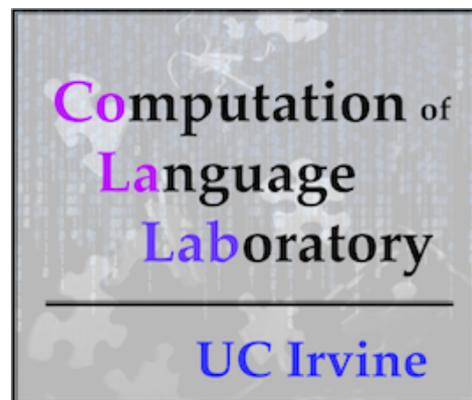


How children are and aren't like adults when interpreting pronouns: A computational cognitive modeling investigation

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Pronoun interpretation



The girls wave at the teacher...

???

...and then **she** leaves.

Pronoun interpretation



The girls wave at the teacher...

pl

???

...and then she leaves.

sg

Agreement mismatch: “she” is singular but “girls” is plural

Pronoun interpretation



The girls wave at **the teacher**...

pl

sg

???

...and then **she** leaves.

sg

Agreement match: both "she" and "teacher" are singular

Pronoun interpretation



The girls wave at the teacher...
sg

...and then **she** leaves.
sg

How to choose?
Use linguistic knowledge of agreement.

Pronoun interpretation



The girl waves at the teacher...

???

...and then **she** leaves.

sg

Pronoun interpretation



The girl waves at the teacher...

sg

???

...and then she leaves.

sg

This could work: both “she” and “girl” are singular.

Pronoun interpretation



The girl waves at **the teacher**...

sg

sg

???

...and then **she** leaves.

sg

But so could this: both “she” and “teacher” are **singular**.

Pronoun interpretation



The girl waves at the teacher...

sg

sg

???

...and then she leaves.

sg

How to choose?

Pronoun interpretation



The girl waves at the teacher...

sg

???

sg

...and then **she** leaves.

sg

How to choose?

Use **contextual** knowledge (who's likely to be leaving)

Maybe the girl is getting ready to leave the classroom.

Pronoun interpretation



The girl waves at the teacher...
subject sg ??? sg
...and then she leaves.
sg

How to choose?

Use linguistic knowledge about connectives.

Maybe pronouns after “and then” tend to refer to the previous subject in this context.

Pronoun interpretation



The girls wave at the teacher...
subject pl ??? sg
...and then she leaves.
sg



Something English-speaking adults have learned:
How to resolve interpretation cue conflicts in context.

Pronoun interpretation



The girls wave at the teacher...
subject pl ??? sg
...and then she leaves.
sg



The need to integrate multiple cues to interpretation doesn't just happen in English, of course.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después **ella** sale.

... *and then* **she** leaves.

sg

Here's the same sentence in Spanish.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg

Spanish also allows the **form** of the pronoun to be null (this means the **agreement** information is on the verb).

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg

Just like English, there are multiple cues available to interpret the pronoun.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg

Spanish-speaking adults also have interpretation preferences.



Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

subject

pl

???

sg

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg



For Spanish-speaking adults...

...the **connective** favors the **subject**.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

subject

pl

sg
???

...y después sale.

... and then PRONOUN leaves.

sg



For Spanish-speaking adults...

...the (singular) agreement (on the verb) indicates the singular object.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave *at the teacher...*
subject pl = sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.
sg



For Spanish-speaking adults...

...the (null) form favors the subject.

Pronoun interpretation



Las niñas saludan a la maestra...
The girls wave at the teacher...
subject pl = sg
???

...y después ∅ sale.
... and then **PRONOUN** leaves.
sg



For Spanish-speaking adults...
...this collection of cues generally causes the pronoun to be interpreted as the singular object (agreement matters the most).

Pronoun interpretation development



Las niñas saludan a la maestra...

The girls wave at the teacher...



pl

= sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.

sg



How do Spanish-learning children develop this ability to interpret pronouns in context?

Pronoun interpretation development



 **PRONOUN** =



Children's ability to interpret a pronoun in an **adult-like** way depends on (at least) two things.



Pronoun interpretation development



PRONOUN



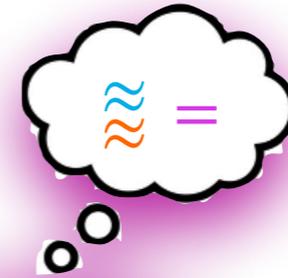
First, children need **adult-like knowledge** of what each cue signals.



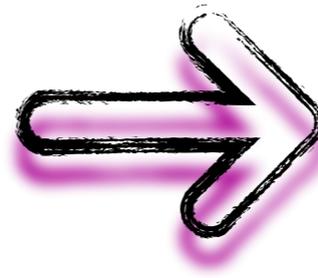
Pronoun interpretation development



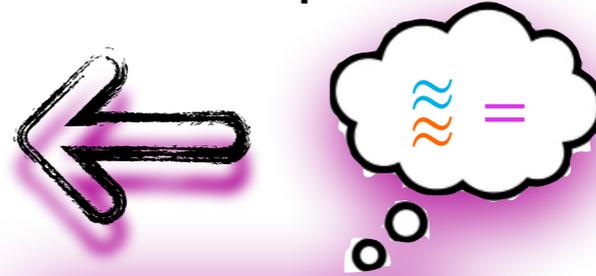
PRONOUN



Second, children need **adult-like ability** to deploy that knowledge in real time.



Pronoun interpretation development



 PRONOUN =



When both of these are adult-like, we should get adult-like pronoun interpretation.



Pronoun interpretation development



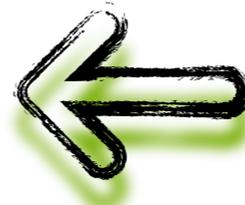
 PRONOUN =



But if we get **non-adult-like** pronoun interpretation, then it could be due to **immature knowledge**, **immature deployment** of that knowledge, or **both!**



Pronoun interpretation development



How do we tell what the differences are between child and adult **pronoun interpretation**? When we understand this better, we'll understand what **children** need to do to become **adults**.



Understanding pronoun interpretation development



The plan, part 1: Get some empirical data on how **children** and **adults** interpret the same **pronoun** in a context where multiple cues are available.

Case study: Mexican Spanish



Understanding pronoun interpretation development



 PRONOUN =

The plan, part 1: Get some empirical data on how **children** and **adults** interpret the same **pronoun** in a context where multiple cues are available.



This will highlight what the **observable differences** are in interpretation behavior.

Understanding pronoun interpretation development



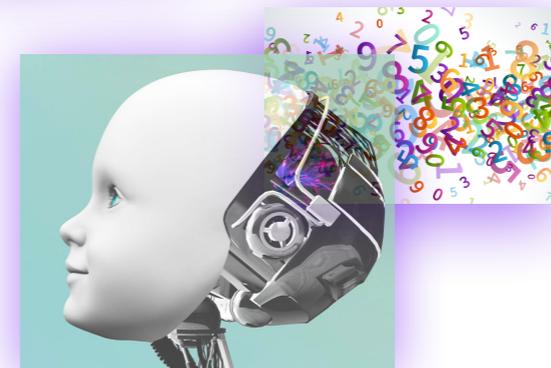
The plan, part 2: Use **computational cognitive modeling** to formally articulate the potential process of pronoun interpretation in the context of these multiple cues.



Understanding pronoun interpretation development



 PRONOUN =



The plan, part 2: Use the **computational cognitive model** to identify the specific differences potentially leading to child and adult pronoun interpretation in context.



Empirical data on pronoun interpretation



Empirical data on pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

sg
???

...y después ∅ sale.

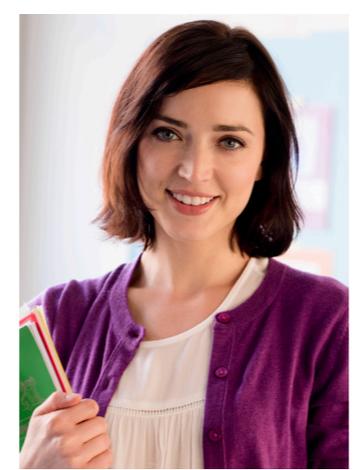
... and then **PRONOUN** leaves.
sg

Children (~3, 4, and ~5) and adults are asked to interpret pronouns in the kind of contexts we saw before.



<=3:	1;11-3;10 (N=33)
4:	4;0-4;11 (N=35)
>=5:	5;0-6;9 (N=29)
Adults	(N=47)

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.
sg

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

===sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.
sg

Rate of subject responses

Choice: Is the pronoun interpreted as the **subject** or the **object**?

We can plot the rate of subject responses.

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

???

sg

...y después ~~Ø~~ sale.

... and then **PRONOUN** leaves.

sg

Rate of subject responses

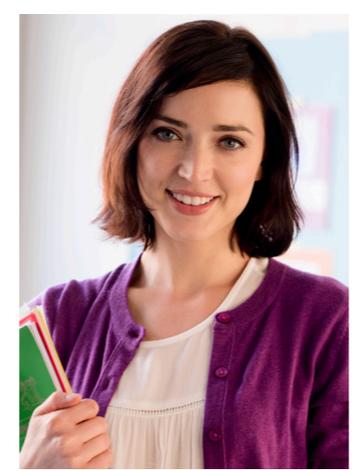
Context:

Does agreement favor the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs

adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

=pl

sg
???

...y después \emptyset salen.

... and then **PRONOUN** leave.

pl

Rate of subject responses

Context:

Does agreement favor the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

==sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.
sg

Rate of subject responses

Context:

Does agreement favor the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...
The girls wave at the teacher...

pl

sg

...y después \emptyset sale.
... and then PRONOUN leaves.

???

sg

Rate of subject responses

Context:
 Does the pronoun form favor the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg



object

???

...y después ella sale.

... and then PRONOUN leaves.

sg

Rate of subject responses

Context:

Does the pronoun form favor the subject or the object?

overt (favors object)

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

... y después \emptyset sale.

... and then **PRONOUN** leaves.

sg

Rate of subject responses

Context:
Does the **connective** favor the **subject** or the **object**?

subject object

Favored by agreement

\emptyset (favors subject) overt (favors object)

Favored by form

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs

adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

sg

...y después \emptyset sale.

... and then **PRONOUN** leaves. sg

???

Rate of subject responses

Context:
Does the **connective** favor the **subject** or the **object**?

y después (favors subject)

subject object

Favored by agreement

\emptyset (favors subject) overt (favors object)

Favored by form

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs

adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

object

???

∅

sale.

...porque

... because

PRONOUN leaves.

sg

Rate of subject responses

Context:

Does the **connective** favor the **subject** or the **object**?

porque (favors object)

subject object

Favored by agreement

∅ (favors subject) overt (favors object)

Favored by form

Empirical data on pronoun interpretation



Favored by connective

y después (favors subject)

porque (favors object)



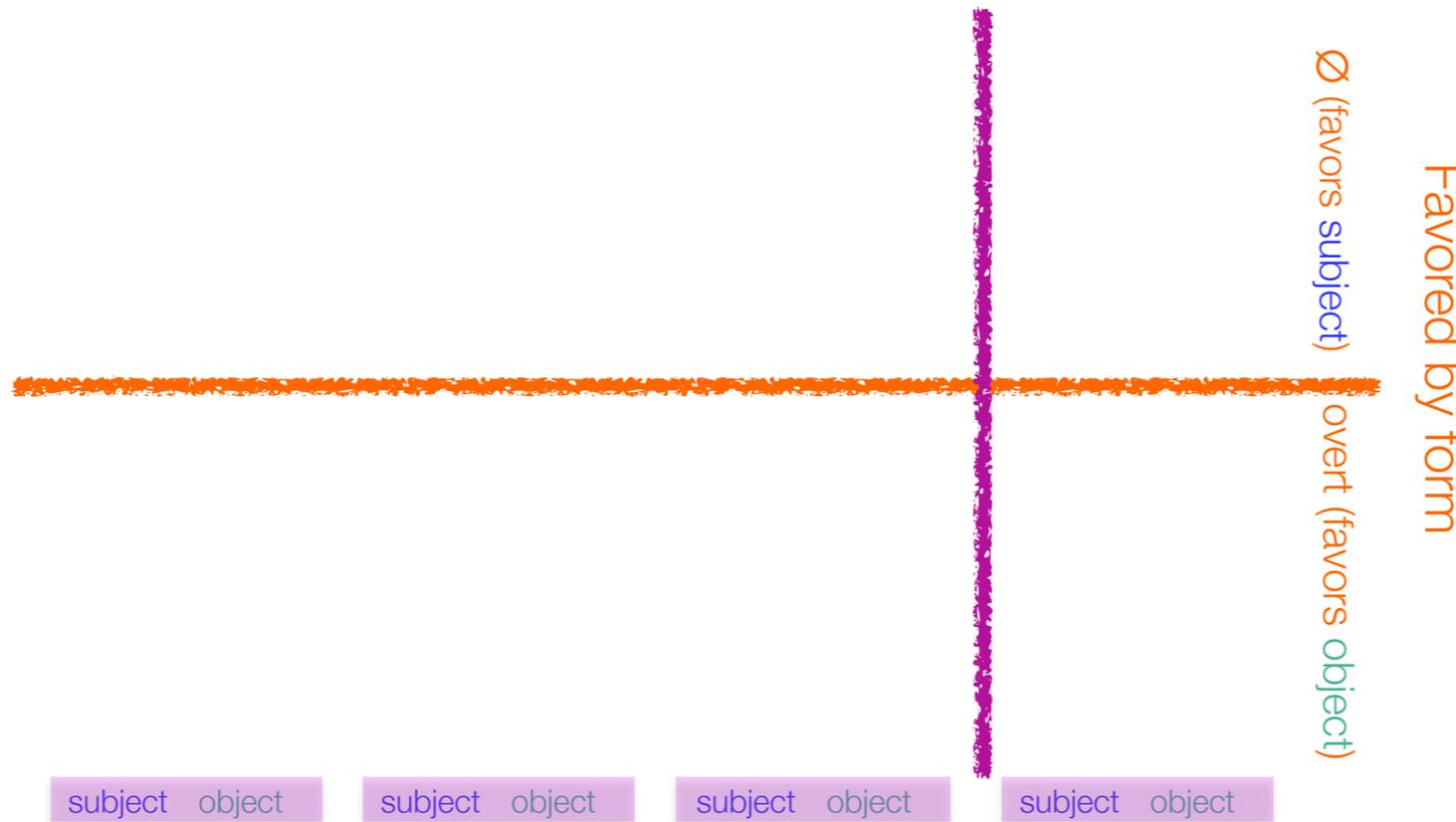
~3yrs

4yrs

~5yrs

adults

Rate of subject responses



Favored by agreement

Empirical data on pronoun interpretation



Favored by connective

y después (favors subject)

porque (favors object)



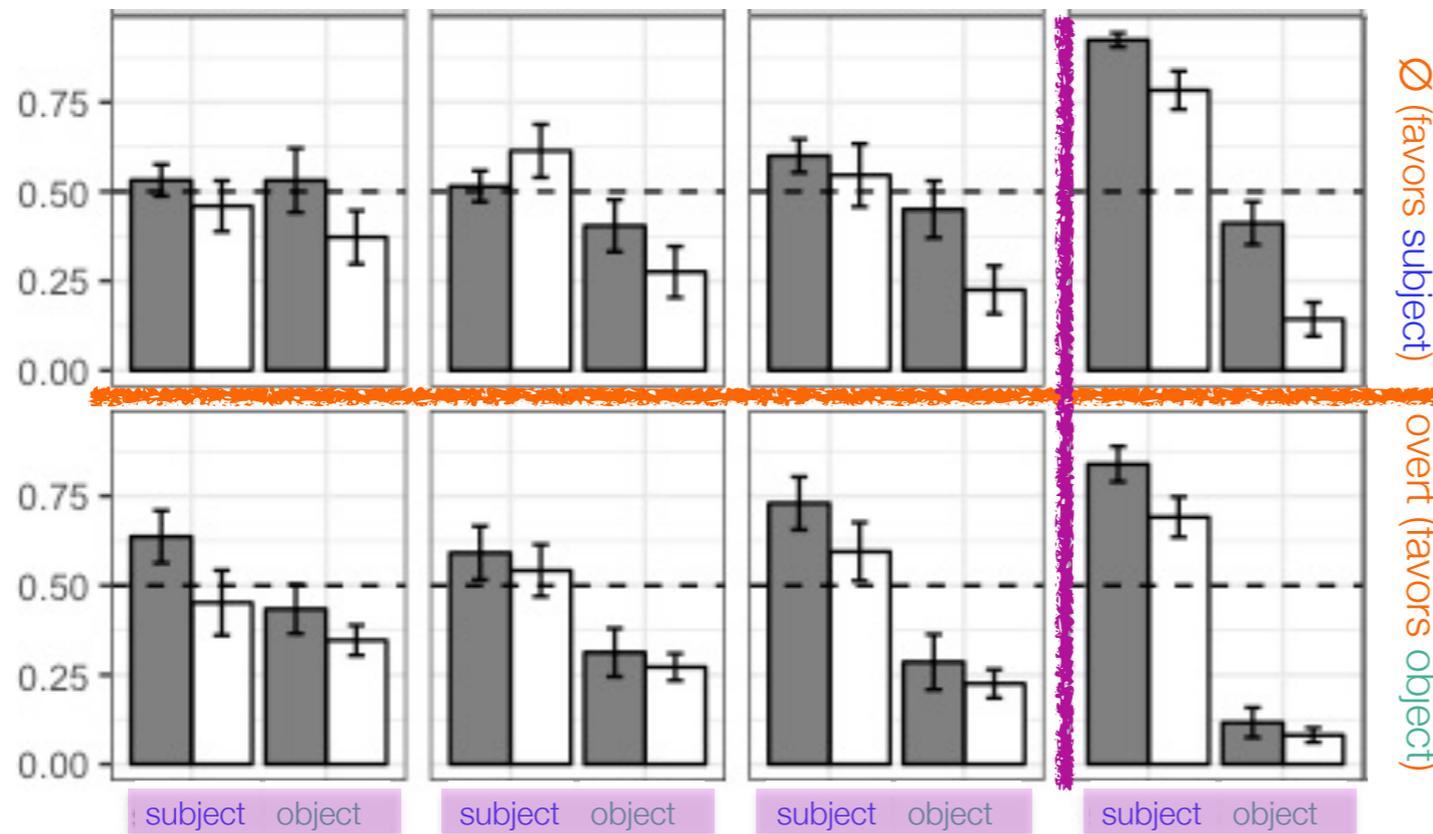
~3yrs

4yrs

~5yrs

adults

Rate of subject responses



∅ (favors subject)

overt (favors object)

Favored by form



Favored by agreement

Empirical data on pronoun interpretation



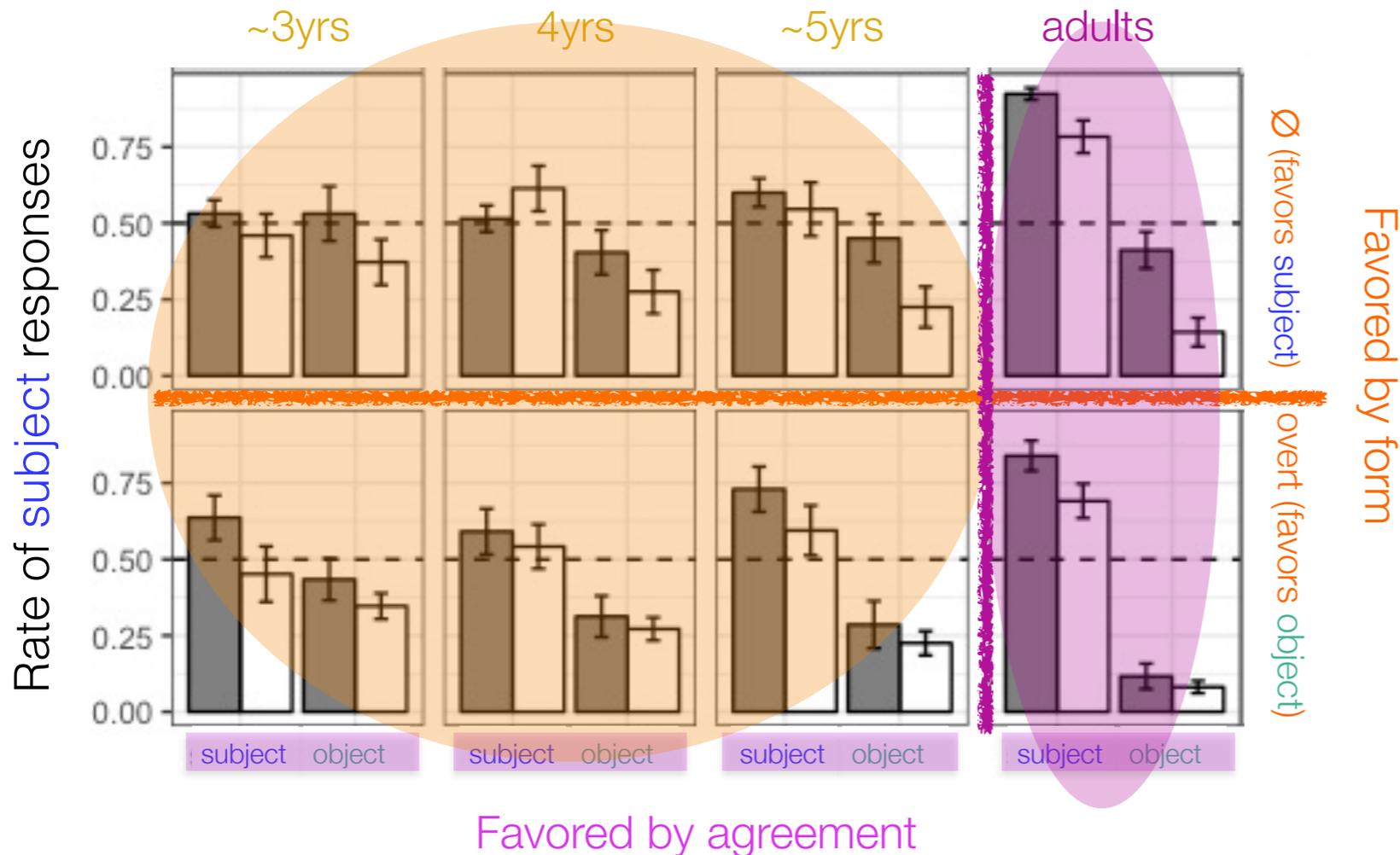
Favored by connective

y después (favors subject)

porque (favors object)



Big qualitative picture:
Children don't behave like adults.



Empirical data on pronoun interpretation



Favored by connective

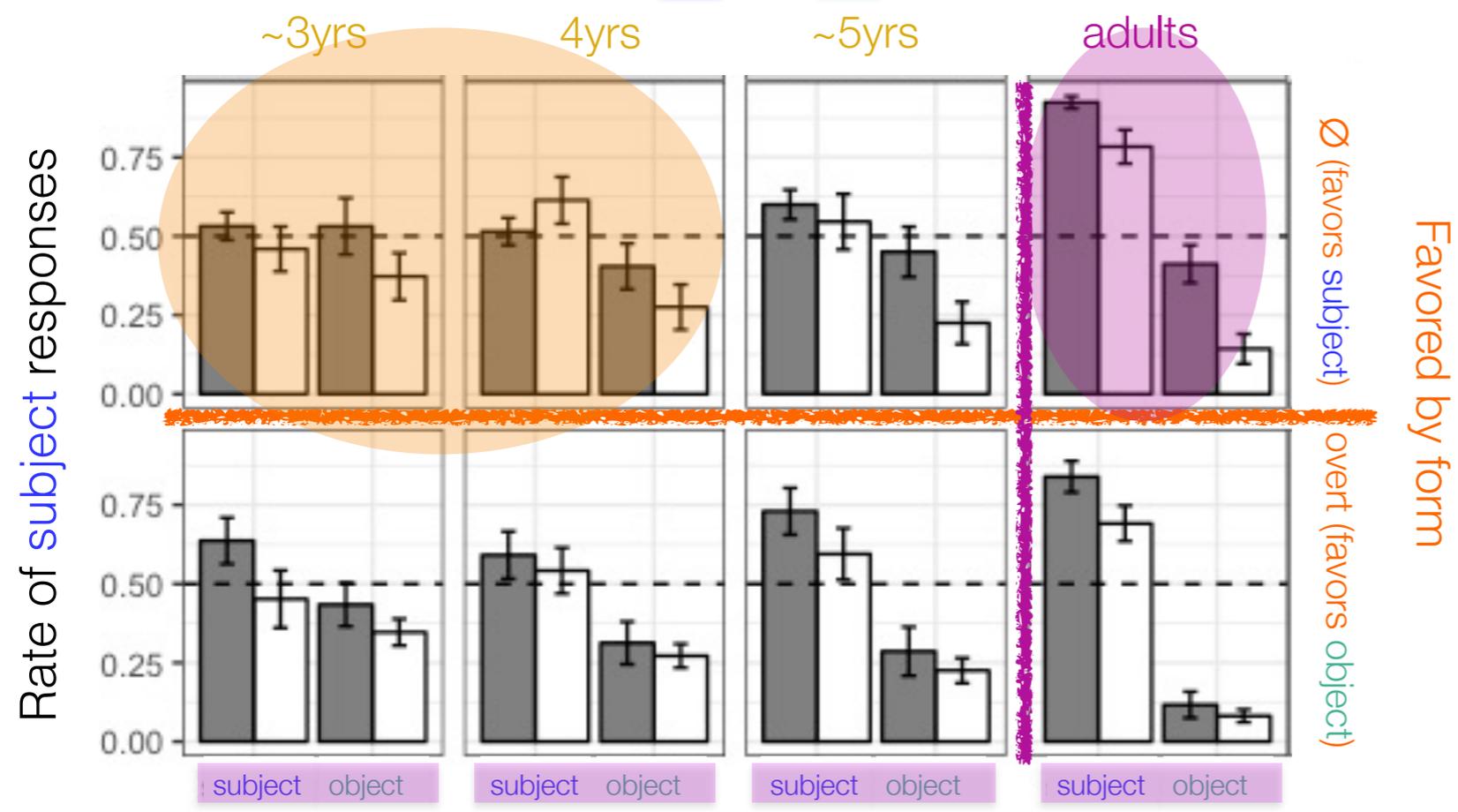
y después (favors subject)

porque (favors object)



Big qualitative picture:
Children don't behave like adults.

Some nuances:
Younger children seem to differ more, especially in certain contexts.



Favored by agreement

Empirical data on pronoun interpretation



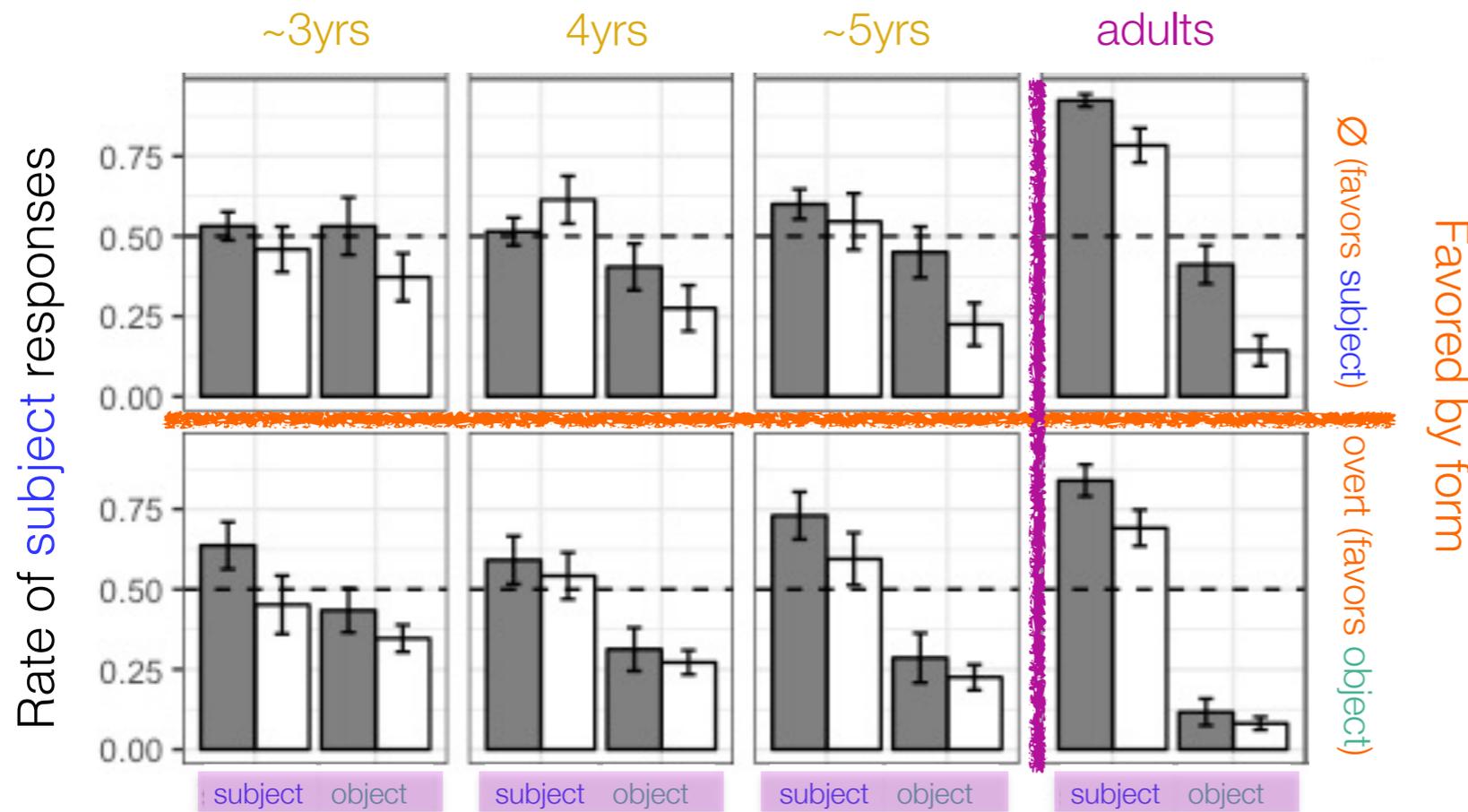
Favored by connective

y después (favors subject)

porque (favors object)

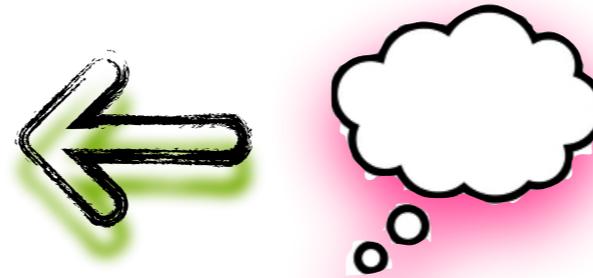


Something needs to change for children to become adult-like — but what?



Favored by agreement

Understanding pronoun interpretation development

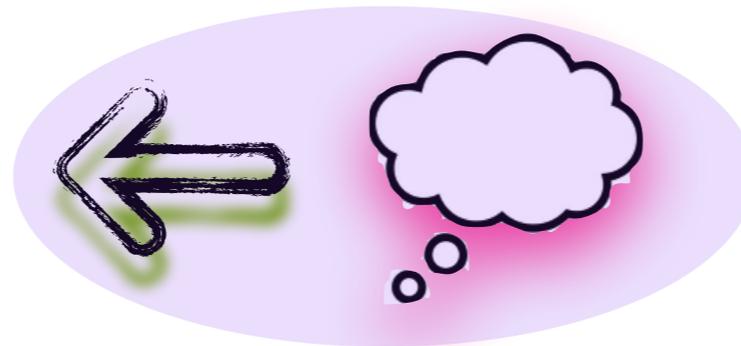


The plan, part 2:

Use **computational cognitive modeling** to formally articulate the potential process of pronoun interpretation in the context of these multiple cues.



Understanding pronoun interpretation development



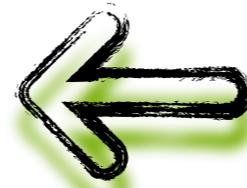
The computational cognitive model formally articulates and implements (what we think are) **relevant aspects** of pronoun **interpretation in context**.



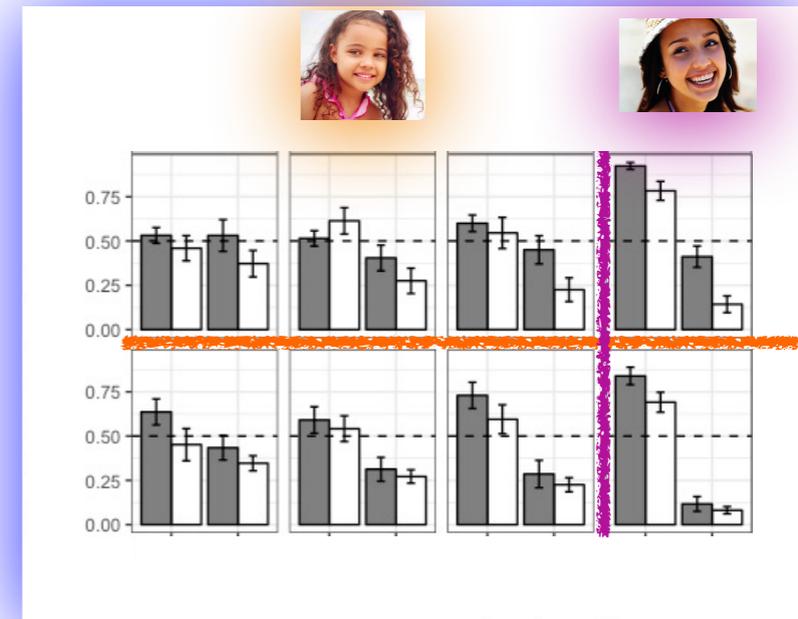
 **PRONOUN** 



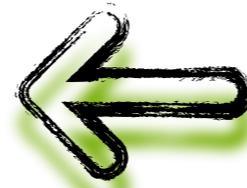
Understanding pronoun interpretation development



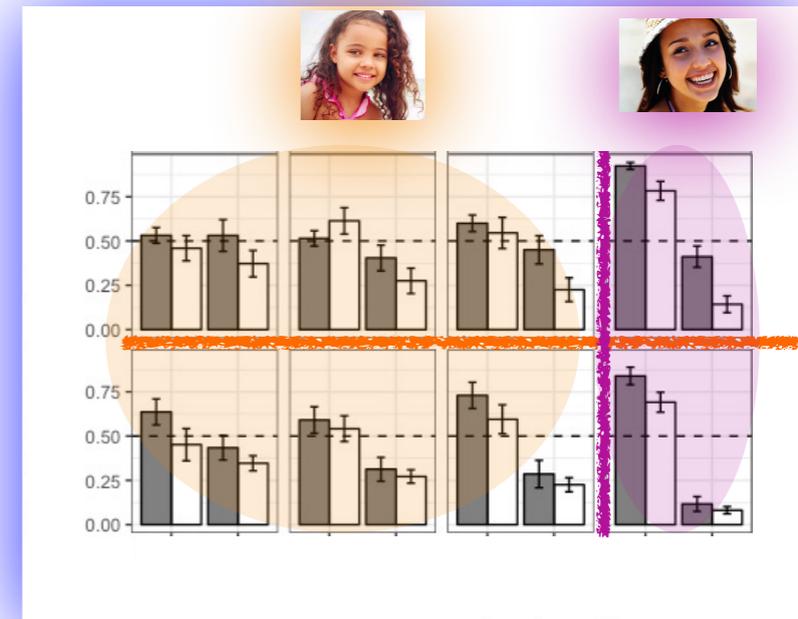
Here: Implement how a modeled listener represents pronoun information and deploys that information in order to predict the probability of a particular interpretation in context.



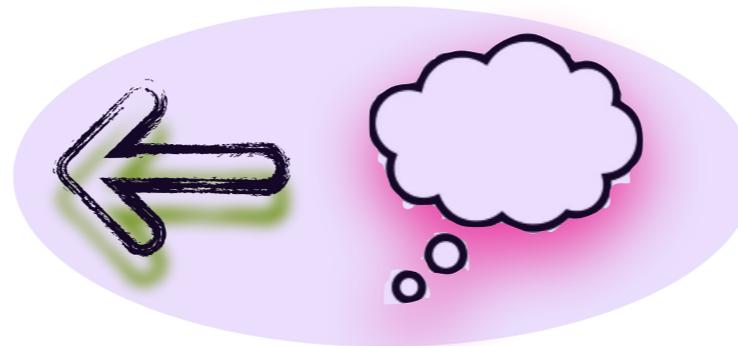
Understanding pronoun interpretation development



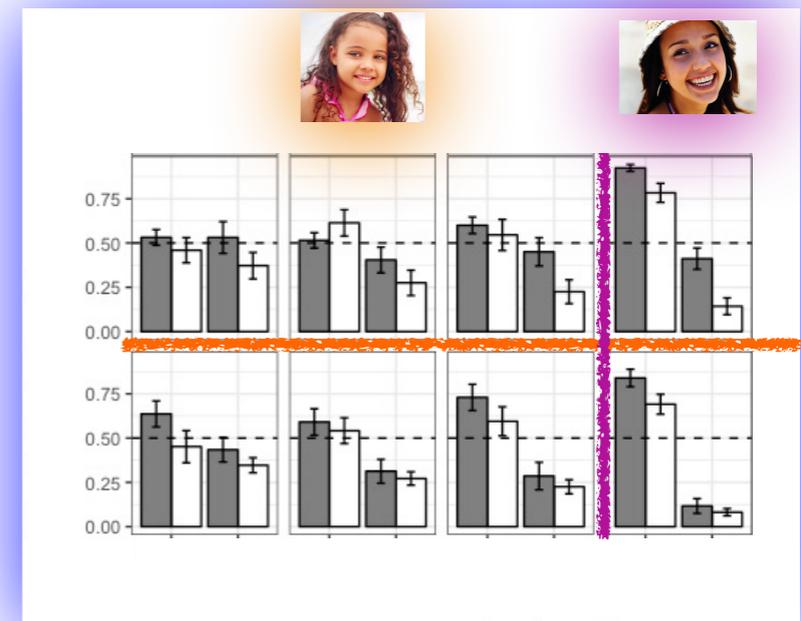
Then, see which options for **representation** and **deployment** best match **child** vs. **adult** pronoun interpretation behavior.



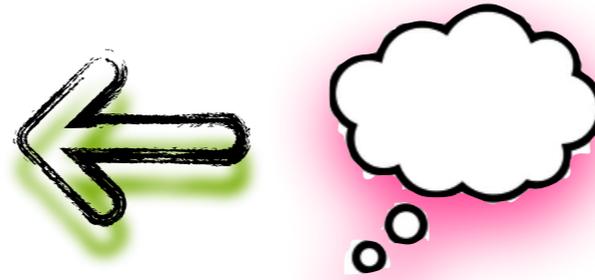
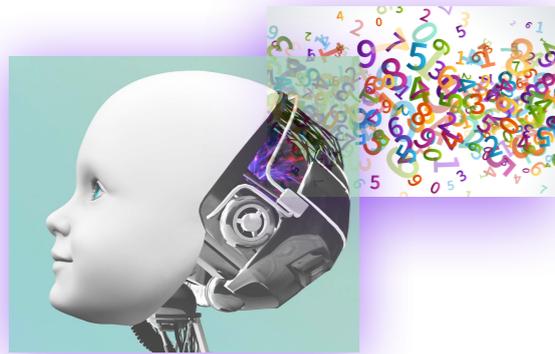
Understanding pronoun interpretation development



This will articulate how children (potentially) differ from adults, and what needs to develop in children for them to become adult-like.

