Individual Differences in Problem Solving and Intelligence

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Abstract

Studies of human problem solving have traditionally used deterministic tasks that require the execution of a systematic series of steps to reach a rational and optimal solution. Most real-world problems, however, are characterized by uncertainty, the need to consider an enormous number of variables and possible courses of action at each stage in solving the problem, and the need to optimize the solution subject to multiple interacting constraints. There are reliable individual differences in people's abilities to solve such realistic problems. It also seems likely that people's ability to solve these difficult problems reflects, or depends on, their intelligence. We report on a study of $N = 101$ adults who completed a series of visual optimization problems (Traveling Salesperson, Minimum Spanning Tree, and Generalized Steiner Tree problems), as well as a cognitive optimization problem (a version of the Secretary problem). We also characterized these individuals along three relevant and important cognitive abilities dimensions, fluid ability, visuo-spatial ability, and cognitive processing speed. Modeling of covariance structures indicated that performance on both types of optimization problems relies on general intelligence and raises the possibility that they can be used to assess intelligence.

Key Words: Traveling Salesperson Problem; Secretary Problem; Human Intelligence; Cognitive abilities

Individual Differences in Problem Solving and Intelligence

This paper describes a study on individual differences in two classes of decision making problems: difficult optimization problems that are presented visually and which can be thought of as perceptual problems; and a difficult sequential decision-making task that involves uncertainty and places demands on working memory and which can be thought of as a cognitive problem.

Recent research has explored whether performance on perceptual optimization problems, particularly the Traveling Salesperson Problem (TSP; Lawler, Lenstra, Rinooy Kan, & Shmoys, 1985), depends on psychometric intelligence (e.g., Vickers, Butavicius, Lee, & Medvedev, 2001; Vickers, Mayo, Heitmann, Lee, & Hughes, 2004). Vickers et al. (2004) reported that in two separate studies performance on TSP correlated with scores on Raven's Advanced Progressive Matrices (Raven, Court, & Raven, 1988) to the extent of about 25 per cent shared variance. Correlations were of similar magnitude for two other perceptual optimization problems, known as the Minimum Spanning Tree Problem (MSTP: Ahuja, Magnanti, & Orlin, 1993) and Generalized Steiner Tree Problem (GSTP: Hwang, Richards, & Winter, 1992). There are no published reports of relationships between sequential decision-making tasks and psychometric intelligence but clear and reliable individual differences in performance on one such task, the full-information version of the Secretary Problem, have been described (Lee, O'Connor, & Welsh, 2004).

Meanwhile, theories on the structure of human intelligence have converged on a model that incorporates at least two strata but arguably three. The first stratum comprises over 60 abilities that are rather narrow and which correspond conceptually to Thurstone's Primary Mental Abilities (Thurstone, 1938, 1947). The pattern of inter-correlations among these first-stratum abilities defines about ten broad abilities at the second stratum. These second-stratum abilities include fluid and crystallized intelligence, as first described by Cattell (1943, 1963), along with others more recently described, primarily by Horn and co-workers (see Horn & Noll, 1997). Controversy remains on the interpretation of the pattern of intercorrelations among these broad second-order abilities. Many researchers and theorists argue for the existence and primacy of a third-stratum general factor (g;

see Carroll, 1993; Jensen, 1998) while others dismiss the importance of *g* (e.g., Stankov, 2002). Deary and Caryl (1997) make the point that whether the broad second-order abilities or the putative third-order general factor are considered most important depends on the particular research question at hand. The point of all this, however, is that it is no longer sufficient to attempt to understand human intelligence, its determinants or consequences, in terms of scores on a single psychometric instrument such as Raven's Matrices.

Here, we examine the relationship of individual differences on difficult perceptual and cognitive problems with psychometric tests chosen to define the three most relevant broad cognitive abilities identified by modern theories of human intelligence: fluid ability (Gf), visuo-spatial ability (Gv), and cognitive processing speed (Gs).

