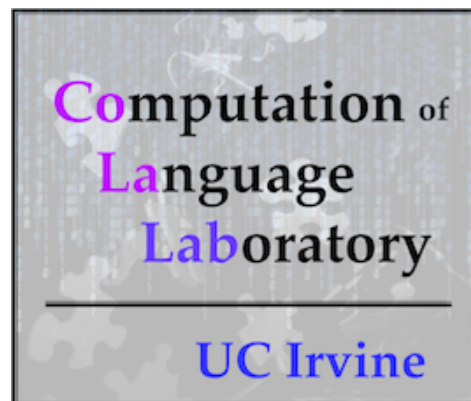


How children are and aren't like adults when it comes to interpreting pronouns: A developmental modeling investigation

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Pronoun interpretation



The girls wave at the teacher...

???

...and then **she** leaves.

Pronoun interpretation



The girls wave at the teacher...

pl

???

...and then she leaves.

sg

Agreement mismatch: "she" is singular but "girls" is plural

Pronoun interpretation



The girls wave at **the teacher**...

pl

sg

???

...and then **she** leaves.

sg

Agreement match: both "she" and "teacher" are singular

Pronoun interpretation



The girls wave at the teacher...
sg

...and then **she** leaves.
sg

How to choose?
Use linguistic knowledge of agreement.

Pronoun interpretation



The girl waves at the teacher...

???

...and then **she** leaves.

sg

Pronoun interpretation



The girl waves at the teacher...

sg

???

...and then she leaves.

sg

This could work: both “she” and “girl” are singular.

Pronoun interpretation



The girl waves at **the teacher**...

sg

sg

???

...and then **she** leaves.

sg

But so could this: both “she” and “teacher” are **singular**.

Pronoun interpretation



The girl waves at the teacher...

sg

sg

???

...and then she leaves.

sg

How to choose?

Pronoun interpretation



The girl waves at the teacher...

sg

???

sg

...and then **she** leaves.

sg

How to choose?

Use **contextual** knowledge (who's likely to be leaving)

Maybe the girl is getting ready to leave the classroom.

Pronoun interpretation



The girl waves at the teacher...

≈ subject

sg

???

sg

...and then she leaves.

sg

How to choose?

Use linguistic knowledge about connectives.

Maybe pronouns after “and then” tend to refer to the previous subject.

Pronoun interpretation



The girls wave at the teacher...
subject pl ??? sg
...and then **she** leaves.
sg

What about when interpretation cues conflict?

Pronoun interpretation



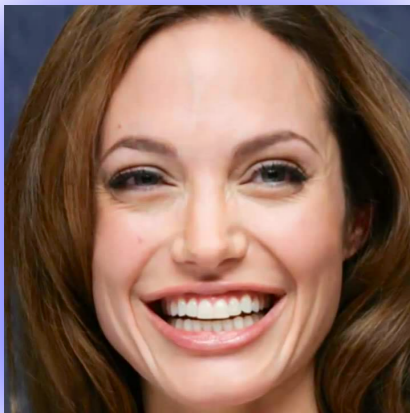
The girls wave at the teacher...
subject pl ??? sg
...and then she leaves.
sg

What about when interpretation cues conflict?
Here, the connective “and then” signals the subject “the girls”,
while the agreement signals the object “the teacher”.

Pronoun interpretation



The girls wave at the teacher...
≈ subject pl ??? sg
...and then **she** leaves.
 sg

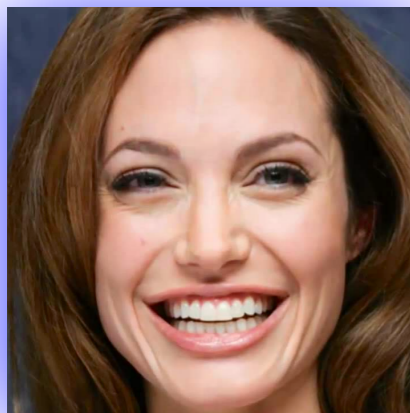


What about when interpretation cues conflict?
Here, English-speaking adults let agreement matter
more than the connective.
So, they interpret “she” as “the teacher”.

Pronoun interpretation



The girls wave at the teacher...
subject pl ??? sg
...and then she leaves.
sg



Something English-speaking adults have learned:
How to resolve interpretation cue conflicts in context.

Pronoun interpretation



The girls wave at the teacher...
subject pl ??? sg
...and then she leaves.
sg



The need to integrate multiple cues to interpretation doesn't just happen in English, of course.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después **ella** sale.

... *and then* **she** leaves.

sg

Here's the same sentence in Spanish.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg

Spanish also allows the **form** of the pronoun to be null (this means the **agreement** information is on the verb).

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg

Just like English, there are multiple cues available to interpret the pronoun.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

???

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg

Spanish-speaking adults also have interpretation preferences.



Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

subject

pl

???

sg

...y después \emptyset sale.

... and then **PRONOUN** leaves.

sg



For Spanish-speaking adults...

...the **connective** favors the **subject**.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

subject

pl

sg
???

...y después sale.

... and then PRONOUN leaves.

sg



For Spanish-speaking adults...

...the (singular) agreement (on the verb) indicates the singular object.

Pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at *the teacher...*
subject pl = sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.
sg



For Spanish-speaking adults...

...the (null) form favors the subject.

Pronoun interpretation



Las niñas saludan a la maestra...
The girls wave at the teacher...
subject pl = sg
???

...y después ∅ sale.
... and then **PRONOUN** leaves.
sg



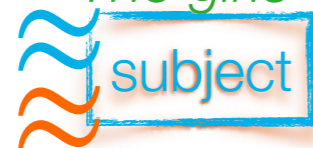
For Spanish-speaking adults...
...this collection of cues generally causes the pronoun to be interpreted as the singular object (agreement matters the most).

Pronoun interpretation development



Las niñas saludan a la maestra...

The girls wave at the teacher...



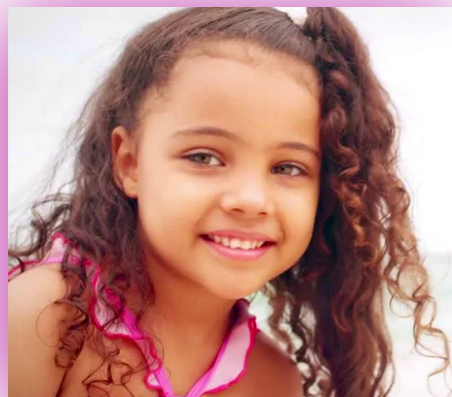
pl

= sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.

sg



How do Spanish-learning children develop this ability to interpret pronouns in context?

Pronoun interpretation development



 **PRONOUN** =



Children's ability to interpret a pronoun in an **adult-like** way depends on (at least) two things.



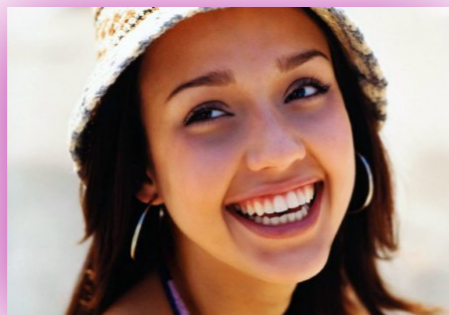
Pronoun interpretation development



PRONOUN



First, children need **adult-like knowledge** of what each cue signals.



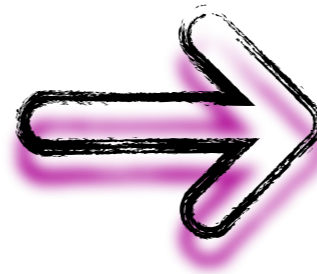
Pronoun interpretation development



PRONOUN



Second, children need **adult-like ability** to deploy that knowledge in real time.



Pronoun interpretation development



 PRONOUN =



When both of these are adult-like, we should get adult-like pronoun interpretation.



Pronoun interpretation development



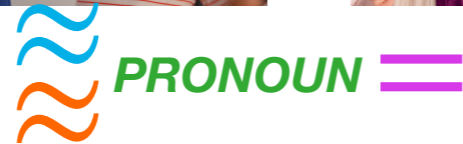
 PRONOUN =



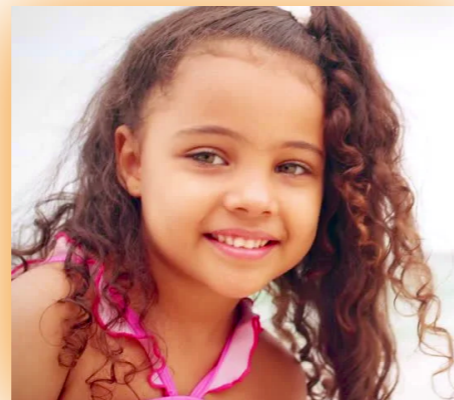
But if we get **non-adult-like** pronoun interpretation, then it could be due to **immature knowledge**, **immature deployment** of that knowledge, or **both!**



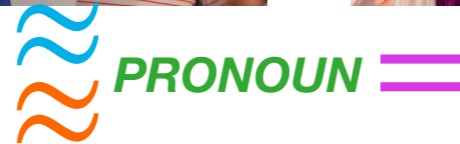
Pronoun interpretation development



How do we tell what the differences are between child and adult **pronoun interpretation**? When we understand this better, we'll understand what **children** need to do to become **adults**.



Understanding pronoun interpretation development



The plan, part 1: Get some empirical data on how **children** and **adults** interpret the same **pronoun** in a context where multiple cues are available.

Case study: Mexican Spanish



Understanding pronoun interpretation development



 PRONOUN =

The plan, part 1: Get some empirical data on how **children** and **adults** interpret the same **pronoun** in a context where multiple cues are available.



This will highlight what the **observable differences** are in interpretation behavior.

Understanding pronoun interpretation development



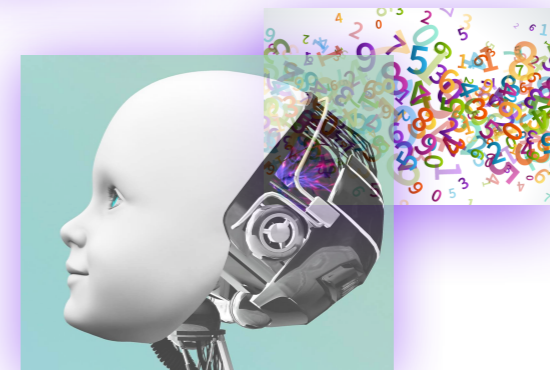
The plan, part 2: Use **computational cognitive modeling** to formally articulate the potential process of pronoun interpretation in the context of these multiple cues.



Understanding pronoun interpretation development



 PRONOUN =



The plan, part 2: Use the **computational cognitive model** to identify the specific differences leading to child and adult pronoun interpretation in context.



Empirical data on pronoun interpretation



Empirical data on pronoun interpretation



Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

sg
???

...y después ∅ sale.

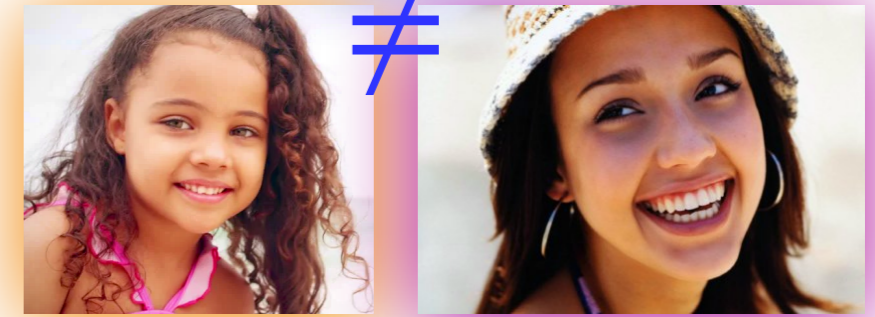
... and then PRONOUN leaves.
sg

Children (~3, 4, and ~5) and adults are asked to interpret pronouns in the kind of contexts we saw before.



<=3:	1;11-3;10 (N=33)
4:	4;0-4;11 (N=35)
>=5:	5;0-6;9 (N=29)
Adults	(N=47)

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

sg
???

...y después ∅ sale.

... and then PRONOUN leaves.
sg

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

===sg
???

...y después ∅ sale.

... and then **PRONOUN** leaves.
sg

Rate of subject responses

Choice: Is the pronoun interpreted as the **subject** or the **object**?

We can plot the rate of subject responses.

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

???

sg

...y después ~~Ø~~ sale.

... and then **PRONOUN** leaves.

sg

Rate of subject responses

Context:

Does agreement favor the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

=pl

???

sg

...y después \emptyset salen.

... and then **PRONOUN** leave.

pl

Rate of subject responses

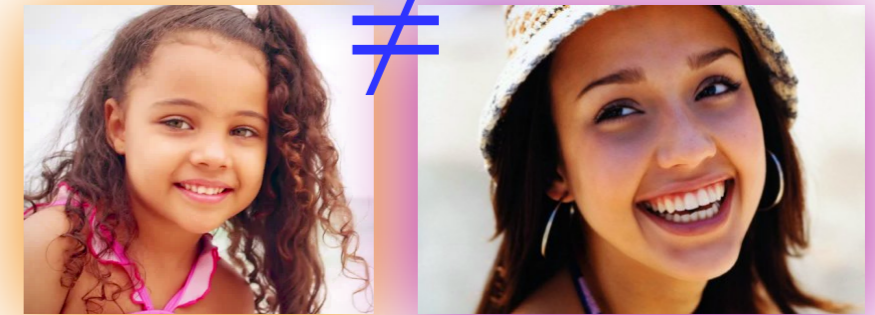
Context:

Does agreement favor the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

==sg
???

...y después ∅ sale.

... and then PRONOUN leaves.

sg

Rate of subject responses

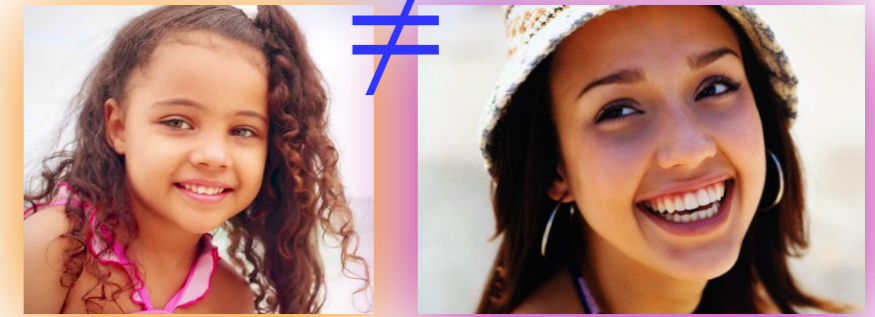
Context:

Does agreement favor the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...
The girls wave at the teacher...

pl

sg

...y después \emptyset sale.
... and then PRONOUN leaves.

???

sg

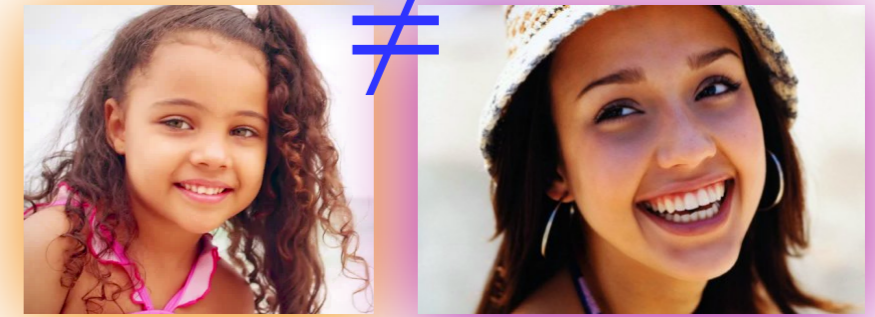
Rate of subject responses

Context:
 Does the pronoun form favor
 the subject or the object?

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

≈ subject pl

??? sg

...y después ∅ sale.

... and then **PRONOUN** leaves. sg

Rate of subject responses

Context:

Does the pronoun **form** favor the **subject** or the **object**?

∅ (favors subject)

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs

adults

Las niñas saludan a la maestra...
The girls wave at the teacher...

pl

sg



object

...y después ella sale.
... and then PRONOUN leaves.

???

PRONOUN

sg

Rate of subject responses

Context:

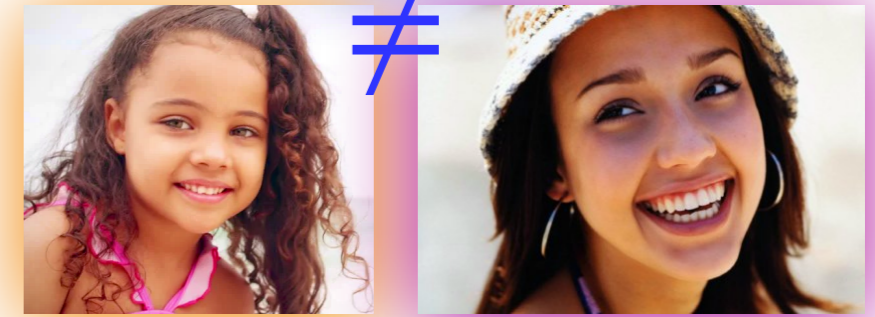
Does the pronoun form favor the subject or the object?

overt (favors object)

subject object

Favored by agreement

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs

adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

... y después \emptyset sale.

... and then PRONOUN leaves.

sg

Rate of subject responses

Context:
Does the **connective** favor the **subject** or the **object**?

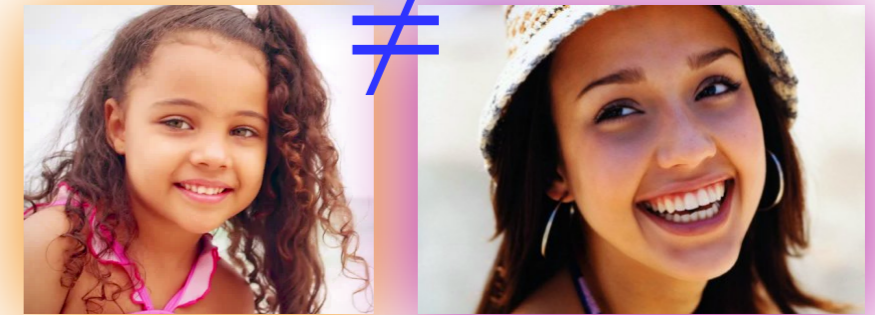
subject object

Favored by agreement

\emptyset (favors subject) overt (favors object)

Favored by form

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs



adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

subject pl

sg

...y después \emptyset sale.

... and then **PRONOUN** leaves. sg

Rate of subject responses

Context:
Does the **connective** favor the **subject** or the **object**?

y después (favors subject)

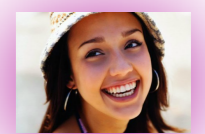
subject object

Favored by agreement

\emptyset (favors subject) overt (favors object)

Favored by form

Empirical data on pronoun interpretation



~3yrs

4yrs

~5yrs

adults

Las niñas saludan a la maestra...

The girls wave at the teacher...

pl

sg

object

???

∅

sale.

...porque

... because

PRONOUN leaves.

sg

Rate of subject responses

Context:

Does the **connective** favor the **subject** or the **object**?

porque (favors object)

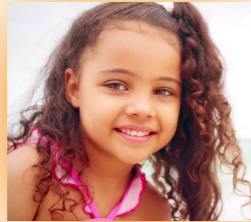
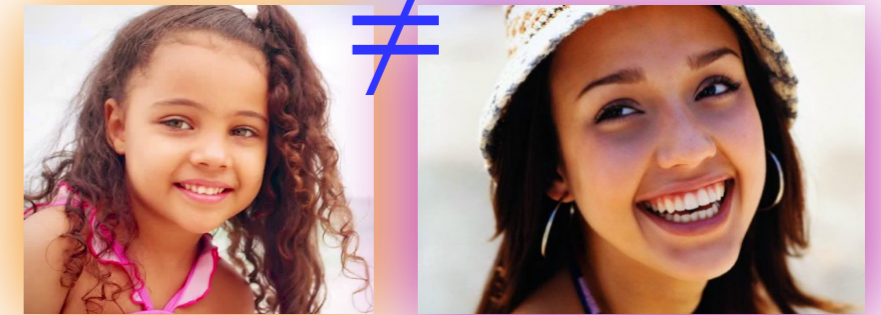
subject object

Favored by agreement

∅ (favors subject) overt (favors object)

Favored by form

Empirical data on pronoun interpretation



Favored by connective

y después (favors subject)

porque (favors object)



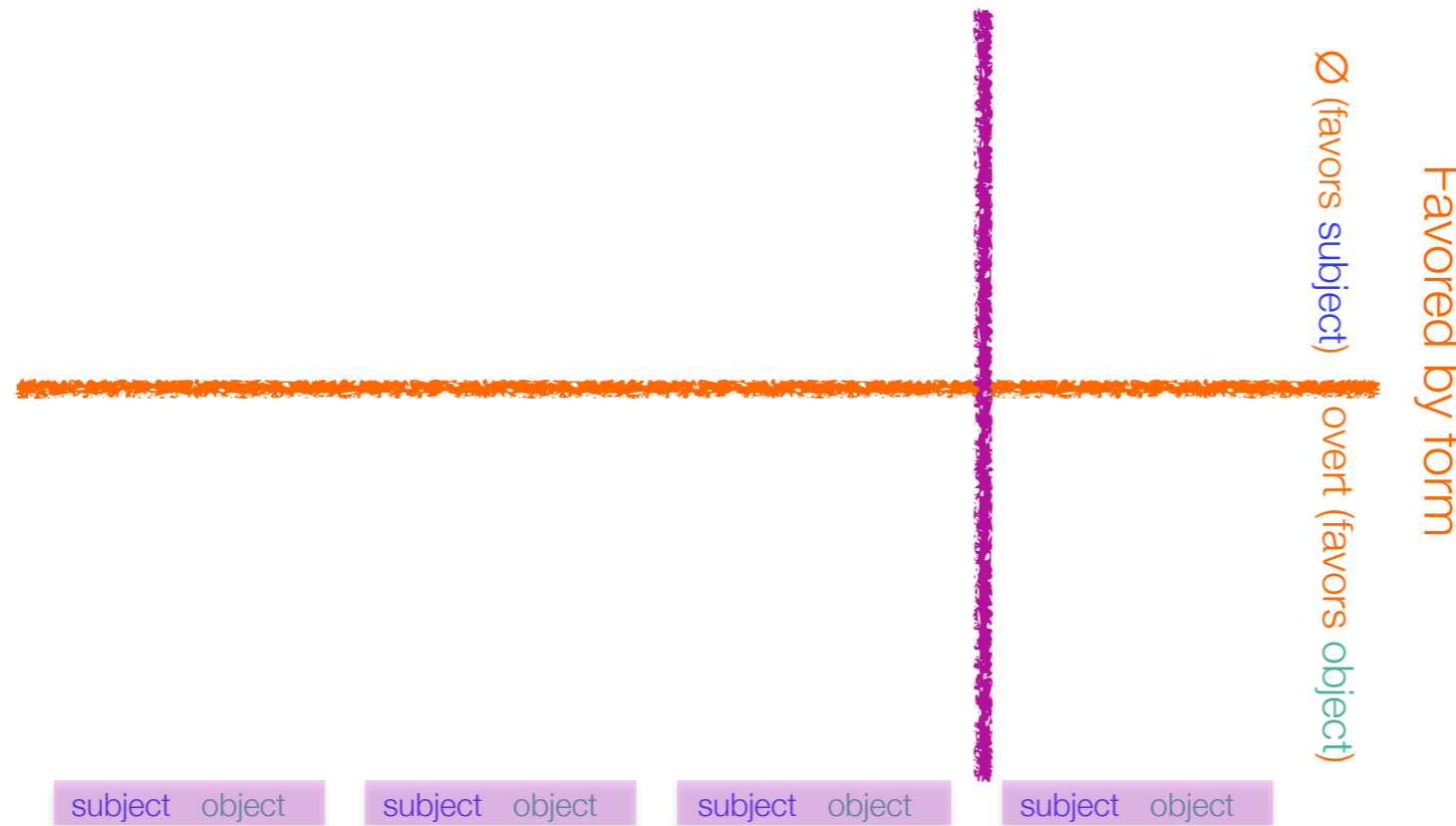
~3yrs

4yrs

~5yrs

adults

Rate of subject responses



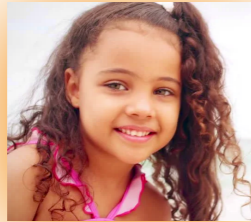
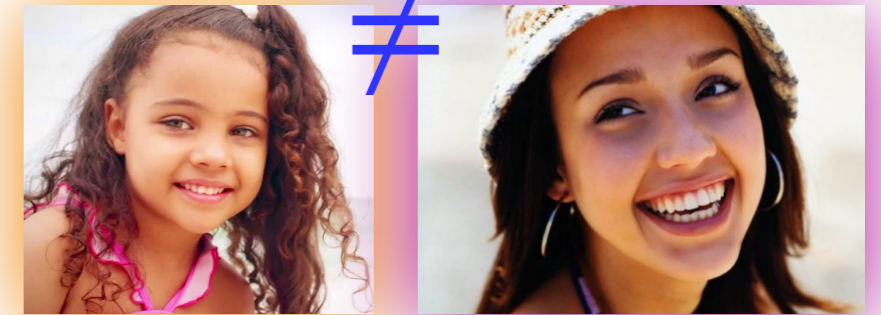
Favored by agreement

∅ (favors subject)
overt (favors object)

Favored by form



Empirical data on pronoun interpretation



Favored by connective

y después (favors subject)

porque (favors object)



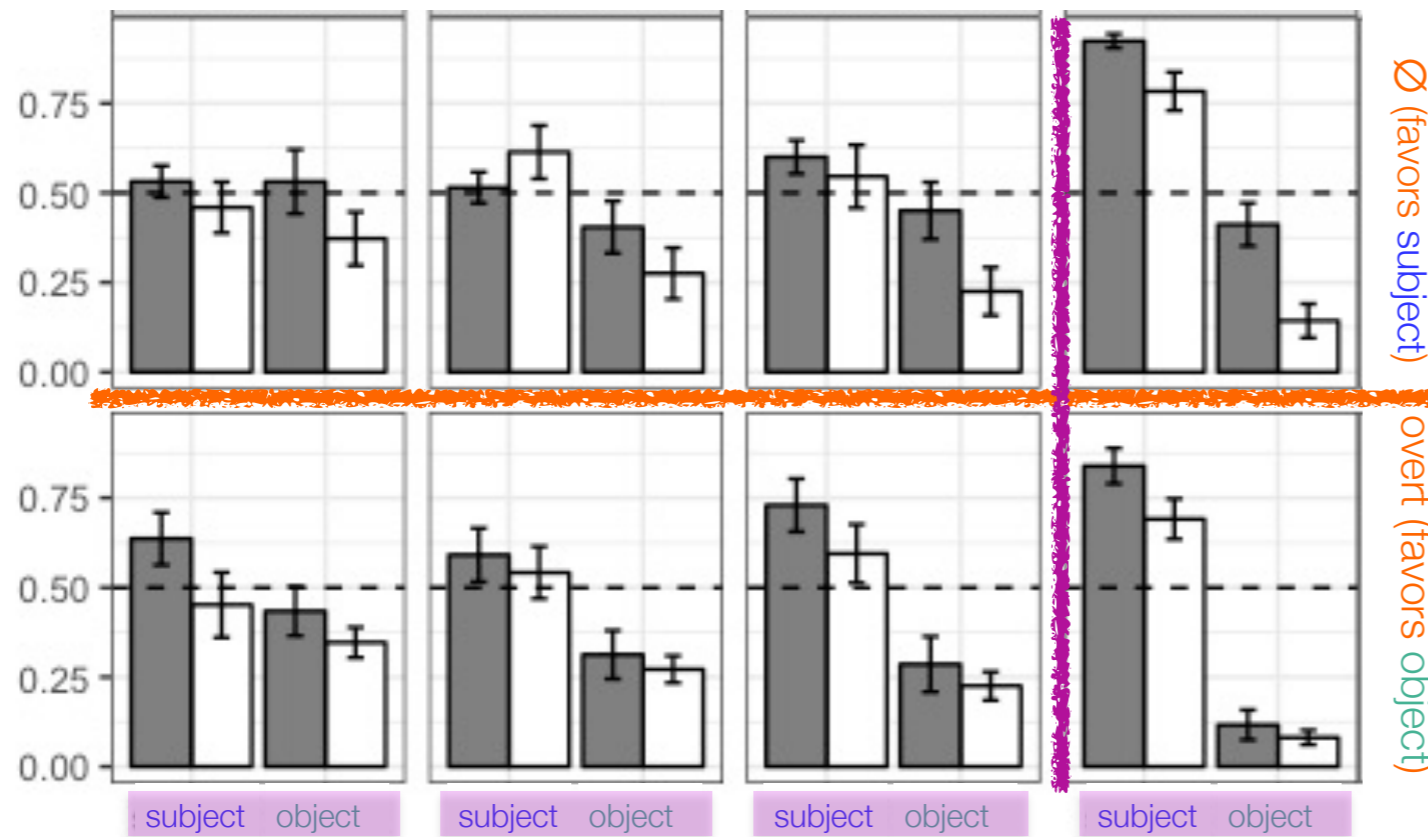
~3yrs

4yrs

~5yrs

adults

Rate of subject responses



∅ (favors subject)

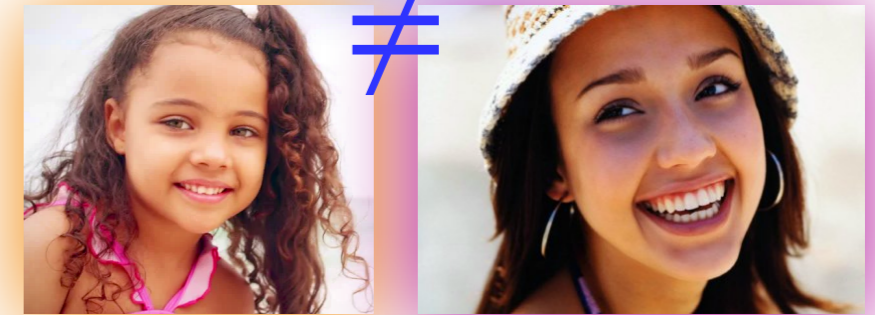
overt (favors object)

Favored by form



Favored by agreement

Empirical data on pronoun interpretation



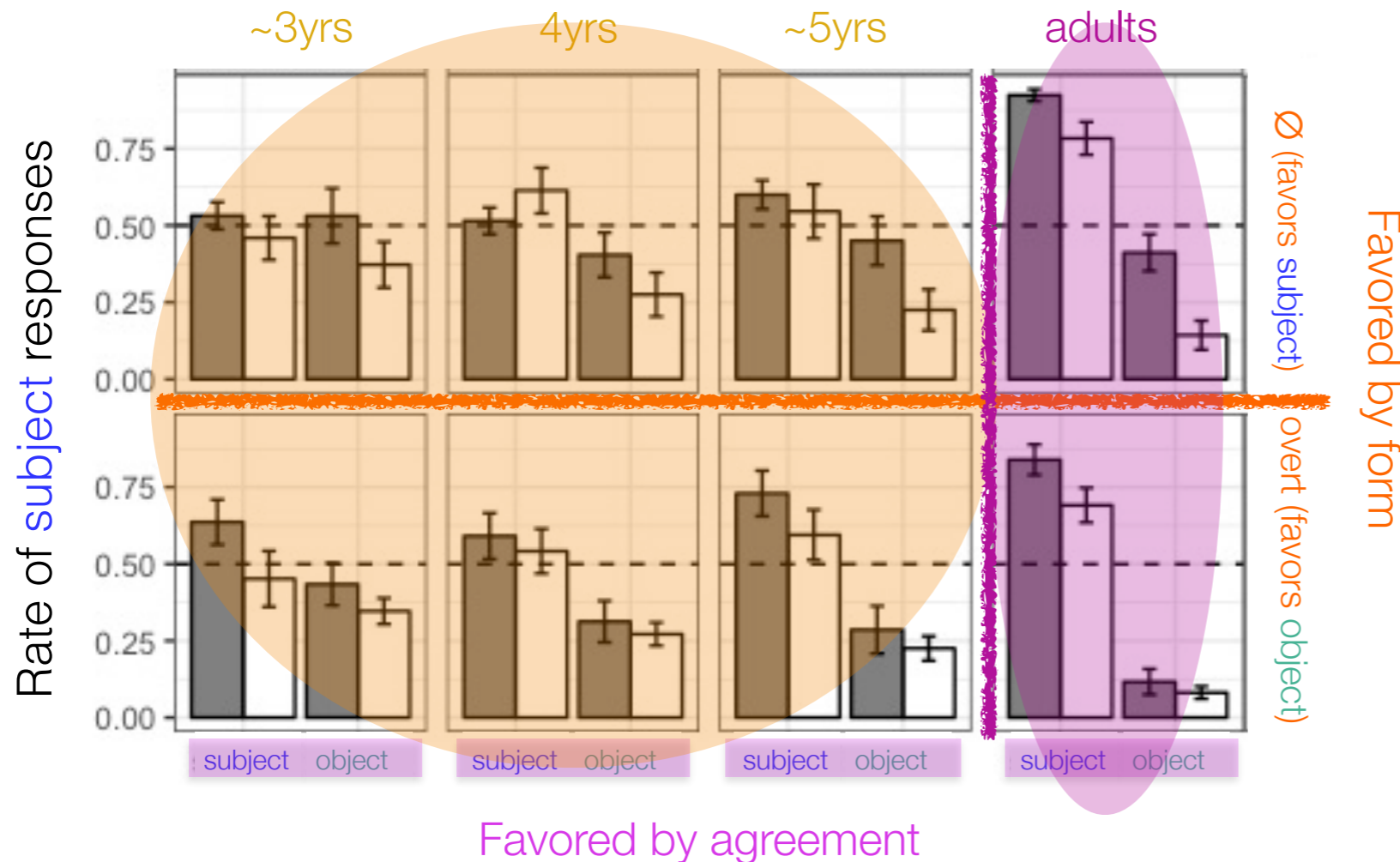
Favored by connective

y después (favors subject)

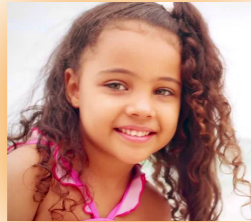
porque (favors object)



Big qualitative picture:
Children don't behave like adults.