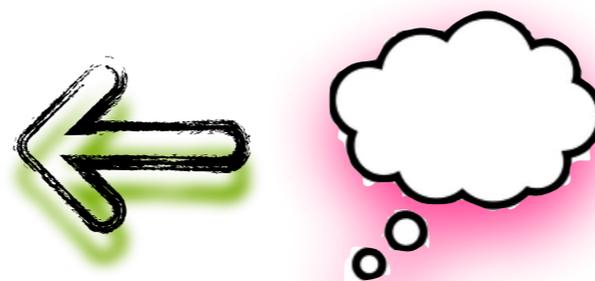


Modeling pronoun interpretation in context



The model uses **Bayesian inference** to implement the cognitive mechanism that combines information to generate a particular **interpretation** in **context**.

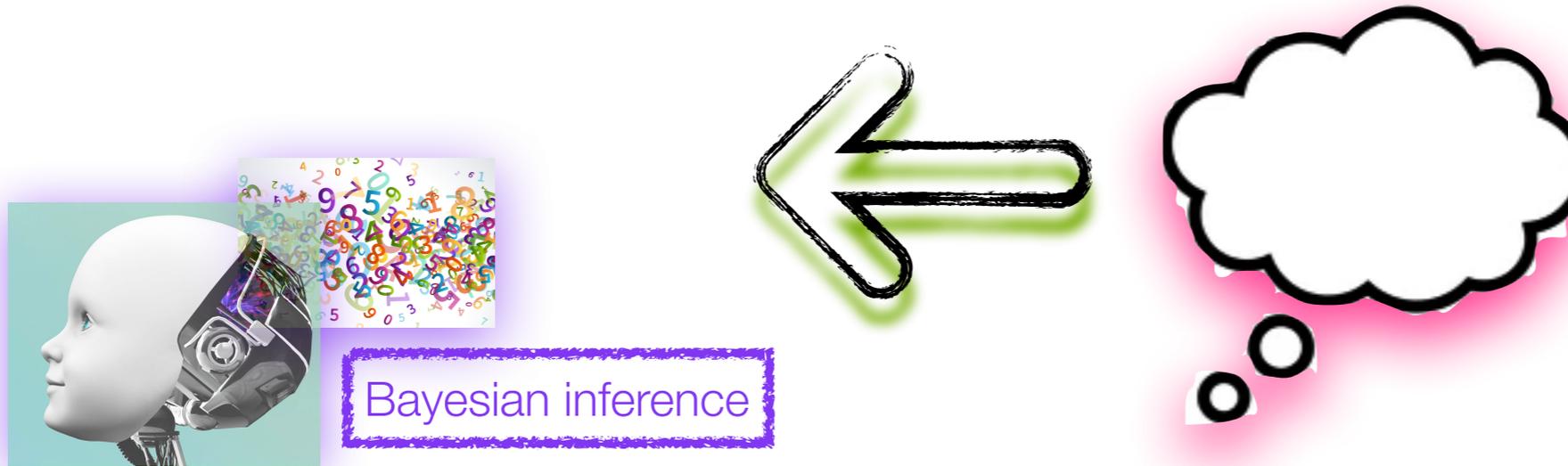
Modeling pronoun interpretation in context



Bayesian inference is commonly used to model human cognition in general and language development in particular, since it matches human behavior quite well (see Pearl in press and b for recent reviews).



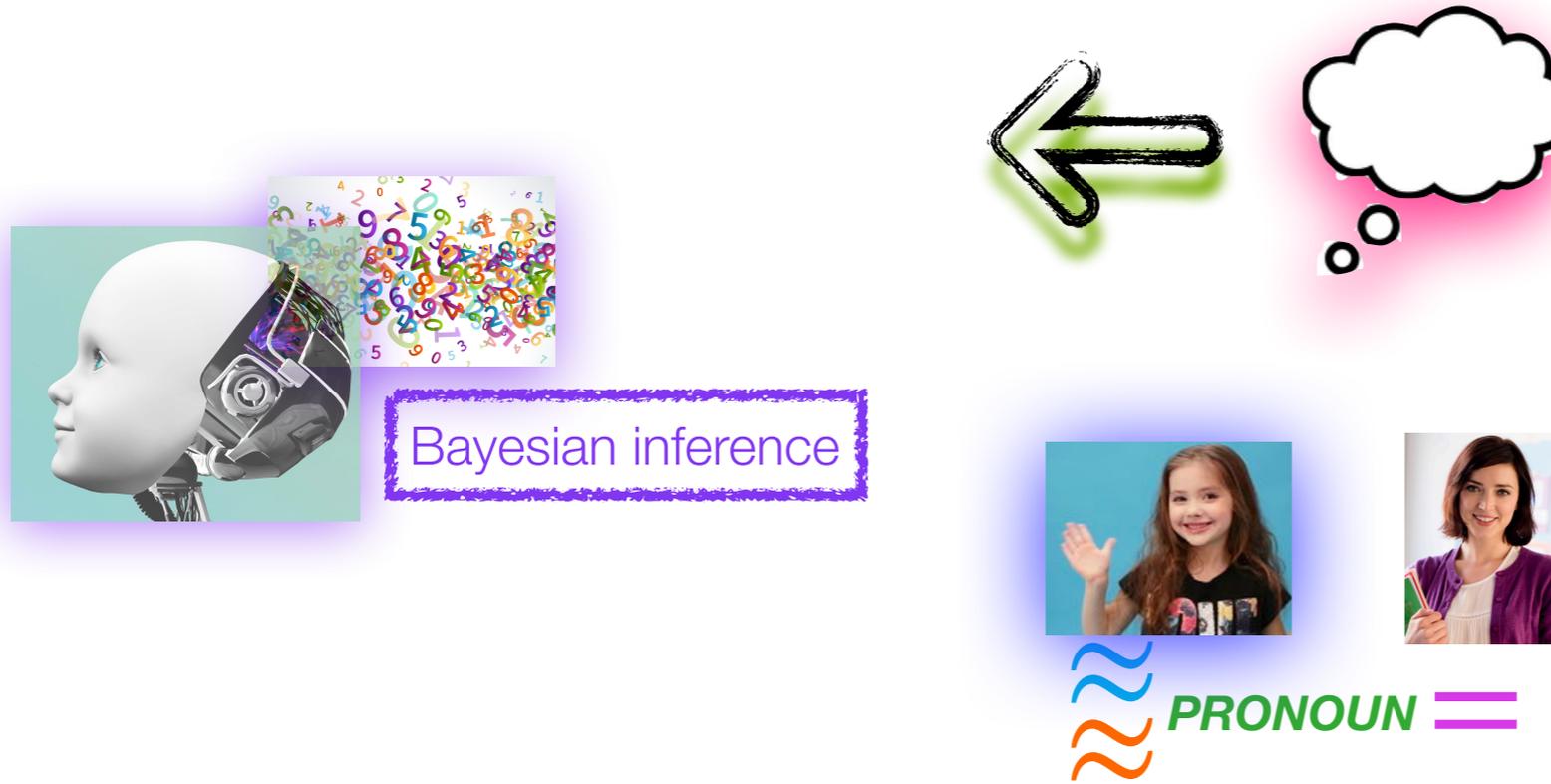
Modeling pronoun interpretation in context



The particular Bayesian model we use is adapted from Gagliardi, Feldman & Lidz (2017), and offers one way to separate out the contributions of information **representation** vs. information **deployment** in the moment.



Modeling pronoun interpretation in context

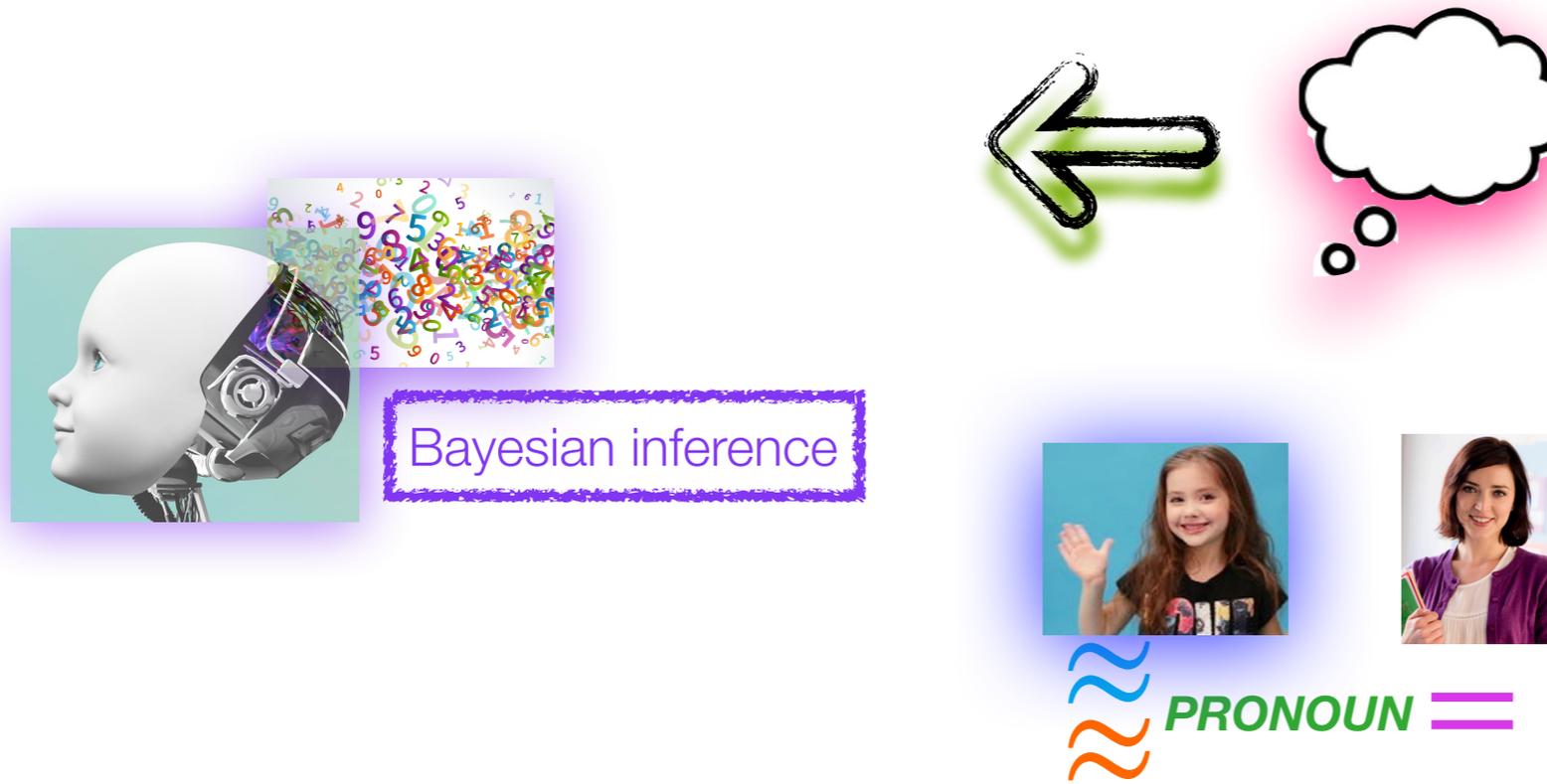


interpretation

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM, CON, MOR} | \alpha_{subj.SG})$$

Interpreting the pronoun as the **subject**, which is **singular**....

Modeling pronoun interpretation in context



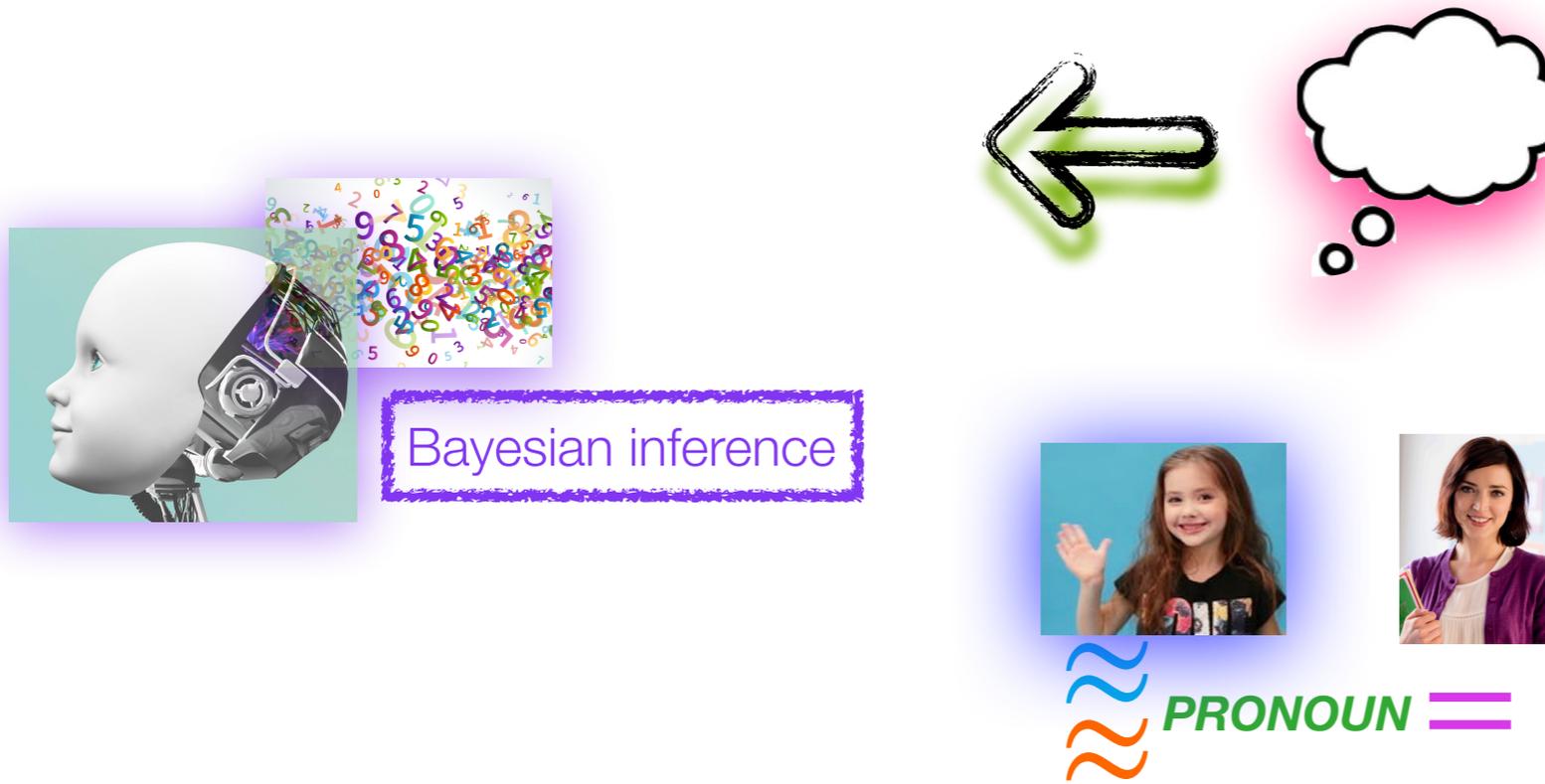
interpretation context

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM, CON, MOR} | \alpha_{subj.SG})$$

∅ ella

...given the particular context involving the pronoun's form,

Modeling pronoun interpretation in context



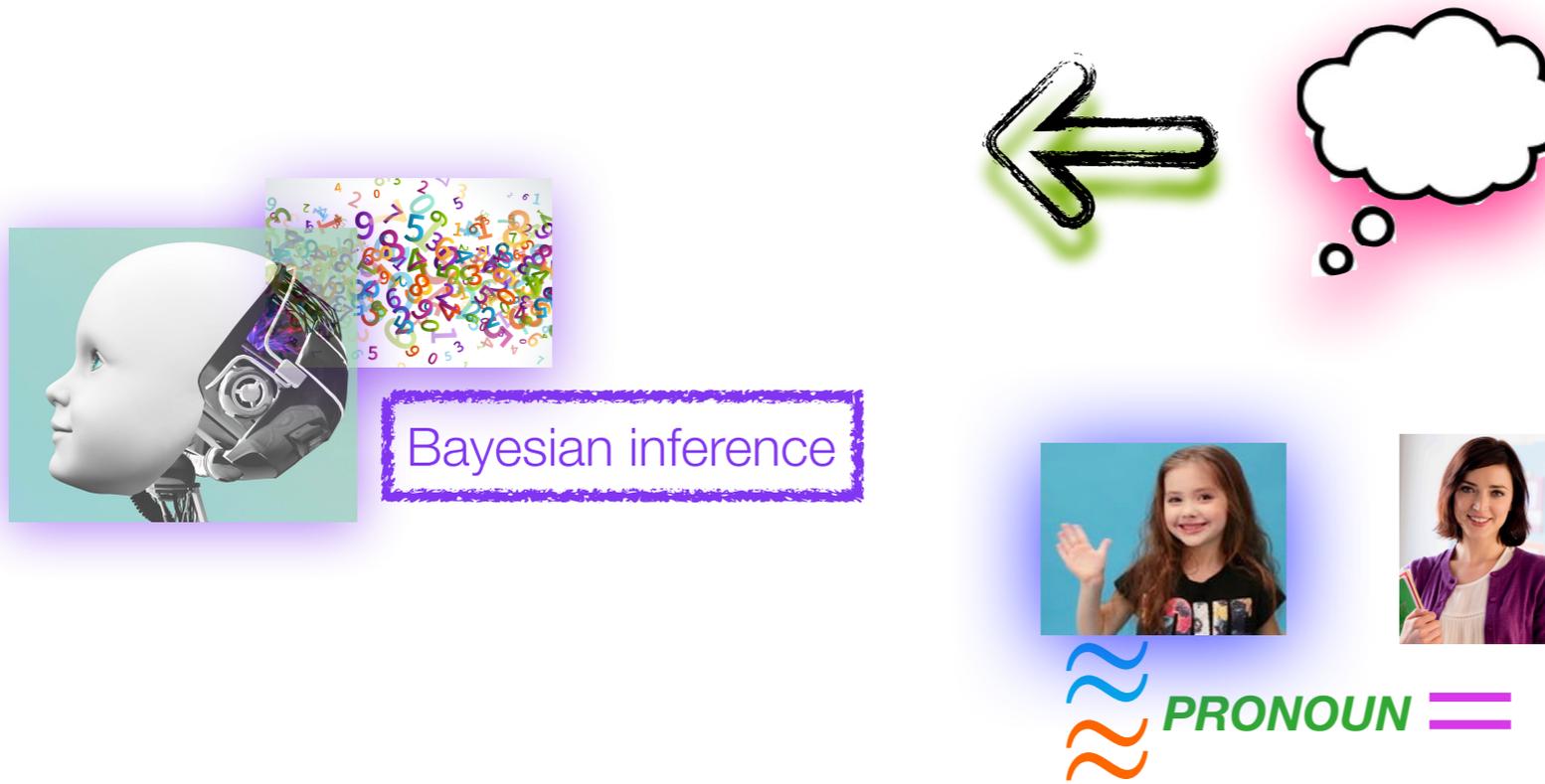
interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM}, \text{CON}, \text{MOR} | \alpha_{subj.SG})$$

y después porque

...given the particular **context** involving
the pronoun's **form**,
the **connective**,

Modeling pronoun interpretation in context

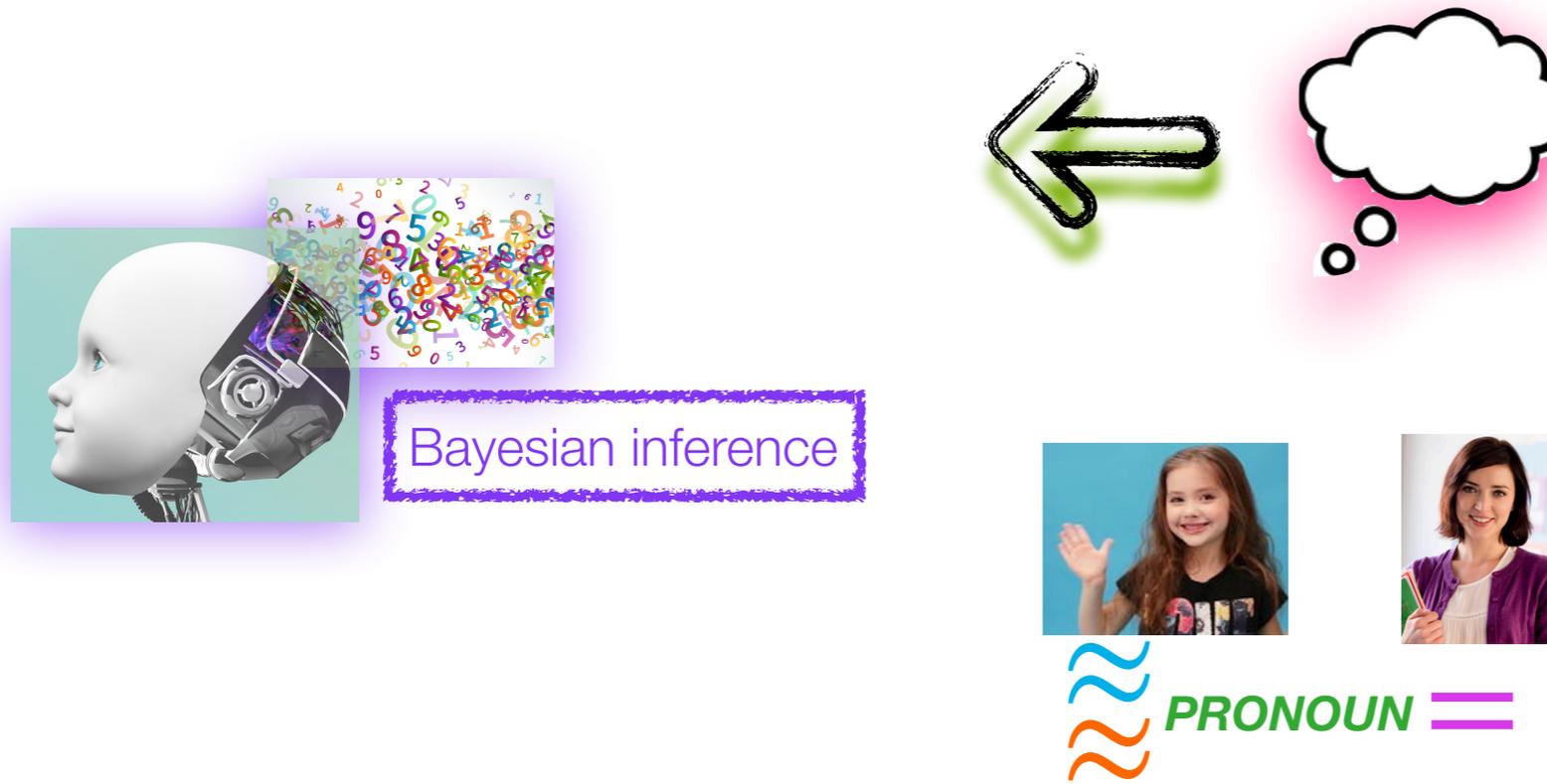


$$\text{interpretation} \quad \text{context}$$
$$p(\alpha_{subj.sg} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.sg}) * p(\text{FORM}, \text{CON}, \text{MOR} | \alpha_{subj.sg})$$

sg *pl*

...given the particular **context** involving
the pronoun's **form**,
the **connective**,
and the **agreement morphology**.

Modeling pronoun interpretation in context

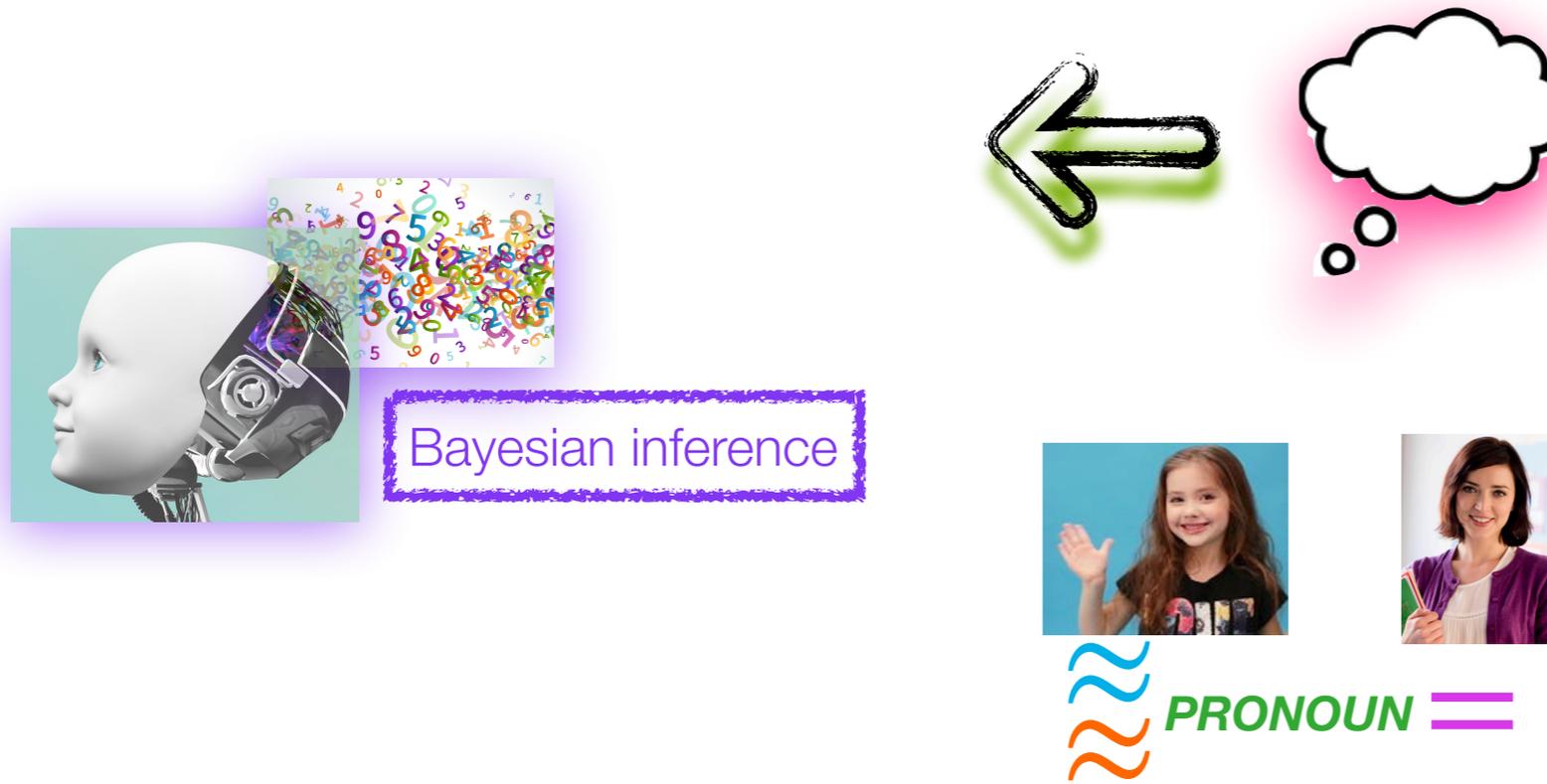


$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM}, \text{CON}, \text{MOR} | \alpha_{subj.SG})$$

interpretation context

This is proportional to the **prior probability** of that interpretation irrespective of this particular context...

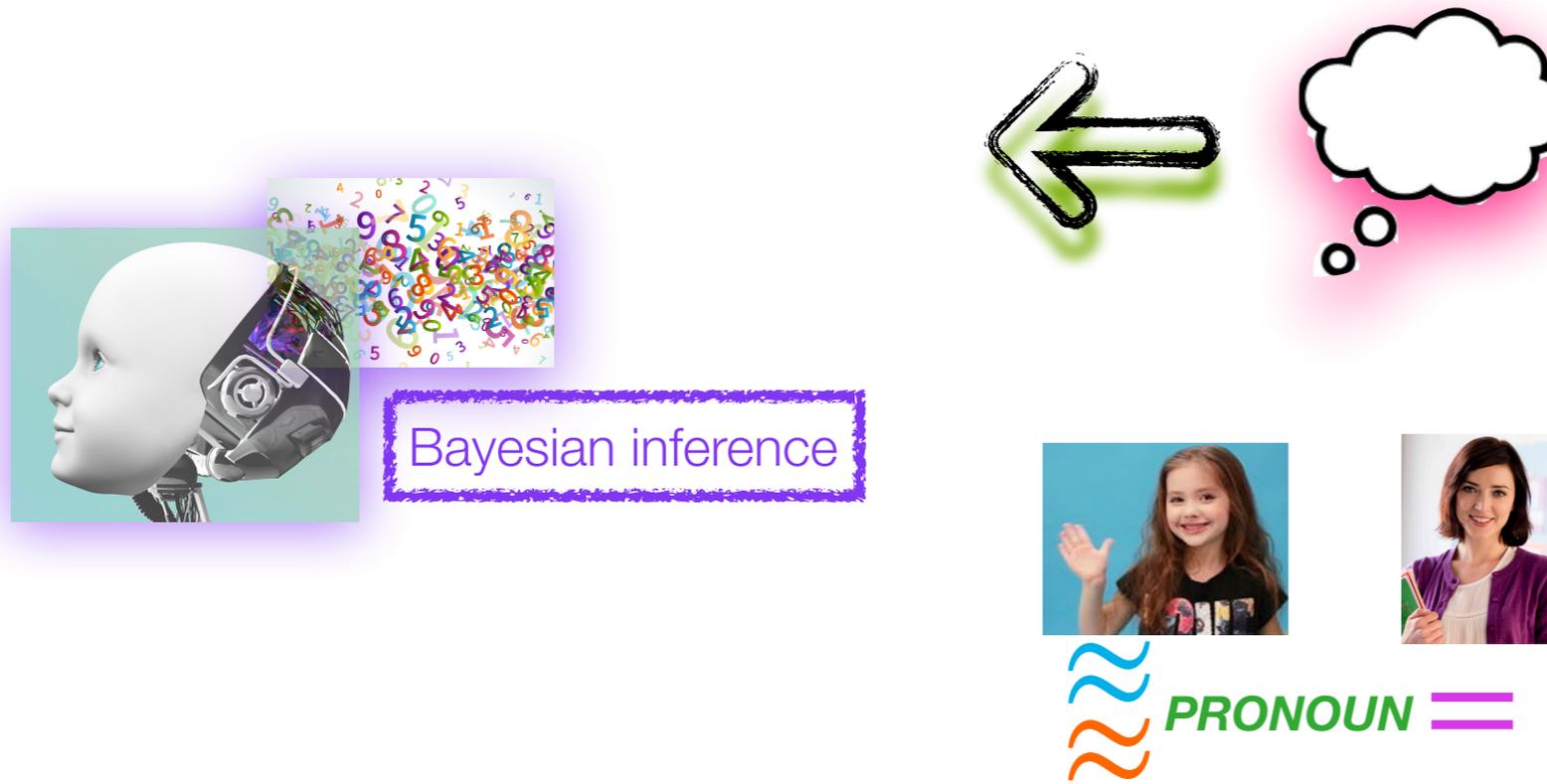
Modeling pronoun interpretation in context



$$\text{interpretation} \quad \text{context}$$
$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM}, \text{CON}, \text{MOR} | \alpha_{subj.SG})$$

...multiplied by the **likelihood** of these context values, given this kind of interpretation (a singular subject).

Modeling pronoun interpretation in context

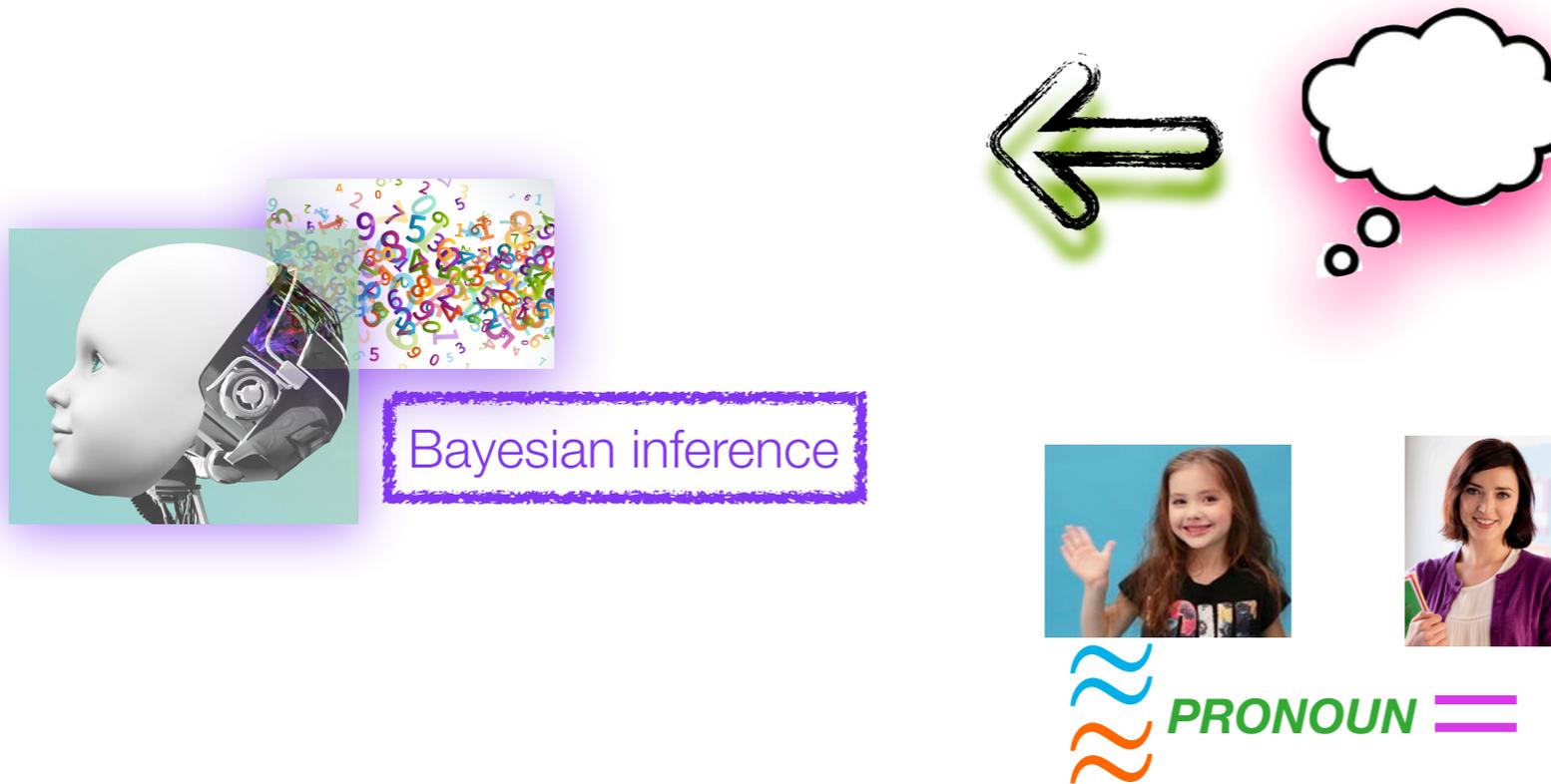


$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

Here, we assume these context values are independent, so we can calculate the likelihood this way.

Not implausible: In visual perception, human behavior is best captured by models assuming features are independent (Vul & Rich 2010).

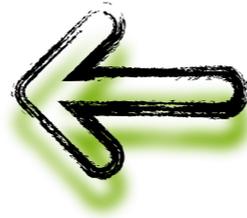
Modeling pronoun interpretation in context



$$\begin{array}{c} \text{interpretation} \\ p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * \\ \text{context} \\ p(\text{CON} | \alpha_{subj.SG}) * \\ p(\text{MOR} | \alpha_{subj.SG}) \end{array}$$

This is the baseline model, which has **accurate representations** of information and **accurately deploys** those representations in the moment.

Modeling pronoun interpretation in context



PRONOUN =



$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



What about a modeled listener who has **inaccurate representations**? This could involve inaccurately representing the prior or the likelihood information, or both.

Modeling pronoun interpretation in context



PRONOUN =

interpretation

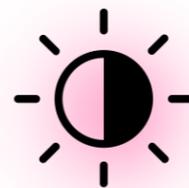
context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



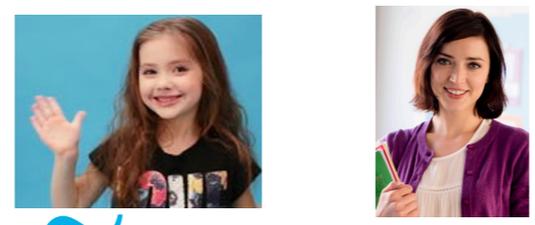
~~inaccurate representations~~

We implement this as a softmax on the true probability (prior or likelihood), with contrast parameter σ .



$$e^{\sigma * \ln(\text{probability})} = \text{probability}^{\sigma}$$

Modeling pronoun interpretation in context



PRONOUN =

interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

✗ ✓ ✗ ✓



✗ inaccurate representations *probability^σ*

About σ :

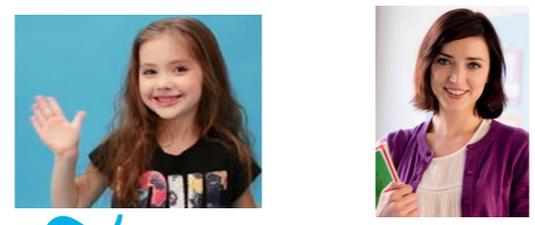
$\sigma < 1$: probability differences are smoothed away.

$\sigma = 1$: probabilities remain accurate.

0.324 vs. 0.676

$\sigma > 1$: probability differences are sharpened.

Modeling pronoun interpretation in context



PRONOUN =

interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



~~inaccurate representations~~ *probability^σ*

About σ : ☀️ ■ ■

$\sigma = 0.5$: probability differences are smoothed away. 0.409 vs. 0.591

$\sigma = 1$: probabilities remain accurate. 0.324 vs. 0.676

$\sigma > 1$: probability differences are sharpened.

Modeling pronoun interpretation in context



PRONOUN =

interpretation

context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



~~inaccurate representations~~

probability^σ

About σ : ☀️



$\sigma < 1$: probability differences are smoothed away.

