Prudence versus Predation and the Gains from Trade*

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Abstract. We analyze a dynamic, two-country model that highlights the various trade-offs each country faces between current consumption and competing investments in its future productive and military capacities as it prepares for a possible conflict in the future. Our focus is on the circumstances under which the effects of current trade between the two countries on the future balance of power render trade unappealing to one of them. We find that a positive probability of future conflict induces the country with less resource wealth to "prey" on the relatively more "prudent" behavior of its larger rival, and more so as conflict becomes more likely. While a shift from autarky to trade always raises the current incomes of both countries, the smaller country realizes the relatively larger income gain from trade and also devotes a relatively larger share of its income gain towards arming. Our analysis shows that the larger country rationally chooses not to trade today when the difference in initial resource wealth is sufficiently large and is more likely to prefer autarky when the probability of future conflict is higher.

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1 Introduction

Despite a resurgence in global tensions in recent years, we live in an era of unprecedented peace between nations. Seventy years have gone by without a repeat of the major conflicts of the early twentieth century; and, since the end of the Cold War, the number of inter-state conflicts has steadily declined to practically zero. The expansion of international trade since World War II, in particular, is credited with ensuring a more secure global order by raising the economic costs of war (Polachek, 1980; Martin et al., 2008; Glick and Taylor, 2010). Nonetheless, global expenditure on defense in 2018 was roughly \$1.8 trillion, a 76% increase in real terms from the post-Cold War lows of the late-1990s. Aside from the West's recent interventions in the Middle East, a major driver of this trend is the rapid expansion of defense spending by emerging economies that have become more integrated into the world trading system in the past few decades.[1] Though the growth of these economies due to trade has been welcome on the whole, the accompanying trend in their defense spending underscores the basic point that wealth created by trade is wealth that can be used to invest in one's military capacity.

Economists have recently begun to expand the scope of international trade theory to explore the importance of arming and conflict (see, e.g., Skaperdas and Syropoulos (2001); Garfinkel et al. (2020) discussed below). These theories, however, are silent on why we continue to observe increases in arming in a seemingly peaceful and continually globalizing world. Furthermore, while "realist" security scholars have long expressed the view that the standard "gains from trade" could be outweighed by the negative consequences for a country's future security if these gains are asymmetrically distributed, trade theory has not examined this argument specifically.[2] This oversight diminishes our understanding of why and when nations expand trade with one another. Even in the present day, policymakers openly regard trade policy as an instrument for achieving security-related goals.[3]

In this paper, with an aim to address these and related issues, we present a simple dynamic theory of trade, investment, and arming, focusing on two countries. Central to both

[1] Data from SIPRI (2019), available at www.sipri.org, show China (212%), Vietnam (188%), Russia (77%) and the former communist nations of Eastern Europe (collectively, 81%) have each increased their defense spending tremendously since 2005 alone, continuing long term trends that soon followed the liberalization of their economies during the 1990s. Recent upward trends are also common across Africa, Asia, and Central and South America.

[2] Scholars writing in the "realist" tradition generally treat dependence on trade with other nations as a source of diminished security, especially if the gains from trade are uneven (see Waltz, 1979; Gilpin, 1981; Grieco, 1990). The opposing "liberal" view argues that the efficiency gains from trade should raise the opportunity cost of war (see Polachek, 1980; Martin et al., 2008). A third view presented in Copeland (2015) combines elements of both views, positing that wars can arise because of uncertainty over future trade (see also Bonfatti and O'Rourke, 2018; Morelli and Sonno, 2017). As will become clearer shortly, the main focus of our analysis in this paper is on the realist view.

[3] For example, in a speech in 2015, then-U.S. Secretary of Defense Ash Carter stated, "you may not expect to hear this from a Secretary of Defense, but in terms of a rebalance in the broadest sense, passing TPP [the Trans-Pacific Partnership] is as important to me as another aircraft carrier" (Carter, 2015).

our motivation and our analysis is the conceit that, while peace prevails in the present, the two countries make these decisions in the shadow of a future conflict that emerges with some known, positive probability. This setting naturally delivers motivations for costly arming in the midst of an ongoing peace—namely, as necessary (and/or opportunistic) preparations made for an uncertain future. More importantly, the nature of the interaction between the two countries, with each having to balance how its decisions today will affect outcomes under both peace and conflict tomorrow, allows us to examine how differences in the initial distribution of resources translate into differences in military power and, ultimately, in preferences toward trade. In doing so, the analysis yields a clear prediction regarding the effectiveness and credibility of security-based arguments for trade restrictions: a sufficiently high threat of conflict can eliminate a larger country's incentive to trade with a smaller rival, but only if the difference in *ex ante* economic size between the two countries is sufficiently large; otherwise, trade in the present remains the best policy for both countries, despite the possibility of future conflict.

Our theory highlights two types of trade-offs that countries face in the midst of an ongoing rivalry. First, both countries must invest some of their current resources to provide for future consumption—a standard "*inter*-temporal" trade-off. Second, they face an "*intra*-temporal" trade-off between two types of investment that support future consumption in distinct ways: "saving", which yields resources for future consumption, and "arming", which determines how these resources will be divided in the event of a future conflict. Of course, countries with larger initial resource endowments are in a better position to satisfy their current consumption needs. As expected, then, a relatively larger country chooses higher levels of arming and saving in equilibrium than its smaller rival. However, the degree to which it enjoys an arming advantage does not depend simply on the difference in endowment sizes (as might be presumed), but rather on how such differences and the likelihood of conflict jointly shape each country's strategic incentives for both arming and saving. In equilibrium, the *ex ante* smaller country allocates a relatively smaller share of its income to saving and a relatively larger share to arming compared with its larger counterpart. Thus, a strictly positive probability of future conflict enables the smaller country to "prey" on the more "prudent" behavior of its counterpart, thereby making it disproportionately powerful as compared with its initial size.

The relevance of trade in this setting derives from its effects on the current income of the two countries. As is true for most static trade models without security concerns—and as we will illustrate using a simple "Armington" example—trade does not reduce the productive efficiency of either country and usually generates real income gains in absolute terms for both. At the same time, we find the intuitive result that the smaller country always gains more from trade than the larger country relative to its initial size. Though unequal distributions of the gains from trade would not ordinarily prevent trade from taking place, our dynamic setting where future security concerns matter highlights a clear set of circumstances under which a larger country will find trade in the present relatively unappealing. More precisely, as the initial size difference between the two countries increases, the larger country's gains from trade become smaller, the smaller country's gains from trade become larger, and—because of how the possibility of conflict affects relative arming choices versus relative saving choices—the smaller country increasingly allocates a larger portion of its income gain towards arming versus saving. To be sure, the presence of dual strategic interactions in both arming and saving makes characterizing the effects of trade in this setting quite complex (and potentially ambiguous). Nonetheless, it is always possible to identify a sufficiently unequal distribution of initial endowments beyond which the larger country prefers autarky to trade.[4] Numerical analysis further clarifies that the larger country's preference for autarky tends to emerge for a wider range of relative endowment sizes when the probability of conflict is higher.[5]

The history of 20th century military rivalries offers numerous episodes that can be related to the tradeoffs highlighted in our theory. These episodes include the steady continuation of trade between Germany and Great Britain (as well as between Germany and its other rivals) in the lead up to World War I, the U.S.'s progressive tightening of economic sanctions against Japan before its entry into World War II, and the U.S.'s aggressive containment policies towards the Soviet Union at the beginning of the Cold War. Though we do not intend to position these historical rivalries as proving our model or being fully illustrated by our model, they share several features that our model can explain. In particular, we can document in each instance how decisions whether to restrict trade reflected discussions surrounding the severity of the threat, the economic losses from restricting trade, and the need to build up arms to maintain the balance of power. The relevant details of these episodes will be explained further in Section 2.