Empirical data on pronoun interpretation



Favored by connective

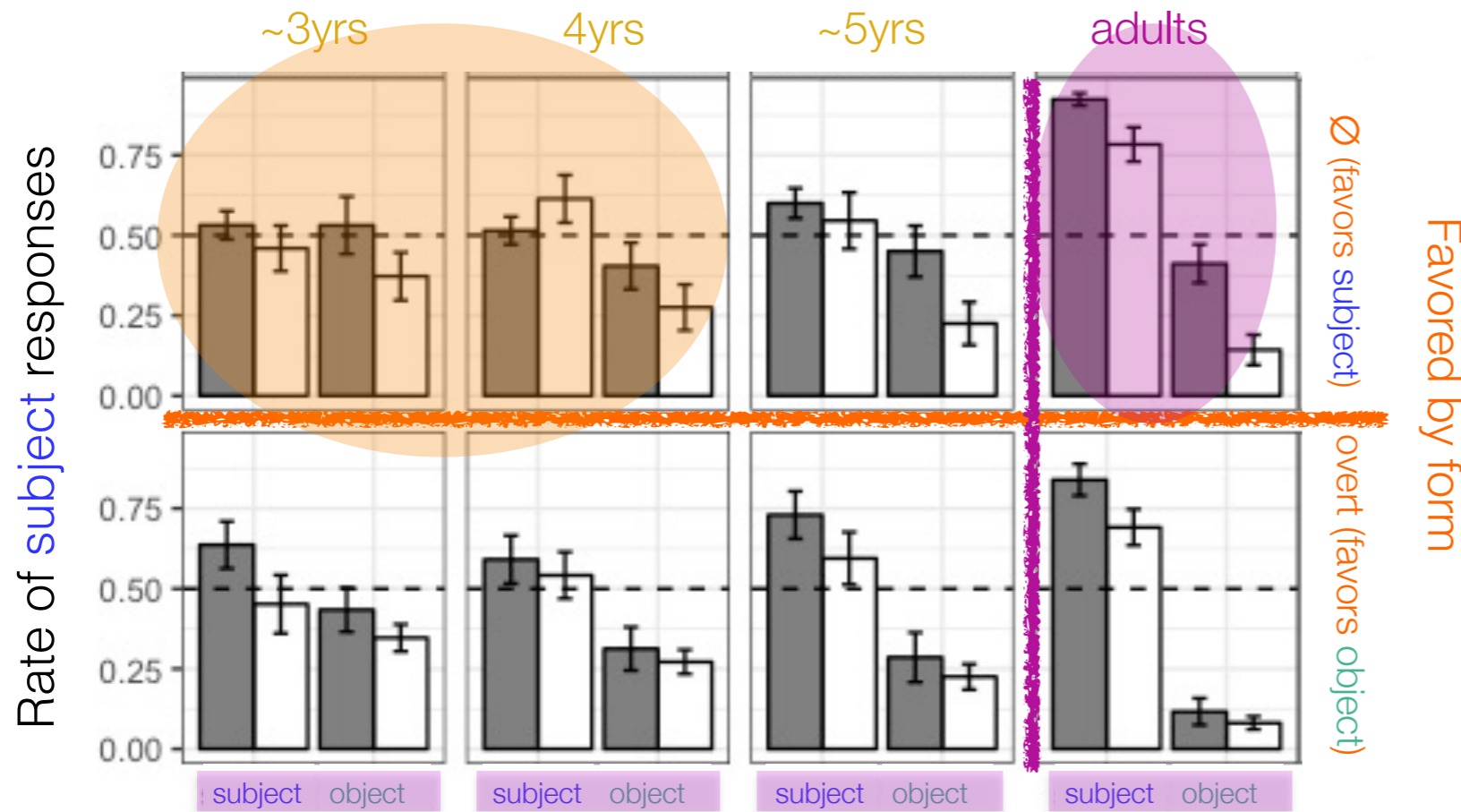
y después (favors subject)

porque (favors object)



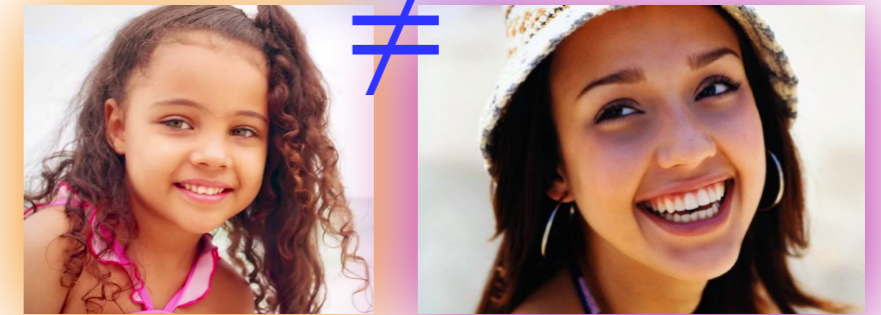
Big qualitative picture:
Children don't behave like adults.

Some nuances:
Younger children seem to differ more, especially in certain contexts.



Favored by agreement

Empirical data on pronoun interpretation



Favored by connective

y después (favors subject)

porque (favors object)

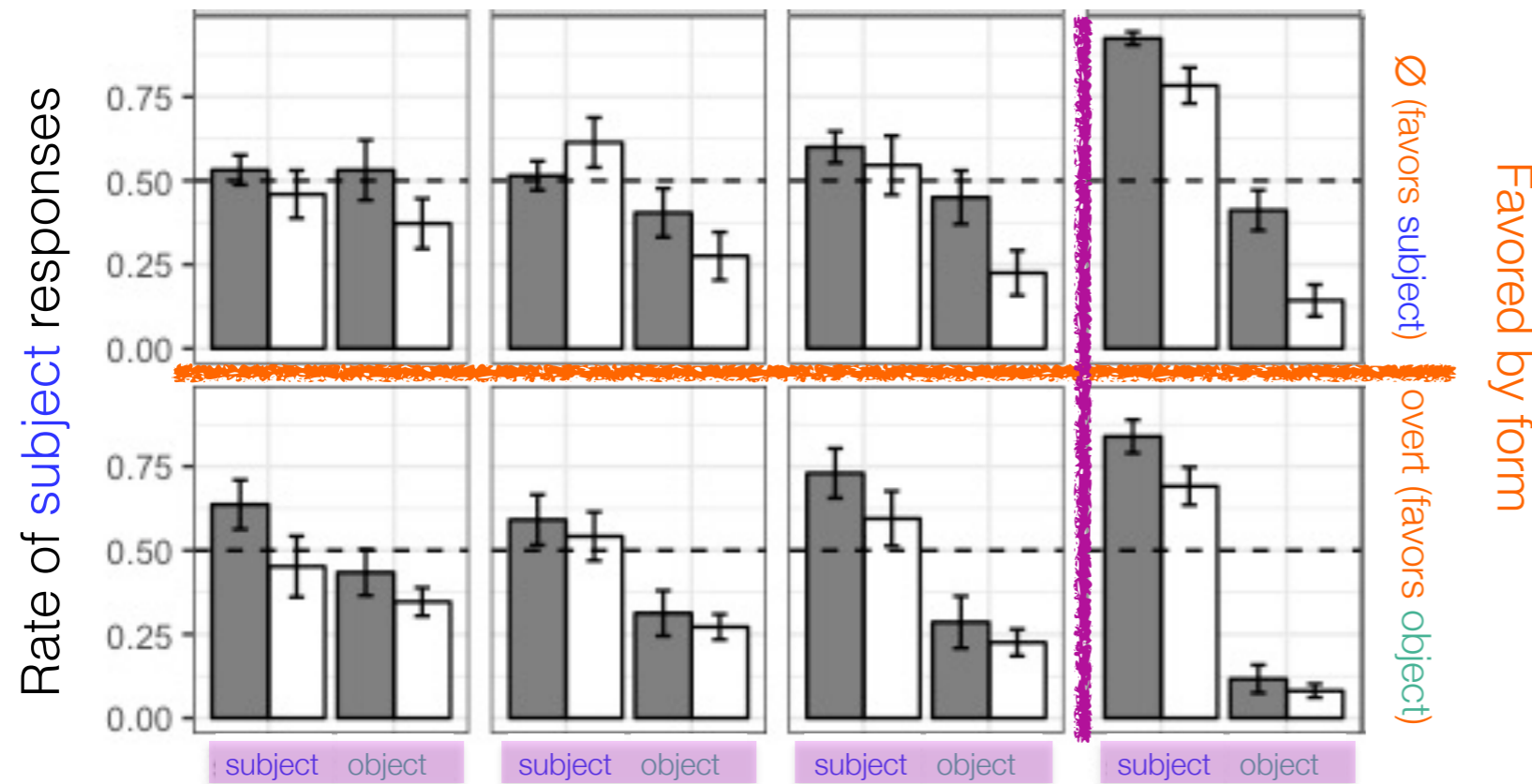


~3yrs

4yrs

~5yrs

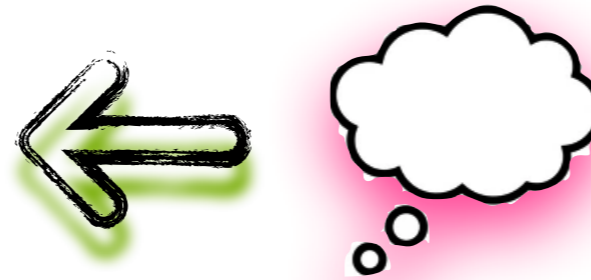
adults



Something needs to change for children to become adult-like — but what?

Favored by agreement

Understanding pronoun interpretation development

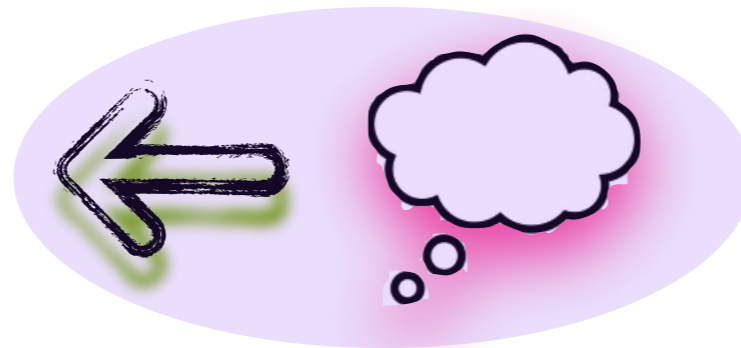


The plan, part 2:

Use **computational cognitive modeling** to formally articulate the potential process of pronoun interpretation in the context of these multiple cues.





Understanding pronoun interpretation development



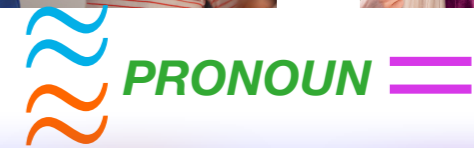
The computational cognitive model formally articulates and implements (what we think are) **relevant aspects** of pronoun **interpretation in context**.



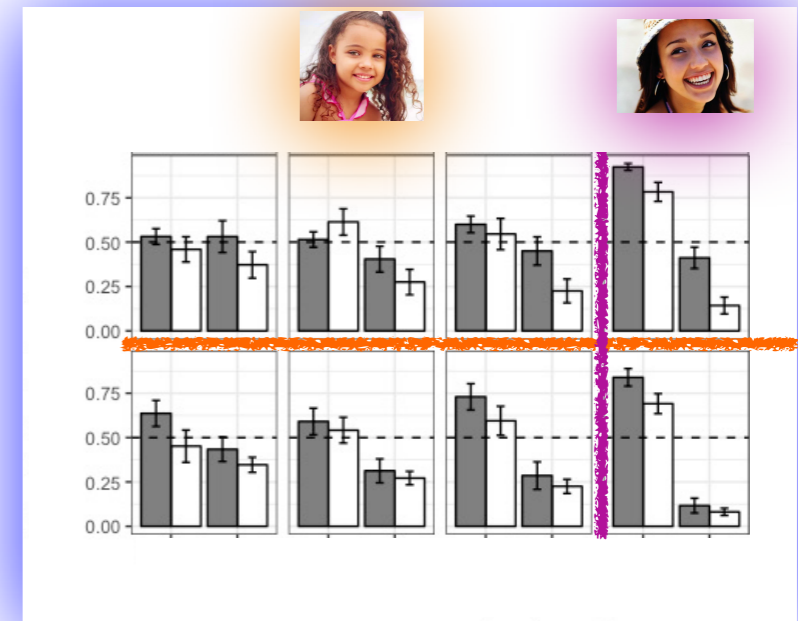
 **PRONOUN** 



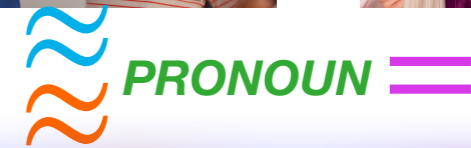
Understanding pronoun interpretation development



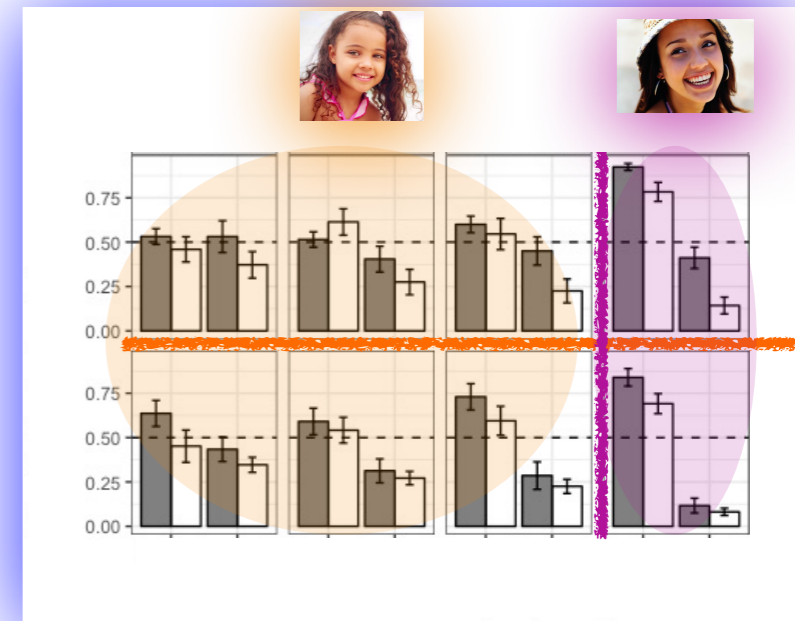
Here: Implement how a modeled listener represents pronoun information and deploys that information in order to predict the probability of a particular interpretation in context.



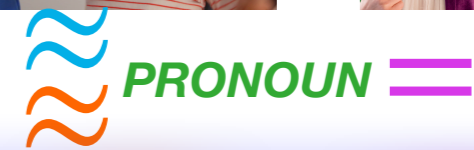
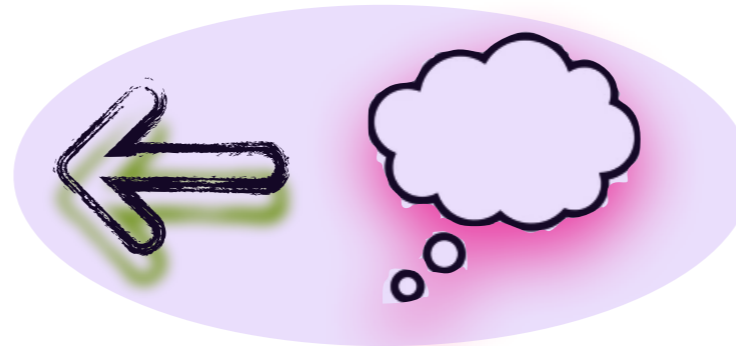
Understanding pronoun interpretation development



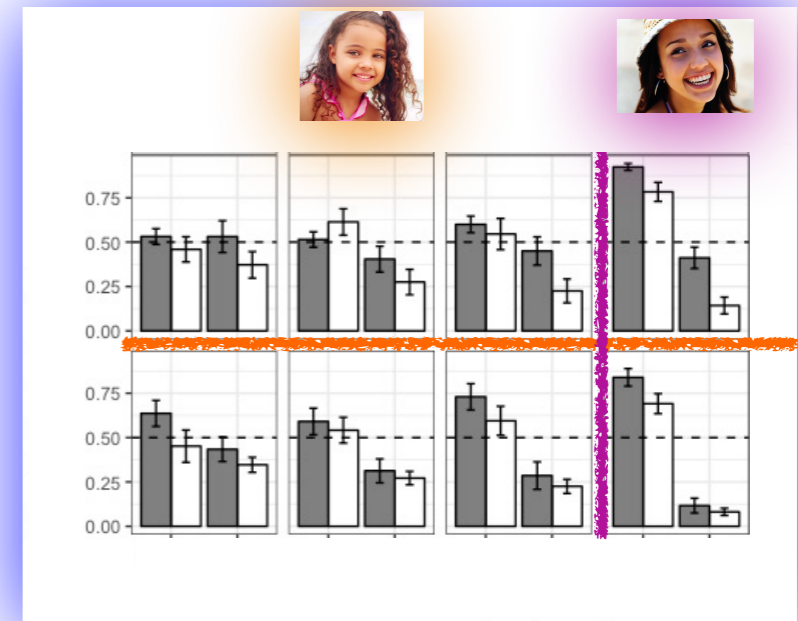
Then, see which options for **representation** and **deployment** best match **child** vs. **adult** pronoun interpretation behavior.



Understanding pronoun interpretation development



This will articulate **how children differ from adults**, and what needs to develop in children for them to become adult-like.



Modeling pronoun interpretation in context



The model uses **Bayesian inference** to implement the cognitive mechanism that combines information to generate a particular **interpretation** in **context**.

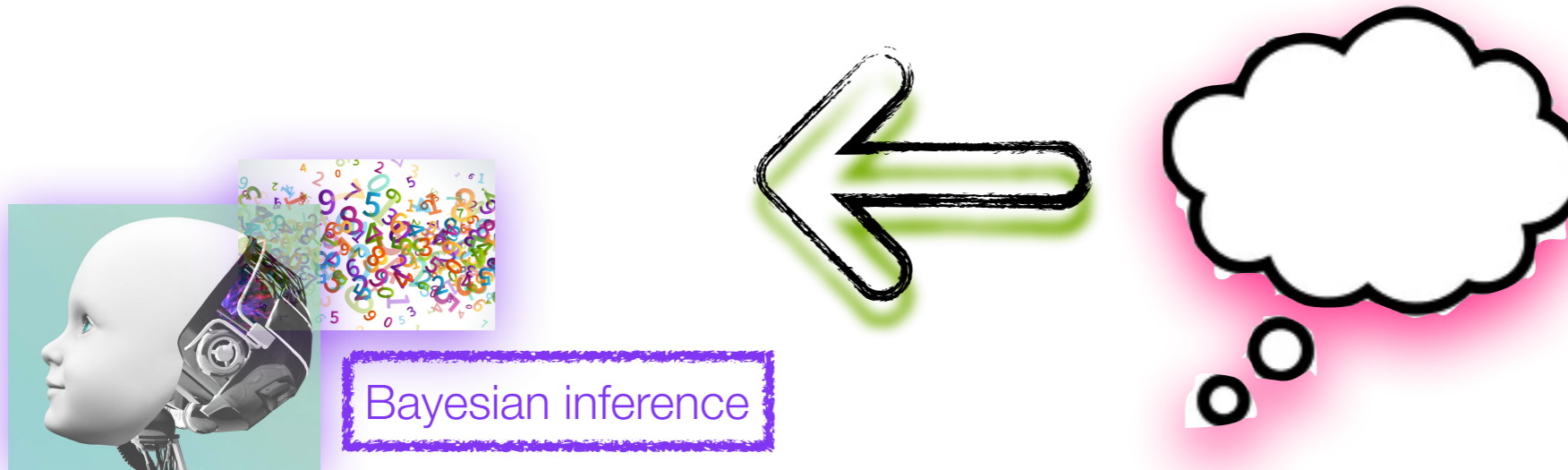
Modeling pronoun interpretation in context



Bayesian inference is commonly used to model human cognition in general and language development in particular, since it matches human behavior quite well (see Pearl in press and b for recent reviews).



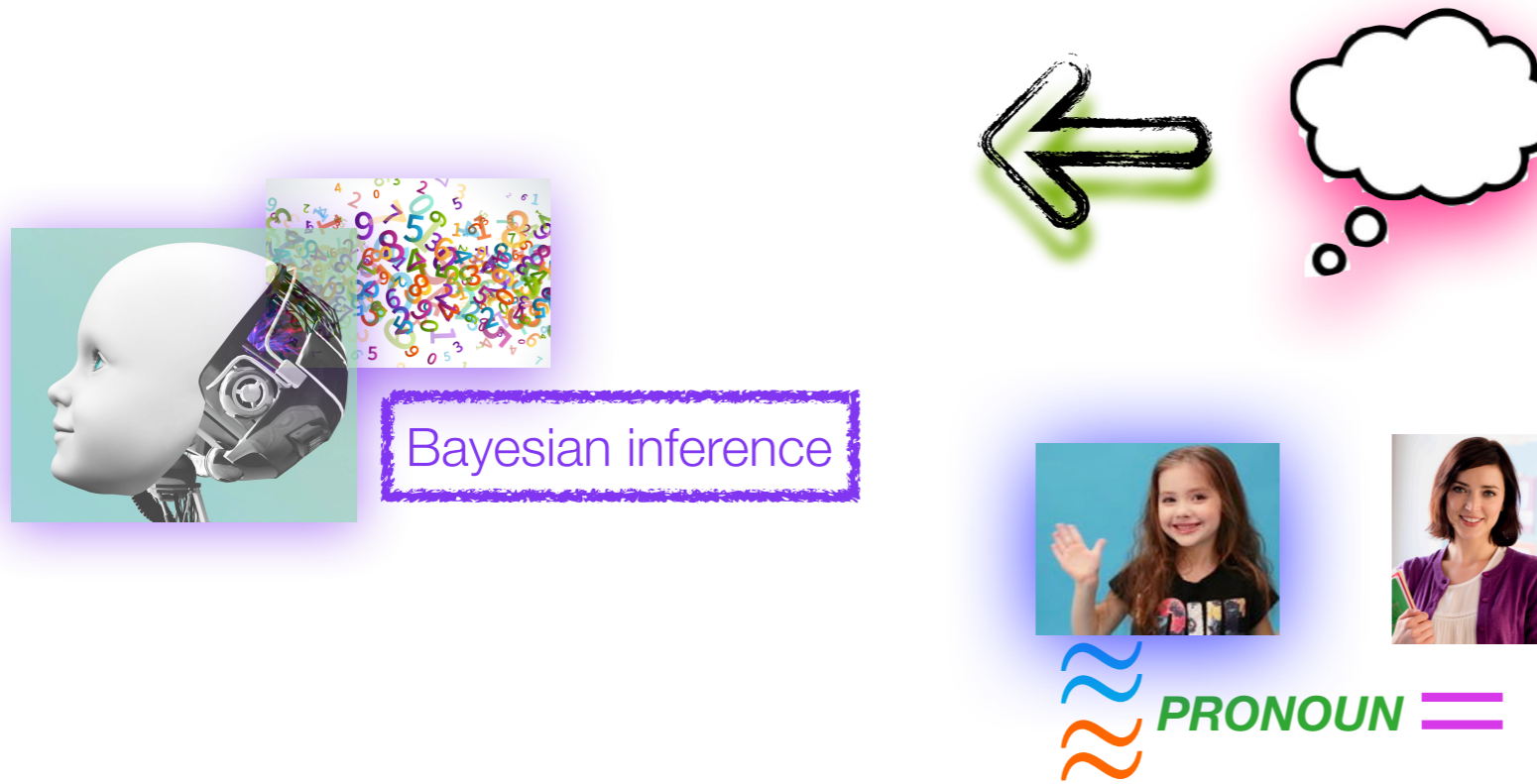
Modeling pronoun interpretation in context



The particular Bayesian model we use is adapted from Gagliardi, Feldman & Lidz (2017), and offers one way to separate out the contributions of information **representation** vs. information **deployment** in the moment.



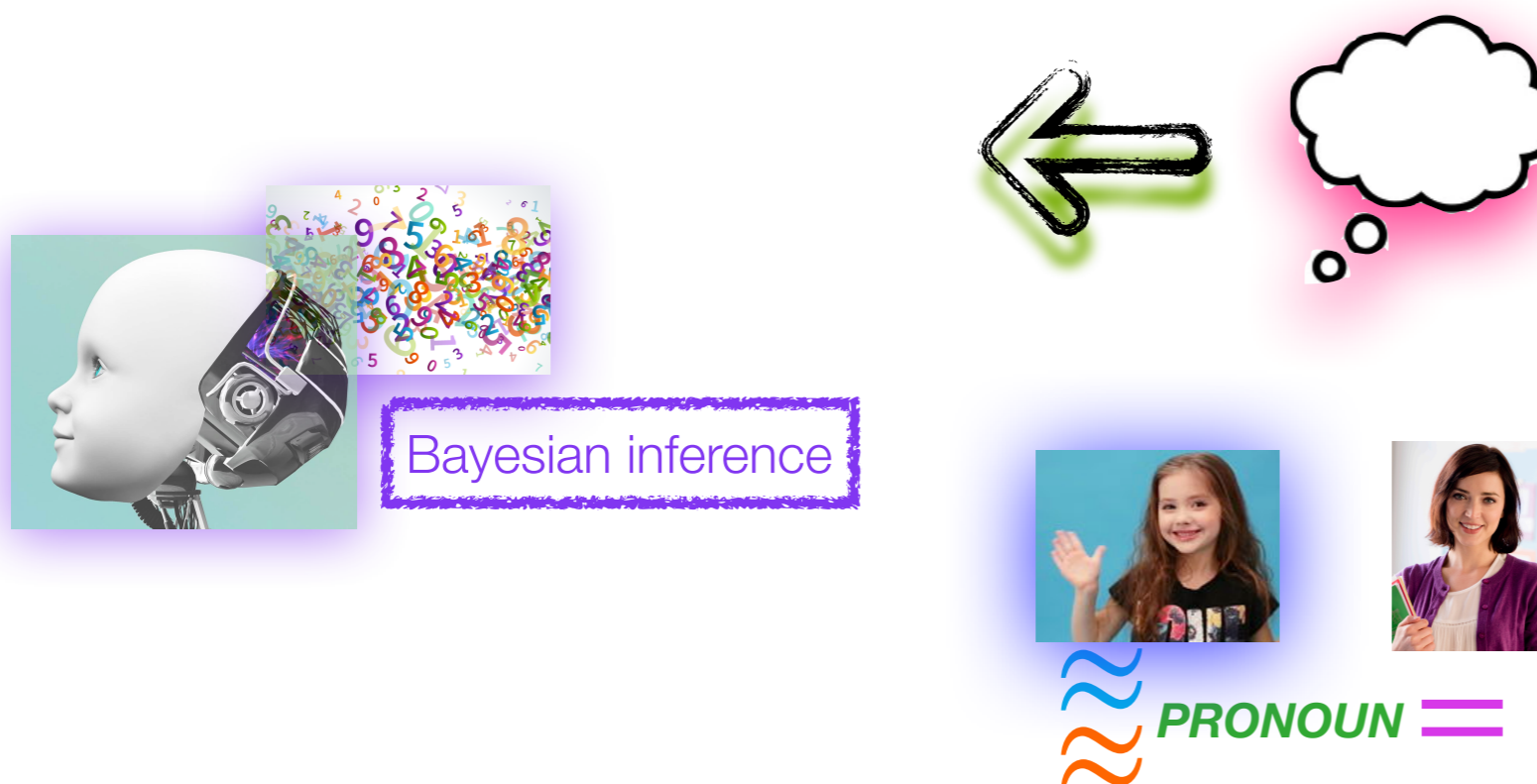
Modeling pronoun interpretation in context



$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM, CON, MOR} | \alpha_{subj.SG})$$

Interpreting the pronoun as the **subject**, which is **singular**....

Modeling pronoun interpretation in context



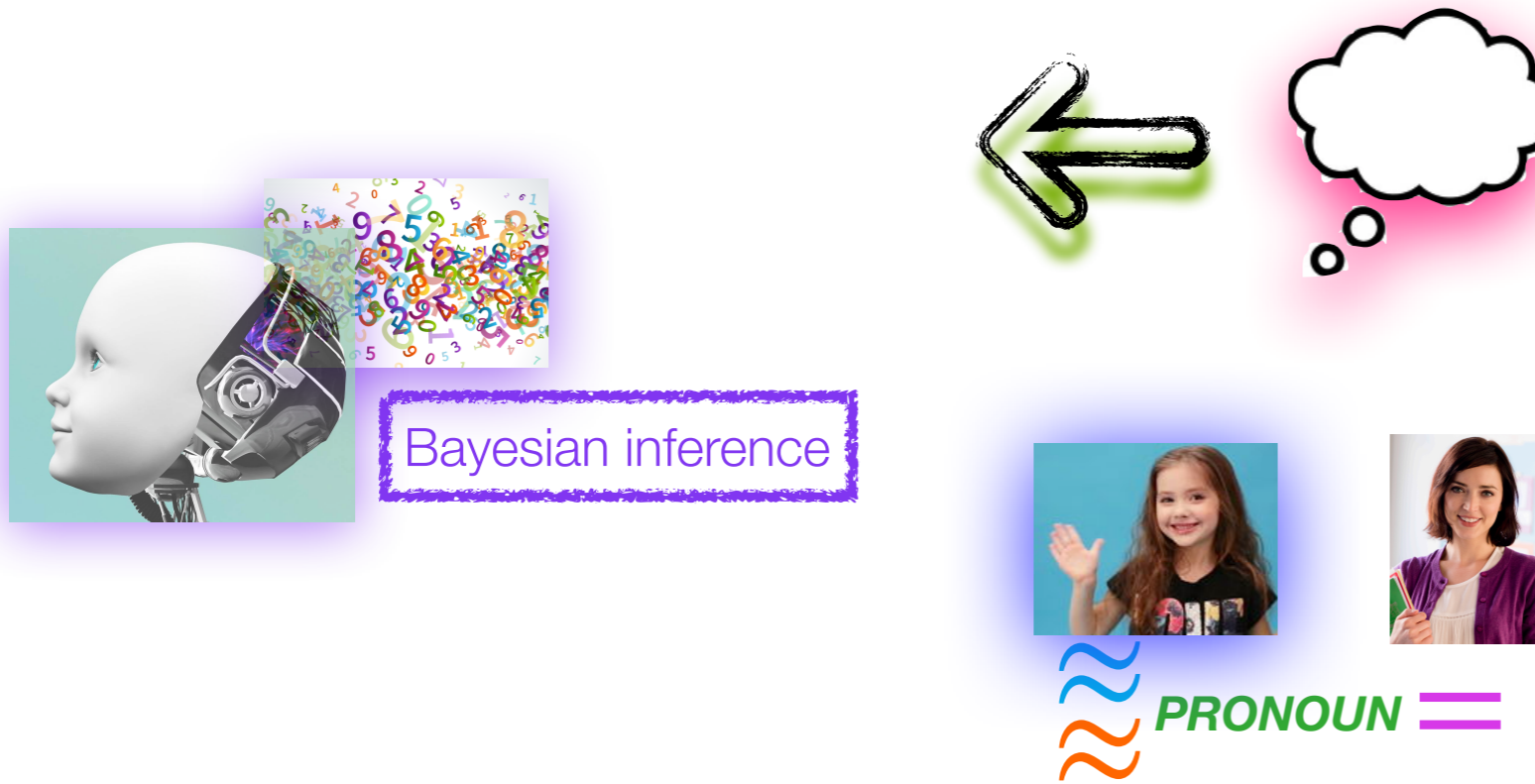
interpretation context

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM, CON, MOR} | \alpha_{subj.SG})$$

∅ ella

...given the particular context involving the pronoun's form,

Modeling pronoun interpretation in context



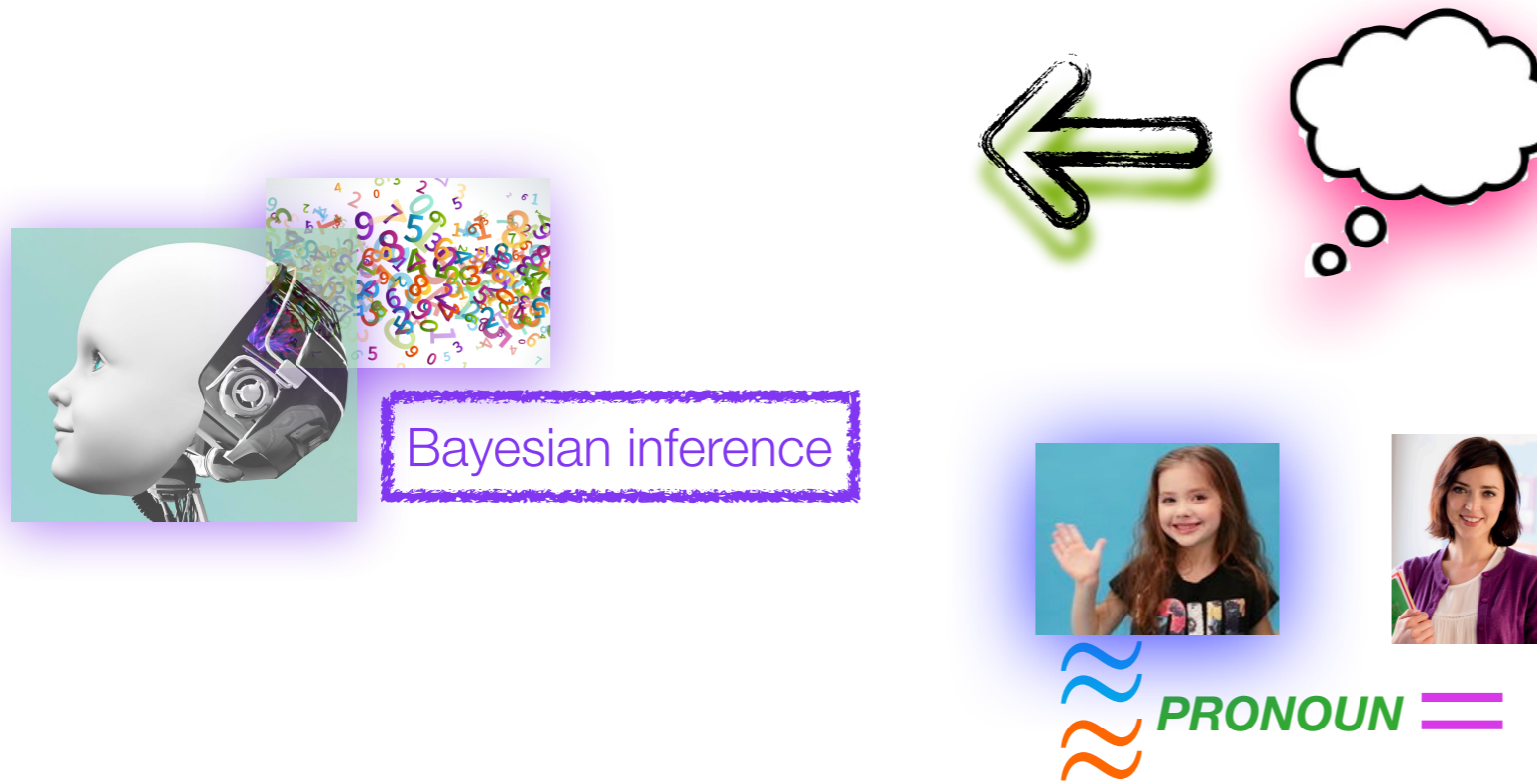
interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM}, \text{CON}, \text{MOR} | \alpha_{subj.SG})$$

y después porque

...given the particular **context** involving
the pronoun's **form**,
the **connective**,