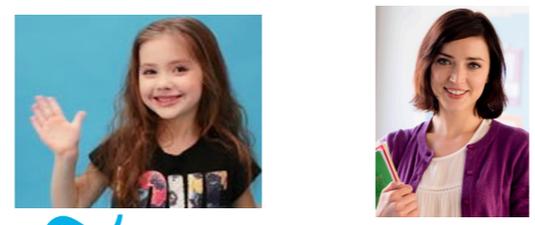
$\sigma = 1$: probabilities remain accurate.

0.324 vs. 0.676

$\sigma = 2$: probability differences are sharpened.

0.187 vs. 0.813

Modeling pronoun interpretation in context



PRONOUN =

probability^σ

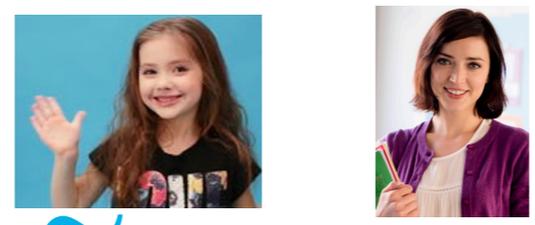
$$\begin{aligned}
 & p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \times \\
 & * p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \times \\
 & * p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \times \\
 & * p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}} \times
 \end{aligned}$$

inaccurate representations



One σ for each information type:
 (in the prior) σ_{α}
 (in the likelihood) σ_{form} , σ_{con} , σ_{mor}

Modeling pronoun interpretation in context



PRONOUN =

probability^σ

$$p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \times p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \times p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \times p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}$$

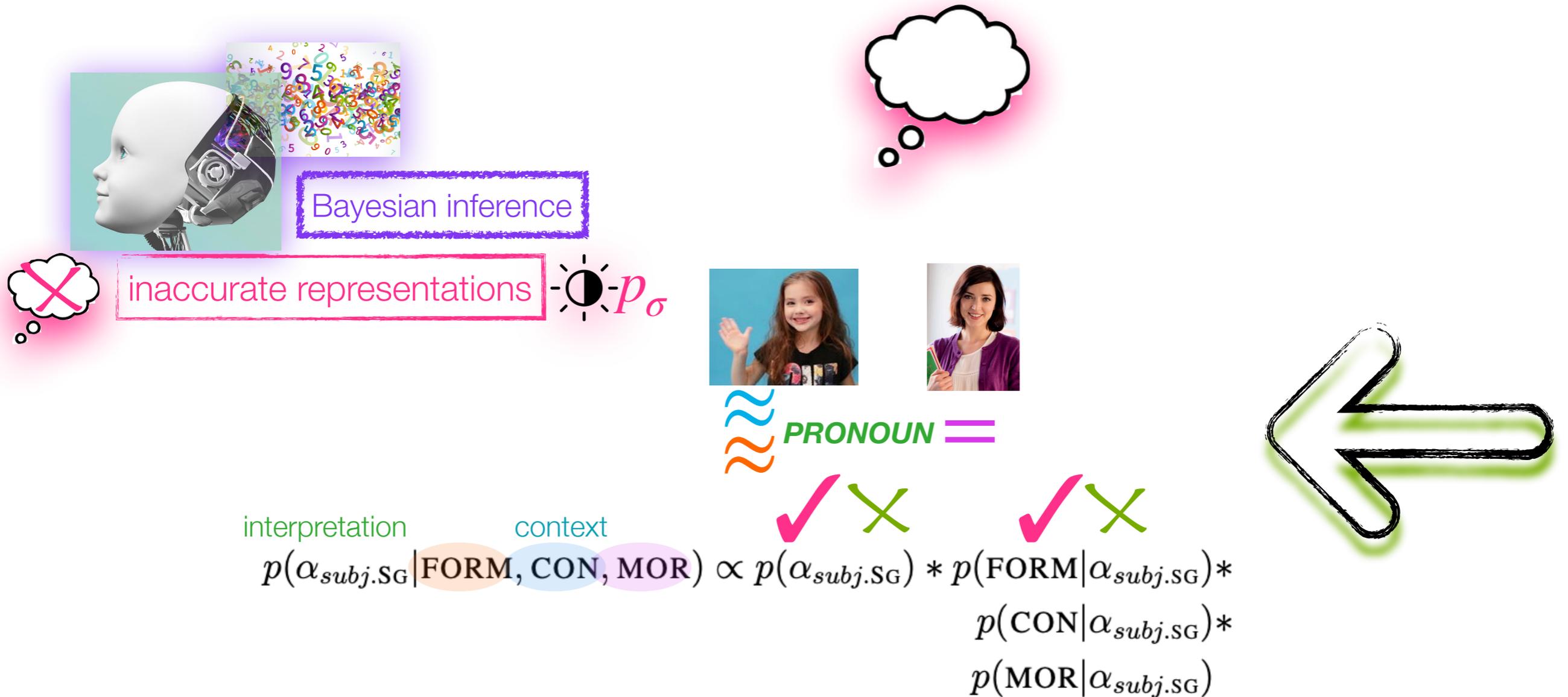
inaccurate representations



We allow $0.00 \leq \sigma \leq 4.00$, and see which σ value combinations best predict **child** and **adult** pronoun interpretation behavior.



Modeling pronoun interpretation in context



What about a modeled listener who has **inaccurate deployment** of information in the representations? This could involve inaccurately deploying the prior or the likelihood information, or both.

Modeling pronoun interpretation in context



Bayesian inference



inaccurate representations p_{σ}



PRONOUN =

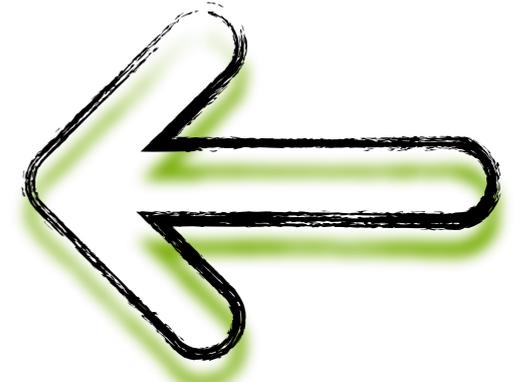


interpretation

context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

~~inaccurate deployment~~



We implement this as ignoring that information. So, for any piece of information, the modeled listener either pays attention to it (and so uses it) or ignores it in the moment.



Modeling pronoun interpretation in context



Bayesian inference



inaccurate representations p_{σ}



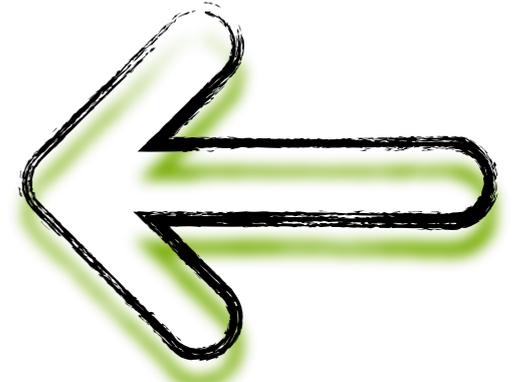
PRONOUN =

interpretation

context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

inaccurate deployment



Not using information means not incorporating it into the inference.

Modeling pronoun interpretation in context



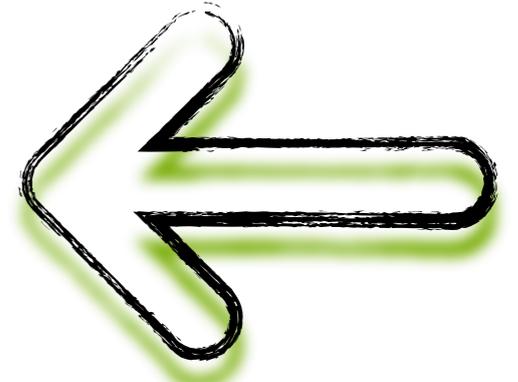
Bayesian inference



inaccurate representations p_σ



PRONOUN =



$$p_{\text{UNIF}}^{\text{interpretation}}(\alpha_{\text{num}}, \alpha_{\text{subj?}} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\text{UNIF}) \times \begin{matrix} \blacksquare & \blacksquare \end{matrix}$$

- * $p(\text{FORM} | \alpha_{\text{num}}, \alpha_{\text{subj?}})$
- * $p(\text{CON} | \alpha_{\text{num}}, \alpha_{\text{subj?}})$
- * $p(\text{MOR} | \alpha_{\text{num}}, \alpha_{\text{subj?}})$

inaccurate deployment



Not using the prior means relying on a uniform (uninformative) prior.

Modeling pronoun interpretation in context

Bayesian inference

inaccurate representations $\odot p_\sigma$

PRONOUN =

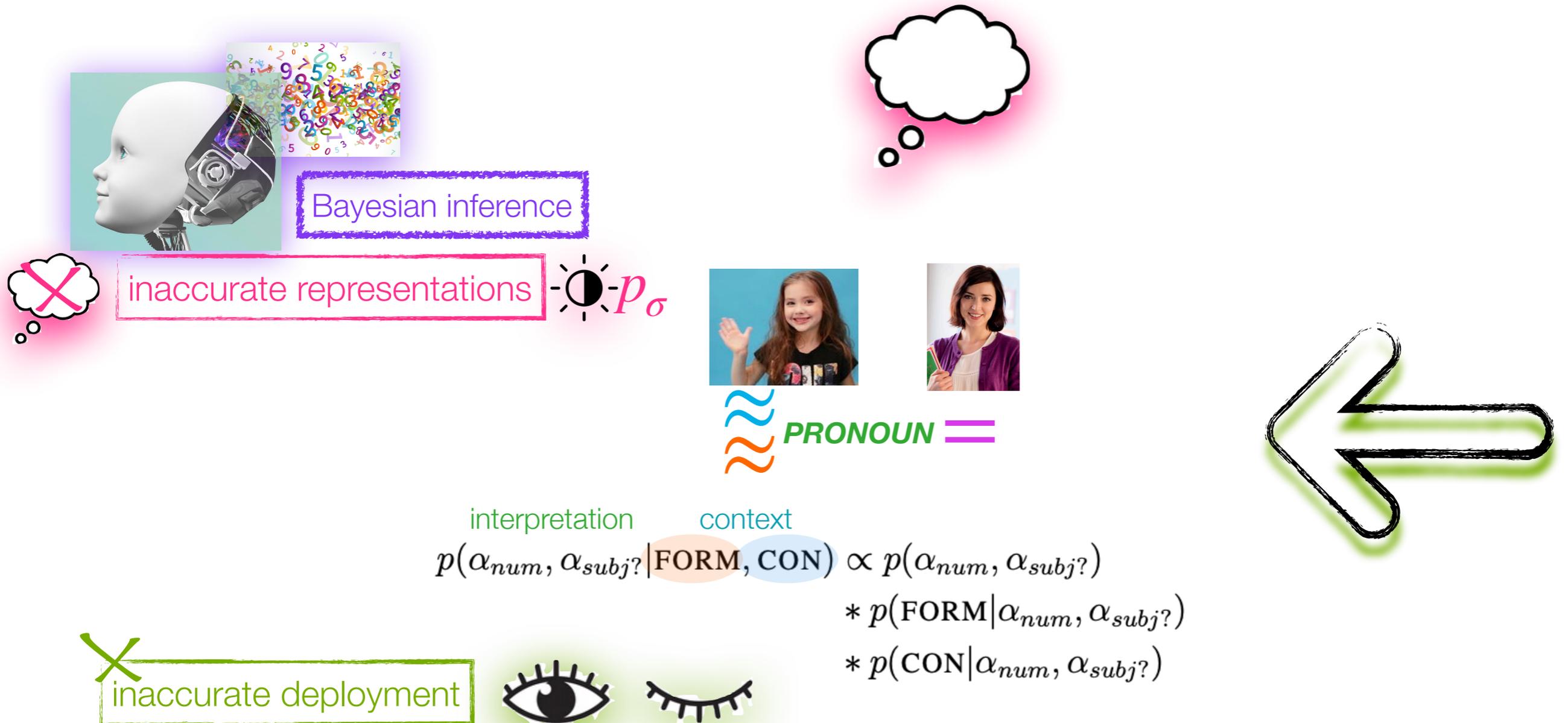
interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

inaccurate deployment

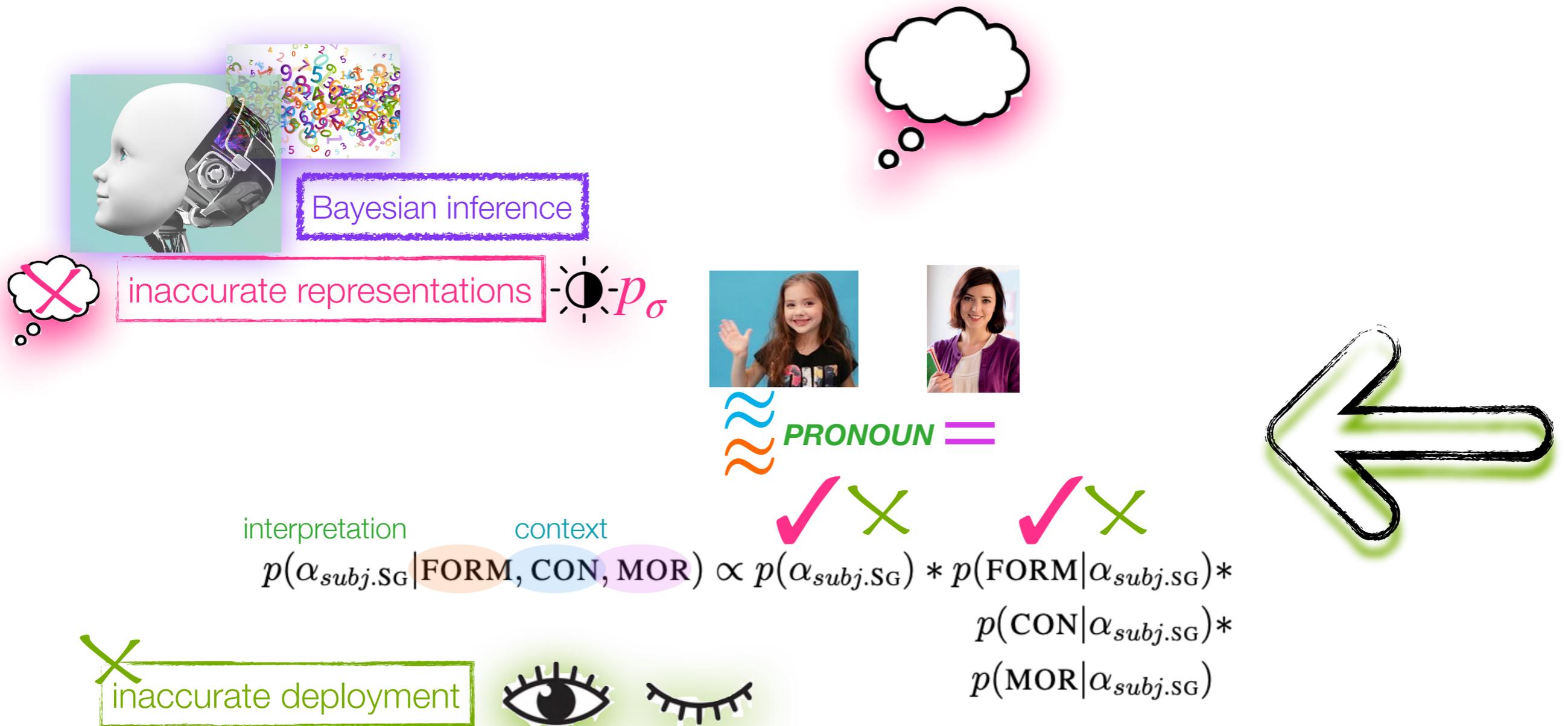
Not using likelihood information for a cue means **not using that cue's information**. For example, ignoring **morphology** information means not using the morphology likelihood.

Modeling pronoun interpretation in context



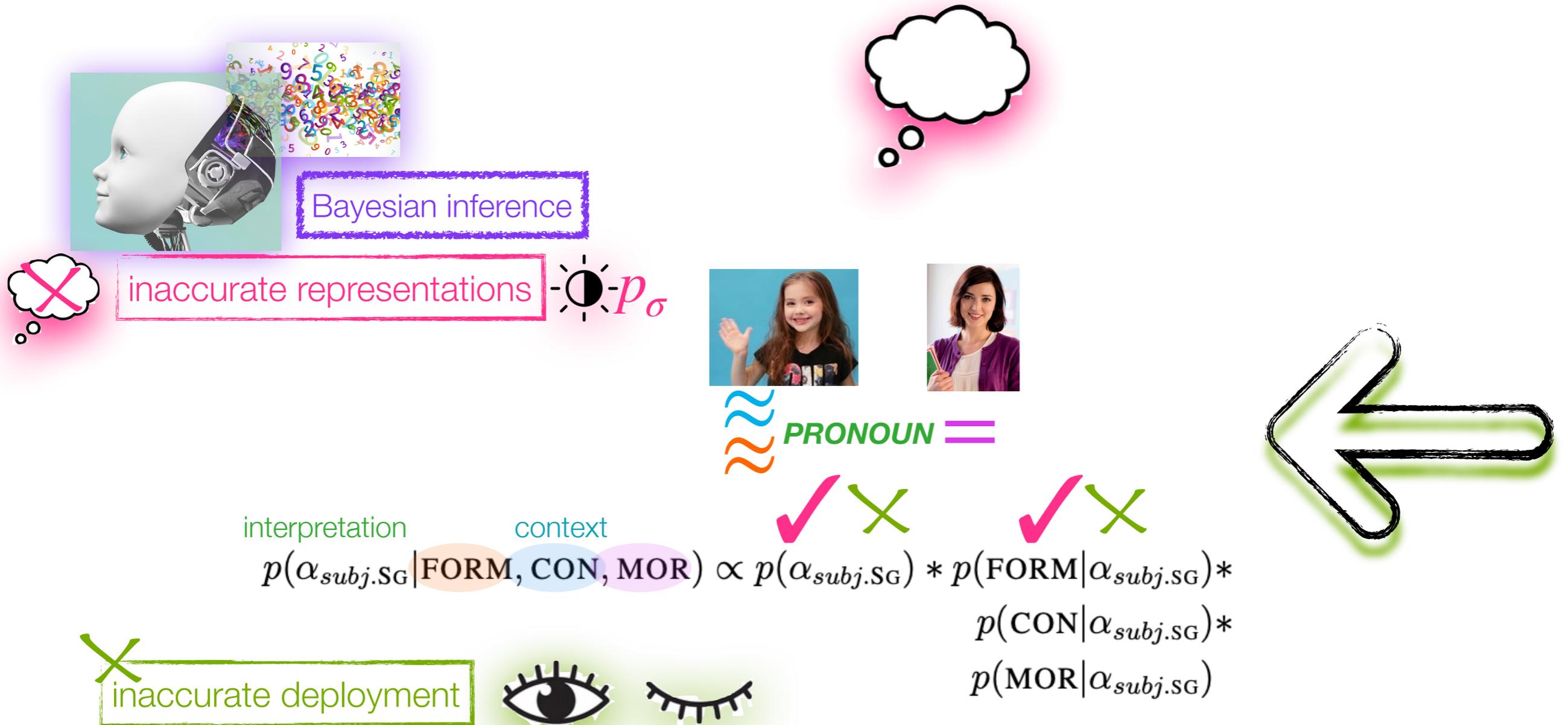
Not using likelihood information for a cue means **not using that cue's information**. For example, ignoring **morphology** information means not using the morphology likelihood.

Modeling pronoun interpretation in context



For any information, the modeled listener could use or not use it in the moment. Use parameter β determines whether a particular information type is used.

Modeling pronoun interpretation in context



Each of the four information types has its own β :

(prior) β_{α}

(likelihood) β_{form} , β_{con} , β_{mor}

Modeling pronoun interpretation in context



Bayesian inference

inaccurate representations



p_{σ}

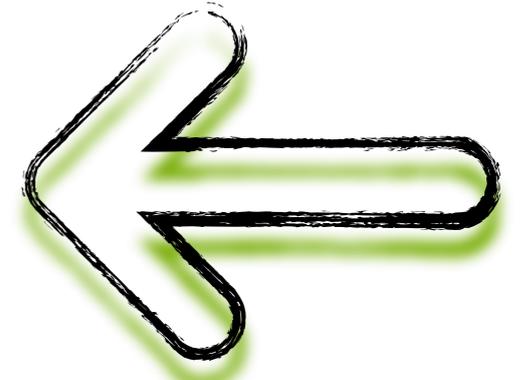
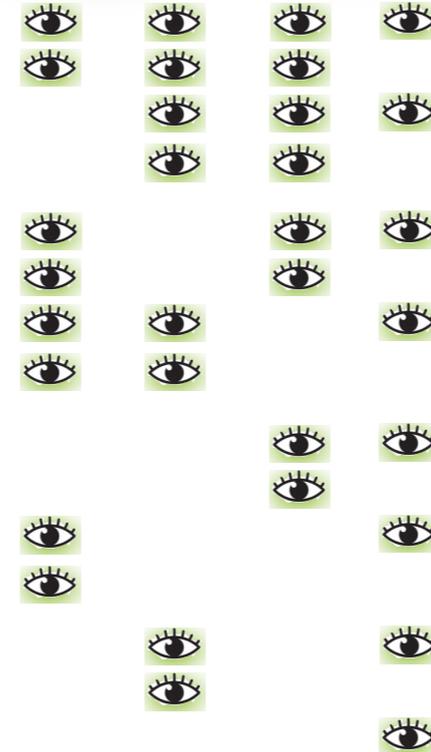
inaccurate deployment



PRONOUN =



β_{form} β_{con} β_{mor} β_{α}



$$\begin{aligned}
 & p_{\beta}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) = \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})
 \end{aligned}$$

This yields 16 possible use combinations for any particular moment, implemented with a mixture model p_{β} .

Modeling pronoun interpretation in context



Bayesian inference

inaccurate representations



p_{σ}

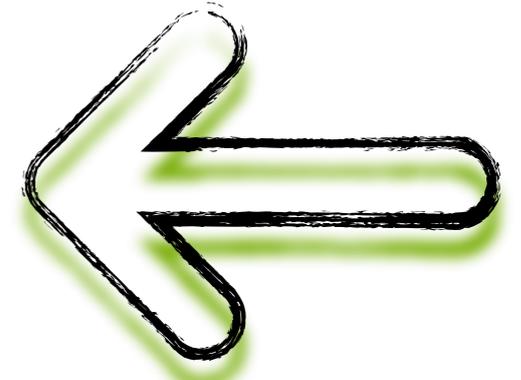
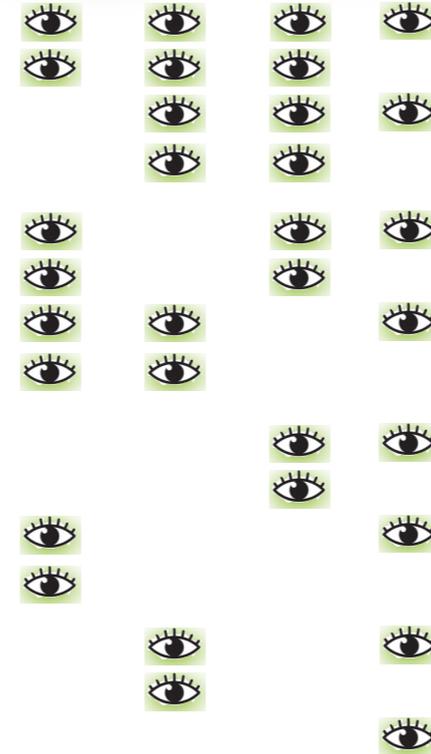
inaccurate deployment



PRONOUN =



β_{form} β_{con} β_{mor} β_{α}



interpretation context

$$\begin{aligned}
 p_{\beta}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) = & \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})
 \end{aligned}$$

We allow $0 \leq \beta \leq 1$, and see which β value combinations best predict child and adult pronoun interpretation behavior.



Modeling pronoun interpretation in context



interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

What about a modeled listener who has both **inaccurate representations** of information and **inaccurate deployment** of those representations?

Modeling pronoun interpretation in context



inaccurate both

$$\begin{aligned}
 p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) &\propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \\
 &* p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \\
 &* p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \\
 &* p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}
 \end{aligned}$$

$$p_{\sigma, \beta}$$

$$\begin{aligned}
 p_{\beta}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) = & \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})
 \end{aligned}$$

We implement this as a combination of the previous two modeled listeners, including σ values for inaccurate representations and β values for inaccurate deployment.

Modeling pronoun interpretation in context



inaccurate both

$$p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \cdot p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \cdot p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \cdot p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}$$

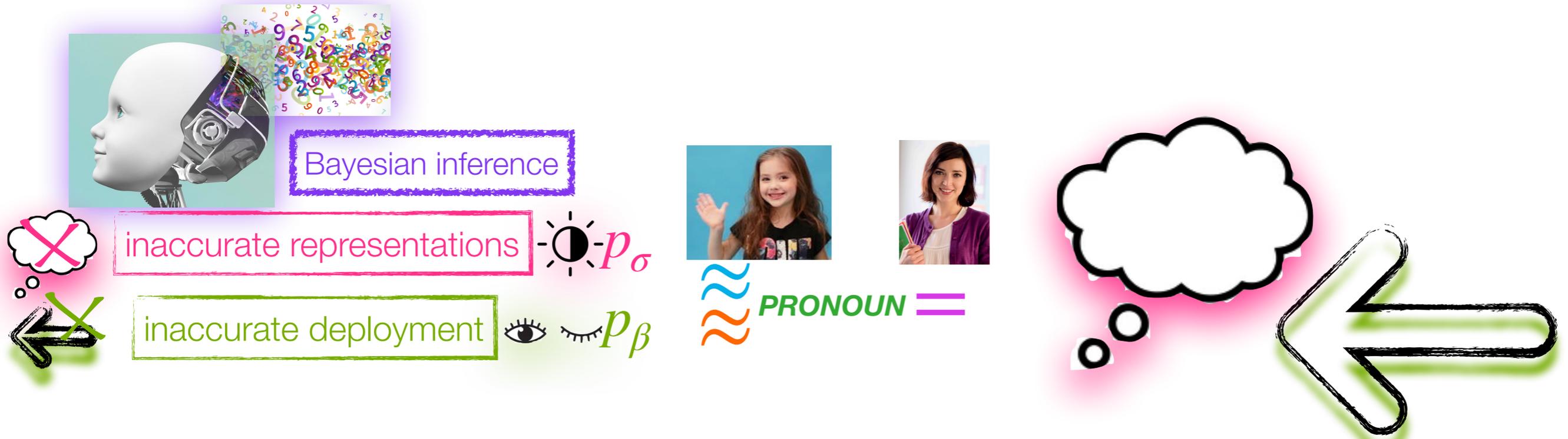
$P_{\sigma, \beta}$

P_{β}

$$p_{\beta}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) = (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})$$

Each term of P_{β} has σ values for all information types that are used.

Modeling pronoun interpretation in context



inaccurate both

$$\begin{aligned}
 P_{\sigma, \beta} &= P_\sigma P_\beta \\
 p_\sigma(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) &\propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_\alpha} \\
 &\quad * p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \\
 &\quad * p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \\
 &\quad * p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}
 \end{aligned}$$

So, $P_{\sigma, \beta}$ has 8 parameter values:

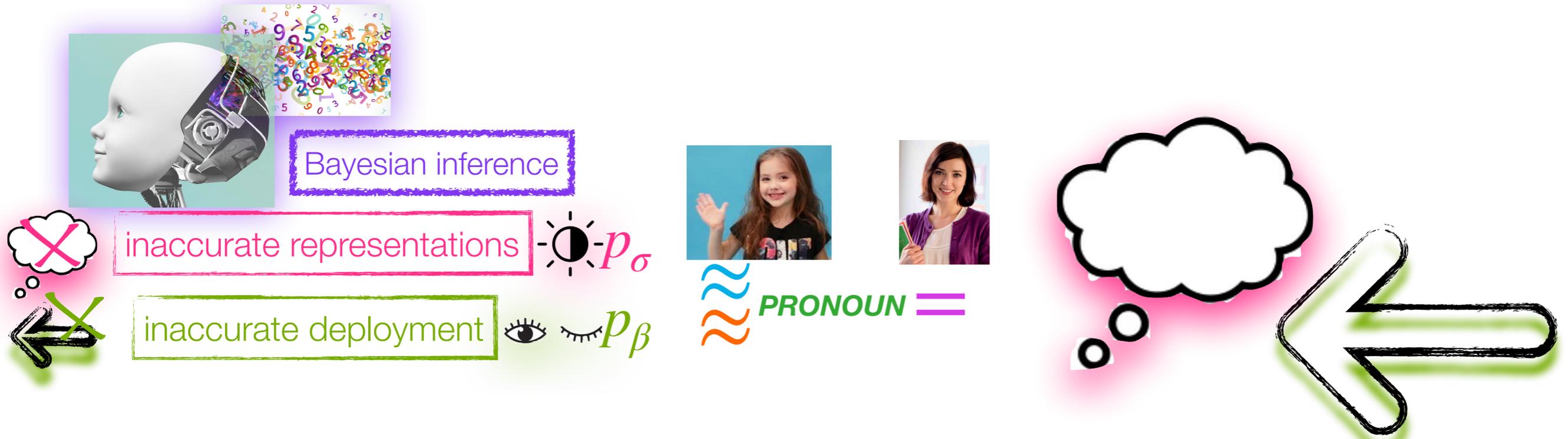
(in the prior) $\sigma_\alpha, \beta_\alpha$

(in the likelihood) $\sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_{form}, \beta_{con}, \beta_{mor}$

$$P_{\sigma, \beta} \quad P_\beta$$

$$\begin{aligned}
 p_\beta(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) = & \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_\alpha) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_\alpha) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_\alpha) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_\alpha) * p_{\text{UNIF}}(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_\alpha) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_\alpha) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_\alpha) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_\alpha) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_\alpha) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_\alpha) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_\alpha) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_\alpha) * p_{\text{UNIF}}(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_\alpha) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_\alpha) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_\alpha) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_\alpha) * p(\text{UNIF})
 \end{aligned}$$

Modeling pronoun interpretation in context



inaccurate both

$$p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}}$$

- * $p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}}$
- * $p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}}$
- * $p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}$

We allow $0.00 \leq \sigma \leq 4.00$ and $0 \leq \beta \leq 1$, and see which σ and β value combinations best predict **child** and **adult** pronoun interpretation behavior.



$$p_{\beta}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) =$$

$$(\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) +$$

$$(1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) +$$

$$(1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) +$$

$$(\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) +$$

$$(\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) +$$

$$(\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) +$$

$$(1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) +$$

$$(1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) +$$

$$(\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) +$$

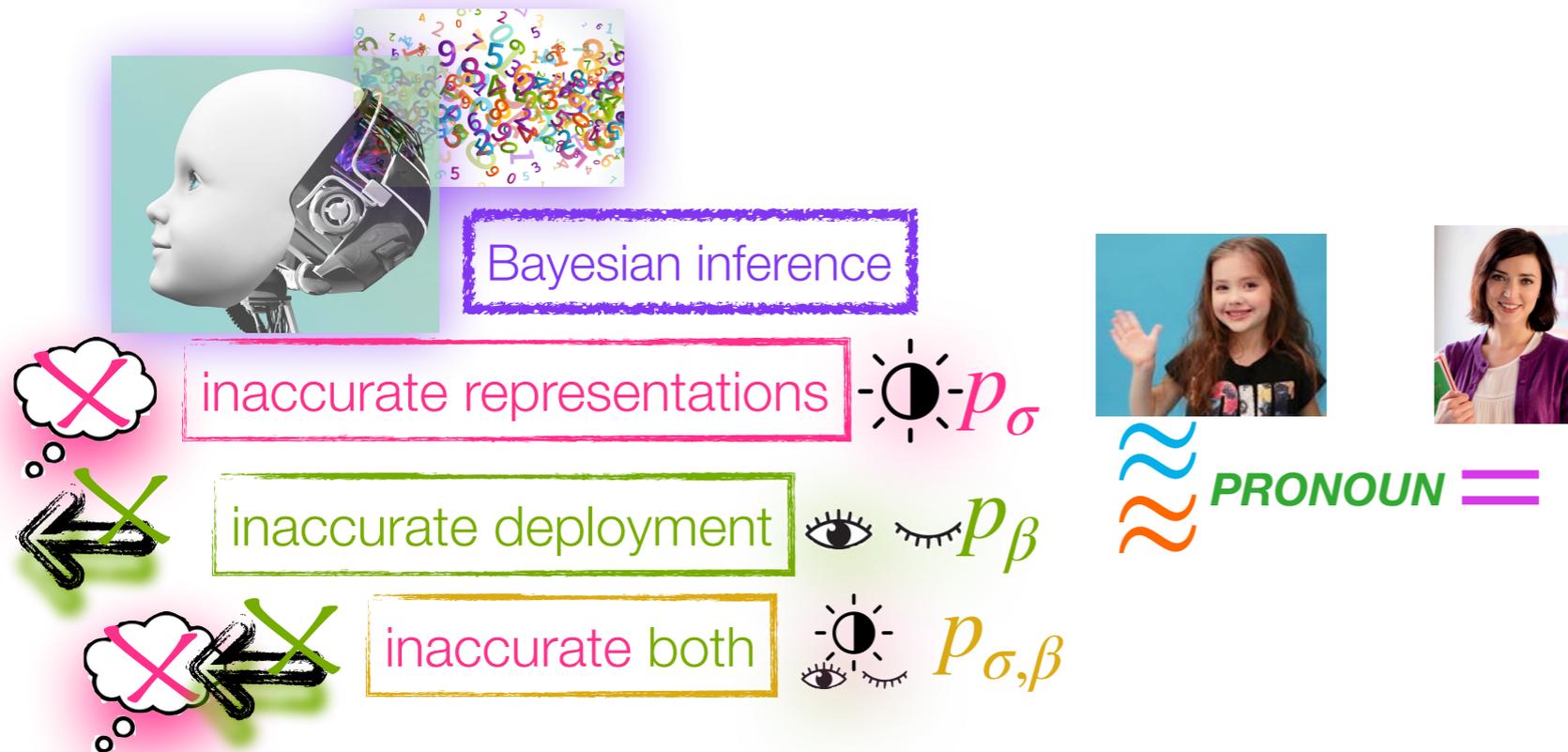
$$(1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) +$$

$$(1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) +$$

$$(1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) +$$

$$(1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})$$

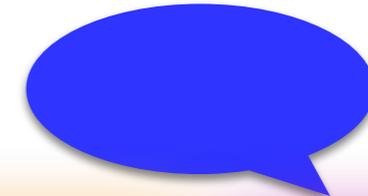
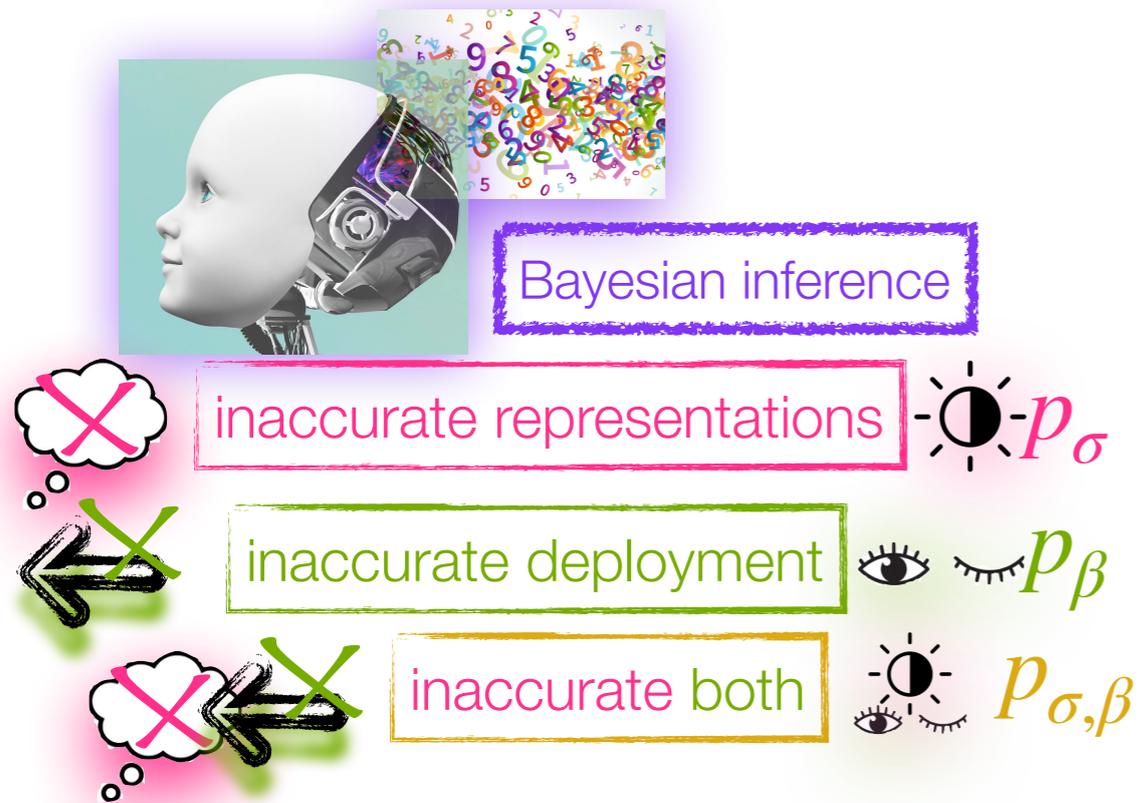
Input to the modeled listener



$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

What **input** is the modeled listener using to represent the various information types?

