Trade Interdependence, Arming and the Choice between War and Peace[†]

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Abstract: We consider a dynamic setting where two countries with competing claims to a resource/asset first arm and then choose whether to resolve their dispute through war or peacefully through settlement. War precludes international trade and can be destructive, but also locks gains and eliminates arming costs in the future. By contrast, a peaceful resolution, possibly supported by arming, avoids destruction and allows for mutually advantageous trade; yet future settlements must be renegotiated under the threat of war. We characterize the conditions under which peace is stable and show that, depending on war's destructiveness, time preferences, and the distribution of resource endowments, greater gains from trade can pacify international tensions, but possibly only for more uneven endowment distributions.

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If an economically self-sufficient man starts a feud against another autarkic man, no specific problems of "war-economy" arise. But if the tailor goes to war against the baker, he must henceforth produce his bread for himself.

Ludwig von Mises (1949, p. 824)

1 Introduction

To what extent does economic interdependence between countries induce more peaceful bilateral relations? The time-honored "liberal peace" hypothesis, advanced and extensively tested by scholars of international relations, states that, because war undermines possibilities for trade, the opportunity cost of war rises as national economies become more highly integrated; with greater interdependence, therefore, we should expect extended peace (e.g., Polachek, 1980).[1] But, while the logic of the liberal peace hypothesis, as spelled out in simple terms by von Mises in the quote above and more generally, addresses the choice between war and peace, it altogether abstracts from arming decisions.[2] Naturally, such decisions are influenced by the anticipation of war or peace. At the same time, however, by influencing the amount of resources available for production of tradable commodities, these decisions condition the potential gains from trade and, consequently, the relative appeal of peace. With an aim to deepen our understanding of the links between international trade and peace, this paper treats the interrelated decisions on arming and conflict initiation as endogenous, studying them in the context of a simple, dynamic general equilibrium model where two large countries dispute the ownership of some resource.[3]

The basic setup of the model is as follows. Once the two contending countries have decided how much of their initial endowments to devote to arming, they choose how to resolve their dispute over the remaining resources in the current period—what we call the "residual" resource. One option involves open conflict (or war), modeled as a winner-take-all contest, with both countries deploying their arms to improve their respective chances of victory and some fraction of the residual resource being destroyed as a result. War also

[1]See Glick and Taylor (2010), for example, who provide evidence that the disruptive effects of war on trade are large. Although the expansion of world trade along with the sharp drop in the frequency of interstate wars witnessed in the decades following the World War period would appear to support the optimism of the liberal peace hypothesis, the evidence based on formal testing is somewhat mixed. Oneal and Russett (1997, 1999), for example, find that the likelihood of war breaking out between two countries depends negatively on the interdependence between them, whereas Barbieri (2002) presents evidence of no significant relation between trade and war. See Copeland (2015) for a comprehensive survey of alternative views and the empirical evidence regarding trade and war in the international relations literature. Morelli and Sonno's (2017) review of that book provides a more compact survey of the relevant literature and presents some new evidence.

[2]Global military spending as a fraction of GDP has fallen considerably since the end of the Cold War, but remains significant at over 2 percent of global GDP (Tian et al., 2024).

[3]The assumption of a materialistic motive for arming and war is consistent with the history of empire building, but is also relevant in current conflicts over resources—e.g., the ongoing dispute between China, Taiwan, Vietnam, the Philippines, Indonesia, Malaysia, and Brunei over control of the Spratly and Paracel islands in the South China Sea (fueled by the suspected abundance of natural resources tied to those islands and the surrounding sea) that might escalate to war despite their trading relations.

destroys the possibility of trade. The other option involves peace, in which case the two countries negotiate a division of the residual resource, with the division depending on their arming choices made in the first stage. Since no arms are actually deployed, this option implies no destruction. Furthermore, it leaves open the opportunity for subsequent trade in the current period. Specifically, each country with its secure resources resulting from the negotiated settlement produces a distinct tradable intermediate good that serves as an input in the production of a consumption good. Trade enhances each country's ability to produce that final good, and herein lie the gains from trade. To keep the analysis simple and succinct, we employ the Armington (1969) model under the standard assumption of symmetric aggregate productivities, which implies that countries' gains from trade depend positively on the degree of heterogeneity between the intermediate inputs they produce (or, equivalently, negatively on the elasticity of substitution) and positively on the evenness of the international distribution of the residual resource.[4] In view of those gains for any positive quantity of guns chosen in the first stage and the possible avoidance of war's destructive effects and assuming that information is complete, countries always have a *short-run* incentive to negotiate a peaceful settlement.

But, when countries take a *longer-run* perspective, settlement need not always emerge as part of equilibrium. The reason is that settlement in the current period concerns the division of resources in that period only. Without the ability to commit today to a division of resources in the future, the two countries might have to arm in each period to settle their ongoing dispute, implying the diversion of additional resources in the future away from the production of consumption goods.[5] The possible appeal of open conflict derives, at least in part, from giving the victor a strategic advantage in future disputes, so that fighting today reduces future arming costs relative to those under settlement.[6] Indeed, depending on the gains from trade, time preferences, and the degree of war's destructiveness, one country could strictly prefer open conflict to peaceful settlement when the resource dispute extends beyond the current period. In such cases, open conflict emerges as the unique subgame perfect, Nash equilibrium. Even when peaceful settlement is Pareto dominant, one or both countries could have an incentive to deviate unilaterally today not only to take control of the aggregate residual resource (net of what is destroyed) but also to reduce future arming costs;

[4]As we explain later on, our analysis remains intact in the context of other models of trade, motivated by differences in technology, including the Eaton and Kortum (2002) trade model in which countries produce and trade a continuum of intermediate goods. Thus, the driving force behind our analysis is not necessarily the manner in which trade (or its cessation) alters countries' patterns of specialization in production per se (as might be inferred from the quote above by von Mises), but rather their gains from trade.

[5]While there always exists a unique outcome under settlement with strictly positive arming by both countries, unarmed peace might also be feasible and, under some conditions, the Pareto preferred outcome. In such cases, this consideration is not relevant.

[6]This logic is reminiscent of that in Garfinkel and Skaperdas (2000) and Skaperdas (2006) mentioned above. Fearon (1995) and Powell (2006) provide similar arguments based on the notion that negotiated settlements for future divisions are not enforceable, but they differ in their emphasis on the importance of exogenous shifts in power. Likewise, while Skaperdas and Syropoulos (1996) do not consider the choice between war and settlement, they emphasize the effect of using military force today to enhance future payoffs.

the presence of such an incentive undermines the subgame perfection of peace.

A central component of our analysis involves identifying the conditions under which peaceful settlement emerges as the *stable* equilibrium, defined as being both Pareto preferred (over open conflict) and subgame perfect. Like Garfinkel and Skaperdas (2000) and Skaperdas (2006) among others who distinguish between the mobilization of resources to produce guns and the decision to use those guns in open conflict in dynamic settings (though without trade), we find that peaceful settlement is more likely to be the stable equilibrium outcome over time when the destructive effects of war are large and when the shadow of the future is weak (i.e., countries discount the future heavily).[7]

The primary contribution of this paper, however, is to characterize the importance of trade openness—which, along the lines of the liberal peace hypothesis, we assume exists under peace but not under war—in each country's optimizing choices. This characterization centers on the per-period relative gains from trade that, as mentioned above, depend positively on the degree of heterogeneity of tradable inputs and the evenness of the initial distribution of resources across the contending countries. Our approach allows us to carry out two tasks: (i) derive the conditions under which peace emerges as a stable equilibrium; and, (ii) show how the gains from trade determine the nature of peace—namely, whether it involves arming or not. In the case of *unarmed peace*, the status-quo distribution of contested resources is preserved, whereas in the case of *armed peace* countries seek to tilt (through arming) the division of contested resources in their favor.

Our analysis establishes that there exists a threshold rate of war's destruction, above which peace is stable for all feasible distributions and even if the gains from trade are negligible. This is not to say, however, that trade is inconsequential. In this case, the magnitude and distribution of the gains from trade determine the form that peace takes: unarmed peace emerges as the stable equilibrium when the gains from trade are sufficiently large for both countries; otherwise, armed peace emerges as the stable equilibrium for all initial resource distributions.

When war is only moderately destructive, trade takes on a greater role in determining the stability of peace. Specifically, if tradable goods are sufficiently heterogeneous, the implied gains from trade alone can render peace stable for all feasible initial resource distributions and again possibly without any arming at all when the distribution is symmetric enough.[8] But, when traded goods are not sufficiently distinct, only armed peace is possible. This possibility itself depends, in addition, on the initial distribution of resource ownership as well as the strength of the shadow of the future. Interestingly, in this case, armed peace Pareto dominates open conflict for sufficiently even and for sufficiently uneven distributions of resources; however, unilateral deviations are profitable for at least one and possibly both countries when the distribution is sufficiently even. Accordingly, armed peace is stable only

[7]See McBride and Skaperdas (2014) who provide experimental evidence in support of this latter prediction, which stands in sharp contrast to Folk-theorem type arguments.

[8]Such a possibility does not require war to be destructive.

for sufficiently uneven resource distributions.

This last finding highlights the value of going beyond to simple logic of the liberal peace hypothesis to explore the importance of the initial distribution of resource ownership and the way this distribution interacts with both the gains from trade and arming incentives to influence the stability of peace. The intuition is as follows. For any given degree of substitutability between traded goods and given arming choices, a perfectly even distribution of residual resources maximizes the global gains from trade. This tendency alone suggests a positive link between the evenness of international resource ownership and the relative appeal of settlement. But, due to strategic complementarity in gun choices, total arming under settlement tends to be higher precisely when the countries are more similar in size. As a result, the possible savings in future arming afforded by declaring war (instead of settling) today tend to be larger. Even when settlement is Pareto preferred to war, these possible savings could render unilateral deviations profitable and, hence, preclude the stability of armed peace.[9] For more uneven distributions, the smaller country has less to gain from war or by deviating from settlement, largely due to its limited resources for arming. Meanwhile, since peace is less costly for more uneven distributions, the larger country has less to gain by wiping out its adversary and foregoing any gains from trade; in addition, due to its relative size advantage, the larger country has some flexibility through its choice of arms to adjust its share of resources so as to realize greater gains from trade (than it could if residual resources were fixed). Thus, settlement with trade remains a possible stable outcome when the initial distribution of resources is sufficiently uneven.

The analysis of this paper is related to earlier work on resource disputes and trade between large countries, starting with Skaperdas and Syropoulos (2002) and more recently Garfinkel et al. (2020) and Garfinkel et al. (2022). Each of these papers considers two large countries that arm to compete for shares of a contested resource, but differ in terms of the question they address regarding trade. More specifically, Skaperdas and Syropoulos (2002) explore, in the context of a static exchange model, how the anticipation of trade influences arming choices made in support of bargaining. Garfinkel et al. (2022) study, within the context of a two-period setting, the effects of trade under peace in the current period when two countries arm in preparation for possible conflict in the next period. Both of these papers point to the possibility that trade can amplify arming incentives to make trade relatively unappealing to at least one country, though for different reasons. In Skaperdas and Syropoulos (2002), the impact of (anticipated) trade on arming is through its effects on the size of the bargaining set and the threatpoint payoffs that can, under some circumstances, make it in the interest of both sides to commit to foreclose on trade. In Garfinkel et al. (2022), the impact of (current) trade on arming for possible future conflict operates through an income channel, with the smaller country enjoying larger relative gains from trade and devoting a sufficiently larger fraction to arming that can render trade unappealing to the more affluent country.

[9] In this case, war emerges as the unique subgame perfect equilibrium.

Garfinkel et al. (2020) characterize the effects of trade as it works through a terms-of-trade channel, finding that trade can dampen arming incentives. Other related papers on arming and trade include Chang and Sellak (2019) and Chang and Wu (2020), though these are partial equilibrium analyses.

In sharp contrast to all of these papers, the present analysis considers explicitly, along side arming choices, the decision of whether to initiate war that would disrupt trade in the current and future periods but would also give the victor an advantage in their future dispute. We view the analysis as complementing the seminal work of Martin et al. (2008), recently generalized by Thoenig (2024), that also studies the choice between war and peaceful settlement when war disrupts trade, but in an incomplete information setting and with an aim to better understand the effects of globalization that expands trading opportunities to additional countries. Importantly, they abstract from arming decisions and, thus, the resource costs of conflict that, as our paper shows, interacts with resource asymmetries across countries to influence the possible gains from trade, the relative appeal of peace, and the nature of peace.[10] Our analysis also complements that of Bonfatti and O'Rourke (2018) who study arming decisions along side the decision to declare war in a trade setting with asymmetric countries. In contrast to Martin et al. (2008), they show war can emerge even when information is complete, provided commitment problems in security policies are present. However, their analysis abstracts from terms-of-trade effects and is more in line with the realist tradition of the international relations literature that, in stark contrast to the liberal peace perspective, suggests that globalization can increase the risk of war due to exogenous future shifts in power or in import dependence.

The analysis of the present paper also contributes to the literature on the choice between war and peace that, while abstracting from trade considerations, distinguishes between the mobilization of resources to arm and the deployment of those arms in open conflict. Relative to Garfinkel and Skaperdas (2000) and Skaperdas (2006), our analysis suggests that the gains from trade normally expand the parameter space for which armed peace is sustainable in a dynamic setting.[11] In contrast to those papers, Powell (1993) and De Luca and Sekeris (2013) explore the possibility of arming as a means to deter the adversary from attacking in order to preserve the status quo. Interestingly, our analysis points to the possibility of unarmed peace that preserves the status quo if either war is sufficiently destructive or if the gains from trade are sufficiently large and the initial distribution of resource endowments is even enough.[12]

[10] Jackson and Nei (2015) take a novel approach to study this issue (one based on a network framework that views alliances as generating both military and trade benefits). However, they too abstract from the endogenous determination of conflict costs.

[11] Jackson and Morelli (2007) also study the choice between war and peace, though they treat arming choices (or countries' military capacity) as exogenous to focus on the role of political bias in the emergence of war.

[12] Also, see Garfinkel and Syropoulos (2021) who briefly consider (as an extension of their baseline model) the importance of trade for the choice between war and peace, but in a single-period setting and with a focus on peace identified with the status quo (and thus no arming). In relation to that analysis, the present paper shows how arming that supports a negotiated division of contested resources differing from the status quo can expand the opportunities for peace even in a multi-period setting.

In the next section, we present a basic model of trade between two countries who dispute ownership claims to a productive resource, and describes the essential features of the two types of conflict resolution they can pursue—namely, open conflict and peaceful settlement. Then, in Section 3, we study the associated outcomes and payoffs and compare them to determine if peace is Pareto preferred to war. Highlighting the importance of trade in Section 4, we examine the conditions under which settlement is immune to unilateral deviations. Section 5 discusses briefly how some of our simplifying assumptions could be relaxed, including the model of trade employed, to make our analysis richer without altering the thrust of our findings. Finally, we offer concluding remarks in Section 6. All technical details are relegated to appendices.

2 A Basic Model of Resource Conflict and Trade

Consider a global economy consisting of two countries ($i = 1, 2$) that interact over two periods. At the beginning of the first period, each country i holds a claim over an asset (a non-labor resource such as land, water, or oil) that generates a stream of services per period of time denoted by R^i, where $R^1 + R^2 = \bar{R}$. However, these claims are not entirely secure. Instead, whatever resource R^i is held initially by country i is available only for the production of "military capacity" or "guns" for short. Each country i devotes G^i ($\leq R^i$) units of its resource to guns, an irreversible and non-contractable investment, to contest the remaining units $X^i = R^i - G^i$ for $i = 1, 2$ that go into a common pool: $\bar{X} \equiv \sum_i X^i = \bar{R} - \bar{G}$, where $\bar{G} \equiv \sum_i G^i$ represents aggregate arming.[13] Once ownership claims over the contested pool are resolved—either through warfare or a peaceful division—each country i produces, on a one-to-one basis with their secure holdings, a distinct and potentially tradable commodity Z^i, used as an intermediate input in the production of a consumption good.[14] Importantly, the technology for producing Z^i in each country i is unique and inalienable.[15] Furthermore, without denying the possible importance of asymmetric information in understanding the

[13]The analysis could be modified to consider the possibility that a fraction $\kappa^i \in [0, 1]$ of X^i is *secure* and the remaining $\left(1 - \kappa^i\right) X^i$ units are subject to appropriation. For $\kappa^i = \kappa$, this modification would allow us to study the implications of various degrees of insecurity including the extreme case of perfect security ("Nirvana"), which arises when $\kappa = 1$ and is the norm in standard neoclassical theory, and other intermediate cases, where $\kappa \in (0, 1)$. Allowing for partial security does not alter our key results; however it is worth mentioning that improvements in security tend to destabilize unarmed peace, but stabilize armed peace. (Details are available from the authors upon request.) We could also modify the analysis to consider $\kappa^i = 1$ while $\kappa^j = 0$ which implies that the contest is over just one country's residual resource. We abstract from this generalization as well. Though interesting in its own right, it introduces a second source of asymmetry that complicates the analysis.

[14]One could also interpret Z^i as a final tradable good of value to consumers.

[15]Put differently, the contest prize (a homogeneous resource) does not include access to the "blueprints" to produce the foreign intermediate good. Trade in our analysis is motivated by differences in technology along the lines of the Ricardian trade model. To bypass some technical issues (e.g., discontinuities in the best-response functions) addressed in Garfinkel et al. 2020 and to highlight the key mechanisms at play here, we focus on the extreme case that conforms to Armington (1969), which amounts to assuming countries produce nationally differentiated goods for trade in world markets. As discussed below in Section 5, the analysis could be couched in alternative trade models, including those that admit the possibility of differences in aggregate productivities across countries that govern how they transform their secure holdings of the resource into Z^i.

emergence of open conflict (with its concomitant disruption of trade as highlighted by Martin et al., 2008), our setting is one with complete information. Hence, as described earlier, our analysis emphasizes instead the problem of commitment in a dynamic framework to understand the emergence of open conflict.

In what follows, we present the details of our framework in three steps. First, we describe the mechanisms of conflict resolution that are available to the contending states. Second, we describe production and possible trade decisions, given the resources *securely* held by each country after their ownership claims have been resolved. Then, we summarize the timing of decisions and describe the extended game.

2.1 Arming and the Resolution of Resource Disputes

From our brief description above, it should be clear that arming is socially costly. That is to say, each country's allocation to build its military capacity G^i reduces the aggregate size of the contested pool, $\bar{X}$. Nevertheless, each country has an incentive to arm. The precise benefit for each country depends on the manner in which they resolve their ownership claims: open conflict or peaceful settlement. In the case of open conflict, which we model as a "winner-take-all" contest, a nation arms to improve its chances of victory. In the case of peaceful settlement conducted under the threat of war, a nation arms to gain leverage in the negotiation process by which the contested resource is divided. There are a variety of ways one can model how arming matters for settlement, based on well-known bargaining protocols, such as Nash bargaining and splitting the surplus.[16] However, we rely on a simple formulation that allows us to highlight the important trade-offs involved without complicating the analysis unnecessarily.

Specifically, we assume that the influence of guns on the outcome under either open conflict or peaceful settlement operates via the following conflict technology:

$$\phi^i = \phi^i(G^i, G^j) = \begin{cases} G^i/\bar{G} & \text{if} \quad \bar{G} > 0 \\ R^i/\bar{R} & \text{if} \quad \bar{G} = 0, \end{cases} \tag{1}$$

for $i, j \in \{1, 2\},\ j \neq i$. In the case of open conflict, ϕ^i represents country i's probability of winning the entire prize; in the case of peaceful settlement, ϕ^i represents the fraction of the contested resource that country i secures in its negotiations with country j.[17] This specification assumes that, when $\bar{G} > 0$, country i's winning probability or share is increasing in its own guns ($\phi^i_{G^i} > 0$) and decreasing in its rival's guns ($\phi^i_{G^j} < 0$). Equation (1) also implies that the conflict technology is symmetric in its arguments and concave in G^i, with

[16] See Anbarci et al. (2002) and Garfinkel and Syropoulos (2018) who study the efficiency properties of rules of division based on these and other protocols in different one-period settings. Under any of these protocols, both the threatpoint and the surplus depend on arming decisions.

[17] One can view settlement modeled this way as a reduced form of some bargaining process in which arming figures prominently in the division of contested resources. In a previous version of this paper, we studied the implications of a division based on an even split of the current-period surplus, but the key insights we find there remain intact with this simpler rule here.

$\phi^i_{G^iG^j} \gtreqless 0$ as $G^i \gtreqless G^j$ for $i \neq j = 1, 2$.[18] However, when $\bar{G} = 0$, each country's winning probability or share is determined by its relative (initial) resource endowment.

Open conflict in the first period has three other important features. First, the deployment of guns can be destructive, leaving only a fraction $\beta \in (0, 1]$ of the common resource pool intact in the first and second periods. If $\bar{G} = 0$ and war is declared, no destruction occurs (i.e., $\beta = 1$). That the destructive effect of war with positive arming persists beyond the period of war could be viewed as reflecting permanent damage to each country's technological apparatus/infrastructure, which reduces their effectiveness to transform the resource secured in the war into the intermediate input. In any case, such destruction tends to detract from the relative appeal of war. Second, whether or not countries arm, open conflict in the first period eliminates opportunities for trade in that period and the next. This assumption, which is clearly extreme, also tends to detract from the relative appeal of open conflict. Our rationale for imposing it here is to capture a salient feature of the liberal peace argument, that greater potential gains from trade imply a larger opportunity cost of war, thereby increasing the likelihood that countries will opt for a peaceful resolution. Finally, open conflict in the first period confers a strategic advantage on the victor in future disputes. In particular, the winner of war in the first period takes control not only of the contested pool after destruction $\beta\bar{X}$, but also of the resource that survives destruction $\beta\bar{R}$ in the next period and without having to arm at that time.[19] This last feature of open conflict is clearly extreme as well. However, it provides a useful benchmark that highlights the potential benefits of open conflict under complete information.[20]

The benefits of arming under peaceful settlement (armed peace) derive from arming's effect to allow a country to secure a larger share of the contested pool $\bar{X}$ in the negotiations. A rationale for "cooperation" arises here because, for any given quantities of guns countries produce within a time period, peaceful settlement (i) supports mutually beneficial bilateral trade and (ii) avoids the destructive consequences of open conflict. Nonetheless, peaceful settlement can be costly. Specifically, as each side tries to leverage its bargaining position by arming, it reduces the resources left for the production and trade of commodities and, thus, the size of the bargaining set, as in the one-period settings of Skaperdas and Syropoulos (2002) and Garfinkel and Syropoulos (2018).[21] Additionally, under the reasonable assumption that countries cannot enter into binding commitments regarding the future division of the resource

[18]This functional form, first introduced by Tullock (1980), belongs to a more general class of contest success functions (CSFs), $\phi^i(G^i, G^j) = h(G^i)/\sum_j h(G^j)$ where $h(\cdot)$ is a non-negative and increasing function, axiomatized by Skaperdas (1996). See Hirshleifer (1989), who explores the properties of two important functional forms of this class, including the "ratio success function" where $h(G) = G^b$ with $b \in (0, 1]$. Though the results to follow remain qualitatively unchanged under this more general specification, we use the one in (1), assuming $b = 1$ for simplicity (and, in our analysis of unilateral deviations from settlement, for tractability).

[19]What we have in mind is that defeat in war undermines the losing side's capacity, organization, and possibly even its will to enter conflict in the future. Put differently, one could view conflict as crippling the losing side's ability to challenge the victor in future conflict. In Section 5, we discuss possible modifications to this assumption that leave our central results qualitatively unchanged.

[20]See Fearon (1995), Garfinkel and Skaperdas (2000), Powell (2006) and Skaperdas and McBride (2014).

[21]The endogeneity of the bargaining set also arises in settings without trade (see Anbarci et al., 2002).

when they settle in the first period, the dispute between the two countries reemerges in the next period, and more arming could be called for at that time. However, unarmed peace, in which the two countries realize the gains from trade over the two periods without arming, might also arise as a possible outcome.

But, since open conflict involves arming only in the first period while settlement need not eliminate the incentive to arm in either period, open conflict could dominate settlement in terms of payoffs for one country. Even if settlement delivers higher *ex-ante* payoffs for both countries, one or both sides could find it optimal to deviate from this outcome unilaterally. We aim to identify the conditions under which settlement is stable (i.e., Pareto preferred and immune to unilateral deviations that produce war).[22] However, before turning to that analysis, we must specify the economic environment, including production and possible trade that play a prominent role in shaping the countries' arming incentives under settlement and, thus, their preferences over open conflict and settlement.

2.2 Production and Possible Trade

With the resources secured in the resolution of the dispute, country i produces on a one-to-one basis Z^i units of its distinct intermediate input. For ease of exposition, we refer to this quantity as country i's "effective endowment," which can in turn be used to produce a consumption good. In the case that the dispute is settled peacefully, Z^i is tradable, allowing each country i to access, through trade, both intermediate inputs. All markets are perfectly competitive, and the final good in each country i is produced according to the following constant elasticity of substitution (CES) technology:

$$F^i = F\left(D_1^i, D_2^i\right) = \left[\sum\nolimits_{j=1,2} (D_j^i)^\theta\right]^{\frac{1}{\theta}} \quad \text{for} \quad \theta \in (0,1],\ i = 1,2, \tag{2}$$

where D_j^i denotes the quantity of intermediate good $j \in \{1,2\}$ employed in country $i \in \{1,2\}$ and $\sigma = \frac{1}{1-\theta} > 1$ is the elasticity of substitution in production. In this model, as in standard trade models, the gains from trade derive from the imperfect substitutability between intermediate inputs (i.e., $\sigma < \infty$). Our assumption that $\theta > 0$ (or $\sigma > 1$), which is needed to ensure that autarky payoffs depend on arming, plays a role similar to the one in models of monopolistic competition, reflecting the value of diversity in productive inputs.

Assume each country i is risk-neutral, aiming to maximize its consumption, F^i shown in (2), given its effective endowment Z^i. Absent trade between the two countries (perhaps because war emerges between them), $D_j^i = 0$ holds for $i \neq j$, implying that each country i produces $F^i = Z^i$ units of the final good that yield the following one-period payoff:

$$w_A^i = Z^i, \text{ for } i = 1,2, \tag{3}$$

where the subscript "A" identifies the autarkic regime.

[22]Our inclusion of the requirement that peace be Pareto preferred to be a stable equilibrium seems appropriate in this context, because the interacting agents are free to communicate during their negotiations.

Turning to the possibility of trade, let p^i_j denote country i's domestic price for good $j = 1, 2$ in any given time period. Then, country i's income derived from the sale of the input it produces equals $Y^i = p^i_i Z^i$. Its choice of inputs that maximizes F^i in (2) subject to its budget constraint, $p^i_i D^i_i + p^i_j D^i_j = Y^i$, implies the following demand functions for good $j = 1, 2$: $D^i_j = \gamma^i_j Y^i / p^i_j$, where $\gamma^i_j \equiv (p^i_j / P^i)^{1-\sigma}$ (with $P^i \equiv [\sum_j (p^i_j)^{1-\sigma}]^{\frac{1}{1-\sigma}}$) denotes country i's expenditure or cost share on good j. Substituting these demand functions back into (2), then, yields country i's indirect payoff:

$$w^i = Y^i / P^i, \text{ for } i = 1, 2. \tag{4}$$

Trade of intermediate goods takes place in the absence of any trade costs. Let π_j be the "world" price of good $j = 1, 2$. Then, $\pi^i \equiv \pi_j / \pi_i$ equals the world relative price of country i's importable that, under perfect competition and free trade, coincides with the domestic relative price of the same good, $p^i \equiv p^i_j / p^i_i$. These prices are endogenously determined through a world market-clearing condition that requires the value of country i's imports to be equal to the value of country j's exports. Using the demand functions shown above with the fact that $Y^i = p^i_i Z^i$, one can verify this condition implies

$$\pi^i = \gamma^i_j Z^i / \gamma^j_i Z^j, \;\text{ for } i, j \in \{1, 2\}, \; j \neq i, \tag{5}$$

where γ^i_j is rewritten as $\gamma^i_j = (\pi^i)^{1-\sigma} / \left[1 + (\pi^i)^{1-\sigma}\right]$. The solution to (5), denoted by π^i_T where "T" indicates the outcome under trade, is given by $\pi^i_T = (Z^i / Z^j)^{1/\sigma}$.

Next, define $\mu^i(\cdot) \equiv [1 + (\pi^i_T)^{1-\sigma}]^{-\frac{1}{1-\sigma}}$ $(= p^i_i / P^i)$. Substituting this definition (together with $Y^i = p^i_i Z^i$) back in (4) enables us to obtain the following expression for country i's one-period payoff under trade:

$$w^i_T = \mu^i(\cdot) Z^i, \text{ for } i = 1, 2. \tag{6}$$

As is the case under autarky, a country's payoff under trade depends on its capacity to produce intermediate good Z^i. However, because $\mu^i(\cdot)$ depends on the world market-clearing price π^i_T and this price depends on (Z^i, Z^j) as just shown, the payoff w^i_T is a function of both countries' output levels (Z^i, Z^j). We describe these dependencies in more detail below. Let us note for now, as revealed by a comparison of w^i_T in (6) with w^i_A in (3), that country i's gains from trade (given both countries' arming choices and thus Z^i) in relative terms are captured by $\mu^i(\cdot) > 1$ that depends inversely on the degree of similarity between the two inputs in the production of the final good (i.e., $\sigma < \infty$).[23] For the sake of generality, we also consider, in Appendix A, the presence of symmetric ("iceberg" type) trade costs, in which case the relative gains from trade are determined jointly by these costs and σ.[24]

[23]See Arkolakis et al. (2012) who discuss the importance of σ in this and other trade models. As noted earlier, we discuss how the analysis could be extended to other models of trade below in Section 5.

[24]Let $\tau \geq 1$ denote this trade cost, where $\tau - 1$ represents the additional quantity of the traded good that must be shipped from an exporter for one unit to arrive at its destination. In this case, $\mu^i(\cdot) = [1 + (p^i)^{1-\sigma}]^{-1/(1-\sigma)}$,

2.3 Timing

The sequence of actions in period $t = 1$ is as follows:

Stage 1. Each country i chooses G^i ($\leq R^i$), treating its rival's decision G^j ($j \neq i$) as given.

Stage 2. The two countries jointly consider the possible division of the contested pool, $\bar{X} = \sum_i (R^i - G^i) = \bar{R} - \bar{G} \geq 0$, in the current period according to ϕ^i in (1).

2a. If both countries agree, each country i's effective endowment becomes $Z^i = \phi^i \bar{X}$.

2b. If at least one country objects, open conflict over $\bar{X}$ ensues. The effective endowments, in this case, are $Z_W = \beta \bar{X}$ for the winner and $Z_L = 0$ for the loser.

Stage 3. If the two sides agree to settle their claims and no deviation from the agreement is recorded, they engage in competitive trade of their intermediate goods, Z^i. War and deviations from settlement foreclose on current and future trade.

What happens in period $t = 2$ depends on the outcome of the two countries' interactions in period $t = 1$. If war breaks out in period $t = 1$ (2b), the defeated side is no longer in contention. Thus, there is no arming in period $t = 2$, and the winner enjoys in that period the stream of benefits associated with controlling $\beta \bar{R}$ units of the services of the primary resource. By contrast, if peaceful settlement arises in period $t = 1$ (2a), the three stages specified above are repeated in period $t = 2$.