Our choice to abstract from the possibility that either country can take actions to try to influence the likelihood of conflict is driven by our perspective in relation to the literature. Theories underlining the "liberal peace" argument that trade can serve as a deterrent to war (e.g., Martin et al., 2008) typically examine how the decision to go to war is affected by exogenous changes in the trade regime.[6] Our approach pursues a kind of converse: even

[4] Remarkably, as shown in the Online Appendix, both this result and the result that the relative gains from trade are inversely related to relative sizes hold not only in our simple Armington setting, but also in many other, more complex trade environments that feature trade costs, incomplete specialization, multiple factors of production, increasing returns and/or heterogeneous firms. To our knowledge, the relationships that we demonstrate between relative sizes and relative gains from trade (independent of the presence of possible conflict) are new results in trade theory.

[5] Our finding that one country might choose not to trade in the present resembles Gonzalez (2005)'s finding that agents might adopt inefficient technologies to discourage future aggression by rivals. Though Gonzalez (2005) also relies on a dynamic model, his work differs from ours in that disincentives for technology adoption arise from how technology adoption shapes future (contested) output, whereas disincentives for trade in our analysis arise from how trade shapes current (secure) output.

[6] The "liberal peace" hypothesis finds some support in the empirical literature—see the recent survey by

acknowledging that trade can be useful for ensuring peace, it is also worth investigating why and when peace could be necessary for ensuring trade.

In shedding light on these issues, our analysis builds on and synthesizes several disparate strands of the relevant literature. The first of these is the "relative gains" argument for restricting economic cooperation articulated by the "realist" school of international relations. In this tradition, as in our model, "cooperation that creates and distributes wealth affects security as well as welfare" (Liberman, 1996); thus, countries concerned about both security and welfare must be strategic in choosing with whom they cooperate and when. Our own formalization of this idea is related to the contributions of Powell (1991) and Gowa (1995) in that we incorporate the linkages between changes in relative wealth, future security threats, and expected payoffs in a unified game-theoretic framework. In this context, a key distinguishing feature of our analysis is that we explicitly model the endogenous relationships between trade, relative wealth, and relative power as well as the conditions under which these relationships could be stronger or weaker.

Second, we share with Skaperdas and Syropoulos (2001), Dal Bó and Dal Bó (2011), Garfinkel et al. (2008, 2015), and Garfinkel et al. (2020) an interest in how trade affects incentives for arming relative tobother, more productive activities. The first four papers focus on a factor-price channel that can render trade by countries involved in conflict unappealing even when the gains from trade are evenly distributed across adversaries. Garfinkel et al. (2020) find, as we do, that trade between adversaries can be relatively unappealing to one country in the presence of sharp resource asymmetries; however, there the mechanism is a terms-of-trade channel.[7] In any case, with an emphasis on how the anticipation of trade influences arming incentives, neither of these two approaches captures the possibility that increases in national income due to trade have direct implications for military spending, as is apparent from the arming statistics cited above. Our analysis, by contrast, centers on an income channel that is more directly relevant for understanding why relative gains from trade might matter.

Third, our model gives rise to a variant of the weak form of Hirshleifer (1991)'s concept

Morelli and Sonno (2017). However, this literature has also highlighted some interesting exceptions relevant to our analysis. Most notably, Hegre (2004) and Morelli and Sonno (2017) respectively find that the peace-promoting effects of trade could be conditional on asymmetries in trade dependence and on asymmetries in resource wealth. In other related work, Seitz et al. (2015) show that, if trade lowers the probability of conflict, the resulting reduction in the need for defense spending can generate substantial economic benefits on top of the usual gains from trade.

[7] Skaperdas and Syropoulos (2001), Dal Bó and Dal Bó (2011), and Garfinkel et al. (2008, 2015) each study extended Hecksher-Ohlin settings with small countries that trade with the rest of the world where changes in relative world prices induce changes in relative factor prices of capital and labor that in turn influence the relative costs of arming versus producing useful output. Garfinkel et al. (2020) analyze a modified Ricardian model of trade between large adversarial countries, intentionally abstracting from both factor-price and income channels to isolate the importance of a terms-of-trade channel that more often induces both countries to reduce their arms production as they internalize the negative price externality; the resulting payoff effect reinforces the traditional gains from trade.

of the “paradox of power”, which states that players with fewer resources devote disproportionately more resources to appropriative activities because they have less to lose and more to gain from distributional conflicts than their larger rivals. While it would be reasonable to conjecture that the presence of this paradox implies a more even distribution of income could be welfare improving for the larger player, Hirshleifer (1991) does not consider this possibility. Our analysis contributes to this line of inquiry by showing that exogenous increases in the size of the smaller player can make both players better off, but only if the initial distribution of resources is sufficiently even. A similar finding also holds for trade. However, because trade has discrete, as opposed to continuous, effects on incomes, our proof of the latter result requires a very different strategy that involves differentiating payoffs under both autarky and trade in the neighborhood of an infinitesimal trading partner. This approach represents a methodological contribution that could also be applied to other settings where trade with a smaller country generates negative externalities, such as via environmental damage or intellectual property theft.

The rest of the paper is organized as follows. The next section provides a discussion of historical military rivalries that evoke the tradeoffs highlighted in our theory. Section 3 describes the basic elements of our model. In Section 4, we then characterize how equilibrium arming, savings, and payoffs respond to (exogenous) changes in first-period incomes. Section 5, closes the model by allowing first-period incomes to be determined by the trade regime and examine each country’s preferences towards trade. We also consider a number of extensions that speak to the robustness of our central theoretical findings. Section 6 concludes. All technical details are provided in the Online Appendix.

2 Historical Examples: Rivalries and Trade

In this section, we review three historical military rivalries that illustrate the tradeoffs that our theoretical analysis aims to highlight: Great Britain and Germany before World War I, the U.S. and Japan before the U.S.’s entry into World War II, and the U.S. and the Soviet Union in World War II’s immediate aftermath. While we would not go so far as to claim that our analysis offers a comprehensive explanation of any of these episodes, our review of them reveals the salience of several key elements found in our theory. In particular, we will document how the choices that countries in these scenarios made (of whether or not to restrict trade) reflected the degree to which their economies benefited from trade with their rivals. The Great Britain/Germany rivalry is useful to focus on first because it shows how countries might continue to allow trade despite their awareness of a growing security threat, whereas the latter two rivalries illustrate cases where the risk of a future conflict with an economically dependent rival induces one country to cut off trade.

Great Britain/Germany. In the decades preceding World War I, Britain increasingly perceived Germany’s naval buildup as a threat to its power and security. One of the interesting elements about this period of Anglo-German relations for our purposes is the role that bilateral trade played in their arms race. As described by Kennedy (1980), the rapid industrialization in Germany that fueled its naval expansion was fed by imports for raw materials and food from Britain's colonies and was financed by London-based banks. At the same time, however, British shipbuilders relied on German sheet metal to build their ships, some of which were exported back to Germany. The British military also benefited from German imports of pig iron, optical equipment, precision tools, automobiles, and even khaki dye for uniforms (Kennedy, 1980; Liberman, 1996). Though Britain remained the world's largest exporter, it had also become the largest importer of iron and steel, with much of it coming from fast-growing Germany. Germany, in turn, had become Britain's second largest export market (Steiner, 1977).[8]

In view of this economic interdependence, it is understandable why Britain continued to trade with Germany even as it had explicitly begun to prepare for a possible war in 1912.[9] To be sure, there had been ample domestic pressure for restrictions on trade via the Tariff Reform movement of Joseph Chamberlain. But, ultimately, this pressure was resisted; as contextualized by Liberman (1996), Britain's government believed that it benefited substantially in absolute terms, if not in relative terms, from freer trade with Germany.[10] Consequently, British-German trade grew unabated right up until the start of the war.[11]

U.S./Japan. The rivalry between the U.S. and Japan, by contrast, illustrates a case where one adversary was significantly more dependent on trade with the other than vice versa. Between 1937 and 1940, Japan relied on imports for 90% of its petroleum consumption, with 66% of its imported petroleum coming from the U.S. Because these petroleum imports were crucial for Japan's war effort—alongside its imports of steel, iron, copper, and other raw materials—the U.S. perceived that economic sanctions would be effective in constraining Japan's threat to its own interests in the Asia-Pacific region (Hosoya, 1968; Saltzman, 2012).