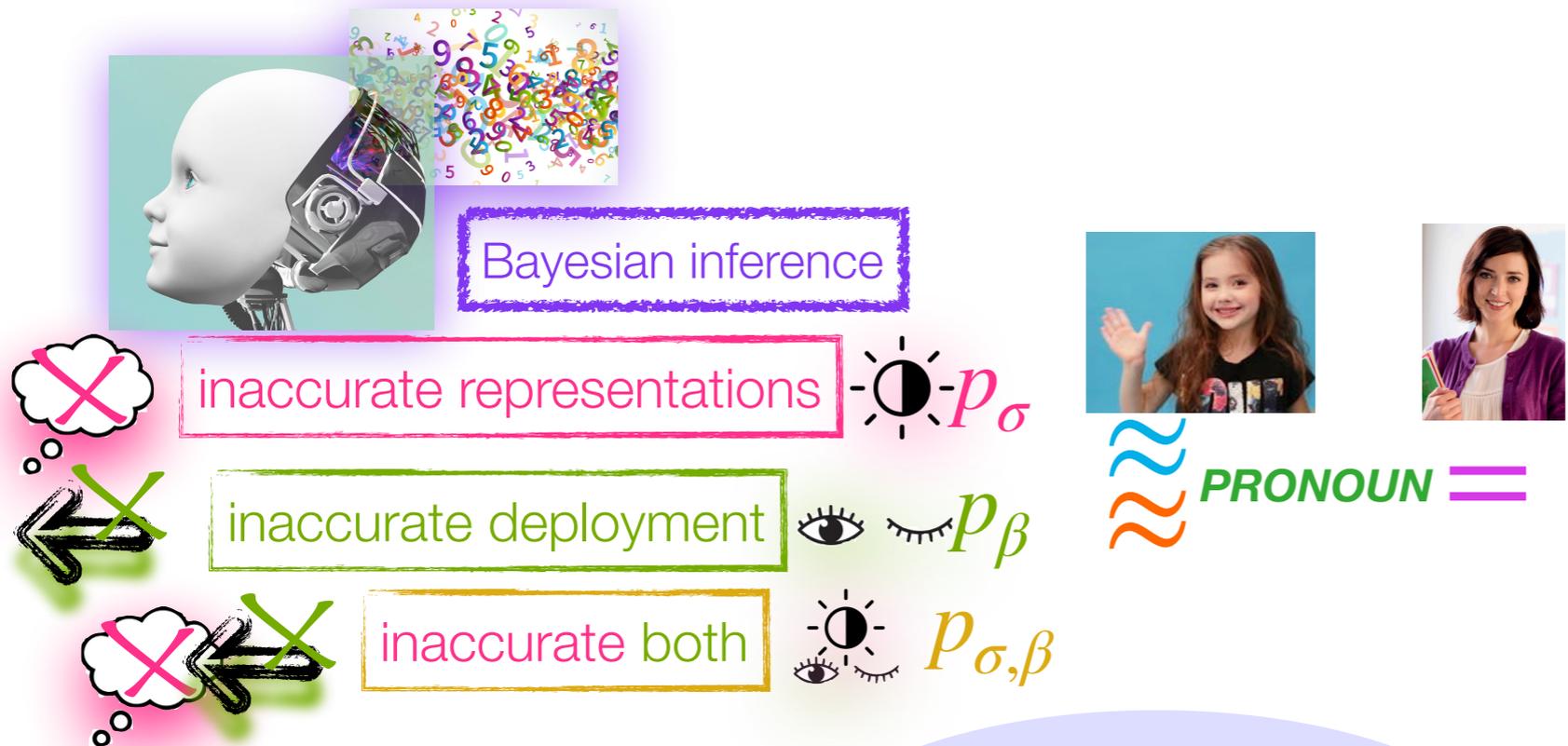
Input to the modeled listener



$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

54,757 utterances of Mexico City spontaneous child-directed speech to children 1;6-5;11 from the Schmitt-Miller corpus (Miller & Schmitt 2012).

Input to the modeled listener



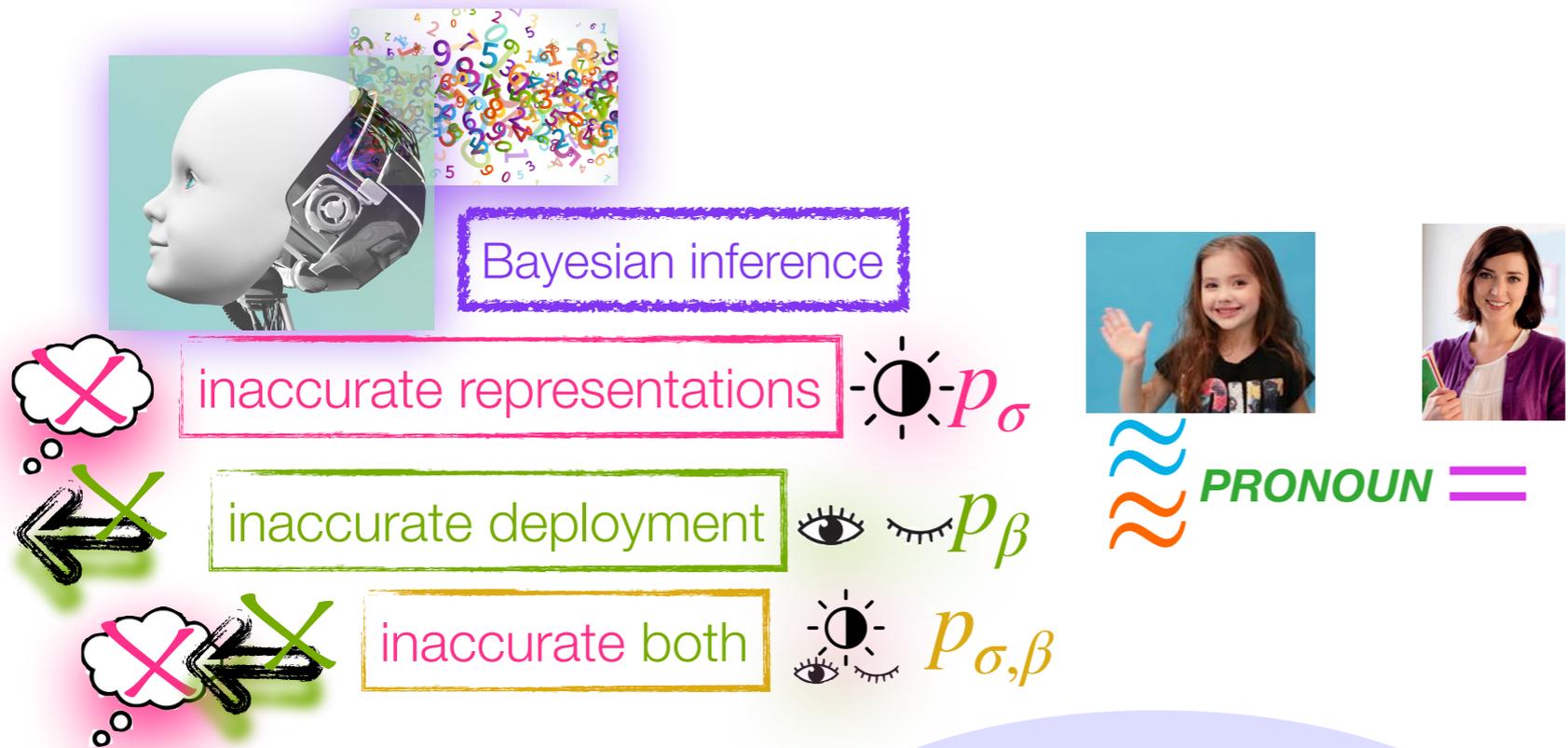
$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
			\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

From this, we estimate the relevant priors and likelihoods.

Input to the modeled listener



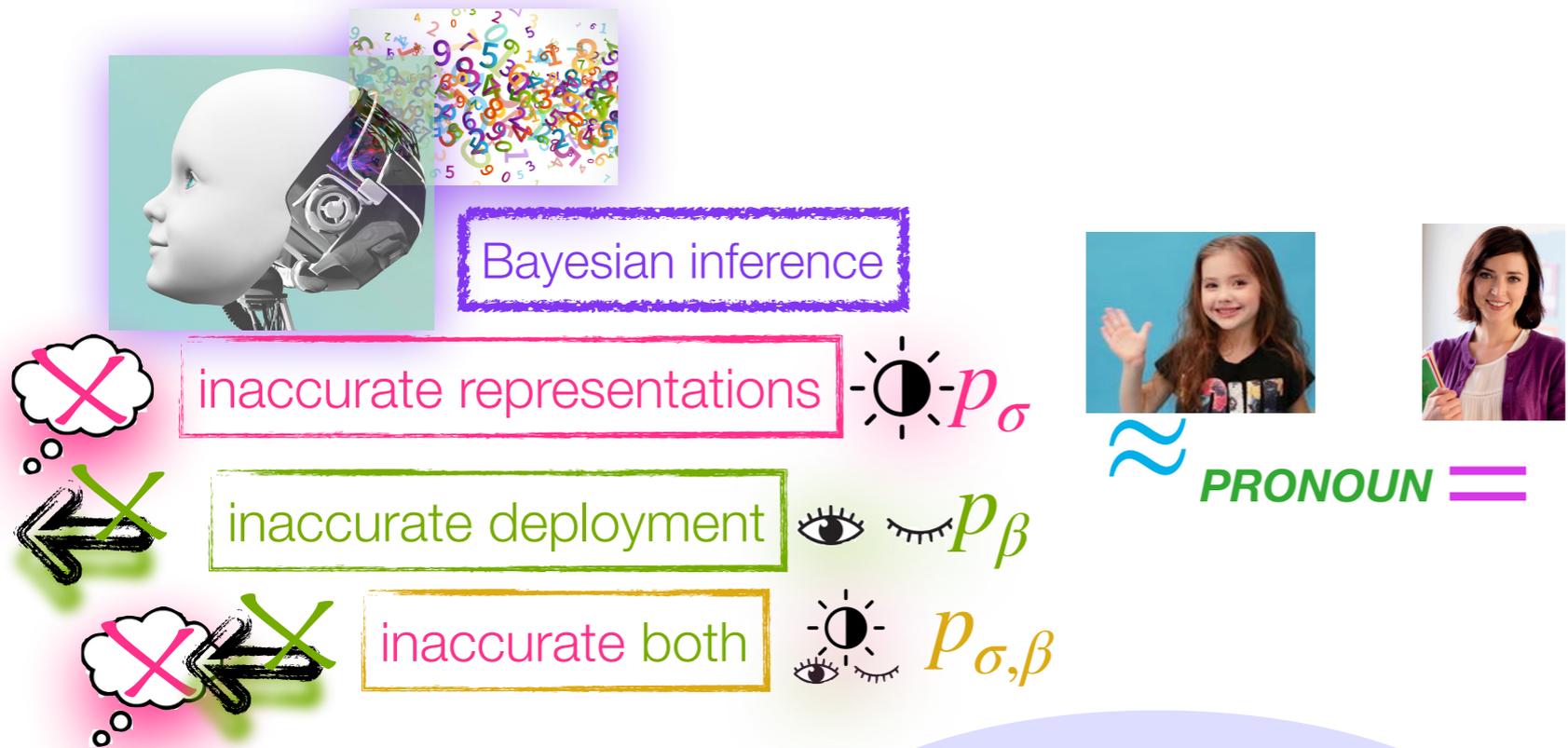
$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
			\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

Singular antecedents generally occur more often.

Input to the modeled listener



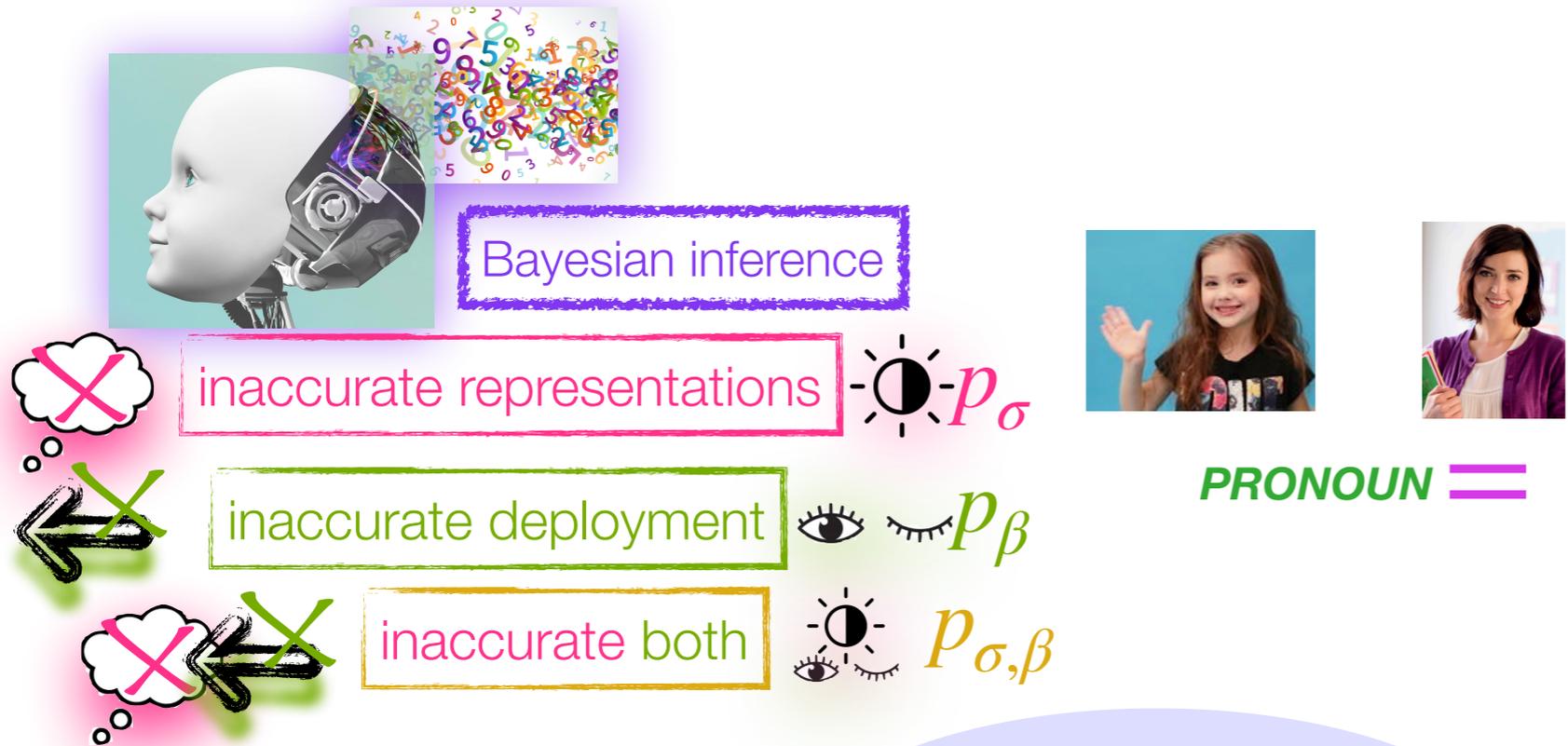
$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



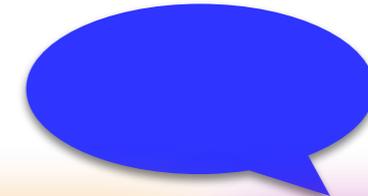
antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
			\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

The null pronoun form is generally used, though some antecedent types use it more often.

Input to the modeled listener



PRONOUN =

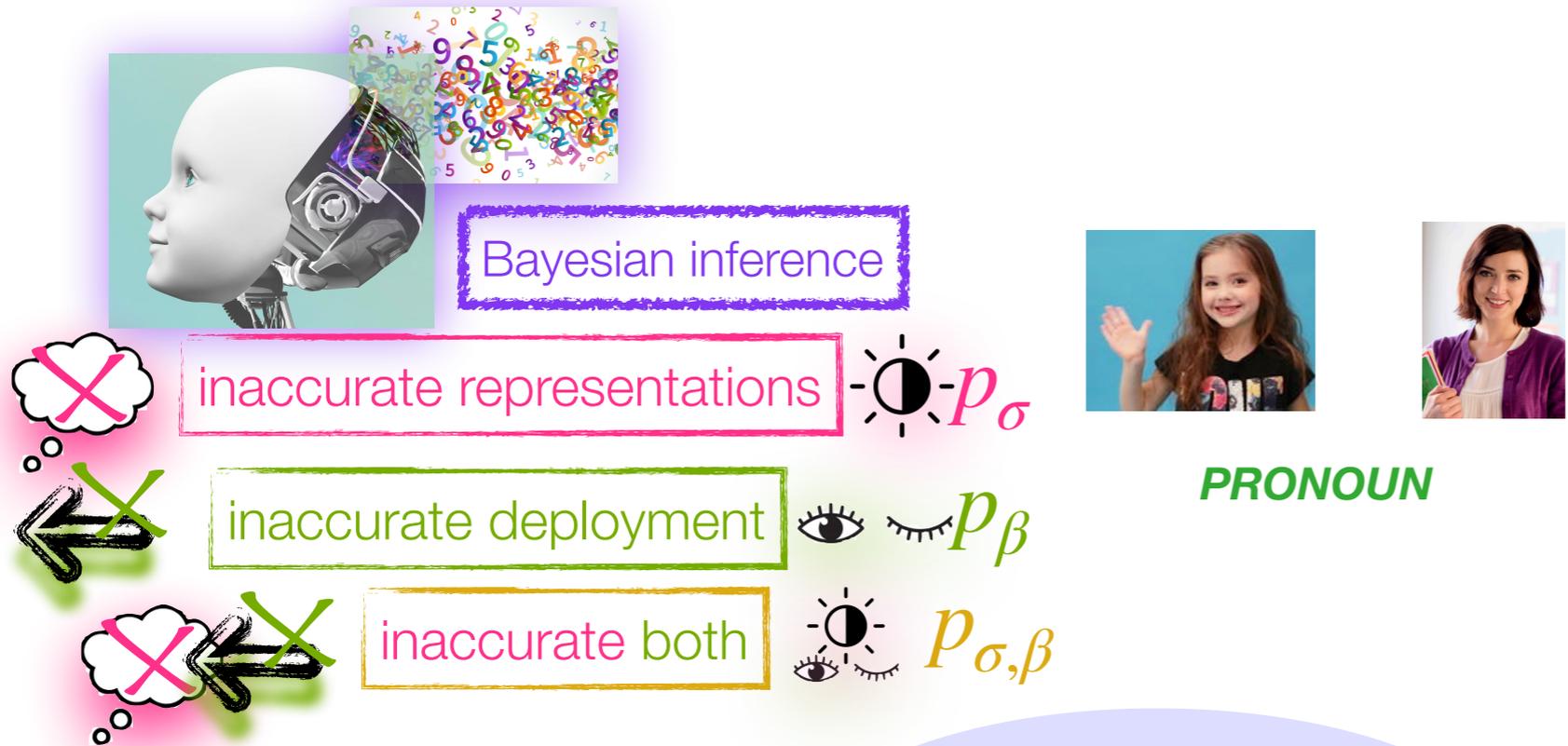


$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

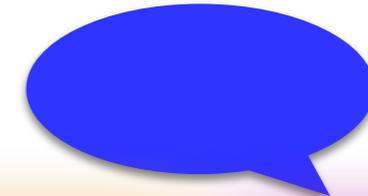
antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
			\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

The connective *porque* is used more often for antecedents that aren't plural subjects, though how much more often varies.

Input to the modeled listener



PRONOUN



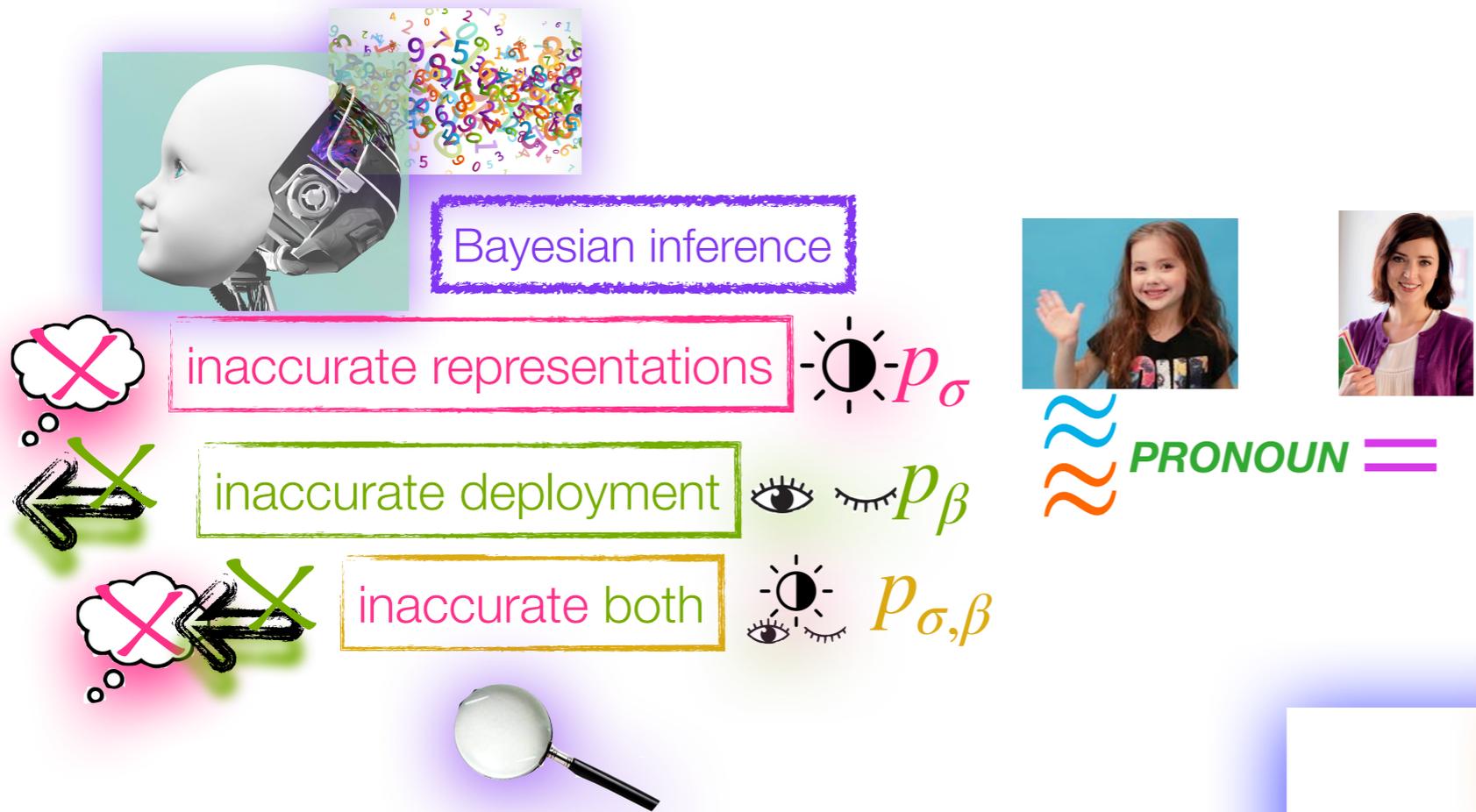
$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
		\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL	
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

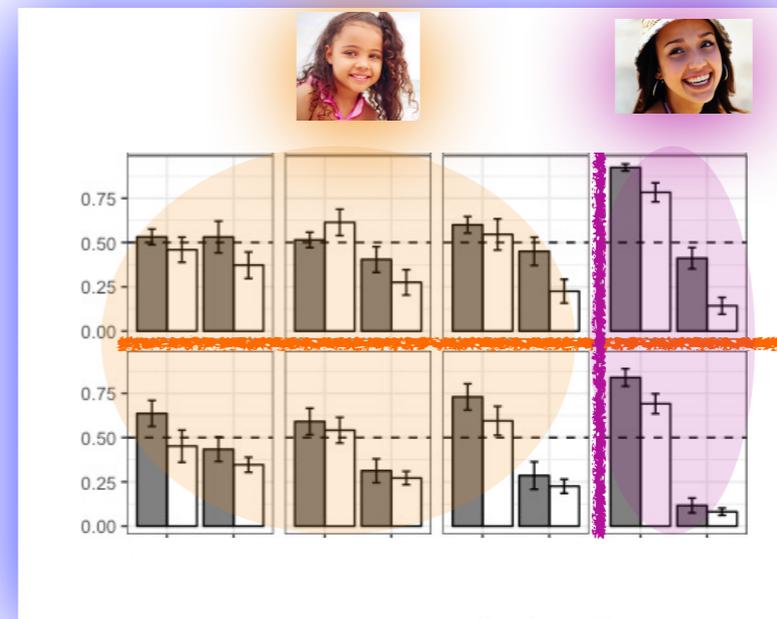
≈
≈
=

Agreement morphology is nearly categorical, with a very strong preference for matching morphology.

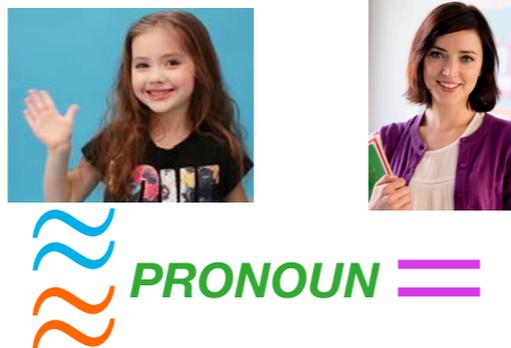
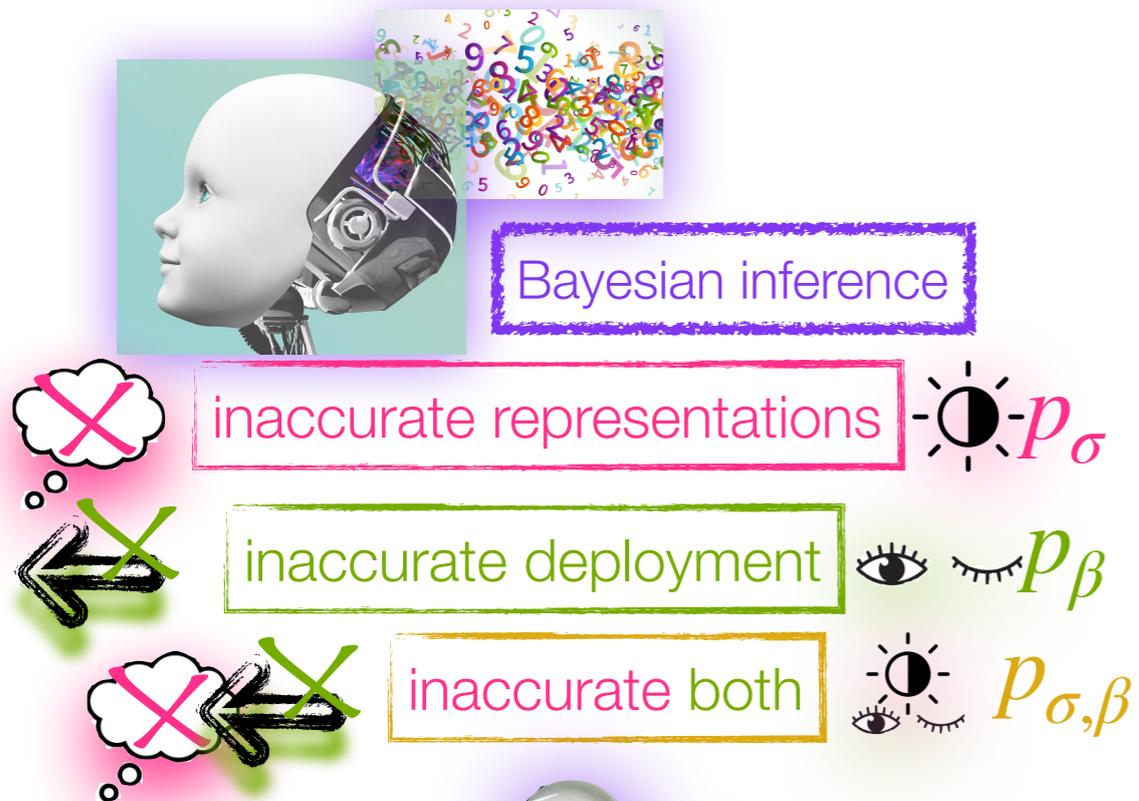
Understanding the development of pronoun interpretation



The plan, part 2: Use the **computational cognitive model** to identify the specific potential differences leading to **child** and **adult pronoun interpretation** in context.

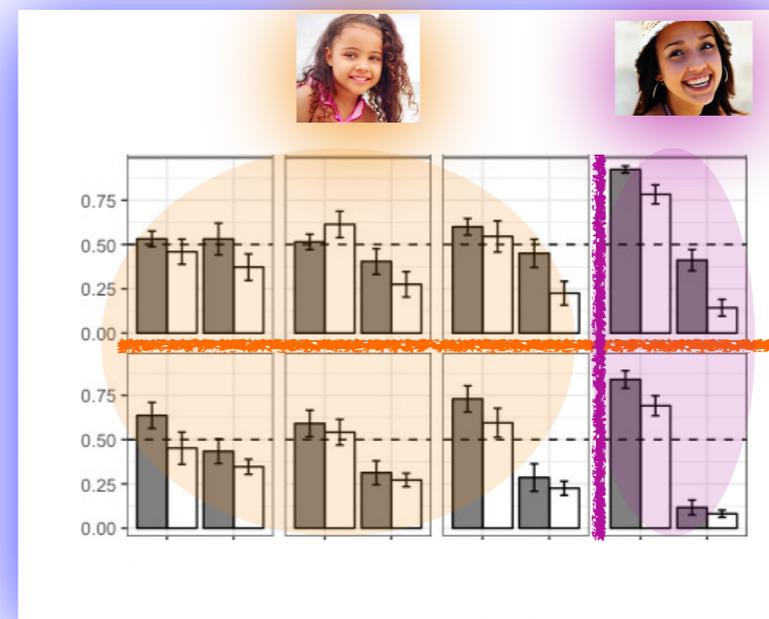


Understanding the development of pronoun interpretation

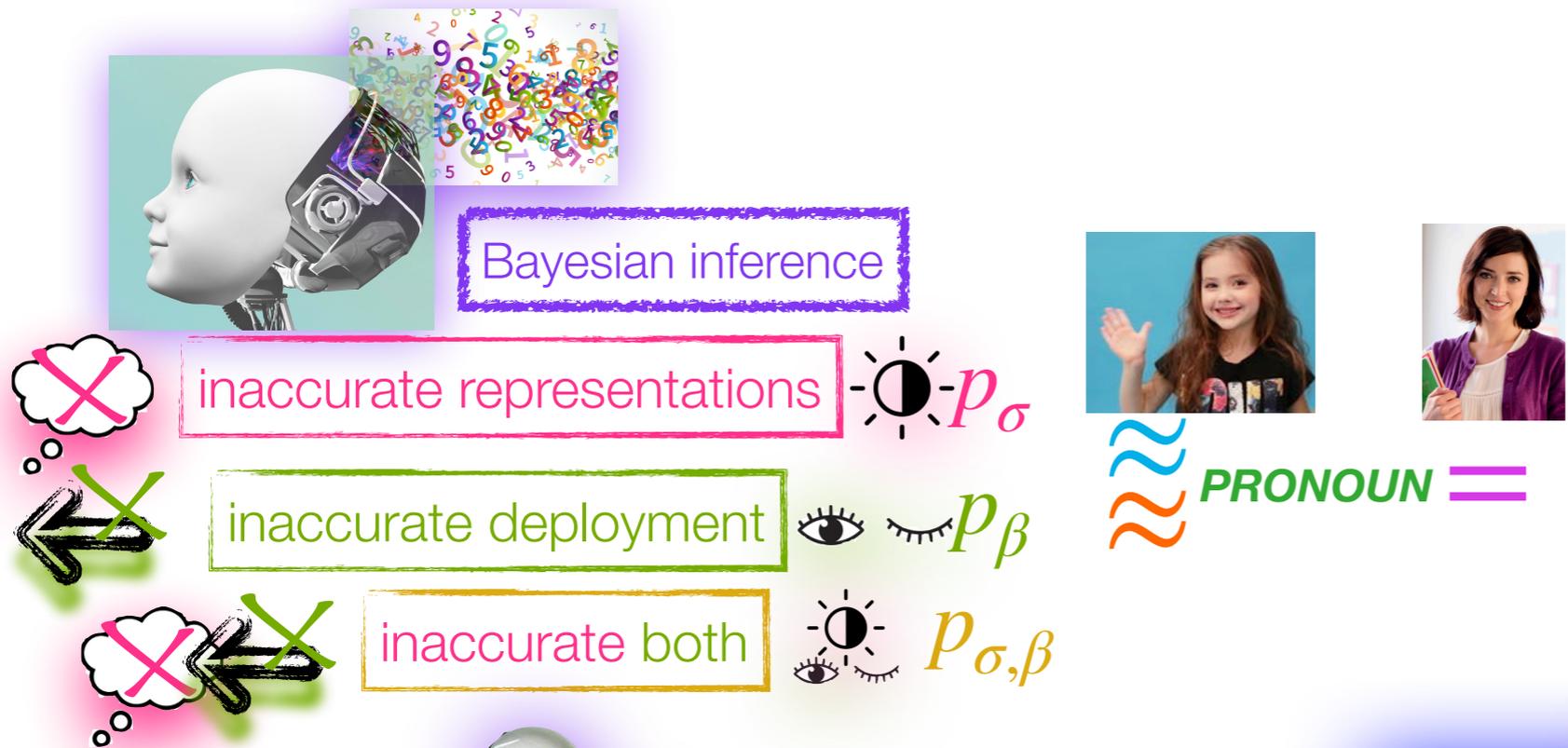


?

Which modeled listener variant
best matches the observed
pronoun interpretation behavior?

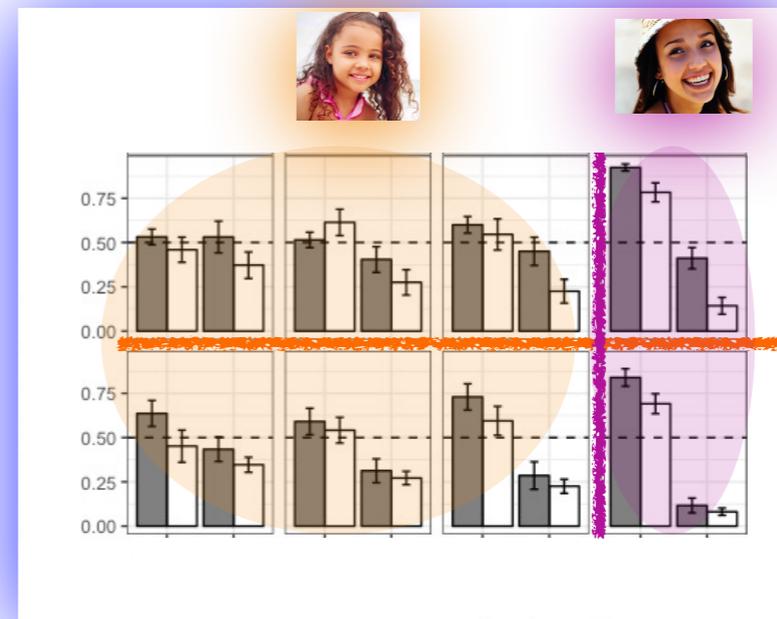


Understanding the development of pronoun interpretation

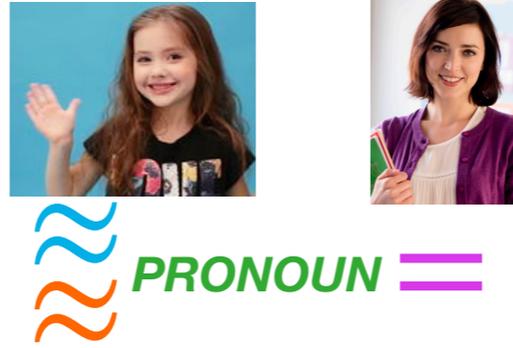
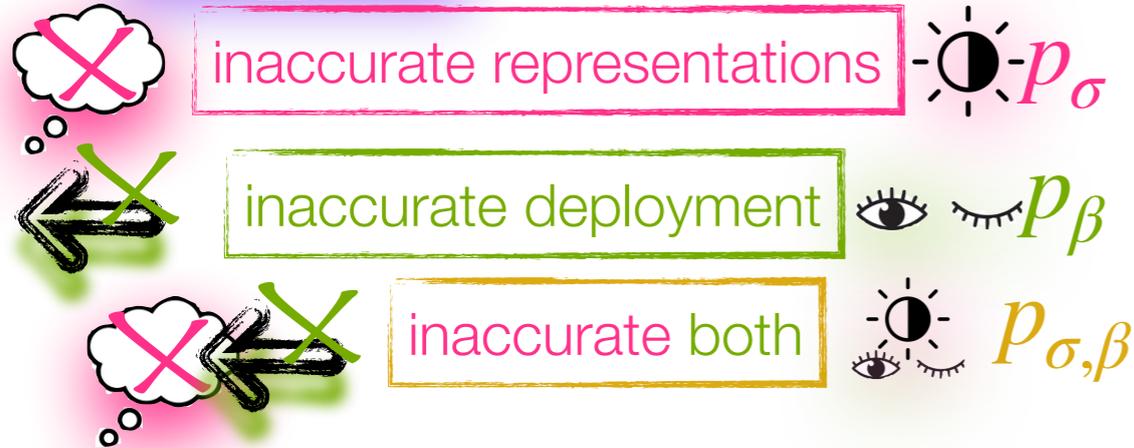
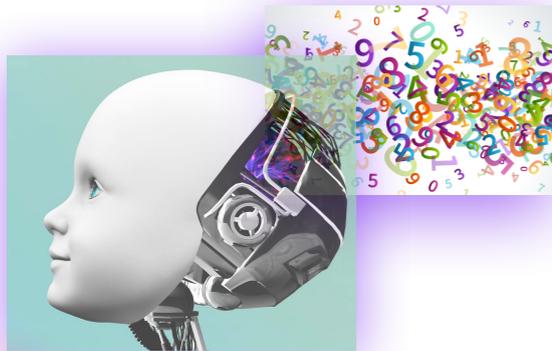


?

Important: Model variants with more parameters have an easier time fitting the data because they have more degrees of freedom.