3 Outcomes under Open Conflict and Settlement

Having specified the model's essential elements, we now study countries' arming decisions and their resulting lifetime payoffs when the two countries anticipate open conflict (or war) and when they anticipate peace. More precisely, in Sections 3.1 and 3.2, we characterize the subgame perfect Nash equilibria of the two-period subgames related to war and peace respectively. As will become evident in Section 3.2, given peace prevails in period $t = 1$, both war and settlement can arise as Nash equilibria in period $t = 2$. But, consistent with the equilibrium concept we apply for the extended game, we select the second-period equilibrium that is Pareto preferred—namely, settlement. In Section 3.3, we then explore how the resulting two-period payoffs under war and peaceful settlement compare to identify the parameter space for which peace in the first period is also Pareto preferred, a necessary (but not sufficient) condition for peace to be a possible stable equilibrium of the extended game. Overall, this characterization lays the foundation for our subsequent analysis in Section 4, where we explore the immunity of peace to unilateral deviations and ultimately the possibility of peace as the stable equilibrium of the extended game.

where $p^i = \tau \pi_T^i$ with π_T^i being defined implicitly by the world market clearing condition $\pi_T^i = (\gamma_j^i Z^i)/(\gamma_i^j Z^j) = (\phi^i \gamma_j^i)/(\phi^j \gamma_i^j)$ and $\gamma_j^i = \gamma_j^i(p^i)$. The results reported in Appendix A are qualitatively unchanged from those reported in the main text, but somewhat richer. See Anderson and and Marcouiller (2005) for a creative modeling and analysis of trade costs. In their setting, these costs emerge due to the appropriation of traded goods (while in transit) by bandits or pirates. The analysis could also be extended to consider the possible use of revenue-generating trade barriers (e.g., import tariffs and export taxes). We abstract from these additional wrinkles for clarity.

3.1 Open Conflict

Let $u^i\left(G^i,G^j\right)$ be country i's expected one-period payoff function under open conflict in the first period. Since war destroys any trading opportunities, equation (3) implies that country i's period $t=1$ payoff contingent on the outcome of the war is linear in its effective resource endowment or intermediate good output level, $Z_W \equiv \beta\bar{X} = \beta(\bar{R}-\bar{G}) \geq 0$ in the case of victory and $Z_L = 0$ in the case of defeat. Thus, country i's expected one-period payoff u^i can be written as follows:

$$u^i \equiv u^i(G^i,G^j) = \phi^i Z_W^i + \left(1-\phi^i\right) Z_L^i = \phi^i\beta\bar{X}, \text{ for } i,j \in \{1,2\},\ j \neq i. \tag{7}$$

The dependence of this payoff on arming by both countries operates through the probability of winning ϕ^i and through the determination of the common pool being contested $\bar{X}$.[25]

Now let $\delta \in [0,1]$ denote the countries' (common) discount factor and U^i country i's expected lifetime payoff under open conflict divided by $1+\delta$, which we refer to as its "average" (per-period) payoff under open conflict (or war). Since country i controls $\beta\bar{R}$ with probability ϕ^i and gets nothing with probability $1-\phi^i$ in period $t=2$, its average payoff is

$$U^i \equiv U^i(G^i,G^j) = \frac{1}{1+\delta}\left[u^i(G^i,G^j) + \phi^i\beta\delta\bar{R}\right], \text{ for } i,j \in \{1,2\},\ j \neq i.$$

Using (7) in the above equation and rearranging terms gives

$$U^i = \frac{\beta}{1+\delta}\phi^i\left(\bar{X}+\delta\bar{R}\right), \text{ for } i=1,2. \tag{8}$$

3.1.1 Incentives to Arm under Open Conflict

The extent to which each country i arms in period $t=1$, in anticipation of war, depends on the solution to $\max_{G^i} U^i$, s.t., $X^i = R^i - G^i \geq 0$ for $i=1,2$. Differentiation of country i's expected payoff U^i in (8) with respect to G^i gives:

$$U^i_{G^i} = \frac{\beta}{1+\delta}\left[\phi^i_{G^i}\left(\bar{X}+\delta\bar{R}\right) - \phi^i\right], \text{ for } i=1,2. \tag{9}$$

The first term inside the square brackets on the right-hand side (RHS) (multiplied by $\beta/(1+\delta)$) represents country i's average discounted marginal benefit to arming. This benefit captures the effect of a marginal increase in G^i to improve country i's probability of winning the war, whereby it can control the output stream of $\beta\bar{X}$ and $\beta\bar{R}$. The second term (again, multiplied by $\beta/(1+\delta)$) represents country i's opportunity cost of arming due to the reduction in the size of the pool $\bar{X}$. Inspection of the RHS in (9) reveals that (i) an increase in the aggregate initial resource ($\bar{R}$) that implies a larger prize and (ii) a stronger shadow of the future (δ) that increases the future valuation of that prize each augment the net marginal benefit to arming. By contrast, an increase in the destructiveness of open conflict ($\beta \downarrow$)

[25] Note that, if in period $t=1$ the countries settled their resource dispute peacefully, u^i in (7) is the payoff that each country would consider when choosing between war and peace in period $t=2$.

has no impact on this marginal condition. Observe that an increase in the rival's guns G^j influences the net marginal benefit of arming to country i through the conflict technology ϕ^i and the current-period prize $\bar{X}$.

Finally, observe the conflict technology (1) implies that $\lim_{G^i \to 0} \phi^i_{G^i}$ is arbitrarily large for G^j arbitrarily close to 0. Accordingly, whenever country i's rival produces a positive quantity of guns, country i's best response is to similarly produce a positive quantity. What if country j produces no guns? In this case, country i could refrain from arming as well, and win all of the contested pool ($\bar{R}$) with probability $R^i/\bar{R}$. However, the specification in (1) also implies that country i could produce an arbitrarily small amount of guns $G^i = \epsilon > 0$ and capture all of the contested pool ($\approx \bar{R}$) with certainty. Assuming that this minuscule deployment of guns generates no destruction, country i always has an incentive to arm in anticipation of open conflict. Since this logic applies to country j, the outcome that emerges when the two countries anticipate open conflict necessarily involves positive arming by both sides.

To proceed, let $B^i_c\left(G^j;\cdot\right)$ denote country i's best response to $G^j > 0$ ($j \neq i$) under open conflict. From the first-order condition (FOC) implied by (9) and the resource constraint that possibly binds in country i's arming choice, one can verify the following best-response functions:

$$B^i_c\left(G^j;\delta,R^i,\bar{R}\right) = \min\{R^i, \widetilde{B}^i_c(G^j)\}, \text{ for } i,j \in \{1,2\},\ j \neq i, \tag{10a}$$

where $\widetilde{B}^i_c(G^j)$ is country i's *unconstrained* best-response function[26] that solves $U^i_{G^i} = 0$:

$$\widetilde{B}^i_c(G^j) \equiv -G^j + \sqrt{(1+\delta)\,\bar{R}G^j}. \tag{10b}$$

The expressions in (10) reveal the importance of the opponent's arming G^j, the aggregate quantity of the initial resource $\bar{R}$, its distribution (R^i, R^j), and the strength of the shadow of the future δ in jointly determining the shape of country i's best-response function $B^i_c(G^j)$. Inspection of (10b), in particular, reveals that country i's incentive to arm in anticipation of open conflict depends positively on its rival's choice G^j when $\widetilde{B}^i_c(G^j) > G^j$ and negatively so when $\widetilde{B}^i_c(G^j) < G^j$. Consistent with our discussion regarding (9), increases in $\bar{R}$ and in the strength of the shadow of the future δ augment country i's arming incentives. Of course, changes in these parameters need not translate into changes in arming choices. Also relevant here are the countries' resource constraints and thus the distribution of initial ownership claims to $\bar{R}$.

Denote the quantity of guns country i produces in this outcome by G^i_c, and define

$$R^c_L \equiv \left[1 - \tfrac{1}{2}(1-\delta)\right]\tfrac{1}{2}\bar{R} \quad \text{and} \quad R^c_H \equiv \left[1 + \tfrac{1}{2}(1-\delta)\right]\tfrac{1}{2}\bar{R}, \tag{11}$$

where "L" ("H") identifies the "low" ("high") endowment threshold in anticipation of open

[26] Here and below, to limit notational cluttering, we suppress the dependence the best-response function on resources and the other parameters of the model.

conflict (c) that together determine the parameter space for which neither country is resource constrained in its arming choice. Clearly, $R_H^c + R_L^c = \bar{R}$ and $R_H^c - R_L^c = \frac{1}{2}(1-\delta)\bar{R} \geq 0$ for $\delta \leq 1$. Using the properties of $B_c^i(G^j)$ implied by (10), together with the aggregate resource constraint $R^i + R^j = \bar{R}$ and (11), leads to the following characterization of security policies when open conflict is anticipated in the second stage in period $t = 1$:

Proposition 1 (Arming under open conflict.) *In the subgame related to open conflict, there exists a Nash equilibrium, with strictly positive arming by both contenders: $G_c^i > 0$ for $i = 1, 2$. Given any $\bar{R}$ such that $R^i + R^j = \bar{R}$, arming decisions and thus winning probabilities are independent of conflict's rate of destruction $(1 - \beta)$, but do depend on the initial distribution of $\bar{R}$ across the two countries and the discount factor (δ) as follows:*

(a) *If $R^i \in [R_L^c, R_H^c]$, then $G_c^i \equiv G_c = R_L^c(\delta)$ and $\phi_c^i = \frac{1}{2}$ for $i = 1, 2$.*

(b) *If $R^i \in (0, R_L^c)$ for $i = 1$ or 2, then $G_c^i = R^i < G_c^j = \widetilde{B}_c^j(R^i, \delta)$ and $\phi_c^i < \phi_c^j$ for $j \neq i$.*

(c) *$dG_c^i/d\delta = 0$ for $R^i \in (0, R_L^c]$, $dG_c^i/d\delta > 0$ for $R^i \in (R_L^c, \bar{R})$ and $d(R_H^c - R_L^c)/d\delta < 0$ with $\lim_{\delta \to 0} R_L^c = \frac{1}{4}\bar{R}$ and $\lim_{\delta \to 0} R_H^c = \frac{3}{4}\bar{R}$, whereas $\lim_{\delta \to 1} R_L^c = \lim_{\delta \to 1} R_H^c = \frac{1}{2}\bar{R}$.*

This proposition establishes that the equilibrium in the arming subgame under open conflict involves strictly positive arming by both countries. Furthermore, an uneven ownership of initial claims to $\bar{R}$ across the two countries matters only insofar as that distribution implies one country is constrained in its production of guns.[27] Specifically, part (a) shows that, when the configuration of initial asset ownership is sufficiently even (i.e., $R^i \in [R_L^c, R_H^c]$ for $i = 1, 2$), the two countries produce an identical amount of guns (i.e., $G_c^i = G_c = R_L^c$ for $i = 1, 2$), and that quantity is invariant to changes in the initial distribution of $\bar{R}$.[28]

However, as shown in part (b), when the configuration of initial ownership claims is sufficiently uneven (i.e., $R^i \in (0, R_L^c]$), the smaller country i's guns choice is constrained by its resource endowment: $G_c^i = R^i$; at the same time, the larger country (j) continues to operate on its unconstrained best-response function shown in (10b), and generally arms by more than its smaller adversary. In such cases, a redistribution of initial resources from the larger country j to the smaller country i relaxes country i's resource constraint, causing it to increase its arming one-for-one with the increase in R^i. The decrease in the larger country's initial resource (R^j) has no direct effect on its arming choice; however, by the strategic complementarity of its best-response function (i.e., $\partial B_c^j(G^i)/\partial G^i > 0$ when $B_c^j(G^i) > G^i$), country j increases its arming in response to country i's more aggressive security policy. As a result, a redistribution of initial resource endowments towards the smaller country results in a new outcome where both countries arm by more. It should be clear, then, that aggregate arming under conflict $\bar{G}_c = G_c^i + G_c^j$ is maximized when neither country is resource

[27] Observe from the definition of R_L^c in (11), at most one country can be resource constrained.

[28] That the outcome is symmetric (i.e., $G_c^i = G_c^j$), even when the contenders have (mildly) uneven resources initially, might seem surprising. However, the result follows from the assumption that they contest the same prize $\bar{X} + \delta\bar{R}$ and the symmetric specification of ϕ^i in (1), implying that $U_{G^i}^i$ shown in (9) can be equal to zero for both countries only if $G^i = G^j$.

constrained: $\bar{G}_c = 2R_L^c$. Conversely, a redistribution of $\bar{R}$ from the smaller (constrained) country i to the larger country j implies less arming by both and thus less aggregate arming: $\bar{G}_c < 2R_L^c$.

Part (c) shows that, while the rate of conflict's destruction $(1-\beta)$ has no effect on arming choices in anticipation of open conflict, an increase in the strength of the shadow of the future (δ) induces greater arming by countries that are not resource constrained, as it increases the value of the contest prize. However, since the aggregate quantity of the initial resource $\bar{R}$ remains unchanged, an increase in δ shrinks the range of initial resource allocations for which both countries are unconstrained, collapsing to a single point at $R_L^c = R_H^c = \bar{R}/2$ as δ approaches 1; at this limit, the dispute over ownership claims results in the full dissipation of total productive resources in period $t = 1$ (i.e., $\bar{X} = 0$). These results are illustrated in Fig. 1(a), which shows country i's arming choice as a function of the distribution of initial resource ownership for alternative values of δ.

3.1.2 Payoffs under Open Conflict

Building on our characterization of arming choices when countries anticipate open conflict, we now examine how their corresponding payoffs, U_c^i $(i = 1, 2)$, depend on the distribution of initial resource ownership (R^i, R^j). When evaluating the effects of exogenous changes in this initial distribution on country i's payoff, it is important to account not only for the direct effects, but also for the possible indirect effects that operate through the conflict technology ϕ^i as they can induce changes in the choices, G_c^i and G_c^j. Of course, by the envelope theorem, when country i's resource constraint is not binding, the effect of a change in its own arming G_c^i on its own payoff U_c^i vanishes; otherwise, exogenous changes in the parameters that enable country i to move closer to its unconstrained optimum improve its payoff. In contrast, a change in country i's rival arming G_c^j always adversely affects its payoff U_c^i, first by reducing its probability of winning the war ϕ^i and second by reducing the overall size of the common resource pool $\bar{X}$.

The next proposition shows how the just described indirect effects of arming decisions combine with the direct effects of changes in war's rate of destruction, the shadow of the future and the distribution of initial resource ownership to influence payoffs, U_c^i.

Proposition 2 (Payoffs under open conflict.) *For all $R^i \in (0, \bar{R})$, expected payoffs under open conflict are decreasing in war's rate of destruction $(1-\beta)$. The payoff effects of changes in the shadow of the future (δ) and in the distribution of initial resource ownership (R^i, R^j) depend on whether one of the country is resource constrained in its arming decision:*

(a) *If $R^i \in [R_L^c, R_H^c]$ for $i = 1$ or 2, then for $i = 1, 2$, $U_c^i = \beta\frac{\bar{R}}{4}$ and*

  (i) $dU_c^i/dR^i = 0$

  (ii) $dU_c^i/d\delta = 0$.

(b) *If $R^i \in (0, R_L^c)$ for $i = 1$ or 2, then*

(i) $dU_c^i/dR^i > 0$, $d^2U_c^i/(dR^i)^2 < 0$ *and* $\lim_{R^i\to 0} U_c^i = 0$, *whereas* $dU_c^j/dR^j > 0$, $d^2U_c^j/(dR^j)^2 > 0$ *and* $\lim_{R^j\to\bar{R}} U_c^j = \beta\bar{R}$.

(ii) $dU_c^i/d\delta < 0$, *while* $dU_c^j/d\delta > 0$.

Since arming is independent of β, changes in that parameter influence payoffs of both countries only directly, and positively so. Fig.1(b) illustrates the remaining parts of the proposition.

The intuition for part (a.i), which shows how country i's expected payoff in anticipation of open conflict depends on the initial distribution of resources when that distribution is sufficiently even, follows from Proposition 1(a) and equation (8). Specifically, in this benchmark case, since countries arm identically, their payoffs are identical; similarly, since their arming choices are invariant to any reallocation of the initial resource (R^i, R^j) within $[R_L^c, R_H^c]$, so too are their payoffs.

The intuition behind part (b.i), which taken as a whole implies that the unconstrained country's expected payoff is greater than that of the constrained country (i.e., $U_c^j > U_c^i$), can be fleshed out by studying the effects of resource redistributions outside the range of $[R_L^c, R_H^c]$, using Proposition 1(b). When country i's resource constraint binds in the production of guns such that $G^i = R^i$, an exogenous shift in the total resource towards that country relaxes its resource constraint, thereby inducing it to arm more and adding to its payoff (i.e., since $U_{G^i}^i > 0$). As previously described, the larger (and unconstrained) opponent j $(\neq i)$ responds by adopting a more aggressive stance in its security policy and that has a negative effect on the smaller country's payoff. In Appendix A, we show that the positive payoff effect due to increases in the smaller country's own arming $G^i = R^i$ dominates the adverse effect due to increases in the arming of its larger rival G^j, with the net marginal effect falling as R^i rises. By contrast, an exogenous shift in the total resource $\bar{R}$ towards the larger and unconstrained country j has no direct effect on that country's payoff. Furthermore, by the envelope theorem, the indirect effect on its payoff due to changes in its own arming G^j vanishes. However, the smaller opponent i $(\neq j)$ behaves less aggressively as its resource endowment falls, and that indirect effect improves country j's payoff. Since, in this case, G^i falls faster than G^j, country j's payoff rises at an increasing rate.

Parts (a.ii) and (b.ii) of the proposition show that the full impact of an increase in the discount factor δ on a country's expected payoff also depends on whether the country's resource constraint on its arming decision is binding or not. Of course, as can be seen from (8), for given arms and thus given $\bar{X}$, an increase in δ has a direct effect to increase each country's payoff. But, as noted in part (c) of Proposition 1, a stronger shadow of the future ($\delta \uparrow$) fuels the arming incentives of an unconstrained country which imparts a negative indirect effect on the opponent. Part (a.ii) shows that, if both countries are unconstrained, the direct and indirect (strategic) effects perfectly offset each other, such that an increase in δ leaves both countries' payoffs unchanged. Turning to part (b.ii), an increase in δ that induces (unconstrained) country j to take a more aggressive stance in its arming policy generates an adverse indirect effect on country i's payoff. In Appendix A, we show that the indirect effect of an increase in δ on the constrained country's payoff U_c^i dominates the direct effect, thus implying $dU_c^i/d\delta < 0$. By contrast, since country i's arming remains unchanged, an increase in δ has no indirect effects on the unconstrained country's (j's) payoff; only the positive direct effect matters, thus implying $dU_c^j/d\delta > 0$.

3.2 Peaceful Settlement

Turning to peaceful settlement, recall that in this case the aggregate residual resource $\bar{X}$ is divided on the basis of the guns produced according to the conflict technology in (1). Thus, country i's effective endowment is given by $Z^i = \phi^i \bar{X}$, where again $\bar{X} = \bar{R} - \bar{G}$. Then, from (6), country i's one-period payoff under settlement equals $w_T^i = \mu^i Z^i = \mu^i \phi^i \bar{X}$, where $\mu^i \equiv [1 + (\pi_T^i)^{1-\sigma}]^{-\frac{1}{1-\sigma}} \geq 1$ (with strict inequality when $\sigma < \infty$) represents country i's relative gains from trade and $\pi_T^i = (Z^i/Z^j)^{1/\sigma} = (\phi^i/\phi^j)^{1/\sigma}$ is the relative price of country i's importable that clears global markets (5). Since an increase in G^i, given G^j, implies an increase in country i's intermediate input Z^i and a reduction of the rival's input Z^j (i.e., an increase in ϕ^i/ϕ^j), the world relative price of country i's importable π_T^i rises as a result.[29]

To study the countries' incentive to arm under settlement that allows for trade, first let $\omega^i \equiv \omega^i\left(\phi^i; \sigma\right) = \mu^i \phi^i$ be the per unit (in terms of the common pool $\bar{X}$) payoff to country i. For simplicity, we rewrite the overall payoff w_T^i as

$$v^i \equiv v^i(\phi^i, \bar{G}; \sigma) = \omega^i \bar{X}, \text{ for } i = 1, 2. \tag{12}$$

This one-period payoff depends negatively on guns through $\bar{X}$. It also depends on guns through the division ϕ^i that affects ω^i directly and indirectly through π_T^i and thus μ^i.[30]

Now suppose that the two countries resolved their resource dispute in period $t = 1$ peacefully, and consider the problem facing country i in $t = 2$, which can be written as:[31]

$$\max_{G^i} v^i(\phi^i, \bar{G}; \sigma), \text{ s.t. } G^i \in [0, R^i], \text{ for } i, j \in \{1, 2\}, \ j \neq i. \tag{13}$$

Let G_s^i denote country i's $(= 1, 2)$ arming solution to the above and v_s^i its associated payoff in anticipation of settlement in $t = 2$. Before moving to the period $t = 1$ problem, observe from u^i in (7) and v^i in (12) with the definition of ω^i that, for any given feasible guns choices of in stage 1 of period $t = 2$ (and thus given $\bar{X}$), $v^i > u^i$ provided $\mu^i > \beta$. Thus, for given

[29]Our analysis to follow pays special attention to how the countries' actions depend on the distribution of resource ownership. In support of that analysis, Lemma A.1 that is presented in Appendix A characterizes how world prices π_T^i depend on the distribution of effective endowments given arming choices. Similarly, Lemma A.2 explores the dependence of expenditure shares $\gamma_j^i = -\pi^i \mu_{\pi^i}^i / \mu^i$ on that distribution. Lemma A.3 explores how, conditioned on the distribution of resource ownership, π_T^i changes with changes in input heterogeneity ($\sigma \downarrow$) and trade costs ($\tau \uparrow$, which are not explicitly considered in the main text).

[30]The properties of ω^i, established in Lemma A.4, are discussed in some detail below. To highlight how the gains from trade alone to country i depend on ϕ^i, Lemma A.5 compares $w_T^i = \omega^i \bar{X}$ and $w_A^i = \phi^i \bar{X}$, for any feasible quantities of guns (and, thus, given $\bar{X}$). In addition, Lemma A.6 characterizes how the world gains from peace given guns depend on the distribution of $\bar{X}$ as well as on the degree of input heterogeneity ($\sigma \downarrow$) and trade costs ($\tau \uparrow$, from which we abstract in the main text).

[31]Recall that, under open conflict in period $t = 1$, the defeated player is out of contention for the remainder of the game. Hence, settlement is feasible in period $t = 2$, only if settlement prevails in period $t = 1$.

guns choices, both countries would choose peace over open conflict in stage 2, provided that either open conflict is destructive ($\beta < 1$) or the relative gains from trade are strictly positive ($\sigma < \infty$ so that $\mu^i > 1$). Accordingly, if peace prevailed in period $t = 1$, it will also prevail in period $t = 2$.[32]

Let us now turn to period $t = 1$ decisions, and suppose they are made in anticipation of settlement in that same period. Since they are also made in anticipation of settlement in the following period, country i's problem in the first stage is to choose G^i to maximize

$$V^i = \frac{1}{1+\delta}\left[v^i\left(\phi^i, \bar{G}; \sigma\right) + \delta v_s^i\right],$$

subject to $G^i \in [0, R^i]$. The stationarity of our setting under peaceful settlement means that the solution to this two-period problem is analytically identical to that described in (13) in period $t = 2$. Accordingly, $V_s^i = v_s^i$ holds for $i = 1, 2$, and it suffices to examine the outcome of the first-period (stage) game in (13).

To explore the effects of G^i on a country i's payoffs given G^j, now consider the direct and indirect effects of the division of resources, ϕ^i. Noting that country i's expenditure share on good j can be written as $\gamma_j^i = -\pi^i \mu_\pi^i / \mu^i$ while using the solution $\pi_T^i = (\phi^i/\phi^j)^{1/\sigma}$ with the fact that $\phi^j = 1 - \phi^i$, we find

$$\omega_{\phi^i}^i = \mu^i \left(1 - \frac{\gamma_j^i/\phi^j}{\sigma}\right), \text{ for } i, j \in \{1, 2\},\ j \neq i, \tag{14}$$

where $\sigma > 1$ by assumption. The first term inside the parentheses (multiplied by μ^i) is the direct effect of shifting a fraction of the common pool to country i. This effect is positive because, at constant prices and for given $\bar{X}$, an increase in ϕ^i expands country i's output of the final good. The second term (also multiplied by μ^i) represents the indirect effect, and is negative since (as discussed above) the implied increase in Z^i and decrease in Z^j generate an adverse terms-of-trade effect (i.e., $\pi_T^i \uparrow$). Differentiating (14) with respect to ϕ^i shows that ω^i is strictly concave in ϕ^i for $\sigma \in (1, \infty)$, reaching a maximum at some $\phi^i \in (\frac{1}{2}, 1)$.[33]

3.2.1 Incentives to Arm under Peaceful Settlement

We can now study countries' incentives to arm under peaceful settlement. Differentiation of (12) with respect to G^i gives

$$v_{G^i}^i = \omega_{\phi^i}^i \phi_{G^i}^i \bar{X} - \omega^i. \tag{15}$$

The second term in the RHS is country i's marginal (i.e., opportunity) cost of arming (MC^i), reflecting the effect of an increase in G^i to reduce the pool of contestable resources, $\bar{X}$. Since

[32] Put differently, given G^i for $i = 1, 2$, choosing peace in period $t = 2$ is Pareto preferred. As will become evident, the effect of the anticipation of settlement in stage 2 on arming choices in stage 1 amplifies that preference for peace over war (see Lemma 1). What's more, as confirmed in Section 4, settlement in period $t = 2$ is immune to unilateral deviations.

[33] See Lemma A.4(b) presented in Appendix A.

$\omega^i = \mu^i\phi^i$, this opportunity cost varies in proportion to the product of the country's share (ϕ^i) of the common pool and its relative gains from trade (μ^i). The first term in the RHS represents country i's marginal benefit to arming (MB^i). This term reflects the impact of an increase in country i's guns on its share of the common pool (captured by $\phi^i_{G^i}\bar{X}$) multiplied by the induced change in its per unit of $\bar{X}$ payoff (captured by $\omega^i_{\phi^i}$) that includes the direct and terms-of-trade effects discussed earlier. Clearly, MB^i is increasing in the size of the common pool $\bar{X}$. In addition, by (1), $\lim_{G^i\to 0}\phi^i_{G^i}$ is arbitrarily large for G^j arbitrarily close to 0 which implies that MB^i becomes arbitrarily large. Thus, as is true under open conflict, when country i's rival produces an arbitrarily small but positive quantity of guns to influence the division of the prize in its favor, country i will always wish to produce a positive quantity of guns as well. But what would country i do if its rival j produced no guns at all ($G^j = 0$)? Would country i necessarily find it appealing to capture all of $\bar{X}$, by producing an infinitesimal quantity of guns as was the case under open conflict?

We study country i's optimal arming decision both for $G^j > 0$ and $G^j = 0$. Starting with the former case, country i's best response under settlement, like under open conflict, depends on the sensitivity of MB^i and MC^i to G^i and the distribution of initial resource ownership. Once again, it is imperative that we take into account the possibility that a country's resource constraint on arming could be binding. As in the case of open conflict, we use a tilde "$\sim$" to identify country i's unconstrained variables and functions.[34]

To proceed, suppose that neither country's arming is constrained by its initial resource endowment and $\widetilde{v}^i_{G^i} = 0$ for $i = 1,2$. Then, from the definitions of MB^i and MC^i and equations (15), (14), and (1), the following must hold:

$$\frac{MB^i}{MB^j} = \frac{MC^i}{MC^j} \;\Rightarrow\; \frac{\omega^i_{\phi^i}\phi^i_{G^i}}{\omega^j_{\phi^j}\phi^j_{G^j}} = \frac{\omega^i}{\omega^j} \;\Rightarrow\; \left(\frac{G^j}{G^i}\right)\left[\frac{1-\frac{\gamma^i_j/\phi^j}{\sigma}}{1-\frac{\gamma^j_i/\phi^i}{\sigma}}\right] = \frac{\phi^i}{\phi^j},$$

for $i \neq j$. Now suppose, $G^j/G^i < 1$ for $i \neq j$ which, by (1), implies $\phi^i/\phi^j > 1$ in the RHS of the last equation. This requires the value of the expression inside the square brackets to exceed 1. Because $\pi^i_T = \left(\phi^i/\phi^j\right)^{1/\sigma}$ is increasing in ϕ^i/ϕ^j, $\pi^i_T > 1$ holds. With the world market-clearing condition (5) this inequality, in turn, implies $\phi^i\gamma^i_j > \phi^j\gamma^j_i$, which can be rewritten as

$$\frac{\gamma^i_j}{\phi^j} > \frac{\gamma^j_i}{\phi^i} \;\Rightarrow\; \frac{\gamma^i_j/\phi^j}{\sigma} > \frac{\gamma^j_i/\phi^i}{\sigma} \;\Rightarrow \frac{1-\frac{\gamma^i_j/\phi^j}{\sigma}}{1-\frac{\gamma^j_i/\phi^i}{\sigma}} < 1,$$

thereby contradicting our supposition that $G^j/G^i < 1$. Since this logic can be applied to the case of $G^j/G^i > 1$, the outcome in security policies when both countries' arming decisions

[34] For example, $\widetilde{v}^i$ and $\widetilde{B}^i_s\left(G^j\right)$ denote country i's unconstrained per period payoff and best-response functions, respectively. When a country i $(= 1,2)$ has an incentive to arm under settlement, its best-response function is $B^i_s\left(G^j;\sigma,R^i,\bar{R}\right) = \min\{R^i, \widetilde{B}^i_s(G^j)\}$, where $\widetilde{B}^i_s(G^j)$ satisfies $v^i_{G^i} = 0$.

are unconstrained by their respective initial resources must be symmetric (i.e., $G^i = G^j$).[35] In turn, we have (i) $\phi^i = \phi^j = \frac{1}{2}$ that implies $\phi^i_{G^i} = 1/2\bar{G}$ for $i = 1, 2$, (where $\bar{G} = G^i + G^j$); (ii) $\pi^i_T = 1$; and (iii) $\gamma^i_j = \gamma^j_i = \frac{1}{2}$. Therefore, country i's relative gains from trade satisfy $\mu^i = \mu \equiv 2^{\frac{1}{\sigma-1}}$ and $\omega^i = \mu/2$ for $i = 1, 2$.

In what follows, we use the transformation $\theta = 1 - \frac{1}{\sigma}$ (from (2)) that positively reflects the degree of substitutability between traded inputs σ. With an application of the results above using (15), one can solve country i's FOC, $\widetilde{v}^i_{G^i} = 0$ for $i = 1, 2$, to find that, when neither country is resource constrained,

$$G^i_s = G_s = \frac{\theta}{2(1+\theta)}\bar{R} \text{ for } i = 1, 2. \tag{16}$$

As revealed by this expression, the level of arming in this symmetric outcome under peaceful settlement is increasing in the degree of substitutability θ, but is independent of the discount factor δ and the rate of destruction $1 - \beta$.[36]

Of course, as in the case of open conflict, a country's arming choice could be resource constrained. Recognizing that possibility, we define the following threshold values:

$$R^s_L \equiv \left[1 - \tfrac{1}{1+\theta}\right] \tfrac{1}{2}\bar{R} \text{ and } R^s_H \equiv \left[1 + \tfrac{1}{1+\theta}\right] \tfrac{1}{2}\bar{R}. \tag{17}$$

As we will see shortly, these thresholds (with the ideas discussed above) enable us to characterize the optimizing security policies under settlement for the entire parameter space.

Before that, let us turn to the possibility of no arming. Suppose, in particular, country i's rival produces zero guns: $G^j = 0$. One option for country i is to produce an infinitesimal but positive quantity $G^i = \epsilon > 0$ to secure $\bar{X}$ ($\approx \bar{R}$) in negotiations. However, since rival j is left with no resources, this option precludes the possibility of trade in intermediate inputs, giving country i a one-period payoff of $v^i_a \equiv v^i(\epsilon, 0) = \omega^i(1; \theta)[\bar{R} - \epsilon] \approx \bar{R}$. The other option for country i that does allow for trade is to produce no guns at all (i.e., $G^i = 0$). By (1), country i's share would be $\phi^i = R^i/\bar{R}$, giving it a one-period payoff of $v^i_u \equiv v^i(0, 0) = \omega^i(R^i/\bar{R}; \theta)\bar{R}$. Since these comparisons are also relevant for country j, the condition for both countries not to arm under settlement is $v^i_u > v^i_a$ or equivalently $\omega^i(R^i/\bar{R}; \theta) > 1$ for $i = 1, 2$. Whether this condition holds or not depends on the magnitude of each country's gains from trade, which hinges on the value of θ and the distribution of resource ownership.