Perceptual Optimization Problems

As described in more detail elsewhere (Vickers et al., 2004), much research into the relationship between problem solving processes and other cognitive abilities has concentrated either on knowledge-lean problems suitable for experimental study, or on complex but domain specific real-world problems. Vickers at al. argued that difficult combinatorial optimization problems have the advantage that while there is often no algorithm that can produce a definitive solution to them in a reasonable time, they can be simply stated and readily understood. Moreover, they can be representative of real-world situations and human performance on them is often close to optimal (e.g., Graham, Joshi, & Pizlo, 2000).

In the TSP, participants are given a set of *n* interconnected nodes which lie on a 2-D Euclidean space and they must devise an itinerary that visits each node exactly once, returns to the starting node, and ensures that the total length of the tour is as short as possible. To arrive at a definitive solution to this problem entails an exhaustive consideration of (n-1)!/2 pathways (see Figure 1a).

In MSTP, participants are required to find the shortest path that directly links all the nodes in an array. This problem has fewer constraints than TSP because the path does not have to be

continuous and closed and a node can be connected by more than two links. The optimal solution is an open, branching path system, which directly links all the nodes and in which branches occur at the nodes (see Figure 1b).

In the elementary three-node Steiner Tree Problem, the shortest path connecting three nodes in a plane can be found by determining the so-called Fermat point and creating a path of links that branch at that point (as illustrated in Fig. 1c). GSTPs contain more than three nodes and the solutions to these problems look like combinations of solutions found in three-node Steiner Tree problems, with additional nodes (P1, P2,…,Pk) as branch points to create the minimum connections between the original nodes, as illustrated in Fig. 1d.

Insert Figure 1 about here

Vickers et al. (2004) showed that performance on versions of these three tasks was characteristic of an individual, with the average intercorrelation among the three tasks implying about 50% shared variance. Moreover, correlations on performance across two rotated versions of these tasks were good with the mean across three tasks being about $r = .7$.

Cognitive Optimization Problem

Many real world decision-making problems are sequential in nature. A series of choices is made available over time and it is often efficient (and sometimes even necessary) to make a selection without waiting to be presented with all of the alternatives. This decision-making scenario has the same essential features as a recreational mathematics problem known as the Secretary Problem (see Ferguson, 1989, for a historical overview). In Secretary Problems, participants are presented with a sequence of possible choices and must decide whether to accept or reject each possibility in turn. The so-called 'full information' version of the problem presents participants with a score from a known distribution for each possibility; the goal is to choose the maximum score in the sequence and any incorrect decision is equally wrong. Gilbert and Mosteller (1966) showed that

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the optimal decision rule for these problems requires choosing the first value that exceeds a threshold level for its position in the sequence.

Because they are not inherently perceptual, Secretary Problems allow consideration of whether results obtained with perceptual optimization problems generalize to cognitively-based problem solving. Secretary Problems also introduce uncertainty and place demands on working memory. While visual problems like TSPs are combinatorially large, the basic information about distances between points is always perceptually available in a complete and certain form to the problem solver. By contrast, the sequences of information in secretary problems are stochastic and presented only temporarily, requiring people to deal with uncertainty and to rely on their memory.

The Current Study

To understand better the previously reported relationship between performance on difficult optimization problems and one, albeit limited, measure of intelligence and to extend that line of research to include a sequential decision-making problem, we administered TSP, MSTP, GSTP, and Secretary Problems as well as marker tests for Gf, Gv, and Gs. We used a sample of sufficient size to model relationships of the problem solving tasks and psychometric tasks using analysis of covariance structures. A series of alternative theoretically plausible models were compared to delineate further the relationships reported by Vickers et al (2001, 2004).

Methods

Participants

There were $N = 101$ participants (58 males; mean age = 25.3, $SD = 7.6$ years) recruited from the general community via advertisements and word-of-mouth. All were paid A$30 at completion of the study.

Materials

Perceptual Optimization Problems Problems were presented, one at a time, in a 15 cm x 15 cm square in the center of a standard 19-inch computer display. Participants could begin at any point by left-clicking on a node (or on a location where they wished to establish a new node) with

the computer mouse. They drew a path by 'dragging' the mouse cursor to a subsequent node and releasing the button, causing a straight line to be drawn between that node and the previously visited node. By right-clicking on a link (or created node) to select it, and then pressing the ''Delete'' key on the keyboard, participants could undo any links or nodes they had drawn. Participants were thus free to connect the nodes in any order, to work alternately from two nodes, or to work on several separated clusters of nodes. If a participant's completed solution was invalid (e.g. because not all TSP nodes had been connected), a warning message was posted on the screen and the participant was obliged to construct a valid solution before proceeding to the next problem.