Modeling pronoun interpretation in context

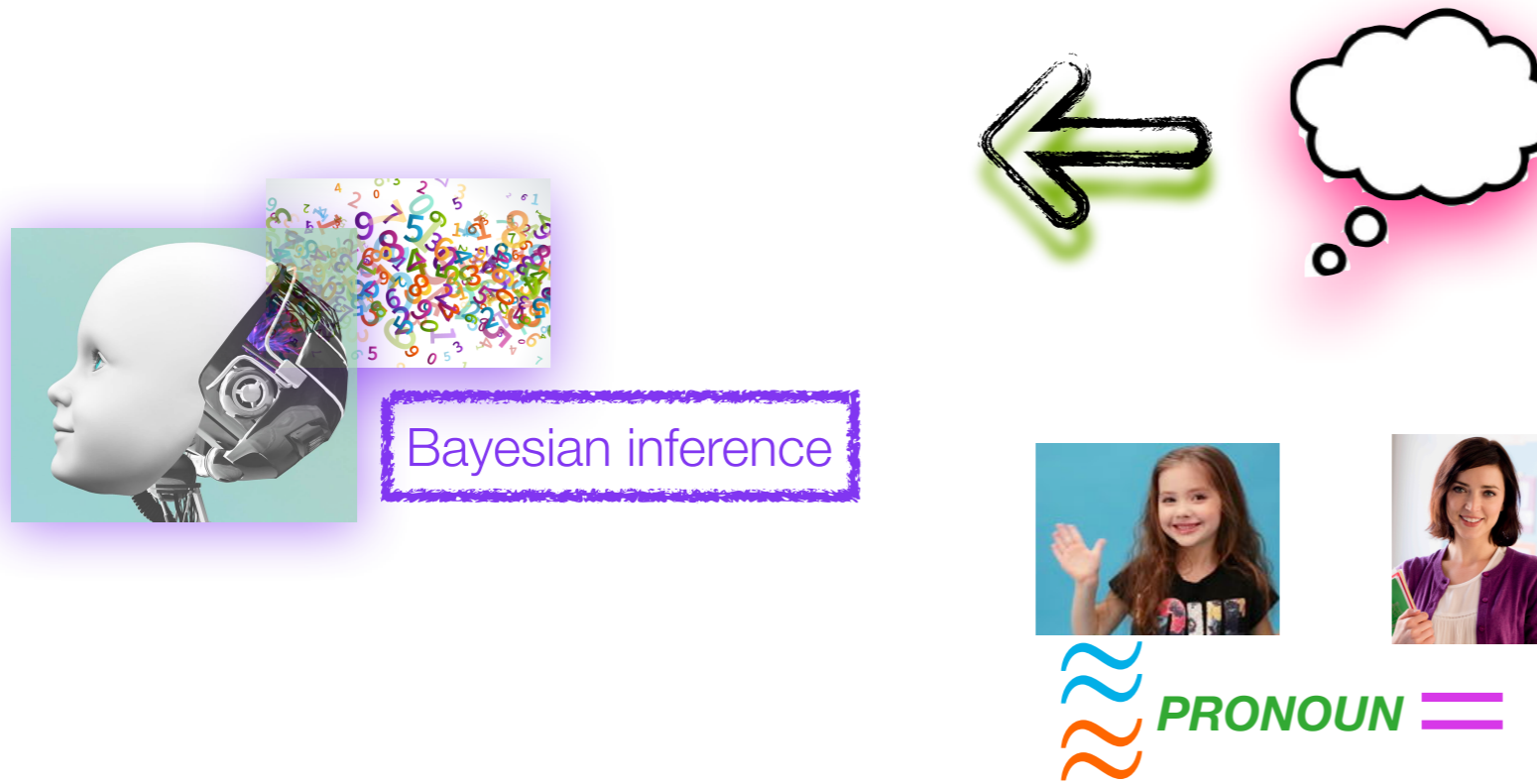


$$\text{interpretation } p(\alpha_{\text{subj.SG}} | \text{FORM, CON, MOR}) \propto p(\alpha_{\text{subj.SG}}) * p(\text{FORM, CON, MOR} | \alpha_{\text{subj.SG}})$$

sg pl

...given the particular **context** involving
the pronoun's **form**,
the **connective**,
and the **agreement morphology**.

Modeling pronoun interpretation in context

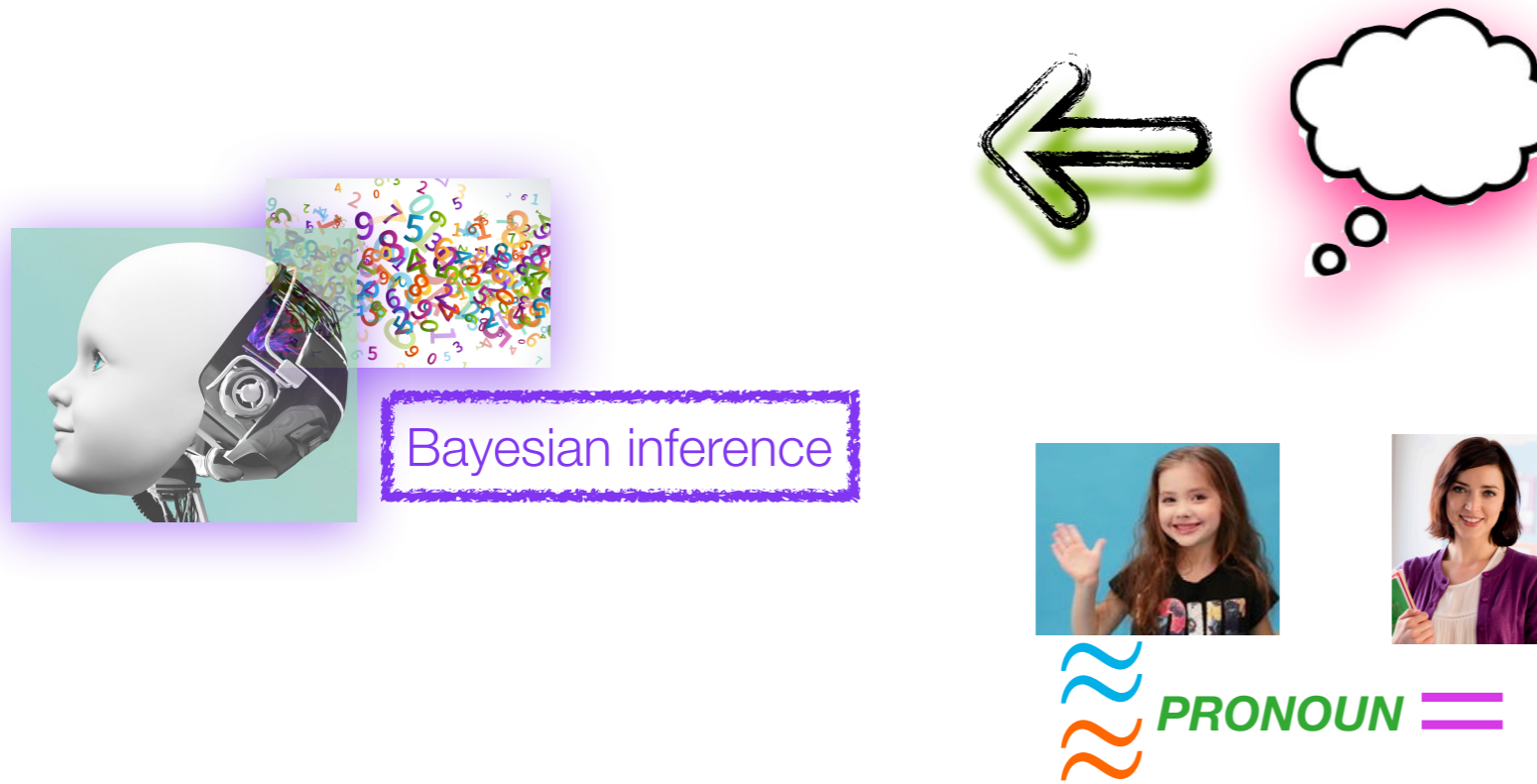


$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM, CON, MOR} | \alpha_{subj.SG})$$

interpretation context

This is proportional to the **prior probability** of that interpretation irrespective of this particular context...

Modeling pronoun interpretation in context

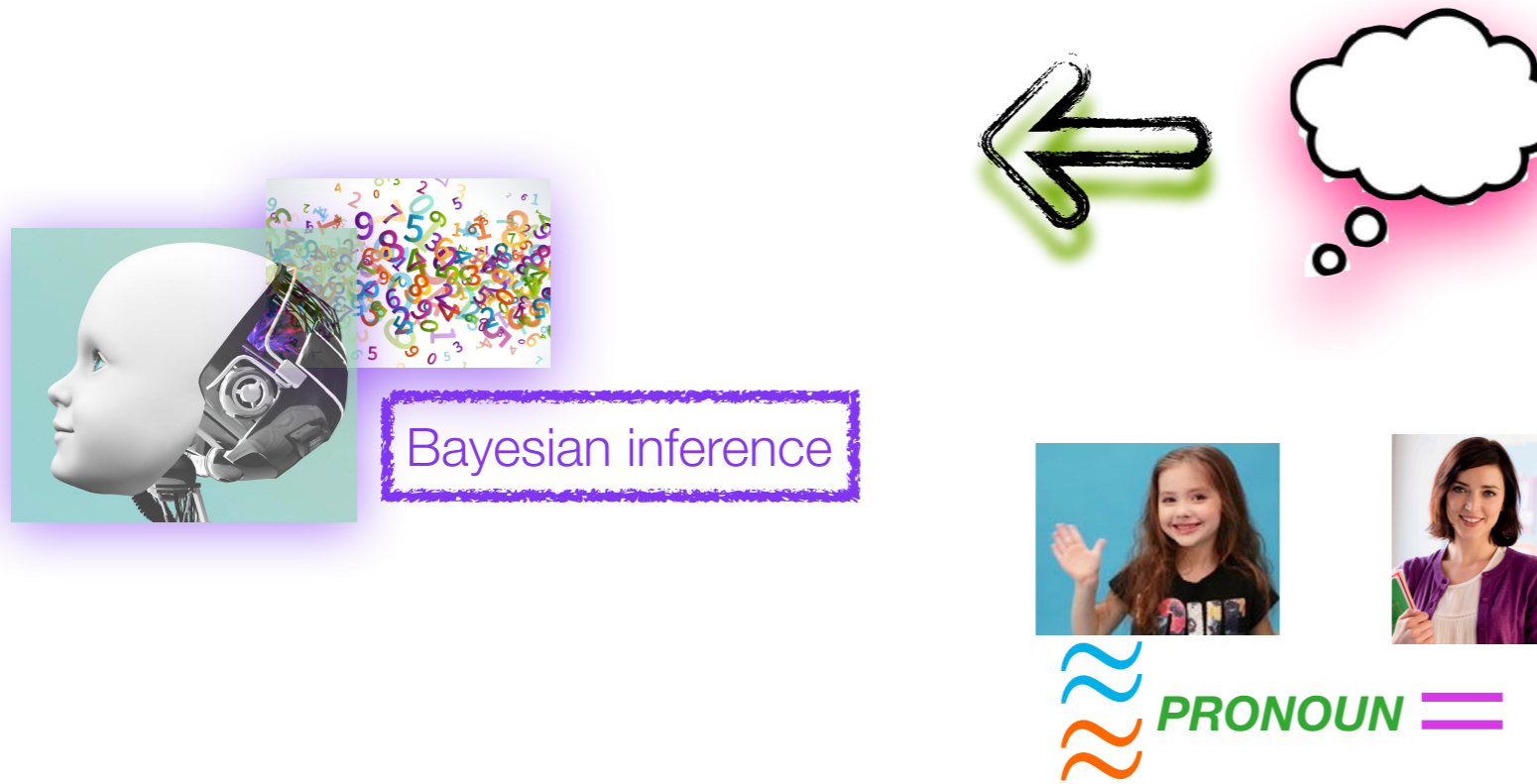


$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM}, \text{CON}, \text{MOR} | \alpha_{subj.SG})$$

interpretation context

...multiplied by the **likelihood** of these context values, given this kind of interpretation (a singular subject).

Modeling pronoun interpretation in context

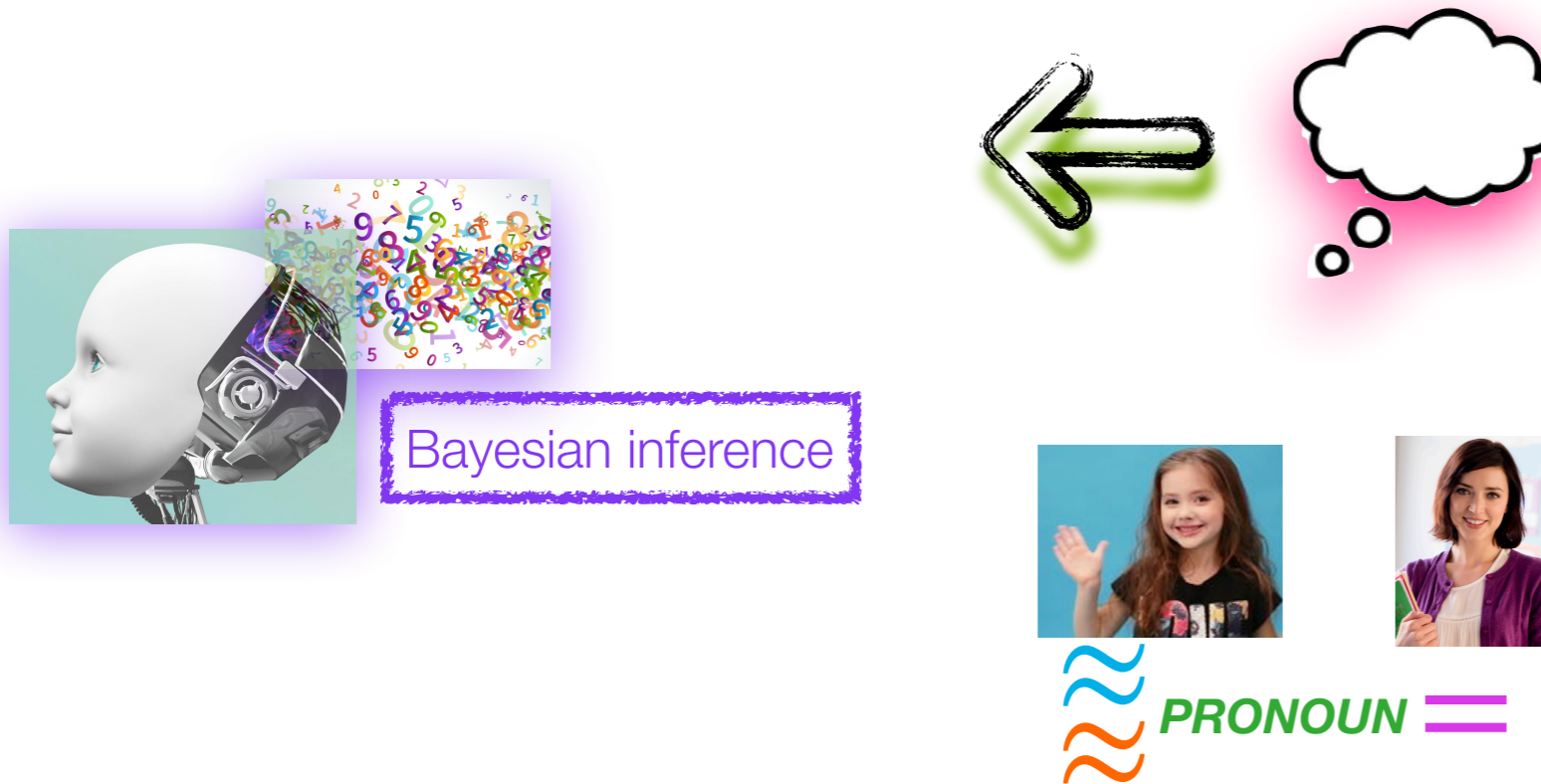


$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

Here, we assume these context values are independent, so we can calculate the likelihood this way.

(For example, the probability of a particular pronoun form, given a certain interpretation, is assumed to be independent from the probability of a particular connective, given a certain interpretation.)

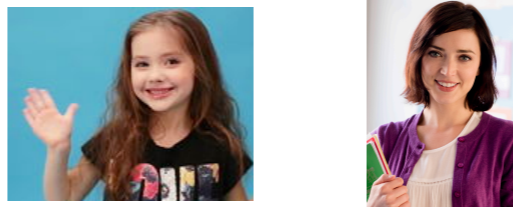
Modeling pronoun interpretation in context



$$\begin{array}{c} \text{interpretation} \\ p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * \\ \text{context} \\ p(\text{CON} | \alpha_{subj.SG}) * \\ p(\text{MOR} | \alpha_{subj.SG}) \end{array}$$

This is the baseline model, which has **accurate representations** of information and **accurately deploys** those representations in the moment.

Modeling pronoun interpretation in context



PRONOUN =



interpretation

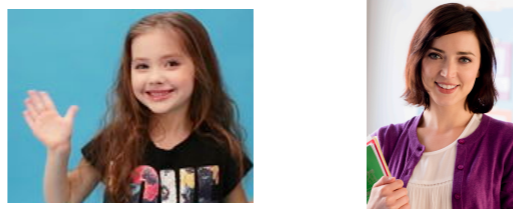
context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



What about a modeled listener who has **inaccurate representations**? This could involve inaccurately representing the prior or the likelihood information, or both.

Modeling pronoun interpretation in context



PRONOUN =

interpretation

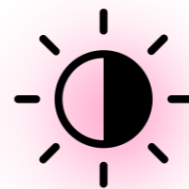
context

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



~~inaccurate representations~~

We implement this as a softmax on the true probability (prior or likelihood), with contrast parameter σ .

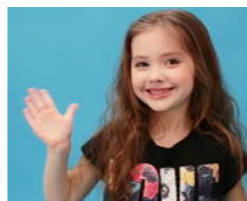


$$e^{\sigma * \ln(\text{probability})} = \text{probability}^{\sigma}$$

Modeling pronoun interpretation in context



Bayesian inference



PRONOUN =



interpretation

context

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



~~inaccurate representations~~

probability^σ

About σ : ☀️



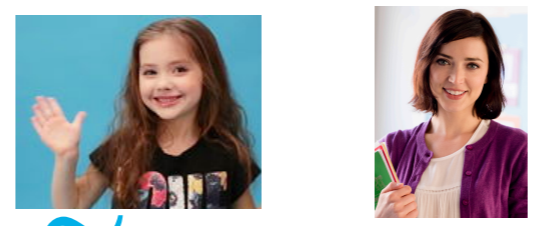
$\sigma < 1$: probability differences are smoothed away.

$\sigma = 1$: probabilities remain accurate.

$\sigma > 1$: probability differences are sharpened.

0.324 vs. 0.676

Modeling pronoun interpretation in context



PRONOUN =

interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

~~inaccurate representations~~ *probability^σ*



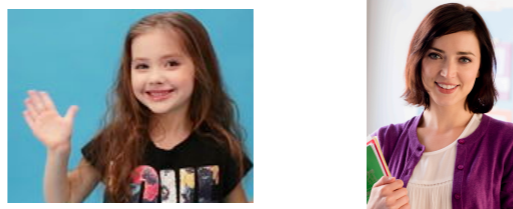
About σ : ☀️ ■ ■

$\sigma = 0.5$: probability differences are smoothed away. 0.409 vs. 0.591

$\sigma = 1$: probabilities remain accurate. 0.324 vs. 0.676

$\sigma > 1$: probability differences are sharpened.

Modeling pronoun interpretation in context



PRONOUN =

interpretation

context

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



~~inaccurate representations~~

probability^σ

About σ : ☀️



$\sigma < 1$: probability differences are smoothed away.

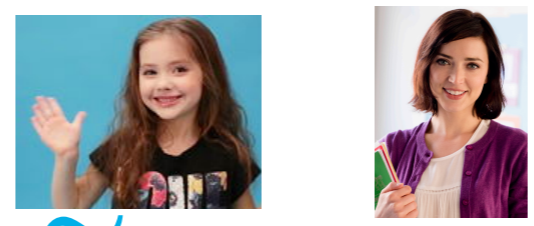
$\sigma = 1$: probabilities remain accurate.

$\sigma = 2$: probability differences are sharpened.

0.324 vs. 0.676

0.187 vs. 0.813

Modeling pronoun interpretation in context



PRONOUN =

probability^σ

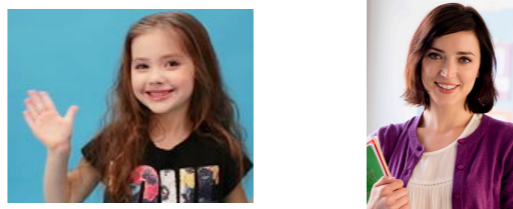
$$\begin{aligned}
 & p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \times \\
 & * p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \times \\
 & * p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \times \\
 & * p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}} \times
 \end{aligned}$$

inaccurate representations



One σ for each information type:
 (in the prior) σ_{α}
 (in the likelihood) σ_{form} , σ_{con} , σ_{mor}

Modeling pronoun interpretation in context



PRONOUN =

probability^σ

$$\begin{aligned}
 & p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \times \\
 & * p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \times \\
 & * p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \times \\
 & * p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}} \times
 \end{aligned}$$

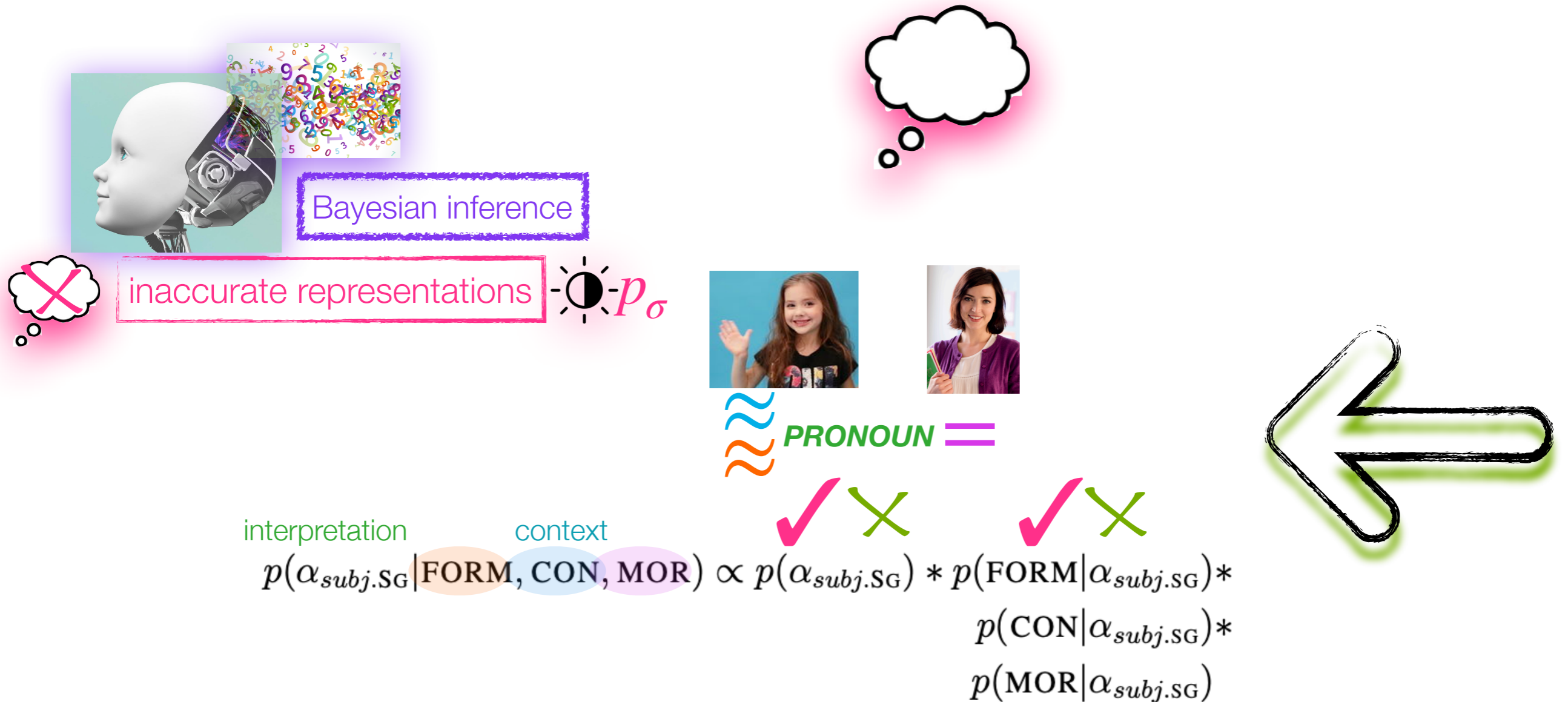
inaccurate representations



We allow $0.01 \leq \sigma \leq 4$, and see which σ value combinations best predict **child** and **adult** pronoun interpretation behavior.



Modeling pronoun interpretation in context



What about a modeled listener who has **inaccurate deployment** of information in the representations? This could involve inaccurately deploying the prior or the likelihood information, or both.

Modeling pronoun interpretation in context



Bayesian inference



inaccurate representations p_σ

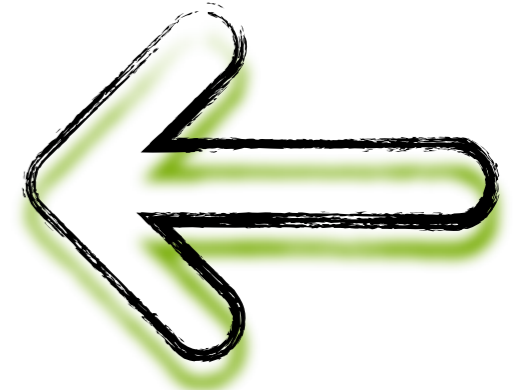


PRONOUN =

$$\begin{array}{c}
 \text{interpretation} \\
 p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * \\
 \text{context} \\
 p(\text{CON} | \alpha_{subj.SG}) * \\
 p(\text{MOR} | \alpha_{subj.SG})
 \end{array}$$

✓ ✗ ✓ ✗

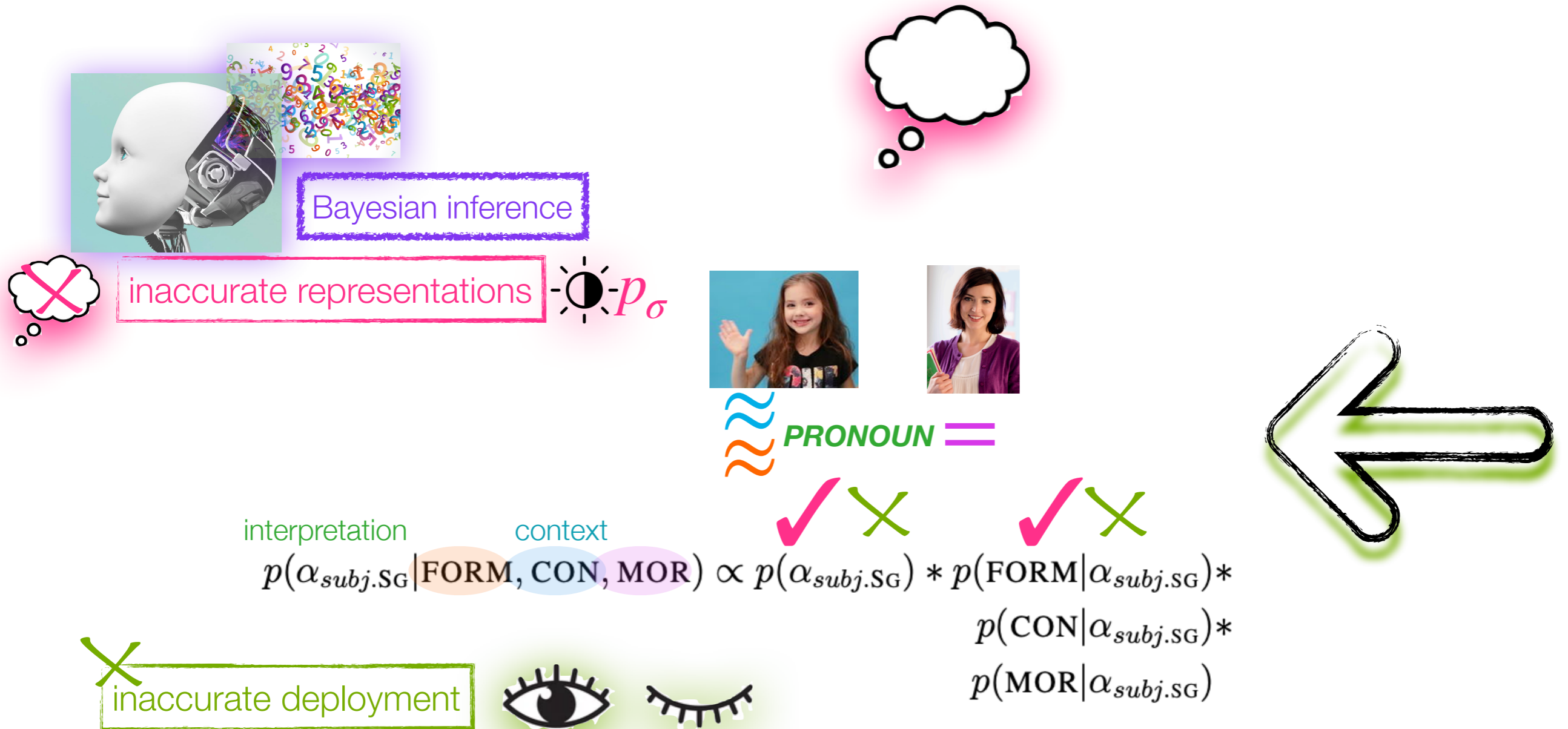
inaccurate deployment



We implement this as ignoring that information. So, for any piece of information, the modeled listener either pays attention to it (and so uses it) or ignores it in the moment.



Modeling pronoun interpretation in context



Not using information means not incorporating it into the inference.

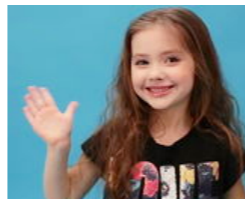
Modeling pronoun interpretation in context



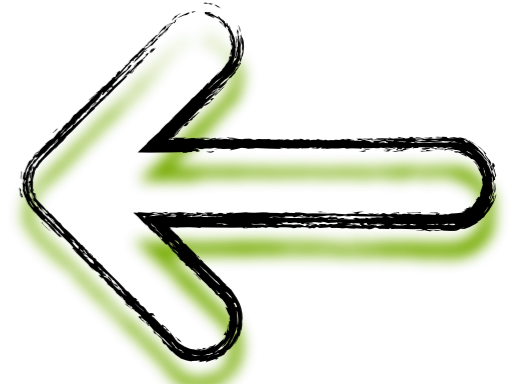
Bayesian inference



inaccurate representations



PRONOUN =



$$p_{\text{UNIF}}^{\text{interpretation}}(\alpha_{\text{num}}, \alpha_{\text{subj?}} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\text{UNIF}) \times$$

$$* p(\text{FORM} | \alpha_{\text{num}}, \alpha_{\text{subj?}})$$

$$* p(\text{CON} | \alpha_{\text{num}}, \alpha_{\text{subj?}})$$

$$* p(\text{MOR} | \alpha_{\text{num}}, \alpha_{\text{subj?}})$$

inaccurate deployment



Not using the prior means relying on a uniform prior.