Building on the ideas above, the next proposition provides a complete characterization of the various arming choices that can emerge in anticipation of settlement.

[35] In a possible extension of the analysis discussed briefly in Section 5 with details provided in online Appendix B, if countries possessed different technologies to convert their secure holdings of resource into their respective intermediate input, their arming choices under peaceful settlement would differ even when neither country is resource constrained; by contrast, their arming choices under open conflict would remain identical.

[36] As shown in an earlier version of the paper, when the division of $\bar{X}$ is based on the split-the-surplus solution, G_s is decreasing in the rate of destruction, because destruction erodes the countries' threatpoint payoffs. Nonetheless, the key insights remain qualitatively unchanged.

Proposition 3 (Arming under peaceful settlement.) *In the subgame related to peaceful settlement, there exist two types of Nash equilibria. In the first, labeled "unarmed peace," both countries choose not to arm (i.e., $G^i = 0$, for $i = 1, 2$). This equilibrium can arise only if* (i) $\theta \in (0, \frac{1}{2}]$ *and* (ii) *the initial configuration of factor ownership is within a set $\mathcal{H}(\theta)$ of sufficiently even resource distributions, where the size of $\mathcal{H}(\theta)$ is decreasing in θ and consists of just one element ($R^i = \bar{R}/2$) when $\theta = \frac{1}{2}$. In the second type, labeled "armed peace," both countries arm (i.e., $0 < G_s^i \leq R^i$, with equality for at most one country). This equilibrium, which can arise for all $\theta \in (0, 1]$ and all distributions of factor ownership, is unique and has the following properties:*

(a) *If $R^i \in [R_L^s, R_H^s]$, then $G_s^i \equiv G_s = R_L^s(\theta)$ and $\phi_s^i = \frac{1}{2}$ for $i = 1, 2$.*

(b) *If $R^i \in (0, R_L^s)$ for $i = 1$ or 2, then $G_s^i = R^i < G_s^j = \widetilde{B}_s^j(R^i)$ and $\phi_s^i < \phi_s^j$ for $j \neq i$.*

(c) *$dG_s^j/d\theta = 0$ for $R^i \in (0, R_L^s]$, $dG_s^j/d\theta > 0$ for $R^i \in (R_L^s, \bar{R})$ and $d\left(R_H^s - R_L^s\right)/d\theta < 0$ with $\lim_{\theta\to 0} R_L^s = 0$ and $\lim_{\theta\to 0} R_H^s = \bar{R}$, whereas $\lim_{\theta\to 1} R_L^s = \frac{1}{4}\bar{R}$ and $\lim_{\theta\to 1} R_H^s = \frac{3}{4}\bar{R}$.*

When the gains from trade are sufficiently large (i.e., $\theta \in (0, \frac{1}{2}]$), the expectation of peaceful settlement supports two distinct Nash equilibria: unarmed peace and armed peace. Recall the best-response property of unarmed peace requires each country i's gains from trade under the status quo (i.e., where $\phi^i = R^i/\bar{R}$ and thus $Z^i = X^i = R^i$) to exceed the payoff it would realize if it captured the entire pool $\bar{X} \approx \bar{R}$ (and thus wiped out the opportunity for trade). This condition implicitly defines the set of sufficiently even resource distributions conditioned on $\theta \in (0, \frac{1}{2}]$, $R^i \in \mathcal{H}(\theta)$, for which neither country would choose to arm, given its rival has not armed.[37] Larger gains from trade ($\theta \downarrow$) amplify the payoffs under the status quo and thus expand the set $\mathcal{H}(\theta)$.

Armed peace, by contrast, can arise for all $\theta \in (0, 1]$ and all endowment distributions. As in the case where both countries anticipate open conflict, uneven factor ownership could result in one country being limited in its arming decision by its resource constraint. Part (a) shows that, when the initial distribution of resource ownership is sufficiently even, the resource constraint on arming binds for neither country and both countries produce equal quantities of guns: $G_s^i = R_L^s$ for $i = 1, 2$.[38] Part (b) shows that, if the initial resource ownership is sufficiently uneven, the less affluent country specializes completely in the production of arms whereas its richer adversary diversifies its production and, at the same, arms by relatively more. As in the case of open conflict, total equilibrium arming $\bar{G} = G_s^i + G_s^j$ rises with resource reallocations to the smaller country, and is maximized at $\bar{G} = 2R_L^s$ when the distribution is sufficiently even.

How does the degree of substitutability θ (which, once again, is inversely related to the

[37]For $R^i \notin \mathcal{H}(\theta)$, it is the less affluent country that prefers to arm when its rival does not, implying that only armed peace is possible under settlement.

[38]Interestingly, as shown in Lemma A.6, this outcome is precisely the one a benevolent social planner would choose for any given $\bar{G} < \bar{R}$. The difference here, of course, is that arming is endogenously determined by policymakers that are motivated by their respective national interests.

magnitude of a country's gains from trade) matter here? Part (c) establishes that, regardless of whether one's rival is resource constrained, an unconstrained country's guns choice depends positively on θ.[39] Furthermore, because increases in θ amplify the countries' incentives to arm and have no effect on $\bar{R}$, they naturally shrink the range of resource endowments for which neither country is resource constrained. Fig. 2(a) illustrates how a country's guns choice under armed peace depends on both θ and the initial distribution of resource ownership.[40]

3.2.2 Payoffs under Peaceful Settlement

Let us now study payoffs under settlement. As argued above, if peace arises in period $t = 1$, then it arises in period $t = 2$, such that a country's average discounted payoff V_s^i coincides with its per period (stationary) payoff v_s^i defined in (12). The next proposition summarizes the salient implications, including details related to both armed and unarmed peace.

Proposition 4 (Payoffs under peaceful settlement.) *Under peaceful settlement, with or without arming, a country's average discounted payoff V_s^i is independent of the discount factor δ and the rate of destruction $1 - \beta$. Under unarmed peace, country i's payoff V_s^i is* (i) *concave in R^i;* (ii) *maximized at a unique $R^i_{\max} \in (\frac{1}{2}\bar{R}, \bar{R})$; and,* (iii) *decreasing in θ. Additionally, $\lim_{R^i \to 0} V_s^i = 0$, $\lim_{R^i \to \bar{R}} V_s^i = \bar{R}$, and $\lim_{R^i \to \bar{R}} dV_s^i/dR^i < 0$. Under armed peace, V_s^i depends on the initial resource ownership and the degree of substitutability θ as follows:*

- (a) *If $R^i \in [R_L^s, R_H^s]$ for $i = 1, 2$, then $V_s^i = V_s^j = 2^{(1/\theta)-2}\bar{R}/(1+\theta)$, with $dV_s^i/d\theta < 0$.*
- (b) *If $R^i \in (0, R_L^s)$ for $i = 1$ or 2, then $\lim_{R^i \nearrow R_L^s} dV_s^i/dR^i \leq 0$, while $dV_s^j/dR^j > 0$ and $d^2V_s^j/(dR^j)^2 > 0$. Furthermore, $dV_s^i/d\theta < 0$ and $dV_s^j/d\theta < 0$.*
- (c) *If $\theta \in (0, 1)$, then $V_s^i > 0$ and $V_s^j > \bar{R}$ for R^i close to 0.*

There exists a critical value of θ, $\widehat{\theta} \approx 0.402$, such that unarmed peace is Pareto preferred to armed peace (i) *for all $R^i \in \mathcal{H}(\theta)$ if $\theta \in (\widehat{\theta}, \frac{1}{2}]$ and* (ii) *for all $R^i \in \widehat{\mathcal{H}}(\theta)$ if $\theta \in (0, \widehat{\theta})$, where $\widehat{\mathcal{H}}(\theta) \subset \mathcal{H}(\theta)$.*

The payoffs under unarmed peace and their properties are precisely what one would expect based on our (static) model of trade if there were no dispute over resource claims and each country i $(= 1, 2)$ maintained its initial resource endowment R^i. We emphasize again, however, that even when countries anticipate peaceful settlement, unarmed peace can arise as a Nash equilibrium only if the gains from trade are sufficiently high and the distribution of factor ownership is sufficiently symmetric (i.e., $\theta \in (0, \frac{1}{2}]$ and $R^i \in \mathcal{H}(\theta)$). By contrast, armed peace is always a possible Nash equilibrium given the anticipation of settlement, such that the associated payoffs characterized in the proposition apply under all parameter values and all feasible distributions of resources.

[39] While an increase in θ (implying lower gains from trade) unambiguously reduces the marginal cost of arming, its effect on the marginal benefit is ambiguous. Nevertheless, in the proof to this part of the proposition, we demonstrate that the effect on the marginal cost dominates.

[40] Ignore the pink curve for now. As shown below, assuming $\delta = 0$ and $\theta = 1$ implies that $G_s^i = G_c^i$.

Part (a) establishes the welfare implications of armed peace when the distribution of initial claims of ownership is sufficiently even. In particular, since the two countries arm identically in this case and, as a consequence, each receives an equal share $\phi_s^i = \frac{1}{2}$ of $\bar{X}$, their payoffs are identical. These payoffs are decreasing in θ due to (i) a direct adverse effect on the gains from trade and (ii) an indirect and adverse effect due to the rival's increased production of arms. Furthermore, any reallocation of $\bar{R}$ across the two countries that keeps their endowments in $[R_L^s, R_H^s]$ leaves arming and thus equilibrium payoffs unchanged.

Part (b) shows what happens in the case where country i is resource-constrained (and thus country j is not). Specifically, while numerical analysis shows that constrained country i's payoff is increasing (decreasing) in the neighborhood of 0 for sufficiently large (small) values of θ, this part of the proposition establishes that it is necessarily decreasing in R^i as R^i approaches R_L^s. Thus, V_s^i likely reaches a local maximum for $R^i \in (0, R_L^s)$. Meanwhile, the unconstrained country's payoff is increasing and convex in its own endowment.[41] The last component of (b) shows that both the constrained country (i) and the unconstrained country (j) benefit as the gains from trade rise (i.e., $\theta \downarrow$). These results with those from part (a) indicate that, for all possible distributions of $\bar{R}$ where each country initially holds a strictly positive amount of the resource, both countries necessarily benefit from enhanced gains from trade. See Fig. 2(b) that illustrates (in green) a country's payoffs under armed peace for various distributions of initial resource ownership and values of θ that negatively influence the gains from trade.[42]

Part (c) establishes that the payoffs for both countries under armed peace are strictly greater than their respective payoffs when one country initially has a claim to nearly all of $\bar{R}$. For additional insight, suppose R^j is infinitesimal but positive. Then, the affluent rival i can also produce an infinitesimal quantity of guns. But, since it is unconstrained by its endowment, it can arm in a way that enables is to obtain $R_{\max}^i$ units of $\bar{X}$ ($\approx \bar{R}$), so that its payoff is (roughly) equal to the highest possible payoff it could obtain under unarmed peace. In essence, under settlement, the unconstrained country (j) finds it appealing to effectively permit the smaller country (i) to produce more of its tradable good and thereby take greater advantage of the opportunities for trade. Since more trade is mutually advantageous, country j's rival (country i) also finds this arrangement appealing.[43]

As discussed earlier in connection with Proposition 3, when $\theta > \frac{1}{2}$, no initial endowment distribution $R^i \in (0, \bar{R})$ exists under which V_{su}^i $(= v_u^i) > \bar{R}$ for both countries; thus, in this case, armed peace is the only possible outcome, as illustrated with the thick green payoff

[41] These results suggest that a redistribution of $\bar{R}$ away from the constrained country (i) towards the unconstrained country (j) could actually be welfare improving in a Pareto sense.

[42] Ignore the pink curve for now.

[43] This reasoning is consistent with our finding discussed above in relation to part (b), that the sign of dV_s^i/dR^i as $R^i \to 0$ could be negative. In particular, an increase in R^i (for R^i initially close to 0) could generate a strong and adverse strategic effect for country i, as the large country j adjusts its guns to ensure it captures a larger share of the residual resource, that dominates the direct and positive effect associated with the increase in R^i. Nonetheless, country i is better off under peace with trade than under war.

curve V^i_{sa} in panel (a) of Fig. 3. The very last component of Proposition 4 Pareto ranks the payoffs under armed peace and unarmed peace when these two outcomes coexist—i.e., when $\theta \in (0, \frac{1}{2}]$ and $R^i \in \mathcal{H}(\theta)$. Panels (b) and (c) show the possibilities.[44] In particular, assuming $\theta \in (\widehat{\theta}, \frac{1}{2})$, panel (b) shows that unarmed peace is possible for $R^i \in \mathcal{H}(\theta) = (R^i_o, \bar{R} - R^i_o)$, where R^i_o solves $V^i_{su} = \bar{R}$. For $R^i \in (0, R^i_o)$, country i's best-response to $G^j = 0$ is to arm by $\epsilon > 0$; likewise, for $R^i \in (\bar{R} - R^i_o, \bar{R})$, country j would arm by $\epsilon > 0$. Moreover, when $R^i \in \mathcal{H}(\theta)$, unarmed peace is Pareto preferred to armed peace. Matters differ when $\theta \in (0, \widehat{\theta})$, as illustrated in panel (c) of the figure. While unarmed peace remains possible for all $R^i \in H(\theta)$, it is Pareto preferred to armed peace only for $R^i \in \widehat{\mathcal{H}}(\theta) \subset \mathcal{H}(\theta)$. When $R^i \in (R^i_o, R^i_{oo})$, where R^i_{oo} $(> R^i_o)$ solves $V^i_{sa} = V^i_{su}$, country i prefers armed peace while country j prefers unarmed peace, and vice versa when $R^i \in (\bar{R} - R^i_{oo}, \bar{R} - R^i_o)$. These more uneven distributions in $\mathcal{H}(\theta)$ but not in $\widehat{\mathcal{H}}(\theta)$ tend to support a higher payoff under armed peace than under unarmed peace only for the less affluent country, despite the effect of arming to reduce the quantity of the resource being contested $\bar{X}$.[45]

3.3 Peace versus War

We now examine how outcomes across peaceful settlement and war differ. Since, as established in Proposition 4 and illustrated in Fig. 3, unarmed peace is Pareto preferred to armed peace for certain resource distributions if the gains from trade are large enough (i.e., $\theta \leq \frac{1}{2}$), we will associate settlement with unarmed peace under these circumstance and with armed peace otherwise.

It should be clear that the possible avoidance of war's destructive effects ($\beta < 1$) and the possible realization of gains from trade ($\theta < 1$) add to the relative appeal of peaceful settlement. The relative appeal of war to a country, by contrast, derives from the possibility of emerging as the winner and capturing the current residual and future resources without having to arm in the future. This consideration is more important when the shadow of the future ($\delta > 0$) is larger. Another factor to consider is how arming incentives in period $t = 1$ compare across the two modes of conflict resolution.

We start with that comparison, focusing initially on the following benchmark case: suppose there are no gains from trade ($\theta = 1$), countries do not value the future ($\delta = 0$), and war is not destructive ($\beta = 1$). Since in the absence of gains from trade we have $\mu^i = 1$, the aver-

[44] In all three panels of the figure, the thickened segments of V^i_{sa} and V^i_{su} indicate that the associated outcomes are immune to unilateral deviations respectively to armed peace and to unarmed peace.

[45] This finding is, in part, related to Hirshleifer's (1991) "paradox of power." Specifically, the payoff-maximizing arming choices under settlement imply a share for the poorer country i that exceeds its initial share of total resources $R^i/\bar{R}$—or, equivalently by (1), its share when neither country arms: $\phi^i(G^i_s, G^j_s) > \phi^i(0, 0)$. Also at play here is the effect of a more symmetric distribution of residual resources (induced by armed peace) to magnify the total gains from trade and more so when θ is smaller. From the less affluent country's perspective, these two effects combined dominate the negative effect of diverting resource away from production to arming. Indeed, starting from an initial distribution of resource ownership that is extremely uneven within $\mathcal{H}(\theta)$ and assuming θ is sufficiently close to zero—where aggregate guns production is relatively small to begin—the gains from trade could be sufficiently larger under armed peace to render it preferable to the more affluent country, too. (This latter possibility is not shown in the figure.)

age payoff under settlement is $v^i = \omega^i \bar{X} \equiv \mu^i \phi^i \bar{X} = \phi^i \bar{X}$ for both $i = 1, 2$. Furthermore, since future payoffs are not valued, the average expected payoff under war is $U^i = u^i = \phi^i \bar{X}$ for both $i = 1, 2$ as well. Consequently, given rival j's arming choice, the unconstrained country (i's) incentives to arm under war and settlement are identical, such that $[R_L^c, R_H^c] = [R_L^s, R_H^s]$ and $G_c^i = G_s^i$ for $i = 1, 2$ and any initial distribution $R^i \in (0, \bar{R})$.[46]

The next lemma shows how departures from this benchmark case matter for equilibrium arming under war and armed peace.

Lemma 1 (A comparison of arming.) *If either $\delta > 0$ or $\theta < 1$, then for any $\beta \in (0, 1]$ the following relations hold under war and armed peace:*

(a) *the resource constraint on arms binds for a larger set of factor allocations under war than under armed peace (i.e., $[R_L^c, R_H^c] \subset [R_L^s, R_H^s]$);*

(b) *$G_c^i \geq G_s^i$, with strict inequality for at least one country.*

In effect, settlement reduces each country i's incentive to arm relative to war in period $t = 1$ given the rival's choice $G^j > 0$.[47] As such, settlement induces lower aggregate arming and, thus, social waste in period $t = 1$ relative to war. But, because guns production represents a recurrent use of resources under armed peace though not under war (or unarmed peace), a comparison of payoffs under these alternative regimes is a bit more involved.

How payoffs under open conflict U_c^i and peaceful settlement V_s^i compare is an interesting question in its own right, but also sheds light on which outcome is more likely to be observed when both are possible. Suppose country j declares "war" and prepares accordingly in the first period. Then, country i's best reply would be to arm for war too.[48] Since the same logic applies to country j, open conflict could be a possible outcome. However, insofar as countries can communicate freely in the process of their negotiations, one would expect them to pursue a mode of conflict resolution that best advances both their mutual and self interests. Thus, whenever both countries' average discounted payoffs under settlement V_s^i exceed those under open conflict U_c^i, open conflict cannot be a stable equilibrium; in this case, the two countries might be able to coordinate on a peaceful outcome.

Let us return to the benchmark case where $\delta = 0$, $\beta = 1$ and $\theta = 1$ (or $\sigma = \infty$ so that $\mu^i = 1$ for $i = 1, 2$). In this case that precludes unarmed peace, (expected) payoffs under war and settlement are equivalent: $U_c^i = V_s^i$ for all $R^i \in (0, \bar{R})$. This equivalence is depicted by the solid green curve in Fig. 2(b) associated with $\theta = 1$. Now suppose the shadow of the future δ increases above zero. Proposition 4 establishes that an increase in δ leaves V_s^i unchanged, whereas Proposition 2 establishes that the increase in δ causes U_c^i to decrease for $R^i \in (0, R_L^c)$, remain unchanged at $U_c^i = V_s^i = \beta \frac{\bar{R}}{4}$ for $R^i \in [R_L^c, R_H^c]$ (while shrinking

[46] As established in Propositions 1 and 3 arming incentives under both open conflict and peaceful settlement are independent of the magnitude of war's destruction $(1 - \beta)$.

[47] That one country might arm identically under war and peace is possible for more uneven resource distributions, where that country is resource constrained in its arming under both modes of conflict resolution.

[48] Since the emergence of war requires only one country to declare it, country i's declaration of "war" or "peace" would not matter given that country j declares "war."

the size of that range), and to increase for $R^i \in (R^c_H, \bar{R})$, as illustrated in the context of Fig. 1(b). Thus, while an increase in δ leaves U^i_c unchanged and equal to V^i_s for sufficiently even distributions of $\bar{R}$, it matters for sufficiently uneven distributions: $U^i_c < V^i_s$ when $R^i \in (0, R^c_L)$ and $U^i_c > V^i_s$ when $R^i \in (R^c_H, \bar{R})$. Fig. 2(b) illustrates this ranking of payoffs with the pink curve representing U^i_c under the extreme assumption that $\delta = 1$, in which case the range $[R^c_L, R^c_H]$ shrinks to a single point, $R^c_L = R^c_H = \bar{R}/2$.[49]

Next, consider a series of decreases in θ (reflecting greater gains from trade), with $\delta = \beta = 1$ fixed in the background. While U^i_c (again, depicted by the pink curve in Fig. 2(b)) is independent of such gains, Proposition 4 established that V^i_s rises at each $R^i \in (0, \bar{R})$, as shown by the upward shift of the green curves in Fig. 2(b). Initial increases in these gains (starting from $\theta = 1$) imply $V^i_s > U^i_c$ for both countries i when the initial distribution of $\bar{R}$ across them is sufficiently even (up to point A in the figure) and when it is sufficiently uneven (beyond point B in the figure). But, once θ falls below a threshold level (associated with point C), settlement Pareto dominates war in a payoff sense for all possible $R^i \in (0, \bar{R})$.[50]

Finally, let us consider the destructiveness of war. Returning to our benchmark assumptions that $\delta = 0$, $\theta = 1$, and $\beta = 1$, an increase in war's destructive effects ($\beta \downarrow$) does not affect V^i_s and reduces U^i_c. Therefore, decreases in β imply $V^i_s > U^i_c$ for all allocations of $R^i \in (0, \bar{R})$. Accordingly, when $\delta = 0$ and $\beta < 1$, settlement is Pareto preferred to war under all resource allocations $R^i \in (0, \bar{R})$ even when there are no gains from trade ($\theta = 1$). For larger values of $\delta > 0$, there exists a threshold rate of destruction $1 - \beta_0$, such that when $\beta < \beta_0$ armed peace payoff is Pareto preferred to war for all $R^i \in (0, \bar{R})$, again even when $\theta = 1$.

The next lemma builds on and extends these ideas:

Lemma 2 (A comparison of payoffs.) *For any given $\delta \in (0, 1]$, there exists a threshold rate of destruction $1 - \beta_0$ where*

$$\beta_0 \equiv \beta_0(\delta) = 1 \Big/ \left(2 - \sqrt{\frac{1}{1+\delta}}\right)^2 \in (0, 1),$$

with $\partial\beta_0/\partial\delta < 0$, and a threshold degree of input substitutability $\theta_0 \equiv \theta_0(\delta, \beta) \in (\frac{1}{2}, 1)$ for $\beta > \beta_0$, with $\partial\theta_0/\partial\delta < 0$ and $\partial\theta_0/\partial\beta < 0$, such that peace is Pareto preferred to open conflict (i.e., $V^i_s > U^i_c$ for $i = 1, 2$) under the following circumstances:

(a) *if $\beta \in (0, \beta_0]$, then for any $R^i \in (0, \bar{R})$;*

(b) *if $\beta \in (\beta_0, 1]$ and*

[49] For less extreme values of $\delta \in (0, 1)$ with $\beta = \theta = 1$, war remains relatively more appealing for country i when its rival (j) is resource constrained under war $R^i \in (R^c_H, \bar{R})$; meanwhile, its constrained rival (j) has a preference for settlement. But, both countries are indifferent between war and peace for $R^i \in [R^c_L, R^c_H]$, where neither country is resource constrained under war.

[50] As noted below, when θ is above this threshold, unarmed peace is not feasible, such that the relevant comparison is between armed peace and war as shown in Fig. 2(b). While this figure shows the extreme cases of $\delta = 0$ and $\delta = 1$, the thrust of the above argument holds true for any $\delta \in [0, 1]$.

(i) $\theta \leq \theta_0$, *then for any* $R^i \in (0, \bar{R})$;

(ii) $\theta > \theta_0$, *then only for sufficiently even and sufficiently uneven international allocations of asset ownership.*

This lemma shows that peace Pareto dominates war in a payoff sense under a variety of conditions. In particular, when war is sufficiently destructive $\beta \leq \beta_0$, peace is Pareto preferred to war for all $R^i \in (0, \bar{R})$ regardless of the size of the gains from trade. The threshold rate of destruction $1 - \beta_0$ is increasing in the salience of the future δ. Even when war is not sufficiently destructive, large enough gains from trade $\theta \leq \theta_0$ similarly render peace payoff dominant over war for both countries and all $R^i \in (0, \bar{R})$. The threshold $\theta_0 > \frac{1}{2}$ is decreasing in both δ and β. Alternatively, when war is not very destructive and the gains from trade are moderate such that $\theta > \theta_0$, peace is not preferred to open conflict by both countries for all $R^i \in (0, \bar{R})$. Instead, as illustrated in Fig. 2(b), peace is Pareto preferred to war only when (i) the international distribution of resource ownership is sufficiently even; or (ii) this distribution is sufficiently uneven.[51]

The relevance of the distribution of resource ownership in the ranking of payoffs across the two modes of conflict resolution underscores the importance of considering arming choices as well as the magnitude of the gains from trade. That is to say, it is not sufficient to consider the gains from trade alone. One can see this most clearly in the case where $\beta \in (\beta_0, 1]$ and the distribution is sufficiently uneven to make country j's resource constraint binding in its arming choice. While the unconstrained country (i) might not gain much from trade due to its relatively larger size, it still prefers peaceful settlement over war since the lower arming in period $t = 1$ and the opportunity for trade in that period and the next period generates enough gains to outweigh the possible benefits of wiping out the rival.

Finally, observe that the conditions stated in the lemma do not distinguish between armed and unarmed peace. Since θ_0 achieves a minimum value ≈ 0.61468 (when $\delta = \beta = 1$) that exceeds the maximum value of θ under which unarmed peace might arise ($\frac{1}{2}$), unarmed peace is not feasible for any $\delta, \beta \leq 1$ when $\theta > \theta_0$, and it suffices to compare payoffs under armed peace with those under open conflict.

4 Unilateral Deviations and the Stability of Peace

Our comparison of peace with open conflict allows us to identify the circumstances under which peace is Pareto preferred to war. But, for peace to arise as a stable equilibrium, it must also be immune to unilateral deviations from it. We address this issue next.

A country i could deviate unilaterally from settlement in one of two ways within a particular time period: (i) Given both countries' gun choices in the first stage under settlement ($G^i = G^i_s$ for $i = 1, 2$) and the expectation the rival country j will declare "peace" in the second stage, country i could simply declare "war" in that stage; (ii) Given $G^j = G^j_s$ in the first stage and the expectation that country j will declare "peace" in the second stage, country

[51] As argued above, given peace prevails in $t = 1$, it Pareto dominates war in a payoff sense also in $t = 2$.

i could choose a different quantity of guns, denoted by $G_d^i \neq G_s^i$ ("d" for deviation), in the first stage and then declare "war " in the second stage. The first type of deviation is relevant when country i's arming decision under settlement G_s^i is limited by its resource endowment R^i ($\leq R_L^s$). The second type of deviation arises when country i's resource constraint on G_s^i is not binding. However, because war inevitably breaks out either way, a country's optimal arming under either type of unilateral deviation is given by its best reply to G_s^j ($j \neq i = 1, 2$) shown in (10), $G_d^i = B_c^i(G_s^j)$, which could be constrained.[52]

With the above in mind, the following point deserves emphasis. From (8), one can verify that the highest possible average payoff a country $i = 1, 2$ might secure for itself under any unilateral deviation is less than $\bar{R}$ for all $R^i \in (0, \bar{R})$. Yet, as we saw in Section 3.2.1, the best-response property of unarmed peace requires that the associated payoff be at least $\bar{R}$. In other words, when unarmed peace is a feasible outcome, it is necessarily an outcome that is immune to the sort of unilateral deviations just described. Therefore, although we focus on the incentives for unilateral deviations from armed peace, our findings to follow apply to peace more generally.

To lay the foundation for our analysis of unilateral deviations in the overall game, let us first consider deviation incentives in the subgame in period $t = 2$ (given that peace prevailed in period $t = 1$). Since $\delta = 0$ in period $t = 2$, we can rewrite country i's payoff under war and peaceful settlement as $u^i = \beta\phi^i\bar{X}$ and $v^i = \mu^i\phi^i\bar{X}$, respectively. Now observe that, for any feasible quantity of guns produced in stage 1 of $t = 2$ and thus $\bar{X}$, both countries would declare "peace" in stage 2 as long as this declaration enables them to avoid destruction ($\beta < 1$) and/or realize the gains from trade ($\theta < 1$, which implies $\mu^i > 1$). In anticipation of that outcome in stage 2, each country i, then, optimally chooses to produce G_s^i in stage 1 of the same period. Hence, a sufficient condition for settlement to be the unique equilibrium of the period $t = 2$ subgame is that $\beta < 1$ and/or $\theta < 1$.[53]

Turning to the extended game, suppose in the first stage of period $t = 1$, each country i anticipates that its rival will choose settlement in the second stage and that its rival j will do the same. Each country i could choose to validate rival j's expectation by producing G_s^i in stage 1 and declaring "peace" in stage 2, or it could deviate in stage 1 by producing $G_d^i = B_c^i(G_s^j)$ shown in (10) and then declaring "war" in stage 2. To characterize the optimizing deviation, we distinguish between four intervals of resource allocations for R^i: (i) $(0, R_L^s)$; (ii) (R_L^s, R_L^d); (iii) (R_L^d, R_H^s); and (iv) $(R_H^s, \bar{R})$, where R_L^d denotes the threshold level of country i's resource, below which it is resource-constrained in its arming under a unilateral deviation.[54] For the first two intervals, country i's optimal unilateral deviation in arming is

[52]Note that a country would not deviate by choosing another quantity of guns $G_d^i \neq G_s^i$, without also declaring war in the second stage. That is to say, if country i anticipates choosing (along with country j) settlement in the second stage, then it's optimal choice of guns would be given by $G_s^i = B_s^i(G_s^j)$.

[53]Even if country i were to adjust its first-stage arming with the intention of declaring "war" in stage 2 (i.e., arm according to $G_d^i = B_c^i(G_s^j) > G_s^i$), it would be better off by declaring "peace" in stage 2, implying that the choice of $G^i = G_d^i$ could not have been optimal to begin with.

[54]Fig. 4, which we will discuss in more detail below, shows these intervals. The inequality $R_L^d > R_L^s$ follows

constrained by its resource endowment R^i. More precisely, in case (i) where $R^i \in (0, R_L^s)$ such that country i would be constrained under settlement, its unilateral deviation involves no adjustment in arming, only a declaration of war: $G_d^i = B_c^i(G_s^j) = G_s^i = R^i$. In case (ii), where $R^i \in (R_L^s, R_L^d)$, country i's optimal deviation entails both a declaration of war and producing a larger quantity of guns as compared with settlement, but only as much as its endowment allows: $G_d^i = B_c^i(G_s^j) = R^i > G_s^i = G_s^j = R_L^s$. For the last two intervals, country i is no longer resource constrained. Specifically, in case (iii) where $R^i \in (R_L^d, R_H^s)$, country i's optimal deviation is given by its unconstrained best-response function under open conflict evaluated at its rival's arming when neither country is resource-constrained under settlement, $G_s^j = G_s = R_L^s$: $G_d^i = \widetilde{B}_c^i(R_L^s) = R_L^d$. In case (iv) where $R^i \in (R_H^s, \bar{R})$, country j's arming is constrained, while country i operates along its best-response function under war $G_d^i = \widetilde{B}_c^i(R^j)$, which equals precisely the amount of its arming under war.