Understanding the development of pronoun interpretation

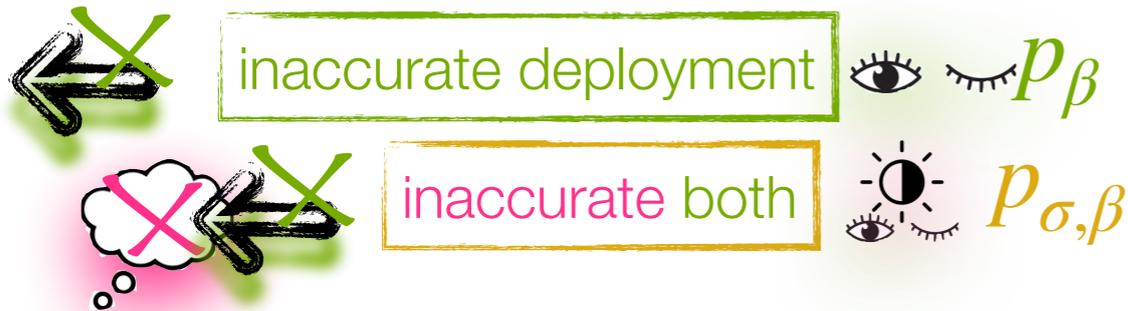
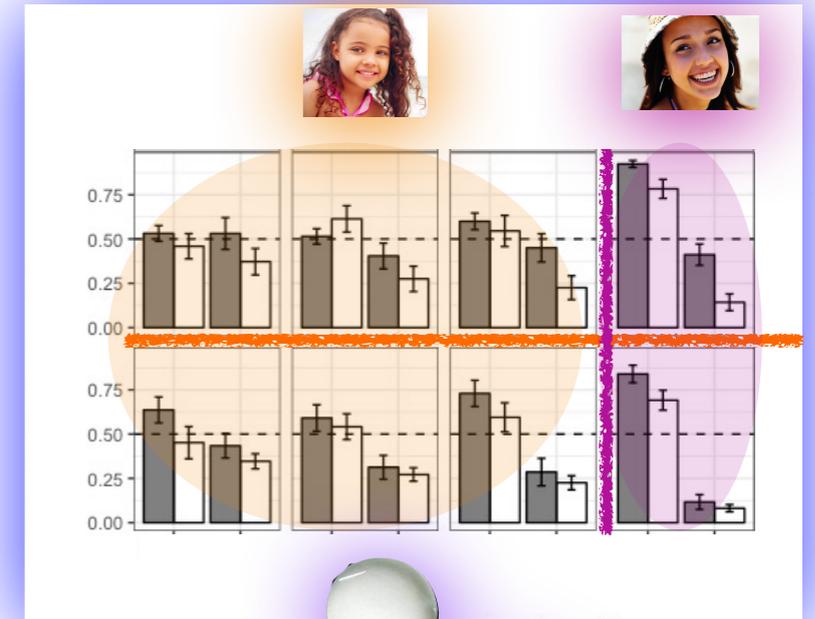


baseline: accurate representations and deployment p

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

0 free parameters

Understanding the development of pronoun interpretation



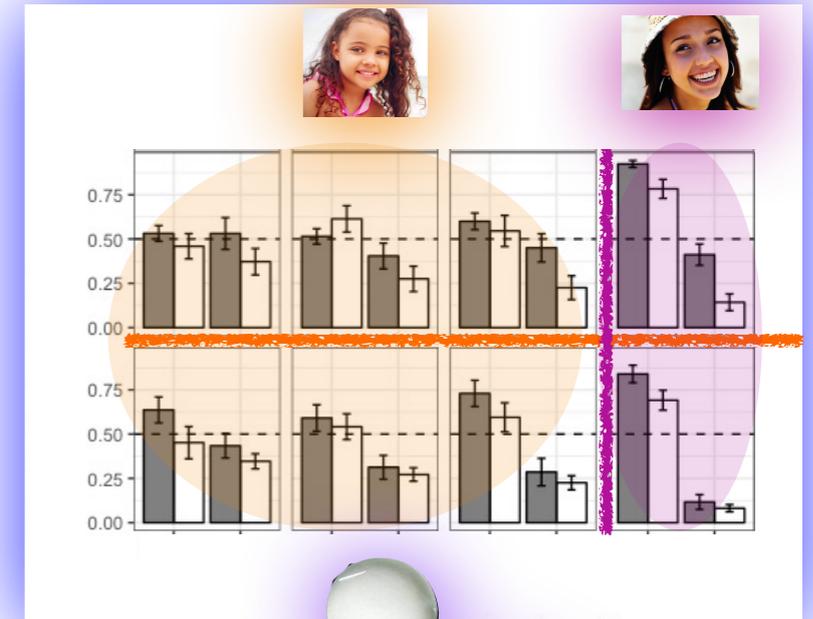
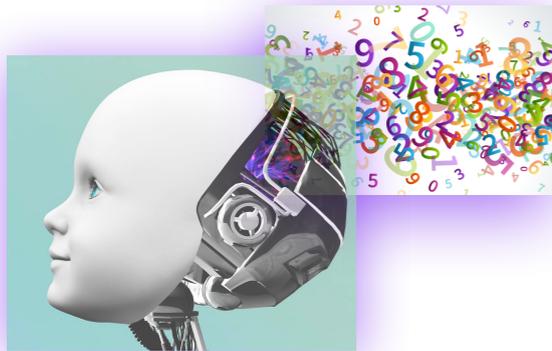
 **PRONOUN** =

baseline: accurate representations and deployment P 0 free parameters

 inaccurate representations  P_{σ}

4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}

Understanding the development of pronoun interpretation



 inaccurate both  $P_{\sigma, \beta}$

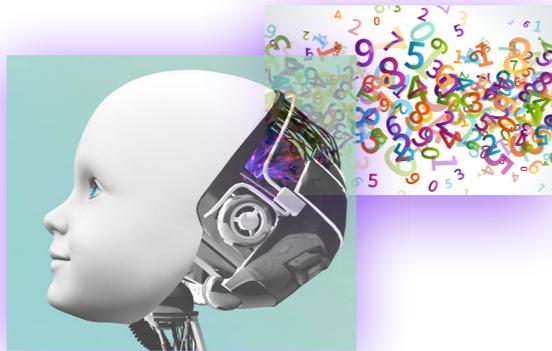
baseline: accurate representations and deployment P 0 free parameters

 inaccurate representations  P_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}

 inaccurate deployment  P_{β}

4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}

Understanding the development of pronoun interpretation



baseline: accurate representations and deployment P 0 free parameters

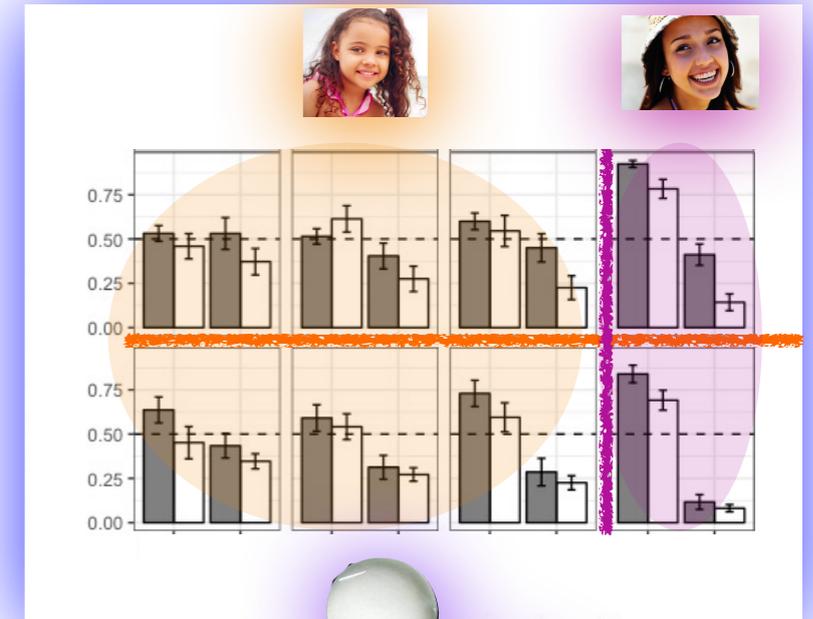
 inaccurate representations  P_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}

 inaccurate deployment  P_{β} 4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}

 inaccurate both  $P_{\sigma, \beta}$

8 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor} , β_{α} , β_{form} , β_{con} , β_{mor}

Understanding the development of pronoun interpretation



We want variants with more parameters to have a **substantially better fit** in order to favor them over variants with fewer parameters.



baseline: accurate representations and deployment P_0 0 free parameters

- 
inaccurate representations

 P_σ 4 free parameters: $\sigma_\alpha, \sigma_{form}, \sigma_{con}, \sigma_{mor}$

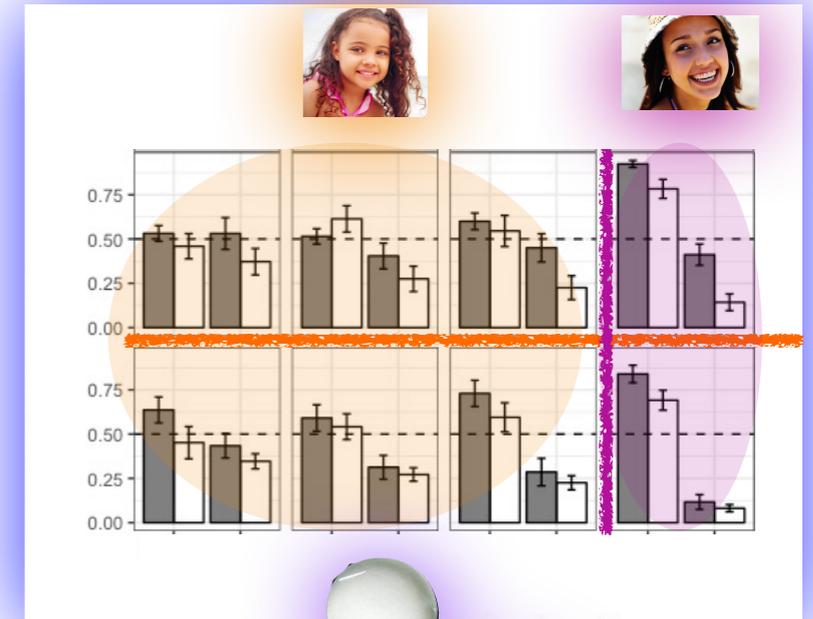
- 
inaccurate deployment

 P_β 4 free parameters: $\beta_\alpha, \beta_{form}, \beta_{con}, \beta_{mor}$

- 
inaccurate both

 $P_{\sigma,\beta}$ 8 free parameters: $\sigma_\alpha, \sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_\alpha, \beta_{form}, \beta_{con}, \beta_{mor}$


Understanding the development of pronoun interpretation



The Bayesian Information Criterion (BIC) is one way to quantify this preference (Schwarz 1978).

baseline: accurate representations and deployment

P 0 free parameters



inaccurate representations



P_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}



inaccurate deployment



P_{β} 4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}



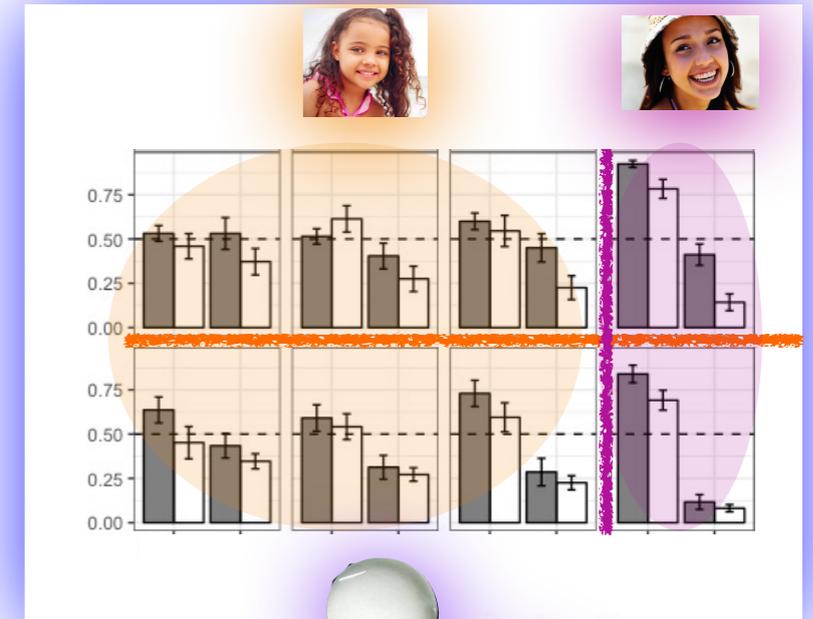
inaccurate both



$P_{\sigma, \beta}$ 8 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor} , β_{α} , β_{form} , β_{con} , β_{mor}



Understanding the development of pronoun interpretation



$BIC = \# \text{ parameters} \cdot \log(|\text{data}|) - 2 \cdot \log(\text{model fit})$

$0 \leq BIC \leq \infty$ (closer to 0 is better)

baseline: accurate representations and deployment

p 0 free parameters



inaccurate representations



p_{σ} 4 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}$



inaccurate deployment



p_{β} 4 free parameters: $\beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$

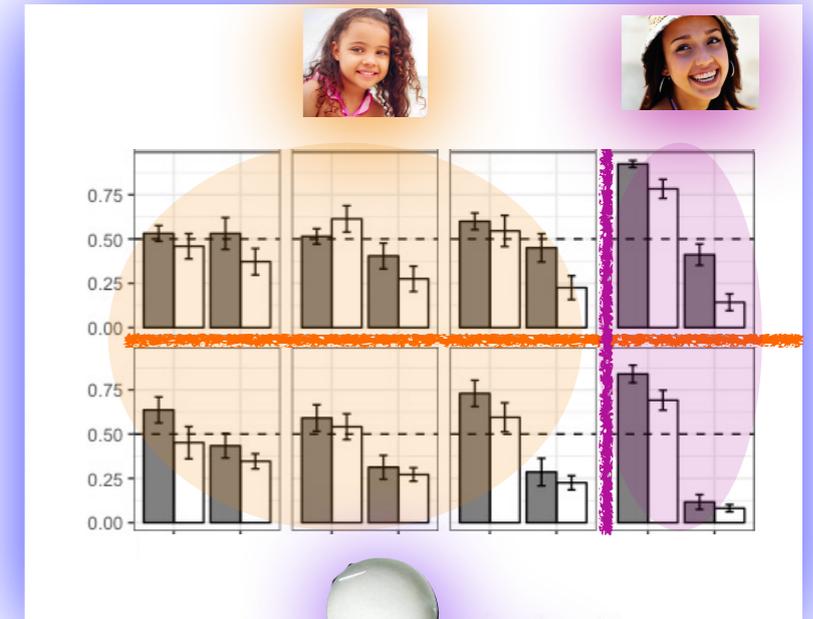


inaccurate both



$p_{\sigma, \beta}$ 8 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$

Understanding the development of pronoun interpretation



more parameters = higher score,
closer to 0 is better

$$\text{BIC} = \# \text{ parameters} \cdot \log(|\text{data}|) - 2 \cdot \log(\text{model fit})$$

$$0 \leq \text{BIC} \leq \infty \text{ (closer to 0 is better)}$$

baseline: accurate representations and deployment P 0 free parameters

~~inaccurate representations~~ P_{σ} 4 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}$

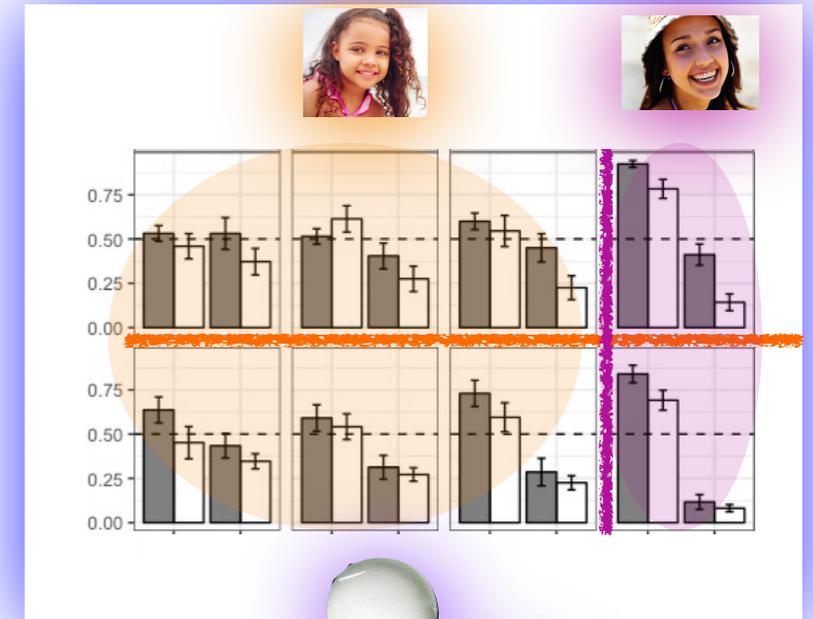
~~inaccurate deployment~~ P_{β} 4 free parameters: $\beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$

~~inaccurate both~~ $P_{\sigma, \beta}$ 8 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$

Understanding the development of pronoun interpretation



model fit = likelihood of data, given model with best-fitting parameter values



BIC = # parameters · log(|data|) - 2 · log(model fit)

$0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

baseline: accurate representations and deployment p 0 free parameters

- 
inaccurate representations

 p_{σ} 4 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}$
- 
inaccurate deployment

 p_{β} 4 free parameters: $\beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$
- 
inaccurate both

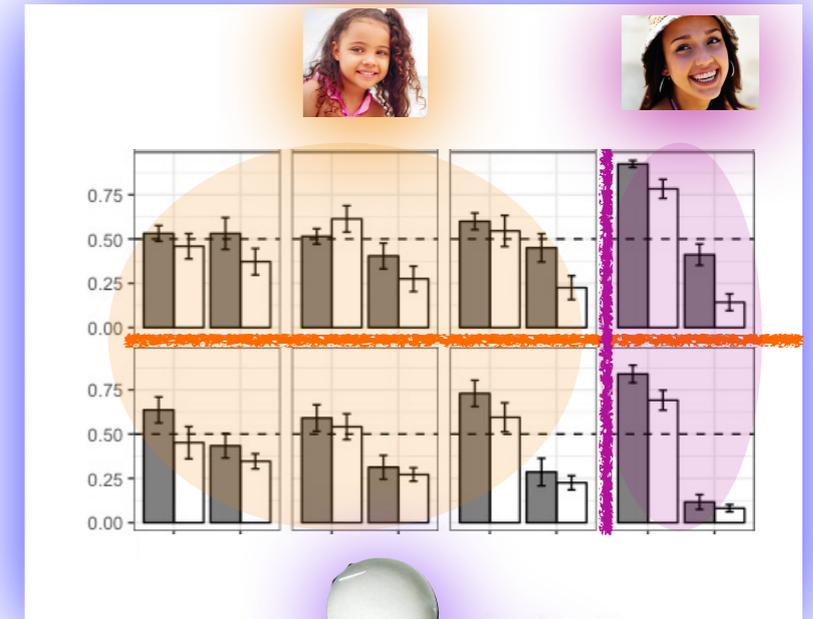
 $p_{\sigma, \beta}$ 8 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$

Understanding the development of pronoun interpretation



$-\infty \leq \log(\text{likelihood}) \leq 0$,
closer to 0 is better

 **PRONOUN** =



$\text{BIC} = \# \text{ parameters} \cdot \log(|\text{data}|) - 2 \cdot \log(\text{model fit})$

$0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

baseline: accurate representations and deployment

P 0 free parameters

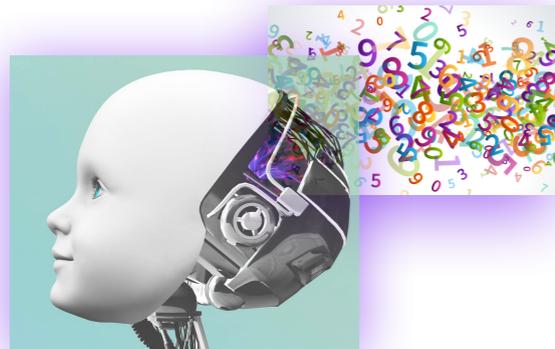
- 
inaccurate representations

 P_{σ} 4 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}$
- 
inaccurate deployment

 P_{β} 4 free parameters: $\beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$
- 
inaccurate both

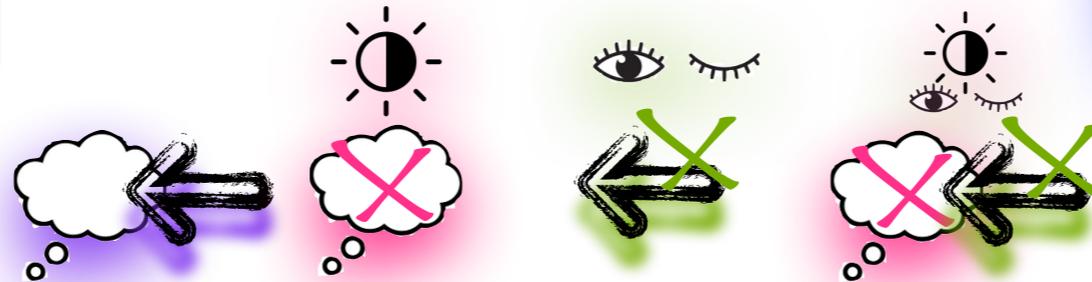
 $P_{\sigma, \beta}$ 8 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$

Understanding the development of pronoun interpretation

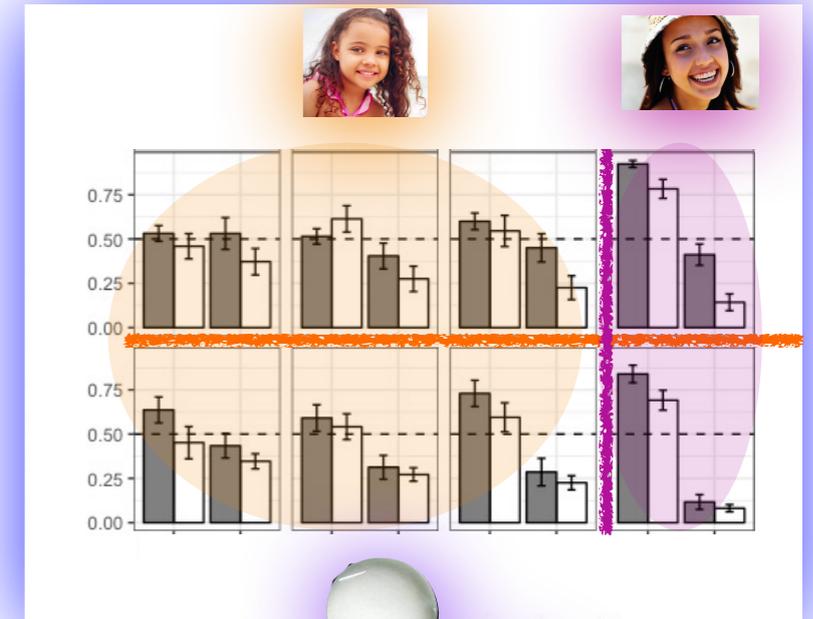


PRONOUN =

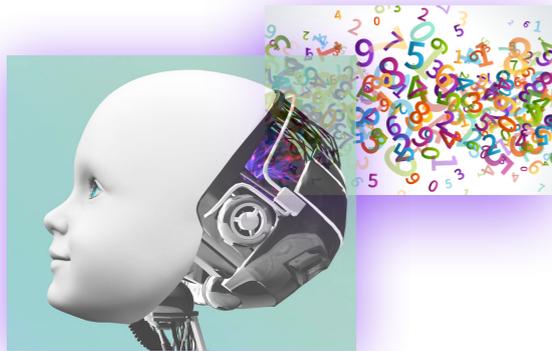
0 ≤ BIC ≤ ∞ (closer to 0 is better)



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3				
4				
≥ 5				
adults				

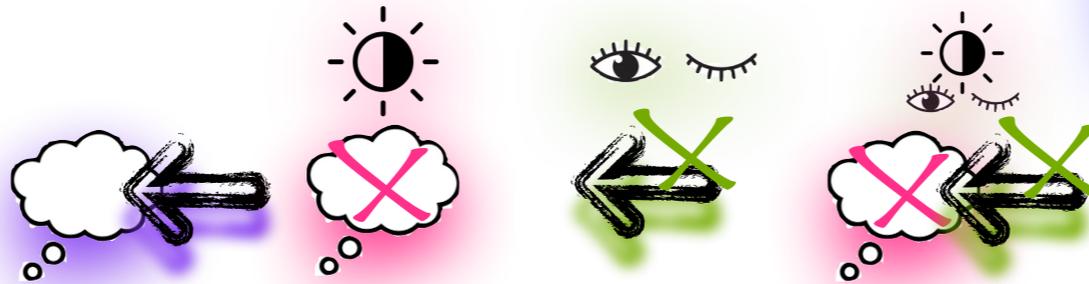


Understanding the development of pronoun interpretation

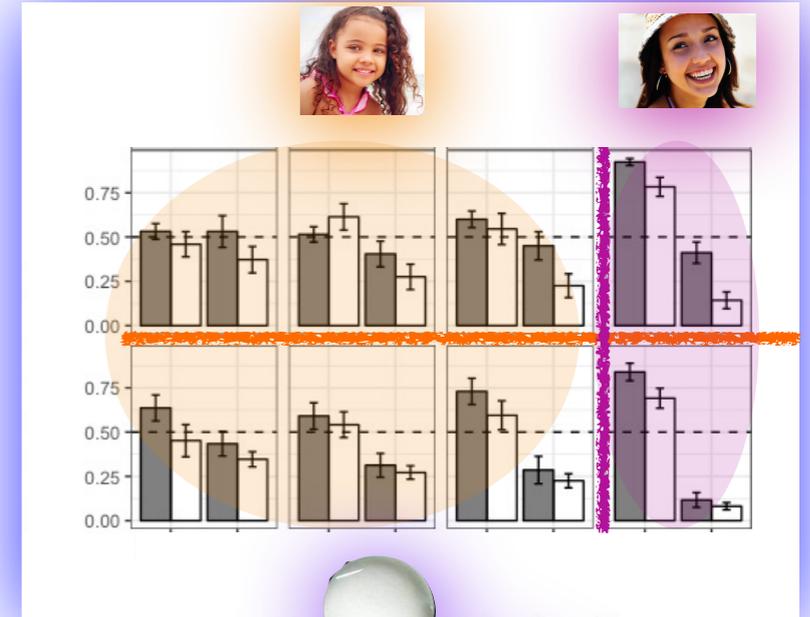


PRONOUN =

$0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4				
≥ 5				
adults				



Kass & Raferty 1995: BIC scores within 2.0 are equivalent

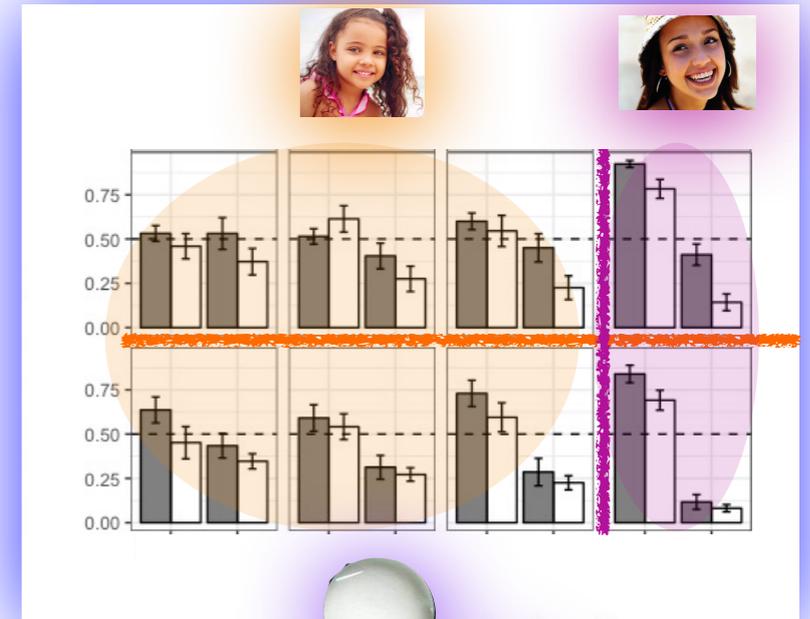
3-year-old pronoun interpretation behavior is best captured by children having inaccurate representations, or inaccurate deployment, but not both inaccurate representations and inaccurate deployment of those representations.



Understanding the development of pronoun interpretation



PRONOUN =



0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent

	baseline	inacc rep	inacc deploy	both inacc
age ≤ 3	2913.00	735.99	735.17	758.11
4				
≥ 5				
adults				

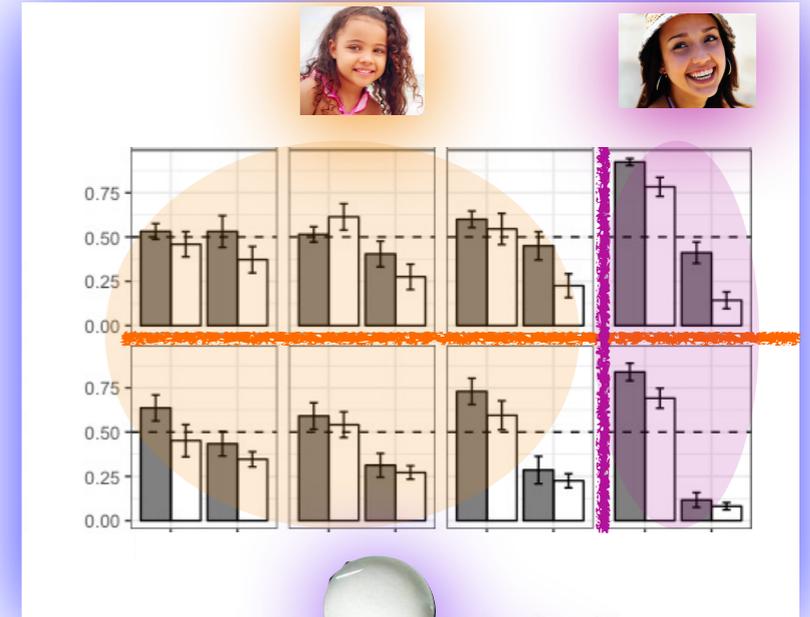
...though all 3 inaccurate variants are far better than the baseline of both accurate representations and accurate deployment.

Understanding the development of pronoun interpretation



PRONOUN =

0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



	baseline	inacc rep	inacc deploy	both inacc
age ≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
≥ 5	1931.33	590.04	590.70	607.12
adults				

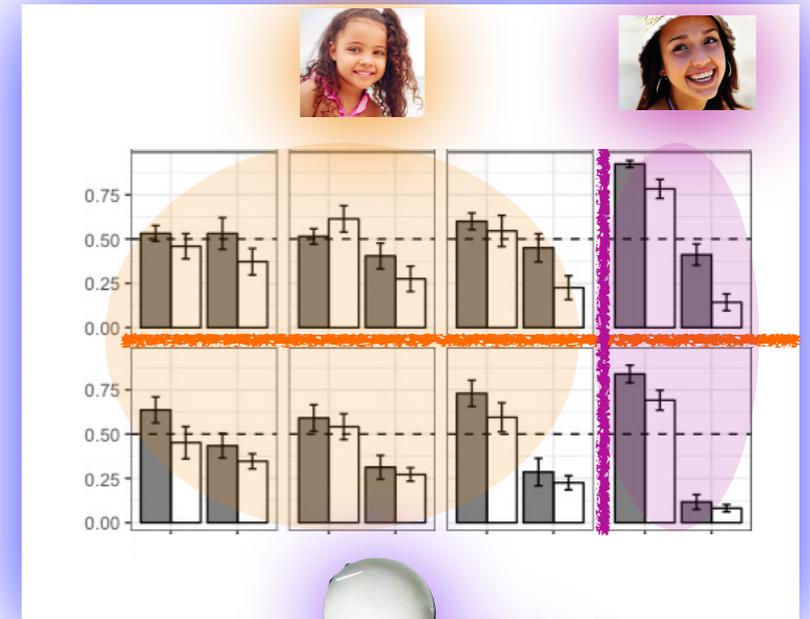
This same pattern holds for 4-year-olds and 5-year-olds, too.

Understanding the development of pronoun interpretation



 **PRONOUN** = 

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
≥ 5	1931.33	590.04	590.70	607.12
adults	1323.82	642.48	646.96	668.58

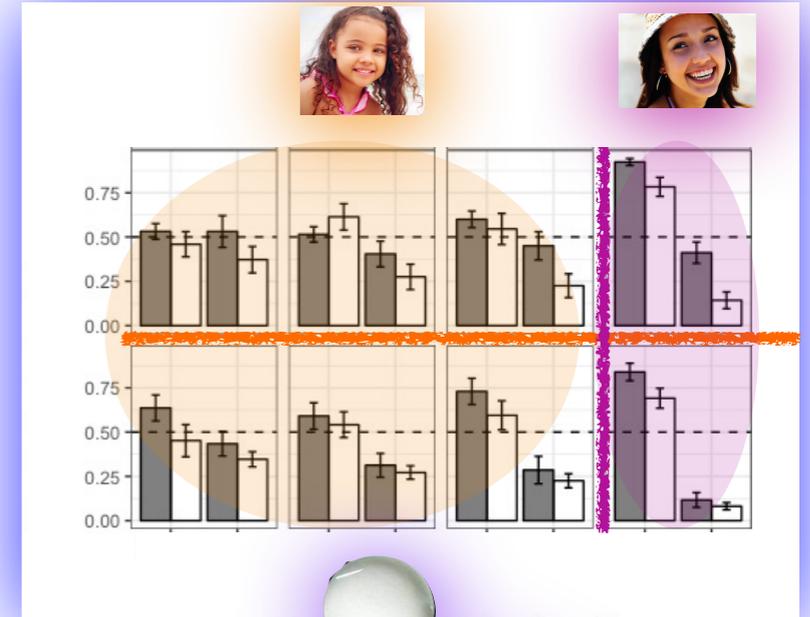
Adult pronoun interpretation behavior is best captured by the model that has inaccurate representations.



Understanding the development of pronoun interpretation



PRONOUN =



0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



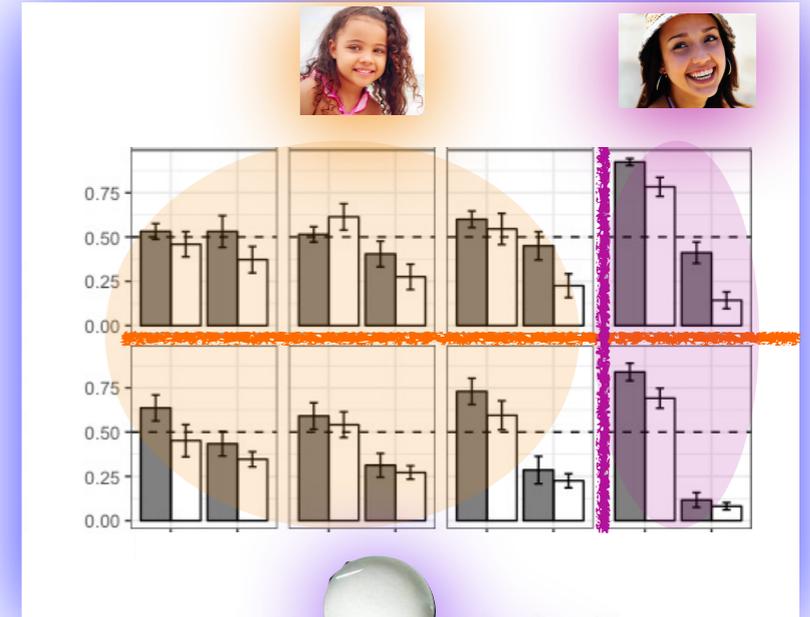
age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
≥ 5	1931.33	590.04	590.70	607.12
adults	1323.82	642.48	646.96	668.58

Like the child models, **all inaccurate variants are far better** than the **baseline** that has both accurate representations and accurate deployment.

Understanding the development of pronoun interpretation



PRONOUN =



0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
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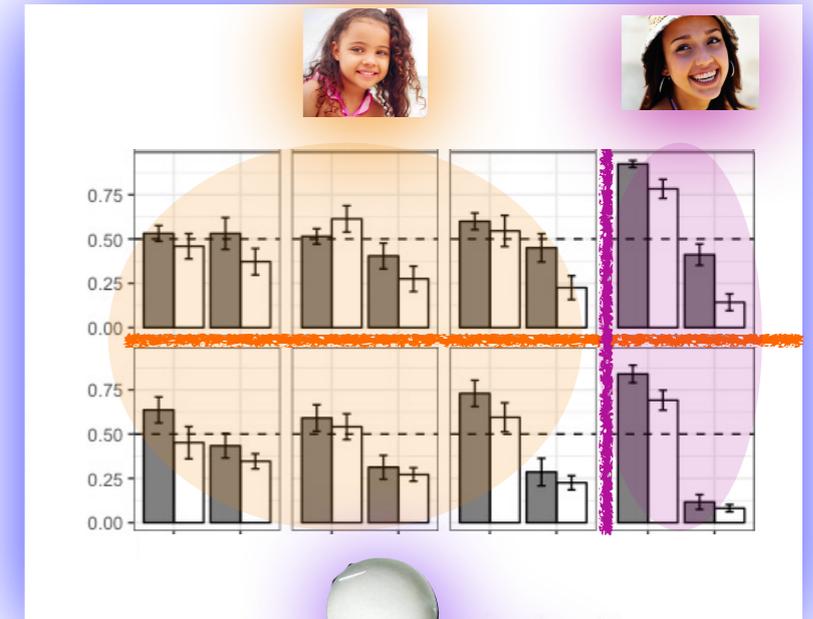
...but within the inaccurate variants, the **inaccurate representations variant is better** than the other two.

Understanding the development of pronoun interpretation



PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
≥ 5	1931.33	590.04	590.70	607.12
adults	1323.82	642.48	646.96	668.58

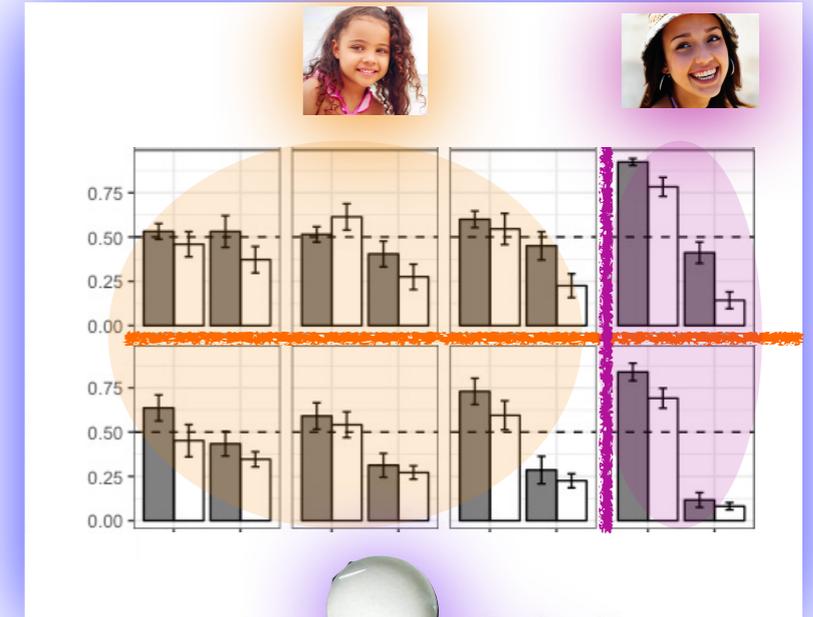
Takeaway: Both **child** and **adult** pronoun interpretation behavior are captured by modeled listeners that are **inaccurate** in some way.