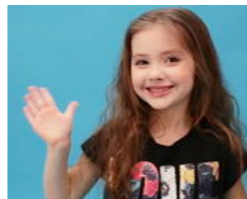
Modeling pronoun interpretation in context



Bayesian inference



inaccurate representations p_{σ}



PRONOUN =



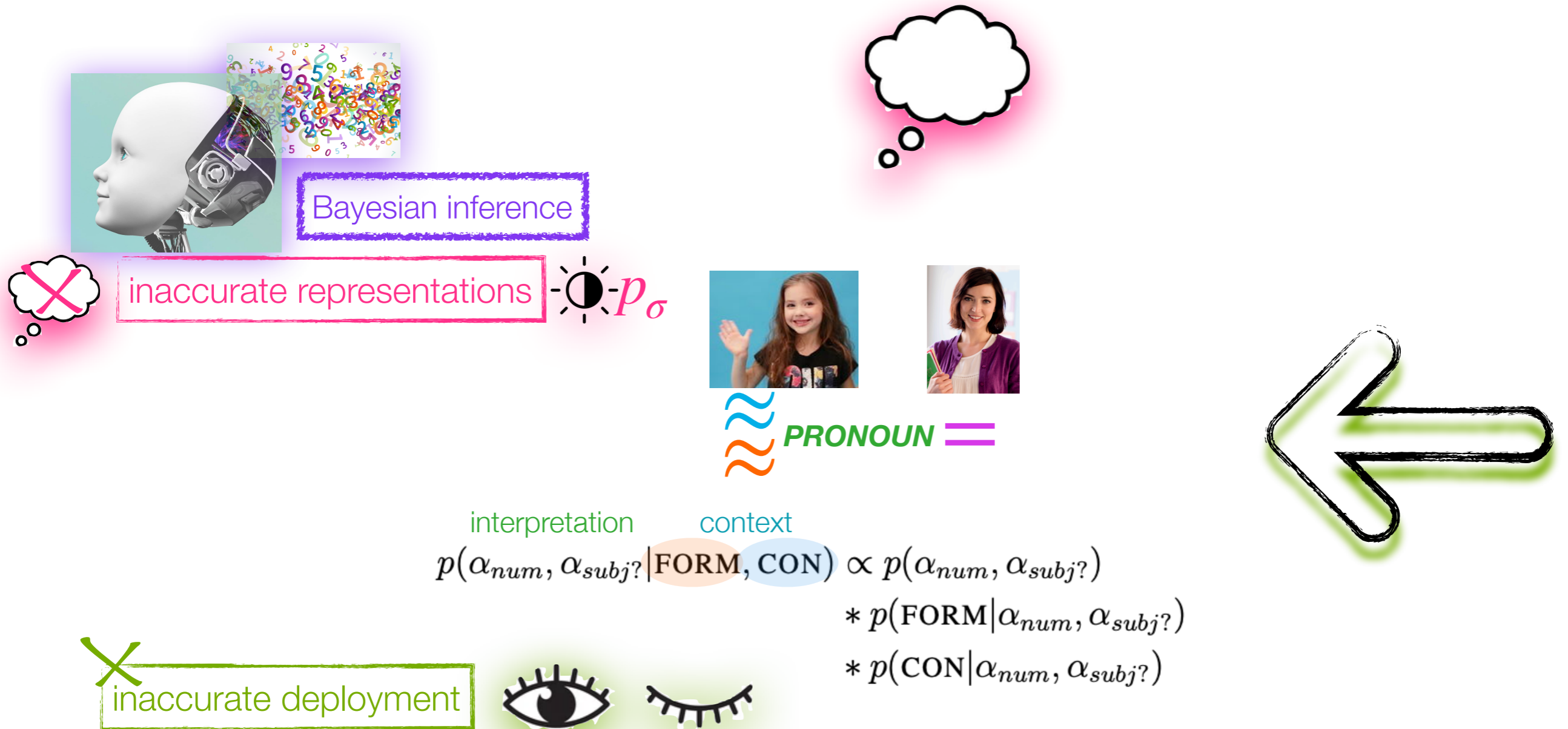
$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

inaccurate deployment



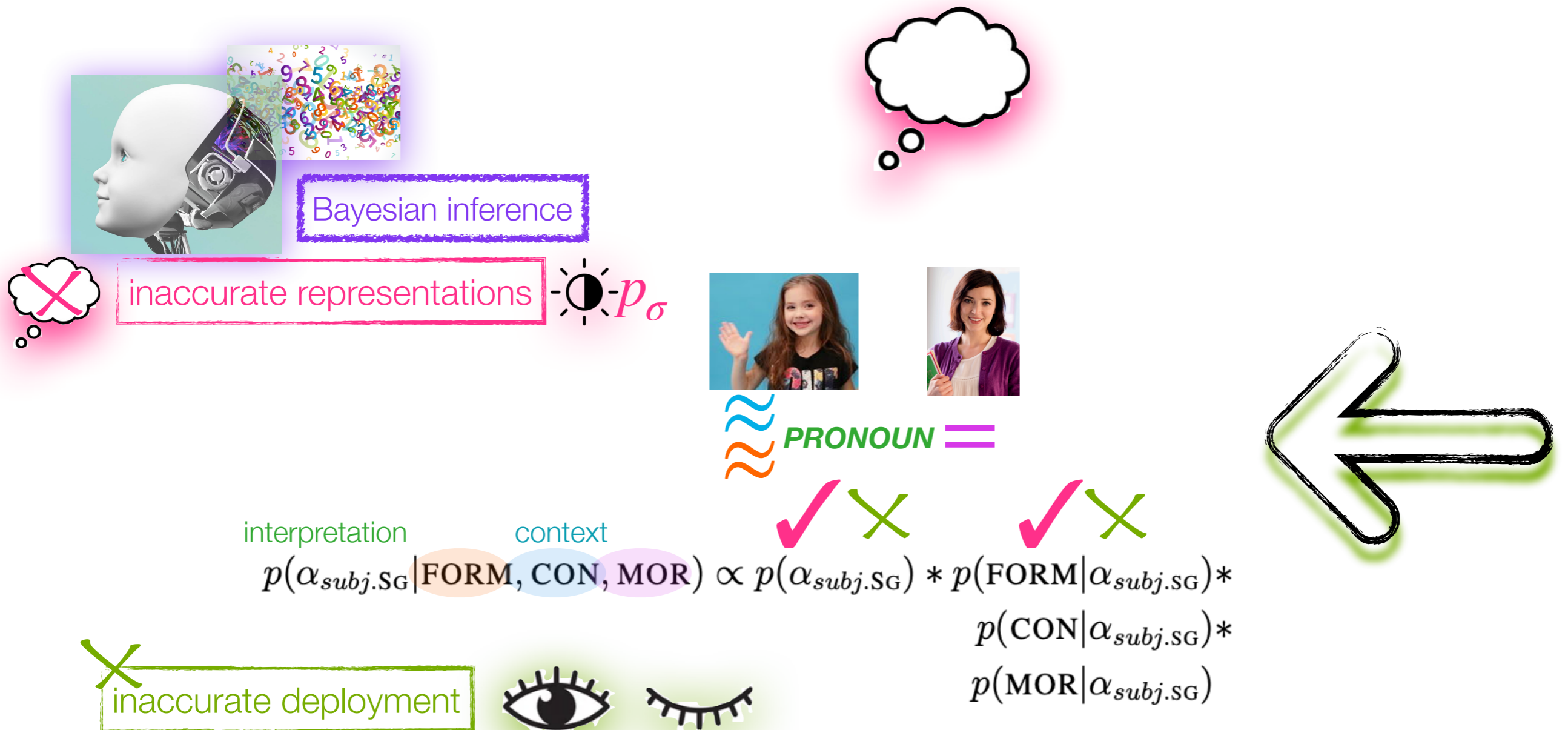
Not using likelihood information for a cue means **not using that cue's information**. For example, ignoring **morphology** information means not using the morphology likelihood.

Modeling pronoun interpretation in context



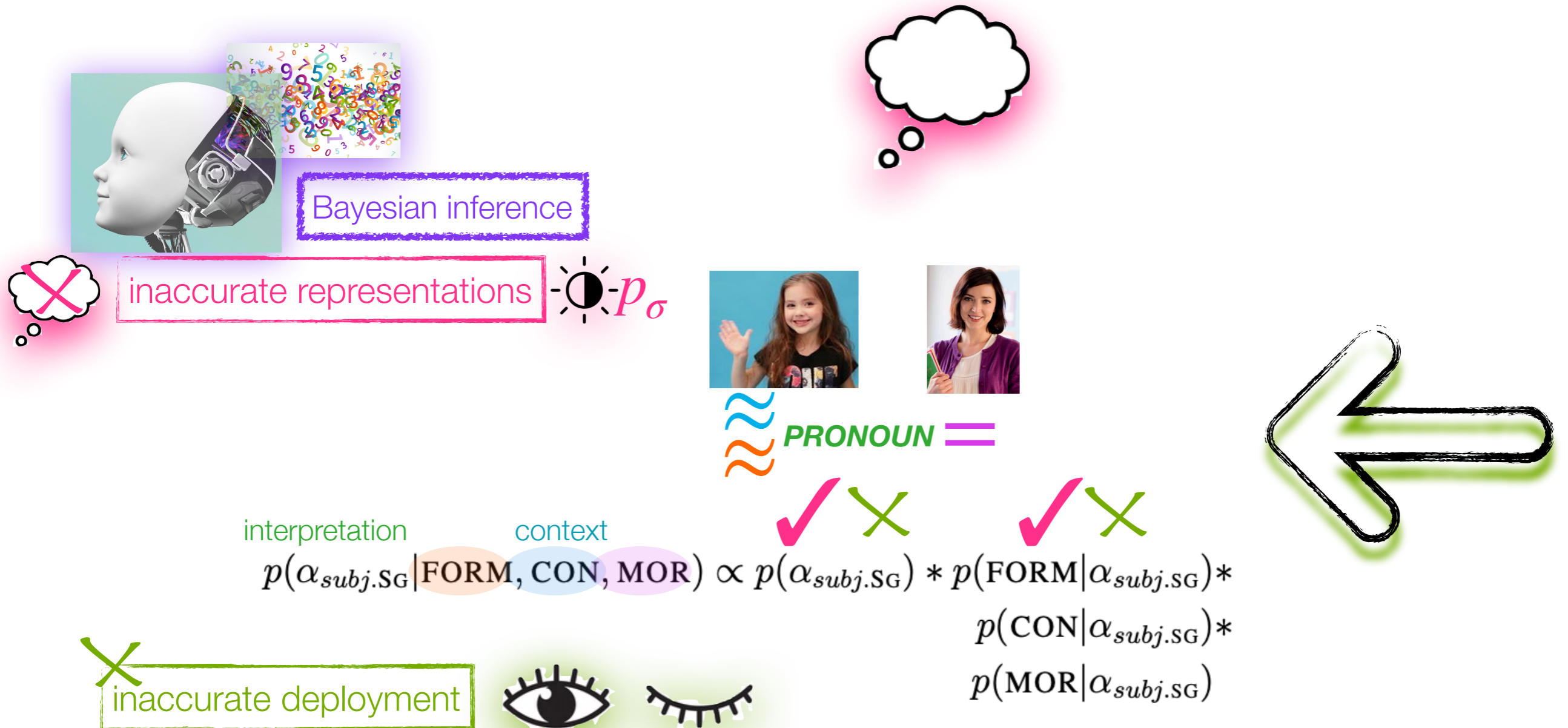
Not using likelihood information for a cue means **not using that cue's information**. For example, ignoring **morphology** information means not using the morphology likelihood.

Modeling pronoun interpretation in context



For any information, the modeled listener could use or not use it in the moment. Use parameter β determines whether a particular information type is used.

Modeling pronoun interpretation in context



Each of the four information types has its own β :

(prior) β_{α}

(likelihood) β_{form} , β_{con} , β_{mor}

Modeling pronoun interpretation in context



Bayesian inference

inaccurate representations



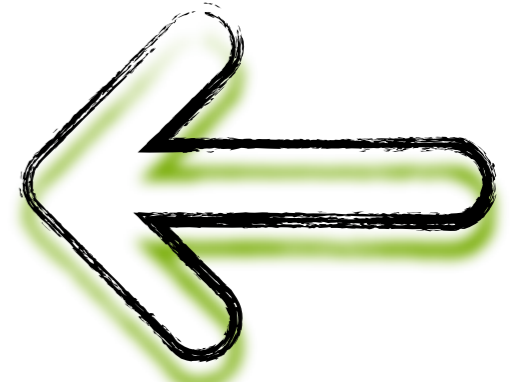
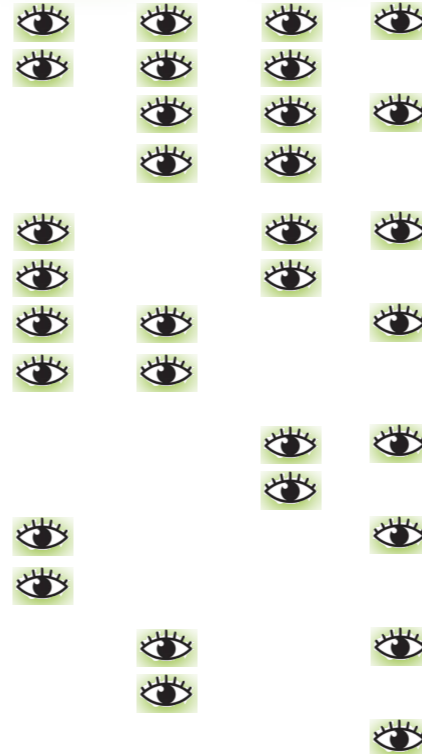
inaccurate deployment



PRONOUN =



β_{form} β_{con} β_{mor} β_{α}



$$\begin{aligned}
 & \text{interpretation} \quad \text{context} \\
 & p_{\beta}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) = \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})
 \end{aligned}$$

This yields 16 possible use combinations for any particular moment, implemented with a mixture model p_{β} .

Modeling pronoun interpretation in context



Bayesian inference

inaccurate representations



p_{σ}

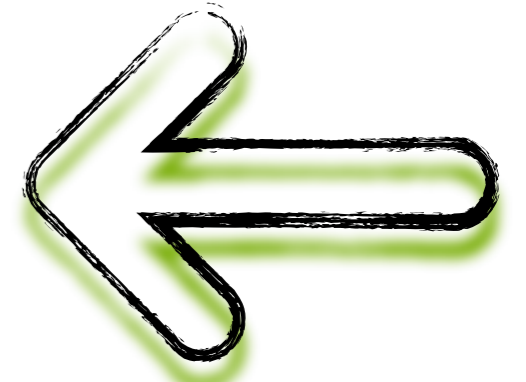
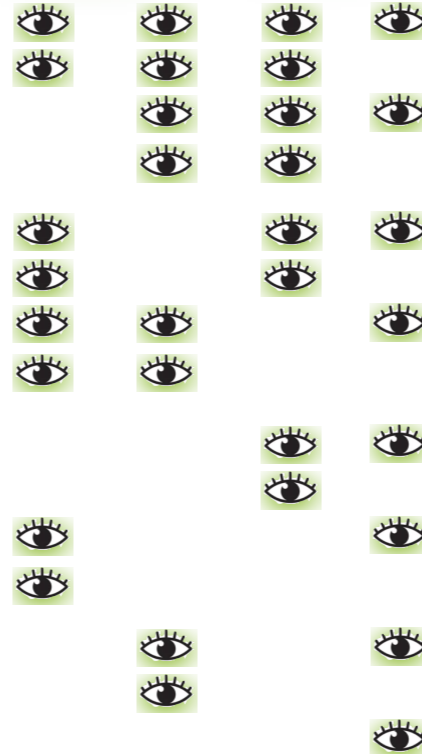
inaccurate deployment



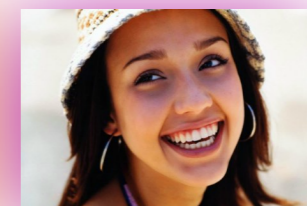
PRONOUN =



β_{form} β_{con} β_{mor} β_{α}

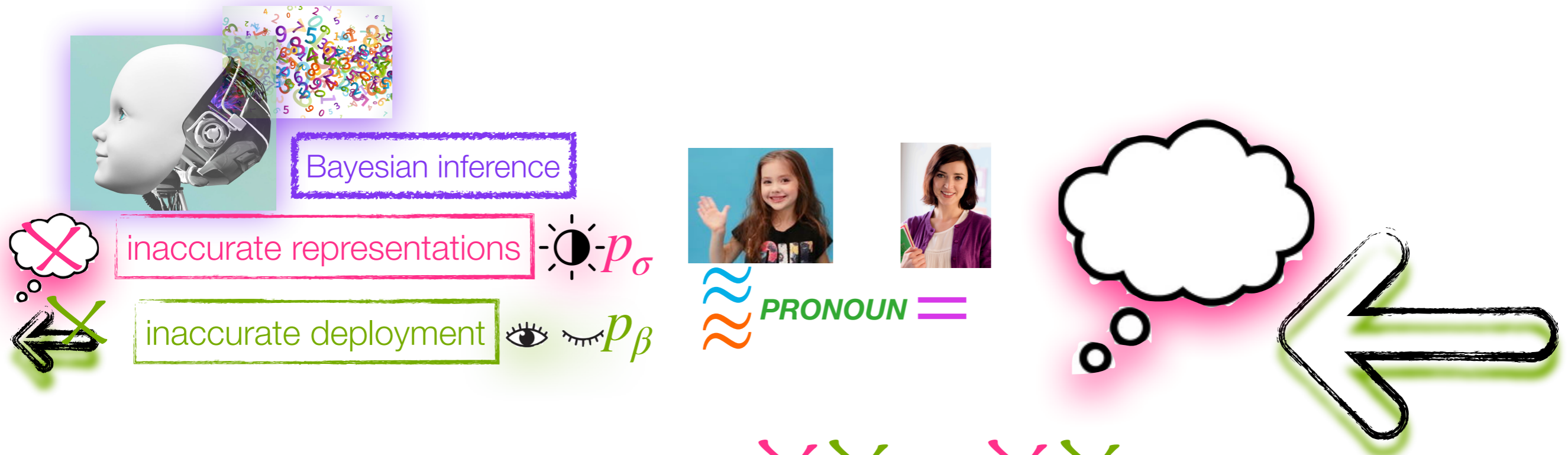


We allow $0 \leq \beta \leq 1$, and see which β value combinations best predict **child** and **adult** pronoun interpretation behavior.



$$\begin{aligned}
 & \text{interpretation} \quad \text{context} \\
 & p_{\beta}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) = \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, } \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})
 \end{aligned}$$

Modeling pronoun interpretation in context

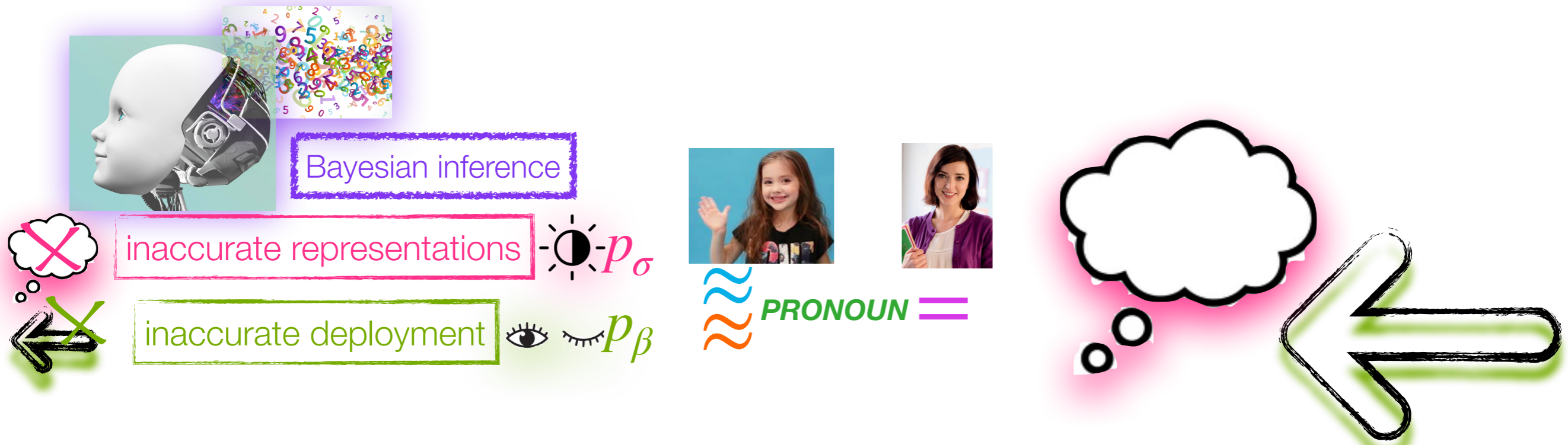


interpretation context

$$p(\alpha_{subj.SG} | \text{FORM}, \text{CON}, \text{MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

What about a modeled listener who has both **inaccurate representations** of information and **inaccurate deployment** of those representations?

Modeling pronoun interpretation in context



inaccurate both

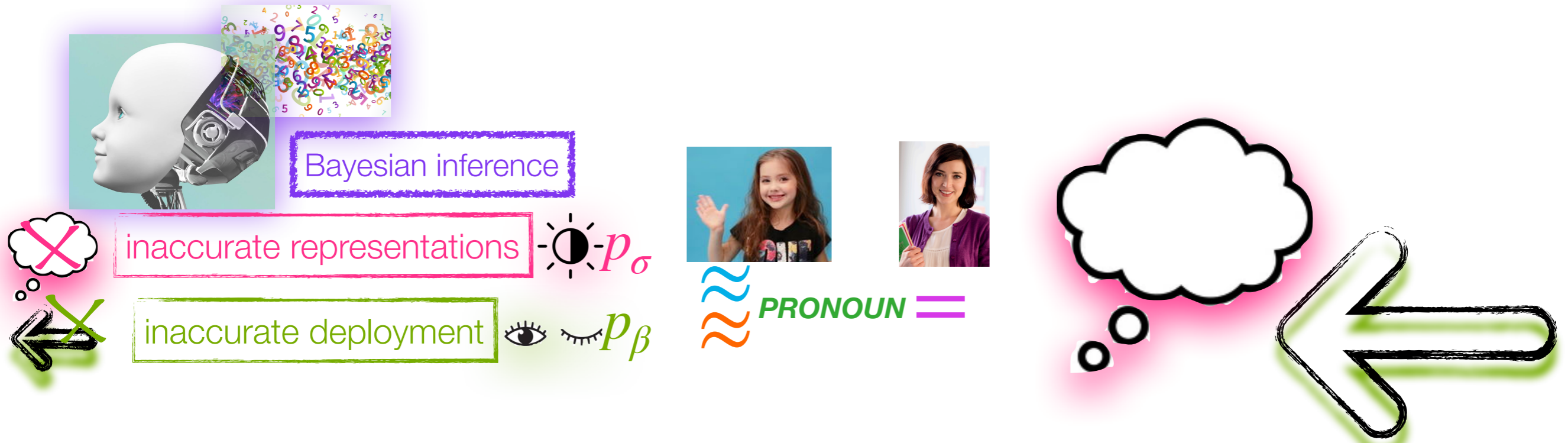
$$\begin{aligned}
 p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) &\propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \\
 &* p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \\
 &* p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \\
 &* p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}
 \end{aligned}$$

$$P_{\sigma, \beta}$$

$$\begin{aligned}
 p_{\beta}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) = & \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})
 \end{aligned}$$

We implement this as a combination of the previous two modeled listeners, including σ values for inaccurate representations and β values for inaccurate deployment.

Modeling pronoun interpretation in context



inaccurate both

$$p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \cdot p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \cdot p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \cdot p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}$$

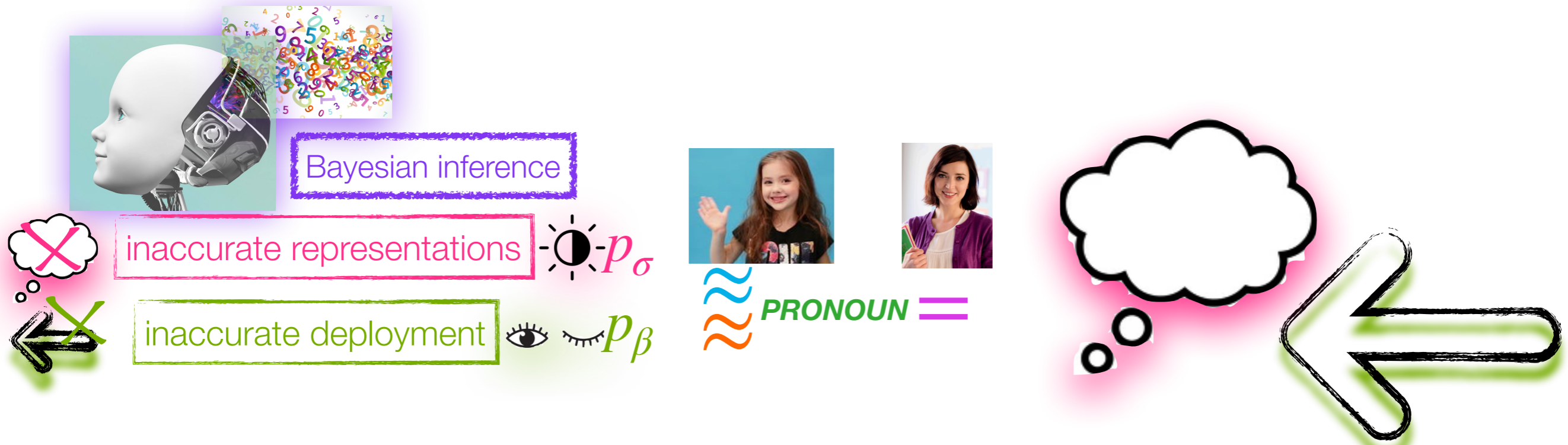
$P_{\sigma, \beta}$

P_{β}

$$p_{\beta}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) = (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})$$

Each term of p_{β} has σ values for all information types that are used.

Modeling pronoun interpretation in context



inaccurate both

$$\begin{aligned}
 p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) &\propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} \\
 &* p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} \\
 &* p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} \\
 &* p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}
 \end{aligned}$$

$$p_{\sigma, \beta}$$

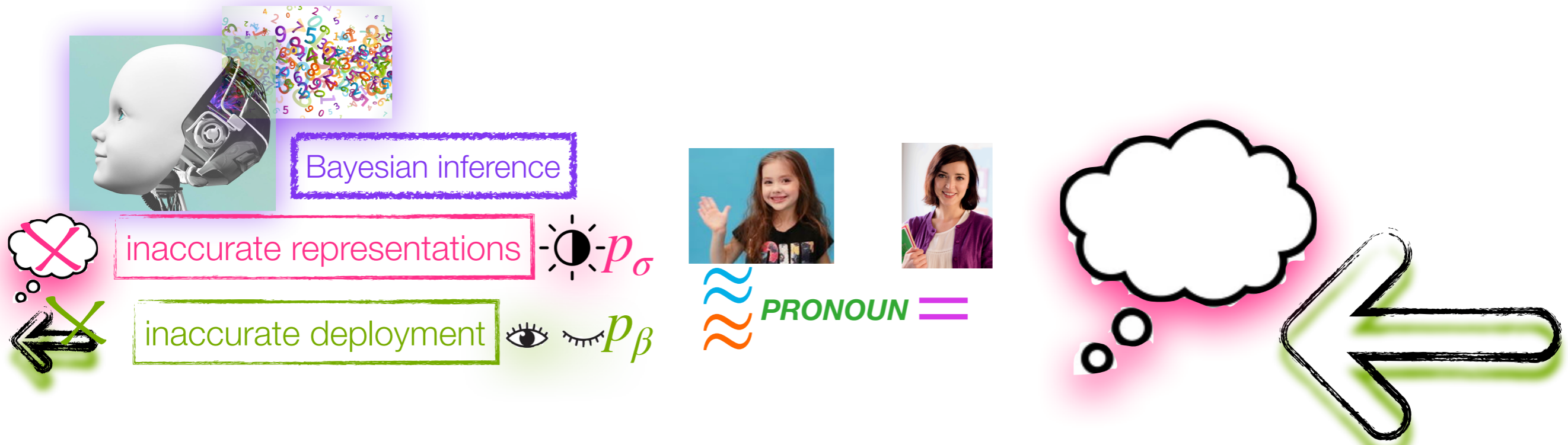
$$\begin{aligned}
 p_{\beta}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) = & \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + \\
 & (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})
 \end{aligned}$$

So, $p_{\sigma, \beta}$ has 8 parameter values:

(in the prior) $\sigma_{\alpha}, \beta_{\alpha}$

(in the likelihood) $\sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_{form}, \beta_{con}, \beta_{mor}$

Modeling pronoun interpretation in context



inaccurate both

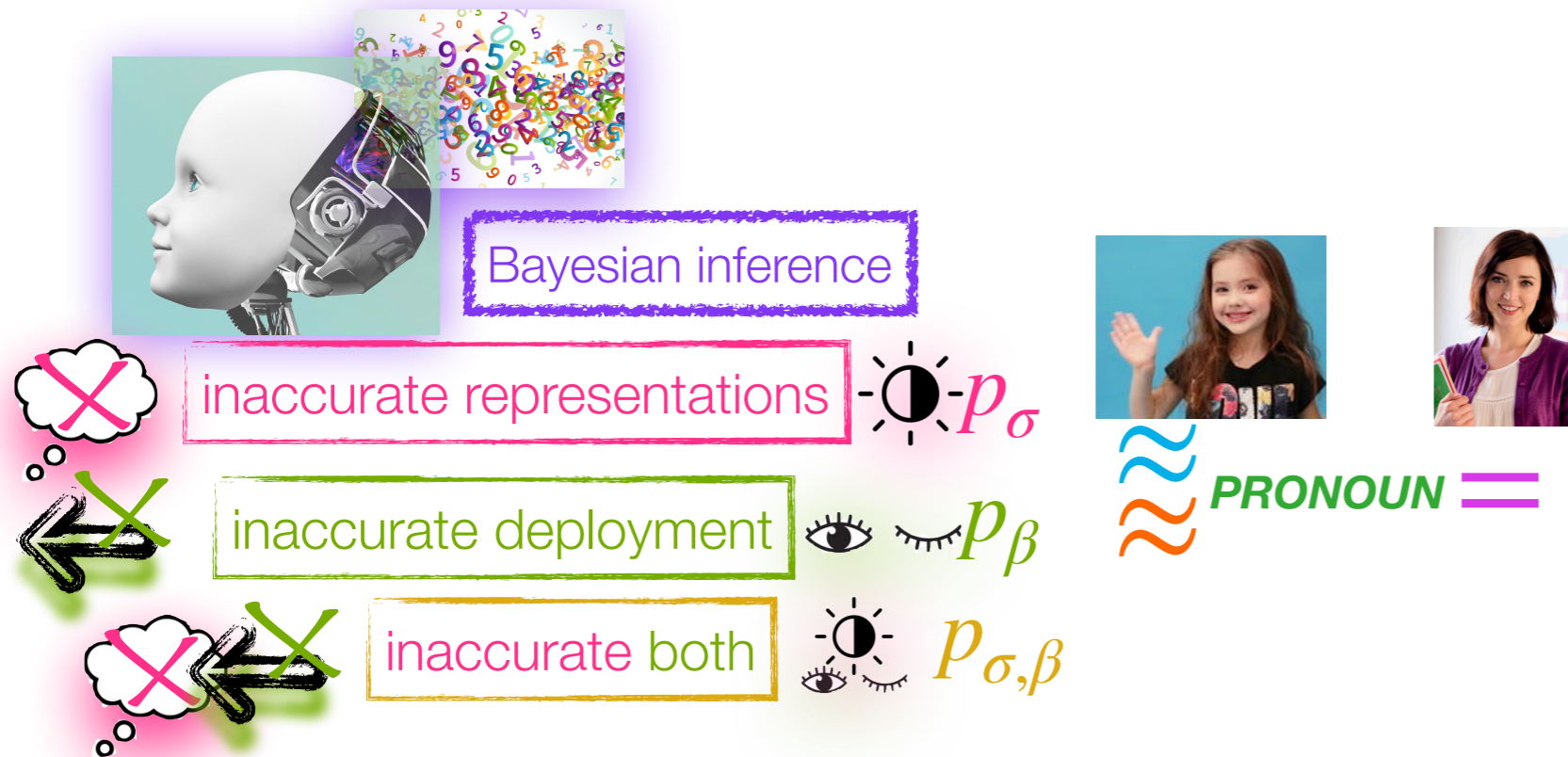
$$p_{\sigma}(\alpha_{num}, \alpha_{subj?} | \text{FORM, CON, MOR}) \propto p(\alpha_{num}, \alpha_{subj?})^{\sigma_{\alpha}} * p(\text{FORM} | \alpha_{num}, \alpha_{subj?})^{\sigma_{form}} * p(\text{CON} | \alpha_{num}, \alpha_{subj?})^{\sigma_{con}} * p(\text{MOR} | \alpha_{num}, \alpha_{subj?})^{\sigma_{mor}}$$

We allow $0.01 \leq \sigma \leq 4$ and $0 \leq \beta \leq 1$, and see which σ and β value combinations best predict **child** and **adult** pronoun interpretation behavior.



$$p_{\beta}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) = (\beta_{form})(\beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p(\alpha | \text{CON, MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM, CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(\beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{MOR}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + (\beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{FORM}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(\beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p_{\text{UNIF}}(\alpha | \text{CON}, \alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(\beta_{\alpha}) * p(\alpha_{num}, \alpha_{subj?}) + (1 - \beta_{form})(1 - \beta_{con})(1 - \beta_{mor})(1 - \beta_{\alpha}) * p(\text{UNIF})$$

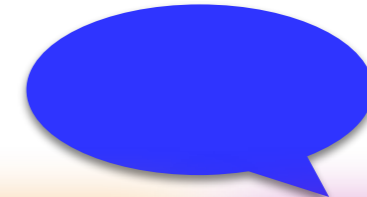
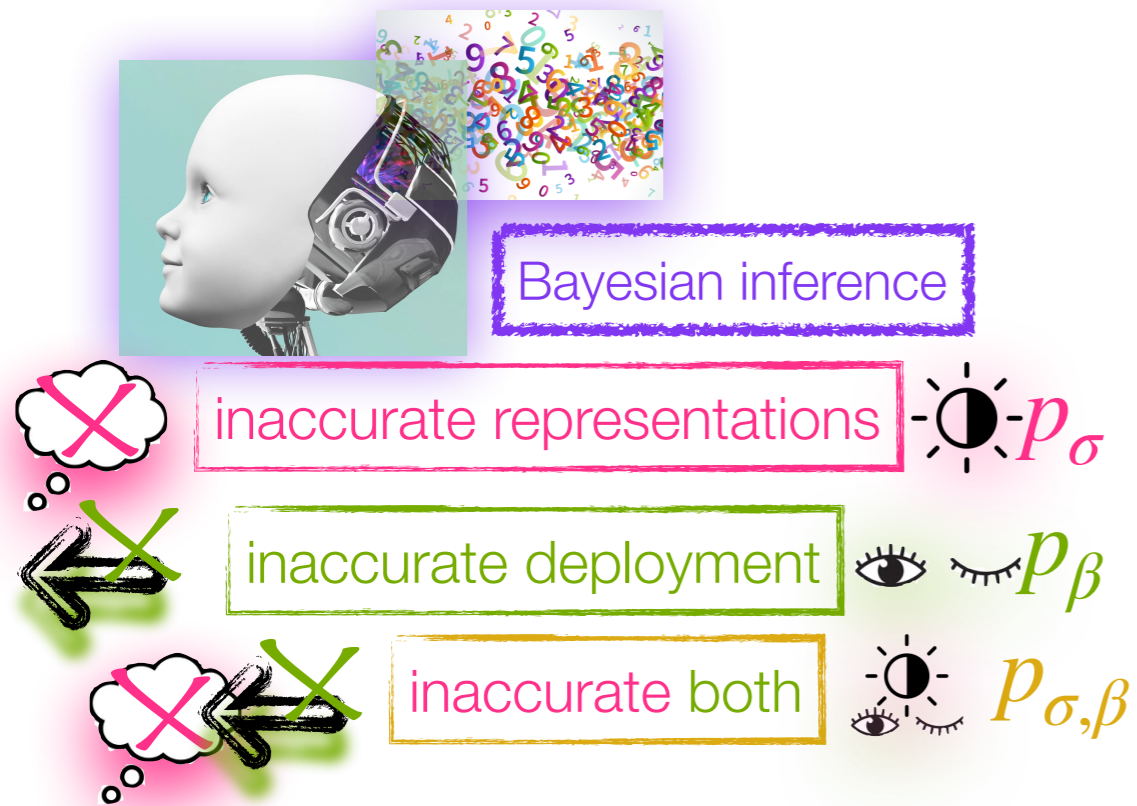
Input to the modeled listener



$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

What **input** is the modeled listener using to represent the various information types?