Participants completed one 30-, 60-, and 90-node TSP and the same for MSTP; and one 15-, 20-, and 25-node GSTP. Participants were given verbal instructions and there were also instructions on the computer screen which could be re-displayed at any time by clicking on a button. There were no time constraints on performance, but participants were asked to complete the tests as quickly and as accurately as they could.

For TSP, participants were asked to connect all the nodes in a continuous tour that visited each node once only, returned to the starting node, and was as short as possible. In the case of MSTP, participants were asked to connect all the noses by make a system of paths between them, using as many paths as necessary, but only between existing nodes, and creating a path system with the shortest possible overall length. For GSTP, participants were asked to connect all the nodes by making a system of paths with the shortest overall length. In this case, however, participants were told that the existing nodes did not have to be linked directly. If they wished, participants could create additional nodes, but they were required to create at least one additional node.

Secretary Problem Each participant completed two sets of problems presented on a computer. The first set consisted of 40 problems of length 5 and the second 40 problems of length 10. Participants did the sets in the same order—length 5, then length 10—but the order of the problems within each set was randomized across participants. For each problem, the participants were told that the values they would see were dollar amounts ranging from zero to one hundred

dollars. They were told the length of the sequence and were instructed to choose the maximum value. It was emphasized that (a) the values were uniformly and randomly distributed between 0.00 and 100.00, (b) a value could only be chosen at the time it was presented, (c) the goal was to select the maximum value, with any selection below the maximum being completely incorrect, and (d) if no choice had been made when the last value was presented, they would be forced to choose this value. As each value was presented, its position in the sequence was shown, together with 'yes' and 'no' response buttons.

Psychometric Tests Participants completed custom computerised adaptations of the Raven Standard Progressive Matrices (SPM; Raven, 1956), Cattell Culture Fair Test Scale 2 Form A (CCF; Cattell & Cattell, 1959), and Spatial Relations from the Woodcock-Johnson Psycho-Educational Battery-Revised (WJ-R; Woodcock & Johnson, 1989). For SPM, participants worked through a series of up to 60 matrices questions; we used a timed version with a 20 min limit. The CCF has four sections, Series, Classification, Matrices and Conditions. Participants worked through examples, followed by the 12, 14, 12, and 8 items for each section, respectively. Each section had a fixed time limit and the total test lasted 12.5 min. Spatial Relations presented participants with a large shape comprised of smaller components. Next to this, were displayed a set of small shapes and participants decided which of these together constituted the large shape. The items become progressively more difficult with the shapes becoming more complex and with more components in each large shape. The test comprised 33 items and finished when the participant answered incorrectly on six items in a row, or when they had completed all items. A fourth computerised test was Picture Swaps, based on a test described by Crawford (1988). Participants are required mentally to swap the order of three pictures presented on the computer screen, according to instructions also presented on the screen, and then to indicate the final order of the pictures. Once they had worked out the answer, they pressed the spacebar on the keyboard to bring up another screen displaying the six possible solutions, numbered 1 to 6. Participants responded via the keyboard numberpad. There were four difficulty levels for the test which differed in the number of

swaps required (1-, 2-, 3-, or 4-swaps). There were six, items for both the one- and two-swap levels, and 12 items for the three- and four-swap levels.

There were also three paper-and-pencil tests, Space Relations from the Differential Aptitude Tests Form W (Bennet, Seashore, & Wesman, 1989), Digit Symbol from the Wechsler Adult Intelligence Scale-III (Wechsler, 1997), and Visual Matching from the WJ-R (Woodcock & Johnson, 1989). Apart from using only the 30 odd-numbered items from Space Relations, these tests were administered according to instructions in the respective manuals. Space Relations had a time limit of 12.5 min, the other two tests had a time limits of 3 min.