Understanding the development of pronoun interpretation



PRONOUN =



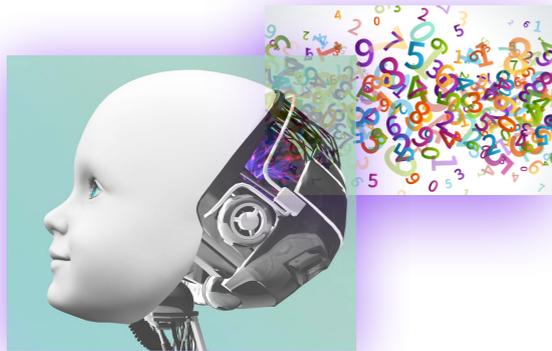
0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent

age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
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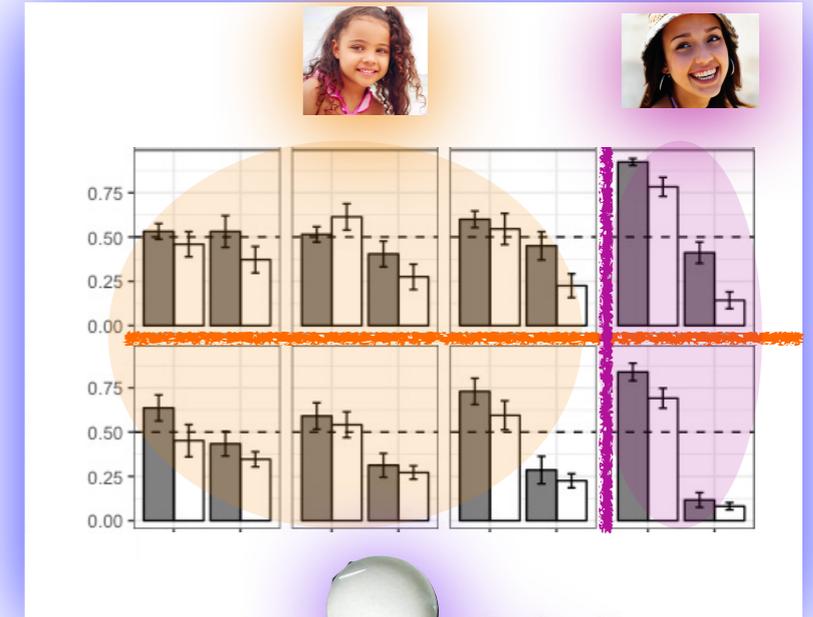
Becoming adult-like doesn't mean becoming accurate!



Understanding the development of pronoun interpretation



PRONOUN =



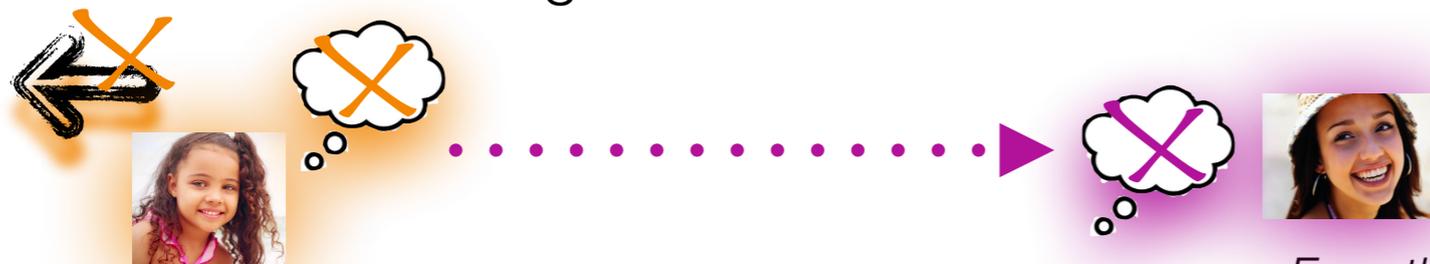
0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
≥ 5	1931.33	590.04	590.70	607.12
adults	1323.82	642.48	646.96	668.58

But what does it mean?

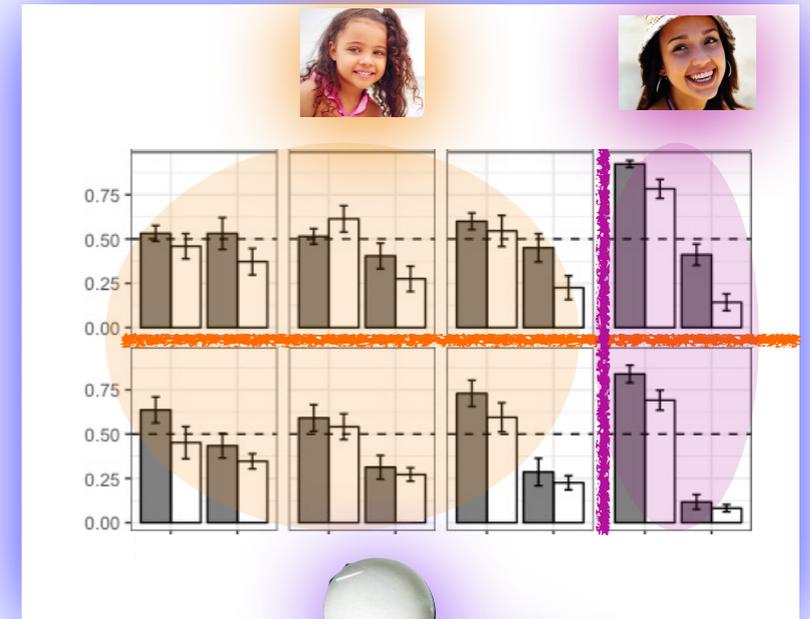
It means learning to become *inaccurate in adult-like ways*.



Understanding the development of pronoun interpretation



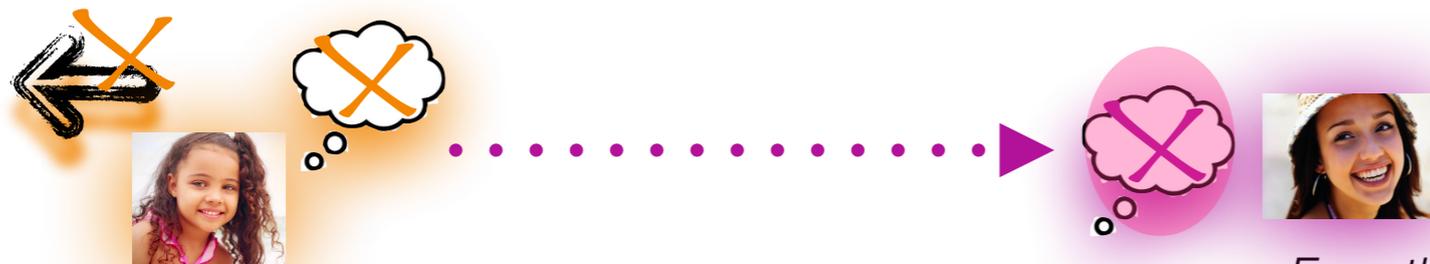
PRONOUN =



0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent

age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
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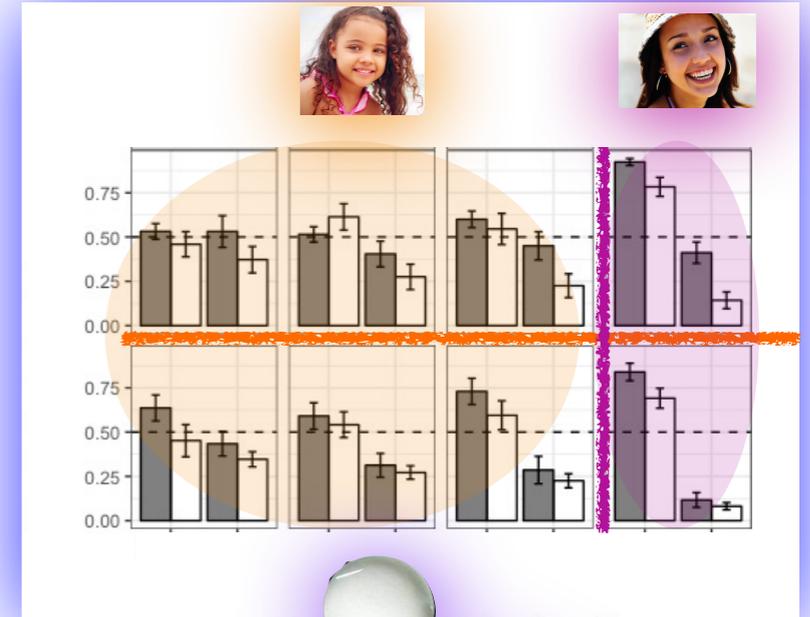
We know that adults *inaccurately represent* the available information.



Understanding the development of pronoun interpretation



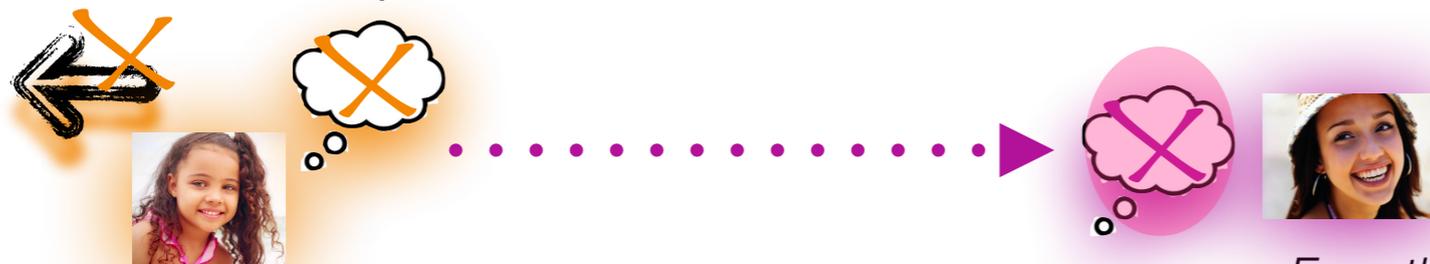
PRONOUN =



0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent

age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
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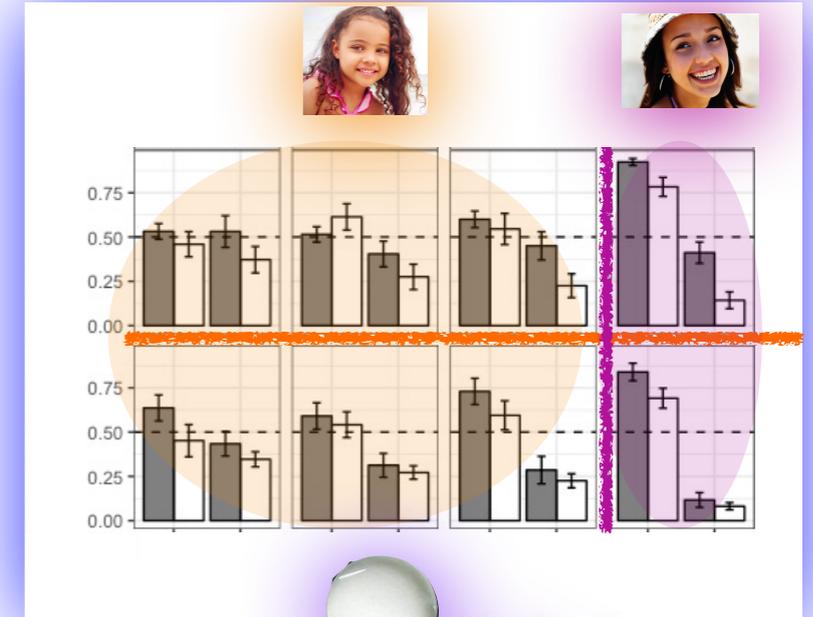
So we can look at the σ values to see how the inaccurate representations are inaccurate — are they too smooth or too sharp?



Understanding the development of pronoun interpretation



PRONOUN =

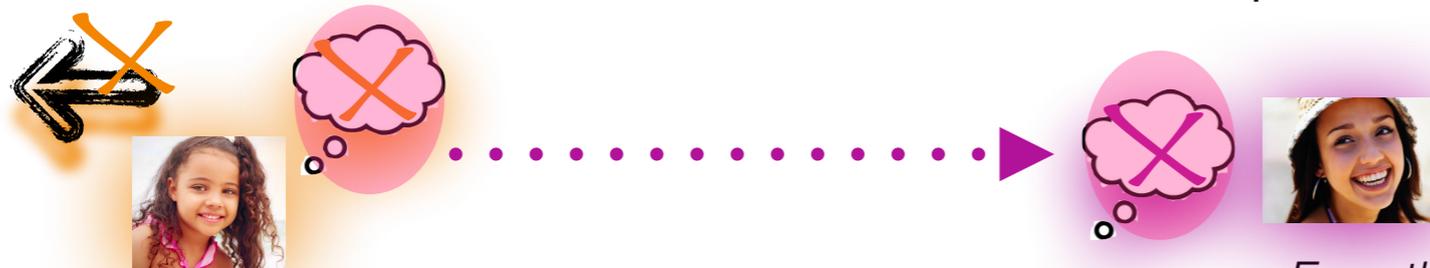


 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
≥ 5	1931.33	590.04	590.70	607.12
adults	1323.82	642.48	646.96	668.58

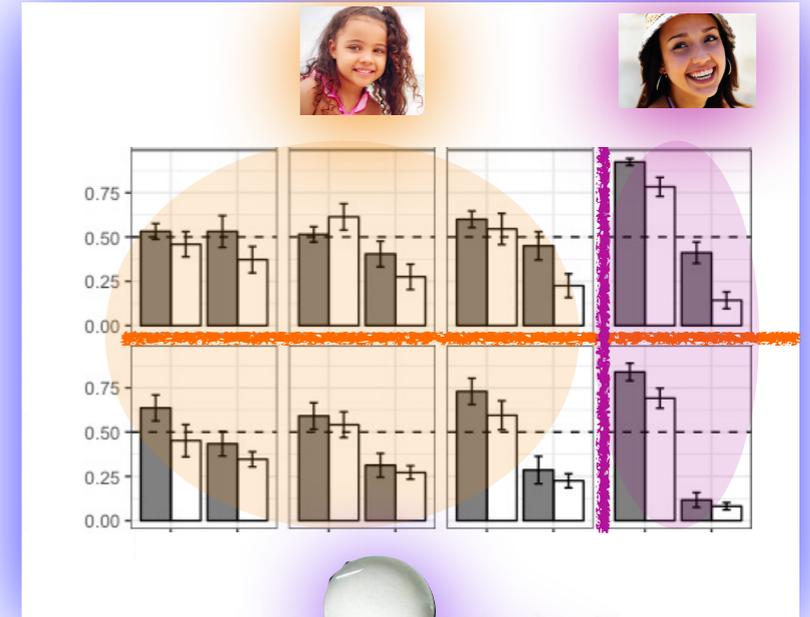
Development: How do these σ values compare to those that the best-fitting child model used that relied on inaccurate representations?



Understanding the development of pronoun interpretation



PRONOUN =



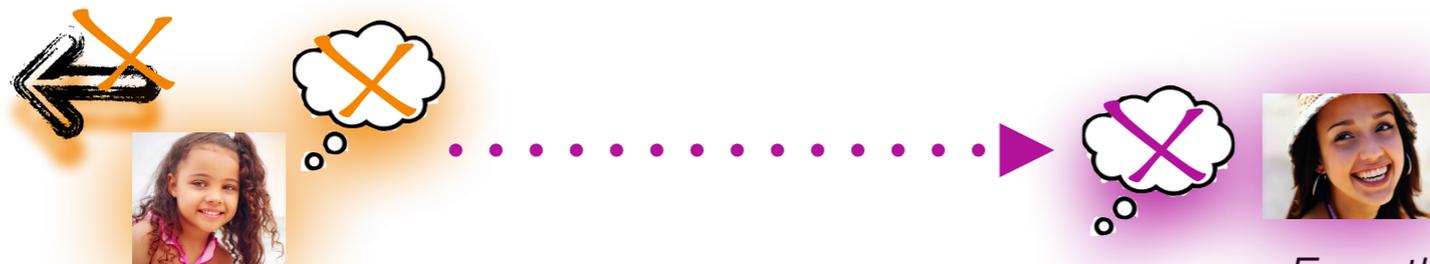
👍 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
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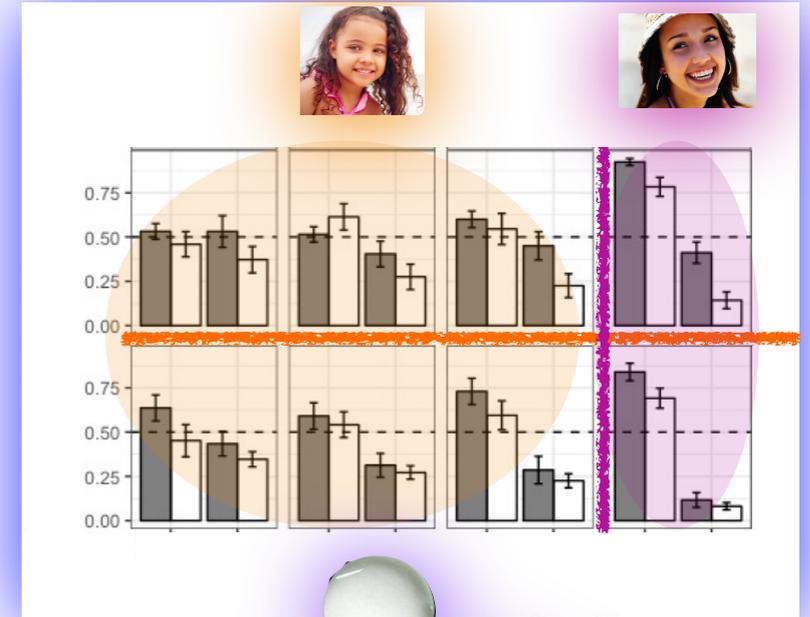
We know that adults accurately deploy their representations.



Understanding the development of pronoun interpretation



PRONOUN =



0 ≤ BIC ≤ ∞ (closer to 0 is better)
 Kass & Raferty 1995: BIC scores within 2.0 are equivalent

age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	2913.00	735.99	735.17	758.11
4	2784.61	752.89	754.06	773.65
≥ 5	1931.33	590.04	590.70	607.12
adults	1323.82	642.48	646.96	668.58

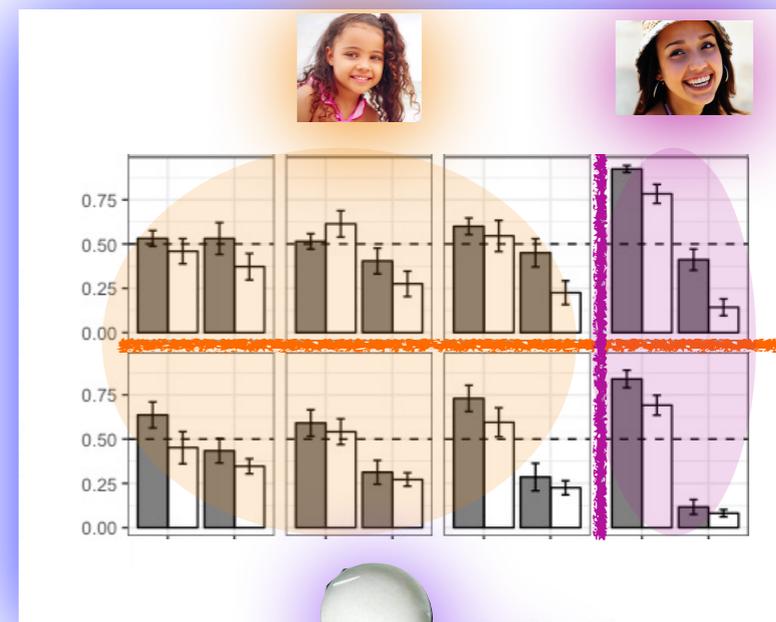
Development: How much <1 are the β values that the best-fitting child model used that relied on inaccurate deployment?



Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



σ_{for} σ_{con} σ_{mor} σ_{α} | β_{for} β_{con} β_{mor} β_{α}

children: inaccurate representations



≤ 3
4
 ≥ 5

children: inaccurate deployment



≤ 3
4
 ≥ 5

adults: inaccurate representations

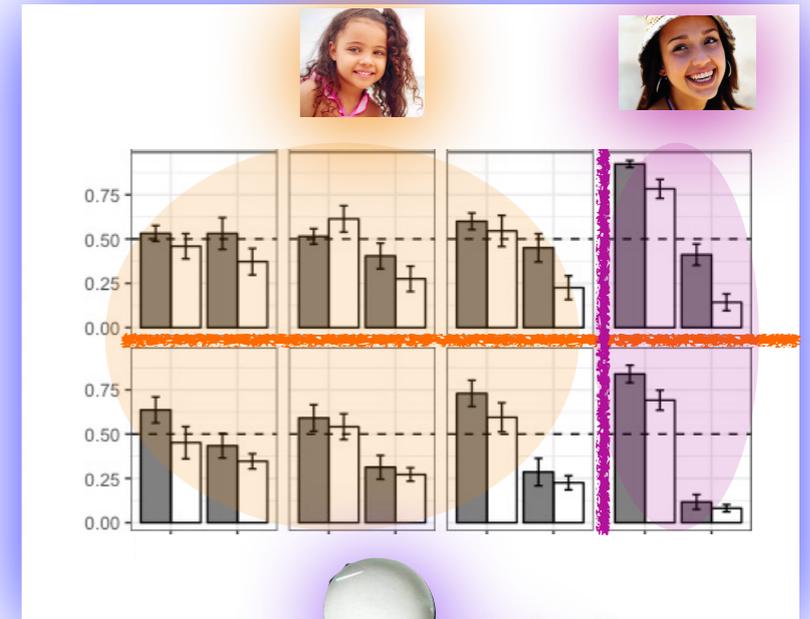


adults

Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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children: inaccurate representations

≤ 3				1	1	1	1
4				1	1	1	1
≥ 5				1	1	1	1

children: inaccurate deployment

≤ 3							
4							
≥ 5							

adults: inaccurate representations

adults				1	1	1	1
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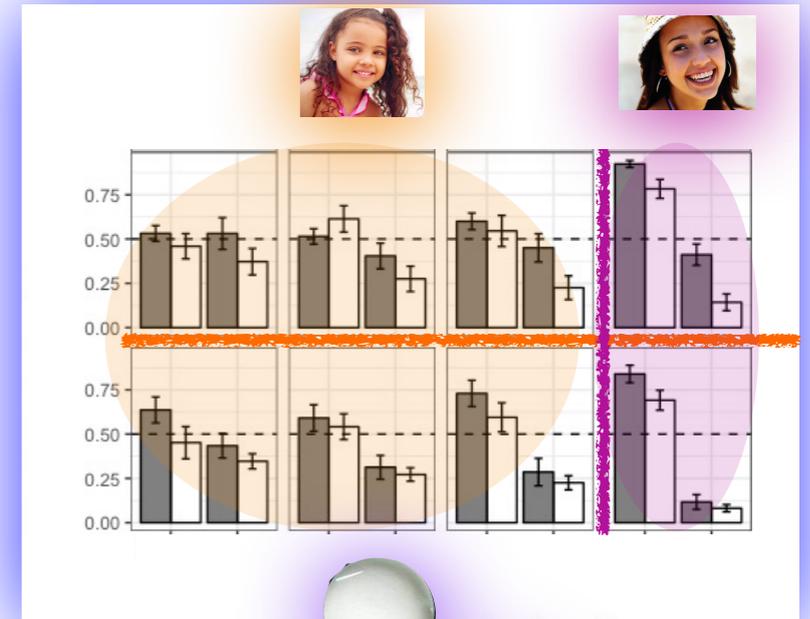


Inaccurate representations, but **accurate deployment**: equivalent to always paying attention to all information ($\beta=1$).

Understanding the development of pronoun interpretation



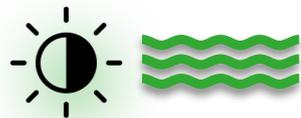
 **PRONOUN** =



$0.00 \leq \sigma \leq 4.00$



$0 \leq \beta \leq 1$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment



≤ 3								
4								
≥ 5								

adults: inaccurate representations



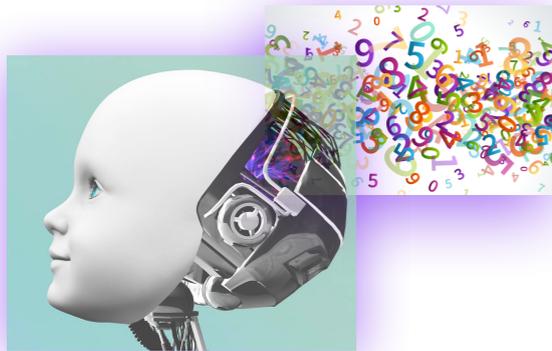
adults	0.25	0.33	0.28	0.00	1	1	1	1
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Both adults and children completely smooth away information about the prior over possible antecedents (σ_{α}).

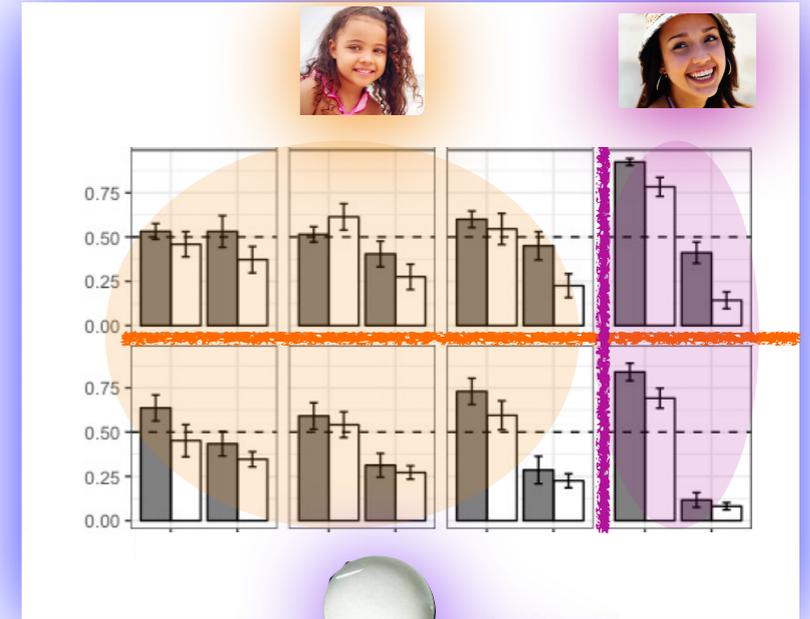
No change needed!

Note: This is functionally the same as never using this information (as if $\beta_{\alpha} = 0.00$).

Understanding the development of pronoun interpretation



PRONOUN =



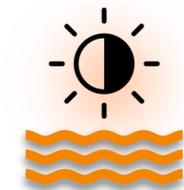
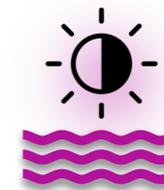
$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1
children: inaccurate deployment								
≤ 3								
4								
≥ 5								
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1



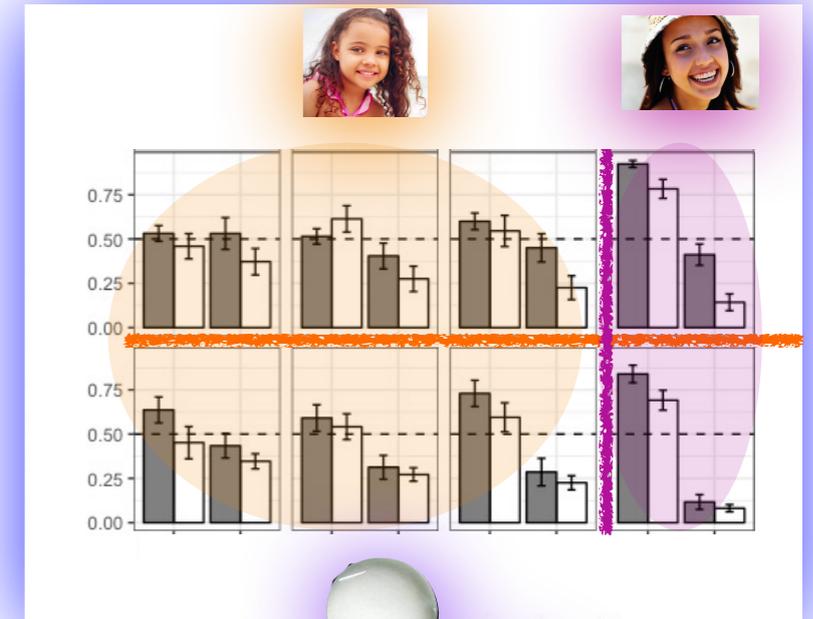
Qualitative similarity:

Both adults and children smooth away information about the form, connective, and agreement morphology ($\sigma < 1$), effectively turning down the contrast.

Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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children: inaccurate representations

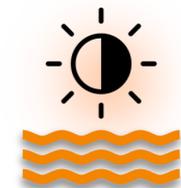
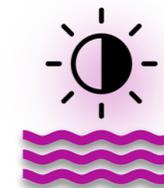
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment

≤ 3								
4								
≥ 5								

adults: inaccurate representations

adults	0.25	0.33	0.28	0.00	1	1	1	1
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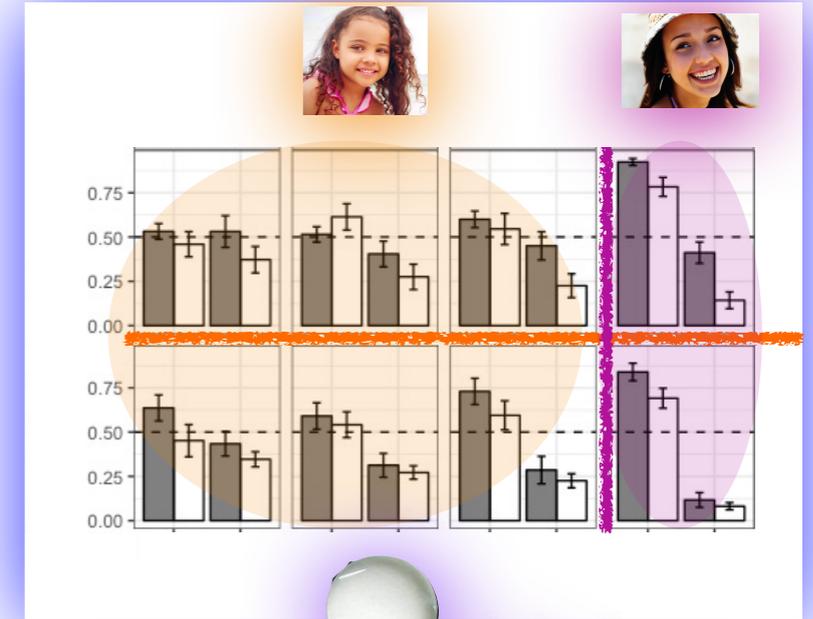
Quantitative differences:
Children smooth more overall
($\sigma = 0.00-0.28$) compared to
adults ($\sigma = 0.25-0.33$)

How much difference does this make?

Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



$$\sigma = 0.25-0.33$$



$$\sigma = 0.00-0.28$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment



≤ 3								
4								
≥ 5								

adults: inaccurate representations



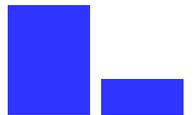
adults	0.25	0.33	0.28	0.00	1	1	1	1
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How much difference does this make?



$$\sigma = .33$$

.75 vs. .25



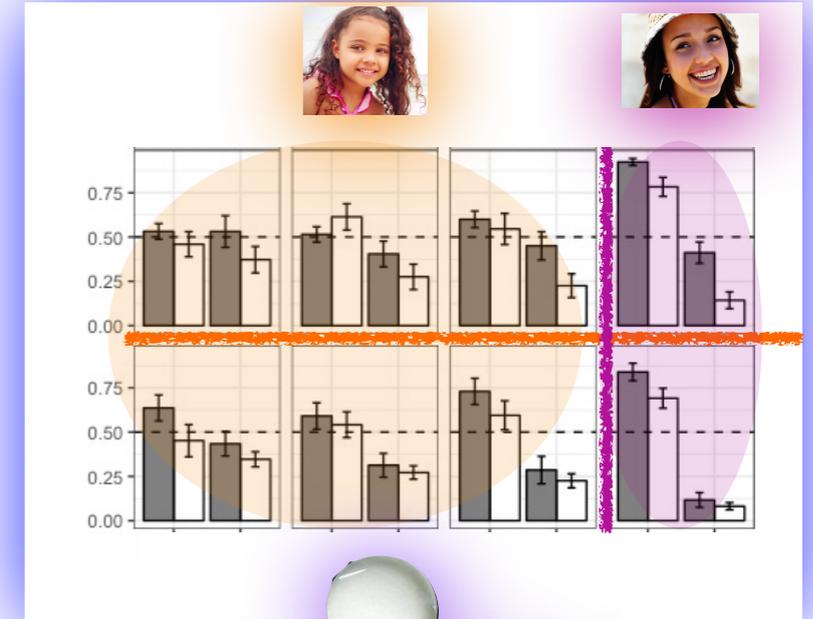
.59 vs. .41



Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



σ_{for} σ_{con} σ_{mor} σ_{α} | β_{for} β_{con} β_{mor} β_{α}

children: inaccurate representations

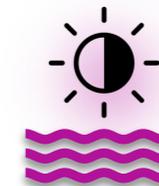
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment

≤ 3								
4								
≥ 5								

adults: inaccurate representations

adults	0.25	0.33	0.28	0.00	1	1	1	1
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$$\sigma = 0.25-0.33$$

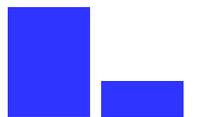


$$\sigma = 0.00-0.28$$

Not a lot for the higher values.



.75 vs. .25



$$\sigma = .33$$

.59 vs. .41



$$\sigma = .28$$

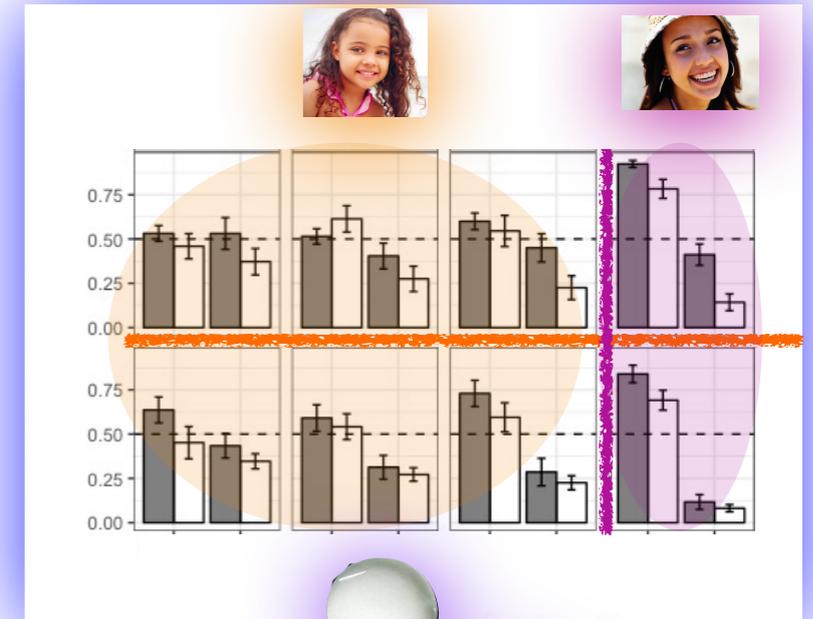
.58 vs. .42



Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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children: inaccurate representations

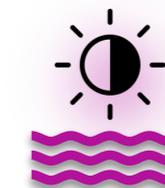
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment

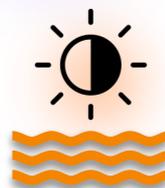
≤ 3								
4								
≥ 5								

adults: inaccurate representations

adults	0.25	0.33	0.28	0.00	1	1	1	1
--------	------	------	------	------	---	---	---	---



$$\sigma = 0.25-0.33$$



$$\sigma = 0.00-0.28$$

More for lower values.



.75 vs. .25



$$\sigma = .33$$

.59 vs. .41



$$\sigma = .11$$

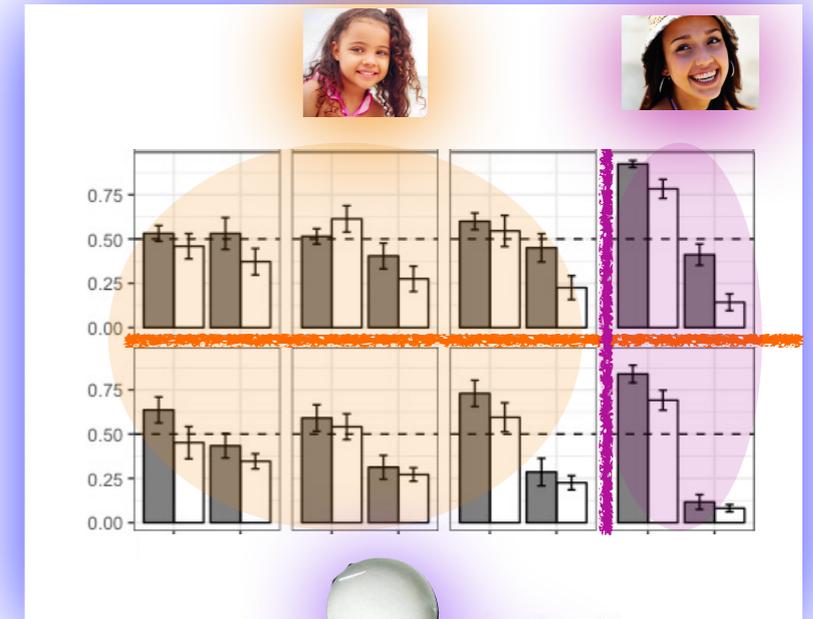
.53 vs. .47



Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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children: inaccurate representations

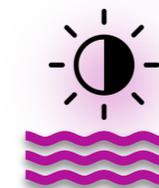
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment

≤ 3								
4								
≥ 5								

adults: inaccurate representations

adults	0.25	0.33	0.28	0.00	1	1	1	1
--------	------	------	------	------	---	---	---	---



$$\sigma = 0.25-0.33$$

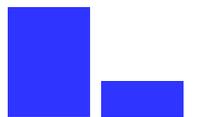


$$\sigma = 0.00-0.28$$

More for lower values.