Next, we ask: when are the unilateral deviations described above profitable? Let $W_d^i \equiv U^i(G_d^i, G_s^j)$ denote the payoff to country i under such deviations. Observe that this payoff satisfies $U^i(G_d^i, G_s^j) \geq U^i(G_s^i, G_s^j)$, as a strict inequality when G_s^i is unconstrained, and is non-increasing in θ, since $U^i(G_d^i, G_s^j)$ is decreasing in the rival's guns choice G_s^j and $dG_s^j/d\theta \geq 0$ as shown in Proposition 3(c). We take as our starting point the special case, illustrated in the two panels of Fig. 4, where there is no discounting of future one-period payoffs ($\delta = 1$), open conflict is not destructive ($\beta = 1$) and $\theta = \theta_0(\delta, \beta) = \theta_0(1, 1)$ as defined in Lemma 2.[55] Panel (a) depicts country i's arming choices under a unilateral deviation (in blue), as well as its arming under peace (in green) and war (in pink), over the range of possible resource distributions. Panel (b) shows the corresponding payoffs, using the same color scheme.[56]

Key here is our starting point: $\theta = \theta_0(1, 1)$. Consider, first, interval (iv), where $R^i \in (R_H^s, \bar{R})$. Since country i is not constrained under either war or a unilateral deviation from peace, its arming choice (given its constrained opponent's choice, $G_s^j = R^j$ $(= G_c^j)$) is the same under both scenarios: $G_d^i = G_c^i = \widetilde{B}_c^i(R^j)$. As a result, $W_d^i(R^i) = U_c^i(R^i)$ for this interval. But, by the definition of $\theta_0(1, 1)$, $V_s^i(R^i) > U_c^i(R^i)$ holds, such that country i has no incentive to deviate unilaterally from peace in interval (iv). Next, consider interval (iii) where $R^i \in (R_d^i, R_H^s)$ and thus $G^j = G_s = R_L^s$. In this case, country i's unilateral deviation is given by $G_d^i = \widetilde{B}_c^i(R_L^s) = R_L^d$, implying that $W_d^i(R^i) = U_c^i(R_H^s)$. At the same time, consistent with our assumption that $\theta = \theta_0$, $V_s^i(R^i) = U_c^i(R_H^s)$ holds. Thus, for interval (iii), we have $W_d^i(R^i) = V_s^i(R^i)$, such that country i has no incentive to deviate unilaterally from peace.

from our finding in Lemma 1 that country i's incentive to arm for any given G^j is higher under open conflict than under settlement.

[55] More precisely, $\theta_0(1, 1)$ is the critical value of θ, conditioned on $\delta = \beta = 1$, that ensures the payoff under armed peace just equals the payoff under open conflict, which is shown in panel (b) of the figure as occurring at $R^i = R_H^s$ (i.e., $V_s^i(R_H^s) = U_c^i(R_H^s)$). Using numerical methods, one can show that $\theta_0(1, 1) \approx 0.61468$.

[56] Our focus on $\delta = 1$, which implies $R_L^s = R_H^c = \bar{R}/2$ as noted earlier and shown in Fig. 4(a), ensures the highest possible payoffs for each country under a unilateral deviation from settlement, allowing us to identify the lowest bound on the gains from trade (given some rate of destruction $1 - \beta$) that would be required for peace to be immune to unilateral deviations from it. Consideration of other values of δ is straightforward and thus omitted.

In such cases, the gains from trade that can be realized under settlement match precisely the expected gains from a unilateral deviation that involve the possibility of emerging as the victor in war and not having to arm in the future.[57]

What about the other intervals where country i is constrained in its arming under a unilateral deviation and possibly under settlement? We already know, from above, that the definition of θ_0 implies $W_d^i(R_L^d) = V_s^i(R_L^d) = V_s^i(R_H^s) = U_c^i(R_H^s)$ when $R^i = R_L^d$. Therefore, country i has no incentive to deviate from settlement at that point. We also know that, as R^i falls into interval (ii) where $R^i \in (R_L^s, R_L^d)$, country i's payoff under settlement remains constant. At the same time, its resource constraint on arming under a unilateral deviation kicks in, becoming increasingly severe to push its deviation payoff $W_d^i(R^i)$ further below $V_s^i(R^i)$ as R^i approaches R_L^s. As R^i falls further, moving into interval (i) where $R^i \in (0, R_L^s)$, so does its deviation payoff, which eventually approaches 0 as R^i approaches 0.[58] While country i's settlement payoff also starts to decline as R^i falls within interval (i), that payoff remains above its deviation payoff, approaching a positive amount due to the large gains from trade $\mu^i > 1$ as R^i becomes very small but remains strictly positive. Accordingly, $V_s^i(R^i) > W_d^i(R^i)$ for $R^i \in (0, R_L^d)$ when $\theta = \theta_0$.

The above establishes that, when $\theta = \theta_0$, neither country has an incentive to deviate from settlement for any resource allocation $R^i \in (0, \bar{R})$. Any decrease in θ below θ_0, implying greater gains from trade for any resource distribution, tilts the balance even more towards settlement. Building on these results, we now describe more generally the robustness of peace, whether unarmed or armed, to unilateral deviations as follows:

Lemma 3 (Immunity to unilateral deviations.) *For any given $\delta \in (0, 1]$, peace is immune to unilateral deviations (i.e., $V_s^i > W_d^i$ for $i = 1, 2$) under the following circumstances based on the thresholds β_0 and θ_0, defined in Lemma 2:*

(a) *if $\beta \in (0, \beta_0]$, then for any $R^i \in (0, \bar{R})$;*

(b) *if $\beta \in (\beta_0, 1]$ and*

  (i) *$\theta \leq \theta_0$, then for any $R^i \in (0, \bar{R})$;*

  (ii) *$\theta > \theta_0$, then only for sufficiently uneven international distributions of resource ownership.*

Comparing the conditions stated in Lemma 2 with those stated above reveals that the unprofitability of a unilaterial deviation from peace is sufficient for peace to be Pareto preferred to war, but not vice versa. In particular, when war is only moderately destructive and the gains from trade are not large enough (i.e., when $\theta > \theta_0$), peace Pareto dominates war in a payoff sense for sufficiently even and uneven resource distributions, but is not immune from unilateral deviations for the more even distributions. In such cases, the overall gains from

[57] More generally, if $\beta < 1$ which would imply a larger value of θ_0 relative to what is drawn in Fig. 4(b), the equality $W_d^i(R^i) = V_s^i(R^i)$ would also reflect the benefit of settlement to avoid war's destruction.

[58] Observe $U_c^i(R^i) < W_d^i(R^i)$ for $R^i \in (0, R_L^s)$ as shown in Fig. 4(b), because the optimizing deviation operates on $B_c^i(G^j)$ but with $G^j = G_s^j < G_c^j$.

peace fall short of the expected benefits of unilaterally deviating from that outcome to the larger country and possibly both countries—namely, to gain an edge in war by arming more and to enjoy (in the case of a victory) the resource savings, associated with not having to arm in the subsequent period, that tend to be higher for more even distributions.[59] By contrast, under armed peace for more uneven resource distributions, the less affluent country realizes greater relative gains from trade and its limited resources makes its chances to win a war relatively small. Meanwhile, the gains from trade to the more affluent country are not very large; but, because arming tends to be smaller for such allocations, the potential future savings from wiping out the adversary are not large enough to make a unilateral deviation profitable.

We now summarize the conditions, based on Lemmas 2 and 3, under which peace emerges in the extended game as the stable equilibrium and the form it takes:

Proposition 5 (Stability of peace.) *Peace arises as a stable equilibrium of the extended game, and is unarmed or armed, depending on the destructiveness of war $1-\beta$, the substitutability of traded goods reflected in θ and the distribution of factor ownership:*

(a) *Suppose either $\beta \leq \beta_0$ given any $\theta \in (0,1)$ or $\beta > \beta_0$ while $\theta \leq \theta_0$. Then, unarmed peace is the stable equilibrium* (i) *for all $R^i \in \mathcal{H}(\theta)$ when $\theta \in (\widehat{\theta}, \frac{1}{2}]$ and* (ii) *for all $R^i \in \widehat{\mathcal{H}}(\theta)$ when $\theta \in (0, \widehat{\theta})$, where $\widehat{\mathcal{H}}(\theta) \subset \mathcal{H}(\theta)$; otherwise, armed peace emerges as the stable equilibrium.*

(b) *Suppose that $\beta > \beta_0$ and $\theta > \theta_0$. Then, unarmed peace is not possible, while armed peace arises as the stable equilibrium but only for sufficiently uneven resource distributions.*

Consistent with the liberal peace hypothesis, part (a) states that, when either war's destructive effects or the gains from trade are sufficiently large, peace is stable under all possible configurations of initial resource distributions. What that hypothesis fails to recognize, however, is how the international distribution of endowments interacts with the gains from trade to shape arming incentives and, through that influence, the stability of peace and the form it takes. Our analysis shows that, whereas unarmed peace tends to be stable for more even distributions (provided $\theta \leq \frac{1}{2}$), armed peace tends to emerge as the stable equilibrium for more uneven distributions. Part (b) of the proposition establishes that, when neither war's destructive effects nor the gains from trade are sufficiently large, only armed peace is possible, and only if the initial distribution of resource ownership is sufficiently uneven. The underlying intuition for these results can be traced back to the fact that, since arming is costly, peace can be costly, too. In particular, because arming incentives under settlement are greatest when initial claims of ownership are distributed more evenly, armed peace is most costly precisely under these same circumstances. This higher cost of armed peace, which implies greater future savings afforded by victory in a war today, matters given $\delta > 0$; and, it

[59] Keep in mind that unarmed peace is not possible for any distribution when $\theta > \theta_0$.

takes on greater weight as the shadow of the future strengthens ($\delta \uparrow$) to make a unilateral deviation more profitable to the larger country. Since unarmed peace involves no such costs, it tends to emerge for more even distributions, provided of course that the gains from trade are sufficiently large (i.e., $\theta < \frac{1}{2}$).

A larger degree of dissimilarity between traded commodities ($\theta \downarrow$) implies greater gains from trade and at the same time less arming under settlement, thereby making it more likely that peace prevails as the stable equilibrium for any given distribution of initial resource ownership. Yet, the stronger is the shadow of the future ($\delta \uparrow$) and/or the lower is the destructiveness of war ($\beta \uparrow$), the less likely it is that peace arises as the stable equilibrium.

5 Generalizations and Qualifications

While our analysis is based on a very simple model of trade, it could be extended to allow each country to produce both tradable inputs through differential access to the relevant technologies as in standard Ricardian type trade models. It could also be extended to the Ricardian framework with a continuum of goods studied by Eaton and Kortum (2002). The payoff function for each country would be nearly identical to what we have here, differing only by a factor that captures the endogenously determined range of goods produced by that country. Similarly, we could allow for the possibility of trading a fixed number of differentiated goods or an endogenously determined number of varieties, as in Krugman (1980). None of these extensions would change the key insights of our analysis.[60]

As analyzed in Appendix A, the baseline model can be extended to allow for the presence of iceberg-type trade costs under peace. Not surprisingly and consistent with the liberal peace hypothesis, the analysis demonstrates that reductions in these costs improve the likelihood of a stable (armed) peace. The intuition here is that such reductions lower the countries' incentives to arm, thereby increasing their payoffs under settlement beyond the added gains from trade for given arms relative to their (hypothetical) payoffs from a unilateral deviation as well as their payoffs under war.

The analysis can also be extended to a multi-country setting, to shed light on the implications of bilateral and multilateral trade liberalization. In an Armington (1969) trade model, with three countries (two rivals and a third neutral country), the result just mentioned, that bilateral trade liberalization between the two rivals alone renders peace more stable, remains intact. Multilateral trade liberalization (i.e., cost reductions in trade with the neutral coun-

[60] One might also extend the analysis to consider the presence of multiple resources as in the Heckscher-Ohlin model—for example, labor which is perfectly secure and land which is partially or completely insecure. Though potentially richer and promising in terms of potential insights, this type of analysis introduces two complications. First, as emphasized in Garfinkel et al. (2015) who study trade and arming only for two small countries, asymmetries in resource endowments, which induce asymmetries in arming choices, render the determination of countries' trade patterns endogenous and complicate the welfare analysis significantly. A second issue, discussed in the context of a different setting by Garfinkel and Syropoulos (2021), arises in the presence of complementarities between these distinct resources in production that affect the (possibly asymmetric) gains from trade and, in turn, the countries' payoffs under settlement and unilateral deviations in ways that remain unclear.

try), in contrast to bilateral trade liberalization, amplifies the rivals' incentives to arm under peace. However, the implied increase in the gains from trade with the third country outweigh the adverse strategic payoff effects of the other rival's increased arming. Accordingly, the rivals' payoffs under armed peace also rise under multilateral trade liberalization. At the same time, their payoffs under an optimal deviation from peace fall due to the greater arming under peaceful settlement. Thus, like bilateral trade liberalization, multilateral trade liberalization can enhance the stability of armed peace.[61]

It is important to emphasize that the above discussion is based on the assumption that conflict and unilateral deviations from peace trigger compete foreclosure of trade between the surviving rival and the neutral country. As such, our finding does not speak directly to Martin, et al.'s (2008) important result that multilateral trade liberalization can destabilize international relations. Nonetheless, we see value in this extension of the model in its clarification of the channels through which trade influences the stability of armed peace. One should be able to see that the ability to trade, not only under peace with the rival and the neutral country, but also under war with the neutral country alone would tend to undermine the stability of peace. This idea brings to the center stage an important geoeconomic issue related to the neutral country and its actual or threatened imposition of trade sanctions. Specifically, it underscores the potential role for the neutral country to support peace between rivals by reducing their future payoffs under conflict and any deviation from peace.

In the context of our baseline model with only two contending countries, one can allow for possible differences in their aggregate productivites (or their abilities to transform their resource into their respective intermediate inputs). As shown in online Appendix B, such differences tend to weaken the stability of peace. On the one hand, relative to the case where this technology is symmetric, an exogenous improvement in one country's aggregate productivity enhances both countries' payoffs under peaceful settlement. On the other hand, for any given initial distribution of resource endowments where neither country is constrained in its arming choice, improvements in one country's technology (relative to the rival's technology) increases its payoffs under war and under a unilateral deviation by more than its payoffs under settlement. This analysis suggests that asymmetries in aggregate productivities could serve as a distinct destabilizing force for international relations.

It is also worth pointing out that a number of our simplifying assumptions used to help isolate the key mechanisms at play in our baseline model are not essential to our central findings, and relaxing them leads to predictable results. For example, extending the model to include more than two periods, or even an infinite horizon, and supposing that the victor in war holds a strategic advantage forever, or just for a finite number of periods (more than two), would amplify the relative appeal of war and thus shrink the parameter space for which peace prevails. Relaxing the assumption that war's destructive effects are exogenous and supposing, in particular, they depend positively on arms deployed could work in the opposite

[61] Details are available from the authors upon request.

direction, adding to the stability of peace. In addition, consideration of rules of division under peace that are less sensitive to the threat-point payoff (e.g., rules based on splitting the surplus) would tend to favor peace.[62] However, with any of these modifications, the implications for how the key parameters matter in determining the stability of peace would not change qualitatively.

Let us consider other modifications to our baseline model that would allow us to address possible objections related to the assumption that the defeated country is put out of contention in the future. One possibility is to relax the assumption that all of the countries' residual resources are contestable. To the extent that a fraction of the countries' resources is secure, the defeated nation would be able to threaten the victor of the first-period war via "rebellion" in future interactions. Accordingly, the victor would have to devote some of its second-period resources to suppress such activity and more generally maintain "law and order," whereby it could protect its own resource and extract its winnings from the losing side. This future expense for the victor would clearly reduce the expected payoff of war relative to peaceful settlement as well as the expected profitability of unilateral deviations, such that the parameter space for which war emerges as the unique equilibrium shrinks.

In a similar but distinct approach, we could suppose instead that the winner of war (e.g., an imperial power) can, in both periods, extract the differentiated intermediate good that the defeated country would have produced with its resources (net of destruction) and traded under peace; but, to do so successfully, the victor may have to incur a sunk cost in each period. Financed with some of the victor's output of the intermediate good it produces, this sunk cost can be interpreted as an investment in facilitating local production, monitoring order, and punishing insurgents. Since the victor appropriates the produced and potentially tradable intermediate good (as opposed to the defeated side's effective resource), one could view this alternative scenario as "colonial" or "forced" trade.[63] Giving the victor access to the other country's distinct intermediate good increases the value of the prize under open conflict relative to what we studied above. However, the investments required of the victor to obtain those goods reduce the value of the prize. Indeed, although such expenditures bring benefits to the imperial power, they could over time cause a significant strain on that country's economy—what Kennedy (1987) has coined "imperial overstretch." Insofar as this effect is likely to dominate, one would expect the amount of equilibrium arming under the anticipation of war to fall. While this indirect effect alone would raise the expected payoff under war, it could be swamped by the direct effect of the required investment by the victor in each period. Hence, under these modifications to our model, war could be less appealing relative to armed peace. These modifications could also reduce the payoff under a unilateral deviation, and thus expand the parameter space under which peace arises as the stable

[62] As mentioned above, in a previous version of this paper, we explored the implications of a rule of division based on the split-the-surplus bargaining protocol.

[63] Under this alternative scenario, the winner's resource net of destruction in the second period would be fully secure, while the loser's resource would be fully insecure.

equilibrium outcome.

6 Concluding Remarks

The liberal peace hypothesis has much intuitive appeal. Greater interdependence between national economies implies larger potential gains from trade; and, insofar as interstate war disrupts the realization of these gains, one would expect potential adversaries to resolve their differences peacefully. In a setting where countries dispute initial resource ownership claims and with a focus on equilibria that are Pareto preferred to war and immune to unilateral deviations, our analysis shows how the endogenous choice of conflict resolution does depend on the potential gains from trade. It goes further, however, in that it links these gains to the elasticity of substitution between traded commodities and the international distribution of resource ownership. Moreover, the analysis emphasizes the fact that peace, insofar as it requires arming, can be costly; and, the magnitude of these costs also depends on the elasticity of substitution between traded goods as well as the distribution of resources. Therefore, the link between trade and conflict initiation is more nuanced than suggested by the liberal peace hypothesis.

An interesting extension of the analysis would be to allow one country to make a pure resource transfer to its rival in advance of their arming decisions. The aim of this line of research would be to sort out the set of conditions under which transfers help promote peace as the stable equilibrium outcome. For example, a transfer from the larger country to the smaller country would, for given guns, augment both countries' gains from trade, and that effect alone would increase the likelihood of peace. However, if the smaller country is resource constrained in its arming prior to the transfer, the countries' arming choices and their threat-point payoffs would also be affected, possibly undermining the stability of peace. Alternatively, a transfer from the smaller country to the larger country that would result in a greater disparity in resources could make peaceful settlement a more likely outcome.

Another potentially fruitful extension left for future research involves the consideration of trade policies. In particular, countries could use trade policies targeting their rivals to influence both the size and the disposition of the surplus under peace. Such an extension would enable us to explore the possible interactions between security and trade policies in dynamic environments.

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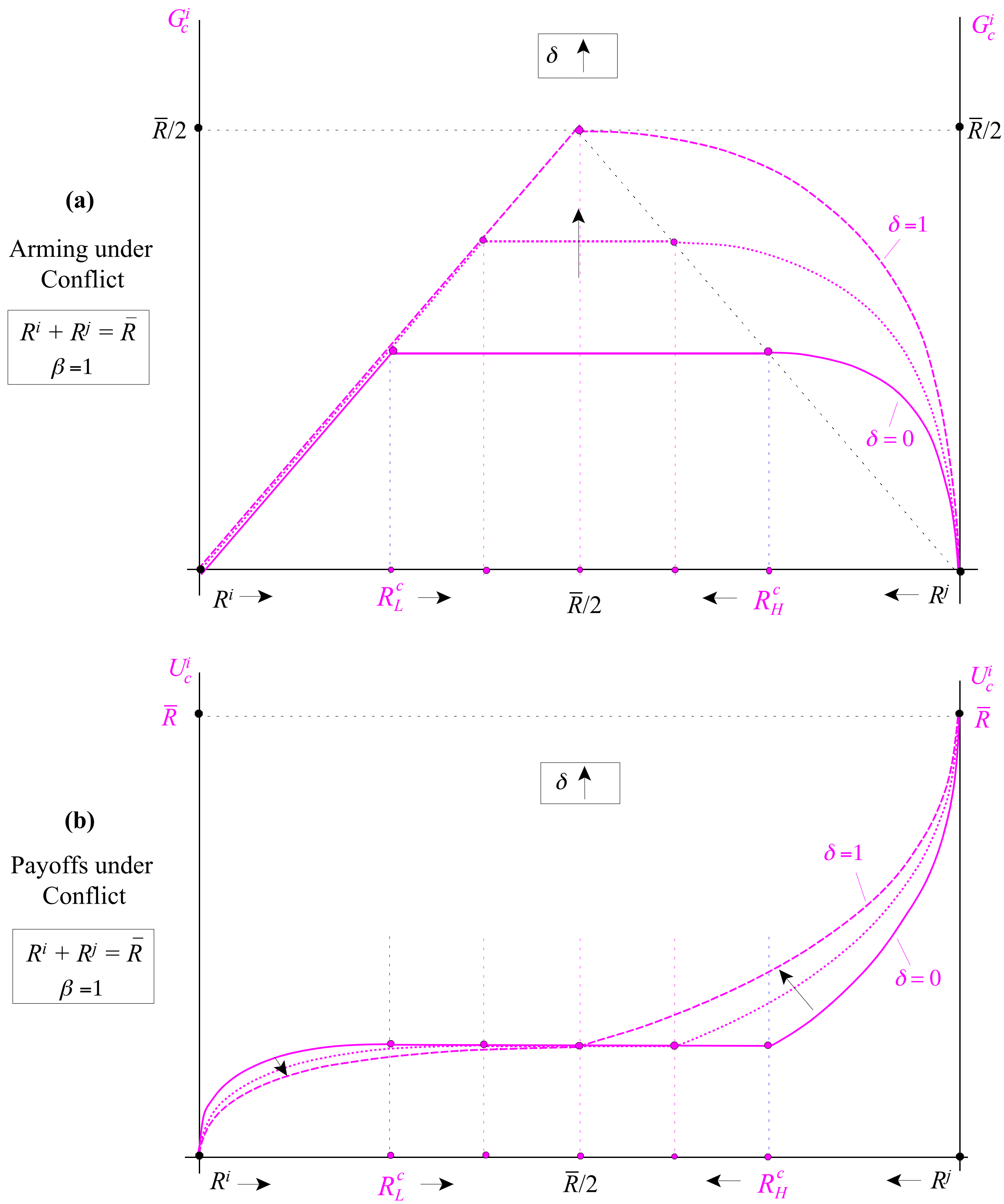

Figure 1: Arming and Payoffs under Open Conflict for Alternative Distributions of Initial Resource Ownership

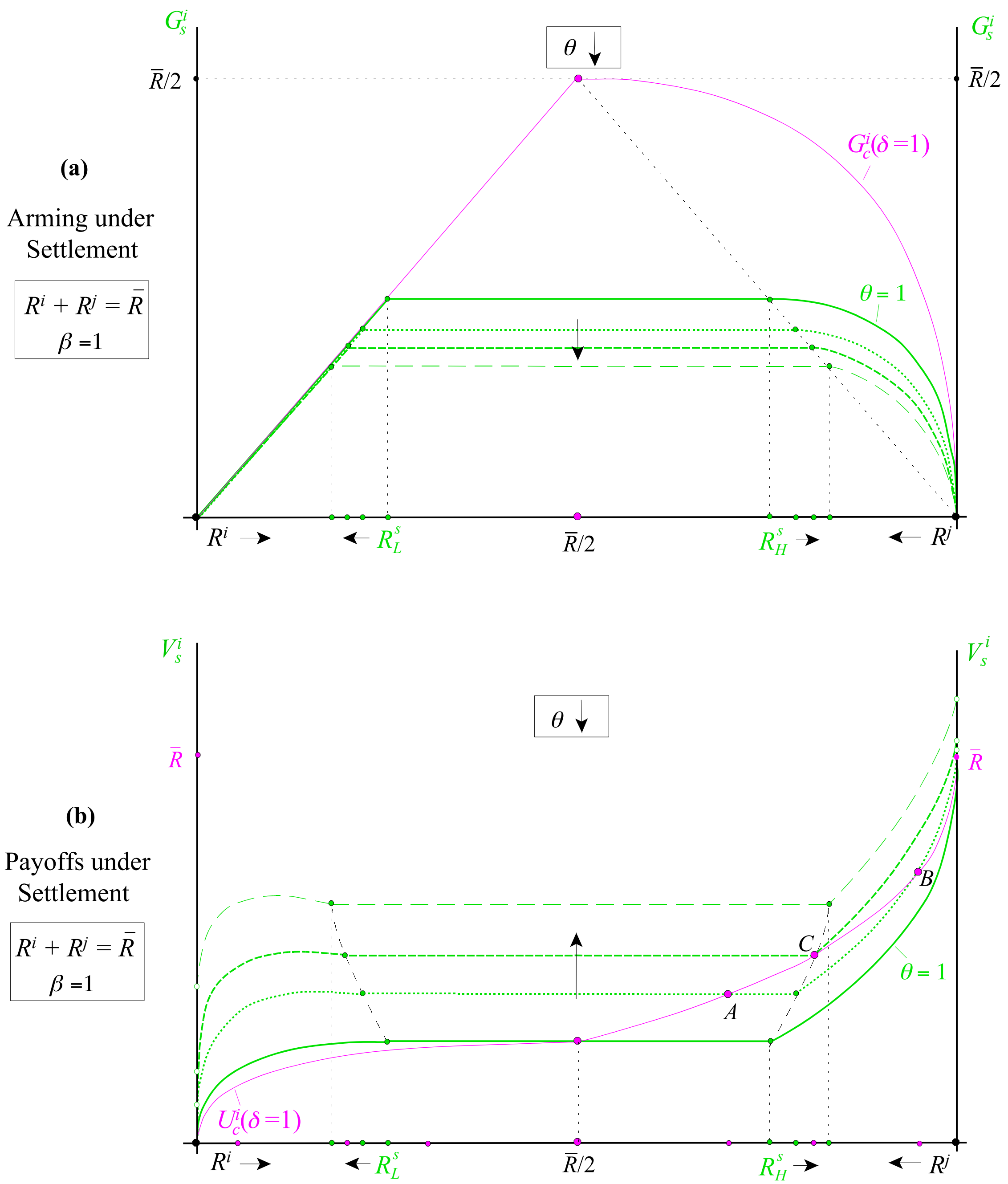

Figure 2: Arming and Payoffs under (Armed) Peaceful Settlement for Alternative Distributions of Initial Resource Ownership

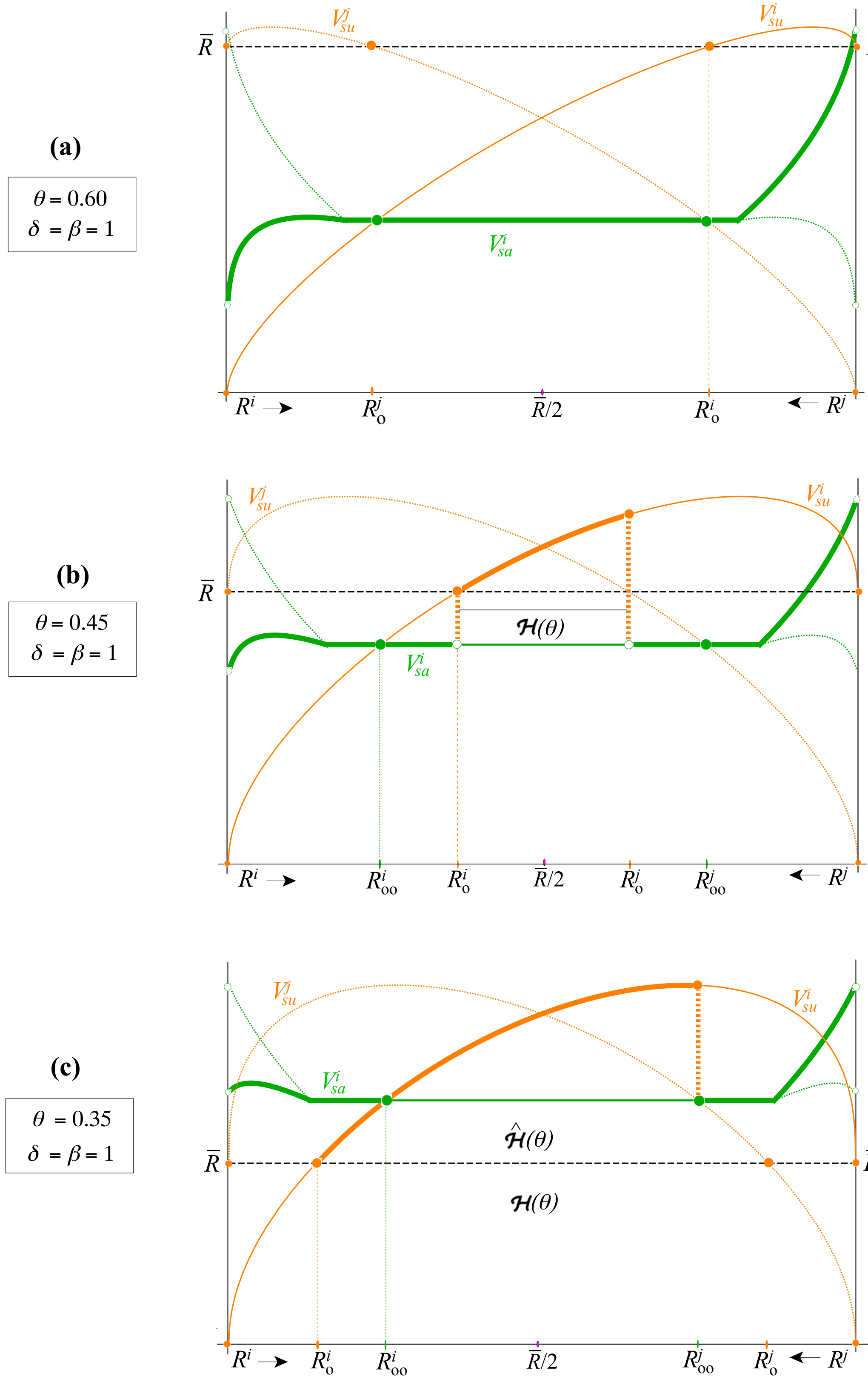

Figure 3: Payoffs under Armed and Unarmed Peace for Alternative Distributions of Initial Resource Ownership

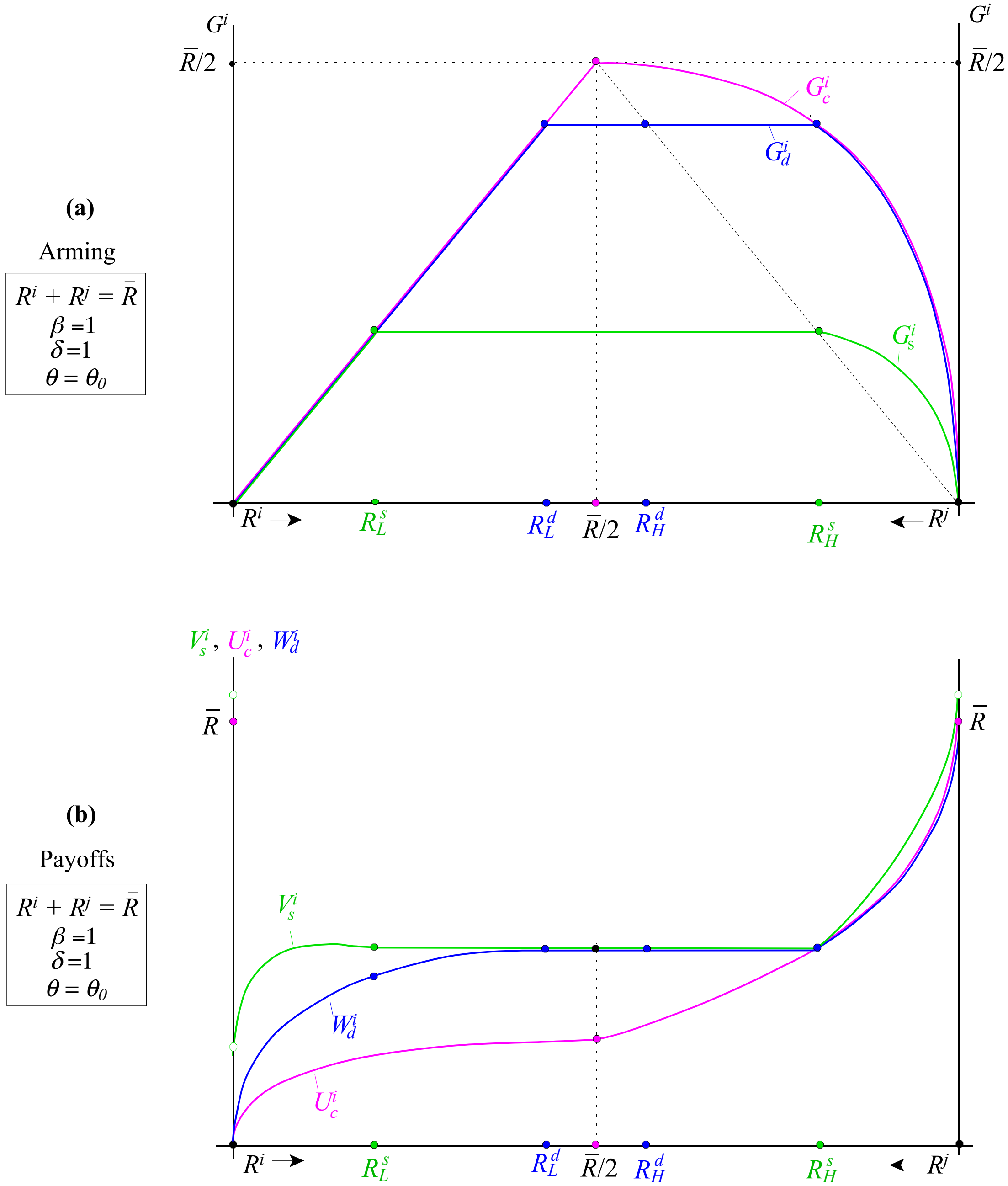

Figure 4: Arming and Payoffs under Open Conflict, Peaceful Settlement, and Unilateral Deviations for Alternative Distributions of Initial Resource Ownership

A Appendix

Proof of Proposition 1. The proof follows from the discussion in the text, along with the best-response functions shown in (10) and the critical values for R shown in (11).