[8] Historical real GDP data from the Maddison Project show that Great Britain was the larger of the two countries in terms of economic size throughout most of this period. Due to its faster growth, Germany briefly caught up to the UK in 1912 and 1913 before falling behind again from 1914 onwards (see Bolt and van Zanden, 2020).

[9] As documented in Williamson (1969), by 1912, British foreign policy had committed to supporting France in the event of an unprovoked attack by Germany. In addition, the resumption of German naval expansion prompted the British to shift their own naval forces from the Mediterranean to the North Sea and to increase their own naval forces. The British and French militaries had been coordinating on a strategic response to a German invasion for 8 years by this point, and the two navies were now also discussing how to coordinate naval deployments.

[10] It is worth adding here that representatives from the British Treasury had been of the view that, even if war were to break out, disrupting trade with Germany would be counter-productive, hurting Britain's economy and, along with that, its war effort (Seligmann, 2017).

[11] According to statistics collected by Liberman (1996), trade between the two countries had nearly doubled since 1900, with annual trade growth actually accelerating (to 10%) between 1912 and 1913. Britain was not alone among Germany's rivals in permitting unfettered trade. Remarkably, Germany's trade with France and Russia (enemy countries far less inclined towards free trade than the British) grew even faster between 1900 and 1913.

The U.S.'s embargo on trade with Japan was progressive in nature and at first proceeded in stages. The abrogation of its commercial treaty with Japan in 1939 served as a warning that severe sanctions might follow. After Japan proceeded to invade French Indochina in 1940, and after peace talks that would have ended the sanctions had failed, the U.S. moved increasingly towards a total embargo on trade and began preparing for war in earnest. Between 1939 and 1941, the U.S. moved the bulk of its fleet to the Pacific, doubled its military spending, froze Japanese-owned assets, and cut its trade with Japan by more than half (Liberman, 1996; Saltzman, 2012). In the context of the theory we will soon describe, these actions are consistent with the U.S. believing that war was now highly possible and that restricting trade with its rival was an effective strategy for enhancing its advantage in the event of a conflict or negotiated settlement.

U.S./Soviet Union. Interestingly, one of the immediate lessons the U.S. took away from its experience with Japan was that it should have moved more quickly to cut off trade. Cain (2005) describes a political environment, in the early days of the emerging post-World War II rivalry with the Soviet Union, where the U.S.'s continued trade with Japan between 1937 and 1940 was seen as a strategic error not to be repeated. The U.S. moved aggressively to design, by 1949, a set of export controls intended to "prevent or delay further increase in the war potential of Eastern European economies" (U.S. Munitions Board, 1949). As discussed in Brawley (2004), new U.S.-led organizations and initiatives such as NATO, the Marshall Plan, and ANZUS were used as a way of binding other major economies to these trade measures in order to enhance their effectiveness. The GATT agreement likewise had the strategic benefit of excluding the Soviet Union from the increased trade that was created.

To synthesize, these episodes illustrate that countries view restrictions on trade as a strategic instrument for maintaining a relative power advantage over their potential future enemies. However, as the Britain-Germany example suggests, they do so with a clear-eyed view of how reduced trade will affect their own economies and militaries, rather than focusing solely on differences in relative gains from trade. Even in the cases of the sanctions against Japan and the Soviet Union, domestic economic considerations were still seen as salient. Liberman (1996) notes that the U.S. State department listed "economic dislocation" as a valid argument against using sanctions against Japan, but ultimately concluded their economic impact on the U.S. would be limited. In the Soviet Union case, the U.S.'s allies began dropping their own sanctions in 1954, once fears of a "hot" war had passed, to realize the benefits from trade with the untapped Soviet market (Mastanduno, 1985).[12] In general, these examples illustrate that decisions to restrict trade with a military rival reflect not only the absolute economic gains from trade trade but also the relative gains

[12] As discussed in Mastanduno (1985), the U.S. continued to view U.S./Soviet Union trade as primarily a "gift" to the Soviet Union rather than a mutual benefit well into the 1970s and 1980s. This view could explain why the U.S. was much slower than its allies to embrace trade with the Soviet Union.

and their implications for security, with the latter becoming less salient when the risk of conflict subsides. Naturally, we do not claim that these are always the most important considerations surrounding trade in any given rivalry. However, that these elements should matter is intuitive, and our analysis will demonstrate how they can be studied together theoretically.

3 A Dynamic Model of Prudence Versus Predation

We consider a two-period model of a world economy that is populated by two countries identified by a superscript $i = 1, 2$. The key feature of our setting is that, while peace always prevails in the first period ($t = 1$), conflict emerges in the second period ($t = 2$) with a strictly positive probability. In the case of peace in period $t = 2$, each country's output, produced using the resources generated from their savings/investments in period $t = 1$, is secure. But, if a conflict emerges, then the combined output of both countries becomes contestable via the force of arms.[13]

Our principal aim in this setting is to explore how the income gains from trade in intermediate inputs in period $t = 1$ influence each country's saving and arming decisions in that same period and how that matters for relative power, growth and, ultimately, national welfare over the two periods. In our initial exposition, we strive to motivate our results for trade in a simple yet general way, deferring formal details regarding trade until Section 5. Accordingly, for now, we generically characterize each country by its possession of an initial resource endowment R^i and a native technology level A^i, such that its final output (or income) under autarky is given by $Y^i = A^i R^i$. The technology parameter A^i is taken to reflect the country's ability to produce intermediate inputs on its own with R^i that it, in turn, employs in the production of final output. To introduce the basis for trade in this general formulation, we suppose there are multiple such inputs, with each country having a comparative advantage in the production of at least one. Then, under trade in inputs, one country—and possibly both—can realize efficiency gains in the production of its final output. That is to say, output under trade can be written instead as $Y^i = T^i R^i$, where $T^i \equiv T^i(R^i, R^j)$ depends on initial resource endowments of both countries as well as technologies and where $T^i \geq A^i$ holds as a strict inequality for at least one country, reflecting the possible gains from trade.[14]

[13]Our assumption that conflict emerges in the future with some positive probability represents an important departure from much of the conflict literature that assumes conflict emerges with certainty. Even in analyses that study the choice between war and peace (e.g., Jackson and Morelli, 2007), war emerges with either probability 1 or probability 0. We view our approach as appealing since it allows us to study both these special cases and intermediate cases where arming is *ex ante* prudent, though not necessarily *ex post*. Furthermore, as discussed below in Section 5.3, our setting can be interpreted as one where (should a dispute arise, which occurs with some probability) countries choose between war and peaceful settlement that amounts to a division of whatever is being contested based on countries' relative arms.