.75 vs. .25



$$\sigma = .33$$

.59 vs. .41



$$\sigma = .02$$

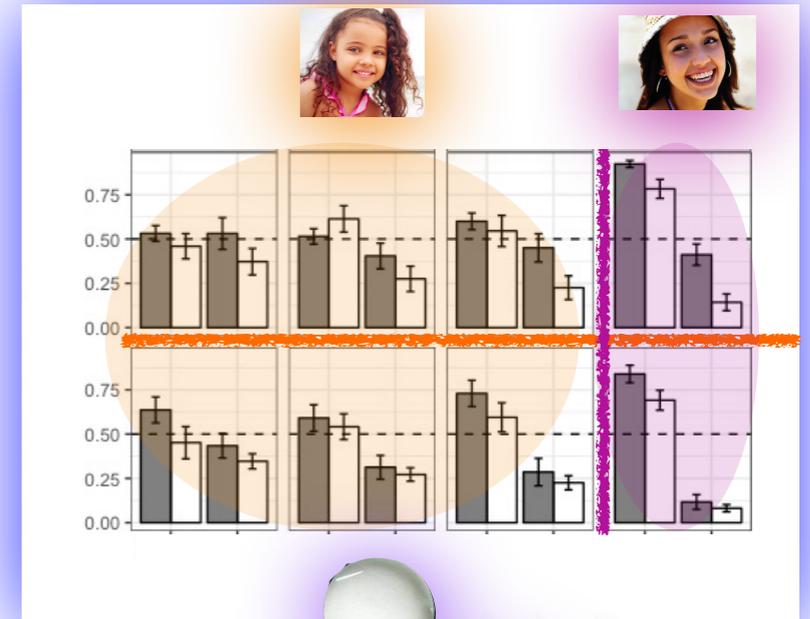
.505 vs. .495



Understanding the development of pronoun interpretation



 **PRONOUN** = 



$0.00 \leq \sigma \leq 4.00$



$0 \leq \beta \leq 1$



σ_{for} σ_{con} σ_{mor} σ_{α} | β_{for} β_{con} β_{mor} β_{α}

children: inaccurate representations

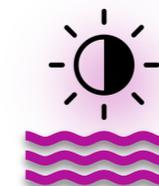
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment

≤ 3								
4								
≥ 5								

adults: inaccurate representations

adults	0.25	0.33	0.28	0.00	1	1	1	1
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$\sigma = 0.25-0.33$



$\sigma = 0.00-0.28$

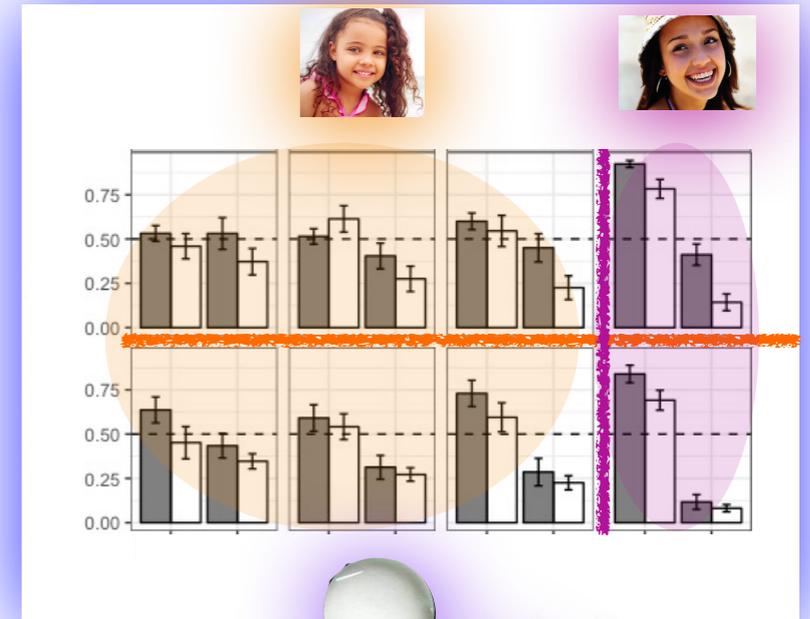
Takeaway:

Though both adults and children may **smooth away information** (turn down the contrast)...

Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



σ_{for} σ_{con} σ_{mor} σ_{α} | β_{for} β_{con} β_{mor} β_{α}

children: inaccurate representations

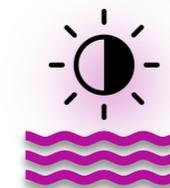
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment

≤ 3								
4								
≥ 5								

adults: inaccurate representations

adults	0.25	0.33	0.28	0.00	1	1	1	1
--------	------	------	------	------	---	---	---	---



$$\sigma = 0.25-0.33$$



$$\sigma = 0.00-0.28$$

Takeaway:

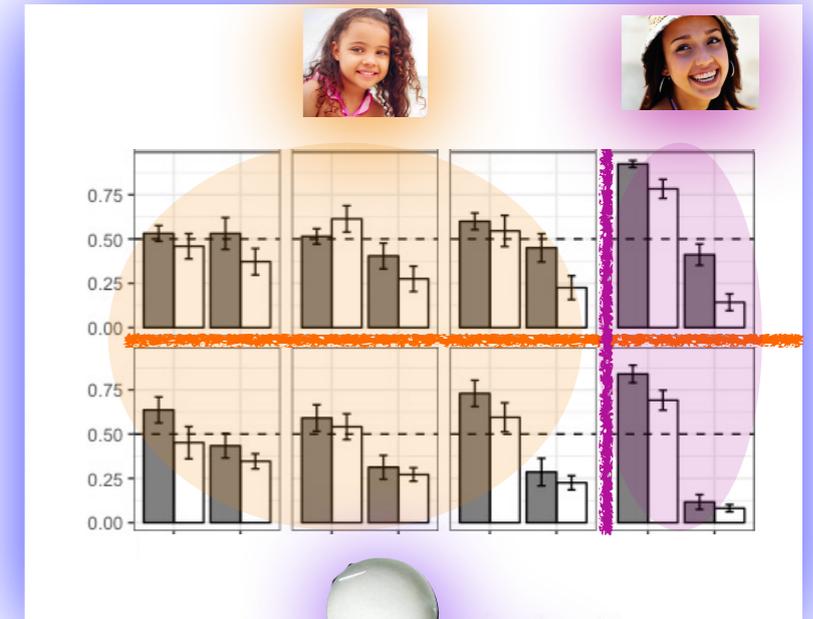
...children may need to learn to smooth away less (turn up the contrast) for pronoun form, connective, and agreement morphology information...



Understanding the development of pronoun interpretation



PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



$$\sigma = 0.25-0.33$$



$$\sigma = 0.00-0.28$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment

≤ 3								
4								
≥ 5								

adults: inaccurate representations

adults	0.25	0.33	0.28	0.00	1	1	1	1
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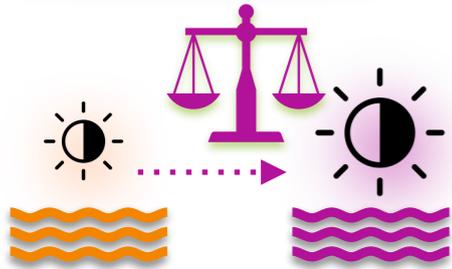
Takeaway:

...and children may need to learn to smooth away about the same amount (0.25-0.33), rather than unevenly (e.g., ≥ 5 : 0.02 for form, 0.11 for morphology, and 0.28 for connective).

Understanding the development of pronoun interpretation

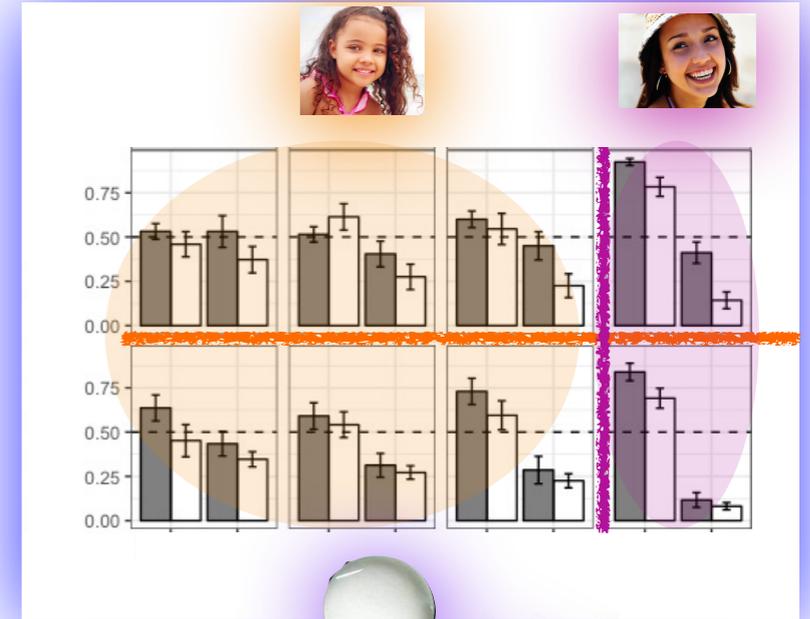


PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$

$$0 \leq \beta \leq 1$$



Or... it could be that children have accurate representations, but *inaccurate deployment*.

This would be a *qualitative difference* from adults.

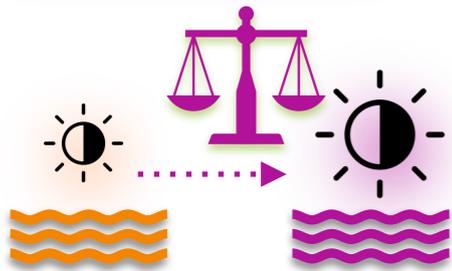


	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1
children: inaccurate deployment								
≤ 3								
4								
≥ 5								
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1

Understanding the development of pronoun interpretation

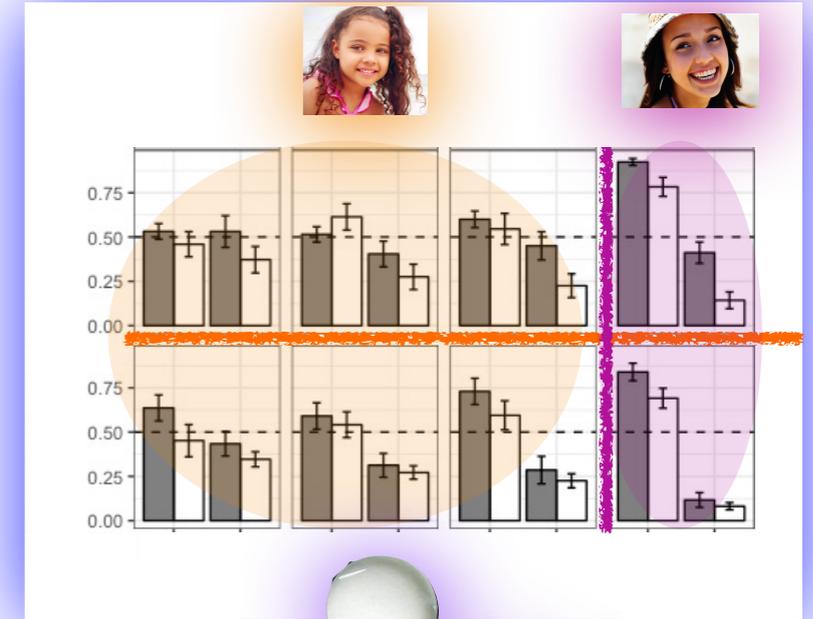


PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$

$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1

children: inaccurate deployment



≤ 3	1	1	1	1				
4	1	1	1	1				
≥ 5	1	1	1	1				

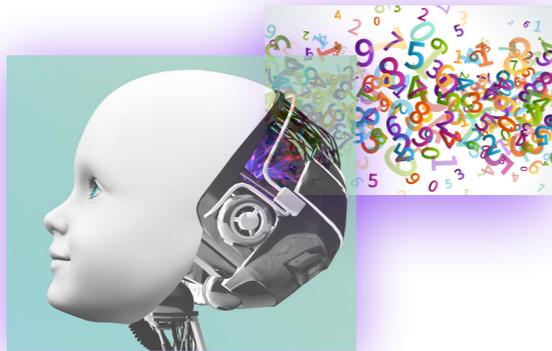
adults: inaccurate representations



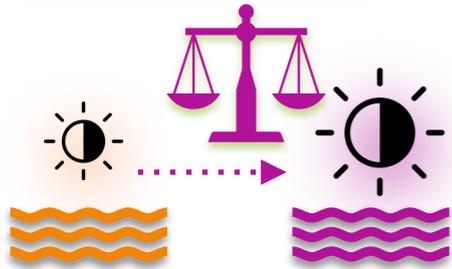
adults	0.25	0.33	0.28	0.00	1	1	1	1
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Accurate representations have $\sigma=1$.

Understanding the development of pronoun interpretation



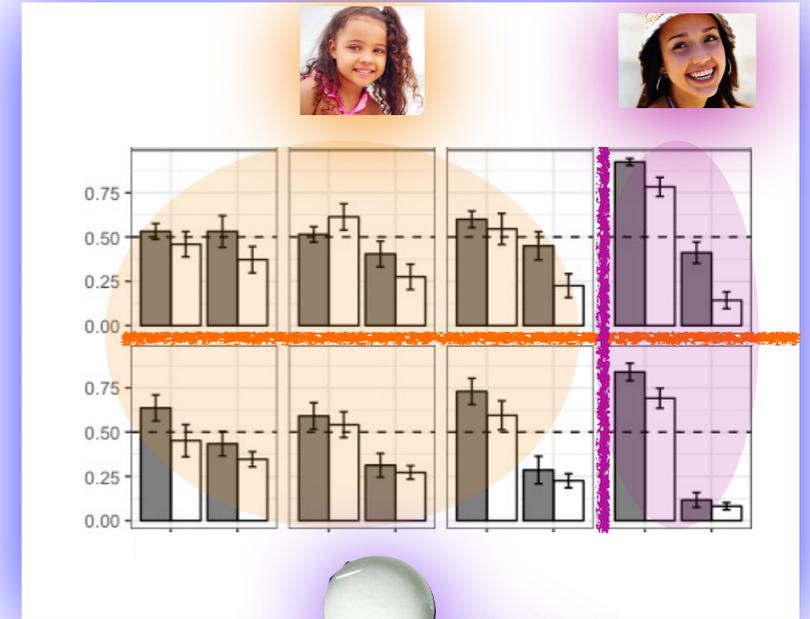
PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1



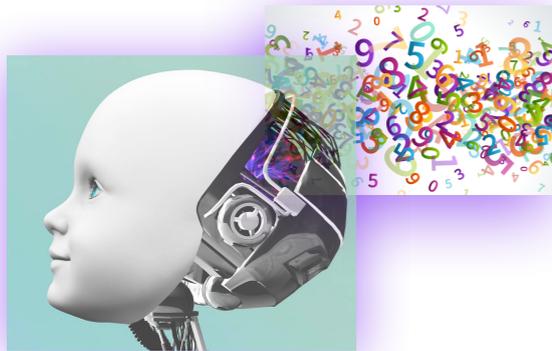
	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate deployment								
≤ 3	1	1	1	1	0.00	0.18	0.10	0.00
4	1	1	1	1	0.00	0.08	0.24	0.00
≥ 5	1	1	1	1	0.00	0.43	0.30	0.00



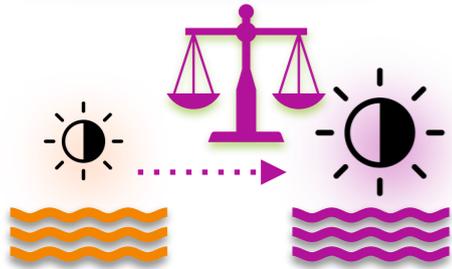
	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1

Inaccurate deployment varies across information types, though all are used <50%.

Understanding the development of pronoun interpretation



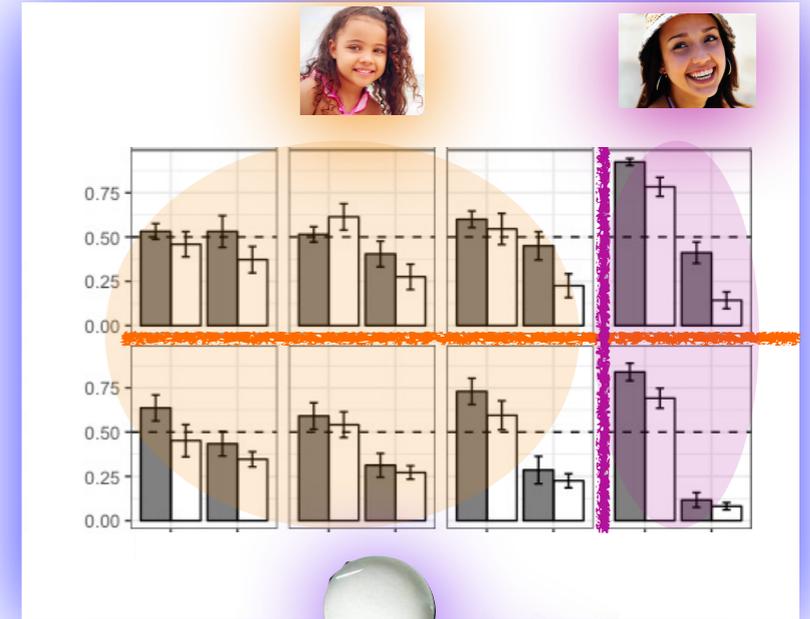
PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



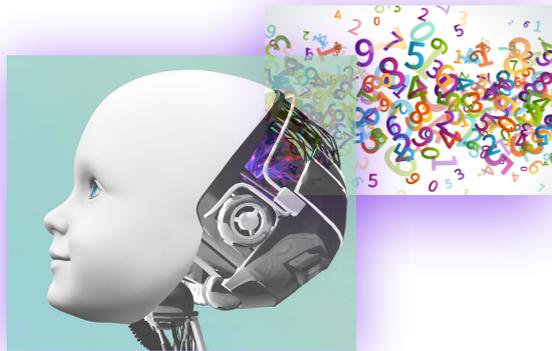
$$0 \leq \beta \leq 1$$



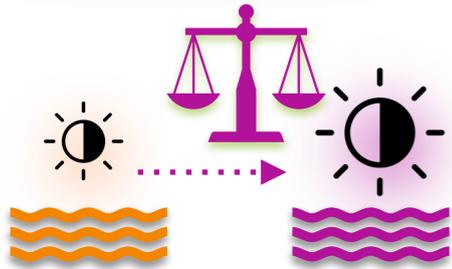
	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
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≥ 5	0.02	0.28	0.11	0.00	1	1	1	1
children: inaccurate deployment								
≤ 3	1	1	1	1	0.00	0.18	0.10	0.00
4	1	1	1	1	0.00	0.08	0.24	0.00
≥ 5	1	1	1	1	0.00	0.43	0.30	0.00
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1

The **connective** and agreement **morphology** are heeded varying amounts.

Understanding the development of pronoun interpretation



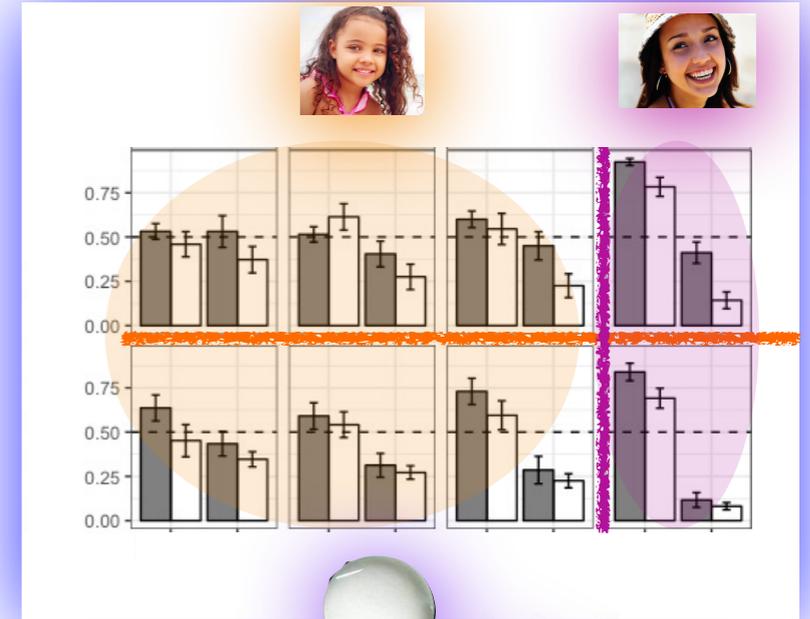
PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1
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≥ 5	1	1	1	1	0.00	0.43	0.30	0.00
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1

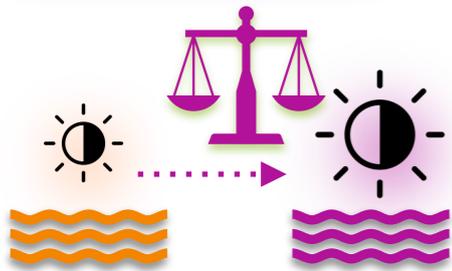


The **prior** over antecedents and **form** are completely ignored ($\beta=0.00$).

Understanding the development of pronoun interpretation

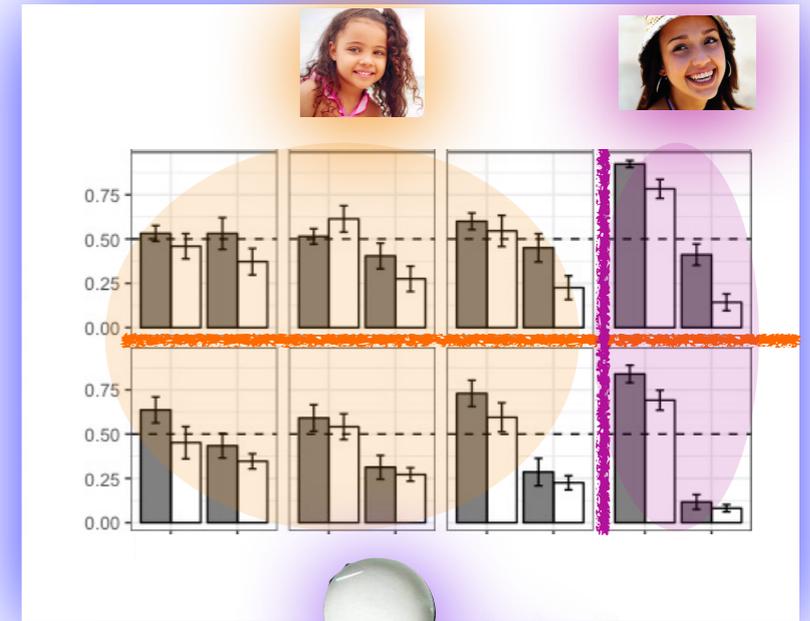


PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$

$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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≥ 5	0.02	0.28	0.11	0.00	1	1	1	1
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4	1	1	1	1	0.00	0.08	0.24	0.00
≥ 5	1	1	1	1	0.00	0.43	0.30	0.00
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1

Note: This is equivalent to smoothing away all information in the representation.

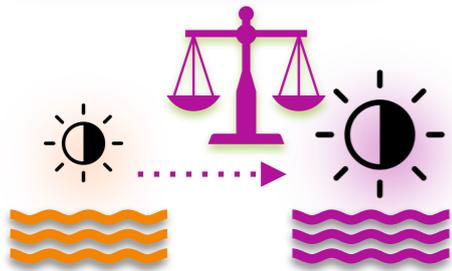


So, a $\beta_{\alpha}=0.00$ accomplishes the same thing as a $\sigma_{\alpha}=0.00$, which is adult-like. Similarity!

Understanding the development of pronoun interpretation



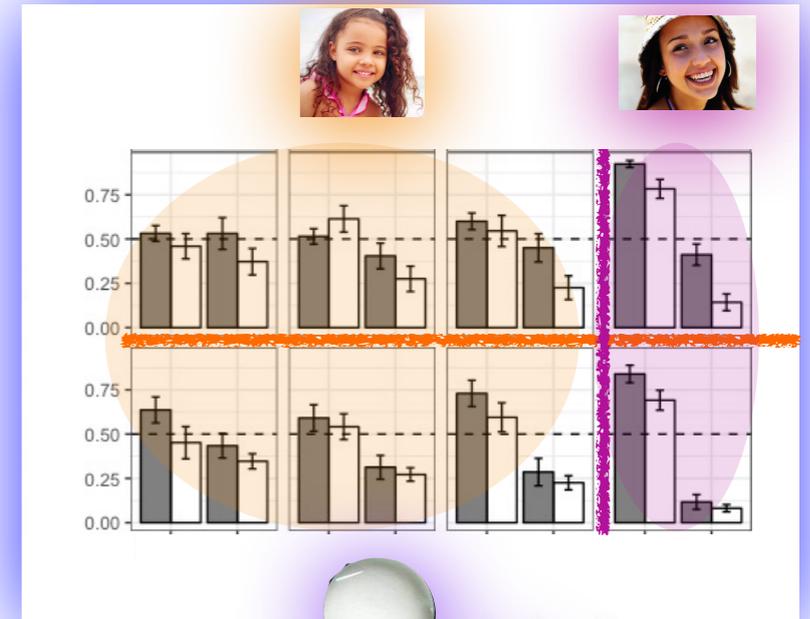
PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$



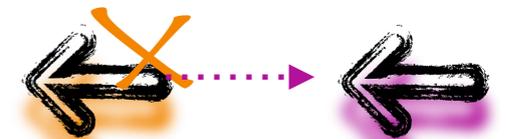
$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1
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4	1	1	1	1	0.00	0.08	0.24	0.00
≥ 5	1	1	1	1	0.00	0.43	0.30	0.00
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1

Takeaway:

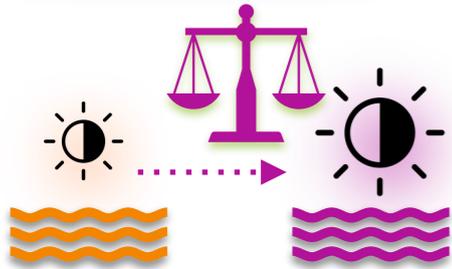
To become adult-like, children would need to switch to using most of their representations accurately (all the time)...



Understanding the development of pronoun interpretation



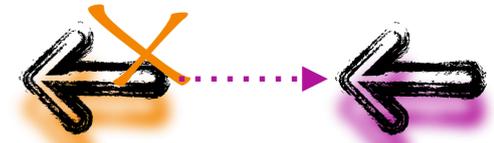
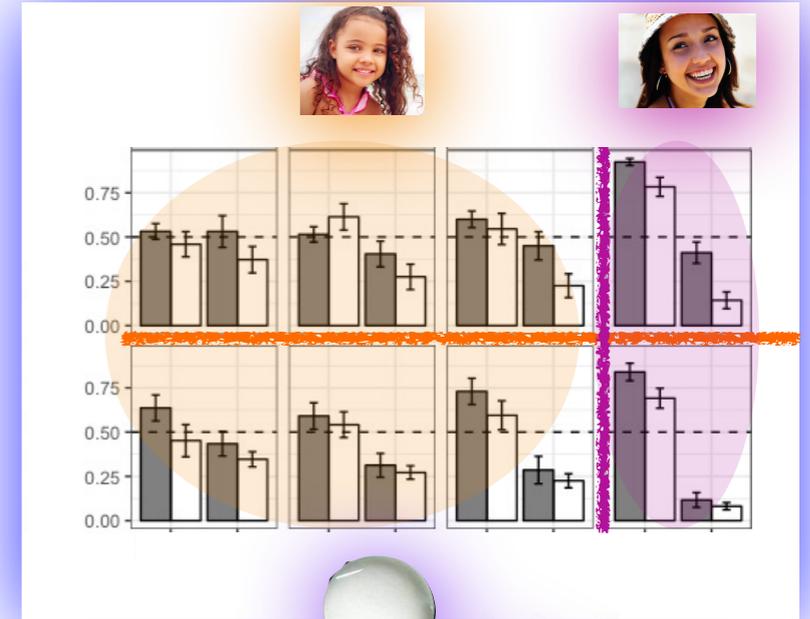
PRONOUN =



$$0.00 \leq \sigma \leq 4.00$$

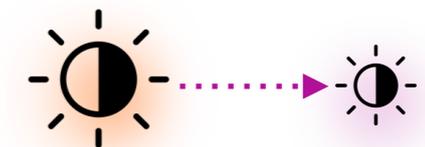


$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.00	0.11	0.04	0.00	1	1	1	1
4	0.00	0.01	0.09	0.00	1	1	1	1
≥ 5	0.02	0.28	0.11	0.00	1	1	1	1
children: inaccurate deployment								
≤ 3	1	1	1	1	0.00	0.18	0.10	0.00
4	1	1	1	1	0.00	0.08	0.24	0.00
≥ 5	1	1	1	1	0.00	0.43	0.30	0.00
adults: inaccurate representations								
adults	0.25	0.33	0.28	0.00	1	1	1	1

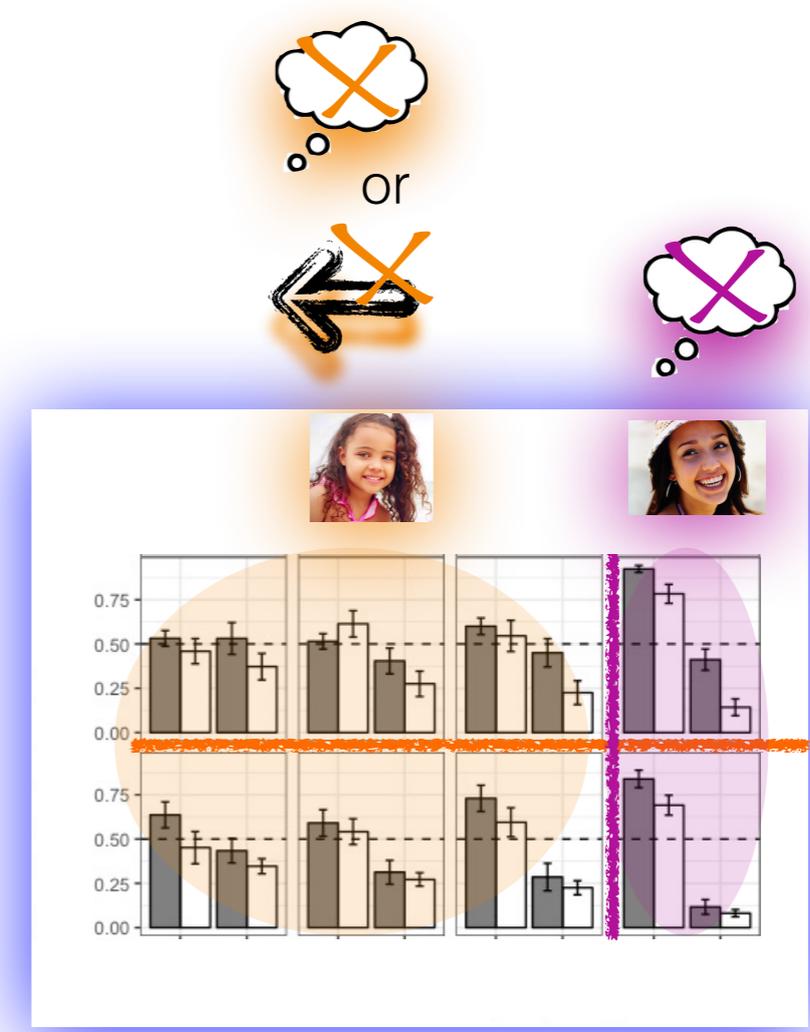
Takeaway:
...and smooth away information (turn down the contrast) for pronoun form, connective, and agreement morphology.



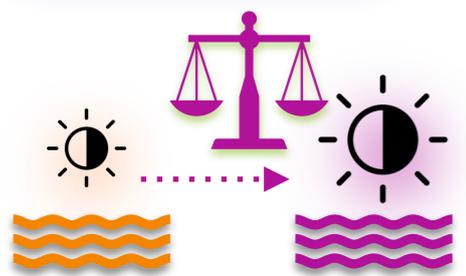
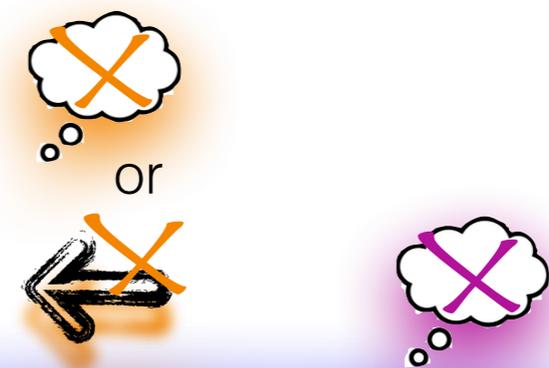
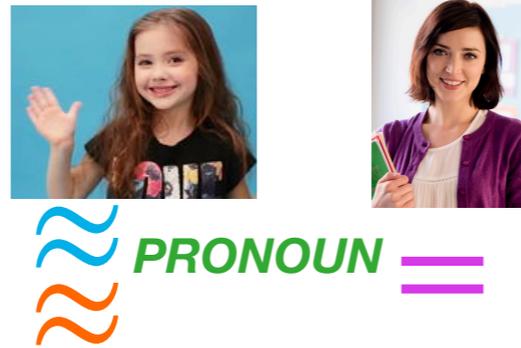
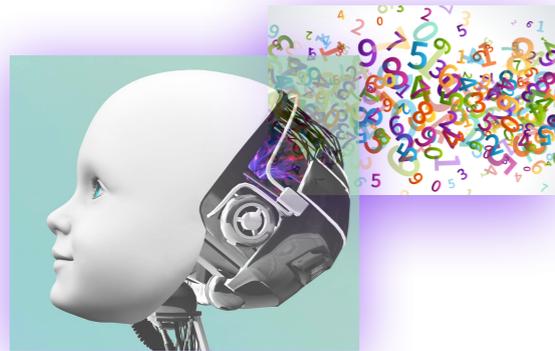
Understanding the development of pronoun interpretation



Takeaway:
 To generate adult-like pronoun interpretation behavior in context, children may need to change both how they **represent** relevant information from their input and how they **deploy** those representations.



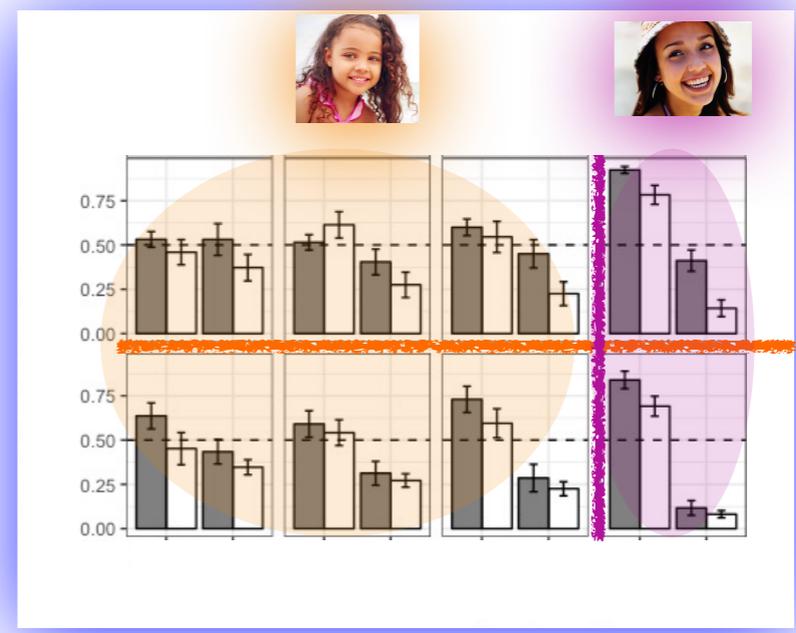
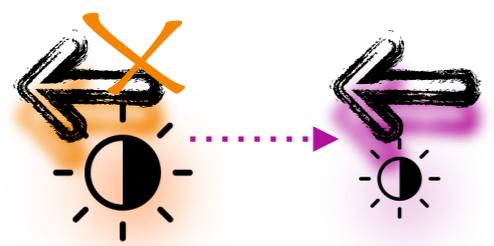
Understanding the development of pronoun interpretation



Takeaway:

In one case, children are **qualitatively different** from adults in two basic ways:

- (1) They use **accurate representations**
- (2) They **deploy** those representations **inaccurately**



Development means changing both of these to be adult-like
 (**turning down the contrast**, **always deploying information**)

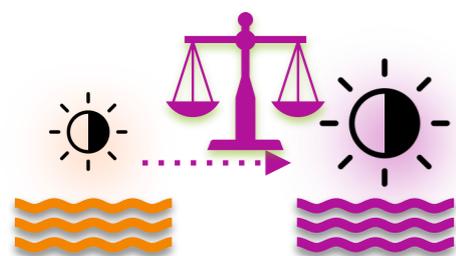
Understanding the development of pronoun interpretation



Takeaway:

In another case, children are **qualitatively similar** to adults:

- (1) They use **inaccurate representations**
- (2) They **deploy** those representations **accurately**

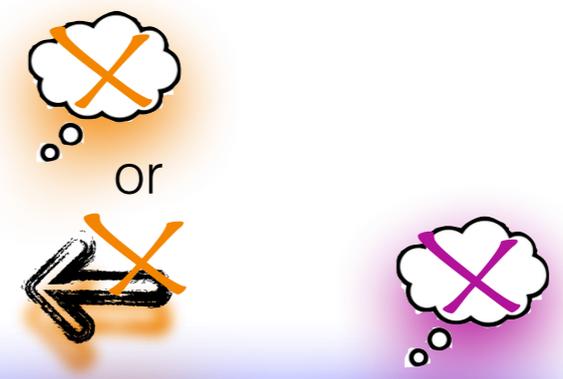
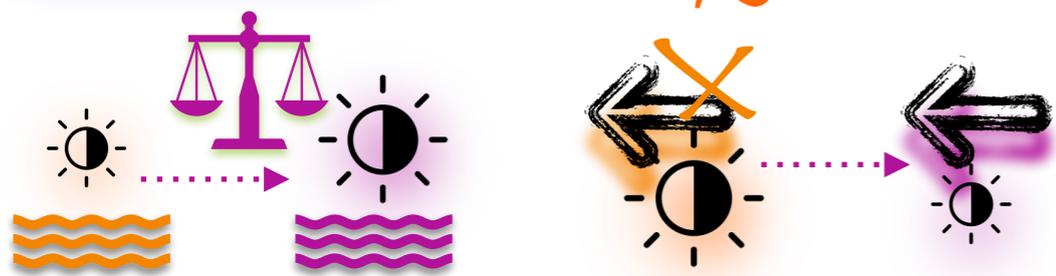


Development means learning to represent information inaccurately in an adult-like way (**turning up the contrast equally**)

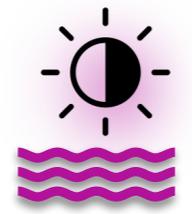
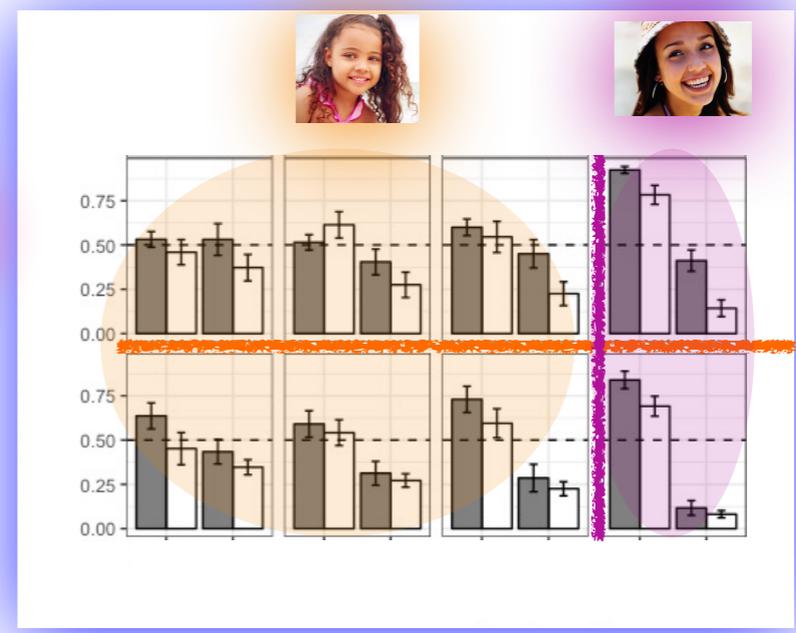
Understanding the development of pronoun interpretation



PRONOUN =



Takeaway:
 Being adult-like **doesn't mean being accurate!**
 Here, the best explanation for adult behavior is representations that are **inaccurate because they smooth away information.**

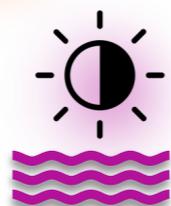
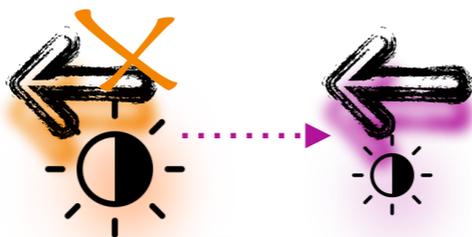
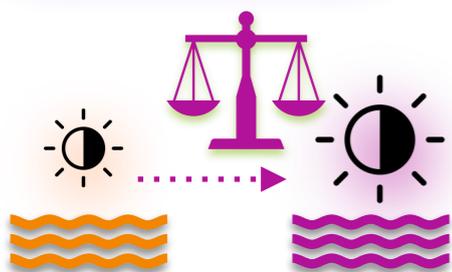


How plausible is this?