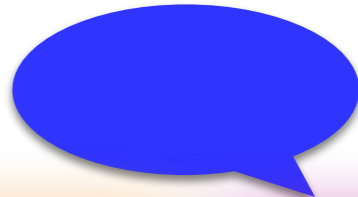
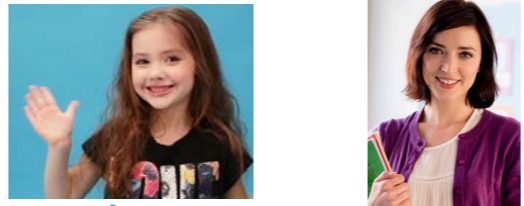
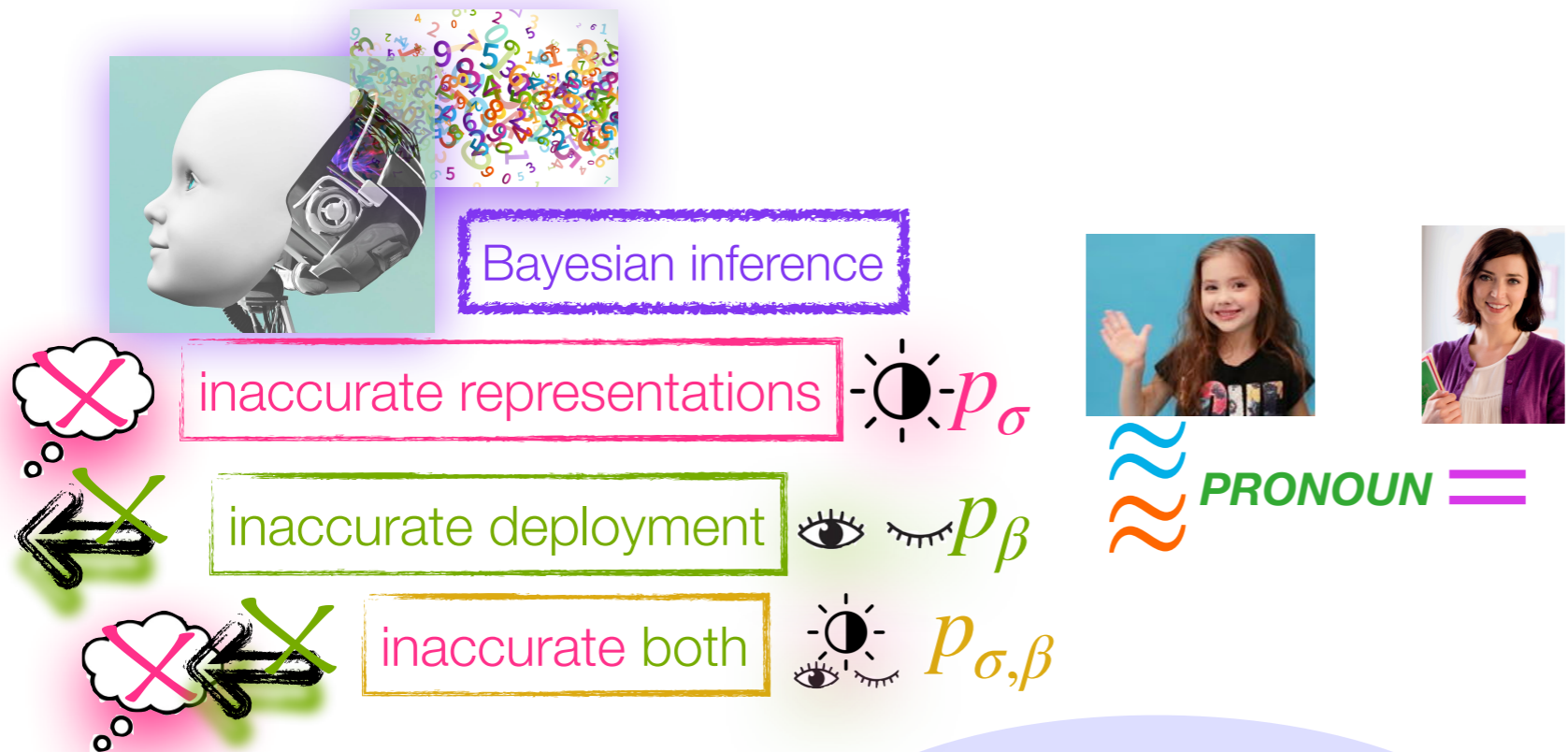
Input to the modeled listener



$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

54,757 utterances of Mexico City spontaneous child-directed speech to children 1;6-5;11 from the Schmitt-Miller corpus (Miller & Schmitt 2012).

Input to the modeled listener

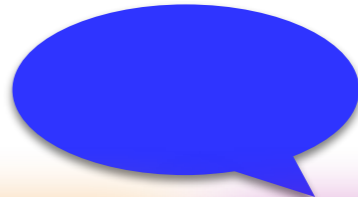
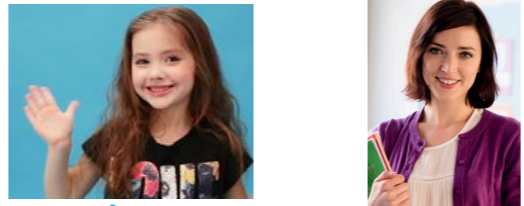
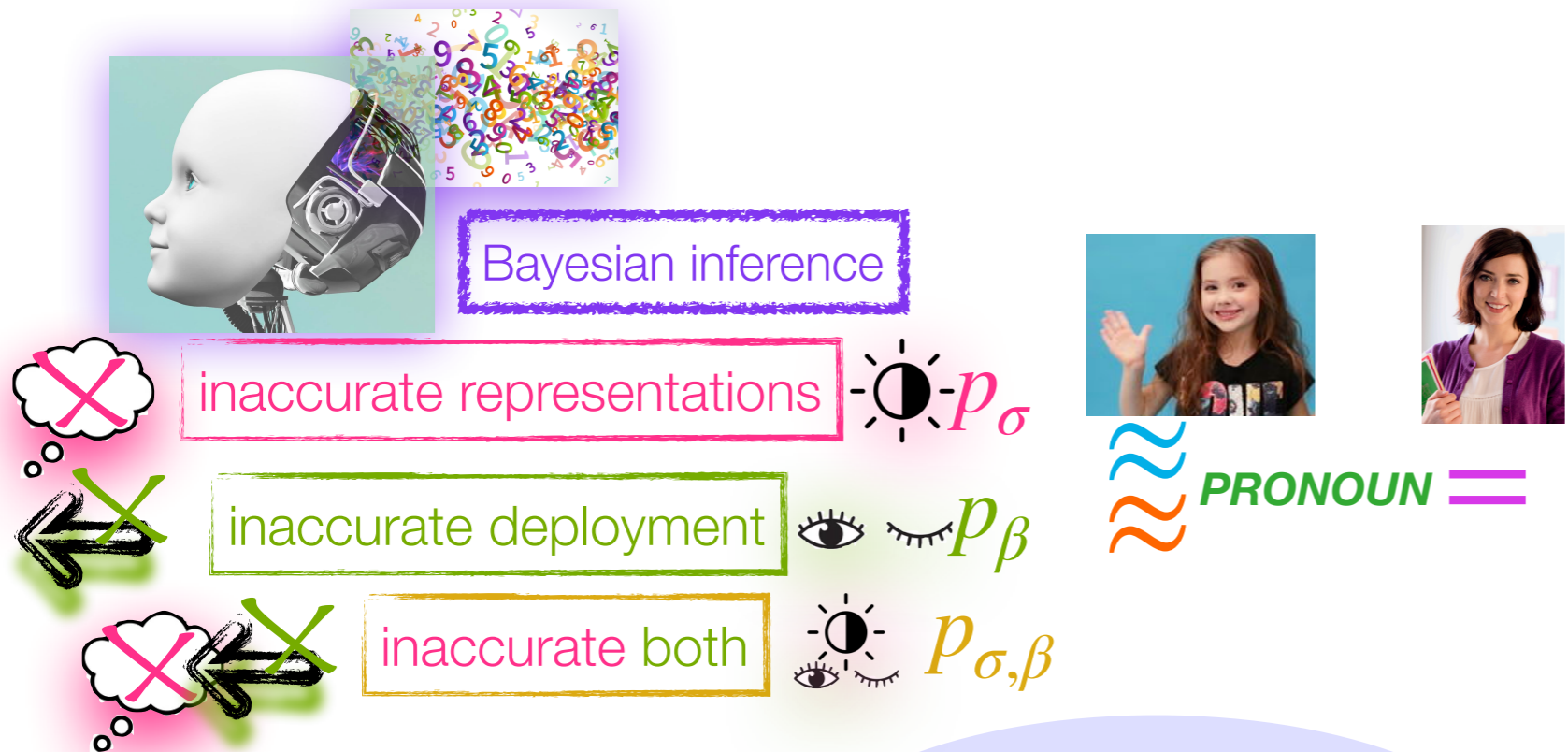


$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
			\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

From this, we estimate the relevant priors and likelihoods.

Input to the modeled listener

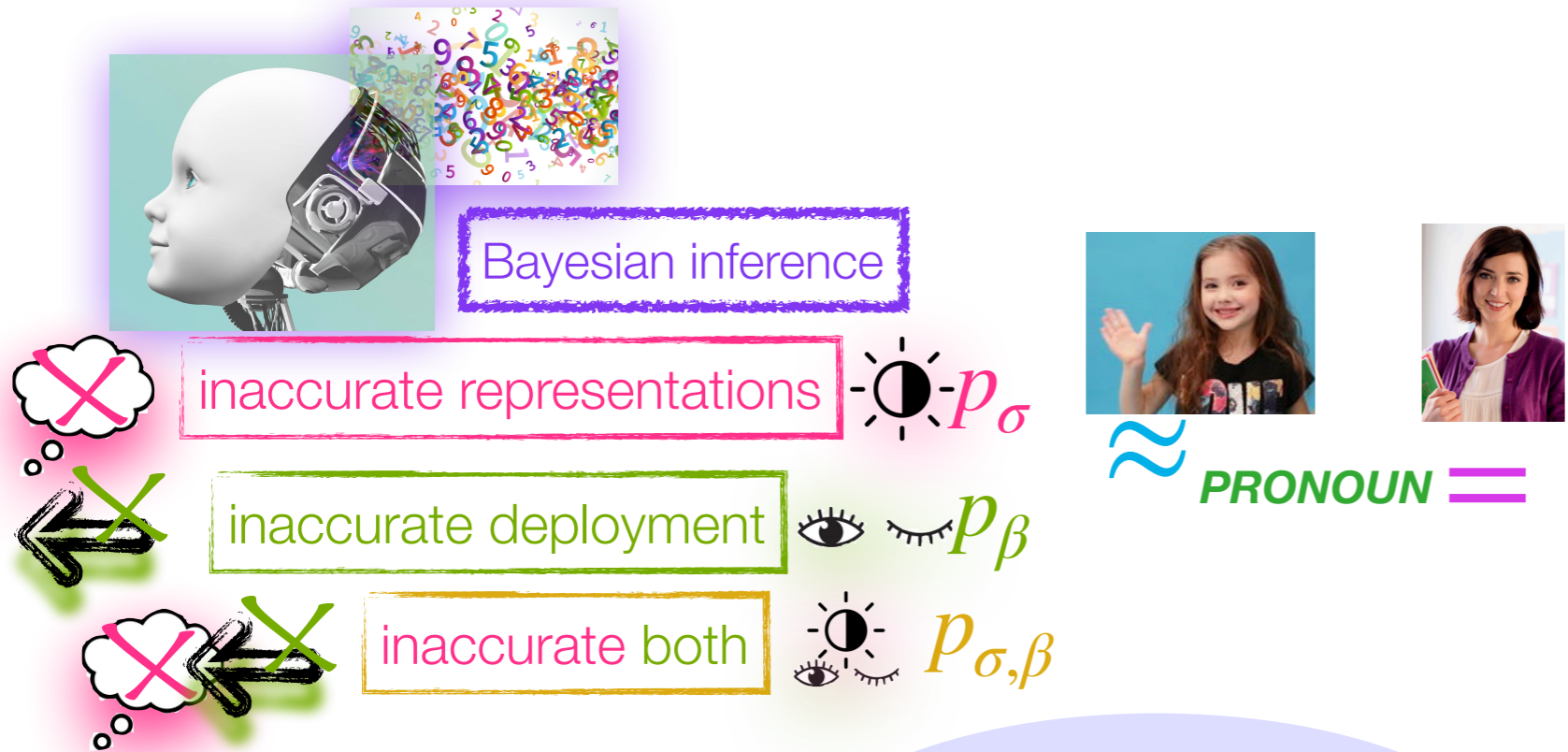


$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
		\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL	
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

Singular antecedents generally occur more often.

Input to the modeled listener



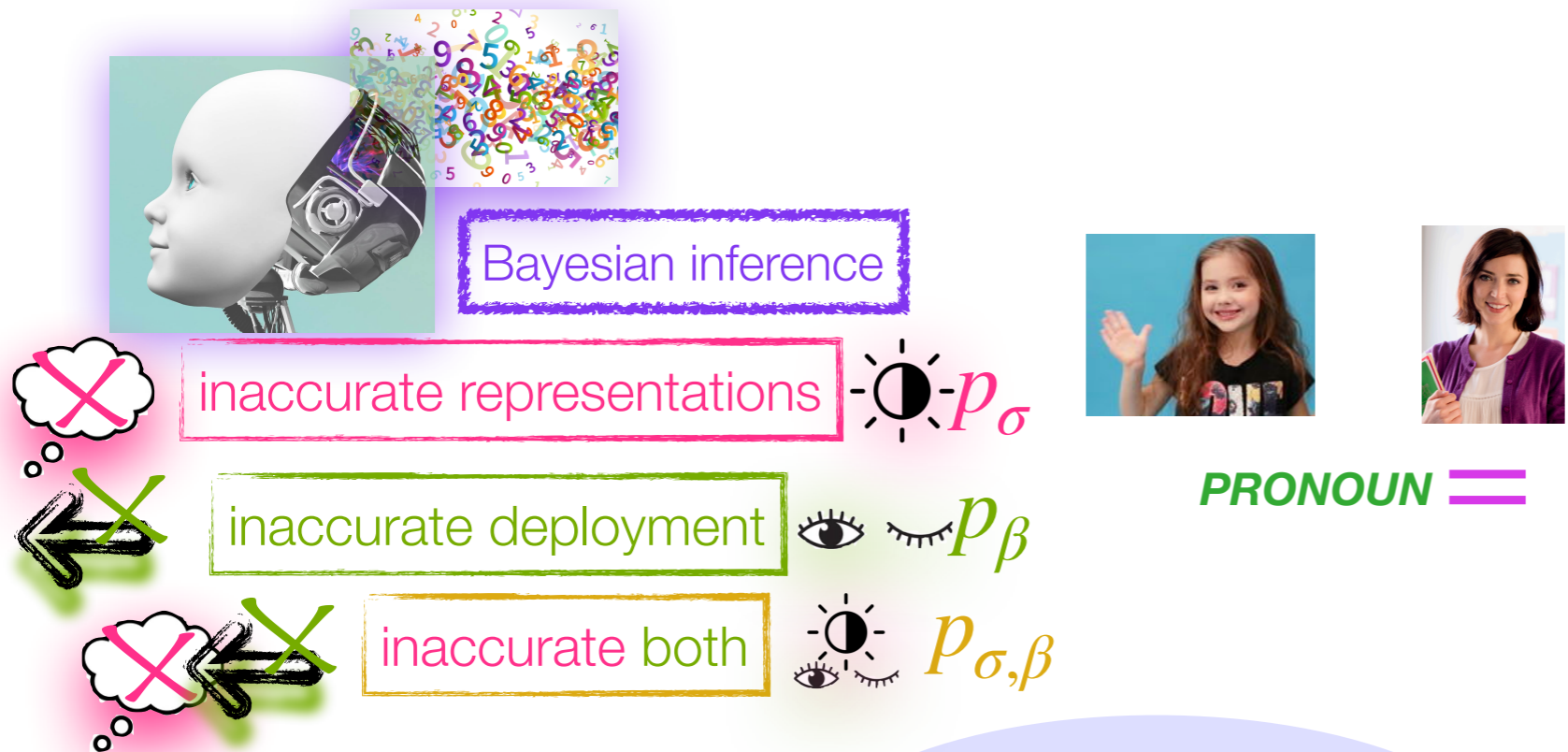
$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$



antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
			\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

The null pronoun form is generally used, though some antecedent types use it more often.

Input to the modeled listener



$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

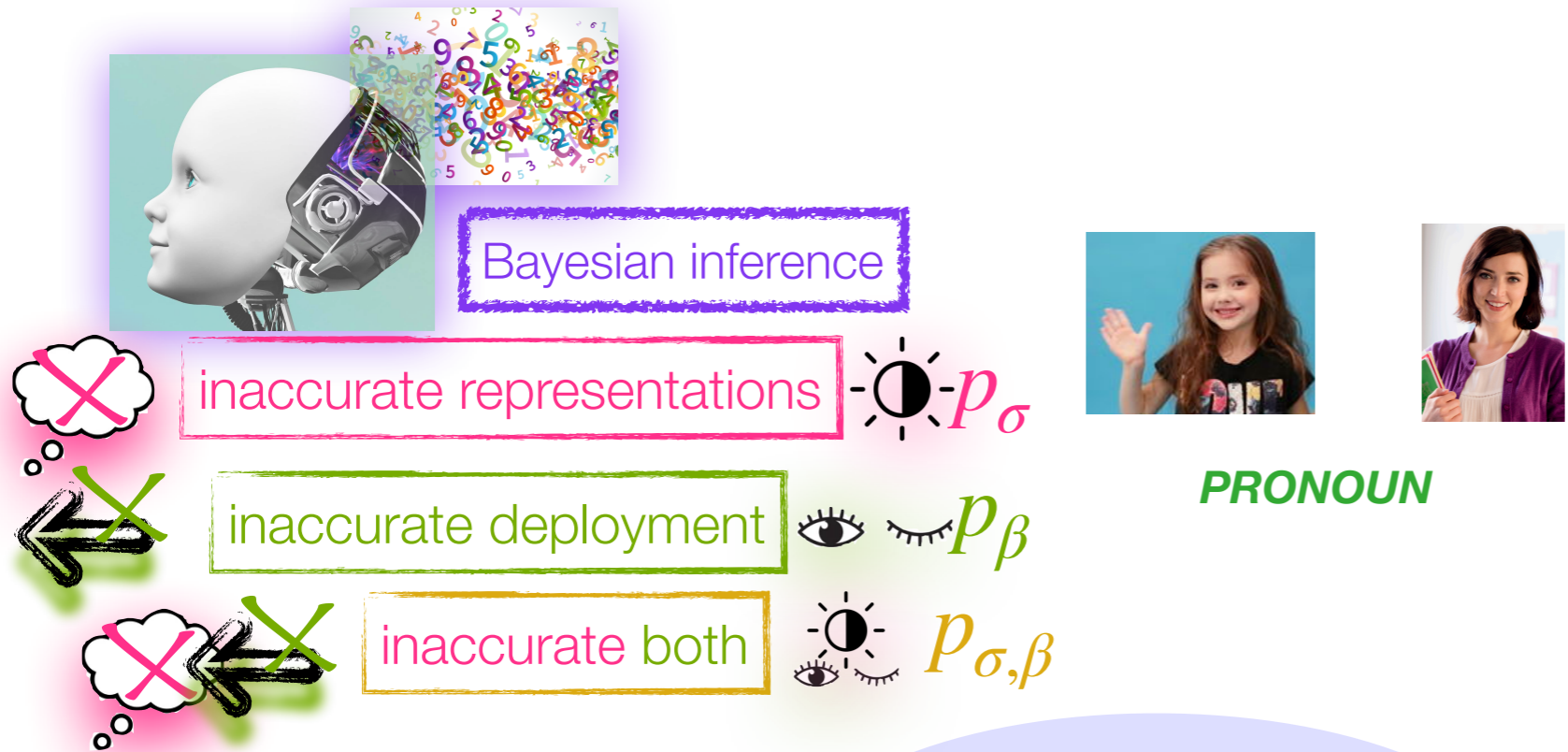


antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
			\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

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The connective *porque* is used more often for antecedents that aren't plural subjects, though how much more often varies.

Input to the modeled listener



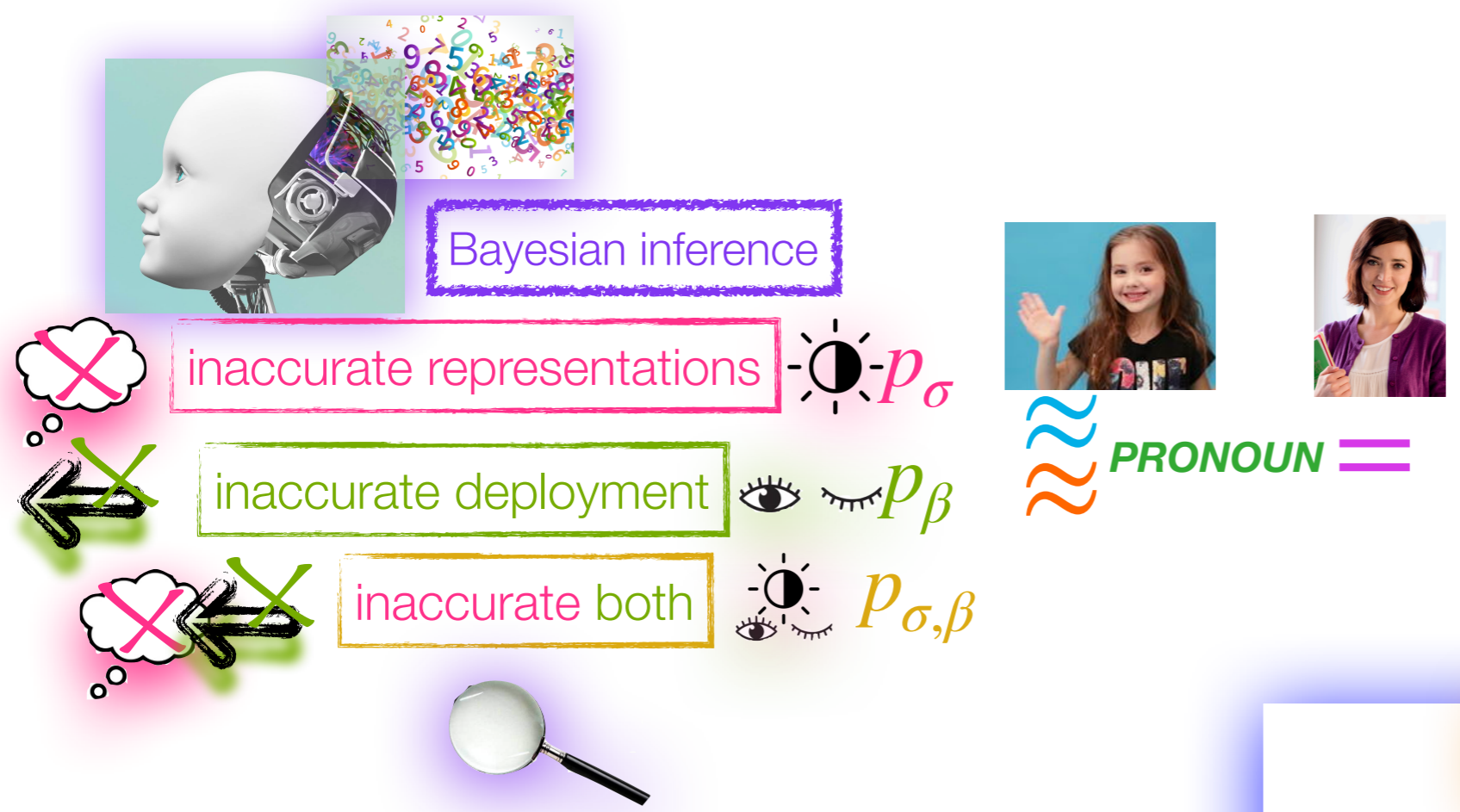
$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

antecedent type		prior	likelihoods					
		$p(\alpha)$	$p(\text{FORM} \alpha)$		$p(\text{CON} \alpha)$		$p(\text{MOR} \alpha)$	
		\emptyset	overt	<i>después</i>	<i>porque</i>	SG	PL	
SUBJ	SG	0.362	0.938	0.062	0.324	0.676	0.998	0.002
	PL	0.071	0.984	0.016	0.750	0.250	0.005	0.995
¬SUBJ	SG	0.438	0.817	0.183	0.132	0.868	0.998	0.002
	PL	0.129	0.959	0.041	0.394	0.606	0.005	0.995

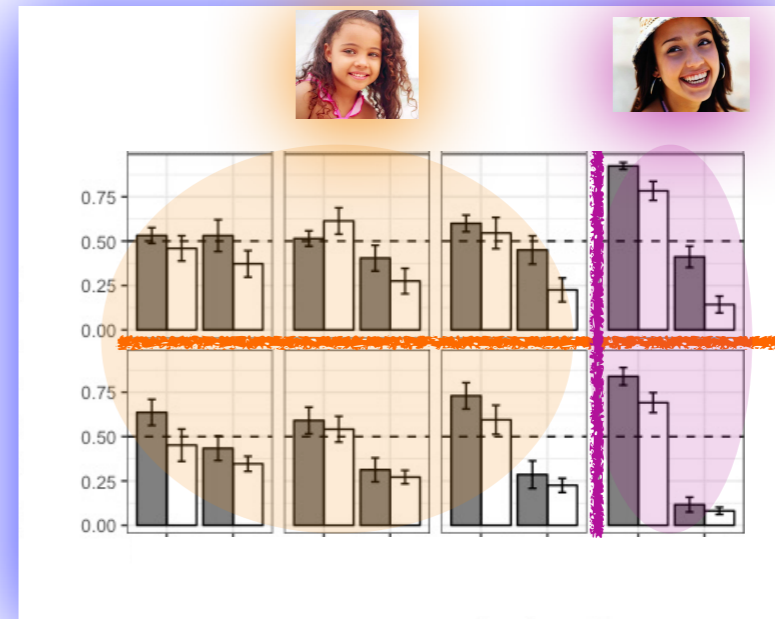
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Agreement morphology is nearly categorical, with a very strong preference for matching morphology.

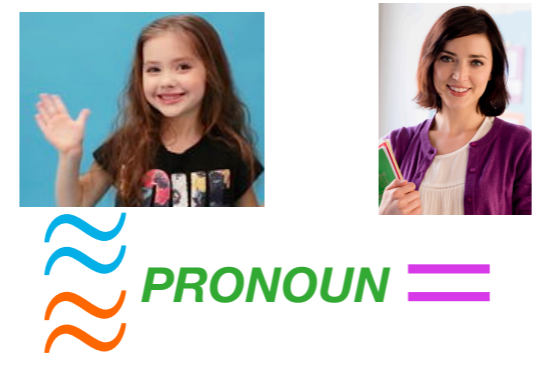
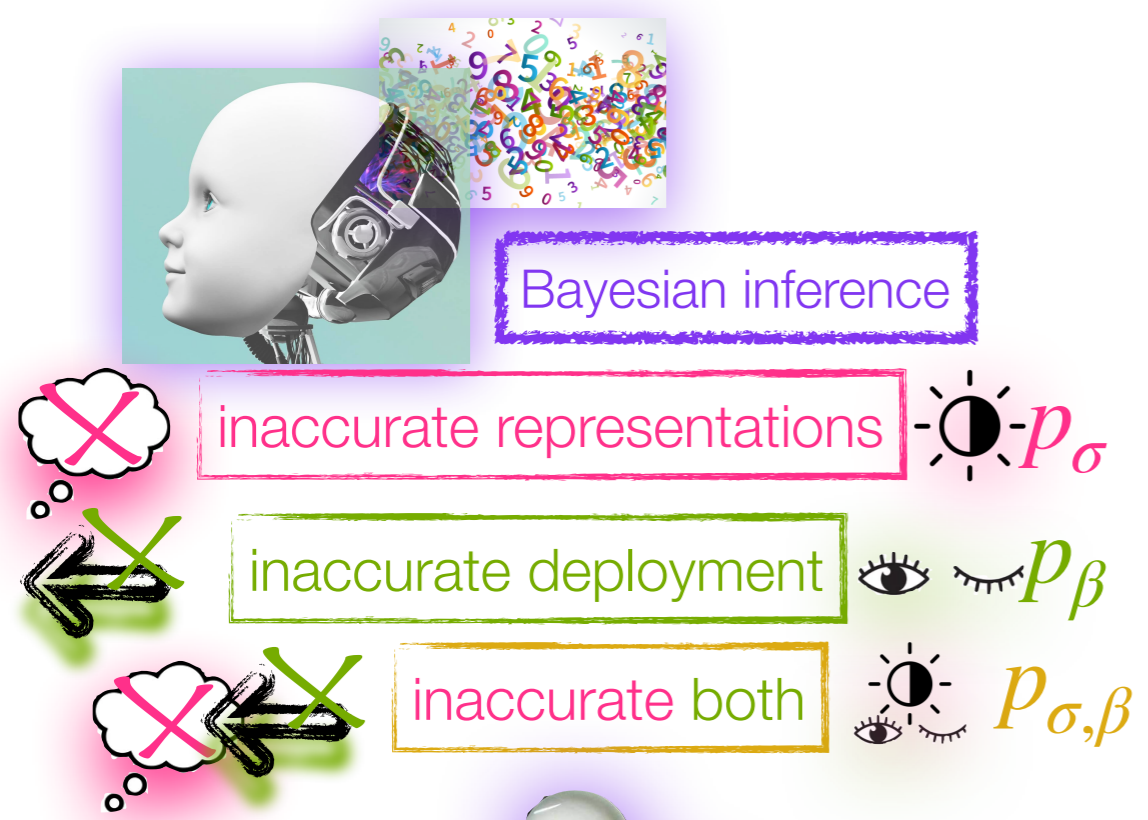
Understanding the development of pronoun interpretation



The plan, part 2: Use the **computational cognitive model** to identify the specific differences leading to **child** and **adult** pronoun interpretation in context.

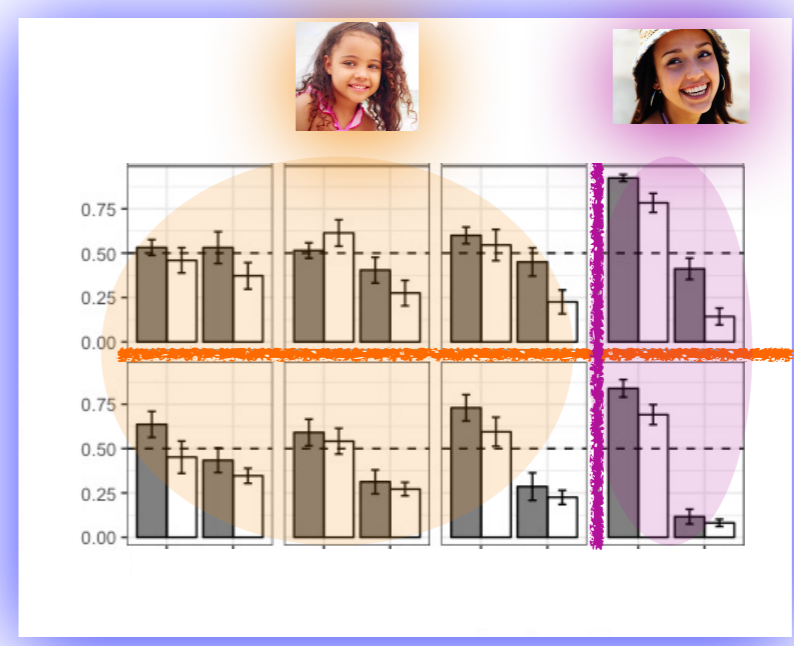


Understanding the development of pronoun interpretation

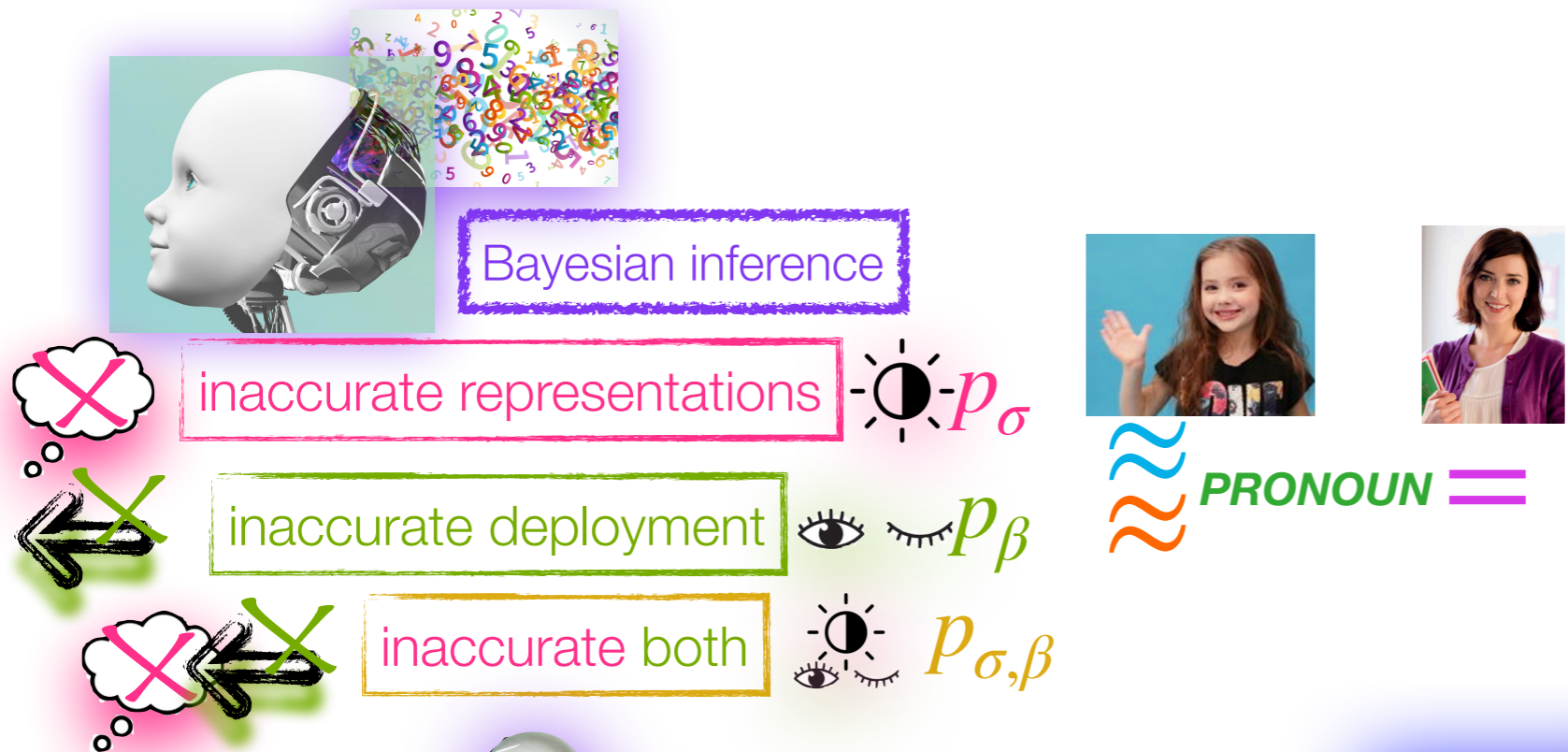


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Which modeled listener variant best matches the observed pronoun interpretation behavior?