[14]In static versions of many trade models, such as the Armington (1969) model, there is an isomorphism between changes in the resource endowment R^i and changes in productivity A^i in terms of their respective

The most salient feature of trade for our current purposes, however, is how these income gains are distributed across countries. Specifically, as we demonstrate below using a simple Armington (1969) trade model and as we can show in other trade models (see Online Appendix D), the smaller of the two countries under autarky can expect to enjoy relatively larger income gains from trade than its larger trading partner:

$$\textit{if} \;\; A^j R^j > A^i R^i, \quad \textit{then} \;\; \frac{T^i R^i}{A^i R^i} > \frac{T^j R^j}{A^j R^j}.$$

This forthcoming result should be kept in mind as we develop the intuition behind our results for trade by first focusing on exogenous changes in relative output.

Setting aside (for now) the decision to trade, a central component of our analysis is how each country subsequently decides to allocate its first-period income Y^i. Specifically, each country divides its output between current consumption C^i and two distinct types of activities that can augment future consumption $\widetilde{C}^i$: "arming", which we denote by G^i, and "saving", which we denote by Z^i. (Throughout, we use a tilde ($\sim$) over a variable to indicate its value in the second period, $t = 2$.) This choice must satisfy the following resource constraint:

$$C^i + G^i + Z^i \leq Y^i, \;\; \text{for} \;\; i = 1, 2. \tag{1}$$

To simplify the exposition, and without altering any of our key results, we will henceforth assume that the two countries have equivalent technologies under autarky: $A^i = A^j = 1$. First-period output in the absence of trade will thus simply be given by $Y^i = R^i$. Turning to period 2, each country i's first-period saving yields $\widetilde{R}^i = Z^i$ units of the productive resource. Assuming that trade is not possible in period $t = 2$, that resource in turn is transformed into $\widetilde{Y}^i = Z^i$ units of second-period output.[15]

From the perspective of period $t = 1$, the output held by each country in period $t = 2$ and, thus, the return from such saving are subject to uncertainty due to the possibility of future conflict. In the baseline version of the model presented here, the weight of this uncertainty is governed by the probability of conflict, denoted by $q \in (0, 1]$. More precisely, in the event that no conflict arises and thus peace prevails, which occurs with probability $1 - q$, country i enjoys its entire output: $\widetilde{C}^i = Z^i$, $i = 1, 2$. By contrast, if a conflict arises, which occurs with probability q, each country's output goes into a contested pool, $Z^i + Z^j$,

influence on the production of final output under autarky, $Y^i = A^i R^i$. Hence, we can write the $T^i(R^i, R^j)$ function as $T^i = T(A^i R^i, A^j R^j)$. Online Appendix D contains remarks on how the relationship between the relative gains from trade and relative endowments depends on technology differences as well as trade costs and other similar parameters.

[15] Although a central objective in this paper is to explore the influence that the trade regime in place in $t = 1$ has on current equilibrium allocations to saving and arming when conflict in the next period possibly materializes, our analysis can be extended to consider the possibility of trade also in $t = 2$. This extension, which is discussed in Section 5.3, reveals that future trade favorably influences preferences for current trade.

$i, j \in \{1, 2\}$, $i \neq j$. In the fuller version of the model presented in Online Appendix A and in some of our extensions discussed in Section 5.3, we allow for the possibility that some of this output is secure even in the event of conflict.

In the case that conflict arises, country i's share ϕ^i of the contested pool in period $t = 2$ depends on arming by both countries (G^i, G^j) chosen in period $t = 1$. This share takes the standard ratio form:

$$\phi^i(G^i, G^j) \equiv \frac{(G^i)^m}{(G^i)^m + (G^j)^m}, \quad \text{if } G^i + G^j > 0 \text{ for } i, j \in \{1, 2\},\ i \neq j, \tag{2}$$

where $m \in (0, 1]$ reflects the effectiveness of arming; if $G^i + G^j = 0$ then $\phi^i = \phi^j = \frac{1}{2}$.[16] This specification implies a country's share is increasing in its own arming (i.e., $\phi^i_{G^i} \equiv \partial\phi^i/\partial G^i = m\phi^i\phi^j/G^i > 0$) and decreasing in the opponent's arming (i.e., $\phi^i_{G^j} \equiv \partial\phi^i/\partial G^j = -m\phi^i\phi^j/G^j < 0$). Furthermore, this conflict technology is symmetric (i.e., $\phi^i(G^i, G^j) = \phi^j(G^i, G^j)$ for any feasible G^i and G^j). The influence of guns on the division of contested output between the two countries can be interpreted as the result of either open conflict (i.e., war without destruction) or a bargaining process with the countries' relative military strength playing a prominent role. Importantly, as discussed below in Section 5.3 (with further details provided in Online Appendix E), our central results to follow remain qualitatively unchanged provided the resolution of conflict—whether it results in a division of contested output as modeled here or is modeled as a "winner-take-all" contest with ϕ^i representing the probability of winning—requires the use of resources to produce arms.

Each country i chooses its allocation of current income Y^i to arming G^i and saving Z^i to maximize expected lifetime utility: $U^i = u(C^i) + \delta E\{u(\widetilde{C}^i)\}$, where $\delta \in (0, 1]$ represents the common discount factor and $u(\cdot)$ has the usual properties that ensure the quasi-concavity of payoff functions and ensure strictly positive allocations to both saving and arming: $u' > 0$, $u'' < 0$, and $\lim_{C\to 0} u'(C) = \infty$. While the results to follow hold under any function $u(C) = C^{1-\rho}/(1-\rho)$ for $\rho > 0$, where ρ is the coefficient of relative risk aversion and $1/\rho$ is the elasticity of intertemporal substitution, we assume logarithmic preferences ($\rho = 1$) to keep the analysis as simple as possible:

$$U^i = \ln C^i + \delta E\left\{\ln \widetilde{C}^i\right\} \ \text{ for } i = 1, 2. \tag{3}$$

This maximization problem for each country i, which takes rival country j's choices as given, is subject to the first-period resource constraint (1), the conflict technology (2) and,

[16] See Skaperdas (1996), who axiomatizes a more general functional form of this conflict technology. The particular form shown in (2) is commonly used in the contest and conflict literatures. We impose the restriction that $m \leq 1$, which is sufficient to ensure that a unique pure-strategy equilibrium exists. Focusing on pure-strategy equilibria allows us abstract from the possibility of multiple equilibria and the issues that arise as a result.

for $i, j \in \{1,2\}$, $i \neq j$, the following:

$$\widetilde{C}^i = \begin{cases} Z^i & \text{with probability } 1-q \\ \phi^i(Z^i + Z^j) & \text{with probability } q. \end{cases} \tag{4}$$

As (4) shows, a country's arming matters for future consumption (through ϕ^i) only in the event of conflict. Thus, when $q = 0$, the model simplifies to a standard consumption/investment savings model (i.e., with $G^i = G^j = 0$), a useful benchmark for highlighting the importance of insecurity and uncertainty for such dynamic problems.[17]

The timing of the extended policy game is as follows. First, at the beginning of period $t = 1$, the two countries' policymakers simultaneously and noncooperatively choose their individually preferred trade regimes. If both countries announce "trade" (T), then the two countries exchange their intermediate goods, and each country i's output level is $Y^i = T^i(R^i, R^j)R^i$ for $i, j \in \{1,2\}$, $i \neq j$; if, however, at least one country announces "autarky" (A), then no trade takes place, and each country i's output level is $Y^i = R^i$. Second, once first-period output levels are determined, each country i chooses G^i and Z^i noncooperatively and simultaneously and consumes the remaining income C^i. In period $t = 2$, each country uses its available resource $\widetilde{R}^i = Z^i$ to produce the intermediate goods and then output $\widetilde{Y}^i = Z^i$. The amount consumed that period by each country depends on whether or not conflict breaks out and of course on both countries' first-period choices as shown in (4).