Procedure

Participants attended the laboratory either individually, or two at a time. Paper-and-pencil tests were completed first in the order described above, followed by TSP, MSTP, GSTP, and Secretary Problem. The computerised psychometric tests were then completed in the order described above. There were two workstations separated by a partition so that participants attending in pairs were isolated from each other when completing the computerised tests. Testing sessions lasted about two hours.

Results

For the perceptual optimization problems, participants' solution lengths for each problem were expressed as a proportion above the benchmark, or best known, solution length (PAB) for that problem. Thus, a PAB score for an optimal solution would be 0 and any solution that exceeded the benchmark would have a positive score. The final measures used for TSP, MSTP, and GSTP in subsequent analyses were the mean PABs for the three problems of each type. For Secretary Problems the measure used was the mean proportion of trials for which the participant chose in accordance with the prediction of the optimal decision rule, averaged across length 5 and length 10 problems.

For SPM, it is known that some items measure visuo-spatial ability while others measure verbal-analytical reasoning (Lynn, Allik, & Irwing, 2004). We therefore created item parcels by

summing across these different types of items. Hereafter, we designate these as SPM-VSp and SPM-VAR, respectively. A similar situation applies to CCF (see Burns & Nettelbeck, 2003). Again, we created item parcels hereafter designated CCF-VSp and CCF-VAR. For Spatial Relations, Picture Swaps, Space Relations, Digit Symbol and Visual Matching, total items correct scores were used. Table 1 shows descriptive statistics for measures used in subsequent analyses. SPM-VSp, CCF-VSp, Spatial Relations and Space Relations were included as measures of Gv; SPM –VAR, CCF-VAR and Picture Swaps as measures of Gf, and Digit Symbol and Visual Matching as measures of Gs.

Insert Table 1 about here

The first set of models to be compared here is as follows: first, a model where all variables load a single latent variable; thus, this model proposes that covariances of all variables depends on a single construct, general intelligence (*g*). This is referred to hereafter as Model 1. Second, a model is proposed where performance on the problem solving tasks used here depends on a 'problem solving ability' different from but related to those currently described in psychometric theories of intelligence. This is referred to as Model 2. The next model allows that the combinatorial optimization problems depend on Gv whereas Secretary Problem depends on Gf. This model, hereafter Model 3, arises *prima facie* on the basis of the requirements of these classes of tasks, as described above. All models were fitted using AMOS 5 (Arbuckle, 2003) or LISREL 8.5 (Joreskög & Sörbom, 2003).

Comparison of these types of models, often referred to generically as structural equation models, should be done using a range of fit criteria, specifically, goodness-of-fit and model complexity should be considered (Kline, 2005). Here we use the Bayesian Information Criterion (BIC; Schwartz, 1978; Raftery, 1995), the Root Mean Square Error of Approximation (RMSEA; Steiger & Lind, 1980) along with its 90% confidence interval, and the likelihood ratio chi-square.

The BIC takes account of sample size and penalizes model complexity; the RMSEA is a parsimony-adjusted index where values less than about .05 indicate close approximate fit and RMSEA greater than or equal to .10 suggests poor fit. The likelihood ratio chi-square tests the hypothesis that the model is correct but it is sensitive to the size of correlations and to sample size; commonly, to overcome these problems, it is divided by the model degrees of freedom and a rule-of-thumb is that this value should be less than about two for a good fitting model. Table 2 shows the fit criteria for the first set of models considered.

Insert Table 2 about here

None of the models fit the data well. Examination of the parameters of the best-fitting model, that is, Model 2, showed that GSTP had an estimated standardized regression weight on the latent variable representing "problem solving" of .98, while TSP, MSTP, and Secretary Problem had weights of .70, .78, and .13, respectively. These estimates suggest misspecification of this model arising because of collinearity of the perceptual optimization problems. Therefore, these three variables were combined by standardizing each and taking the mean. This new measure is hereafter referred to as Perceptual Optimization Problems. This variable was incorporated into a new model, hereafter Model 4 (see Figure 2), based on Model 3. It can be seen in Figure 2 that Model 4 is better than any of the others considered so far. It should also be noted that this model can be equivalently represented with a higher-order general factor instead of correlated first-order factors.