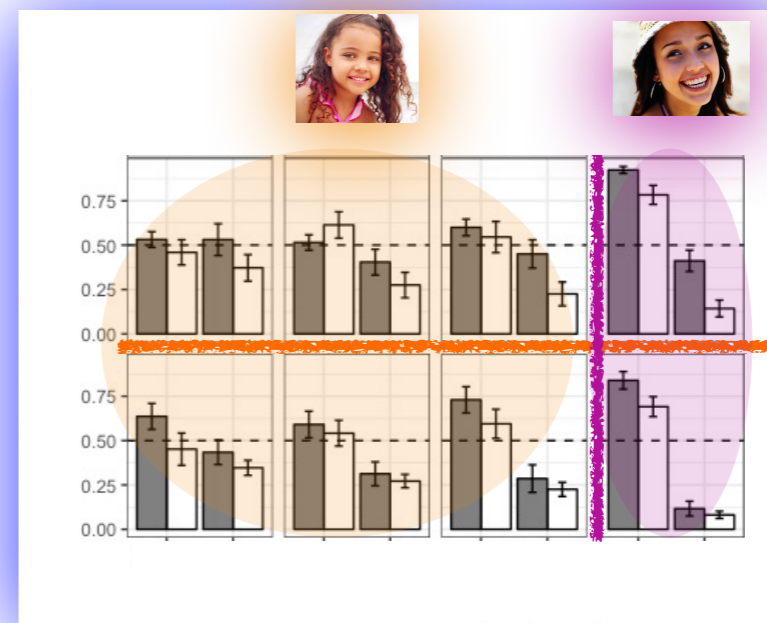


Understanding the development of pronoun interpretation

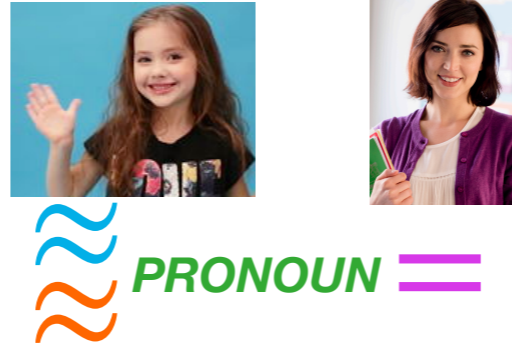
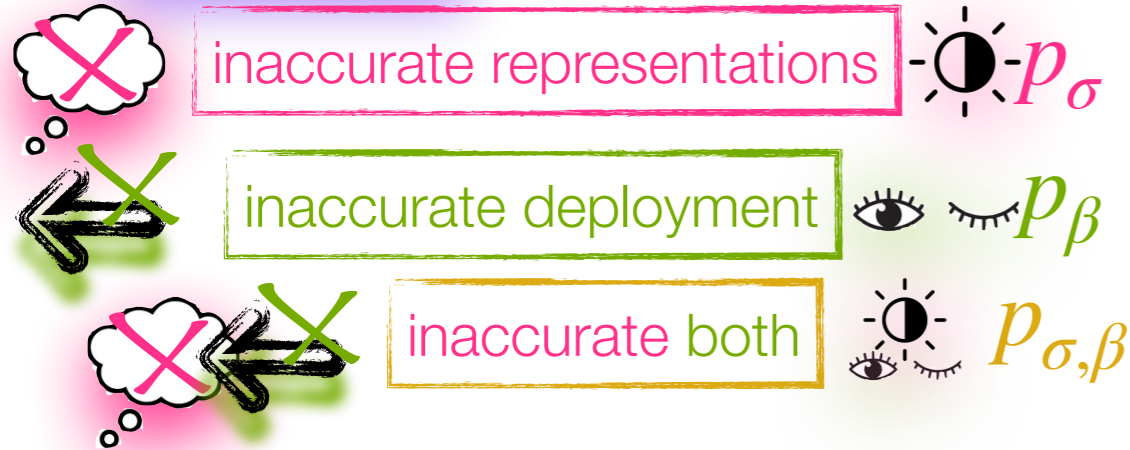
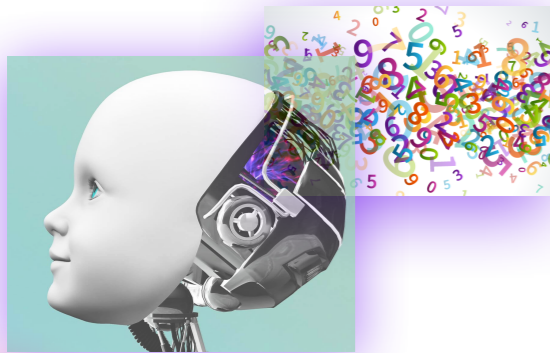


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Important: Model variants with more parameters have an easier time fitting the data because they have more degrees of freedom.



Understanding the development of pronoun interpretation

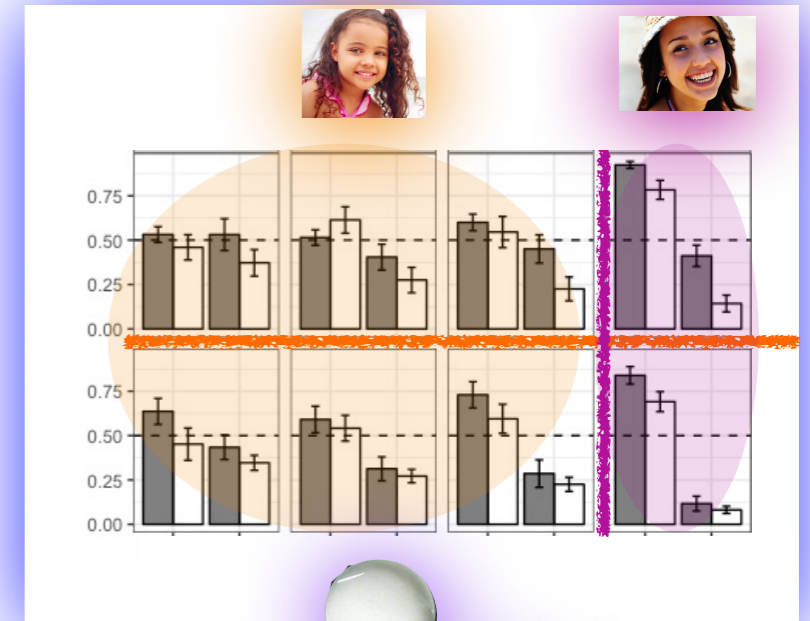


baseline: accurate representations and deployment p

$$p(\alpha_{subj.SG} | \text{FORM, CON, MOR}) \propto p(\alpha_{subj.SG}) * p(\text{FORM} | \alpha_{subj.SG}) * p(\text{CON} | \alpha_{subj.SG}) * p(\text{MOR} | \alpha_{subj.SG})$$

0 free parameters

Understanding the development of pronoun interpretation



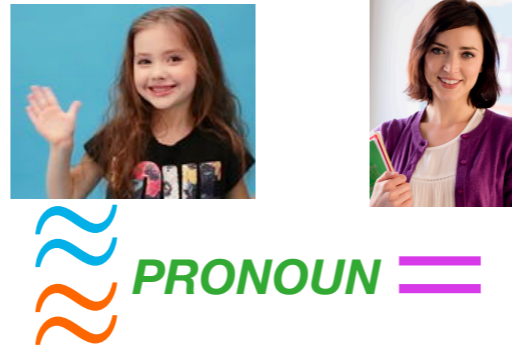
 PRONOUN =

baseline: accurate representations and deployment P 0 free parameters

 inaccurate representations P_{σ}

4 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}$

Understanding the development of pronoun interpretation



 inaccurate both  $P_{\sigma, \beta}$

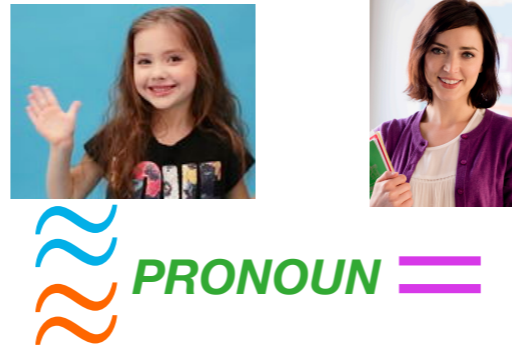
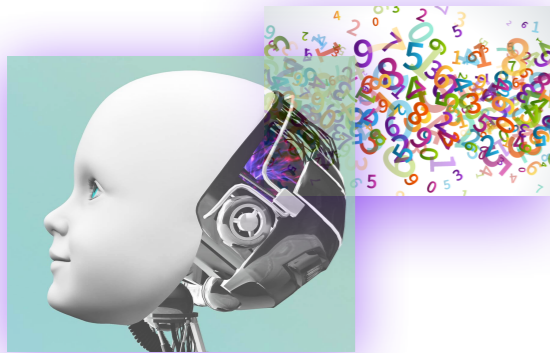
baseline: accurate representations and deployment P 0 free parameters

 inaccurate representations  P_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}

 inaccurate deployment  P_{β}

4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}

Understanding the development of pronoun interpretation



baseline: accurate representations and deployment P 0 free parameters

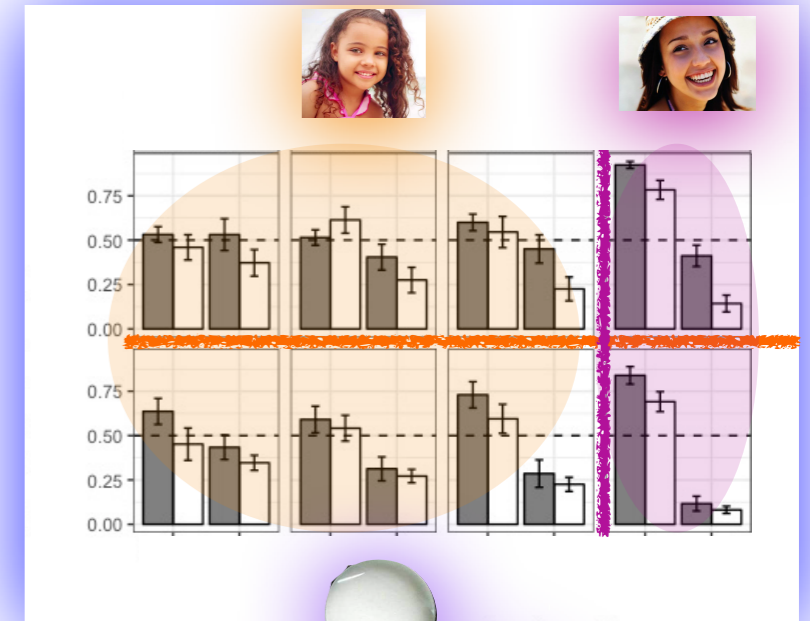
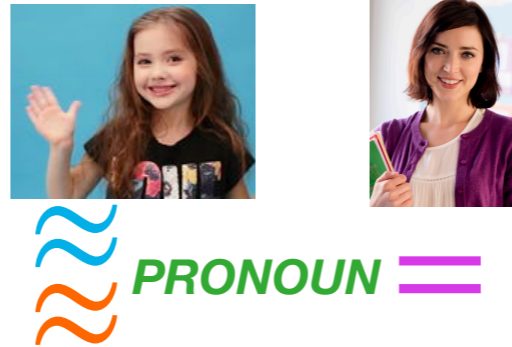
 inaccurate representations  P_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}

 inaccurate deployment  P_{β} 4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}

 inaccurate both  $P_{\sigma,\beta}$

8 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor} , β_{α} , β_{form} , β_{con} , β_{mor} Forsythe & Pearl 2019, in prep

Understanding the development of pronoun interpretation



We want variants with more parameters to have a **substantially better fit** in order to favor them over variants with fewer parameters.

baseline: accurate representations and deployment

P 0 free parameters



inaccurate representations



P_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}



inaccurate deployment



P_{β} 4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}



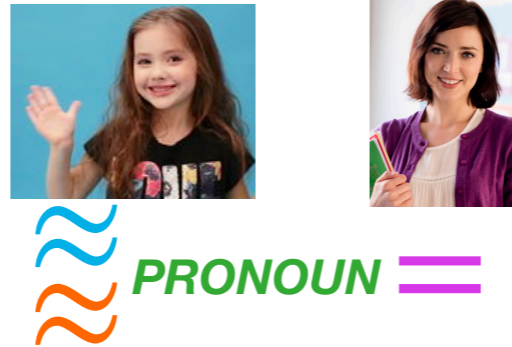
inaccurate both



$P_{\sigma, \beta}$ 8 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor} , β_{α} , β_{form} , β_{con} , β_{mor}









Understanding the development of pronoun interpretation



The Bayesian Information Criterion (BIC) is one way to quantify this preference (Schwarz 1978).

baseline: accurate representations and deployment P 0 free parameters

- 
inaccurate representations

 P_{σ} 4 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}$
- 
inaccurate deployment

 P_{β} 4 free parameters: $\beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$
- 
inaccurate both

 $P_{\sigma, \beta}$ 8 free parameters: $\sigma_{\alpha}, \sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_{\alpha}, \beta_{form}, \beta_{con}, \beta_{mor}$



Understanding the development of pronoun interpretation



$$\text{BIC} = \# \text{ parameters} \cdot \log(|\text{data}|) - 2 \cdot \log(\text{model fit})$$

$$0 \leq \text{BIC} \leq \infty \text{ (closer to 0 is better)}$$

baseline: accurate representations and deployment

p 0 free parameters



inaccurate representations



p_σ

4 free parameters: $\sigma_\alpha, \sigma_{form}, \sigma_{con}, \sigma_{mor}$



inaccurate deployment



p_β

4 free parameters: $\beta_\alpha, \beta_{form}, \beta_{con}, \beta_{mor}$



inaccurate both

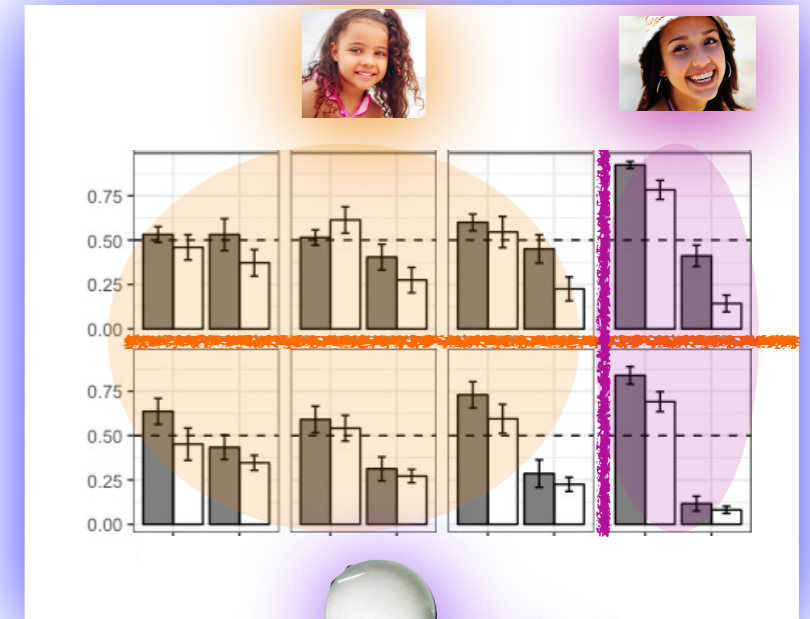


$p_{\sigma,\beta}$

8 free parameters: $\sigma_\alpha, \sigma_{form}, \sigma_{con}, \sigma_{mor}, \beta_\alpha, \beta_{form}, \beta_{con}, \beta_{mor}$



Understanding the development of pronoun interpretation



more parameters = higher score,
closer to 0 is better

$$\text{BIC} = \# \text{ parameters} \cdot \log(|\text{data}|) - 2 \cdot \log(\text{model fit})$$

$$0 \leq \text{BIC} \leq \infty \text{ (closer to 0 is better)}$$



baseline: accurate representations and deployment

p 0 free parameters



inaccurate representations



p_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}



inaccurate deployment



p_{β} 4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}



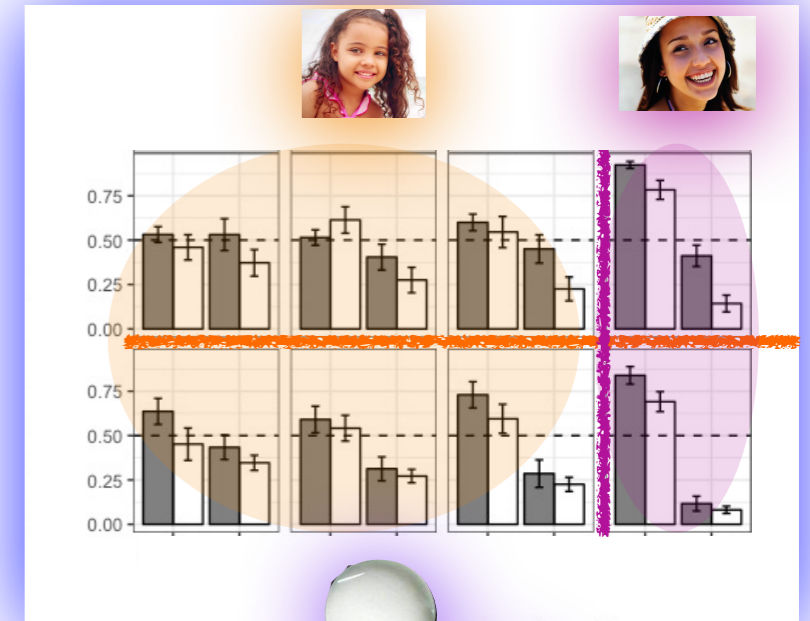
inaccurate both



$p_{\sigma, \beta}$ 8 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor} ,
 β_{α} , β_{form} , β_{con} , β_{mor}



Understanding the development of pronoun interpretation



model fit = likelihood of data, given model with best-fitting parameter values

 **PRONOUN** =

$$\text{BIC} = \# \text{ parameters} \cdot \log(|\text{data}|) - 2 \cdot \log(\text{model fit})$$

$$0 \leq \text{BIC} \leq \infty \text{ (closer to 0 is better)}$$

baseline: accurate representations and deployment

p 0 free parameters



inaccurate representations



p_{σ}

4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}



inaccurate deployment



p_{β}

4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}



inaccurate both



$p_{\sigma, \beta}$

8 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor} ,

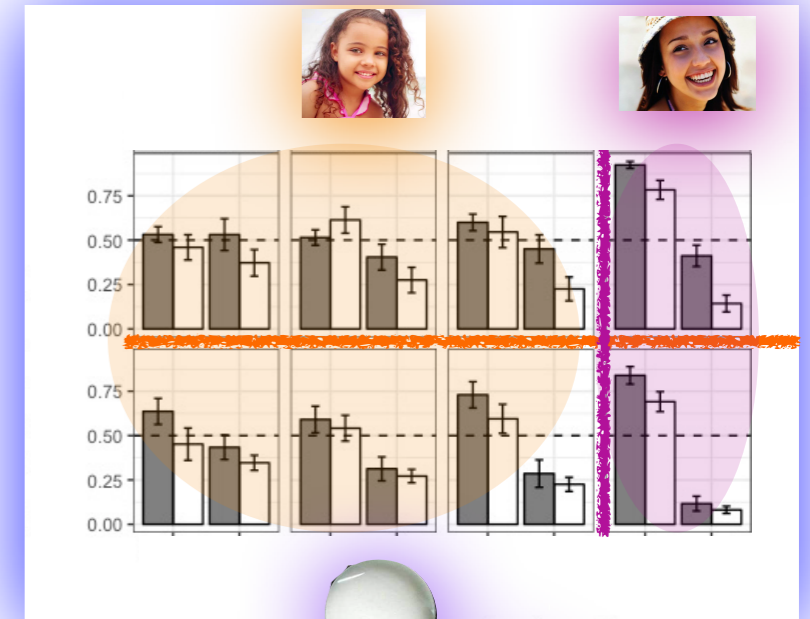
β_{α} , β_{form} , β_{con} , β_{mor}

Understanding the development of pronoun interpretation



$-\infty \leq \log(\text{likelihood}) \leq 0$,
closer to 0 is better

 **PRONOUN** =



BIC = # parameters · log(|data|) - 2 · log(model fit)

$0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

baseline: accurate representations and deployment

p 0 free parameters



inaccurate representations



p_{σ} 4 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor}



inaccurate deployment



p_{β} 4 free parameters: β_{α} , β_{form} , β_{con} , β_{mor}

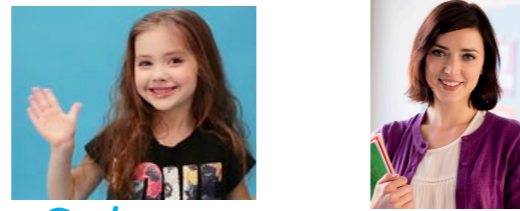


inaccurate both



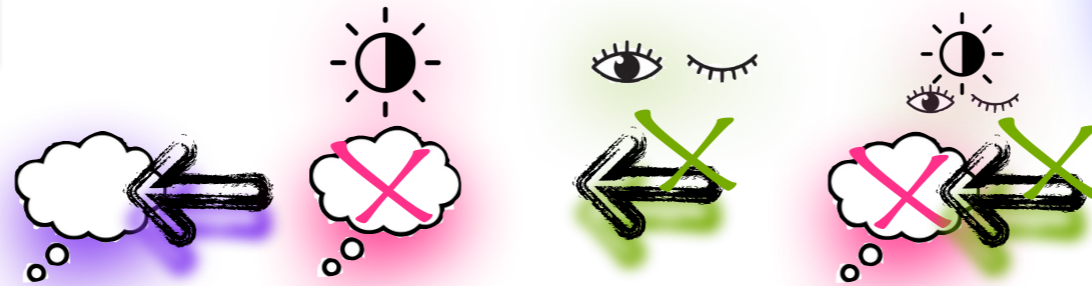
$p_{\sigma, \beta}$ 8 free parameters: σ_{α} , σ_{form} , σ_{con} , σ_{mor} ,
 β_{α} , β_{form} , β_{con} , β_{mor}

Understanding the development of pronoun interpretation



PRONOUN =

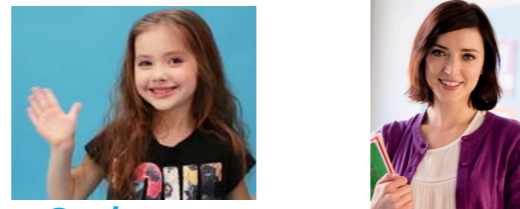
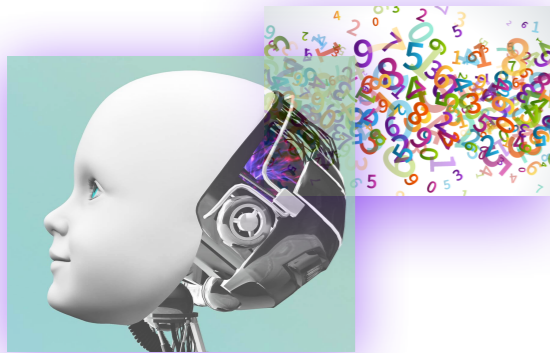
0 ≤ BIC ≤ ∞ (closer to 0 is better)



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3				
4				
≥ 5				
adults				

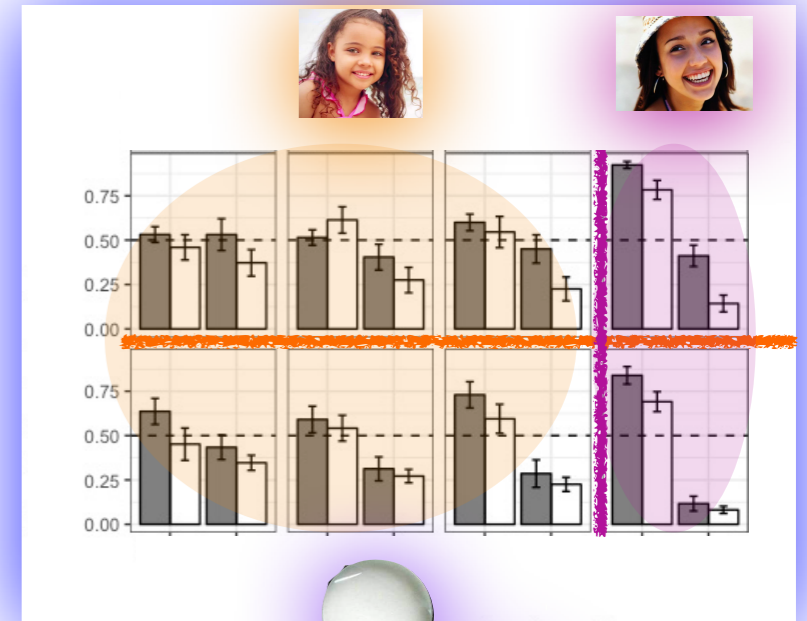
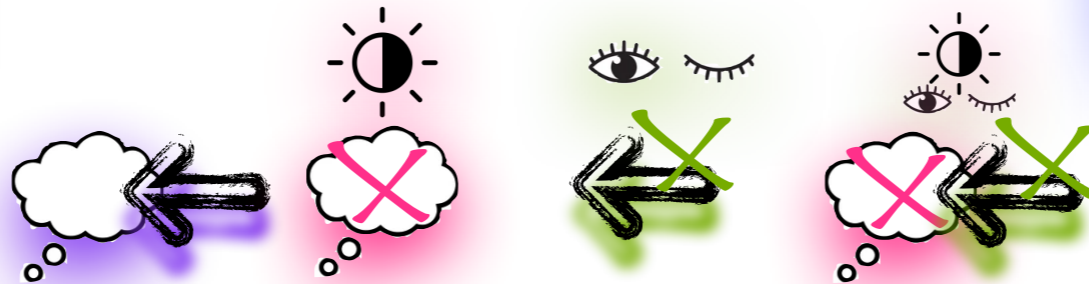




Understanding the development of pronoun interpretation



PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

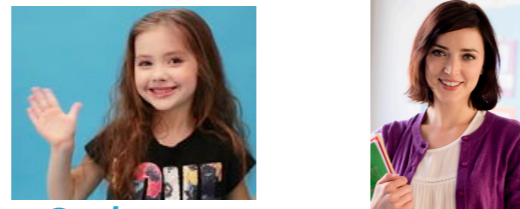


age	baseline	inacc rep	inacc deploy	both inacc
 ≤ 3	3548.40	1450.67	1453.05	1473.04
4				
≥ 5				
 adults				


3-year-old pronoun interpretation behavior is best captured by children having inaccurate representations, but accurate deployment

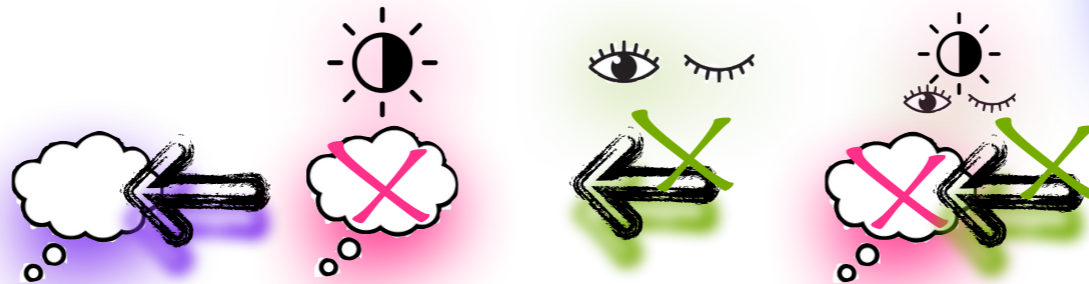


Understanding the development of pronoun interpretation

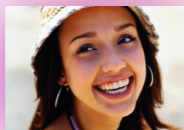


PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4				
≥ 5				

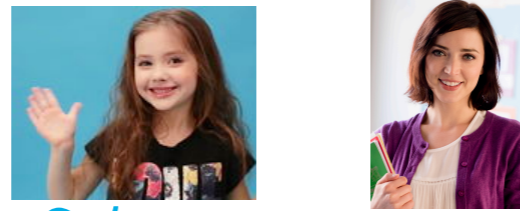


adults


...though all 3 inaccurate variants are far better than the baseline of both accurate representations and accurate deployment.

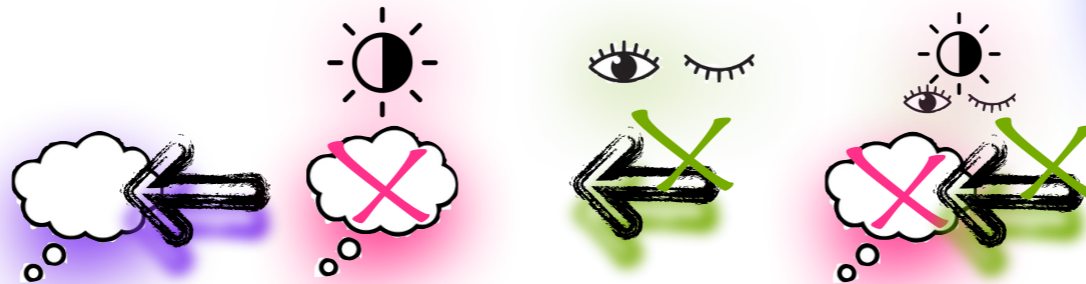


Understanding the development of pronoun interpretation



PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

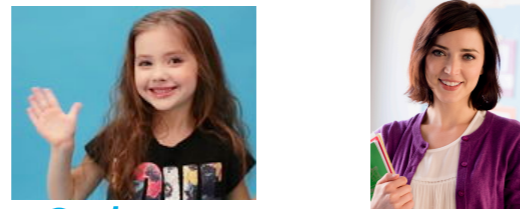


age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults				

This same pattern holds for 4-year-olds and 5-year-olds, too.

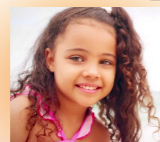
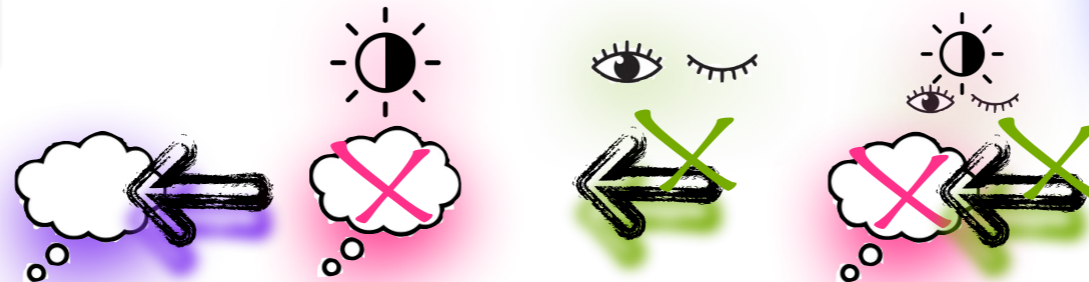
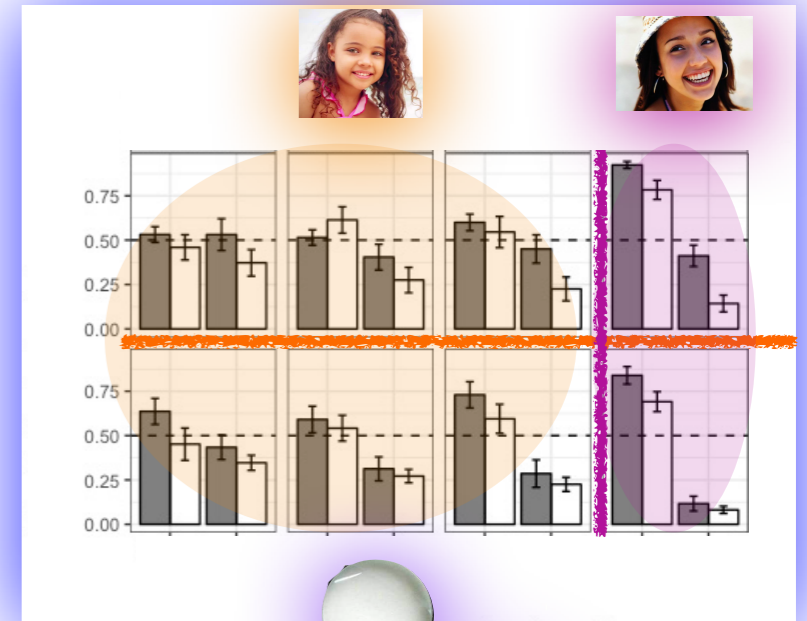


Understanding the development of pronoun interpretation



PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

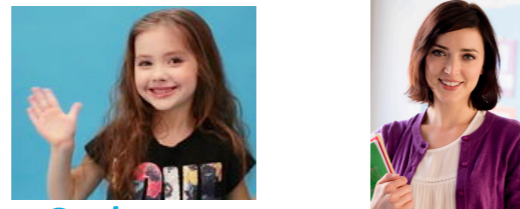


age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults	2402.53	1637.24	1612.84	1277.71


Adult pronoun interpretation behavior is best captured by the model that has both inaccurate representations and inaccurate deployment.