A key difference between the interaction we have just described and that in standard models of distributive conflict is that each player has more than one instrument it can use to influence payoffs. In standard conflict models, each player is viewed as choosing its quantity of arms only; given the player's initial resources, those choices determine residually the size of the prize. In the present setup, while saving choices alone determine the size of the prize, these choices are jointly determined with arming choices. Our characterization of this more complex problem in the next section—in particular, the "equilibrium in shares" approach described in Section 4.1—therefore represents a methodological contribution of our work to the conflict literature even before considering our central question regarding trade.

4 Equilibrium Arming, Saving and Payoffs Given Income

Given the dynamic structure of the model, we find the subgame perfect equilibrium by solving the model backwards. Specifically, in this section, we characterize the Nash equilibrium of the simultaneous-move subgame in arming and saving and the associated discounted payoffs given Y^i for $i = 1, 2$, deferring until the next section our discussion of trade. Using equation (1) as an equality (due to non-satiation) together with (4), we can rewrite country

[17] We could also modify the model so that conflict, when it arises, destroys a fraction of the contested pool of output. We do not consider this possibility here, because it does not substantively alter our conclusions.

i's expected, two-period payoff (3) as follows:

$$U^i = \ln\left(Y^i - G^i - Z^i\right) + \delta\left[q\ln\left(\phi^i[Z^i + Z^j]\right) + (1-q)\ln\left(Z^i\right)\right], \tag{5}$$

for $i, j \in \{1, 2\}$, $i \neq j$, where $\phi^i = \phi^i(G^i, G^j)$ is shown in (2) and where $Y^i = R^i$ under autarky and $Y^i = T^i(R^i, R^j)R^i$ under free trade. Country i's choices of arming G^i and saving Z^i in an interior solution, then, satisfy the following first-order conditions (FOCs):

$$U^i_{G^i} = \delta\left[\frac{q\phi^i_{G^i}}{\phi^i}\right] - \frac{1}{Y^i - G^i - Z^i} = 0 \tag{6a}$$

$$U^i_{Z^i} = \delta\left[\frac{q}{Z^i + Z^j} + \frac{1-q}{Z^i}\right] - \frac{1}{Y^i - G^i - Z^i} = 0, \tag{6b}$$

for $i, j \in \{1, 2\}$, $i \neq j$, which is a system of four equations in four unknowns.

The second terms shown in the expressions for $U^i_{G^i}$ and $U^i_{Z^i}$ in (6a) and (6b) respectively represent the marginal costs to country i of arming (MC^i_G) and saving (MC^i_Z) that arise as such activities reduce current consumption, $C^i = Y^i - G^i - Z^i > 0$. Because G^i and Z^i constitute competing uses of $t = 1$ output and they displace the same quantity of current consumption, their marginal costs are identical (i.e., $MC^i_G = MC^i_Z$) and always reflect the *inter*-temporal trade-off between present and future consumption. In addition, both MC^i_G and MC^i_Z are increasing and convex in G^i and Z^i (respectively) and are decreasing in country i's $t = 1$ output Y^i; that is, $\lim_{G^i+Z^i \to Y^i} MC^i_J = \infty$ and $\partial MC^i_J/\partial Y^i < 0$ for $J = G, Z$.

The first term shown in the expression for $U^i_{G^i}$ in (6a) represents country i's expected, discounted marginal benefit of producing an additional gun (MB^i_G). This benefit derives from the effect of increased arming to expand country i's share of the contested output and thereby augment its future consumption $\widetilde{C}^i$ in the event of conflict. Accordingly, MB^i_G depends positively on the probability of conflict q and the discount factor δ. Next, observe from (2) that $\phi^i_{G^i} = m\phi^i\phi^j/G^i$. Thus, country i's marginal benefit of arming simplifies as $MB^i_G = \delta qm\phi^j/G^i$, which clearly is decreasing in country i's own arming G^i and increasing in the other country's arming G^j. Noting again that MC^i_G is increasing in G^i but is independent of G^j, it follows that country i's payoff is strictly concave in G^i (i.e., $U^i_{G^iG^i} < 0$) and that the two countries' arming choices are strategic complements (i.e., $U^i_{G^iG^j} > 0$).

Country i's expected marginal benefit of saving (MB^i_Z) is captured by the first term shown in the expression for $U^i_{Z^i}$ in (6b). Unlike MB^i_G, this expected benefit derives from two distinct sources, one that matters only in the event of conflict and one that matters only in the event of peace. If conflict arises, increases in savings affect the total pie of insecure future output to be contested. If instead peace prevails, each country's savings then convert entirely to future consumption. Not surprisingly, then, MB^i_Z falls with increases in the likelihood of conflict q and rises with increases in the discount factor δ. Further inspection also reveals MB_Z^i is decreasing in the country's own saving Z^i, and, when $q < 1$, $\lim_{Z^i \to 0} MB_Z^i = \infty$. Thus, provided the probability of future peace is strictly positive ($q < 1$), both countries choose strictly positive savings: $Z^i > 0$ for $i = 1, 2$. Since MC_Z^i is increasing in Z^i, these properties imply country i's payoff is strictly concave in Z^i (i.e., $U^i_{Z^i Z^i} < 0$). Furthermore, since MB_Z^i is decreasing in Z^j while MC_Z^i is independent of Z^j, the countries' savings choices are strategic substitutes (i.e., $U^i_{Z^i Z^j} < 0$).

4.1 Equilibrium in Shares

Building on the relationships outlined above, we can define and characterize the equilibrium implied by (6). In view of the complexity of this strategic environment, we first reduce the dimensionality of the problem in order to obtain what we call an "equilibrium in shares" representation. This approach enables us to illuminate how country "size" translates into "power" as well as how this relationship is moderated by changes in the probability of conflict q, thereby paving the way for our upcoming analysis of how the trade regime matters for equilibrium outcomes and payoffs.

To proceed, define the share that country i contributes to the (potentially) contested pool of future output as

$$\theta^i(Z^i, Z^j) \equiv \frac{Z^i}{Z^i + Z^j} \quad \text{if } Z^i + Z^j > 0 \text{ for } i, j \in \{1, 2\},\ i \neq j, \tag{7}$$

where $\theta^j = 1 - \theta^i$. One can easily verify that $q < 1$ ensures $\theta^i, \theta^j > 0$, $\theta^i_{Z^i} = \theta^i \theta^j / Z^i > 0$, and $\theta^i_{Z^j} = -\theta^i \theta^j / Z^j < 0$. The definition of θ^i allows us to characterize relative saving choices across countries in terms of a single endogenous parameter: $Z^i/Z^j = \theta^i/\theta^j = \theta^i/(1-\theta^i)$ for $i, j \in \{1, 2\}$, $i \neq j$. Similarly using the conflict technology in (2), we can write relative arming choices as a function of ϕ^i: $G^i/G^j = (\phi^i/\phi^j)^{1/m} = (\phi^i/(1-\phi^i))^{1/m}$ for $i, j \in \{1, 2\}$, $i \neq j$. Using these two relationships, we then transform (6) (a system of four equations in four unknowns) into a system of two equations in just two unknowns—specifically, the appropriative and contributive shares, ϕ^i and θ^i.