Insert Figure 2 about here

The final model to be considered, hereafter Model 5, is suggested by examination of Figure 2 and has the interesting property of allowing all variables to load on a single latent variable,

representing g, while at the same time allowing Digit Symbol and Visual Matching to load on an orthogonal latent variable representing Gs.

Thus, Model 5 allows that in this sample and with these variables, Gf and Gv cannot be distinguished from each other (this is a not uncommon outcome, see Danthiir, Roberts, Pallier & Stankov, 2001), or from g (see also Gustaffson & Undheim, 1996). By contrast Gs is well marked. Model 5 has BIC = 167.7, RMSEA = .07. CI90 = [.02, .10], and $\chi2(43) = 61.6$, p = .03. While this model is marginally less plausible than Model 4, because of the orthogonality constraint imposed, it shows the loadings of all variables on g after Digit Symbol and Visual Matching have been residuals for Gs (see Table 3).

Insert Table 3 about here

Discussion

The main question of interest here was on the relationship between individual differences in performance on difficult optimization problems and constructs described in modern psychometric theories of human intelligence. The two classes of problem solving tasks we used had the very desirable characteristic that they are representative of real-world decision making scenarios. Definitions of intelligence invariably make reference to problem solving, along with the ability to reason, plan, think abstractly, comprehend complex ideas, learn quickly and learn from experience (see e.g., Neisser et al., 1996) but a common criticism of traditional intelligence testing is that it fails to capture the requirements of real-world problem solving.

When difficult optimization problems were jointly examined with a test of intelligence, robust relationships were found (Vickers et al. 2004). Here, we acknowledged that human intelligence is better described in terms of a hierarchical model of related abilities with constructs being most narrow at the foot of the hierarchy and very general at the apex. We sampled abilities with the aim of defining fluid ability (Gf), which includes reasoning abilities and working memory

and is arguably indistinguishable from *g*; visuo-spatial ability (Gv), that is, abilities in apprehending the forms, shapes and positions of objects and forming and manipulating mental representations of those forms, shapes and positions; and cognitive processing speed (Gs) which many view as fundamental to individual differences in intelligence.

We found that both a single general factor model (Model 1) and a model incorporating a separate problem solving ability (Model 2) were inadequate to account for the data we observed. A model (Model 4) that allowed a variable representing the perceptual optimization problems to load Gv, and the Secretary Problem to load Gf, with both Gv and Gf correlated with each other and Gs, provided a good account of the data. Not reported here were variations on this model with the optimization and secretary problems loading either on the alternate or on both latent variables (i.e., Gv and Gf); these were poorer fitting models. Thus, at this level of analysis, both classes of optimization problems were shown as being relatively good measures of Gv or Gf, respectively.

The final model considered comprised two orthogonal latent variables. All variables loaded on general ability but the two cognitive speed measures also loaded on Gs. This arrangement allowed us to assess the *g*-loadings of all measures. The main outcome was that the variable representing the combinatorial optimization problems had a loading of about .5 on *g*, while Secretary Problem had a loading of about .4. These loadings are high enough to encourage us to speculate that both of these types of problems could prove useful as measures of cognitive ability.

Much work is required before, for example, TSP or Secretary Problem could be routinely used to assess intelligence; clearly, performance on both types of problem should be validated against criteria other than just intelligence tests so as to determine their predictive validity. Such problems may fill a gap in current test batteries because they are demonstrably related to real-world problem solving; and performance on them does not depend on acculturated knowledge.

Finally, we suggest that studies on the relationships between problems like TSP and Secretary Problem and intelligence should consider: (i) the developmental trajectories of performance on these problem solving tasks in comparison to those of other cognitive abilities; and

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(ii) relationships between solution times on problem solving tasks and performance on so-called elementary cognitive tasks, including psycho-motor speed, which correlate substantially with cognitive abilities. Of promise also is the application of models on TSP and Secretary Problem performance to a better understanding of individual differences in human intelligence.

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Acknowledgements

Table 1

Descriptive statistics and Pearson correlations for optimization problems, Secretary

Problem, and psychometric measures for N = 101 participants.