Proof of Proposition 2. That U_c^i for $i = 1, 2$ depends positively on β for all $R^i \in (0, \bar{R})$ follows from the finding in Proposition 1, that equilibrium arming is independent of β, and the fact that an increase β implies more of the residual resource remains after war for employment by the victor in the production of its intermediate good.

Part (a). From Proposition 1(a), when neither country is resource constrained in its arming decision, we have the symmetric solution $G_c = R_L \equiv \frac{1}{4}(1+\delta)\bar{R}$ implying $\phi^i = \phi^j = \frac{1}{2}$ and $\bar{X} = \bar{R} - 2G_c = \frac{1}{2}(1-\delta)\bar{R}$. Then, the specification for average payoffs in (8) gives $U_c^i = \beta\frac{\bar{R}}{4}$, which is independent of the distribution of $\bar{R}$ and time preferences δ.

Part (b). Turning to the case of sufficiently uneven distributions that imply country i is resource constrained, while country j is not, Proposition 1(b) shows that $G_c^i = R^i$ and, with (10b), implies

$$G_c^j = \widetilde{B}_c^j(R^i) = -R^i + \sqrt{(1+\delta)\bar{R}R^i} > R^i.$$

Appropriately differentiating the expression above, with $d\bar{R} = 0$, gives:

$$\frac{dG_c^j}{dR^i} = \frac{G_c^j - R^i}{2R^i} > 0 \tag{A.1a}$$

$$\frac{dG_c^j}{d\delta} = \frac{R^i}{\bar{G}_c}\frac{\bar{R}}{2} > 0, \tag{A.1b}$$

where $\bar{G}_c = R^i + G_c^j = \sqrt{(1+\delta)\bar{R}R^i}$.

A change in a country's own initial resource affects its payoff shown in (8) directly through its effect on $\bar{R}$. But, since our focus here is on changes in the distribution of $\bar{R}$ such that $dR^i = -dR^j$, we need to consider only the indirect effects as follows:

$$\frac{dU_c^i}{dR^i} = U_{G^i}^i + U_{G^j}^i\frac{dG_c^j}{dR^i} = \frac{\beta}{1+\delta}\left(\left[\phi_{G^i}^i\left(\bar{X}+\delta\bar{R}\right) - \phi^i\right] + \left[\phi_{G^j}^i\left(\bar{X}+\delta\bar{R}\right) - \phi^i\right]\frac{dG_c^j}{dR^i}\right)$$
$$= \frac{\beta}{1+\delta}\left[\frac{G_c^j - R^i}{2R^i}\right] > 0 \tag{A.2a}$$

$$\frac{dU_c^j}{dR^i} = U_{G^i}^j\frac{dG_c^i}{dR^i} + U_{G^j}^j\frac{dG_c^j}{dR^i} = \frac{\beta}{1+\delta}\left(\left[\phi_{G^i}^j\left(\bar{X}+\delta\bar{R}\right) - \phi^j\right] + [0]\frac{dG_c^j}{dR^i}\right)$$
$$= -\frac{\beta}{1+\delta}\left[\frac{G_c^j}{R^i}\right] < 0. \tag{A.2b}$$

The above expressions can be obtained using country j's FOC based on (9), $U_{G^j}^j = 0$ to eliminate $(\bar{X} + \delta\bar{R})$, (A.1a) and the properties of the conflict technology in (1). To establish the limit results, we use (8) keeping in mind that, while $\lim_{R^i \to 0} G^i = \lim_{R^i \to 0} G^j = 0$, $\lim_{R^i \to 0} \phi^j = \lim_{R^i \to 0} \{1 - \sqrt{R^i/(1+\delta)\bar{R}}\} = 1$.

Differentiation of (A.2a) and (A.2b) with respect to R^i while using (A.1a) gives

$$\frac{d^2 U_c^i}{(dR^i)^2} = -\frac{\beta}{1+\delta}\left[\frac{\bar{G}_c}{4(R^i)^2}\right] < 0 \tag{A.3a}$$

$$\frac{d^2 U_c^j}{(dR^i)^2} = \frac{\beta}{1+\delta}\left[\frac{\bar{G}_c}{2(R^i)^2}\right] > 0. \tag{A.3b}$$

Equation (A.3a) reveals that the constrained country's average payoff U_c^i is concave in R^i. Furthermore, since $dR^j = -dR^i$ implies, by (A.3b), $d^2U_c^j/(dR^j)^2 = d^2U_c^j/(dR^i)^2 > 0$, it follows that the unconstrained country's equilibrium average payoff is convex in its own endowment R^j, thereby completing the proof of (b.i).

Turning to part (b.ii), we examine the influence of δ on payoffs in (8). Accounting for both the direct and indirect welfare effects, we have for the constrained country i:

$$\begin{aligned}\frac{dU_c^i}{d\delta} = U_\delta^i + U_{G^j}^i \frac{dG_c^j}{d\delta} &= \beta\left[\frac{\phi^i(\bar{R}-\bar{X})}{(1+\delta)^2} + \frac{\phi_{G^j}^i\left(\bar{X}+\delta\bar{R}\right) - \phi^i}{1+\delta}\left(\frac{dG_c^j}{d\delta}\right)\right] \\ &= \frac{\beta\phi^i}{(1+\delta)^2}\left[\bar{G}_c - \tfrac{1}{2}\left(1+\delta\right)\bar{R}\right].\end{aligned} \tag{A.4}$$

The last expression above makes use of the definitions $\bar{X} = \bar{R} - \bar{G}_c$, $\phi^i = G_c^i/\bar{G}_c = R^i/\bar{G}_c$, and $\phi^j + \phi^i = 1$, along with the implication of (1) that $\phi_{G^j}^i = -\phi_{G^j}^j$ in country j's FOC (which requires $\phi_{G^j}^j\left(\bar{X}+\delta\bar{R}\right) = \phi^j$) and (A.1b). To evaluate the sign of (A.4), observe that when neither country's arming decision is constrained by its initial resource endowment, the aggregate quantity of guns under open conflict $\bar{G}_c$ equals $2R_L^c = \frac{1}{2}(1+\delta)\bar{R}$. But, assuming that country i is resource constrained, we have $G_c^i = R^i < R_L^c$, which implies $G_c^j > R^i$. Thus, owing to strategic complementarity exhibited by the unconstrained country's best-response function, it follows that $\bar{G}_c < 2R_L^c$, such that the expression shown in (A.4) is negative. As such, the direct (and positive) effect of an increase in δ on U_c^i is dominated by the indirect (and negative) effect of δ on the unconstrained rival's arming (which rises), implying that the constrained country's average discounted payoff necessarily falls. For the unconstrained country j, the direct effect of an increase in δ on U_c^j is strictly positive, while there is no indirect effect since $G_c^i = R^i$. As such, an increase in δ necessarily augments the unconstrained country's average discounted payoff U_c^j. ||

Some Comparative Statics under Trade. For the sake of generality here, we allow for the possible presence of symmetric trade costs.[64] Accordingly, let $\tau \in [1, \infty)$ be the quantity of a good that must be shipped from country i for one unit to arrive in country j ($\neq i$). Let a "ˆ" over variables denote percent change (e.g., $\widehat{x} \equiv dx/x$). The definitions of expenditure

[64]The analysis can be easily extended to accommodate asymmetric trade costs.

shares $\gamma_j^i = (p^i)^{1-\sigma} / \left[1 + (p^i)^{1-\sigma}\right]$ and domestic prices $p^i = \tau\pi^i$ imply

$$\widehat{\gamma}_j^i = -(\sigma - 1)\gamma_i^i(\widehat{\pi}^i + \widehat{\tau}) - \gamma_i^i \ln\left(p^i\right) d\sigma. \tag{A.5}$$

Now let the subscript "T" indicate the equilibrium values under trade and logarithmically differentiating (5) to find, after simplifying,

$$\widehat{\pi}_T^i = \frac{1}{\Delta}\left\{\widehat{Z}^i - \widehat{Z}^j + (\sigma - 1)(\gamma_j^i - \gamma_i^j)\widehat{\tau} - \left[\gamma_i^i \ln(p_T^i) - \gamma_j^j \ln(p_T^j)\right] d\sigma\right\}, \tag{A.6a}$$

where

$$\Delta = 1 + (\sigma - 1)(\gamma_i^i + \gamma_j^j) > 0 \tag{A.6b}$$

is the Marshall-Lerner condition for stability, which is clearly satisfied for $\sigma > 1$ since that implies $\Delta > 1$.[65] Hence, an increase in country i's effective endowment Z^i affects its terms of trade adversely. Exactly the opposite is true for an increase in Z^j. Equation (A.6a) also reveals that the effect of an increase in trade costs τ on country i's terms of trade π_T^i depends qualitatively on the ranking of the two countries' expenditure shares of their respective importables. Similarly, the impact of the elasticity of substitution σ on π_T^i depends on the manner in which internal prices compare internationally. As we will see shortly, both rankings depend on the distribution of $\bar{X}$ or, equivalently, the effective endowments, Z^i and Z^j.

To gain some understanding of how the division of $\bar{X}$ matters not only for equilibrium prices, but also for the countries' payoffs and their gains from trade, we suppose for now that $Z^i = \lambda^i \bar{X}$ for $i = 1, 2$, where $\lambda^i \geq 0$ (implying $\lambda^j = 1 - \lambda^i \leq 1$) is an arbitrary division of the common pool $\bar{X}$ $(= \bar{R} - \bar{G} > 0)$ including $\lambda^i \equiv \phi^i$. Additionally, keep $\bar{R}$, G^i, and G^j (and thus $\bar{G}$ and $\bar{X}$) fixed in the background. The next two lemmas describe how exogenous changes in λ^i affect π_T^i and γ_j^i respectively.

Lemma A.1 *Country i's terms of trade π_T^i depends on the division λ^i of a given $\bar{X}$ for $i, j \in \{1, 2\},\ i \neq j$, as follows:*

(a) $\partial\pi_T^i/\partial\lambda^i > 0$ *and, for* $\lambda^i > \lambda^j$, $\partial^2\pi_T^i/(\partial\lambda^i)^2 \geq 0$.

(b) $\lim_{\lambda^i \to 0} \pi_T^i = 0$, $\lim_{\lambda^i \to \frac{1}{2}} \pi_T^i = 1$ *and* $\lim_{\lambda^i \to 1} \pi_T^i = \infty$.

(c) *If* $\lambda^i \gtreqqless \frac{1}{2}$ *then* $\pi_T^i \gtreqqless 1$ *and* $p_T^i \gtreqqless p_T^j$.

Proof: *Part (a).* Because the supply of country i's intermediate input is $Z^i = \lambda^i \bar{X}$, we have $\widehat{Z}^i - \widehat{Z}^j = \widehat{\lambda}^i - \widehat{\lambda}^j = \left(\frac{1}{\lambda^i} + \frac{1}{\lambda^j}\right) d\lambda^i$ for any given $\bar{X}$. Since $\Delta > 0$, applying this result to (A.6a) gives $\partial\pi_T^i/\partial\lambda^i = 1/(\lambda^i\lambda^j\Delta) > 0$, which proves the first portion of part (a).

To prove the convexity of π_T^i in λ^i for $\lambda^i \geq \lambda^j$, we differentiate $\partial\pi_T^i/\partial\lambda^i$ shown above with respect to λ^i, keeping in mind that $\lambda^j = 1 - \lambda^i$ and using the definition of Δ in (A.6b)

[65] In the special case of free trade (i.e., $\tau = 1$ and $p_T^i = \pi_T^i$), $p_T^j = 1/p_T^i$, $\gamma_i^i + \gamma_j^j = 1$ and, from (A.6b), $\Delta = \sigma > 1$ hold.

and the facts that $p^i = \tau\pi^i$, $\pi^j = 1/\pi^i$ and $\pi^i \left(\partial\gamma^i_j/\partial\pi^i\right) = -(\sigma-1)\,\gamma^i_i\gamma^i_j$ for $i,j \in \{1,2\}$ and $i \neq j$:

$$\begin{aligned}
\frac{\partial^2 \pi^i_T}{(\partial\lambda^i)^2} &= \frac{\lambda^i - \lambda^j}{(\lambda^i\lambda^j)^2\,\Delta} + \frac{(\sigma-1)}{\lambda^i\lambda^j\Delta^2}\left[\pi^i_T\left(\frac{\partial\gamma^i_j}{\partial\pi^i}\right) + \pi^j_T\left(\frac{\partial\gamma^j_i}{\partial\pi^j}\right)\right]\frac{\partial\pi^i_T/\partial\lambda^i}{\pi^i_T} \\
&= \frac{\lambda^i - \lambda^j}{(\lambda^i\lambda^j)^2\,\Delta} + \frac{(\sigma-1)^2}{\pi^i_T\,(\lambda^i\lambda^j)^2\,\Delta^3}\left(\gamma^j_j\gamma^j_i - \gamma^i_i\gamma^i_j\right).
\end{aligned}$$

The first term in the last line of the expression above is non-negative due to our assumption that $\lambda^i \geq \lambda^j$. Hence, it is sufficient to show that the second term is non-negative as well. Using the definitions of the expenditure shares in terms of domestic prices gives

$$\begin{aligned}
\gamma^j_j\gamma^j_i - \gamma^i_i\gamma^i_j &= \frac{(p^j)^{1-\sigma}}{[1+(p^j)^{1-\sigma}]^2} - \frac{(p^i)^{1-\sigma}}{[1+(p^i)^{1-\sigma}]^2} = \frac{\left[(p^i)^{\sigma-1} - (p^j)^{\sigma-1}\right]\left[\left(p^ip^j\right)^{\sigma-1} - 1\right]}{[1+(p^i)^{\sigma-1}]^2\,[1+(p^j)^{\sigma-1}]^2} \\
&= \frac{(p^j)^{\sigma-1}\left[\left(\pi^i_T\right)^{2(\sigma-1)} - 1\right]\left[\tau^{2(\sigma-1)} - 1\right]}{[1+(p^i)^{\sigma-1}]\,[1+(p^j)^{\sigma-1}]}.
\end{aligned}$$

Since $\sigma > 1$ and $\tau \geq 1$, the desired result follows from part (c) of the lemma (shown below), thereby establishing the convexity of π^i_T in λ_i for $\lambda^i > \lambda^j$.

Part (b). The expenditure shares can be written, using $p^i = \tau\pi^i$ and noting $\pi^j = 1/\pi^i$, as

$$\gamma^i_j = \frac{1}{1+\tau^{\sigma-1}\left(\pi^i\right)^{\sigma-1}} \quad \text{and} \quad \gamma^j_i = \frac{\left(\pi^i\right)^{\sigma-1}}{\left(\pi^i\right)^{\sigma-1} + \tau^{\sigma-1}}, \tag{A.7}$$

where $\widehat{\gamma}^i_j = -(\sigma-1)\left(1-\gamma^i_j\right)\widehat{\pi}^i$. Substituting these expressions along with $Z^i = \lambda^i\bar{X}$ and $Z^j = (1-\lambda^i)\bar{X}$ into the world market clearing condition (5), after some rearranging, gives

$$\left[\frac{1+\tau^{\sigma-1}\left(\pi^i_T\right)^{\sigma-1}}{\tau^{\sigma-1} + \left(\pi^i_T\right)^{\sigma-1}}\right]\left(\pi^i_T\right)^{\sigma} = \frac{\lambda^i}{1-\lambda^i}. \tag{A.8}$$

To proceed, we study the behavior of π^i_T on the LHS of the condition above as λ^i varies on the RHS. Now observe that the RHS behaves as follows: (i) $\lim_{\lambda^i\to 0}\text{RHS} = 0$, (ii) $\lim_{\lambda^i\to\frac{1}{2}}\text{RHS} = 1$, and (iii) $\lim_{\lambda^i\to 1}\text{RHS} = \infty$. Clearly, the limits of the LHS must match the respective limits of the RHS in all three cases. In what follows, keep in mind that, for any finite $\tau \geq 1$ and $\sigma > 1$, $\tau^{\sigma-1}$ in the LHS is finitely positive. In case (i), the expression inside the square brackets of the LHS is finitely positive for all $\pi^i_T \geq 0$. Therefore, $\lim_{\lambda^i\to 0}\text{LHS} = 0$ only if $\lim_{\lambda^i\to 0}\pi^i_T = 0$. Similarly, in case (ii), $\lim_{\lambda^i\to\frac{1}{2}}\pi^i_T = 1$ because no other value of π^i_T ensures $\lim_{\lambda^i\to\frac{1}{2}}\text{LHS} = 1$. Lastly, in case (iii), $\lim_{\lambda^i\to 1}\pi^i_T = \infty$ because the expression inside the square brackets of the LHS is finitely positive for all $\tau \geq 1$ and $\pi^i_T \geq 0$ (including the case of $\pi^i_T \to \infty$).

Part (c). This part follows readily from the first component of part (a) and part (b). ||

Lemma A.2 *For $i, j \in \{1, 2\}$ and $i \neq j$, the division λ^i of $\bar{X}$ has the following implications for the expenditure shares:*

(a) $\lim_{\lambda^i \to \frac{1}{2}} \gamma^i_j \leq \frac{1}{2}$, $\lim_{\lambda^i \to 1} \gamma^i_j = 0$ *and* $\lim_{\lambda^i \to 1} \gamma^i_j / \lambda^j = \infty$.

(b) *If* $\lambda^i \gtreqless \frac{1}{2}$ *then* $\gamma^i_i \gtreqless \gamma^j_j$ *and* $\gamma^i_j \lesseqgtr \gamma^j_i$.

Proof: *Part (a).* The first component of part (a) follows from the second component of Lemma A.1(b), the definition of γ^i_j in (A.7), and the assumptions that $\tau \geq 1$ and $\sigma > 1$. The second component follows from the third component of Lemma A.1(b), which implies $\lim_{\lambda^i \to 1} \left(\pi^i_T\right)^{\sigma-1} = \infty$ and (A.7). The last component of part (a) follows by rewriting (5) as $\gamma^i_j / \lambda^j = \pi^i_T \gamma^j_i / \lambda^i$ and by noting that the limit of the RHS is

$$\lim_{\lambda^i \to 1} \left(\frac{\gamma^j_i}{\lambda^i}\right) \times \lim_{\lambda^i \to 1} \left(\pi^i_T\right) = \left[\frac{\lim_{\lambda^i \to 1}(\gamma^j_i)}{\lim_{\lambda^i \to 1}(\lambda^i)}\right] \times \lim_{\lambda^i \to 1} \left(\pi^i_T\right) = \left[\frac{1}{1}\right] \times \infty,$$

which implies $\lim_{\lambda^i \to 1} \text{LHS} = \lim_{\lambda^i \to 1}(\gamma^i_j / \lambda^j) = \infty$. Thus, the convergence of γ^i_j to 0 is slower that the convergence of λ^j to 0 as $\lambda^i \to 0$.

Part (b). The two components of this part follow from straightforward calculations using Lemma A.1(b) and the expressions for the expenditure shares in (A.7). $||$

Lemma A.3 *If* $\lambda^i \gtreqless \frac{1}{2}$ *then* $d\pi^i_T / d\tau \lesseqgtr 0$ *and* $d\pi^i_T / d\sigma \lesseqgtr 0$.

Proof: This lemma follows from (A.6a), which shows how π^i_T depends on τ and σ, with Lemmas A.1(c) and A.2(b), conditional on the division of $\bar{X}$. It suggests that larger trade costs and a greater distinction between traded commodities impart a home bias in favor of the country with the larger effective endowment. $||$

To identify the effect of changes in countries' effective endowments Z^i and Z^j on country i's payoff w^i_T under trade, we use (6) along with (A.6a) and the fact that $p^i \mu^i_{p^i} / \mu^i = -\gamma^i_j$:

$$\widehat{w}^i_T = \widehat{Z}^i - \gamma^i_j \widehat{\pi}^i_T = \left(1 - \frac{\gamma^i_j}{\Delta}\right) \widehat{Z}^i + \frac{\gamma^i_j}{\Delta} \widehat{Z}^j.$$

Since $0 < \gamma^i_j < 1$ whereas $\Delta > 1$, we have $0 < \gamma^i_j / \Delta < 1$, which implies w^i_T unambiguously rises with increases in country i's effective endowment Z^i. As such, immiserizing growth (due to an adverse terms-of-trade effect) does not arise in this context. Similarly, an increase in country j's effective endowment Z^j increases w^i_T because of a favorable (to country i) terms-of-trade effect.[66]

Again, keeping in mind that λ^i captures an arbitrary division of $\bar{X}$ (including ϕ^i), let $Z^i = \lambda^i \bar{X}$ so that $w^i_T = \omega^i \bar{X}$ where $\omega^i \equiv \mu^i \lambda^i$. We now explore how the division λ^i of

[66] Equi-proportionate increases in Z^i and Z^j would cause both countries' welfare to rise proportionately because they expand each country's income, while leaving world prices unchanged.

the common pool $\bar{X}$ and the quantity of guns $\bar{G}$ affect w_T^i. Naturally, $dw_T^i/d\bar{G} = -\omega^i$ and $dw_T^i/d\lambda^i = \bar{X}\omega_{\lambda^i}^i$. One can show (from the definition of ω^i and (A.6a)) that

$$\omega_{\lambda^i}^i = \mu^i \left(1 - \frac{\gamma_j^i/\lambda^j}{\Delta}\right), \tag{A.9}$$

which simplifies to (14) for $\Delta = \sigma$ in the case of free trade and $\lambda^i = \phi^i$ studied in the main text. In addition, $dw_T^i/d\xi = \bar{X}\omega_\xi^i$ for $\xi \in \{\tau, \sigma\}$, so that the dependence of ω^i, not just on λ^i and $\bar{G}$, but also on trade costs and the elasticity of substitution are important. After some algebra, using the facts that $p^i = \tau\pi^i$ and $p^i\mu_{p^i}^i/\mu^i = -\gamma_j^i$, along with (A.6a) and the expression for $\Delta > 0$, we find:

$$\begin{aligned} \omega_\tau^i &= \lambda^i \mu_{p^i}^i p_\tau^i = \omega^i \left(p^i \mu_{p^i}^i/\mu^i\right)\left(\tau p_\tau^i/p^i\right)\frac{1}{\tau} \\ &= -\frac{\omega^i\gamma_j^i}{\tau\Delta}\left[1 + 2(\sigma-1)\left(1-\gamma_i^j\right)\right] < 0. \end{aligned} \tag{A.10}$$

In short, the (direct) effect of an increase in trade costs on a country's payoff under trade, keeping λ^i and $\bar{G}$ fixed, is non-positive.

The effect of σ on ω^i is a bit more involved as in this case we have

$$\begin{aligned} \omega_\sigma^i &= \lambda^i\left\{\mu_\sigma^i + \mu_{p^i}^i p_\sigma^i\right\} = \omega^i\left\{\mu_\sigma^i/\mu^i + \left(p^i\mu_{p^i}^i/\mu^i\right)\left(p_\sigma^i/p^i\right)\right\} \\ &= \omega^i\left\{-\frac{1}{(\sigma-1)^2}\left[(\sigma-1)\gamma_j^i \ln p^i + \ln\left[1+(p^i)^{1-\sigma}\right]\right]\right. \\ &\quad \left.+\frac{\gamma_j^i}{\Delta}\left[\gamma_i^i \ln p^i - \gamma_j^j \ln p^j\right]\right\}. \end{aligned}$$

Using (A.6b) and the properties of logarithms, the above equation can be rewritten (after some additional algebra) as

$$\begin{aligned} \omega_\sigma^i &= -\frac{\omega^i}{\Delta(\sigma-1)^2}\left\{\Delta\ln[1+(p^i)^{1-\sigma}] - \gamma_j^i \ln\left(p^i\right)^{1-\sigma}\right. \\ &\quad \left. - (\sigma-1)\gamma_j^i\gamma_j^j \ln(p^i)^{1-\sigma} - (\sigma-1)\gamma_j^i\gamma_j^j \ln(p^j)^{1-\sigma}\right\} \\ &= -\frac{\omega^i}{\Delta(\sigma-1)^2}\left\{\Delta\ln[1+(p^i)^{1-\sigma}] - \gamma_j^i\ln(p^i)^{1-\sigma} + 2(\sigma-1)^2\gamma_j^i\gamma_j^j \ln\tau\right\}. \end{aligned} \tag{A.11}$$

Inspection of the RHS of the last expression reveals that $\omega_\sigma^i < 0$ for the following reasons: (i) $\Delta > \gamma_j^i$ implies $\Delta\ln[1+(p^i)^{1-\sigma}] - \gamma_j^i \ln(p^i)^{1-\sigma} > 0$; and (ii) $\tau \geq 1$ implies the last term in the curly brackets is positive. Since $\theta = 1 - \frac{1}{\sigma}$, we also have $\omega_\theta^i < 0$.

Let us now study in finer detail the dependence of ω^i on the division λ^i of $\bar{X}$. In particular, starting with $\lambda^i = 0$, let us ask how arbitrary reallocations of $\bar{X}$ from country j to country i ($i \neq j$) affect ω^i and ω^j. Going back to (A.9), one can see that the direct effect of such a resource transfer is to increase (decrease) the recipient's (donor's) output and thus (given prices) its payoff. However, working against this direct effect, the transfer also causes an indirect effect that improves (worsens) recipient (donor) country's terms-of-trade.[67] The presence of this trade-off raises the following questions. Is there an optimal division, $\lambda^i_{\max}$, of the common pool that maximizes country i's payoff ω^i? If so, what are its properties? Furthermore, is it possible for resource transfers to immiserize both the recipient and the donor countries?[68] These questions are of interest in their own right. But, as we will see later, they are of special interest in the context of the resource disputes we are studying due to their consequences for arming and the division of the common pool that, in turn, have important implications for countries' preferences over war and peace.

Since $\sigma = 1/(1-\theta)$, we use σ and θ interchangeably. The next lemma summarizes several noteworthy properties of ω^i.

Lemma A.4 *For any given guns, the payoff ω^i ($\equiv \mu^i\lambda^i$) depends on the division λ^i of $\bar{X}$, the degree of substitutability $\theta \in (0,1)$ and trade costs $\tau \in [1,\infty)$ as follows:*

(a) (i) $\lim_{\lambda^i\to 0}\omega^i = 0$ *and* $\lim_{\lambda^i\to 1}\omega^i = 1$.

(ii) $\lim_{\lambda^i\to\frac{1}{2}}\omega^i \gtreqqless \lim_{\lambda^i\to 1}\omega^i = 1$ *as* $\theta \lesseqqgtr \breve{\theta}(\tau)$, *where* $\breve{\theta}'(\tau) < 0$, $\breve{\theta}(1) = \frac{1}{2}$ *and* $\lim_{\tau\to\infty}\breve{\theta}(\tau) = 0$.

(iii) $\lambda^i \gtreqqless \frac{1}{2} \;\Rightarrow\; \omega^i \gtreqqless \omega^j$.

(b) ω^i *is strictly concave in* $\lambda^i \in (0,1)$ *and attains a maximum* $\lambda^i_{\max} = \arg\max_{\lambda^i}\omega^i \in (\frac{1}{2},1)$, *which is increasing in θ and τ, with* $\lim_{\theta\to 0}\lambda^i_{\max} = \frac{1}{2}$ *and* $\lim_{\theta\to 1}\lambda^i_{\max} = \lim_{\tau\to\infty}\lambda^i_{\max} = 1$.

(c) $\omega^i_{\lambda^i} < 0$ *and* $\omega^j_{\lambda^i} < 0$ *for all* $\lambda^i \in (\lambda^i_{\max}, 1)$, $i,j \in \{1,2\}$ *and* $i \neq j$.

(d) $\partial\omega^i/\partial\xi < 0$ *for* $\xi \in \{\theta,\tau\}$.

(e) $\partial\left(\lambda^i\omega^i_{\lambda^i}/\omega^i\right)/\partial\theta > 0$ *if* $\pi^i_T \geq 1$ *and* $\partial\left(\lambda^i\omega^i_{\lambda^i}/\omega^i\right)/\partial\tau > 0$.

Proof: *Part (a).* For any $\lambda^i \in (0,1)$, given $\sigma > 1$ ($\theta \in (0,1]$) and $\tau < \infty$, we have

$$\mu^i \equiv \left[1 + \tau^{1-\sigma}\left(\pi^i_T\right)^{1-\sigma}\right]^{1/(\sigma-1)} > 1. \tag{A.12}$$

From Lemma A.1(b), we know that (i) $\lim_{\lambda^i\to 1}\pi^i_T = \infty$, which implies $\lim_{\lambda^i\to 1}\mu^i = 1$. Then, the definition of ω^i ($\equiv \mu^i\lambda^i$) readily implies $\lim_{\lambda^i\to 1}\omega^i = 1$. To prove $\lim_{\lambda^i\to 0}\omega^i = 0$, we

[67]Once again, for now guns $\bar{G}$ and thus $\bar{X}$ are kept fixed in the background. One can also think of such reallocations as a resource gift from country j (the donor) to country i (the recipient). Amano (1966) addresses this terms-of-trade issue in a variety of contexts. However, he does not study the welfare implications of resource transfers for both donor and recipient countries. Garfinkel et al. (2020) examine a variant of this issue in the context of a modified Ricardian model of trade and conflict.