To derive the first of these equations, we proceed in two steps. First, we form the *relative* marginal benefits of arming and saving, given respectively by

$$\frac{MB_G^j}{MB_G^i} = \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}+1} \qquad \text{and} \qquad \frac{MB_Z^j}{MB_Z^i} = \frac{\theta^i}{\theta^j}\left[\frac{1 - q + q\theta^j}{1 - q + q\theta^i}\right].$$

Keeping in mind that $\phi^j = 1 - \phi^i$ and $\theta^j = 1 - \theta^i$, one can easily see that the expressions above depend only on ϕ^i, θ^i, and q. Second, we exploit the fact that, in any equilibrium, the marginal benefits of arming and saving must equalize for each country (since $MC_G^i = MC_Z^i$ implies $MB_G^i = MB_Z^i$ for $i = 1, 2$). Thus, the following equalities must also hold in equilibrium:

$$\frac{MB_G^j/MB_Z^j}{MB_G^i/MB_Z^i} = \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}+1} \frac{\theta^j}{\theta^i}\left[\frac{1-q+q\theta^i}{1-q+q\theta^j}\right] = \frac{MC_G^j/MC_Z^j}{MC_G^i/MC_Z^i} = 1. \tag{8}$$

Rewriting (8), we obtain the first of two conditions that define an equilibrium:

$$S^i\left(\phi^i,\theta^i;q\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}}\left[\frac{\left(1-q+q\theta^i\right)\left(\phi^i/\theta^i\right)}{\left(1-q+q\theta^j\right)\left(\phi^j/\theta^j\right)}\right] - 1 = 0. \tag{9}$$

The first term in the expression for $S^i(\phi^i,\theta^i;q)$ represents the ratio of the relative marginal benefits of arming and saving across countries, whereas the second term reflects the ratio of the relative marginal costs of arming and saving across countries that must equal 1. Since $\phi^i + \phi^j = 1$ and $\theta^i + \theta^j = 1$, the condition in (9) implicitly defines a relationship between θ^i and ϕ^i that we henceforth refer to as the "S^i-contour" or, alternatively, as "schedule S^i". The lemma below describes the key properties of this schedule, named for its "S" shape as depicted in Fig. 1.

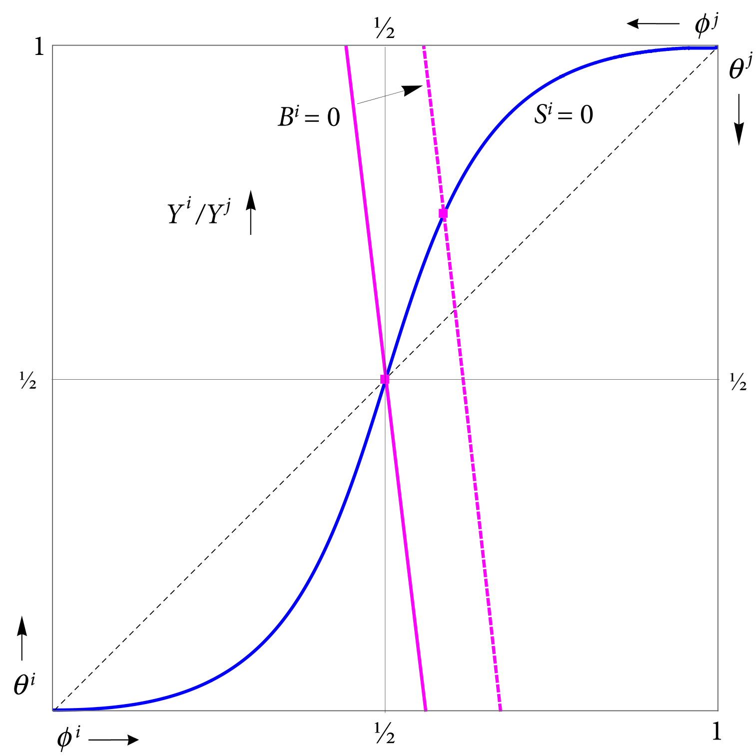

Figure 1: The Determination of Countries' Equilibrium Shares in Appropriative and Productive Investments

Lemma 1 *The $S^i(\phi^i,\theta^i;\cdot) = 0$ condition in (9) implicitly defines a continuous and increasing relationship between θ^i and ϕ^i that holds true in equilibrium. This relationship is* *characterized as follows*:

(a) $S^i_{\phi^i} > 0$, $S^i_{\theta^i} < 0$ *and* $d\theta^i/d\phi^i|_{S^i=0} = -S^i_{\phi^i}/S^i_{\theta^i} > 0$;

(b) $\lim_{\phi^i\to 0} d\theta^i/d\phi^i|_{S^i=0} = \lim_{\phi^i\to 1} d\theta^i/d\phi^i|_{S^i=0} = 0$ *and* $\lim_{\phi^i\to\frac{1}{2}} d\theta^i/d\phi^i|_{S^i=0} > 1$;

(c) *if* $\phi^i \gtreqless \frac{1}{2}$, *then* $\theta^i \gtreqless \phi^i$;

(d) $\lim_{\phi^i\to 0} \theta^i/\phi^i|_{S^i=0} = 0$ *and* $\lim_{\phi^i\to 1} \theta^i/\phi^i|_{S^i=0} = 1$.

Part (a) establishes that the S^i-contour is increasing over the entire range of values of ϕ^i, with points to the right (left) and below (above) the contour implying $S^i > 0$ ($S^i < 0$). Yet, from part (b), the contour is "flat" at the endpoints. Part (c) points out that the less powerful country's contributive share to the potentially contested pool of future income is not only less than that of its relatively more powerful rival but also less than its own appropriative share. Finally, combined with parts (b) and (c), part (d) establishes that the contour starts at $(\phi^i, \theta^i) = (0, 0)$, crosses the midpoint $(\phi^i, \theta^i) = (\frac{1}{2}, \frac{1}{2})$ where it is steeper than 1, and ends up at $(\phi^i, \theta^i) = (1, 1)$.

For some intuition regarding the ϕ^i/θ^i relationship along the S^i-contour, recall that it represents a balance between the countries' marginal benefits of arming relative to the marginal benefits of saving and their respective relative marginal costs. As shown in (8), because the ratio of the two countries' marginal costs is fixed at 1, adjustments in ϕ^i/θ^i along the contour are due solely to changes in the ratio of countries' relative marginal benefits. To dig a little deeper, consider the midpoint of the S^i-contour, at $(\phi^i, \theta^i) = (\frac{1}{2}, \frac{1}{2})$ where $\phi^i/\theta^i = \phi^j/\theta^j = 1$. Now consider how the ratio of relative marginal benefits would change if ϕ^i and θ^i increased proportionately (i.e., if we moved NE along the 45° line in the figure). Since ϕ^i and θ^i increase (and thus ϕ^j and θ^j decrease), while $\phi^i/\theta^i = \phi^j/\theta^j$ remain unchanged at 1, the ratio of marginal benefits in (8) rises above 1, implying $S^i > 0$ and that we have traveled below schedule S^i. Therefore, starting at $\phi^i = \theta^i = \frac{1}{2}$, an increase in a country's appropriative share ϕ^i must be accompanied by a greater increase in its contributive share θ^i (such that $\theta^i > \phi^i$) to keep the value of the ratio of marginal benefits equal to 1 and thus remain on the S^i-contour, as emphasized in Lemma 1(c). However, part (b) establishes that this tendency becomes less pronounced as ϕ^i approaches 1.[18]

As shown in the definition in (9) and as we discuss in detail below, the shape of the S^i-contour also depends on the probability of conflict q. But, it does not depend on income levels Y^i and Y^j or on the discount factor δ; thus, while any equilibrium in (ϕ^i, θ^i) must lie somewhere on the S^i-contour, determining its exact location requires a second condition capturing the influence of these other variables on relative arming and saving decisions.

[18] Consideration of asymmetries, either in the conflict technology (e.g., $\phi^i = \beta^i G^i/(\beta^i G^i + \beta^j G^j)$ for $\beta^i, \beta^j > 0$) or in production technologies would mainly alter the point where the S-contour meets the 45° line without affecting its behavior at the extremes. As such, allowing for such asymmetries does not affect any of the key limit results that we focus on.

