edition, Berkeley and Los Angeles.