[68]In the standard trade literature that considers pure *income* transfers between two trading partners, this possibility does not arise. In fact, stability of the world trading equilibrium necessarily implies that the recipient enjoys a welfare improvement while the donor suffers a welfare loss. Prior work in this area (e.g., Brecher and Bhagwati, 1982; Bhagwati et al. 1983) also emphasized the idea that, indeed, transfers could worsen the recipient's welfare in the presence of distrortions. Grossman (1984) argued that, when goods are already traded freely, trade in factors can be welfare-reducing. However, his analysis was in the context of factor movements that require earnings in the host country to be transferred back to the country of origin. Moreover, he did not study the possible existence of immisering factor movements in the Pareto sense.

rewrite ω^i as

$$\omega^i = \left(\lambda^i/\pi_T^i\right)\left[\left(\pi_T^i\right)^{\sigma-1} + \tau^{1-\sigma}\right]^{1/(\sigma-1)}.$$

Since as established in Lemma A.1(b) $\lim_{\lambda^i\to 0}\pi_T^i = 0$ and $\tau \in [1,\infty)$ by assumption, the expression inside the square brackets is finitely positive as $\lambda^i \to 0$. Let us rearrange the world market clearing condition (5), using $Z^i = \lambda^i\bar{X}$ for $i = 1,2$, as $\lambda^i/\pi_T^i = \lambda^j\gamma_i^j/\gamma_j^i$. Lemma A.2(a) implies $\lim_{\lambda^i\to 0}\gamma_j^i = 1$ and $\lim_{\lambda^i\to 0}\gamma_i^j = 0$. Thus, since $\lambda^j \to 1$ in this limit, we have $\lim_{\lambda^i\to 0}\left(\lambda^i/\pi_T^i\right) = 0$, thereby completing the proof of (a.i).

To prove part (a.ii), recall that, by Lemma A.1(b), $\lim_{\lambda^i\to 1/2}\pi_T^i = 1$, which implies

$$\lim_{\lambda^i\to 1/2}\omega^i = \tfrac{1}{2}\left[1+\tau^{1-\sigma}\right]^{1/(\sigma-1)}.$$

Equating the expression above to $\lim_{\lambda^i\to 1}\omega^i = 1$ and rearranging terms yields $g(\sigma,\tau) = 1+\tau^{1-\sigma} - 2^{\sigma-1} = 0$, which implicitly defines the critical value of σ as a function of τ, $\breve{\sigma}(\tau)$, (or, equivalently, $\breve{\theta}(\tau)$) introduced in the lemma. One can now verify the following: $\breve{\sigma}(1) = 2$ and $\lim_{\tau\to\infty}\breve{\sigma}(\tau) = 1$ (which imply $\breve{\theta}(1) = \frac{1}{2}$ and $\lim_{\tau\to\infty}\breve{\theta}(\tau) = 0$); and, by the implicit function theorem, we have $\breve{\sigma}'(\tau) = -g_\tau/g_\sigma < 0$ (and thus $\breve{\theta}'(\tau) < 0$).

Part (a.iii) follows from parts (a.i) and (a.ii) and part (b) that follows.

Part (b). Using the results from Lemma A.2(a) that $\lim_{\lambda^i\to 1}\gamma_j^i = 0$ while $\lim_{\lambda^i\to 1}\gamma_i^j = 1$ in (A.6b) readily implies $\lim_{\lambda^i\to 1}\Delta = \sigma > 1$. But, Lemma A.2(a) also establishes $\lim_{\lambda^i\to 1}\gamma_j^i/\lambda^j = \infty$. Thus, the expression inside the parentheses in (A.9) becomes negative as $\lambda^i \to 1$ (i.e., $\lim_{\lambda^i\to 1}\omega_{\lambda^i}^i < 0$), so that $\omega^i \to 1$ from above as $\lambda^i \to 1$. Yet, ω^i is continuous in λ^i and from part (a), we know that $\lim_{\lambda^i\to 0}\omega^i = 0$; therefore, ω^i attains a maximum at some λ^i, denoted by $\lambda_{\max}^i$, that solves $\omega_{\lambda^i}^i = 0$ in (A.9).

We now prove that $\omega_{\lambda^i\lambda^i}^i < 0$, which implies that ω^i is strictly concave in λ^i and thereby establishes the uniqueness of $\lambda_{\max}^i < 1$. Differentiation of $\omega_{\lambda^i}^i$ in (A.9) gives

$$\begin{aligned}\omega_{\lambda^i\lambda^i}^i = \mu^i\Bigg\{&\frac{p^i\mu_{p^i}^i}{\mu^i}\left(\frac{p_{\lambda^i}^i}{p^i} - \frac{p_{\lambda^j}^i}{p^i}\right)\left(1-\frac{\gamma_j^i}{\lambda^j\Delta}\right) - \frac{\gamma_j^i}{\left(\lambda^j\right)^2\Delta} - \frac{p^i\left(\partial\gamma_j^i/\partial p^i\right)}{\lambda^j\Delta}\left(\frac{p_{\lambda^i}^i}{p^i} - \frac{p_{\lambda^j}^i}{p^i}\right)\\ &+\frac{\gamma_j^i}{\lambda^j\Delta^2}\left[p^i\Delta_{p^i}\left(\frac{p_{\lambda^i}^i}{p^i} - \frac{p_{\lambda^j}^i}{p^i}\right) + p^j\Delta_{p^j}\left(\frac{p_{\lambda^i}^j}{p^j} - \frac{p_{\lambda^j}^j}{p^j}\right)\right]\Bigg\}.\end{aligned}$$

Using the facts that $p^i\mu_{p^i}^i/\mu^i = -\gamma_j^i$, $p_{\lambda^i}^i/p^i - p_{\lambda^j}^i/p^i = 1/(\lambda^i\lambda^j\Delta)$, $p^i(\partial\gamma_j^i/\partial p^i) = -(\sigma-1)\gamma_i^i\gamma_j^i$, and $p^i\Delta_{p^i} = (\sigma-1)^2\gamma_i^i\gamma_j^i$ for $i \neq j$ enables us to transform the above expression into:

$$\begin{aligned}\omega_{\lambda^i\lambda^i}^i &= \mu^i\left[-\frac{\gamma_j^i}{\lambda^i\lambda^j\Delta}\left(1-\frac{\gamma_j^i}{\lambda^j\Delta}\right) - \frac{\gamma_j^i}{\left(\lambda^j\right)^2\Delta} + \frac{(\sigma-1)\gamma_i^i\gamma_j^i}{\lambda^i\left(\lambda^j\Delta\right)^2} + \frac{\gamma_j^i(\sigma-1)^2}{\lambda^i\left(\lambda^j\right)^2\Delta^3}\left(\gamma_i^i\gamma_j^i - \gamma_j^j\gamma_i^j\right)\right]\\ &= -\frac{\gamma_j^i\omega^i}{\left(\lambda^i\lambda^j\Delta\right)^2}\left[\gamma_i^i A + \gamma_j^j B\right], \qquad \text{(A.13a)}\end{aligned}$$

where

$$A = 1 + \frac{(\sigma-1)^2}{\Delta}\left(\gamma_i^i + \gamma_j^j - 1\right) \quad \text{and} \quad B = \frac{\sigma(\sigma-1)}{\Delta}. \tag{A.13b}$$

We now establish that $A > 0$ by showing that $\gamma_i^i + \gamma_j^j - 1 \geq 0$. Using the definition of the expenditure shares as a function of internal prices, we calculate the following:

$$\begin{aligned}\gamma_i^i + \gamma_j^j - 1 &= \frac{1}{1+(p^i)^{1-\sigma}} + \frac{1}{1+(p^j)^{1-\sigma}} - 1 = \frac{(p^i p^j)^{\sigma-1} - 1}{[1+(p^i)^{\sigma-1}]\,[1+(p^j)^{\sigma-1}]} \\ &= \frac{\tau^{2(\sigma-1)} - 1}{[1+(p^i)^{\sigma-1}]\,[1+(p^j)^{\sigma-1}]} \geq 0,\end{aligned}$$

for $i, j \in \{1,2\}$ and $i \neq j$, since $\tau \geq 1$. Thus, $A > 0$. In addition, $B > 0$ holds, because $\sigma > 1$. Hence, $\omega^i_{\lambda^i\lambda^i} < 0$ holds for $i = 1, 2$.

Having already shown above that $\lambda^i_{\max} < 1$, we now prove that $\lambda^i_{\max} > \frac{1}{2}$. Evaluating $\omega^i_{\lambda^i}$ at $\lambda^i = \frac{1}{2}$ gives $\omega^i_{\lambda^i}\big|_{\lambda^i=\frac{1}{2}} = 2\mu^i(\frac{1}{2} - \gamma_j^i/\Delta) > 0$, where the sign follows from the finding that $\gamma_j^i \leq \frac{1}{2}$ (by Lemma A.2(a)), while $\Delta > 1$. Therefore, $\lim_{\lambda^i \to \frac{1}{2}} \omega^i_{\lambda^i} > 0$.

The influence of $\xi \in \{\theta, \tau\}$ on $\lambda^i_{\max}$ can be studied by applying the implicit function theorem to (A.9): $d\lambda^i_{\max}/d\xi = -\omega^i_{\lambda^i\xi}/\omega^i_{\lambda^i\lambda^i}$, where $\omega^i_{\lambda^i\lambda^i} < 0$. By differentiating $\omega^i_{\lambda^i}$ with respect to ξ and evaluating the resulting expression at $\lambda^i = \lambda^i_{\max}$, one can find that $\omega^i_{\lambda^i\xi} > 0$ for $\xi \in \{\theta, \tau\}$, which proves that $d\lambda^i_{\max}/d\xi > 0$. The very last two components of part (b) follow readily by the taking the appropriate limits of (A.9) and finding $\lambda^i_{\max}$ that solves $\omega^i_{\lambda^i} = 0$.[69]

Part (c). This part follows from part (b). It is interesting in that it indicates that giving the larger economy an even bigger share in the specified range hurts both countries.

Part (d). This part follows from equations (A.10) and (A.11).

Part (e). To prove this part, which helps to prove Proposition 3(c) below, we pre-multiply (A.9) by λ^i and divide by ω^i. Doing so gives $\lambda^i\omega^i_{\lambda^i}/\omega^i = 1 - \gamma_j^i/(\lambda^j\Delta)$, where $\Delta > 1$ is shown in (A.6b). Therefore, $\text{sign}\{\partial(\lambda^i\omega^i_{\lambda^i}/\omega^i)/\partial\xi\} = \text{sign}\{-\partial(\gamma_j^i/\Delta)/\partial\xi\}$ where $\xi \in \{\sigma, \tau\}$. Differentiating logarithmically the expression inside the brackets gives

$$d\Upsilon \equiv -\widehat{\gamma}_j^i + \widehat{\Delta} = -\widehat{\gamma}_j^i + \frac{\sigma-1}{\Delta}\left(\gamma_i^i\widehat{\gamma}_i^i + \gamma_j^j\widehat{\gamma}_j^j\right) + \frac{\gamma_i^i + \gamma_j^j}{\Delta}d\sigma.$$

Noting that $\gamma_i^i\widehat{\gamma}_i^i = -\gamma_j^i\widehat{\gamma}_j^i$ for $i, j \in \{1,2\}$ and $i \neq j$, recall that the expressions for $\widehat{\gamma}_j^i$ were defined in (A.5) and keep in mind that $p^i = \tau\pi^i$. Changes in σ or in τ affect $\widehat{\gamma}_j^i$ directly and indirectly through the world market-clearing price π_T^i. The latter effect was described in (A.6a). Putting these ideas together gives (after substantial simplification and rearrangment)

$$d\Upsilon/d\sigma = \frac{1}{\Delta}C_1 \ln\left(\pi_T^i\right) + C_2 \ln(\tau) + \frac{\gamma_i^i + \gamma_j^j}{\Delta} \tag{A.14}$$

[69]For the limit as $\tau \to \infty$, the last component of Lemma A.2(a) implies $\omega^i_{\lambda^i} > 0$ for all $\lambda^i \in (0,1]$.

where

$$C_1 = \gamma_i^i + \frac{(\sigma-1)\left(\gamma_i^i\gamma_j^i - \gamma_j^j\gamma_i^j\right)}{\Delta} > 0$$

$$C_2 = \frac{2(\sigma-1)\gamma_j^j\gamma_i^j}{\Delta} + C_1\left[1 - \frac{(\sigma-1)\left(\gamma_i^i - \gamma_j^j\right)}{\Delta}\right] > 0.$$

The signs of C_1 and C_2 can be easily established by using the definition of Δ in (A.6b). Clearly, then, the sign of the first term in (A.14) term is non-negative if $\pi_T^i \geq 1$. Similarly, the sign of the second term is also non-negative because $\tau \geq 1$. Of course, the last term is positive. Since $\theta = 1 - \frac{1}{\sigma}$, this establishes the first component of part (e).

In the special case of free trade ($\tau = 1$) studied in the text, the second term vanishes and only the first and the third terms in (A.14) matter. Furthermore, $C_1 = \gamma_i^i$ because $\gamma_i^i\gamma_j^i - \gamma_j^j\gamma_i^j = 0$ in this case, $\gamma_i^i + \gamma_j^j = 1$ and $\Delta = \sigma$. Consequently, $d\Upsilon/d\sigma = \frac{1}{\sigma}\left(1 + \gamma_i^i \ln\left(\pi_T^i\right)\right)$. For future purposes, also note that $\pi_T^i = 1$ implies $d\Upsilon/d\sigma = \frac{1}{\sigma} > 0$.

Turning to the impact of trade costs on $\lambda^i\omega_{\lambda^i}^i/\omega^i$, one can show that

$$d\Upsilon/\widehat{\tau} = (\sigma-1)\left[\frac{2(\sigma-1)\gamma_j^j\gamma_i^j}{\Delta} + C_1\left(1 - \frac{\gamma_i^i - \gamma_j^j}{\Delta}\right)\right]. \tag{A.15}$$

The RHS of the above expression is positive; therefore, increases in trade costs result in increases in $\lambda^i\omega_{\lambda^i}^i/\omega^i$, as stated in the second component of this part of the lemma. ||

Next we turn to compare the countries' payoffs under trade with the payoffs under autarky for any division (λ^i) of $\bar{X}$ across the two regimes. Since $w_T^i = \omega^i\bar{X}$ and $w_A^i = \lambda^i\bar{X}$, this comparison can be understood by studying the behavior of ω^i relative to that of λ^i.[70]

Lemma A.5 *For any given $\bar{X} > 0$, payoffs under trade (w_T^i) and under autarky (w_A^i) have the following properties:*

(a) (i) $w_T^i > w_A^i$ *for all* $\lambda^i \in (0,1)$.

(ii) $\lim_{\lambda^i \to 0} w_T^i = \lim_{\lambda^i \to 0} w_A^i = 0$.

(iii) $\lim_{\lambda^i \to 1} w_T^i = \lim_{\lambda^i \to 1} w_A^i = \bar{X}$.

(b) $\lim_{\lambda^i \to \frac{1}{2}} w_T^i \gtreqless \bar{X}$ *as* $\theta \lesseqgtr \breve{\theta}(\tau)$, *where* $\breve{\theta}(1) = \frac{1}{2}$ *and* $\breve{\theta}'(\tau) < 0$.

Proof: Each part follows readily from Lemma A.4. ||

The importance of this lemma for our analysis is twofold. First, when guns are exogenously determined (a situation that serves as a valuable benchmark), trade dominates autarky in payoffs for all possible allocations of the common pool except in the extreme cases where $\lambda^i = 0$ and $\lambda^i = 1$. Second, if θ is sufficiently close to 0 (so that the gains from trade are

[70] One should keep in mind, though, that our upcoming comparison of open conflict and peaceful settlement will be complicated by the fact that arming incentives differ across these regimes, implying that both the size of the common pool $\bar{X}$ and its division $\lambda^i = \phi^i$ will be endogenous and thus will also differ.

sufficiently high), a country enjoys a higher payoff under an even split of the common pool relative to a situation in which it controls the entire pool. As will become evident, this finding helps explain the emergence of an equilibrium under settlement with less and, under some circumstances, no arming at all.

The next lemma characterizes the global gains from peace per unit of $\bar{X}$, given by

$$\Omega(\lambda^i;\theta,\tau,\beta) \equiv S/\bar{X} = \omega^i + \omega^j - (u^i + u^j)/\bar{X} = \omega^i + \omega^j - \beta,$$

as it depends on the distribution λ^i of $\bar{R}$:

Lemma A.6 *For any feasible quantity of guns $\bar{G}$ that yields a common pool of non-negligible size $\bar{X} = \bar{R} - \bar{G} > 0$, the global gains from peace function, $\Omega(\lambda^i;\theta,\tau,\beta)$, is strictly concave in λ^i and maximized at $\lambda^i = \frac{1}{2}$. Furthermore,*

(a) $\lim_{\lambda^i \to 0} \Omega = \lim_{\lambda^i \to 1} \Omega = 1 - \beta$

(b) $\partial\Omega/\partial(\xi) < 0$ *for* $\xi \in \{\theta,\tau,\beta\}$.

Proof: The proof follows in a straightforward way from the properties of the individual components of Ω, studied in Lemma A.4. In particular, the strict concavity of Ω in λ^i is due to the fact that it is the sum of two strictly concave functions ω^i and ω^j. The reason a benevolent social planner would choose $\lambda^i = \frac{1}{2}$ is threefold: (i) the production function $F(\cdot,\cdot)$ of the final good (2) is symmetric across countries $i = 1, 2$; (ii) the technologies of countries' respective intermediate goods are identical; and (iii) the rate of destruction $1 - \beta$ is fixed. Part (a) is fairly obvious: if all of $\bar{X}$ is allocated to a single country, there are no gains from trade. Nonetheless, peace could still generate global gains through the avoidance of destruction. Part (b) follows from Lemma A.4(d) and the definition of Ω above. ||

Henceforth, we identify λ^i with ϕ^i (i.e., $\lambda^i \equiv \phi^i$). However, as noted earlier, our analysis goes through if other division rules (e.g., splitting the surplus) are considered. Moreover, while in the main text we focus on free trade ($\tau = 1$), in the proofs that follow we continue to consider the possibility of costly trade ($\tau > 1$) as well as free trade ($\tau = 1$).

Proof of Proposition 3. We break the proof of this proposition in two parts. First, we establish the results related to unarmed peace and then we proceed with the results that relate to armed peace.

Unarmed peace. Since $G^i = 0$ for $i = 1, 2$, a country's payoff under unarmed peace is given by $V_s^i = \omega^i \bar{R}$. Because $\bar{R}$ remains fixed, the behavior of V_s^i as the distribution of ownership claims changes is governed by the behavior of ω^i, where now $\omega^i = \mu^i \phi^i$ and $\phi^i = R^i/\bar{R}$. But, Lemma A.4 (whose proof hinges on Lemmas A.1–A.3) has already described the behavior of ω^i for any division λ^i including $\lambda^i = \phi^i = R^i/\bar{R}$.

As discussed in the main text, the emergence of unarmed peace as a potential outcome requires $V_s^i \geq \bar{R}$ for $i = 1, 2$. To see that this condition is satisfied under the circumstances stated in the proposition, note that by part (b) of Lemma A.5 (where $\bar{X} = \bar{R}$, $\lambda^i = \phi^i$ and $w_T^i = V_s^i$) we have $V_s^i \geq \bar{R}$ at $R^i = \frac{1}{2}\bar{R}$ for both $i = 1, 2$ if $\theta \in (0, \breve{\theta}(\tau)]$ (with equality if $\theta = \breve{\theta}(\tau)$), where $\breve{\theta}(1) = \frac{1}{2}$ under free trade. Since V_s^i is concave and increasing in R^i for $R^i \leq \frac{1}{2}\bar{R}$ (by part (b) of Lemma A.4), there exists a unique value R_e^i of R^i that ensures $V_s^i(R_e^i, \theta, \tau) = \bar{R}$ and satisfies $R_e^i(\theta, \tau) \leq \frac{1}{2}\bar{R}$ for $\theta \in (0, \breve{\theta}(\tau)]$ (with equality if $\theta = \breve{\theta}(\tau)$). It also follows that $V_s^i \geq \bar{R}$ for any $R^i \in [R_e^i(\theta, \tau), \bar{R}]$ ($i = 1, 2$). Thus, $V_s^i \geq \bar{R}$ for $i = 1, 2$ if $R^i \in \mathcal{H}(\theta, \tau) = [R_e^i(\theta, \tau), \bar{R} - R_e^i(\theta, \tau)]$.

By part (d) of Lemma A.4, we have $\omega_\theta^i < 0$ which implies $\partial V_s^i / \partial\theta < 0$. As such, an application of the implicit function theorem to the condition that defines R_e^i readily implies $dR_e^i/d\theta > 0$; therefore, the size of relevant interval, now written as $\mathcal{H}(\theta, \tau)$ to reflect its dependence on $\tau \geq 1$, is decreasing in θ. Likewise, Lemma A.4(d) shows that $\omega_\tau^i < 0$, which implies $dV_s^i/d\tau < 0$. Accordingly, as one can easily verify, $\partial R_e^i/\partial\tau > 0$ holds, so that the size of $\mathcal{H}(\theta, \tau)$ also depends negatively on τ.

Armed peace. We first prove that $\widetilde{v}^i$ is strictly quasi-concave in G^i which implies the existence of a best-response function $\widetilde{B}_s^i(G^j)$, for $G^j > 0$. This property also establishes the existence of an equilibrium in arming of the subgame associated with peace. We then prove that the equilibrium is unique.

Our earlier finding that $\lim_{\phi^i \to 0} \omega^i = 0$ (Lemma A.4(a)) together with the properties of the CSF in (1) and with the expression in (15) imply $\lim_{G^i \to 0} v_{G^i}^i > 0$. Hence, for $G^j > 0$, country i produces a positive quantity of guns. Now, G^i is either constrained or unconstrained by country i's initial resource endowment R^i. We start with the latter case, noting that, by the definition of the unconstrained payoff function $\widetilde{v}^i$, $G^i < R^i$ and $\widetilde{v}_{G^i}^i = 0$.

We now show that the solution to this FOC, represented by $\widetilde{B}_s^i(G^j)$, is unique. Differentiating $\widetilde{v}_{G^i}^i$ in (15) with respect to G^i and simplifying the resulting expression yields (after some rearrangement)

$$\widetilde{v}_{G^i G^i}^i = -\left[\left(-\frac{\omega_{\phi^i\phi^i}^i}{\omega_{\phi^i}^i} - \frac{\phi_{G^i G^i}^i}{\phi_{G^i}^i \phi_{G^i}^i}\right)\phi_{G^i}^i \bar{X} + 2\right]\omega_{\phi^i}^i \phi_{G^i}^i, \tag{A.16a}$$

where $\phi_{G^i}^i > 0$ from (1) and $\omega_{\phi^i}^i > 0$ by Lemma A.4(b) and the FOC for arming at an interior solution. Additionally, the expression inside the square brackets is positive because $-\omega_{\phi^i\phi^i}^i > 0$ holds by Lemma A.4(b), and from (1) $-\phi_{G^i G^i}^i / \left(\phi_{G^i}^i \phi_{G^i}^i\right) = 2/\phi^j > 0$ holds. Thus, $\widetilde{v}_{G^i G^i}^i < 0$ for $i = 1, 2$ holds, implying $\widetilde{v}^i$ is strictly quasi-concave and $\widetilde{B}_s^i(G^j)$ is unique.[71] This finding also implies the existence of an equilibrium in arming when neither country's choice is constrained by its endowment (for additional details see Garfinkel et al. 2020 and Garfinkel et al. 2015).

[71] One can also show that $\widetilde{B}_s^i(G^j) > G^j$ for sufficiently small values of G^j.

Differentiating $\widetilde{v}^i_{G^i}$ in (15) with respect to G^j gives (after some rearrangement),

$$\widetilde{v}^i_{G^iG^j} = \left[\left(-\frac{\omega^i_{\phi^i\phi^i}}{\omega^i_{\phi^i}} - \frac{\phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}}\right)\phi^i_{G^i}\bar{X} + 1 + \frac{\phi^i_{G^i}}{\phi^i_{G^j}}\right]\omega^i_{\phi^i}\left(-\phi^i_{G^j}\right), \tag{A.16b}$$

where $-\phi^i_{G^j} > 0$ for $i \neq j$ and $\omega^i_{\phi^i} > 0$. Turning to the expression inside the square brackets, we have already noted that $-\omega^i_{\phi^i\phi^i}/\omega^i_{\phi^i} > 0$. From the conflict technology (1), we have $-\phi^i_{G^iG^j}/\left(\phi^i_{G^i}\phi^i_{G^j}\right) = 1/\phi^j - 1/\phi^i$, which is non-negative for $G^i \geq G^j$. Finally, we have $1 + \phi^i_{G^i}/\phi^i_{G^j} = 1 - \phi^j/\phi^i$ which, once again, is non-negative for $G^i \geq G^j$. Bringing these results together yields $\widetilde{v}^i_{G^iG^j} > 0$ for $G^i \geq G^j$.

We now turn to $\widetilde{B}^i_s(G^j)$. An application of the implicit function theorem to $\widetilde{v}^i_{G^i} = 0$ shows $\partial\widetilde{B}^i_s/\partial G^j = -\widetilde{v}^i_{G^iG^j}/\widetilde{v}^i_{G^iG^i}$ or more precisely, using (A.16),

$$\frac{\partial\widetilde{B}^i_s}{\partial G^j} = -\left(\frac{\phi^i_{G^j}}{\phi^i_{G^i}}\right)\frac{\left[\left(-\frac{\omega^i_{\phi^i\phi^i}}{\omega^i_{\phi^i}} - \frac{\phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}}\right)\phi^i_{G^i}\bar{X} + 1 + \frac{\phi^i_{G^i}}{\phi^i_{G^j}}\right]}{\left[\left(-\frac{\omega^i_{\phi^i\phi^i}}{\omega^i_{\phi^i}} - \frac{\phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}}\right)\phi^i_{G^i}\bar{X} + 2\right]}. \tag{A.17}$$

By the preceding discussion, the expression above is positive for $G^i \geq G^j$. Thus, $\widetilde{B}^i_s(G^j)$ exhibits strategic complementarity for $G^j > 0$ and all $G^i \geq G^j$.[72] Considering, in particular, the point where $G^i = G^j$ gives $-\phi^i_{G^j}/\phi^i_{G^i} = 1$, $\phi^i_{G^iG^j} = 0$, and $1 + \phi^i_{G^i}/\phi^i_{G^j} < 2$. Therefore, $\partial\widetilde{B}^i_s/\partial G^j\big|_{G^i=G^j} \in (0,1)$, which implies that the symmetric equilibrium (i.e., the solution to $\widetilde{v}^i_{G^i}(G^i, G^i) = 0$) is unique. Part (a) below provides an explicit expression for this solution for $\tau \geq 1$.

To establish existence and uniqueness of equilibrium when one country is resource constrained, observe that the quantity of arms produced by country i when its resource constraint does not bind, $\widetilde{G}^i_s$ $(i = 1, 2)$, is no greater than $\frac{1}{4}\bar{R}$ $(\geq R^s_L)$. Now suppose that $\widetilde{G}^i_s > R^i$, so that country i is resource constrained. By the definition of a country's best-response function, we have $B^j_s(G^i;\cdot) = \min\{R^j, \widetilde{B}^j_s(G^i)\}$ for any $G^i > 0$. In particular, if $B^i_s = R^i$, then $B^j_s = \widetilde{B}^j_s(R^i)$. Because $(\widetilde{B}^j_s(R^i), R^i)$ lies on both countries' best-response functions, neither country has an incentive to deviate from it. Thus, this point represents a Nash equilibrium of the subgame in arming in anticipation of settlement. Uniqueness of equilibrium follows from the definition of the best-response functions whose shape ensures that they intersect only once (excluding, of course, the no-arming equilibrium).

Part (a). This part in the case of free trade (i.e., $\tau = 1$) follows readily from the discussion in the text and the definitions of the threshold values R^s_L and R^s_H in (17). Even allowing for costly trade (i.e., $\tau \geq 1$), our arguments in the text continue to imply $G^i_s = G_s$. Using that symmetry result in the FOCs $\widetilde{v}^i_{G^i} = 0$ for $i = 1, 2$, then, delivers the following solution:

$$G_s = \frac{1}{4}\left[1 - \frac{1-\theta}{1+\theta}\tau^{-\frac{\theta}{1-\theta}}\right]\bar{R}, \tag{A.18}$$

[72]The property holds more generally, even for values of $G^i < G^j$ but sufficiently close to G^j.

with the implied thresholds, $R_L^s = G_s$ and $R_H^s = \bar{R} - G_s$. Observe further that $G_s = R_L^s \to \frac{1}{4}\bar{R}$ as $\tau \to \infty$ as well as when $\theta \to 1$.

Part (b). That the unconstrained country j will produce more guns than its constrained rival i follows from the strategic complementarity of G^j for $G^i \leq G^j$ and the fact that $R^i < R_L^s$. It then follows that $\phi_s^i < \phi_s^j$.

Part (c). Turning to the dependence of the unconstrained country j's optimal arming on $\xi \in \{\theta, \tau\}$, we start by noting that $\widetilde{B}_s^j\left(R^i; \theta, \tau\right)$ is implicitly defined by the FOC, $\widetilde{v}_{G^j}^j = 0$. Thus, an application of the implicit function theorem yields $dG_s^j/d\xi = -\widetilde{v}_{G^j\xi}^j / \widetilde{v}_{G^jG^j}^j$. We have already shown that $\widetilde{v}_{G^jG^j}^j < 0$, implying $\text{sign}\{dG_s^j/d\xi\} = \text{sign}\{\widetilde{v}_{G^j\xi}^j\}$ holds. To proceed, we rewrite $\widetilde{v}_{G^j}^j$, using (15), as

$$\widetilde{v}_{G^j}^j = \omega^j \left[\left(\phi^j \omega_{\phi^j}^j / \omega^j \right) \left(\phi_{G^j}^j / \phi^j \right) \bar{X} - 1 \right] = 0.$$

Given $\omega^j > 0$, the expression inside the square brackets must equal 0. Furthermore, since $(\phi_{G^j}^j/\phi^j)\bar{X}$ does not depend on $\xi \in \{\theta, \tau\}$, we must have $\text{sign}\{\widetilde{v}_{G^j\xi}^j\} = \text{sign}\{\partial(\phi^j \omega_{\phi^j}^j/\omega^j)/\partial\xi\}$, which tells us how the ratio of the marginal benefit to arming to the marginal cost under armed peace responds to changes in the gains from trade determined jointly by the degree of substitutability $\theta \in (0, 1]$ and trade costs $\tau \geq 1$. As shown above, the effects of θ and τ on the marginal cost of arming (i.e., ω_θ^i and ω_τ^i) are negative (see the discussion in relation to (A.11) and (A.10)), while the effect on the marginal benefit to arming is ambiguous. However, Lemma A.4(e) confirms that an unconstrained country's marginal benefit to arming rises relative to its marginal cost of arming with an increase in $\xi \in \{\theta, \tau\}$. Specifically, in the case of θ, because country j is unconstrained, Lemma A.1(c) implies $\pi_T^i \geq 1$, which implies from Lemma A.4(e) that $\widetilde{v}_{G^j\theta}^j > 0$; in the case of τ, Lemma A.4(e) implies that we always have $\widetilde{v}_{G^j\tau}^j > 0$. Thus, an unconstrained country's arming under settlement is increasing in $\xi \in \{\theta, \tau\}$.

Turning to the effect of θ and τ on the thresholds, we appropriately differentiate G_s in (A.18) to find

$$\begin{aligned}
\frac{\partial G_s}{\partial \theta} &= \frac{\tau^{-\frac{\theta}{1-\theta}}}{2\left(1-\theta^2\right)} \left[\frac{1-\theta}{1+\theta} + \frac{1}{2}\ln(\tau) \right] \bar{R} > 0 \\
\frac{\partial G_s}{\partial \tau} &= \frac{\theta \tau^{-\frac{1}{1-\theta}}}{4\left(1+\theta\right)} \bar{R} > 0,
\end{aligned}$$

Thus, we have $dR_L^s/d\xi = -dR_H^s/d\xi = dG_s/d\xi > 0$ and $d(R_H^s - R_L^s)/d\xi < 0$.