1	TSP	1	2	3	4	5	6	7	8	9	10	11	12	13
2	MSTP	.54												
3	GSTP	.69	.78											
4	SecProb	-.14	-.10	-.13										
5	SPM-VSp	-.27	-.22	-.39	.21									
6	SPM-VAR	-.36	-.29	-.43	.23	.52								
7	CCF-VSp	-.25	-.20	-.23	.14	.40	.38							
8	CCF-VAR	-.31	-.20	-.31	.24	.28	.42	.48						
9	SpatRel	-.36	-.27	-.35	.25	.45	.49	.50	.32					
10	PicSwaps	-.25	-.19	-.28	.37	.58	.35	.39	.31	.46				
11	SpaceRel	-.38	-.11	-.29	.20	.30	.32	.47	.51	.35	.42			
12	DigSym	-.09	-.12	-.10	.15	.26	.33	.13	.32	.08	.34	.19		
13	VisMat	-.14	-.25	-.14	.19	.34	.33	.23	.28	.15	.37	.22	.63	
	Mean	.093	.061	.140	.711	19.1	6.98	20.4	6.50	20.5	28.4	25.9	85.5	42.5
	SD	.041	.073	.104	.126	2.31	3.75	3.26	2.22	4.63	7.62	4.84	15.4	5.54

Note: TSP is Traveling Salesperson Problem; MSTP is Minimum Spanning Tree Problem; GSTP is Generalized Steiner Tree Problem; SecProb is Secretary Problem; SPM-VSp is Standard Progressive Matrices-Visuo-Spatial; SPM-VAR is Standard Progressive Matrices-Verbal Analytic Reasoning; CCF-VSP is Cattell Culture Fair-Visuo-Spatial; CCF-VAR is Cattell Culture Fair-Verbal Analytic Reasoning; SpatRel is Spatial Relations; PicSwaps is Picture Swaps; SpaceRel is Space Relations; DigSym is Digit Symbol; VisMat is Visual Matching

Table 2

Fit criteria for three models showing Bayesian Information Criterion (BIC), Root Mean Square Error of Approximation (RMSEA) and its 90% CI, and Likelihood Ratio Chi-Square. For comparison purposes, the relevant statistics are shown for the saturated model, where no constraints are imposed and fit is perfect; and for the null model where population correlations are assumed to be zero.

Model	BIC	RMSEA CI$_{90}$	χ^2 (df)	p
1	356.5	.16 [.14, .18]	236.5 (65)	<.001
2	239.5	.07 [.04, .10]	91.8 (59)	.004
3	314.1	.14 [.11, .16]	180.3 (62)	<.001
Saturated	420.0	-	0 (0)	-
Null	596.6	.24 [.22, .26]	536.6 (78)	<.001

Note: For comparison purposes, the relevant statistics are shown for the saturated model, where no constraints are imposed and fit is perfect; and for the null model where population correlations are assumed to be zero.

Table 3

Maximum likelihood estimates of loadings on two orthogonal latent variables.

	g	Gs
OpProb	.492	—
SecProb	.368	—
SPM-VSp	.642	—
SPM-VAR	.668	—
CCF-VSp	.617	—
CCF-VAR	.601	—
SpatRel	.662	—
PicSwaps	.649	—
SpacRel	.594	—
DigSym	.357	.828
VisMat	.433	.575

Note: See Table 1 for variable names except OpProb is mean of standardized scores for TSP, MSTP and GSTP. All estimated loadings differ from zero $p < .001$. Loadings fixed to zero marked as —.

Figure Captions

Figure 1. (a) 50-node TSP array; (b) 50-node MTSP array. For both (a) and (b) the optimal solution is shown as solid connecting edges. (c) the Fermat point (P) for a simple Steiner Tree problem as an open circle, with the optimal solution shown by broken lines. (d). 15-node GSTP with the interpolated open points (open circles) and the optimal tree solution (broken lines) found by a computer algorithm.