To derive this second condition, we solve for each country i's arming and saving decisions, G^i and Z^i, from the FOCs in (6), in order to obtain:

$$G^i = \frac{\gamma^i}{1+\gamma^i+\zeta^i} Y^i \quad \text{and} \quad Z^i = \frac{\zeta^i}{1+\gamma^i+\zeta^i} Y^i, \text{ for } i = 1, 2, \tag{10}$$

where

$$\gamma^i = \gamma^i\left(\phi^i\right) \equiv \delta q m \left(1-\phi^i\right) \geq 0 \quad \text{(with equality when } \phi^i = 1) \tag{11a}$$

$$\zeta^i = \zeta^i\left(\theta^i\right) \equiv \delta\left(q\theta^i + 1 - q\right) > 0, \tag{11b}$$

represent weights that jointly determine spending on arming and saving respectively per unit of income spent on current consumption.[19] Clearly, the income share that country i channels into arming and the income share it channels into saving, shown in (10), depend on both ϕ^i and θ^i through the relationships shown in (11). To proceed, observe from (10) that the ratio G^i/G^j can be written as a function of the two countries' expenditure shares and recall that the specification of ϕ^i in (2) implies $G^i/G^j = (\phi^i/\phi^j)^{1/m}$ for $i, j \in \{1, 2\}$, $i \neq j$. Together, these implications give us our second equilibrium condition:

$$B^i\left(\phi^i, \theta^i; Y^i/Y^j, q\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{1/m} - \frac{\gamma^i\left(1+\gamma^j+\zeta^j\right)}{\gamma^j\left(1+\gamma^i+\zeta^i\right)}\left(\frac{Y^i}{Y^j}\right) = 0. \tag{12}$$

Since $\phi^i + \phi^j = 1$ and $\theta^i + \theta^j = 1$, the above equation, with the definitions of γ^i and ζ^i for $i, j \in \{1, 2\}$, $i \neq j$ shown in (11), implicitly defines another relationship between ϕ^i and θ^i, which we call the "B^i-contour" or "schedule B^i". The next lemma characterizes its shape.

Lemma 2 *The $B^i(\phi^i, \theta^i; \cdot) = 0$ condition in (12) defines implicitly a continuous and decreasing relationship between ϕ^i and θ^i that holds true in equilibrium. Specifically, $B^i_{\phi^i} > 0$, $B^i_{\theta^i} > 0$ and thus $d\theta^i/d\phi^i|_{B^i=0} = -B^i_{\phi^i}/B^i_{\theta^i} < 0$.*

Schedule B^i is the negatively sloped curve in Fig. 1 that, drawn for $Y^i = Y^j$, goes through the midpoint where $\phi^i = \theta^i = \frac{1}{2}$.[20] Points to the right (left) and above (below) the curve imply $B^i > 0$ ($B^i < 0$).

Observe that, like the definition of schedule S^i, the definition of schedule B^i uses both countries' FOCs. However, its derivation relies more directly on the two countries' arming decisions, which in turn explains why the ratio of incomes $y^i \equiv Y^i/Y^j$ and (through the parameters γ^i and ζ^i) the discount factor δ appear in the second term of the expression for B^i shown in (12). Observe further, as revealed by inspection of (12) using (11), the shape

[19] In the fuller version of the model where output is partially secure in the event of conflict, each weight depends on both ϕ^i and θ^i. Either way, the share of income spent on first-period consumption is always given by $1/(1+\gamma^i+\zeta^i)$, with the weights always being equal to $\gamma^i = G^i MB^i_G$ and $\zeta^i = Z^i MB^i_Z$.

[20] That $\phi^i = \theta^i = \frac{1}{2}$ is a point on schedule B^i when $Y^i = Y^j$ can be confirmed using equation (11) with the condition $B^i = 0$ in (12).

and location of the B^i-contour also depend on the probability of conflict q as well as on the relative first-period incomes, y^i.

Using Lemmas 1 and 2, we now turn to the determination of shares, ϕ^i and θ^i, and their properties in a pure-strategy equilibrium.

Proposition 1 (Equilibrium in Shares.) *Suppose* $q \in (0,1)$, $\delta \in (0,1]$ *and* $y^i \in (0,\infty)$. *Then, a unique equilibrium* $(\phi^{i*}, \theta^{i*}) \in (0,1) \times (0,1)$ *in appropriative and contributive shares* $(i = 1,2)$ *exists, with the following properties*:

(a) $d\phi^{i*}/dy^i > 0$ *and* $d\theta^{i*}/dy^i > 0$*; furthermore,* $\theta^{i*} \gtreqqless \phi^{i*} \gtreqqless \frac{1}{2}$ *as* $y^i \gtreqqless 1$;

(b) $\lim_{y^i \to 0} \phi^{i*} = \lim_{y^i \to 0} \theta^{i*} = 0$, $\lim_{y^i \to 0} \theta^{i*}/\phi^{i*} = 0$, *and* $\lim_{y^i \to \infty} \theta^{i*}/\phi^{i*} = 1$;

(c) $\partial\phi^{i*}/\partial q \lesseqqgtr 0$, $\partial\phi^{i*}/\partial\delta \lesseqqgtr 0$ *and* $\partial\theta^{i*}/\partial\delta \lesseqqgtr 0$ *as* $y^i \gtreqqless 1$.

Fig. 1 illustrates the equilibrium shares in appropriative and productive investments, depicted by the intersection the B^i- and S^i-contours that have been derived from the FOCs for arming and saving. As illustrated in the figure, when $Y^i = Y^j$ (or $y^i = 1$), the intersection of the two schedules occurs at the midpoint where the two countries are equally powerful as well as equal contributors to future income ($\phi^{i*} = \theta^{i*} = \frac{1}{2}$). An increase in Y^i given Y^j (equivalently, an increase in y^i) relaxes country i's *inter*-temporal trade-off between current and future consumption, thereby reducing its marginal costs of arming and saving and causing the B^i-contour to shift rightward. The equilibria induced by such changes in Y^i then trace out the S^i-contour, reflecting changes in the relative marginal benefits of arming and saving as country i changes in size. Thus, as pointed out in part (a), when country i is initially larger (i.e., $Y^i > Y^j$), the B^i-contour intersects the S^i-contour to the right and above the midpoint, implying it is more powerful than country j and an even bigger relative contributor to future income (i.e., $\theta^{i*} > \phi^{i*} > \frac{1}{2}$). While the smaller country is less powerful in equilibrium (i.e., $\phi^{j*} < \phi^{i*}$) and contributes less to future output (i.e., $\theta^{j^*} < \theta^{i*}$), it obtains a larger share of that future output in the event of conflict relative to its contribution (i.e., $\theta^{j*} < \phi^{j*} < \frac{1}{2}$). This latter result, which reflects the smaller country's ability to "prey" on the more prudent behavior of its larger rival when future conflict is possible, is reminiscent of (though distinct from) the weak form of Hirshleifer (1991)'s "paradox of power."[21] Part (b) of the proposition, which characterizes the relative limiting behaviors of ϕ^{i*} and θ^{i*}, establishes that as size differences become infinitely large, the smaller country's contributive share vanishes faster than its power share.