The remaining points of part (c) can be found by evaluating (A.18) at the appropriate limits: $\lim_{\theta \to 0} G_s = 0$, whereas $\lim_{\theta \to 1} G_s = \frac{1}{4}\bar{R}$ that imply the corresponding limits of the threshold values as stated in the proposition. In addition, one can find that the limits of the threshold values of the resources as $\tau \to \infty$ match those as $\theta \to 1$, as expected since $\lim_{\theta \to 1} \mu^i = \lim_{\tau \to 1} \mu^i = 1$—i.e., there are no relative gains from trade. $\qquad ||$

Proof of Proposition 4. The independence of average discounted payoffs from δ under settlement, in general, is due to the stationarity of the structure of the model under this form of conflict resolution. The independence from β is due to the fact that there is no destruction under settlement and our assumption that the division of the common pool is on the basis of the conflict technology (1). Payoffs under unarmed peace are derived from the definition of $V_s^i = v_s^i$ in (12) with $G^i = G^j = 0$, which implies $\phi^i = R^i/\bar{R}$ and $\bar{X} = \bar{R}$. The characterization of V_s^i under unarmed peace then follows from the properties of ω^i described in Lemma A.4.

Let us turn to the characterization of V^s under armed peace.

Part (a). Since $v^i = \omega^i \bar{X}$, where $\bar{X} = \bar{R} - 2G_s$, and since $\phi^i = \frac{1}{2}$ such that $\pi_T^i = 1$ and $p_T^i = \tau$, we have $V_s^i = v_s^i$ equals a constant for $i = 1, 2$ and all $R^i \in [R_L^s, R_H^s]$. Furthermore, we have $dv_s^i/d\xi = \omega_\xi^i \bar{X} - 2\omega^i\,(dG_s/d\xi)$ for $\xi \in \{\theta, \tau\}$. But, we know that $\omega_\xi^i < 0$ from Lemma A.4(d) and $dG_s/d\xi > 0$ from the proof of Proposition 3(c). It follows that $dv_s^i/d\xi < 0$ for $\xi \in \{\theta, \tau\}$. To find the solution for V_s^i in this symmetric outcome, first note that the findings above imply $\omega^i = \phi^i \mu^i = \frac{1}{2}(1 + \tau^{-\frac{\theta}{1-\theta}})^{\frac{1-\theta}{\theta}}$. Then, with this expression, the definition of $v_s^i = \omega^i \bar{X}$ and the value of $\bar{X} = \bar{R} - 2G_s$ implied by the solution for G_s in (A.18), we obtain

$$V_s^i = \frac{1}{4}\left(1 + \tau^{-\frac{\theta}{1-\theta}}\right)^{\frac{1-\theta}{\theta}} \left[1 + \frac{1-\theta}{1+\theta}\tau^{-\frac{\theta}{1-\theta}}\right] \bar{R},$$

which simplifies to the value of V_s^i reported in the proposition when $\tau = 1$.

Part (b). First note from the definition of payoffs that, for $R^i \in (0, R_L^s)$, endowment reallocations that keep $\bar{R}$ unchanged affect the countries' payoffs (regardless of whether one country is resource constrained) solely through their impact on both countries' guns choices. Focusing on the constrained country i's arming (which in this part requires $G_s^i = R^i$ and implies $G_s^j = \widetilde{B}_s^j(R^i)$), we have:

$$\begin{aligned}
\frac{dV_s^i}{dR^i} &= v_{G^i}^i + v_{G^j}^i \left[\frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right] = -v_{G^j}^i \left[-\frac{v_{G^i}^i}{v_{G^j}^i} - \frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right] \\
&= \left[\omega_{\phi^i}^i \phi_{G^j}^j \bar{X} + \omega^i\right] \left[\frac{\omega_{\phi^i}^i \phi_{G^i}^i \bar{X} - \omega^i}{\omega_{\phi^i}^i \phi_{G^j}^j \bar{X} + \omega^i} - \frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right],
\end{aligned} \tag{A.19}$$

for $i, j \in \{1, 2\}$ and $i \neq j$, where the last expression was obtained by noting $\phi_{G^j}^i = -\phi_{G^j}^j$, which implies $-v_{G^j}^i = (\omega_{\phi^i}^i \phi_{G^j}^j \bar{X} + \omega^i) > 0$, and adjusting the signs of the expressions appropriately. Since $\omega_{\phi^i}^i \phi_{G^j}^j \bar{X} + \omega^i > 0$, the expression above implies

$$\operatorname{sign}\left\{\frac{dV_s^i}{dR^i}\right\} = \operatorname{sign}\left\{\frac{\omega_{\phi^i}^i \phi_{G^i}^i \bar{X} - \omega^i}{\omega_{\phi^i}^i \phi_{G^j}^j \bar{X} + \omega^i} - \frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right\}. \tag{A.20}$$

Now suppose $R^i \to R_L^s$, which implies $V_{G^i}^i \to 0$ and thus $(\omega_{\phi^i}^i \phi_{G^i}^i \bar{X} - \omega^i) \to 0$ in (A.20). But, by the proof to Proposition 3 (see equation (A.17) and the related discussion), we know that

$\lim_{R^i \to R^s_L} d\widetilde{B}^j_s(R^i)/dR^i \in (0,1)$. Thus, the expression inside the brackets in the RHS of (A.20) is negative, as required. One can further show that $\text{sign}\{\lim_{R^i\to 0} dV^i_s/dR^i\}$ is determined by the gains from trade that jointly depends on θ and τ.[73]

Let us turn to the endowment effects for the unconstrained country j. As R^j increases and R^i falls to keep $\bar{R}$ unchanged, country j operates along $\widetilde{B}^j_s(R^j)$. Hence, we have

$$\frac{dV^j_s}{dR^j} = \widetilde{v}^j_{G^i}\left(\frac{dR^i}{dR^j}\right) = -\widetilde{v}^j_{G^i},$$

which implies, since $\widetilde{v}^j_{G^i} = \omega^j_{\phi^j}\phi^j_{G^i}\bar{X} - \omega^j < 0$, that country j's payoff rises along its best-response function $\widetilde{B}^j_s(R^i)$ as R^j increases given $\bar{R}$. Differentiating the above expression with respect to R^j (while again keeping in mind that $dR^i = -dR^j$) gives

$$\begin{aligned}
\frac{d^2V^j_s(\widetilde{B}^j_s(R^i),R^i)}{(dR^j)^2} &= \left[-\widetilde{v}^j_{G^iG^j}\frac{d\widetilde{B}^j_s(R^i)}{dR^i} - \widetilde{v}^j_{G^iG^i}\right]\left(\frac{dR^i}{dR^j}\right) \\
&= \widetilde{v}^j_{G^iG^j}\left[\frac{d\widetilde{B}^j_s(R^i)}{dR^i}\right] + \widetilde{v}^j_{G^iG^i} = \widetilde{v}^j_{G^jG^i}\left[-\frac{\widetilde{v}^j_{G^jG^i}}{\widetilde{v}^j_{G^jG^j}}\right] + \widetilde{v}^j_{G^iG^i} \\
&= -\Theta^j/\widetilde{v}^j_{G^jG^j}, \text{ where } \Theta^j \equiv \left(\widetilde{v}^j_{G^jG^i}\right)^2 - \widetilde{v}^j_{G^jG^j}\widetilde{v}^j_{G^iG^i}. \qquad \text{(A.21)}
\end{aligned}$$

Since $\widetilde{v}^j_{G^jG^j} < 0$, the sign of the above expression coincides with the sign of Θ^j. We need to show that $\Theta^j > 0$. We bypass the problem of not knowing the sign of $\widetilde{v}^j_{G^iG^i}$ by proceeding as follows. Utilizing the fact that $\omega^j_{\phi^j\phi^j} < 0$ (Lemma A.4(b)), define

$$\Gamma^j \equiv -\left(\omega^j\omega^j_{\phi^j\phi^j}\right)/(\omega^j_{\phi^j})^2 > 0,$$

and note that country j's FOC can be rewritten as $\bar{X}\phi^j_{G^j} = \omega^j/\omega^j_{\phi^j}$. We can now rewrite the expressions for $\widetilde{v}^j_{G^jG^i}$ and $\widetilde{v}^j_{G^jG^j}$ in (A.16a)–(A.16b) for country j and compute the expression for $\widetilde{v}^j_{G^iG^i}$ as follows:

$$\widetilde{v}^j_{G^jG^i} = \left[\Gamma^j + 2 - \bar{X}\phi^j_{G^jG^i}/\phi^j_{G^i}\right]\left(-\omega^j_{\phi^j}\phi^j_{G^i}\right) \qquad \text{(A.16a}'\text{)}$$

$$\widetilde{v}^j_{G^jG^j} = \left[\Gamma^j + 2 - \bar{X}\phi^j_{G^jG^j}/\phi^j_{G^j}\right]\left(-\omega^j_{\phi^j}\phi^j_{G^i}\right)\left(\phi^j_{G^j}/\phi^j_{G^i}\right) \qquad \text{(A.16b}'\text{)}$$

$$\widetilde{v}^j_{G^iG^i} = \left[\Gamma^j + \left(\phi^j_{G^j}/\phi^j_{G^i}\right)\left(2 - \bar{X}\phi^j_{G^iG^i}/\phi^j_{G^i}\right)\right]\left(-\omega^j_{\phi^j}\phi^j_{G^i}\right)\left(\phi^j_{G^i}/\phi^j_{G^j}\right). \qquad \text{(A.16c}'\text{)}$$

Now define $r \equiv -\phi^j_{G^j}/\phi^j_{G^i}$ $(= \phi^j_{G^j}/\phi^i_{G^i} > 0)$. Substituting the above expressions into the definition of Θ^j in (A.21) and simplifying the resulting expression give

$$\begin{aligned}
\Theta^j = \left(\omega^j_{\phi^j}\phi^j_{G^i}\right)^2 \Bigg\{ &\left[\Gamma^j + 2 - \bar{X}\phi^j_{G^jG^i}/\phi^j_{G^j}\right]^2 \\
&- \left[\Gamma^j + 2 - \bar{X}\phi^j_{G^jG^j}/\phi^j_{G^j}\right]\left[\Gamma^j - r\left(2 - \bar{X}\phi^j_{G^iG^i}/\phi^j_{G^i}\right)\right]\Bigg\}.
\end{aligned}$$

[73] For example, numerical analysis reveals that when $\tau = 1$, $\lim_{R^i\to 0} dV^i_s/dR^i > 0$ if θ is large enough, whereas $\lim_{R^i\to 0} dV^i_s/dR^i < 0$ if θ is sufficiently small.

Next, noting that $\bar{G} \equiv G^i + G^j$, we can apply the properties of the conflict technology in (1) to find

$$-\phi^j_{G^j G^i}/\phi^j_{G^j} = (1-r)/\bar{G}$$
$$-\phi^i_{G^j G^j}/\phi^j_{G^j} = 2/\bar{G}$$
$$-\phi^j_{G^i G^i}/\phi^j_{G^i} = 2/\bar{G}.$$

Letting $x \equiv \bar{X}/\bar{G}$, one can apply the above relations to the expression for Θ^j immediately above to obtain

$$\begin{aligned}\Theta^j &= \left(\omega^j_{\phi^j}\phi^j_{G^i}\right)^2 \left\{\left[\Gamma^j + 2 + (1-r)\,x\right]^2 - \left[\Gamma^j + 2\,(1+x)\right]\left[\Gamma^j - 2r\,(1+x)\right]\right\} \\ &= \left(\omega^j_{\phi^j}\phi^j_{G^i}\right)^2 (1+r)\left[2\Gamma^j + 4(1+x) + (1+r)\,x^2\right] > 0.\end{aligned}$$

The positive sign of Θ^j proves that the unconstrained country j's payoff V^j_s rises at an increasing rate in R^j as that country operates along its best-response function.

We now prove that $dV^i_s/d\xi < 0$ and $dV^j_s/d\xi < 0$ ($i \neq j = 1, 2$) for $\xi \in \{\theta, \tau\}$. Focusing on the resource constrained country i we have

$$\frac{dV^i_s}{d\xi} = v^i_\xi + v^i_{G^j}\frac{dG^j_s}{d\xi}. \tag{A.22}$$

The first term in the RHS of (A.22) captures the direct effect of ξ on V^i_s and is present under all circumstances. The second term is a strategic effect due to the fact that country i's rival (j) is unconstrained by its initial endowment. The first term is negative because $v^i_\xi = \omega^i_\xi \bar{X}$ and $\omega^i_\xi < 0$ by Lemma A.4(d). Additionally, $v^i_{G^j} < 0$ and $dG^j_s/d\xi < 0$ by Proposition 3(c). Clearly, then, the constrained country i's payoff falls with increases in the degree of substitutability in traded goods θ and trade costs τ.

The impact of $\xi \in \{\theta, \tau\}$ on V^j_s is solely due to a direct effect (because $G^i_s = R^i$) which, once again, by Lemma A.4(d), we know is negative.

Part (c). Suppose $R^i = 0$ initially, so that there is no trade and country i is inconsequential. Since $\omega^j = 1$ in this case and there is no need for arming, we have $V^j_s = \bar{R}$. Now suppose $R^i = \varepsilon > 0$ is infinitesimal, so that $R^j = \bar{R} - \varepsilon$ is arbitrarily close to (but less than) $\bar{R}$. Because country i produces an infinitesimal quantity of guns $G^i_s = R^i = \varepsilon$, country j will be able to induce a division (with a larger but also infinitesimal G^j) that brings it arbitrarily close to its optimum $\phi^j_{\max} < 1$ (Lemma A.4(b)). That is to say, with a division of $\bar{R}$ that allows for trade, the unconstrained country realizes a higher payoff $V^j_s(\phi^j_{\max}, \cdot) > \bar{R}$. Likewise, the smaller country i realizes some gains from trade, so that $V^i_s > 0$.

Finally, we turn to the very last part of Proposition 4 regarding the critical value of θ, below which it is not possible to Pareto rank unarmed and armed peace for more uneven distributions within $\mathcal{H}(\theta)$. As suggested by our discussion in the text and by panel (c) of

Fig. 3, such a possibility arises when $V_{sa}^i > \bar{R}$ for $R^i \in [R_L^s, R_H^s]$. Recall from part (a) of the proposition that, in this case, V_{sa}^i $(= V_s^i = V_s^j)$ is decreasing in θ for all $\theta \in (0,1)$. Furthermore, abstracting from trade costs, the expression for V_{sa}^i shown in part (a) implies $V_{sa}^i|_{\theta=\frac{1}{2}} = \frac{2}{3}\bar{R}$ and $V_{sa}^i|_{\theta=\frac{1}{3}} = \frac{3}{2}\bar{R}$ for $i = 1,2$. Hence, there exists a $\theta \in (\frac{1}{3}, \frac{1}{2})$, labeled $\widehat{\theta}$ (≈ 0.402), such that $\widehat{\mathcal{H}}(\widehat{\theta}) = \mathcal{H}(\widehat{\theta})$. For $\theta \in (0, \widehat{\theta})$, $\widehat{\mathcal{H}}(\theta)$ is a proper subset of $\mathcal{H}(\theta)$.[74] ||

Proof of Lemma 1. We take as our starting point, the benchmark case where $\delta = 0$, $\theta \to 1$ or $\tau \to \infty$ that imply (from (11) and (A.18)) that $R_L^c = R_L^s = \frac{1}{4}\bar{R}$ and $R_H^c = R_H^s = \frac{3}{4}\bar{R}$, and thus $[R_L^c, R_H^c] = [R_L^s, R_H^s]$. In what follows, we prove both parts (a) and (b), focusing first on the case where neither country is resource constrained and then on the case where one country is resource constrained.

When both countries are unconstrained by their endowments, $G_c^i = G_c = R_L^c$ rises and R_H^c falls with increases in δ (from Proposition 1(c)), whereas $G_s^i = G_s = R_L^s$ falls and R_H^s rises with decreases in θ and τ (from the proof to Proposition 3(c)). It follows that $G_s < G_c$ and $[R_L^c, R_H^c] \subset [R_L^s, R_H^s]$ for $\delta \in (0,1]$ and/or any $\theta \in (0,1)$ with $\tau \in [1,\infty)$ in this case.

Now suppose that country i's arming is constrained under armed peace: $G_s^i = R^i < R_L^s$. The argument above assuming $\delta > 0$ and/or $\theta \in (0,1)$ with $\tau \in [1,\infty)$ implies that country i's arming will also be constrained under open conflict: $G_c^i = R^i < R_L^s < R_L^c$. However, if country i's arming is constrained under open conflict, it need not (though could) be constrained under armed peace. Thus, $G_s^i \leq G_c^i$ holds.

Since country j's arming is unconstrained by its endowment under armed peace (as well as under conflict), $\widetilde{v}_{G^j}^j = \omega_{\phi^j}^j \phi_{G^j}^j \bar{X} - \omega^j = 0$ holds for $G^i > 0$. This FOC defines $\widetilde{B}_s^j(G^i)$ implicitly and can rewritten as $(\phi^j \omega_{\phi^j}^j/\omega^j)(\phi_{G^j}^j \bar{X}) = \phi^j$. Next, we use this expression with (9) to evaluate $U_{G^j}^j$ at $G^j = \widetilde{B}_s^j(G^i)$ for any $\delta \in [0,1]$:

$$
\begin{aligned}
U_{G^j}^i\big|_{G^j=B_s^j(G^i)} &= \frac{\beta}{1+\delta}\left[\phi_{G^j}^j\left(\bar{X}+\delta\bar{R}\right)-\phi^j\right]\Bigg|_{G^j=B_s^j(G^i)} \\
&= \frac{\beta}{1+\delta}\phi_{G^j}^j\bar{X}\left[1+\delta\bar{R}/\bar{X}-\phi^j\omega_{\phi^j}^j/\omega^j\right].
\end{aligned}
$$

Returning to our benchmark case where (i) $\delta = 0$ and (ii) $\theta \to 1$ or $\tau \to \infty$ (either of which implies $\mu^j = 1$ and thus $\omega^j = \phi^j$), the expression above equals 0, so that $G_s^j = G_c^j$ for any $G^i > 0$. It then follows, from Proposition 1(c) and the proof to Proposition 3(c), that any departures from our benchmark—namely, $\delta \in (0,1]$ or $\theta \in (0,1)$ with $\tau \in [1,\infty)$—imply $G_s^j = \widetilde{B}_s^j(G_s^i) < G_c^j = \widetilde{B}_c^j(G_c^i)$. ||

Proof of Lemma 2. Let us return again to our benchmark case, where (i) $\delta = 0$ and (ii) $\theta \to 1$ or $\tau \to \infty$, implying $V_s^i = U_c^i$ for $i = 1,2$ and all $R^i \in (0,\bar{R})$. One can easily infer, from Propositions 2 and 4, that, for any $\delta \in [0,1]$, the smaller country has a strict preference for settlement when $\beta \in (0,1)$ and/or $\theta \in (0,1)$ (with $\tau < \infty$). These propositions also

[74] Note that our use of a "^" to designate this value of θ does not indicate a percentage change in θ.

imply that, when $\delta \in (0, 1]$ with $\beta = 1$ and $\theta = 1$ (or $\tau \to \infty$), the smaller country strictly prefers peace over war if it is resource constrained and is indifferent between the two modes of conflict resolution otherwise. Thus, to identify the conditions under which peace is Pareto preferred to war, it suffices to focus on the preferences of the larger country.

Part (a): Accordingly, suppose $R^i \geq \frac{1}{2}\bar{R}$, so that now country i is at least as large as country j ($\neq i$), and let δ rise to a positive level in $(0, 1]$ while $\beta = 1$ and $\theta = 1$ (or $\tau \to \infty$). By Proposition 2, we know U_c^i will rise for all $R^i \in (R_H^c, \bar{R})$, while Proposition 4 implies V_s^i will remain unchanged. As such, $V_s^i = U_c^i$ for $R^i \in \left[\frac{1}{2}\bar{R}, R_H^c\right]$ and $V_s^i < U_c^i$ for $R^i \in (R_H^c, \bar{R})$.

Maintaining the assumptions of a positive discount factor ($\delta \in (0, 1]$) and no trade under armed peace ($\theta = 1$ or $\tau \to \infty$), now suppose β falls marginally below 1. Since U_c^i falls proportionately with β (Proposition 2) for any $R^i \in [\frac{1}{2}\bar{R}, \bar{R})$ and V_s^i is independent of β (Proposition 4), U_c^i will intersect V_s^i at two resource distributions, $R^i = R_A^i$ and $R^i = R_B^i$, where $\frac{1}{2}\bar{R} < R_A^i < R_H^s < R_B^i < \bar{R}$. Importantly, by construction, these points are unique. In particular, uniqueness of R_A^i holds, because it is determined by the intersection of a flat V_s^i and an upward sloping U_c^i as $R_A^i < R_H^s$. Uniqueness of R_B^i holds because the higher discount factor and reduced value of β decrease and flatten U_c^i relative to V_s^i (which remains unaffected) for all relevant values of R^i; therefore, $dV^i/dR^i\big|_{R^i=R_B^i} > dU_c^i/dR^i\big|_{R^i=R_B^i}$. Additionally, these values of R^i have the following implications: (i) $V_s^i \geq U_c^i$ for any $R^i \in [\frac{1}{2}\bar{R}, R_A^i] \cup [R_B^i, \bar{R})$ (with equality at R_A^i and R_B^i); and (ii) $V_s^i < U_c^i$ for $R^i \in (R_A^i, R_B^i)$. Because U_c^i is continuously decreasing in β, R_A^i will rise and R_B^i will fall with reductions in β. Eventually, we reach a threshold value β_0 of destruction that implies $R_A^i = R_B^i = R_H^s$ and $V_s^i(R_H^s) = U_c^i(R_H^s)$. For $\beta < \beta_0$, we have $V_s^i > U_c^i$ for all $R^i \in (\frac{1}{2}\bar{R}, \bar{R})$.

To identify β_0, we evaluate U_c^i and V_s^i at $R^i = R_H^s$. By Lemma 1, rival country j is resource constrained under war but not under peace at that initial distribution: $G_c^j = G_s^j = R_L^s < R_L^c$. Furthermore, our maintained assumption that $\theta = 1$ (or $\tau \to \infty$) implies $R_L^s = \frac{1}{4}\bar{R}$. Starting with the case of conflict, we can use (10b) to calculate $G_c^i = \widetilde{B}_c^i(R_L^s)$, $\bar{X}_c = \bar{R} - G_c^i - R_L^s$, and then $\phi_c^i = \phi^i(G_c^i, R_L^s)$, which in turn give

$$U_c^i(R_H^s) = \frac{\beta}{1+\delta}\phi_c^i\left[\bar{X}_c + \delta\bar{R}\right] = \frac{1}{4}\beta\bar{R}\left[2 - \sqrt{\frac{1}{1+\delta}}\right]^2. \tag{A.23}$$

The stationarity of payoffs under armed peace implies $V_s^i(R_H^s) = v_s^i(R_H^s)$, where $v_s^i = \omega^i\bar{X}_s$, and $\bar{X}_s = \bar{R} - G_s^i - G_s^j$. Since neither country is constrained under peace at $R^i = R_H^s$, we have $G_s^i = G_s = R_L^s = \frac{1}{4}\bar{R}$ so that $\phi^i = \phi = \frac{1}{2}$ and $\bar{X}_s = \frac{1}{2}\bar{R}$. Since $\theta = 1$ (or $\tau \to \infty$) further implies that $\mu^i = 1$ holds, we have $\omega^i = \frac{1}{2}$, and

$$V_s^i(R_H^s) = \tfrac{1}{4}\bar{R}. \tag{A.24}$$

Setting $U_c^i(R_H^s)$ in (A.23) equal to $V_s^i(R_H^s)$ in (A.24) and solving for β gives

$$\beta_0 \equiv \beta_0(\delta) = \left(2 - \sqrt{\frac{1}{1+\delta}}\right)^{-2}, \tag{A.25}$$

as claimed in the lemma. One can easily verify that $d\beta_0/d\delta < 0$ for all $\delta \in [0,1)$ with $\beta_0(0) = 1$ and $\beta_0(1) \approx 0.598$. Even in the absence of trade ($\theta = 1$ and/or $\tau \to \infty$), $\beta < \beta_0$ implies that armed peace is Pareto preferred to war for all endowment distributions. What's more, this ranking of payoffs remains intact if $\theta < 1$ and $\tau < \infty$, since V_s^i rises continuously with decreases in θ and/or in τ (see the proof to Proposition 4), while U_c^i remains unchanged. This completes the proof to part (a).

Part (b): Let us now suppose $\beta \in (\beta_0(\delta), 1]$ for $\delta \in (0,1]$ so that, by our preceding analysis, R_A^i and R_B^i satisfy $\frac{1}{2}\bar{R} < R_A^i < R_H^s < R_B^i < \bar{R}$. For any finite trade cost τ, gradually allow θ to fall below 1. By Proposition 4, we know V_s^i will rise due to the direct increase in the gains from trade and possibly the benefits of reduced arming, whereas U_c^i remains unchanged. As a result, R_A^i will rise and R_B^i will fall, such that these two points move towards each other. For the reasons mentioned above, R_A^i remains unique as θ continues to fall further below 1. Furthermore, we can show numerically that, as θ falls, R_B^i remains unique as well, implying that $dV^i/dR^i\big|_{R^i=R_B^i} > dU_c^i/dR^i\big|_{R^i=R_B^i}$ must continue to hold, too, for $\theta < 1$.[75] With additional reductions in θ, R_A^i and R_B^i eventually converge to R_H^s. (By the same logic, provided $\theta < 1$, continuous reductions in τ (> 1) will raise V_s^i thereby causing R_A^i and R_B^i to move towards each other and converging R_H^s.)

At the point of convergence $R_A^i = R_B^i = R_H^s$, there exists a threshold value of θ for any finite $\tau \in [1,\infty)$, denoted by θ_0, that ensures $V_s^i(R_H^s)\big|_{\theta=\theta_0} = U_c^i(R_H^s)\big|_{\theta=\theta_0}$. To characterize this threshold, let us reevaluate $V_s^i(R_H^s)$ and $U_c^i(R_H^s)$, using (17) and explicitly taking into account that $R_H^s = \bar{R} - R_L^s$ depends on τ and θ as discussed in connection with (A.18). In particular, since $R^i = R_H^s$ (and thus $R^j = R_L^s$) neither country is constrained under settlement, implying from Proposition 3(s) that $G_s^i = G_s^j = G_s$ and $\phi^i = \frac{1}{2}$ and from Lemma A.4(a) that $\omega^i = \omega^j = \omega_s$. Let $k \equiv k(\theta,\tau) = \frac{1-\theta}{1+\theta}\tau^{-\frac{\theta}{1-\theta}} \in [0,1)$. Then, from (A.18), we have $G_s = R_L^s = \frac{1}{4}(1-k)\bar{R}$ and $\bar{X}_s = \frac{1}{2}(1+k)\bar{R}$. Therefore,

$$V_s^i(R_H^s) = \omega_s \bar{X}_s = \omega_s \tfrac{1}{2}(1+k)\bar{R}. \tag{A.26}$$

For future reference, let us evaluate the expression above at $\theta = \breve{\theta}(\tau) \in (0, \frac{1}{2}]$ for finite τ, defined in Lemma A.4(a) as the value of θ which ensures, given the concavity of ω^i, that $\omega^i \geq 1$ for all $\phi^i \in [\frac{1}{2}, 1)$. Because $\omega_s^i\big|_{\phi^i=1/2,\theta=\breve{\theta}(\tau)} = 1$, we have

$$V_s^i(R_H^s)\big|_{\theta=\breve{\theta}(\tau)} = \tfrac{1}{2}(1+k)\bar{R}. \tag{A.26$'$}$$

Next, we evaluate U_c^i at $R^i = R_H^s$ following the same strategy as we used to calculate (A.23),

[75] Recall that, if there is no destruction ($\beta = 1$), then $R_B^i = \bar{R}$ for $\delta \in (0,1]$; thus, the inequality still holds.

but in this case using the definition of $k(\theta, \tau)$ with $\theta < 1$ and a finite value for $\tau \geq 1$:

$$U_c^i\left(R_H^s\right) = \tfrac{1}{4}\beta\bar{R}\left(2 - \sqrt{\frac{1-k}{1+\delta}}\right)^2. \tag{A.27}$$

Our preceding analysis suggests that $V_s^i(R_H^s)/U_c^i\left(R_H^s\right)\big|_{\theta=1} < 1$, for $\beta > \beta_0$ with $\delta \in (0,1]$. From Proposition 4, we also know that V_s^i rises relative to U_c^i as θ falls. Now observe, from (A.26′) and (A.27), that

$$\left.\frac{V_s^i(R_H^s)}{U_c^i\left(R_H^s\right)}\right|_{\theta=\breve{\theta}(\tau)} = \frac{\frac{1}{2}\left(1+k\right)\bar{R}}{\frac{1}{4}\beta\bar{R}\left(2 - \sqrt{\frac{1-k}{1+\delta}}\right)^2} > \frac{\frac{1}{2}\left(1+k\right)}{\left(1-\frac{1}{2}\sqrt{1-k}\right)^2},$$

where, of course, k assumes a particular value in $(0,1)$ when $\theta = \breve{\theta}\left(\tau\right)$. But, the RHS of the above expression exceeds 1 for all $k \in (0,1)$; therefore, there exists a $\theta_0(\tau) < \breve{\theta}(\tau)$ that ensures $V_s^i(R_H^s)$ rises above $U_c^i\left(R_H^s\right)$ before θ falls to $\breve{\theta}(\tau)$. Put differently, $V_s^i < U_c^i$ at R_H^s when $\theta > \theta_0(\tau)$ and $V_s^i > U_c^i$ at R_H^s when for $\theta = \breve{\theta}(\tau)$. This finding and the fact that $dV^i/dR^i\big|_{R^i=R_H^s} > dU_c^i/dR^i\big|_{R^i=R_H^s}$ implies that the threshold $\theta_0(\tau)$ is unique.[76]

The implications of $\theta \gtreqless \theta_0$ detailed in part (b) of the lemma follow readily. Furthermore, one can confirm the effects of increases in β and γ on θ_0, as stated at the beginning of the lemma, by applying the implicit function theorem to $V_s^i(R_H^s)\big|_{\theta=\theta_0} = U_c^i(R_H^s)\big|_{\theta=\theta_0}$ and using the fact that $dV_s^i(R_B^i)/dR^i > dU_c^i(R_B^i)/dR^i$ including at $R_B^i = R_H^s$. ||

Proof of Lemma 3. Having demonstrated that peace is immune to unilateral deviations when $\theta \leq \theta_0(\delta, \beta)$ in the text, we focus here on what happens when θ rises above θ_0 such that, from Proposition 2, $V_s^i(R_H^s) < U^i(R_H^s)$ holds. Fig. A.1(a) shows equilibrium arming for country $i = 1$ under conflict, settlement and the optimizing unilateral deviation for all distributions of initial resource ownership $R^1 \in (0, \bar{R})$ when $\beta = \delta = 1$—i.e., the conditions that are most favorable to deviations from peace as well as to war; Fig. A.1(b) shows the corresponding payoffs.[77]

To start, suppose $R^i \geq R_H^s$. In this case, where country j is constrained under settlement, $G_s^j = R^j$ holds. Accordingly, under a unilateral deviation, country i operates on its unconstrained best-response function under conflict $G_d^i = \widetilde{B}_c^i(R^j)$. As such, we have $W_d^i(R^i) = U_c^i(R^i)$ for all $R^i \geq R_H^s$.[78] Thus, for this range of endowments, country i has an incentive to deviate from peace precisely under those conditions when it finds war relatively more appealing.

When $R^i \in (R_L^d, R_H^s)$, country j' arming is unconstrained by its resource endowment under settlement and produces $G_s^j = R_L^s$. Meanwhile, country i, in a unilateral deviation,

[76] We can show analytically that this inequality holds. Details are available on request.

[77] Again, we use green for values under armed peace, pink for values under war and blue for values under a unilateral deviation from peace by country $i = 1$.