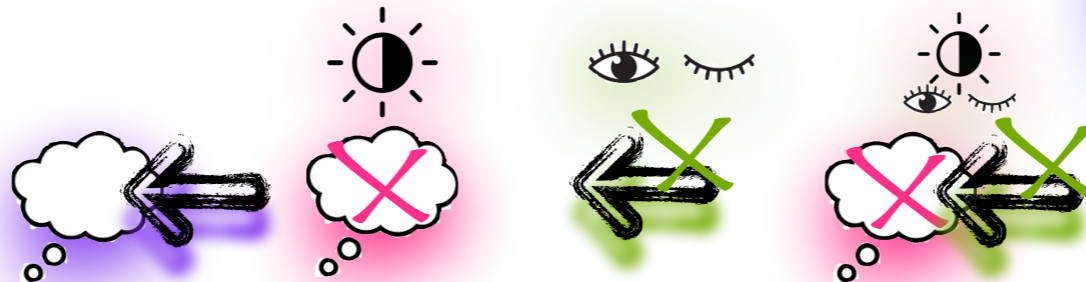


Understanding the development of pronoun interpretation



PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

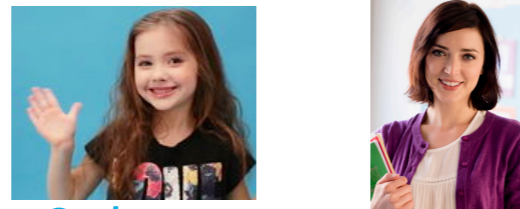


?


age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults	2402.53	1637.24	1612.84	1277.71

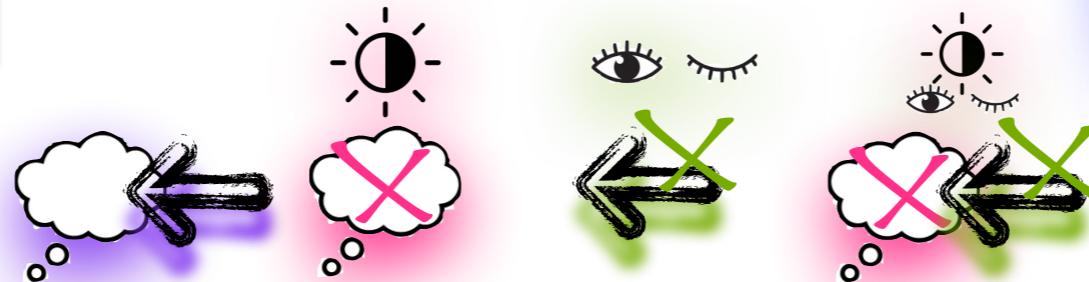
Like the child models, **all inaccurate variants are far better** than the **baseline** that has both accurate representations and accurate deployment.

Understanding the development of pronoun interpretation

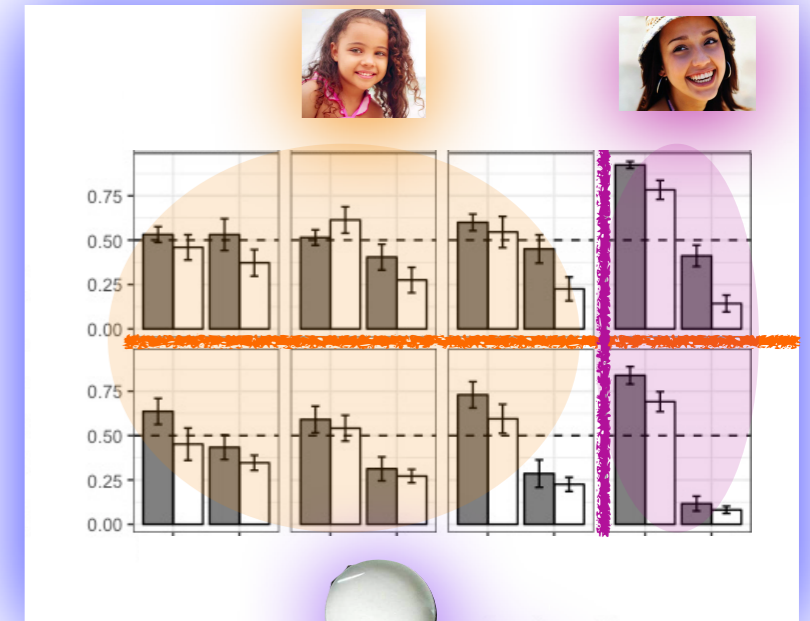


PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

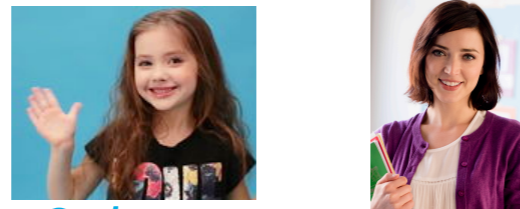


age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults	2402.53	1637.24	1612.84	1277.71



...but within the inaccurate variants, the **both inaccurate variant is much better** than the other two.








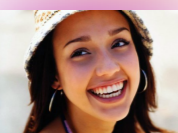
Understanding the development of pronoun interpretation



PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

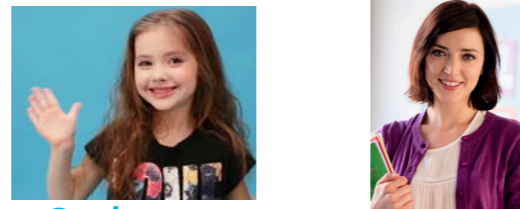


					
age	baseline	inacc rep	inacc deploy	both inacc	
  ≤ 3	3548.40	1450.67	1453.05	1473.04	
4	3497.57	1510.55	1515.28	1532.71	
≥ 5	2528.07	1216.34	1219.04	1239.78	
  adults	2402.53	1637.24	1612.84	1277.71	


Takeaway: Both child and adult pronoun interpretation behavior are captured by modeled listeners that are inaccurate in some way.











Understanding the development of pronoun interpretation



PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

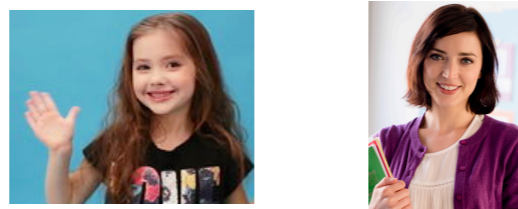


					
age	baseline	inacc rep	inacc deploy	both inacc	
 	≤ 3	3548.40	1450.67	1453.05	1473.04
	4	3497.57	1510.55	1515.28	1532.71
	≥ 5	2528.07	1216.34	1219.04	1239.78
 	adults	2402.53	1637.24	1612.84	1277.71

Becoming adult-like doesn't mean becoming more accurate!

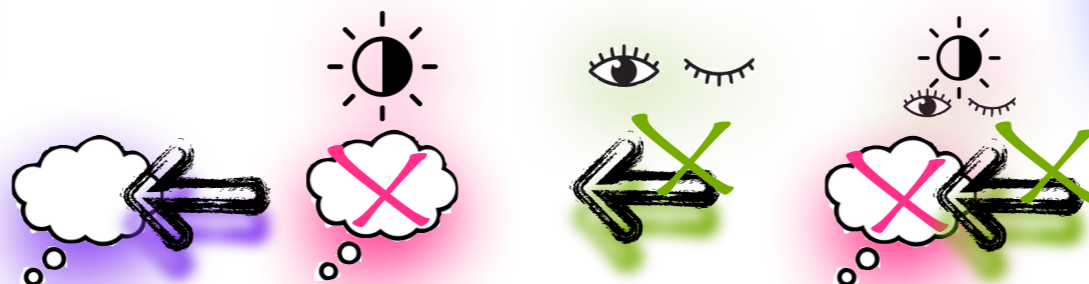


Understanding the development of pronoun interpretation

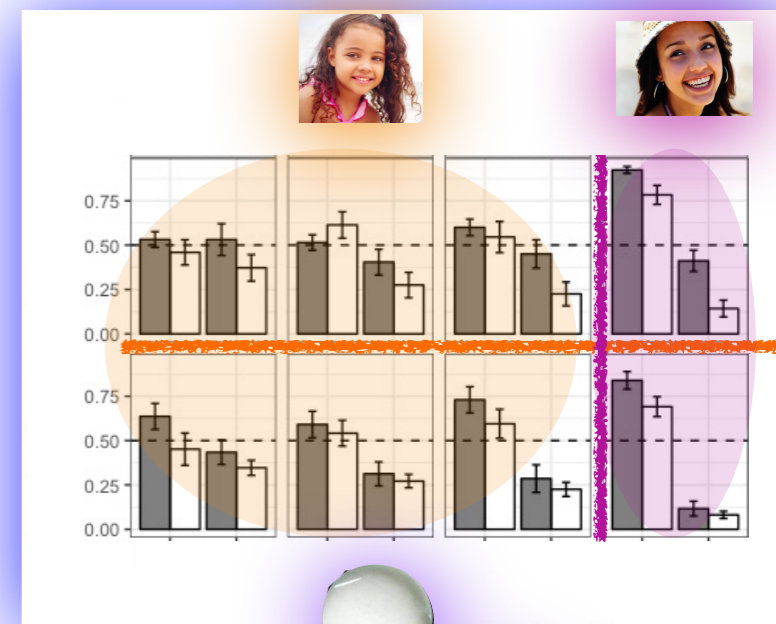


PRONOUN =

0 ≤ BIC ≤ ∞ (closer to 0 is better)



age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults	2402.53	1637.24	1612.84	1277.71

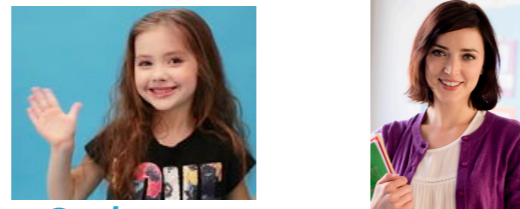


But what does it mean?

It means learning to become *inaccurate in adult-like ways*.

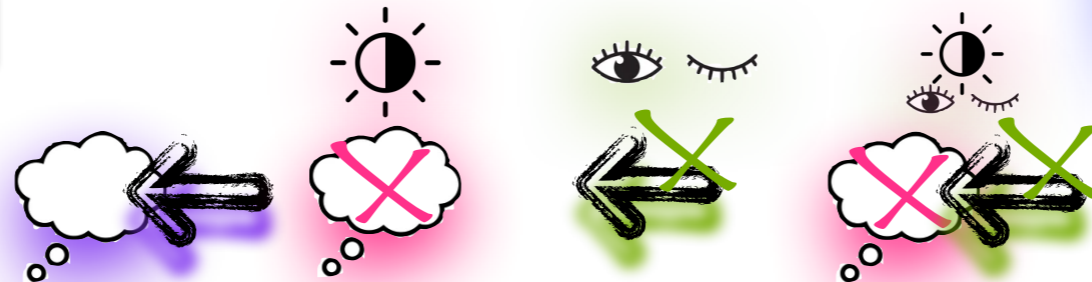


Understanding the development of pronoun interpretation

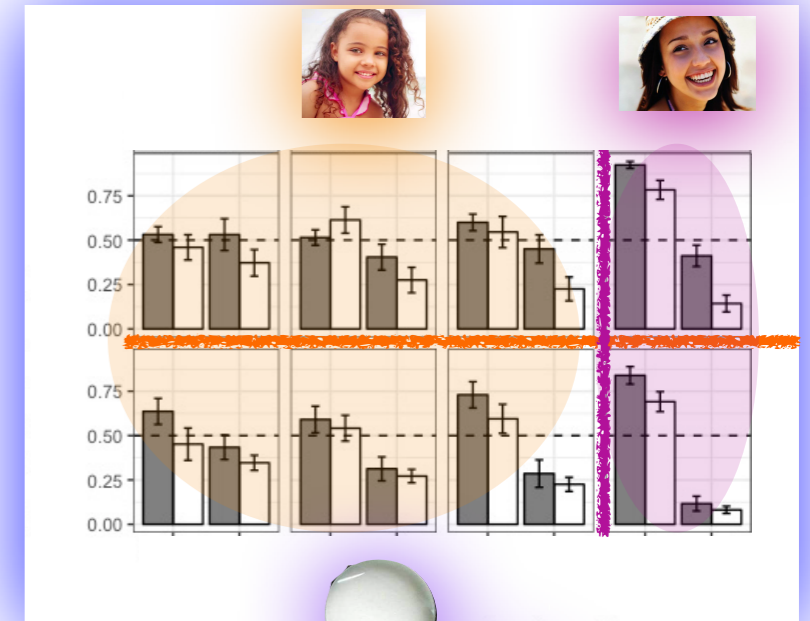


PRONOUN =

0 ≤ BIC ≤ ∞ (closer to 0 is better)



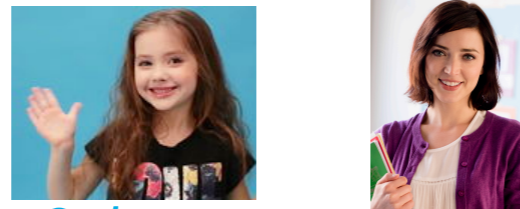
age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults	2402.53	1637.24	1612.84	1277.71




We know that adults *inaccurately deploy* their representations.

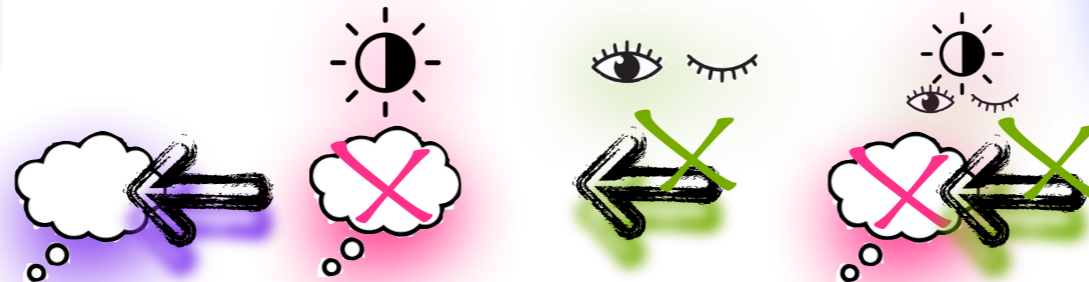


Understanding the development of pronoun interpretation

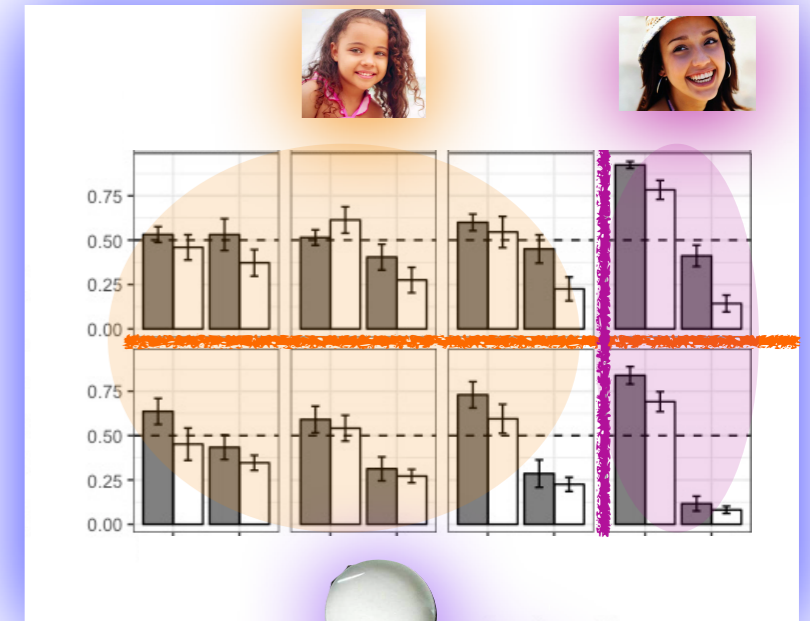


PRONOUN =

 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)



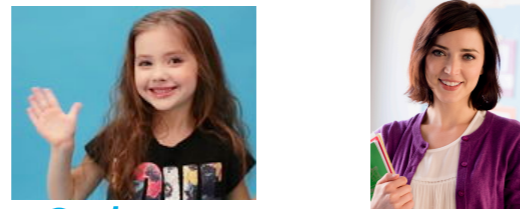
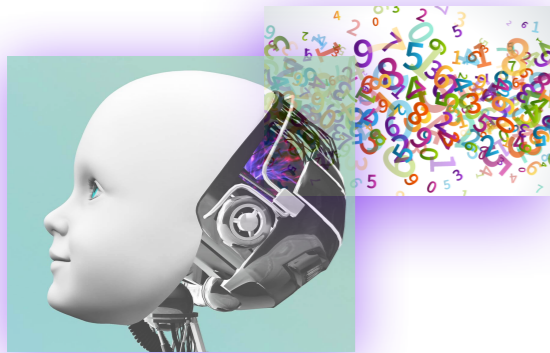
age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults	2402.53	1637.24	1612.84	1277.71



So we can look at the β values to see which information is being ignored and how often it's being ignored.




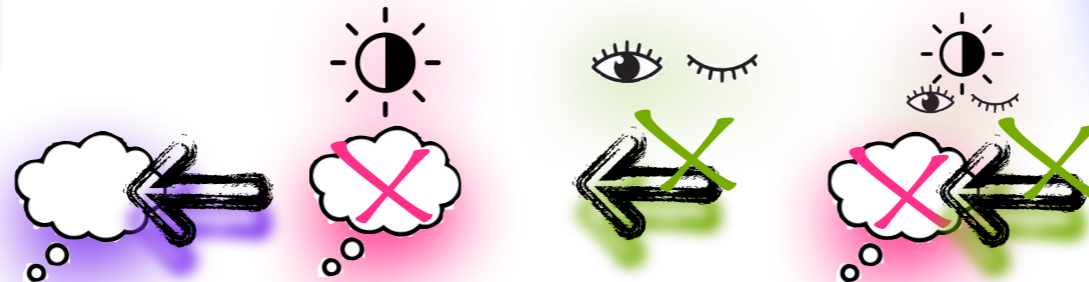
Understanding the development of pronoun interpretation



PRONOUN =

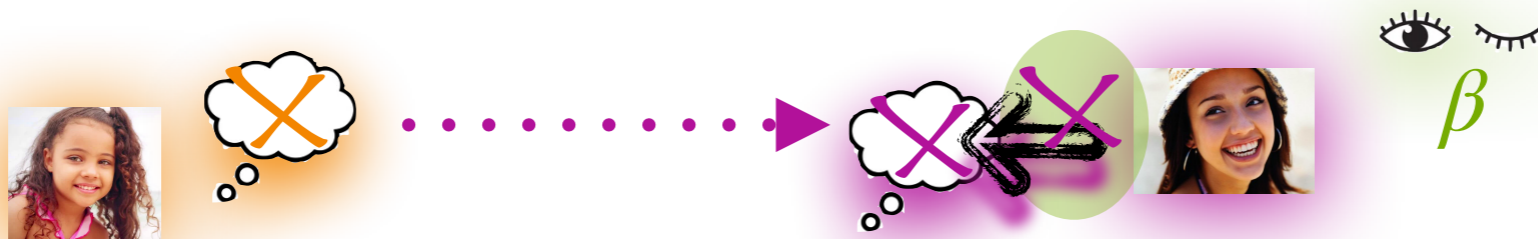


 $0 \leq \text{BIC} \leq \infty$ (closer to 0 is better)

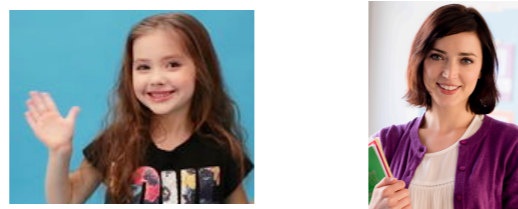


age	baseline	inacc rep	inacc deploy	both inacc
≤ 3	3548.40	1450.67	1453.05	1473.04
4	3497.57	1510.55	1515.28	1532.71
≥ 5	2528.07	1216.34	1219.04	1239.78
adults	2402.53	1637.24	1612.84	1277.71

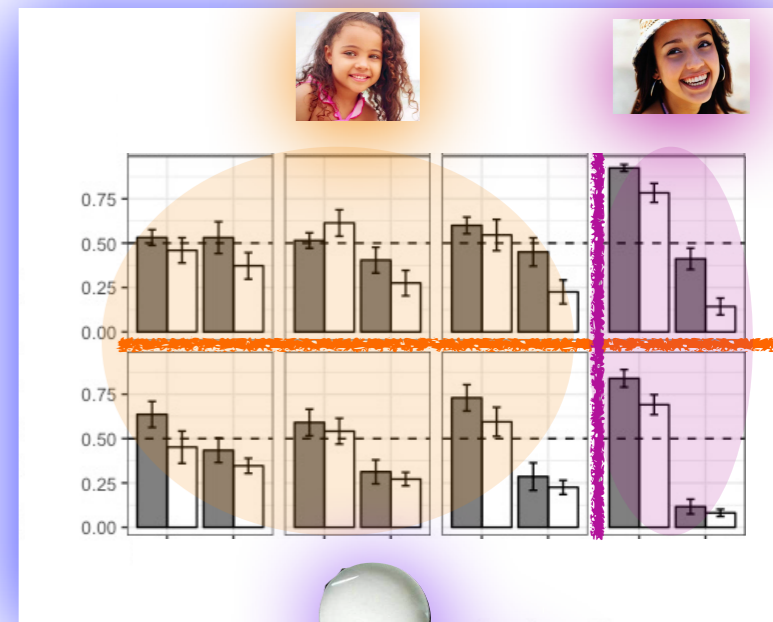
We can also look at the σ values to see how the inaccurate representations are inaccurate — are they too smooth or too sharp?



Understanding the development of pronoun interpretation



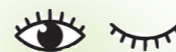
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$

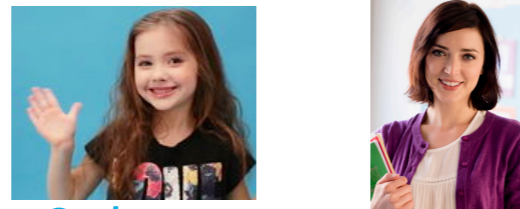


	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3					1	1	1	1
4					1	1	1	1
≥ 5					1	1	1	1
adults: both inaccurate								
adults								



All children are best represented by **accurate deployment**, which is equivalent to always paying attention to all information ($\beta=1$).

Understanding the development of pronoun interpretation



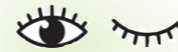
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$

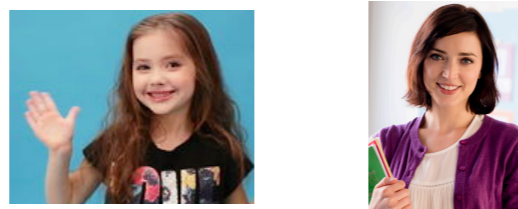


	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3					1	1	1	1
4					1	1	1	1
≥ 5					1	1	1	1
adults: both inaccurate								
adults					0.25	0.40	0.80	0.98

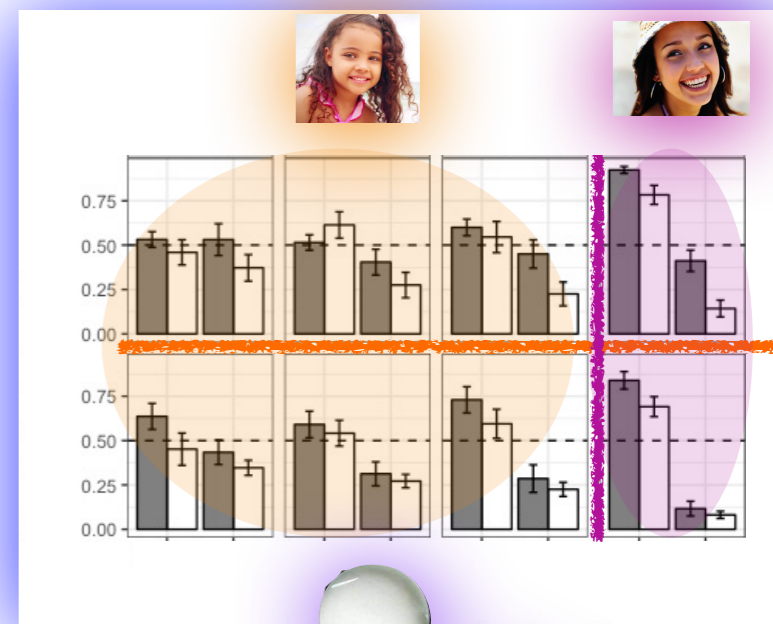


Adults pay much more attention to the **prior** over possible antecedents (β_{α}) and **agreement morphology** (β_{mor})...

Understanding the development of pronoun interpretation



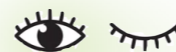
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
----------------	----------------	----------------	-------------------	---------------	---------------	---------------	------------------

children: inaccurate representations

≤ 3				1	1	1	1
4				1	1	1	1
≥ 5				1	1	1	1

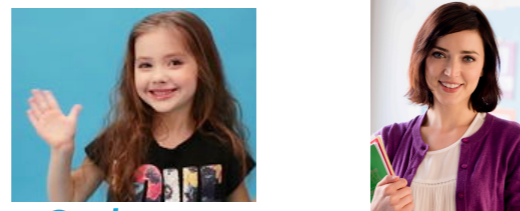
adults: both inaccurate

adults				0.25	0.40	0.80	0.98
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...and much less attention to the pronoun **form** (β_{α}) and the **connective** (β_{con}).

Understanding the development of pronoun interpretation



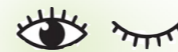
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
----------------	----------------	----------------	-------------------	---------------	---------------	---------------	------------------

children: inaccurate representations

≤ 3				1	1	1	1
4				1	1	1	1
≥ 5				1	1	1	1

adults: both inaccurate

adults				0.25	0.40	0.80	0.98
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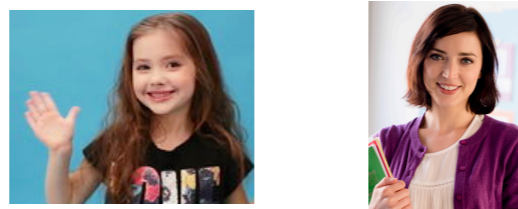
Becoming adult-like:

Learn how much to ignore certain information

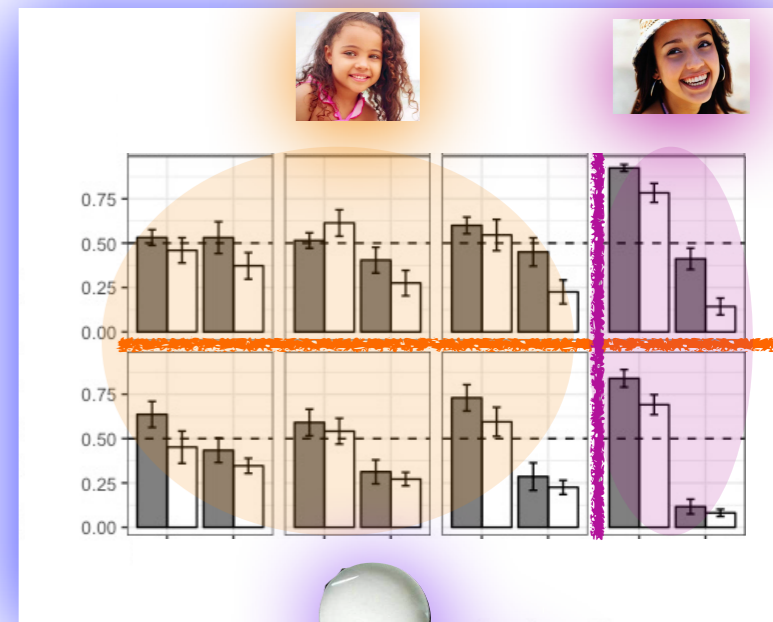
(the pronoun form and connective more often, sometimes the agreement morphology, and occasionally even the prior).



Understanding the development of pronoun interpretation



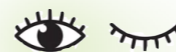
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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children: inaccurate representations

≤ 3					1	1	1	1
4					1	1	1	1
≥ 5					1	1	1	1

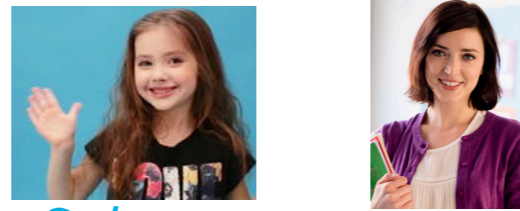
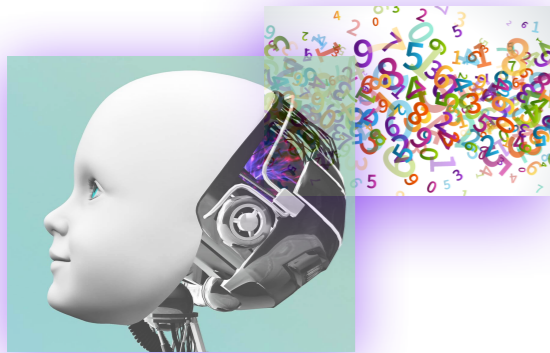
adults: both inaccurate

adults					0.25	0.46	0.8	0.98
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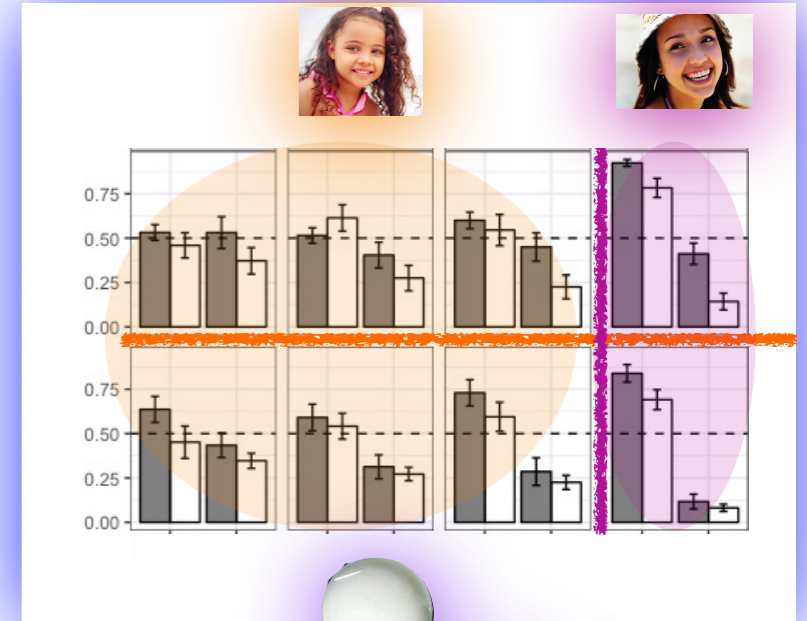


What about the **representations**? Both child and adult representations are inaccurate, but **how are they inaccurate**?

Understanding the development of pronoun interpretation



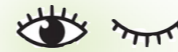
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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children: inaccurate representations

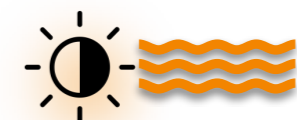
≤ 3	0.01	0.23	0.02	0.01	1	1	1	1
4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

adults: both inaccurate

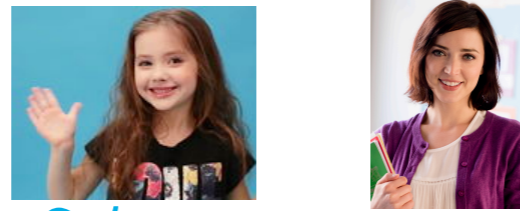
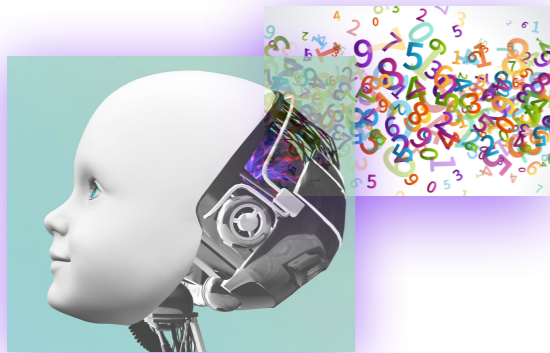
adults	0.25	0.46	0.81	0.98				
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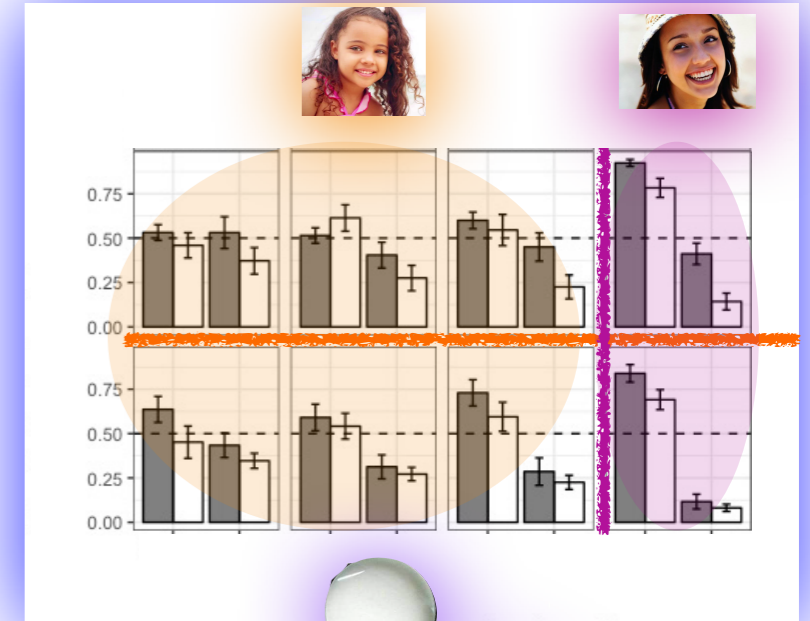
Child representations, with σ values often much < 1 , seem to smooth away probability differences, effectively turning down the contrast.