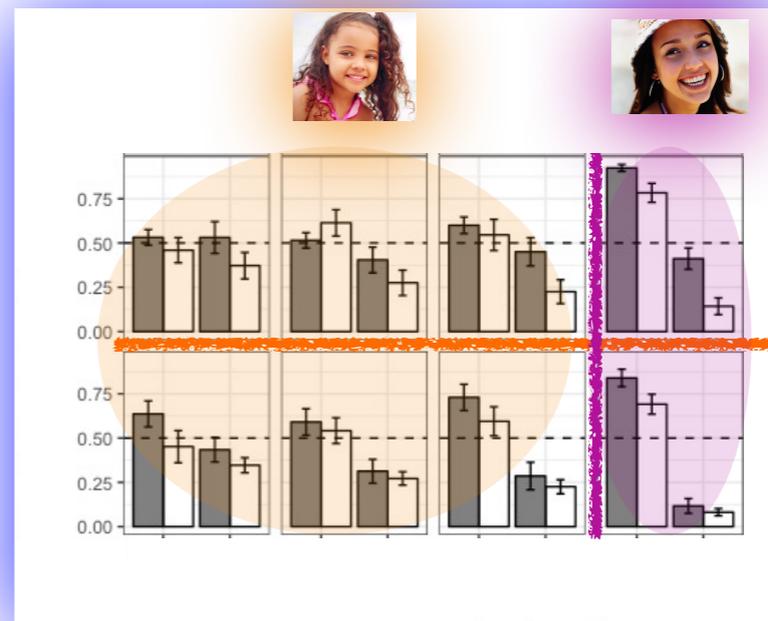
Understanding the development of pronoun interpretation



PRONOUN =



How plausible is this?

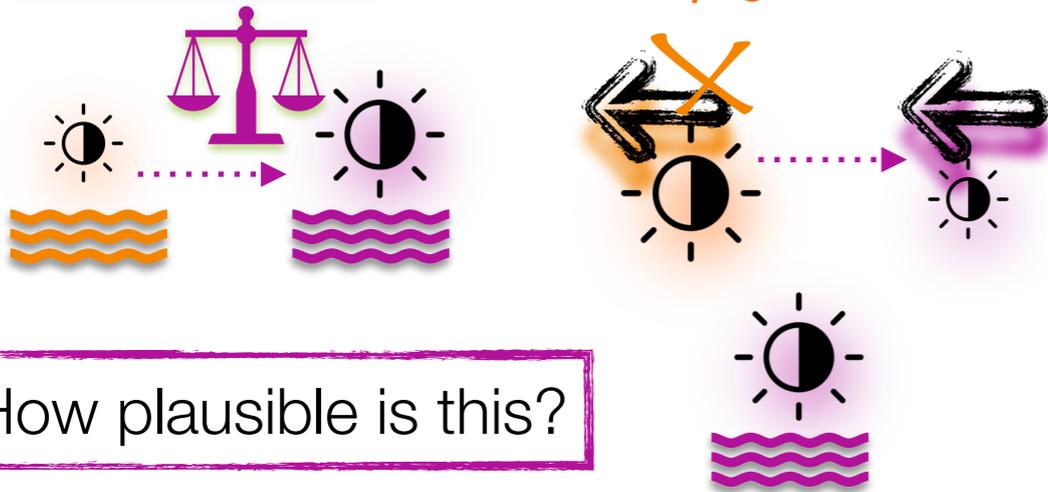


Adults smoothing away information:

In adult decision-making studies (Kahneman & Tversky 1979), adults “interpret” representations, rather than using them accurately

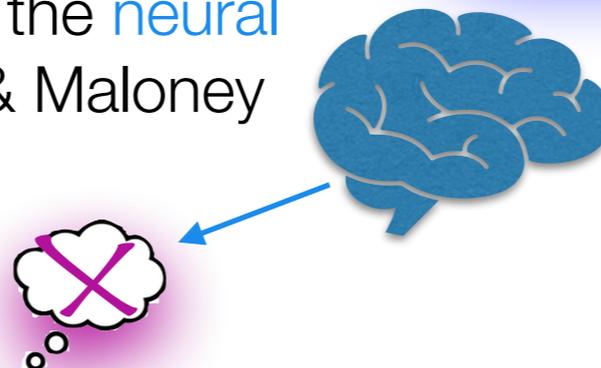


Understanding the development of pronoun interpretation



Adults smoothing away information:

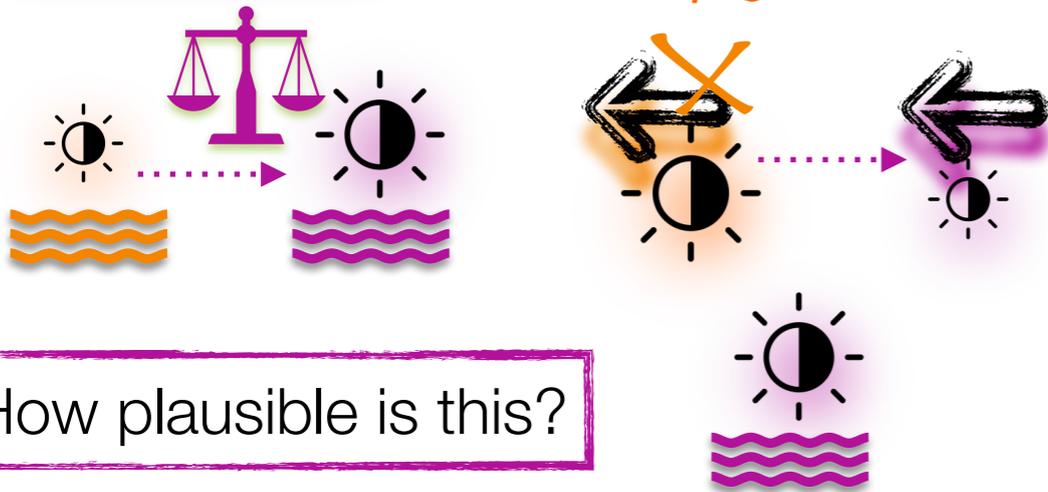
More recently, adult decision-making studies have found limits on the “dynamic range of the neural representation of probability” (Zhang & Maloney 2012, Zhang et al. 2020)



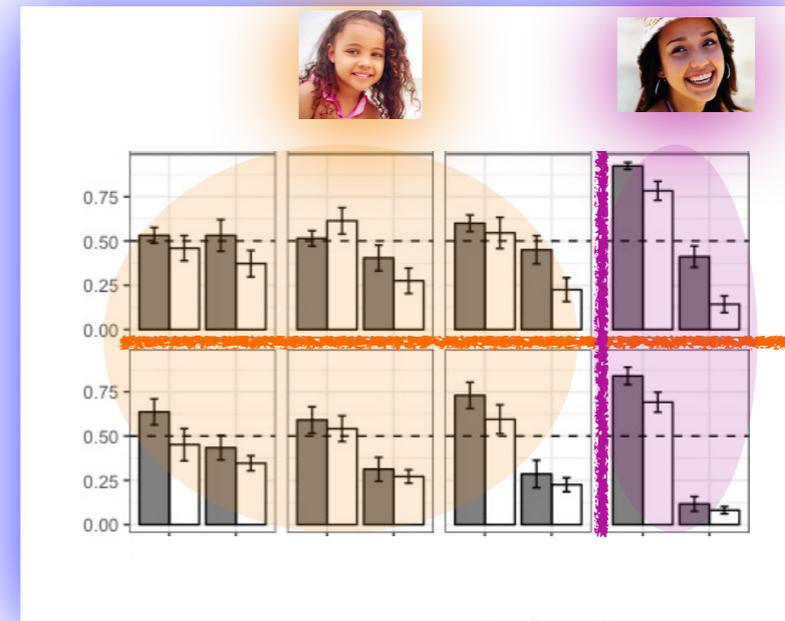
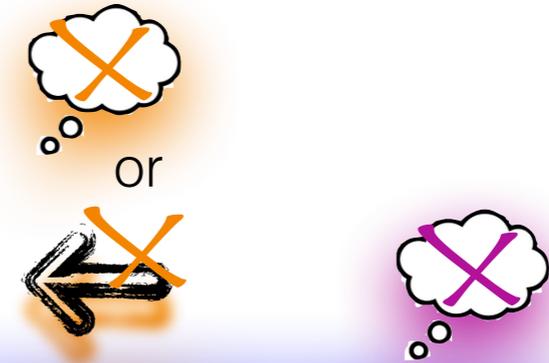
Understanding the development of pronoun interpretation



PRONOUN =



How plausible is this?



Adults smoothing away information:

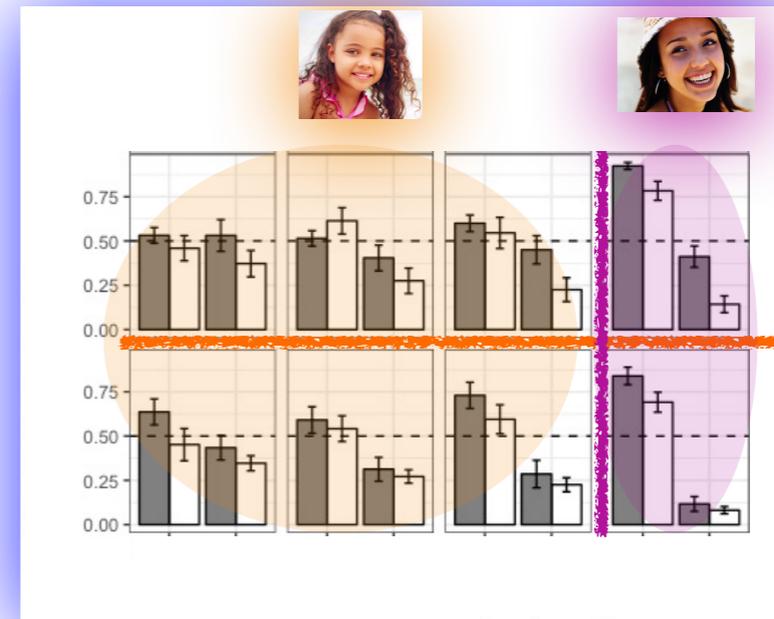
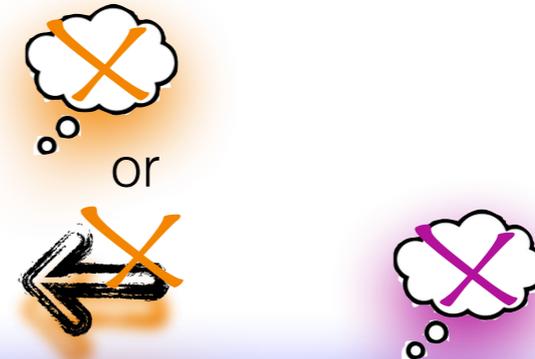
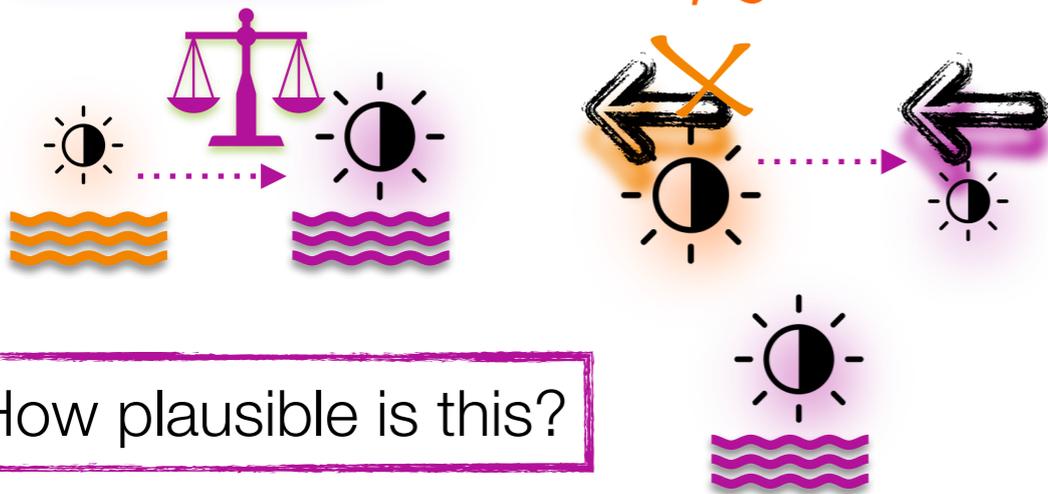
Zhang & Maloney 2012, Zhang et al. 2020:
This neural limitation causes endpoints (near 0 and 1) to get smoothed away into something between 0.16 to 0.80

antecedent type		prior	p(FORM α)				p(MOR α)	
		p(α)	∅	overt	descriptive	que	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.016	0.750	0.250	0.005	0.995	
¬SUBJ	SG	0.438	0.868	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

Understanding the development of pronoun interpretation



PRONOUN =



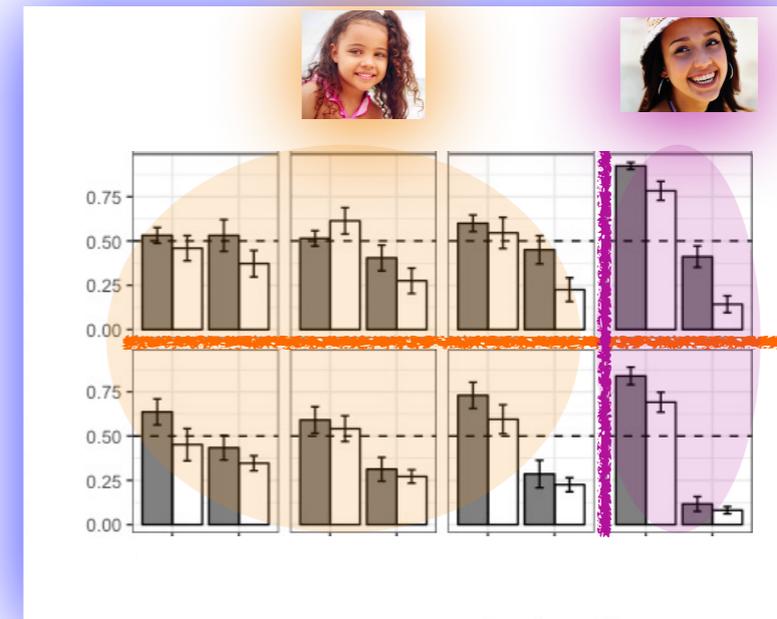
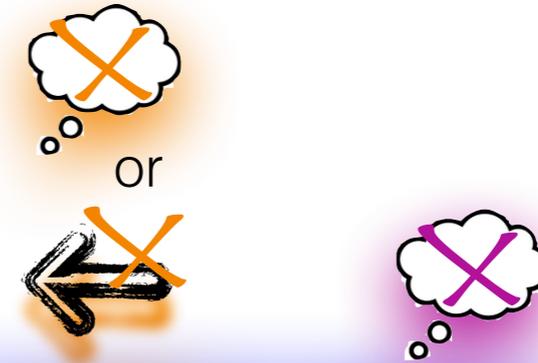
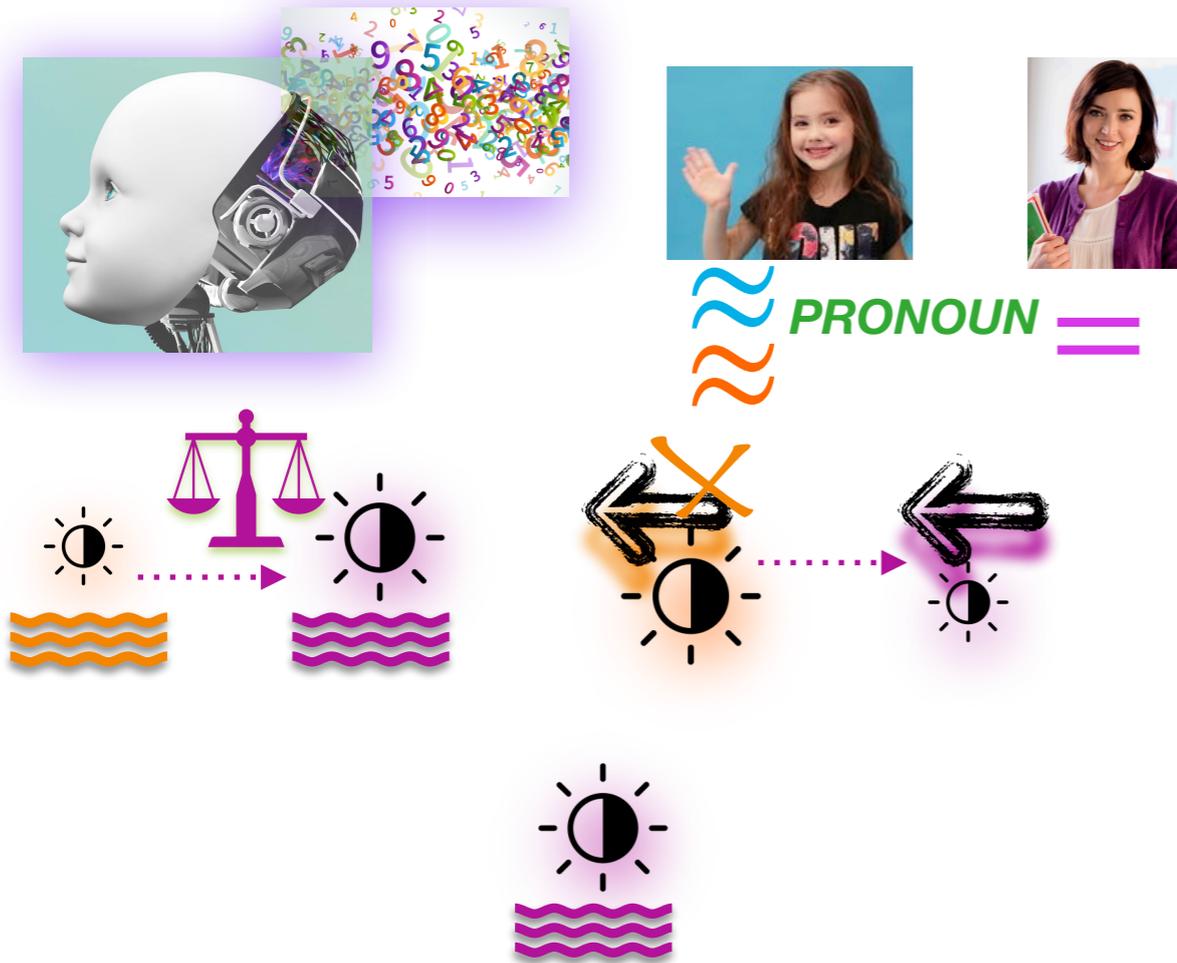
Adults smoothing away information:

Zhang & Maloney 2012, Zhang et al. 2020:
...which is what these sigma values do.

	σ_{for}	σ_{con}	σ_{mor}
adults	0.25	0.33	0.28

		prior	1					
antecedent type		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{MOR} \alpha)$			
			\emptyset	overt	<i>desp</i>	<i>que</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.016	0.750	0.250	0.005	0.995	
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Understanding the development of pronoun interpretation



So actually, adults having inaccurate representations where they smooth away information this amount is pretty plausible!

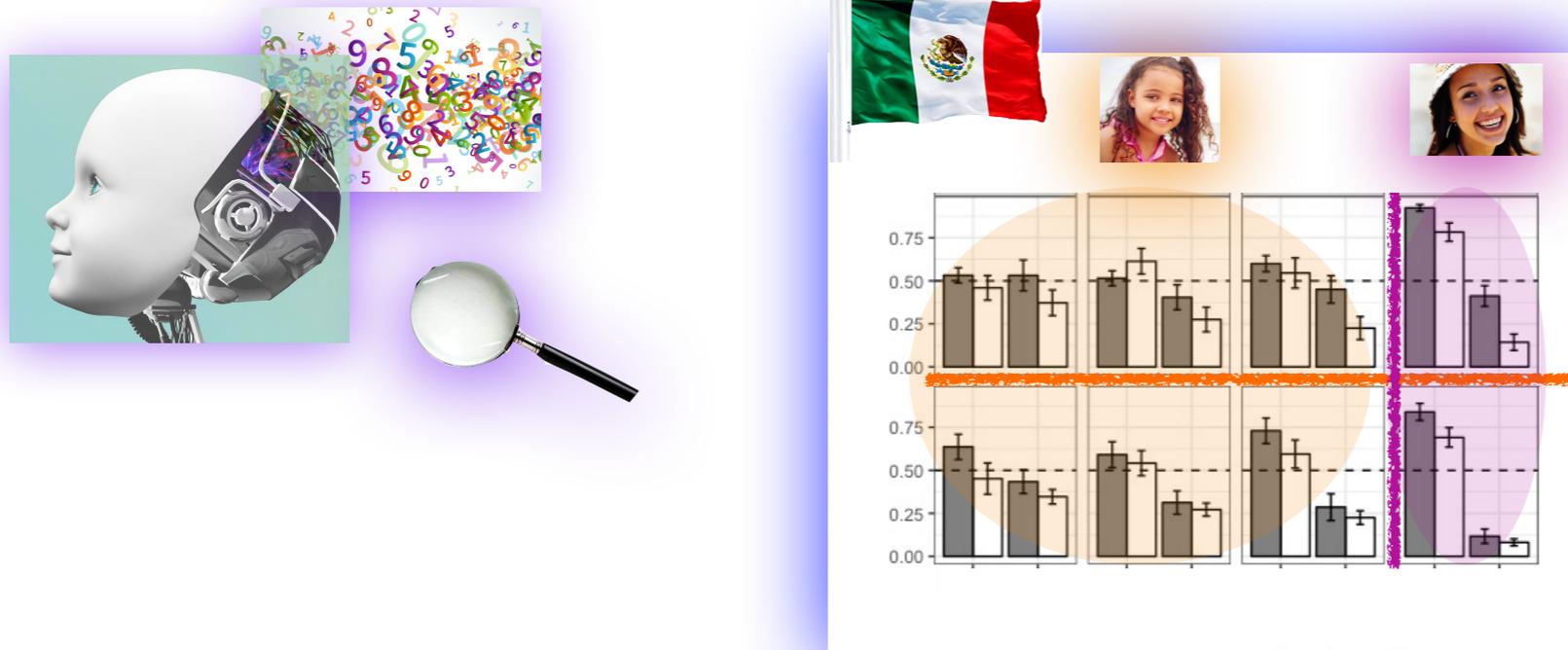


The big picture



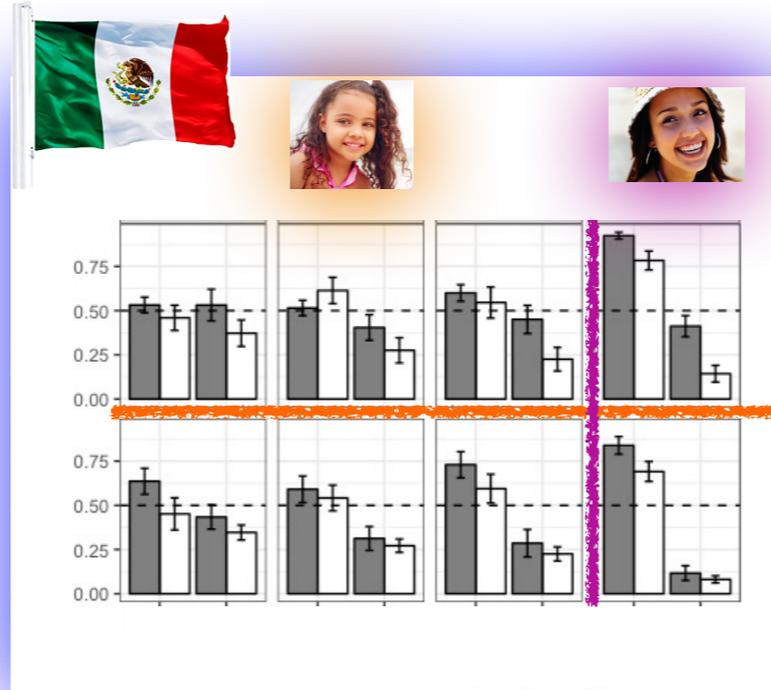
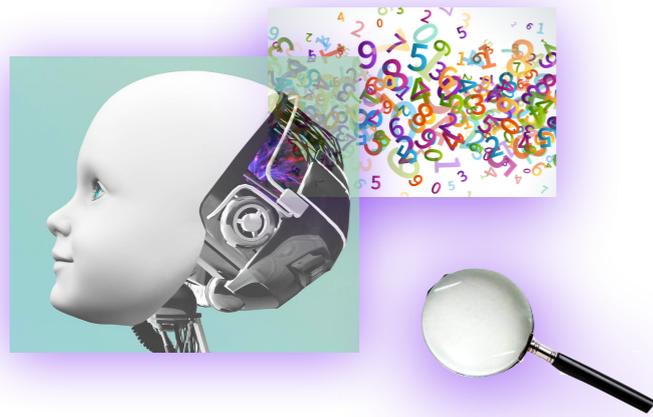
More generally, this case study demonstrates how we can use [computational cognitive modeling](#)...

The big picture



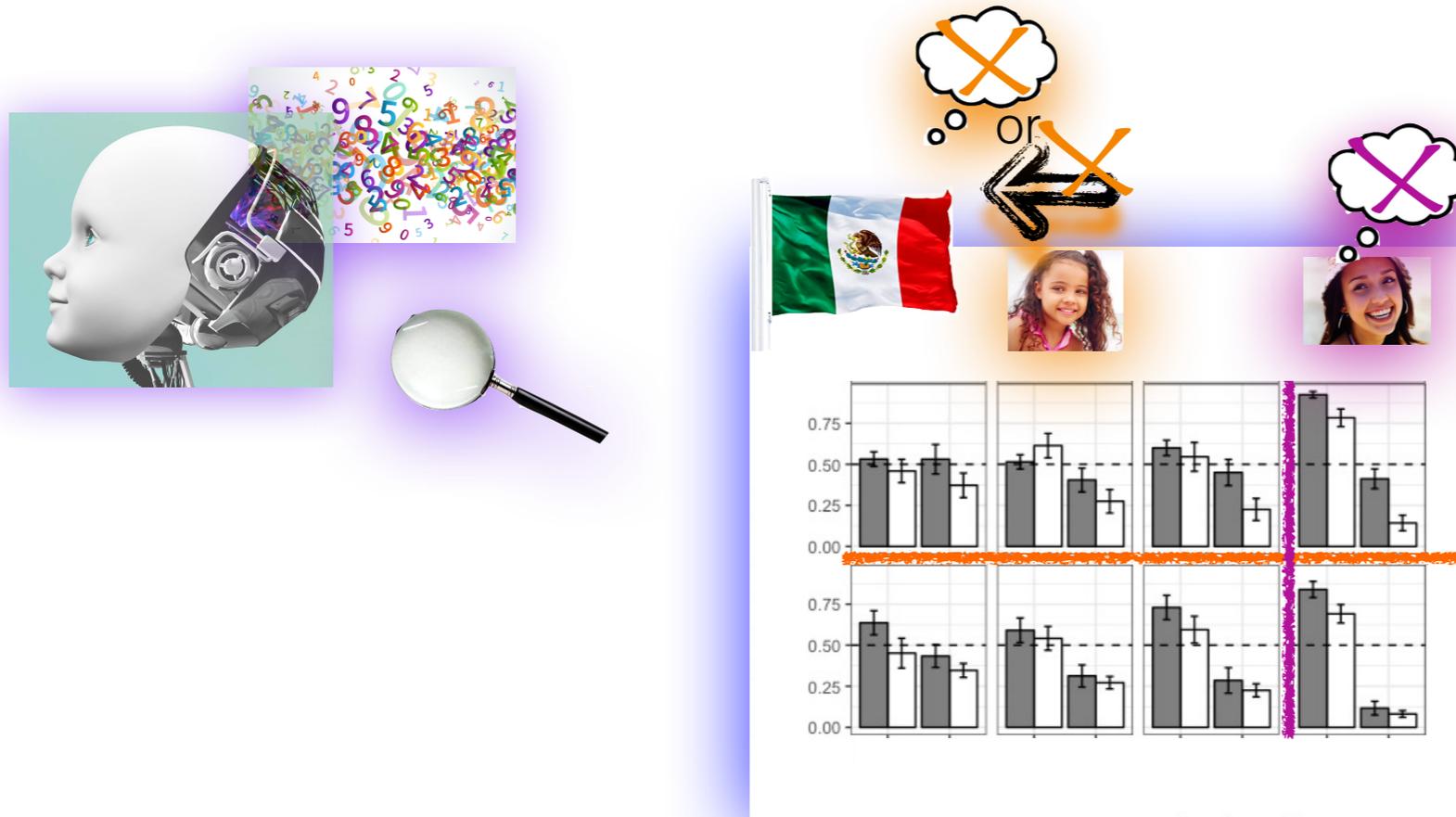
More generally, this case study demonstrates how we can use [computational cognitive modeling](#), grounded in [empirical data](#)...

The big picture



More generally, this case study demonstrates how we can use [computational cognitive modeling](#), grounded in [empirical data](#), to better understand how [children](#) and [adults](#)...

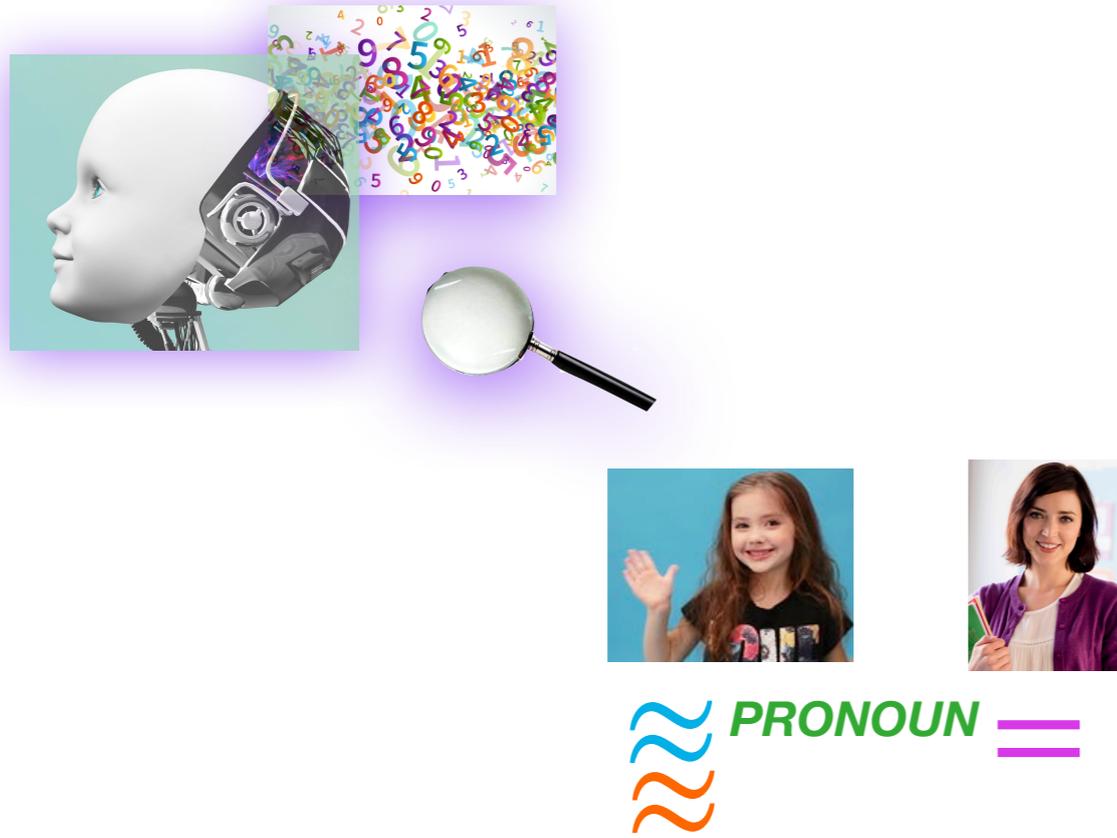
The big picture



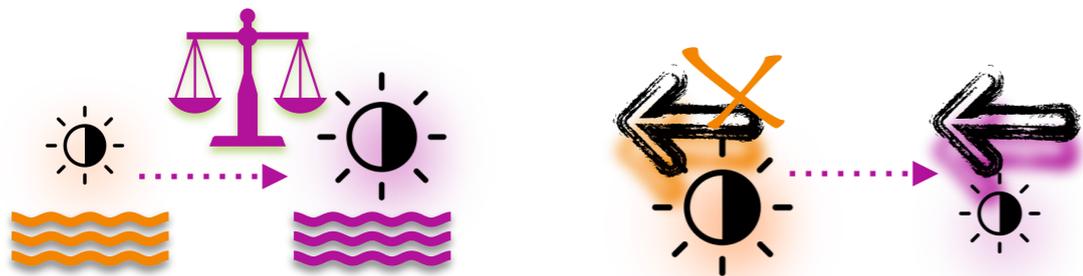
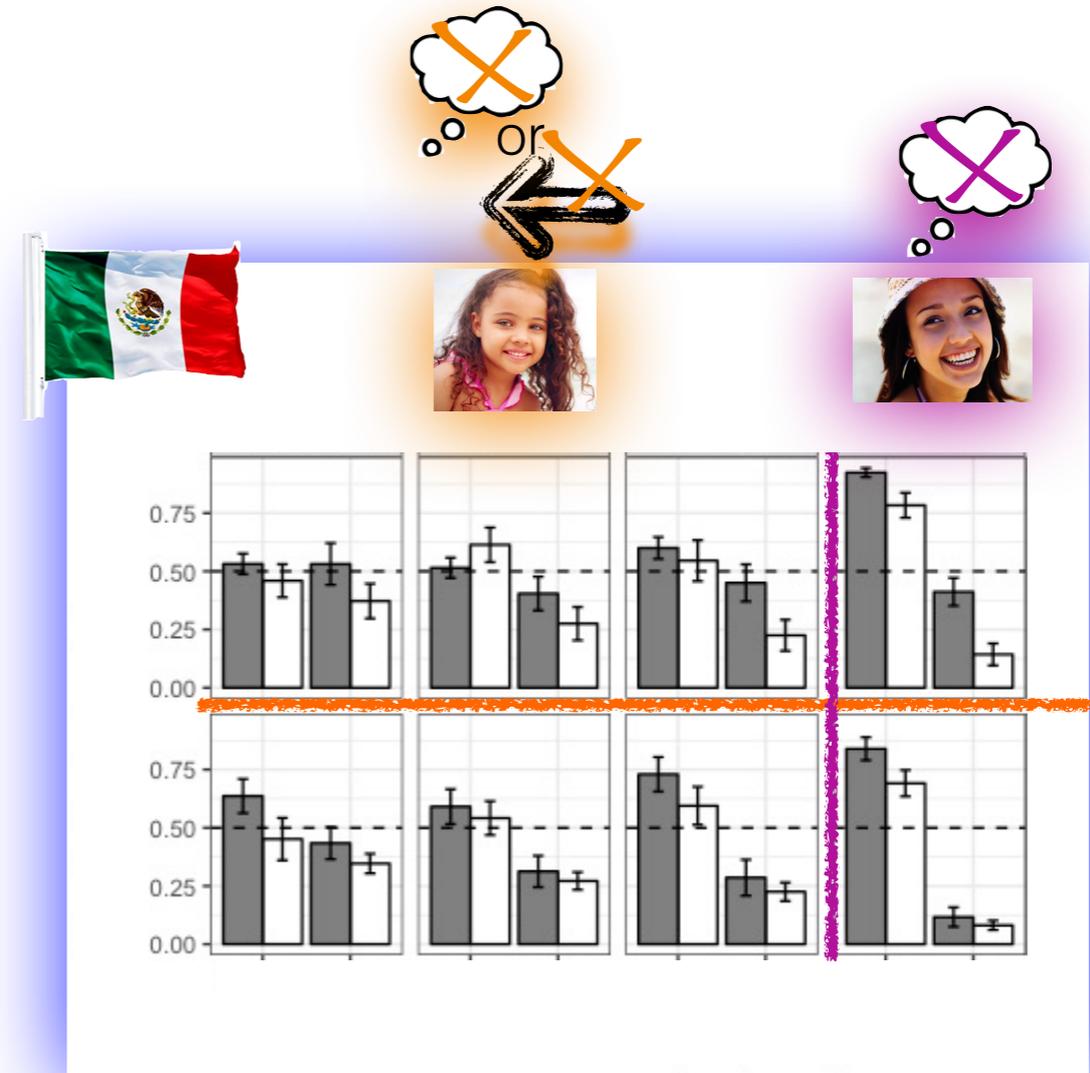
More generally, this case study demonstrates how we can use **computational cognitive modeling**, grounded in **empirical data**, to better understand how **children** and **adults** can solve complex linguistic tasks (like interpreting pronouns in a context with multiple, potentially conflicting, cues).



The big picture



This helps us better understand **what children may need to do to become adult-like** (and it seems to be about learning to be inaccurate the adult way).

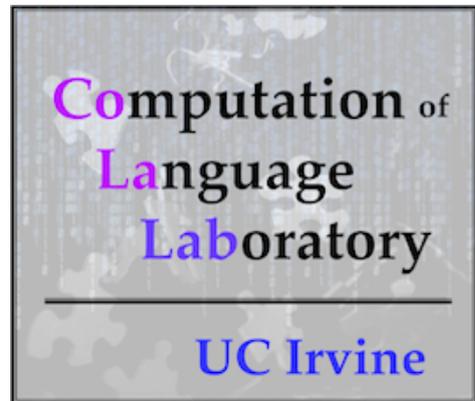


Thank you!

Hannah Forsythe



UMaryland CLIP Colloquium 2021
UCI QuantLang Collective



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