Figure 2. Model 4 showing maximum likelihood estimates of standardized regression weights and fit criteria. See Tables 1 & 3 for variable names.
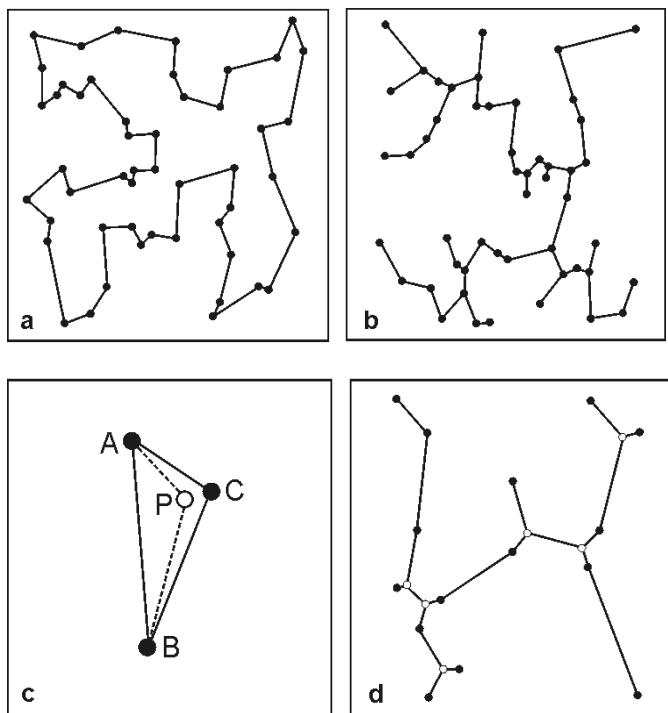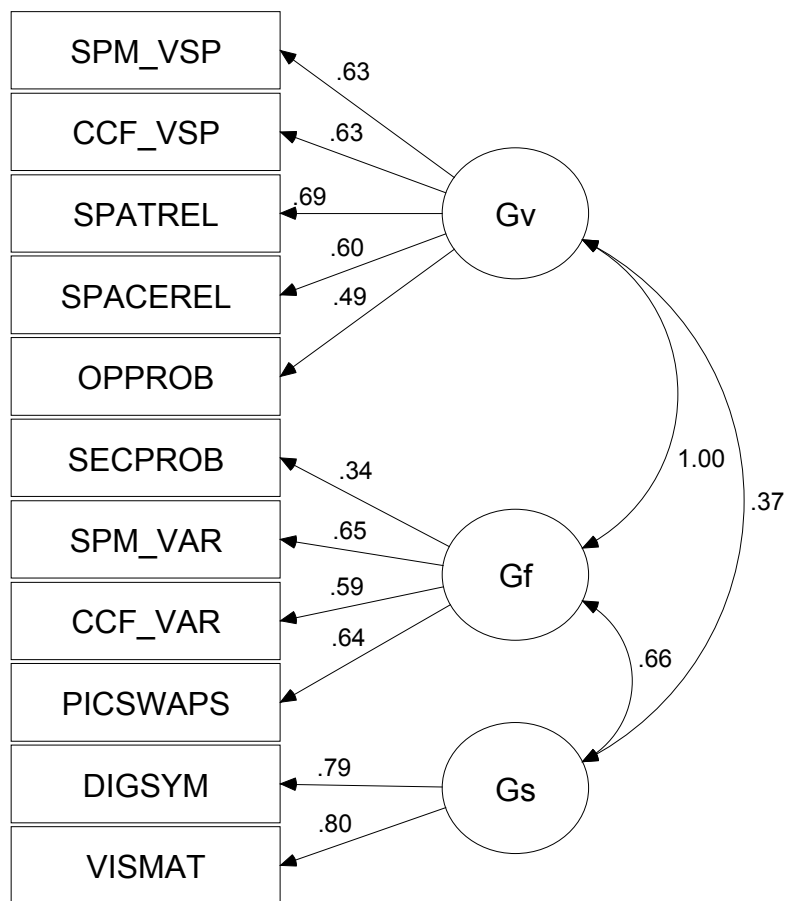
Figure 1 Burns et al.

BIC = 168.00
RMSEA = .05; CI90 = [.00, .09]
CHI SQ(41) = 52.62, p = .11

Figure 2 Burns et al.