Understanding the development of pronoun interpretation



PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



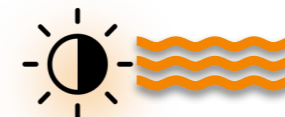
$$0 \leq \beta \leq 1$$



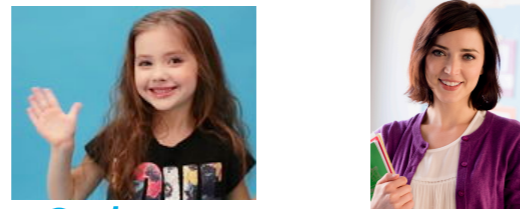
	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.01	0.23	0.02	0.01	1	1	1	1
4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1
adults: both inaccurate								
adults					0.25	0.46	0.81	0.98



$\sigma = .23$.75 vs. .25
 .56 vs. .44



Understanding the development of pronoun interpretation



PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
--	----------------	----------------	----------------	-------------------	---------------	---------------	---------------	------------------

children: inaccurate representations

≤ 3	0.01	0.23	0.02	0.01	1	1	1	1
4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

adults: both inaccurate

adults	0.25	0.46	0.81	0.98				
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.75 vs. .25

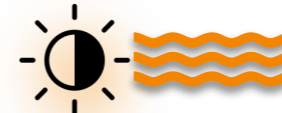


$$\sigma = .23$$

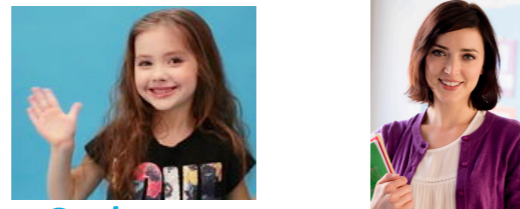
.56 vs. .44

$$\sigma = .10$$

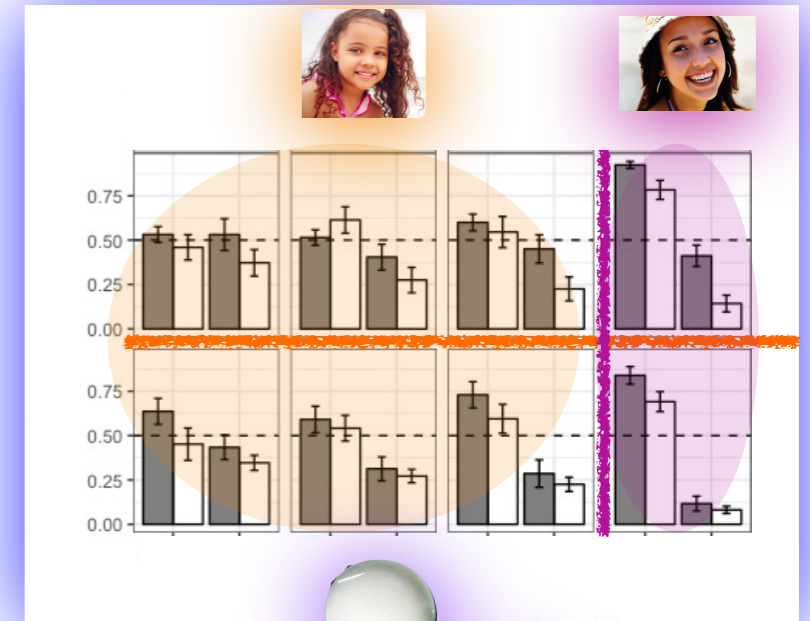
.53 vs. .47



Understanding the development of pronoun interpretation



PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$

$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
--	----------------	----------------	----------------	-------------------	---------------	---------------	---------------	------------------

children: inaccurate representations

≤ 3	0.01	0.23	0.02	0.01	1	1	1	1
4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

adults: both inaccurate

adults	0.25	0.46	0.8	0.98
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.75 vs. .25



$$\sigma = .23$$

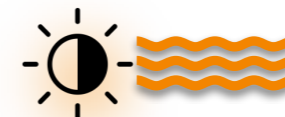
.56 vs. .44

$$\sigma = .10$$

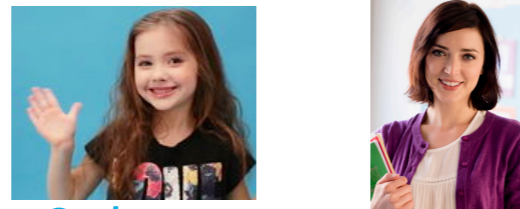
.53 vs. .47

$$\sigma = .01$$

.502 vs. .498



Understanding the development of pronoun interpretation



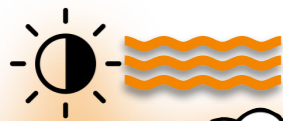
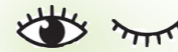
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



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	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

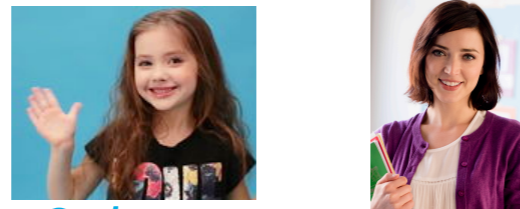
adults: both inaccurate

adults	4.00	3.00	4.00	0.01	0.25	0.46	0.81	0.98
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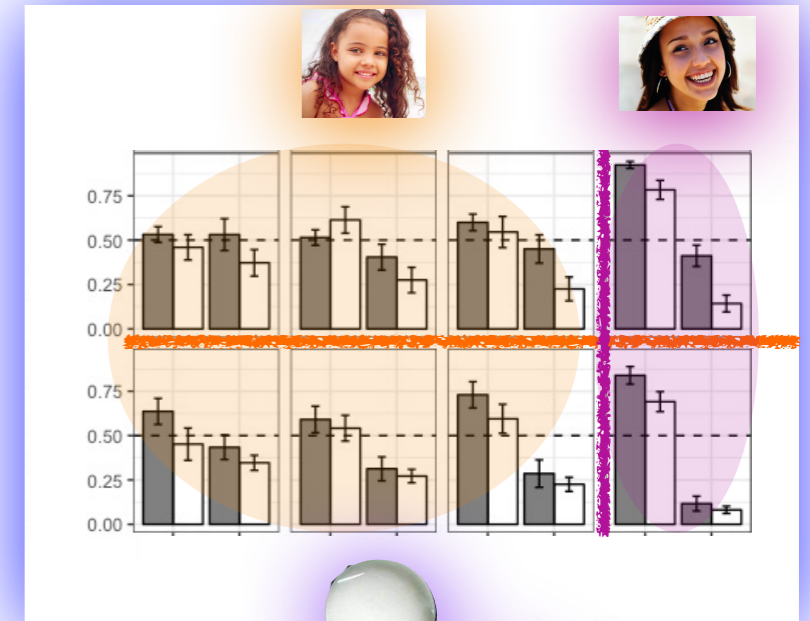


Adults do this same extreme smoothing for the prior information.

Understanding the development of pronoun interpretation



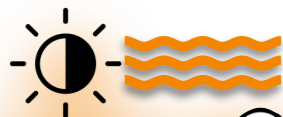
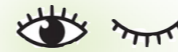
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



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	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

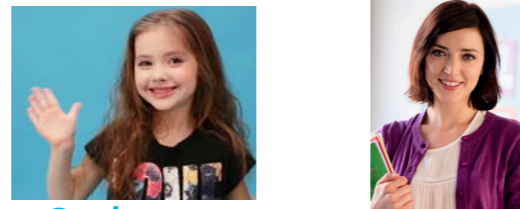
adults: both inaccurate

adults	4.00	3.00	4.00	0.01	0.25	0.46	0.81	0.98
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But adults **substantially sharpen** all the other information ($\sigma = 3.00, 4.00$)

Understanding the development of pronoun interpretation



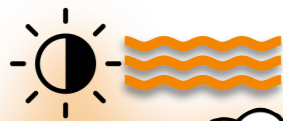
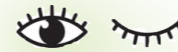
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
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4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

adults: both inaccurate

adults	4.00	3.00	4.00	0.01	0.25	0.46	0.81	0.98
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.75 vs. .25

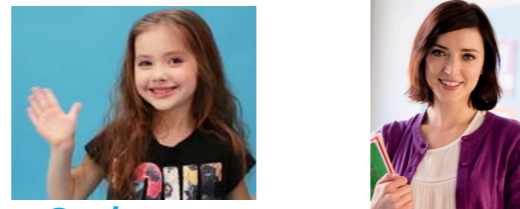


$\sigma = 3.00$

.96 vs. .04



Understanding the development of pronoun interpretation



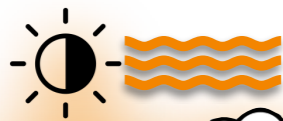
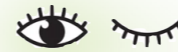
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



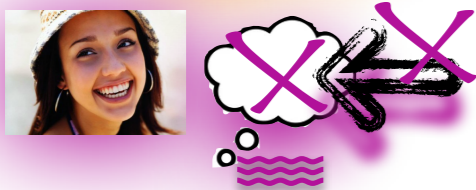
$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
children: inaccurate representations								
≤ 3	0.01	0.23	0.02	0.01	1	1	1	1
4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

adults: both inaccurate

adults	4.00	3.00	4.00	0.01	0.25	0.46	0.81	0.98
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.75 vs. .25

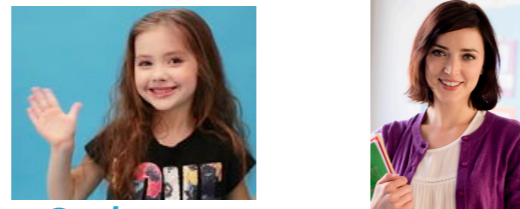


$\sigma = 3.00$.96 vs. .04

$\sigma = 4.00$.99 vs. .01



Understanding the development of pronoun interpretation



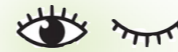
PRONOUN =



$$0.01 \leq \sigma \leq 4.00$$



$$0 \leq \beta \leq 1$$



	σ_{for}	σ_{con}	σ_{mor}	σ_{α}	β_{for}	β_{con}	β_{mor}	β_{α}
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children: inaccurate representations

≤ 3	0.01	0.23	0.02	0.01	1	1	1	1
4	0.01	0.07	0.10	0.01	1	1	1	1
≥ 5	0.01	0.11	0.10	0.01	1	1	1	1

adults: both inaccurate

adults	4.00	3.00	1.00	0.01	0.25	0.46	0.81	0.98
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Becoming adult-like: Children need to learn to sharpen certain perceived representations, rather than smooth away probability differences.

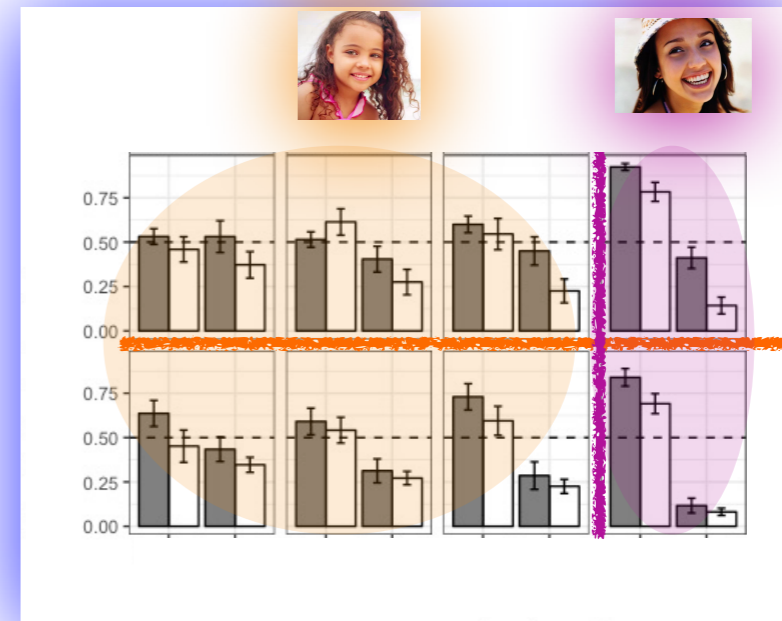
Understanding the development of pronoun interpretation



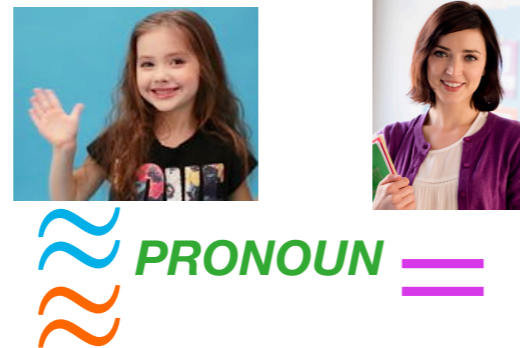
 **PRONOUN** = 

Takeaway:

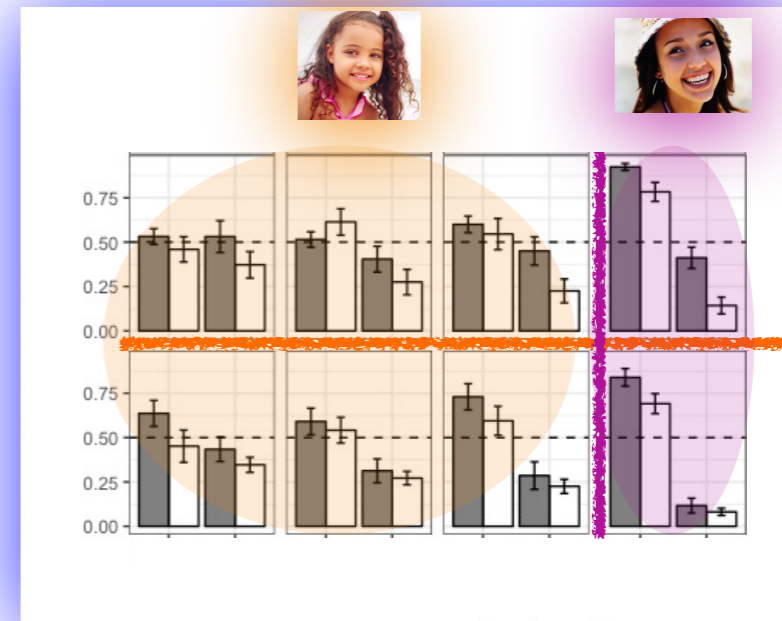
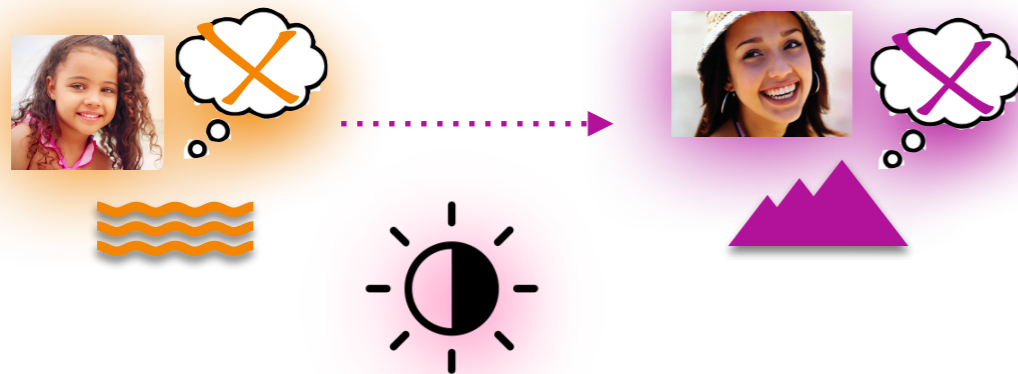
To generate adult-like pronoun interpretation behavior in context, children need to change both how they **represent** relevant information from their input and how they **deploy** those representations.



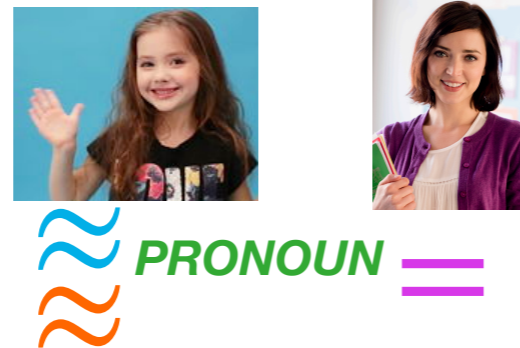
Understanding the development of pronoun interpretation



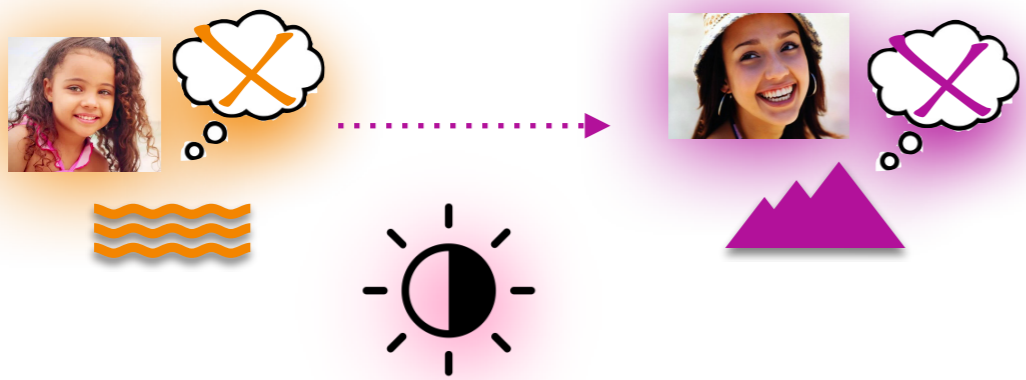
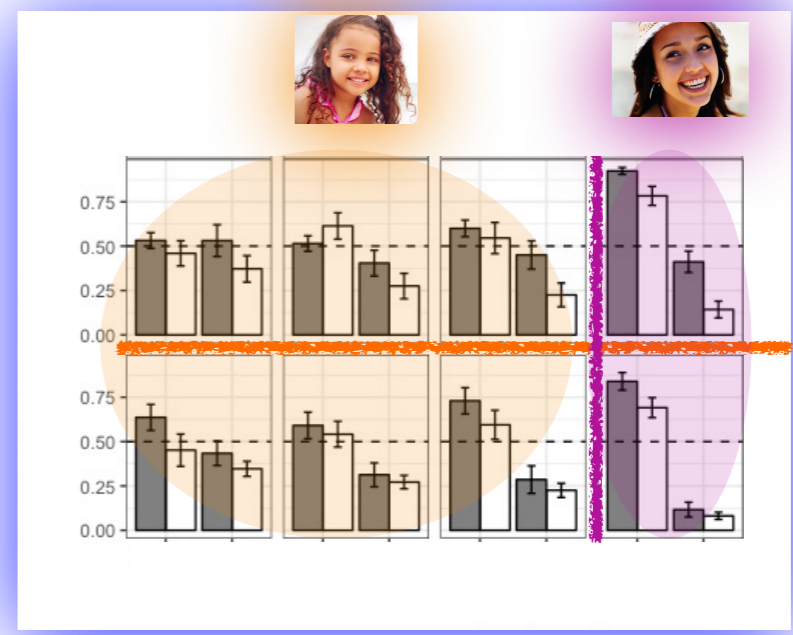
Takeaway:
Child representations are overly smooth — to become adult-like, many representations must sharpen perceived probability differences.



Understanding the development of pronoun interpretation

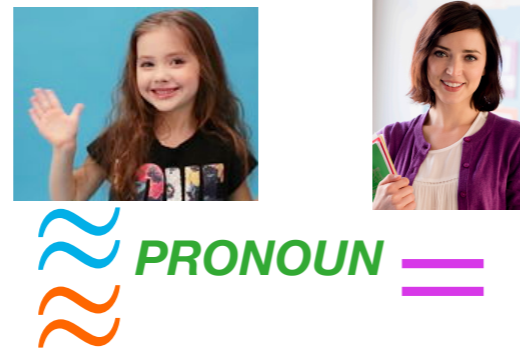


Takeaway:
Child representations are overly smooth — to become adult-like, many representations must sharpen perceived probability differences.

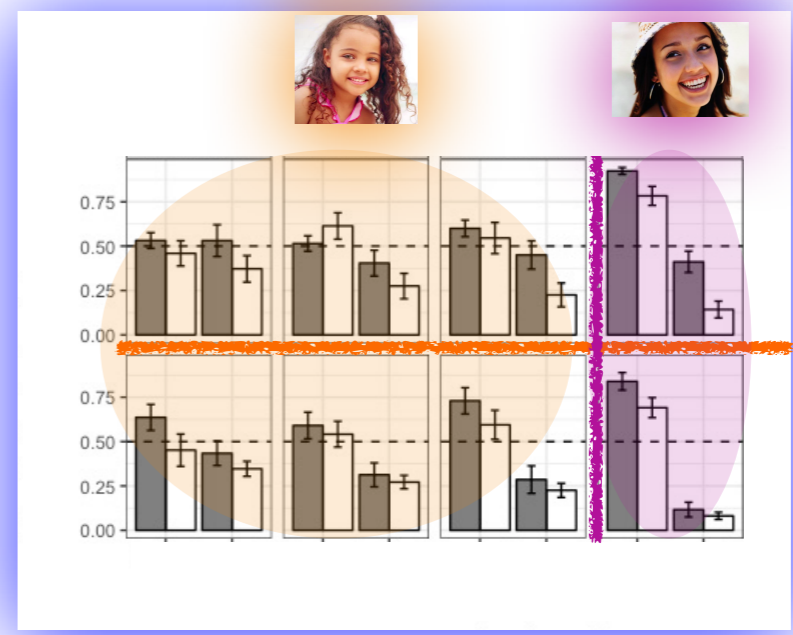
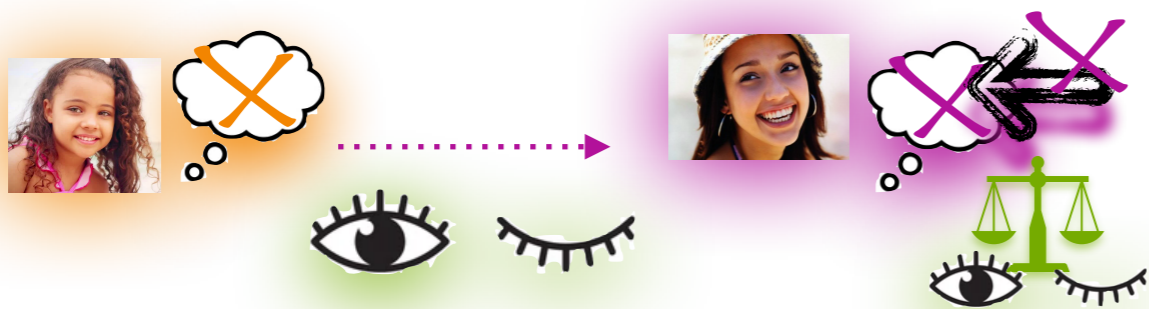


Important: Being adult-like doesn't mean being accurate!

Understanding the development of pronoun interpretation



Takeaway:
Adult deployment is selective — to become adult-like, children must learn how to **selectively ignore information** like adults.



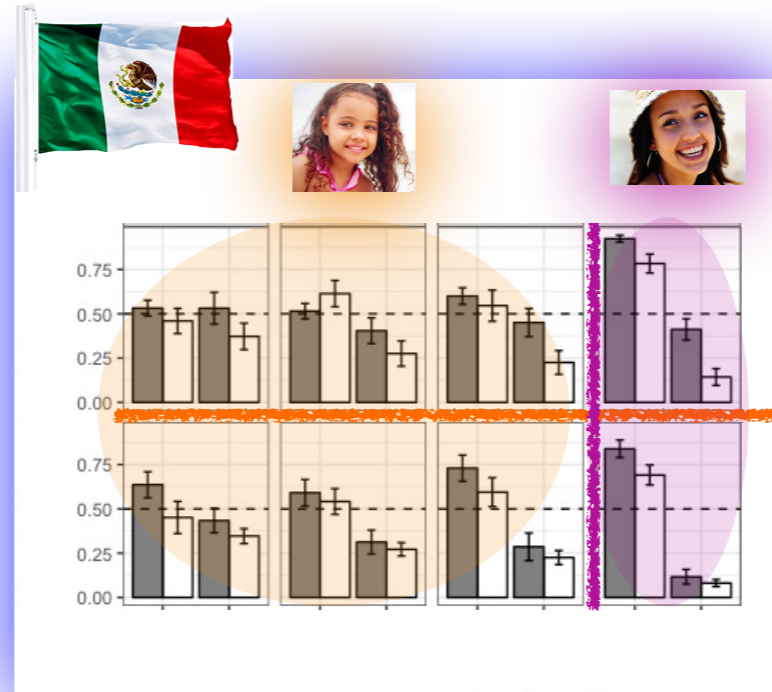
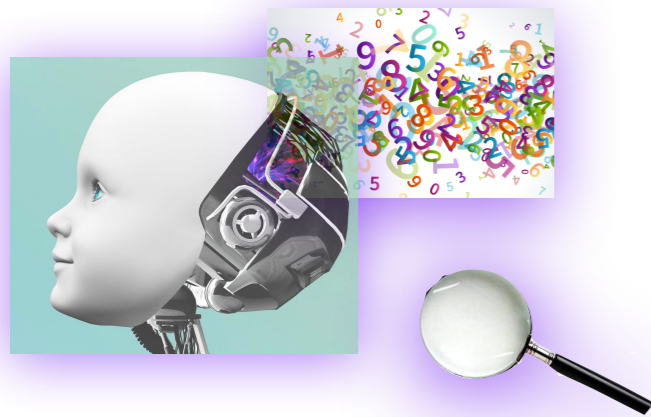
Important: Being adult-like doesn't mean being accurate!

The big picture



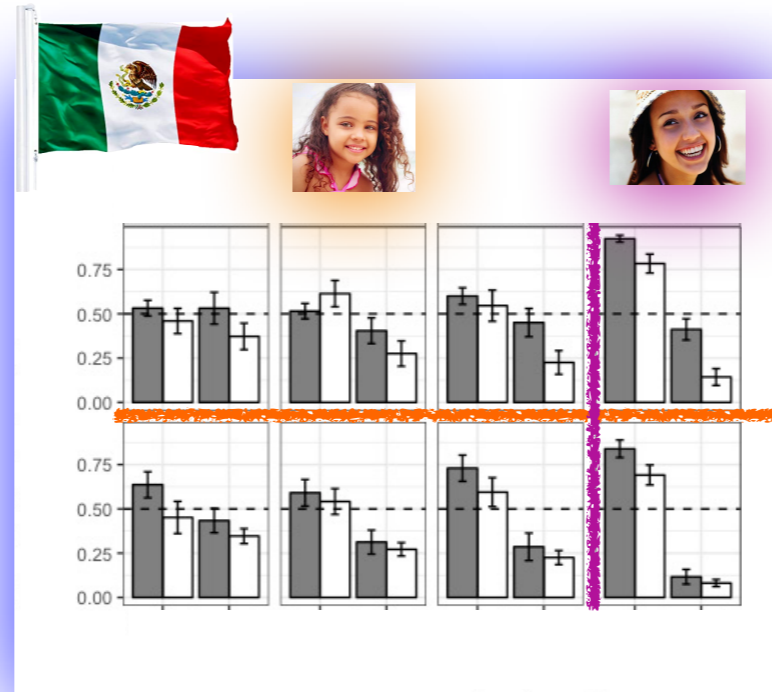
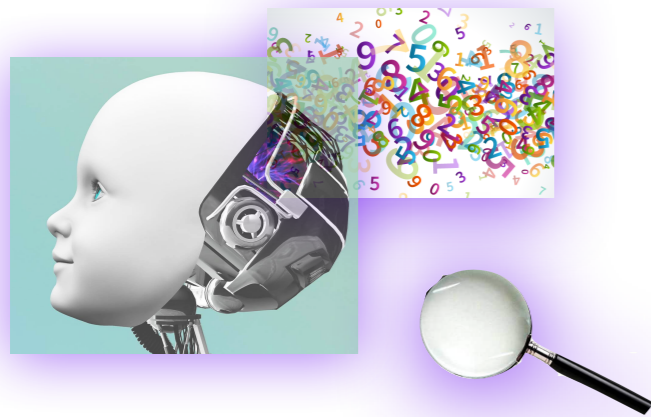
More generally, this case study demonstrates how we can use [computational cognitive modeling](#)...

The big picture



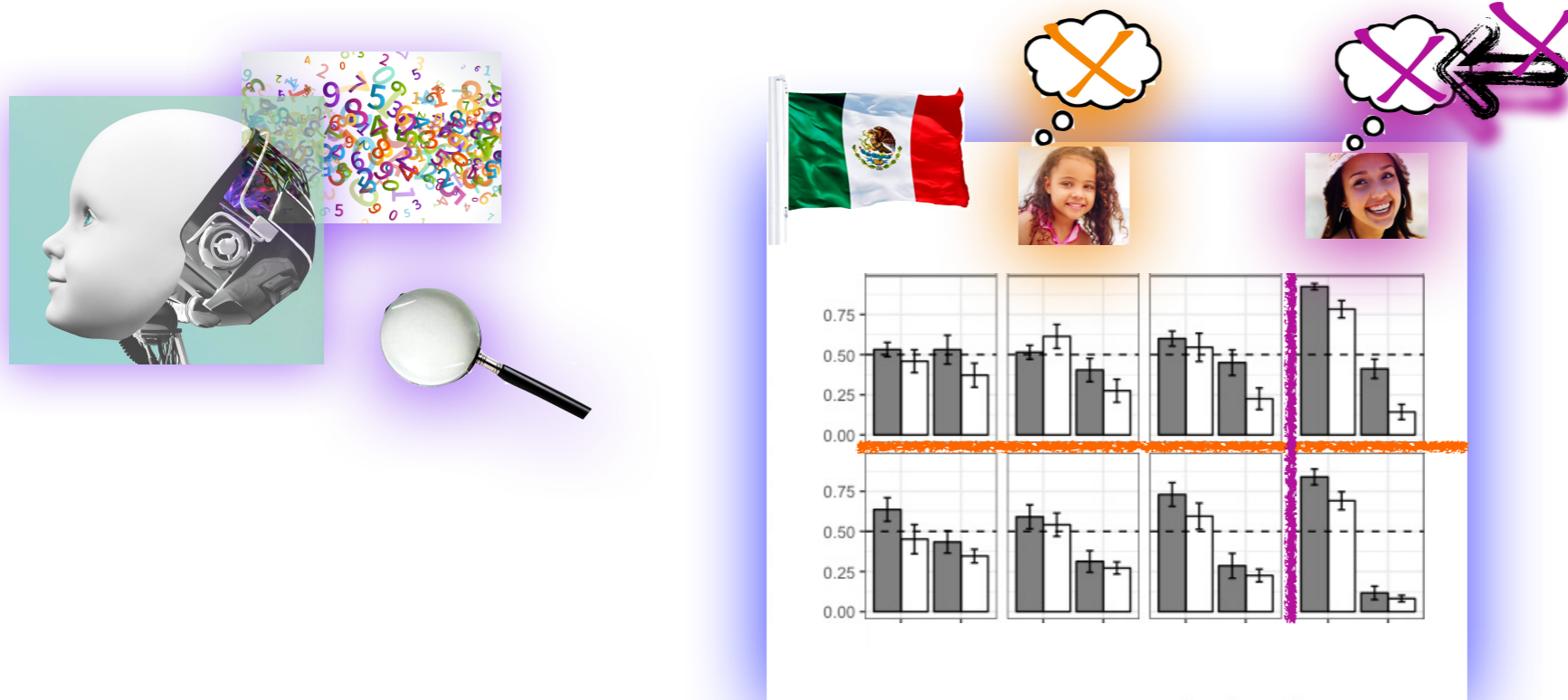
More generally, this case study demonstrates how we can use [computational cognitive modeling](#), grounded in [empirical data](#)...

The big picture



More generally, this case study demonstrates how we can use [computational cognitive modeling](#), grounded in [empirical data](#), to better understand how [children](#) and [adults](#)...

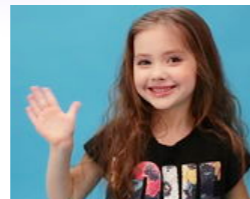
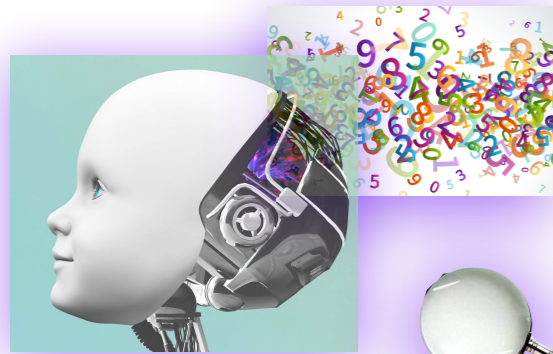
The big picture



More generally, this case study demonstrates how we can use **computational cognitive modeling**, grounded in **empirical data**, to better understand how **children** and **adults** can solve complex linguistic tasks (like interpreting pronouns in a context with multiple, potentially conflicting, cues).



The big picture



This helps us better understand **what children need to do to become adult-like** (and it seems to be about learning the right helpful biases).

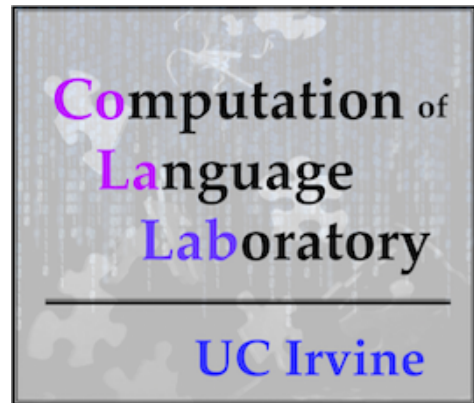


Thank you!

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