[78] Since $U_c^i(R^i)$ is independent of θ (and τ) this segment of $W_d^i(R^i)$ is also independent of θ (and τ).

would continue to operate on its best-response function under open conflict $G_d^i = \widetilde{B}_c^i(R_L^s) = R_L^d$ $(< \bar{R}/2)$, such that $W_d^i(R^i) = U_c^i(R_H^s)$ for such allocations.[79] But, our consideration of $\theta > \theta_0$ (for a given finite $\tau \geq 1$) implies $V_s^i(R^i) = V_s^i(R_H^s) < U_c^i(R_H^s)$ for $R^i \in (R_L^d, R_H^s)$.[80] Accordingly, for $R^i \in (R_L^d, R_H^s)$, $W_d^i(R^i) > V_s^i(R^i)$ holds, and at least one of the countries and possibly both (specifically, when $R^i \in (R_L^d, R_H^d)$) will have an incentive to deviate from settlement.

As R^i falls below R_L^d, $W_d^i(R^i)$ falls too due to the tightening of country i's resource constraint whose effect dominates any favorable strategic effect (for $R^i \leq R_s^L$), and approaches 0 as $R^i \to 0$. Although $V_s^i(R^i)$ also eventually falls with decreases in $R^i < R_L^s$, this payoff approaches some positive amount (by Proposition 4) as $R^i \to 0$. Thus, there exists at least one intersection where $W_d^i(R^i) = V_s^i(R^i)$ for $R^i < R_L^d$.[81] As R^i falls below that the intersection, $W_d^i(R^i)$ falls below $V_s^i(R^i)$, such that for sufficiently uneven initial distributions of $\bar{R}$ the smaller country has no incentive to deviate from settlement.

Considering the larger country's perspective, we also know (from the proof of Lemma 2) that there exists a unique intersection between $V_s^i(R^i)$ and $U_c^i(R^i)$, labeled R_B^i, at some $R^i > R_H^s$ when $\theta > \theta_0$ (for finite $\tau \geq 1$). Then $W_d^i(R^i) = U_c^i(R^i) \geq V_s^i(R^i)$ for all $R^i \in (R_H^s, R_B^i]$ and $W_d^i(R^i) < V_s^i(R^i)$ for all $R^i \in (R_B^i, \bar{R})$. Thus, for sufficiently large values of R^i implying sufficiently low values of R^j, neither country has an incentive to deviate unilaterally from peace. ||

[79] Recall that a larger value of θ (given finite $\tau \geq 1$) implies increased arming under settlement by the opponent in this range: $G_s^j = R_L^s \uparrow$, which implies $R_H^s = \bar{R} - R_L^s \downarrow$. Therefore, an increase in θ (for given $\tau \geq 1$) implies a downward shift in $W_d^i(R^i)$ for $R^i < R_H^s$, with the flat segment meeting $U_c^i(R^i)$ at a new, lower value of R_H^s, illustrated by the blue dot at that resource allocation in Fig. A.1(b). The black dot to the right of that on $U_c^i(R^i)$ at the intersection of that payoff and the dashed black line curve represents the point where $W_d^i(R^i)$ converges to $U_c^i(R^i)$ at a lower value of θ such as θ_0.

[80] A larger θ adversely affects V_s^i, as fully described in Proposition 4. In addition, one can show that, for a given increase in θ, the flat segment of $V_s^i(R^i)$ (for $R^i \in (R_L^s, R_H^s)$) falls by more than the flat segment of $W_d^i(R^i)$ (for $R^i \in (R_L^d, R_H^s)$.

[81] At that intersection, where arming by the two countries are identical under the two modes of operation (i.e., $G_d^i = G_s^i = R^i$ and $G^j = G_s^j = \widetilde{B}_s^j(R^i)$), the gains realized under settlement (and the avoidance of destruction when $\beta < 1$) equal the savings by not having to arm in the next period. That this intersection occurs at R_L^s in Fig. A.1 is merely a coincidence. It could occur above or below R_L^s.

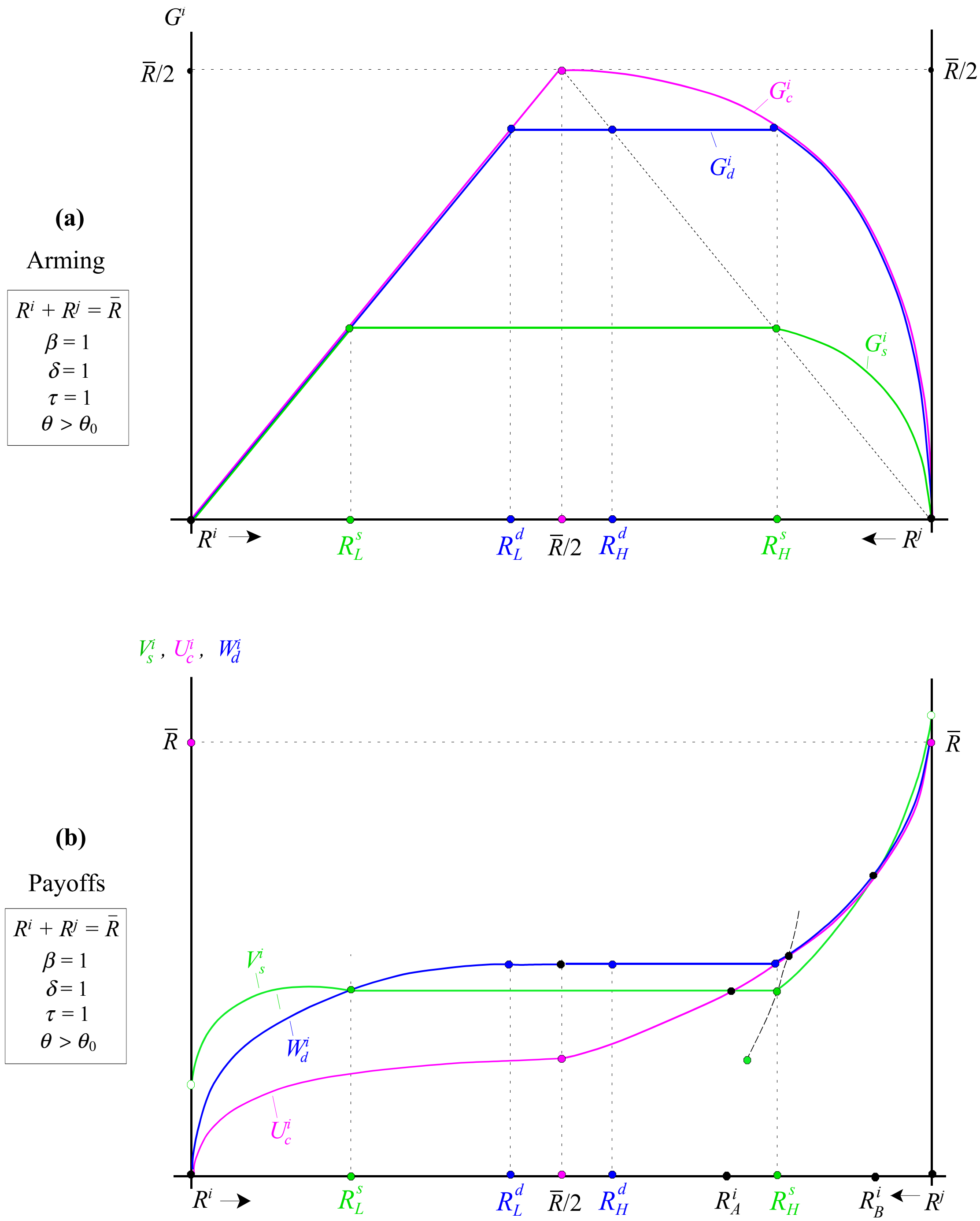

Figure A.1: Arming and Payoffs under Conflict, Settlement, and Unilateral Deviations for Alternative Distributions of Initial Resource Ownership and Smaller Gains from Trade

B Online Appendix: The Stability of Settlement with Asymmetric Aggregate Productivities

In what follows, we provide brief notes on the extended version of the model mentioned in Section 5 of the text that allows the two countries to differ with respect to their aggregate productivities—i.e., their abilities to transform their *ex post* resource into their respective intermediate goods. Specifically, under settlement, country i holds $\phi^i \bar{X}$ units of the resource, where ϕ^i specified in (1) of the main text indicates country i's share of the common pool resource $\bar{X} = \bar{R} - \bar{G}$ secured in the negotiation, and then produces $Z^i = A^i \phi^i \bar{X}$ units of its intermediary input for $i = 1, 2$.[1] After characterizing how arming incentives for each country i depend on both A^i and A^j under settlement and trade, we establish that this additional source of asymmetry leaves unchanged our finding that peaceful settlement can be unstable when the international distribution of resources is sufficiently even such that neither country is resource constrained in its arming choices. More interestingly, the analysis suggests that this additional asymmetry serves as an independent source of instability.

As in the baseline version of the trade model where $A^1 = A^2 = 1$, we assume inputs are transformed into goods for final consumption according to a standard CES technology, as shown in (2) of the main text, with $\theta = (\sigma - 1)/\sigma \in (0, 1]$ that positively reflects the constant elasticity of substitution $\sigma \in (1, \infty)$. Furthermore, we assume this production function is identical across countries. As previously defined, p^i_j denotes country i's domestic price for input $j = 1, 2$, and π^j_j $(= p^j_j)$ denotes its world price. Then, country i's income that comes from the sale of the intermediate input it produces equals $Y^i = p^i_i Z^i = \pi_i Z^i$. Furthermore, abstracting from trade costs, the world and domestic prices coincide, such that $p^j_i = p^i_i = \pi_i$.[2]

With a maintained focus on free trade, we follow the logic spelled out in the text to find the equilibrium world price of country i's imported good in units of the good it produces:

$$\pi^i_T = \frac{\pi_j}{\pi_i} = \left(\frac{Z^i}{Z^j}\right)^{\frac{1}{\sigma}}, \; i, j \in \{1, 2\}, j \neq i, \tag{B.1}$$

which is precisely the expression for the equilibrium relative world price we found in the baseline model. However, in this context, we have $Z^i = A^i \phi^i \bar{X}$. Using that specification and the definition of ϕ^i in (1) one can find $\pi^i_T = \left(A^i \phi^i / A^j \phi^j\right)^{\frac{1}{\sigma}} = \left(A^i G^i / A^j G^j\right)^{\frac{1}{\sigma}}$, which shows the importance of relative aggregate productivities, A^i/A^j, and of relative aggregate arming choices, G^i/G^j.

To proceed, we now turn to one-period payoffs under free trade. As in the baseline model, country i's indirect utility can be written as $w^i = Y^i/P^i$, where as previously defined in the

[1] As described below in section B.2.2, production is similarly governed by the aggregate productivity parameters under open conflict and autarky.

[2] One could allow for trade costs, τ^i_j (≥ 1), which represents the number of units of input j that must be shipped from country j to country i. Assuming the same technology across countries for producing the good for final consumption (2), the trading equilibrium requires $p^i_j = \tau^i_j \pi_j$ for $i, j \in \{1, 2\}$, $j \neq i$. To keep our analysis focused, however, we assume free trade (or $\tau^i_j = 1$).

text P^i denotes country i's price index that is given by $P^i = [\sum_{j=1}^{2}(p_j^i)^{1-\sigma}]^{1/(1-\sigma)}$. We can rewrite country i's indirect utility as follows:

$$w^i = \frac{p_i^i Z^i}{\left[\sum_{j=1}^{2}(p_j^i)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} = \frac{A^i \phi^i \bar{X}}{\left[\sum_{j=1}^{2}(\pi_j/\pi_i)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} = \frac{A^i \phi^i \bar{X}}{\left[1+(\pi_T^i)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}},$$

The expressions above reveal that the effects of various shocks on country i's payoff are transmitted through its normalized price index P^i/π_i, evaluated at the equilibrium relative prices of the tradable intermediate inputs, and its production of the intermediate input Z^i.

To explore more carefully how the technology parameters matter in this context, we use (B.1) in the above expression for w^i to obtain country i's per period (or average discounted), indirect payoff under settlement with free trade ($v^i = V^i$) for $i = 1, 2$:

$$v^i = \frac{A^i \phi^i \bar{X}}{\left[\sum_{j=1}^{2}\left(\frac{Z^i}{Z^j}\right)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{1-\sigma}}} = \frac{A^i \phi^i \bar{X}}{\left[\sum_{j=1}^{2}\left(\frac{A^i\phi^i}{A^j\phi^j}\right)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{1-\sigma}}}$$

Recalling the definition $\theta \equiv (\sigma-1)/\sigma$, further manipulation of the above shows

$$v^i = \frac{(A^i\phi^i)^\theta}{\sum_{j=1}^{2}(A^j\phi^j)^\theta}\left[\sum_{j=1}^{2}\left(A^j\phi^j\right)^\theta\right]^{1/\theta}\bar{X}.$$

Now, let us define $\omega^i = \gamma^i Q$ where

$$\gamma^i = \left(A^i\phi^i\right)^\theta / \left[\sum_{j=1}^{2}\left(A^j\phi^j\right)^\theta\right], \tag{B.2a}$$

$$Q = \left[\sum_{j=1}^{2}\left(A^j\phi^j\right)^\theta\right]^{1/\theta}. \tag{B.2b}$$

Then, we can rewrite the last expression above for v^i more compactly as

$$v^i = \omega^i \bar{X}, \;\; i \in \{1, 2\}. \tag{B.3}$$

As in the baseline model, ω^i represents country i's per unit payoff in terms of the common pool of resource after arming decisions, $\bar{X}$. ω^i depends positively on the country's expenditure share on the input it produces γ^i and positively on Q, which can be thought of as the per unit transformation of the common pool resource into goods for final consumption, with $\gamma^i = \frac{\partial Q/\partial A^i}{Q/A^i}$. Noting that $\phi^j/\phi^i = G^j/G^i$, we may also write v^i in (B.3) as

$$v^i = A^i\phi^i\bar{X}\left[\sum_{j=1}^{2}\left(\frac{A^jG^j}{A^iG^i}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}, \;\; i \in \{1, 2\}. \tag{B.4}$$

The expression above reveals that an increase in country i's productivity (A^i), holding G^i and G^j fixed, has two effects. First, it amplifies the value of the resource secured by the country ($\phi^i\bar{X}$). Second, it worsens country i's terms of trade as reflected in the expression in the square brackets. From the expressions in (B.2) that show $\partial\gamma^i/\partial A^i > 0$ and $\partial Q/\partial A^i > 0$, it follows that the combined effect of an increase in A^i on v^i is positive: $\partial v^i/\partial A^i > 0$. The expression in (B.4) also shows that an increase in A^i includes only an improvement in its terms of trade for country $j \neq i$, implying $\partial v^j/\partial A^i > 0$. Of course, this discussion of the payoff effects of a technological improvement is incomplete as it has kept guns choices fixed in the background.

B.1 Impact of Technology on Equilibrium Guns Choices under Free Trade

We now turn to the optimizing guns choices under settlement and trade and study how they depend on the productivity parameters A^i for $i = 1, 2$. Using (B.4) along with the definition of γ^i in (B.2a), we can write country i's FOC for arming at an interior solution as follows:

$$v^i_{G^i} = v^i \left[\frac{\phi^i_{G^i}}{\phi^i} - \frac{1}{\bar{X}} - \frac{1-\gamma^i}{\sigma G^i} \right] = 0, \ i \in \{1, 2\}. \tag{B.5}$$

The first term inside the square brackets is the marginal benefit of arming that reflects the effect of a unit increase in G^i (given G^j) to increase country i's share of the common pool resource $\bar{X}$. The second term reflects the opportunity cost of arming as resources are diverted away from producing Z^i. The third term reflects the marginal price effect of a unit increase in i's arming as it induces a deterioration of the country's terms of trade that can be viewed as an extra opportunity cost. An increase in A^i that would induce a smaller expenditure share on imports (i.e., $\gamma^i \uparrow$) reduces this opportunity cost with no direct effects on the other terms in the brackets.

Applying the specification for the conflict technology (1) to (B.5) shows that, at an interior optimum for $i = 1, 2$, $G^i = (\sigma\phi^j - \gamma^j)\bar{X}/\sigma$ $(i \neq j)$, which requires $\sigma\phi^j - \gamma^j \geq 0$. Aggregating across countries gives $\bar{G}_s = G^i_s + G^j_s = (1 - 1/\sigma)\,\bar{X} = \theta\bar{X}$. Since $\bar{X} = \bar{R} - \bar{G}$, it follows that $\bar{G}_s = \frac{\theta}{1+\theta}\bar{R}$, which is increasing in the aggregate resource $\bar{R}$ and equal to the value of $\bar{G}_s$ obtained in the baseline model, implying that $\bar{X}_s = \frac{1}{1+\theta}\bar{R}$. Clearly, then, the aggregate quantity of guns, while positively related to $\bar{R}$, is independent of the technology parameters A^i for $i = 1, 2$. That is not to say, however, the technology has no influence on arming choices.

To characterize the influence of A^i at an interior equilibrium, we first differentiate the system of equations in (B.5) appropriately to find

$$\begin{aligned}
v^i_{G^iG^i} &= -\frac{1}{\bar{X}^2} - \frac{\phi^j}{(G^i)^2} - \frac{\phi^i\phi^j}{(G^i)^2} + \frac{\gamma^j}{\sigma\,(G^i)^2} + \frac{(\sigma-1)\,\gamma^i\gamma^j}{\sigma^2\,(G^i)^2} < 0 \\
v^i_{G^iG^j} &= -\frac{1}{X^2} + \frac{\phi^i\phi^j}{G^iG^j} - \frac{(\sigma-1)\,\gamma^i\gamma^j}{\sigma^2 G^iG^j} > 0 \\
v^i_{G^iA^i} &= \frac{(\sigma-1)\gamma^i\gamma^j}{\sigma^2 A^iG^i} > 0 \\
v^j_{G^jA^i} &= -\frac{(\sigma-1)\gamma^i\gamma^j}{\sigma^2 A^iG^j} < 0.
\end{aligned}$$

The last two expressions imply $G^i v^j_{G^j A^i} = -G^j v^i_{G^i A^i} > 0$, with the inequality following from our assumption that $\sigma > 1$. Turning to the inequality in the second line, observe that $v^i_{G^i G^j} = v^j_{G^j G^i}$. To sign that expression we use the implication of the FOCs noted above, that $G^i = \left(\sigma\phi^j - \gamma^j\right)\bar{X}/\sigma$ $(i \neq j)$, to rewrite it as

$$v^i_{G^i G^j} = \frac{\phi^j\gamma^i + \phi^i\gamma^j - \gamma^i\gamma^j}{\sigma G^i G^j} = \frac{\phi^i\phi^j + \left(\phi^i - \gamma^i\right)^2}{\sigma G^i G^j} > 0,\ i \neq j.$$

If $v^i_{G^i G^i} < 0$, then the inequality immediately above implies G^i and G^j are strategic complements. Finally, to confirm that $v^i_{G^i G^i} < 0$ holds, observe that

$$\begin{aligned} G^i v^i_{G^i G^i} &= \left(G^i v^i_{G^i G^i} + G^j v^i_{G^i G^j}\right) - G^j v^i_{G^i G^j} \\ &= -\left(\frac{\bar{X} + \bar{G}}{\bar{X}^2}\right) - G^j v^i_{G^i G^j} = -\left(\frac{\bar{R}}{\bar{X}^2}\right) - G^j v^i_{G^i G^j} < 0. \end{aligned}$$

Thus, the payoff functions are quasi-concave and an equilibrium in arming exists.

Next, note that the following determinant is positive:

$$\begin{aligned} D &\equiv G^i v^i_{G^i G^i} G^j v^j_{G^j G^j} - G^j v^i_{G^i G^j} G^i v^j_{G^j G^i} = \\ &= \left(\frac{\bar{R}}{\bar{X}^2} + G^j v^i_{G^i G^j}\right)\left(\frac{\bar{R}}{\bar{X}^2} + G^i v^j_{G^j G^i}\right) - G^j v^i_{G^i G^j} G^i v^j_{G^j G^i} > 0 \\ &= \frac{\bar{R}}{G^i G^j \sigma^2 \bar{X}^3}\left(\sigma^2 G^i G^j + \bar{G}\bar{X}\left[\sigma^2\phi^i\phi^j - (\sigma - 1)\gamma^i\gamma^j\right]\right) > 0. \end{aligned}$$

The last line is useful for subsequent calculations. Although the expression in the square brackets hints at the presence of a possible ambiguity, it is possible to show that the expression is positive.[3] The second line shows clearly that $D > 0$ such that, with the preceding analysis, it ensures uniqueness of the interior equilibrium in which arms are strategic complements. Perhaps more importantly for our current purposes, the above paves the way for simple comparative statics in A^i.[4]

Let a hat "$\widehat{\ }$" denote percent change. Now consider the following system of equations:

$$\begin{aligned} G^i v^i_{G^i G^i}\widehat{G}^i + G^j v^i_{G^i G^j}\widehat{G}^j + A^i v^i_{G^i A^i}\widehat{A}^i &= 0 \\ G^i v^j_{G^j G^i}\widehat{G}^i + G^j v^j_{G^j G^j}\widehat{G}^j + A^i v^j_{G^j A^i}\widehat{A}^i &= 0. \end{aligned}$$

The solution to the above system is given by

$$G^j_s \widehat{G}^j_s / \widehat{A}^i = -G^i_s \widehat{G}^i_s / \widehat{A}^i = -\frac{G^i G^j \bar{X}\left(\sigma - 1\right)\gamma^i\gamma^j}{\sigma^2 G^i G^j + \bar{G}\bar{X}\left[\sigma^2\phi^i\phi^j - (\sigma - 1)\gamma^i\gamma^j\right]} < 0. \tag{B.6}$$

Based on this equation and the proceeding analysis, we can now characterize the equilib-

[3] Specifically, one can use (B.10) below to eliminate γ^i from the expression and show that it is positive.

[4] As in the baseline model, a sufficiently uneven distribution of initial resources (R^i and R^j) could imply that one country is resource constrained. However, to highlight the importance of asymmetric aggregate productivies on the feasibility of peaceful settlement, we abstract from that possibility here.

rium in arming under settlement (and free trade) and show how it differs from that in the baseline model. As noted earlier, under the maintained assumption that neither country is resource constrained in arming, aggregate arming in this setting is independent of aggregate productivities, A^i for $i = 1, 2$; furthermore, while aggregate arming depends on the aggregate resource endowment, the international distribution of $\bar{R}$ is inconsequential for the countries' individual arming choices as long as the resource constraint for arming remains unbinding. Now suppose that $A^i = A^j$, which implies $G_s^i = G_s^j$ as in the baseline model. According to (B.6), an exogenous increase in A^i would cause country i's arming to rise and country j's arming to fall, leaving $\bar{G}$ unchanged: $dG_s^i/dA^i = -dG_s^j/dA^i > 0$. Thus, in contrast to the baseline model, arming choices will differ even when neither country is resource constrained, with the more productive country also being the more powerful.

B.2 Impact of Technology on Payoffs and the Stability of Peaceful Settlement

We now turn to the implications of an exogenous change in one country i's productivity A^i on the prospects for peace. To do so we need to identify the effect on the average discounted payoff under peaceful settlement $V_s^i = v_s^i$ relative to the payoff under a unilateral deviation W_d^i as well under conflict U_c^i. Our starting point is where initially $A^i = A^j$, neither country is resource constrained, and neither country has an incentive to deviate unilaterally from peaceful settlement.

B.2.1 Payoff Effects Under Peaceful Settlement and Free Trade

Differentiation of the payoff function in (B.4) gives

$$\widehat{V}^i = \widehat{v}^i = \widehat{A}^i - \left(\frac{\gamma^j}{\sigma}\right)\widehat{A}^i + G^j\left(-\frac{1}{\bar{X}} - \frac{\phi^j}{G^j} + \frac{\gamma^j}{\sigma G^j}\right)\widehat{G}_s^j, \ \ i \neq j = 1, 2. \tag{B.7}$$

The above expression is consistent with our earlier discussion of the impact of A^i on v^i. In particular, the first and second terms capture the direct and terms-of-trade effects of A^i on v^i, respectively. The last term captures the strategic effect of G^j that is brought about by the increase in A^i. (Once again, keep in mind that this effect will be present only if the resource constraint on j's arming is not binding.)

At an interior solution, this strategic effect contains conflicting terms that travel through $\bar{X}$ (the first and negative term in the parentheses), through the share ϕ^i (the second and negative term) and the world price (the last and positive term). To sort out the sign of this term, we use country j's FOC from (B.5), which implies $-\frac{1}{\bar{X}} - \frac{\phi^j}{G^j} = \frac{\gamma^i}{\sigma G^j} - \frac{1}{G^j}$, to rewrite (B.7) as

$$\widehat{V}^i = \widehat{A}^i - \left(\frac{\gamma^j}{\sigma}\right)\widehat{A}^i - \left(\frac{\sigma - 1}{\sigma}\right)\overset{(-)}{\left(\widehat{G}_s^j/\widehat{A}^i\right)}\widehat{A}^i, \ \ i \neq j = 1, 2. \tag{B.8}$$

It follows from the above that strategic effect of increasing A^i on V^i is positive. Thus, in light of the fact that the sum of the first and second terms is positive, we can unambiguously state that V_s^i is increasing in A^i. However, as suggested above and as shown more formally below, the payoff under conflict is also increasing in A^i: $\widehat{U}_c = \widehat{A}^i$. Thus, to be able to compare $\widehat{V}_s^i$ and $\widehat{U}_c^i$ we must identify the sign of the sum of the terms-of-trade effect (second term in (B.8)) and the strategic effect (third term in (B.8)).

Let us define the following that combines the second and third terms in (B.8)):

$$\Psi^i \equiv -\frac{\gamma^j}{\sigma} - \left(\frac{\sigma-1}{\sigma}\right)\overset{(-)}{\left(\widehat{G}_s^j/\widehat{A}^i\right)}. \tag{B.9}$$

We proceed to demonstrate that $\Psi^i < 0$. Substituting the comparative static effect on $\widehat{G}_s^j$ from (B.6) in Ψ^i above allows us to rewrite it as

$$\Psi^i = \frac{\gamma^j}{\sigma}\left(-1 + \frac{G^i\bar{X}\left(\sigma-1\right)^2\gamma^i}{\sigma^2 G^i G^j + \bar{G}\bar{X}\left[\sigma^2\phi^i\phi^j - (\sigma-1)\gamma^i\gamma^j\right]}\right).$$

Now recall that $G^i = \left(\sigma\phi^j - \gamma^j\right)\bar{X}/\sigma$ at the interior equilibrium. Substituting this value in Ψ^i above allows us to rewrite it solely as a function of σ, ϕ^i, and γ^i:

$$\Psi^i = \frac{\gamma^j}{\sigma}\left(-1 + \frac{\left(\sigma\phi^j - \gamma^j\right)\left(\sigma-1\right)^2\gamma^i}{\sigma\left(\sigma\phi^i - \gamma^i\right)\left(\sigma\phi^j - \gamma^j\right) + (\sigma-1)\left[\sigma^2\phi^i\phi^j - (\sigma-1)\gamma^i\gamma^j\right]}\right).$$

Next, the values of G^i and G^j obtained from the FOCs can help define the equilibrium value of ϕ^i implicitly:

$$\frac{G^i}{G^j} = \frac{\phi^i}{\phi^j} = \frac{\sigma\phi^j - \gamma^j}{\sigma\phi^i - \gamma^i} \quad\Longrightarrow\quad \gamma^i = \phi^j + \sigma(\phi^i - \phi^j). \tag{B.10}$$

Using (B.10), we can eliminate γ^i in Ψ^i and simplify it as follows:

$$\Psi^i = -\frac{\gamma^j\sigma(\sigma-1)\phi^j\left[\sigma\phi^i + (\sigma-1)^2\left(1-2\phi^i\right)\right]}{\left[2\sigma(\sigma-1)+1\right]\phi^i\phi^j + \sigma(\sigma-1)^2(\phi^i-\phi^j)^2}.$$

So, it all boils down to the sign of $\Psi_0^i \equiv \sigma\phi^i + (\sigma-1)^2\left(1-2\phi^i\right)$, which is linear in ϕ^i.[5]

To sign Ψ_0^i, we use the requirement that $0 < \gamma^i < 1$ that, with (B.10), enables us to identify the lower and upper bounds for ϕ^i. It is easy to verify that these bounds are $\underline{\phi}^i \equiv \frac{\sigma-1}{2\sigma-1}$ and $\bar{\phi}^i \equiv \frac{\sigma}{2\sigma-1}$, respectively. Then, evaluating Ψ_0^i at these bounds of ϕ^i gives: (i) $\Psi_0^i(\underline{\phi}^i) = \sigma - 1 > 0$, and (ii) $\Psi_0^i(\bar{\phi}^i) = 1 > 0$. It is now easy to see that $\Psi_0^i > 0$ (and thus $\Psi^i < 0$) for all $\phi^i \in (\underline{\phi}^i, \bar{\phi}^i)$. In short, the adverse terms-of-trade payoff effect of an increase in A^i dominates the positive strategic payoff effect. Hence, although $\widehat{V}_s^i > 0$, $\widehat{V}_s^i < \widehat{A}^i$ holds.

B.2.2 Payoff Effects under Conflict and under Unilateral Deviations

Having established the dependence of payoffs under settlement and trade on A^i, we now turn to the payoffs under conflict and autarky. Under autarky, equation (2) in the main text with

[5] As can be verified, $\partial\Psi_0^i/\partial\phi^i \gtreqless 0$ as $2 \gtreqless \sigma$.

$D_j^i = 0$, implies that country i, if victorious in the first-period conflict which occurs with probability ϕ^i, would produce $\beta A^i \bar{X}$ in that period and $\beta A^i \bar{R}$ in the next period, where as defined in the main text $\beta \in (0, 1]$ represents the survival rate of resources in the event of conflict. Thus, we can write country i's average discounted payoffs, where $\delta \in (0, 1]$ represents the countries' common discount factor, as

$$U^i = \frac{\beta}{1+\delta} A^i \phi^i \left(\bar{X} + \delta \bar{R}\right), \ i \in \{1, 2\}. \tag{B.11}$$

Inspection of U^i reveals that, since there are no international technology spillovers under conflict, country i's equilibrium arming choice G_c^i will be independent of A^i and A^j for $i \in \{1, 2\}$ $i \neq j$. As such, $\widehat{U}_c^i = \widehat{A}^i$, as noted earlier. Clearly, then, an increase in A^i reduces the payoff under settlement relative to that under conflict and autarky for country i.

However, for our purposes, we must compare the effects of a change in technology on the payoff under settlement with trade relative to the effects on payoffs under an optimizing unilateral deviation, $W_d^i(G_d^i, G_s^j) = U^i(G_d^i, G_s^j)$. What's more, since G_s^j depends on the technology parameter A^i, so will the optimizing deviation by country i and its resulting payoffs. Using (B.11), one can verify that relevant change in that payoff is:

$$\widehat{W}_d^i = \widehat{A}^i + \frac{G_s^j \phi_{G^j}^i}{\phi^i} \widehat{G}_s^j - \left(\frac{G_s^j}{\bar{X} + \delta \bar{R}}\right) \widehat{G}_s^j.$$

The first term represents the direct payoff effect of an increase in A^i, whereas the second and third terms show the strategic payoff effects induced by a change in G_s^j. Specifically, the second term captures the effect that travels through the conflict technology ϕ^i, and the third term captures the effect that travels through the common pool resource in the first period. Since $\widehat{G}_s^j < 0$ and $\phi_{G^j}^i < 0$, both terms are positive, implying that the effect on the deviation payoff is greater than that on the payoff under conflict: $\widehat{W}_d^i > \widehat{U}_c^i = \widehat{A}^i$. More to the point of this appendix, we have $\widehat{W}_d^i > \widehat{A}^i > \widehat{V}_s^i$. Thus, the introduction of an additional source of asymmetry—namely, differences in aggregate productivity—does not reverse but instead reinforces the finding in the baseline model that armed peace can be unstable for relatively even distributions of initial resource endowments. This analysis also suggests that differences in aggregate productivities across countries can serve as a distinct source of instability in international relations.[6]

[6] We could also calculate the effects of an increase in A^i on the relevant payoffs for country j. But, this is not necessary here.