The first component of part (c) reveals how the influence of the probability of future conflict q on each country's *intra*-temporal trade-off between arming and saving weighs on the balance of power. Specifically, it establishes that a deterioration of international relations ($q \uparrow$) tends to diminish differences in power. Since, as mentioned earlier, the

[21] In fact, as shown below, the weak form of the paradox of power that states $Y^j/Y^i < \phi^{j*}/\phi^{i*} < 1$ holds in our setting (see footnote 25).

marginal benefit of arming MB^i_G shown as the first term in the expression for $U^i_{G^i}$ in (6a) is increasing in q while the marginal benefit of saving MB^i_Z shown as the first term in the expression for $U^i_{Z^i}$ in (6b) is decreasing in q, an increase in q raises MB^i_G/MB^i_Z or equivalently reduces the opportunity cost of arming for each country i. The result that an increase in q reduces the disparity in power across countries given income levels suggests that the opportunity cost of arming falls by more for the smaller country.

To tease out some intuition here, observe that, due to the symmetry of the conflict technology (2), the marginal benefit of arming MB^i_G depends symmetrically on country i's own arms G^i and those of its rival G^j. The marginal benefit of saving MB^i_Z, meanwhile, is nearly symmetric across countries i. The sole difference appears in the second term, $(1-q)/Z^i$. Underscoring the importance of saving for the possibility of peace, this term governs the relationship between differences in country size and the *intra*-temporal trade-off. To fix ideas, suppose $Y^i > Y^j$, which implies by part (a) of the proposition that $\theta^i > \frac{1}{2}$ and thus $Z^i > Z^j$. Accordingly, all else the same, this second term is smaller for the larger country (i), which means its opportunity cost of shifting resources from saving to arming is smaller, thereby giving it a military advantage. As the probability of conflict rises ($q \uparrow$), both $(1-q)/Z^i$ and $(1-q)/Z^j$ fall, but the latter falls by more, thereby weakening the larger country's military advantage.[22] Although the larger country remains more powerful, this result suggests that a greater likelihood of future conflict amplifies the smaller country's predatory stance through its more aggressive arming relative to its saving that contributes to future income.[23]

Finally, the last two components of part (c) of the proposition show that an increase in the discount factor δ tends to reduce differences in power ϕ^{i*} and in contributive shares θ^{i*} across countries. As discussed earlier, an increase in δ magnifies the marginal benefits of both arming and saving for each country. This magnification effect is larger for the smaller

[22]In the limit as $q \to 1$, both MB^i_G and MB^i_Z become symmetric across i, meaning that both FOCs in (6) can be satisfied as strict equalities (required for an interior solution) for both countries only when $G^i = G^j$, which implies $\phi^i = \phi^j = \frac{1}{2}$. In this special case, the larger country necessarily saves more, such that $C^i = C^j$ despite differences in first-period incomes; and, since $\phi^i = \frac{1}{2}$ for $i = 1, 2$ and $q = 1$ by assumption, $\widetilde{C}^i = \widetilde{C}^j$ also holds, such that two countries enjoy identical payoffs in any interior solution, again despite differences in first-period incomes. However, this possibility arises only when those differences in income are not too pronounced. Otherwise, a corner solution arises in which the smaller country does not save at all and its appropriative share is less than $\frac{1}{2}$.

[23]The effects of an increase in q on the *intra*-temporal trade-off can be visualized in the setting of Fig. 1 as a counterclockwise rotation of the S^i-contour around the midpoint $(\frac{1}{2}, \frac{1}{2})$, with the endpoints unchanged, thereby making the relationship between saving (and thus country size) and power less linear. At the same time, the B^i-contour also rotates in a counterclockwise direction around the point where it intersects the 45° line when $m = 1$ or above (below) that intersection when $Y^i < Y^j$ ($Y^i > Y^j$) and $m < 1$. When $Y^i = Y^j$, the curves rotate as just described, but around their intersection at the midpoint, such that ϕ^{i*} is not affected. Part (a) of Proposition B.1, presented in Online Appendix B, states further that θ^{i*} is similarly independent of q when $Y^i = Y^j$. Although we cannot pin down the influence of this parameter on θ^{i*} for all Y^i and Y^j, part (b) of Proposition B.1 shows that, in the case of an extreme asymmetry as $Y^i \to 0$ for given $Y^j \in (0, \infty)$, θ^i is decreasing in q.

country, however, causing it to become more aggressive and, at the same time, more prudent relative to its larger rival.[24]

In sum, Proposition 1 tells us how the countries' appropriative and contributive shares (or relative arming and saving) are related to relative incomes, as well as how the distribution of power adjusts to changes in the probability of a future conflict and in time preferences. But, it leaves unanswered the question of how such changes influence the arming and saving decisions of each country in levels. For example, while we know that an increase in country i's relative income makes that country more powerful, it is unclear whether each country devotes more or less resources to arming. Similarly, while we know that country i's saving rises relative to that of its rival, we do not know yet whether the two countries save more or less.

Nonetheless, an appealing feature of our "equilibrium in shares" approach is that the share variables ϕ^{i*} and θ^{i*} pin down the fractions of current income allocated to arming and saving via (11) with (10), allowing us to recover equilibrium spending choices by each country i on G^{i*} and Z^{i*} as functions of ϕ^{i*} and θ^{i*}. Then, having identified the effects of changes in relative income Y^i/Y^j on ϕ^{i*} and θ^{i*}, we can characterize their effects on G^{i*} and Z^{i*}. This characterization not only allows us to flesh out further the implications of Proposition 1, but also prepares the groundwork for our study of the effects of trade on equilibrium arming, saving and payoffs.

4.2 Income Changes and Equilibrium Arming, Saving, and Payoffs

We now turn to examine the implications of changes in one or both countries' incomes for their equilibrium choices and payoffs. Letting a caret ($\wedge$) over variables denote percent changes (e.g., $\widehat{x} \equiv dx/x$), the following proposition characterizes these effects.

Proposition 2 (Equilibrium Arming and Saving.) *Suppose* $q \in (0,1)$, $\delta \in (0,1]$, *and* $y^i \in (0,\infty)$. *Then, an exogenous change in the countries' incomes with* $0 \leq \widehat{Y}^j < \widehat{Y}^i$ *for* $j \neq i$ *imply the following responses in saving and arming:*

$$\widehat{Z}^{j*} < \widehat{Y}^j < \widehat{G}^{j*} < \widehat{G}^{i*} < \widehat{Y}^i < \widehat{Z}^{i*}.$$

Generally speaking, an increase in country i's income (given Y^j) generates positive, direct effects on country i's own arming and saving primarily by reducing the marginal cost of both activities and thereby directly relaxing the country's *inter*-temporal trade-off—i.e., between present and future consumption. At the same time, there are further, indirect effects reflecting how changes in country i's arming and saving levels induce the rival country (j) to adjust its own arming and saving as well as how these adjustments feed back into

[24] The effect of an increase in δ can be visualized as a counterclockwise rotation of the B^i-contour around the point where it intersects the 45° line when $m = 1$ or above (below) that intersection when $m < 1$ and $Y^i < Y^j$ ($Y^i > Y^j$).

country i's choices. That arming and saving do not increase proportionately with the change in income for either country reflects the combined influence of these indirect effects.[25]

For greater clarity, let us suppose that $\widehat{Y}^j = 0$. This case is especially relevant for our upcoming analysis, since a shift from autarky to trade implies $\widehat{Y}^j = 0$ when country i is infinitesimal. As the proposition shows, a given increase in country i's first-period income induces an increase in both its saving and arming, with a larger (percentage) change in saving.[26] The reasoning here for the larger effect on country i's saving builds on the set of strategic interactions we discussed earlier in connection with the FOCs (6). Specifically, the countries' saving choices are always strategic substitutes, and their arming choices are always strategic complements.[27] Thus, when $\widehat{Y}^j = 0$, country j responds to the increases in G^{i*} and Z^{i*} induced by an increase in Y^i by shifting resources from saving to arming.[28] The reduction in country j's saving induces country i to increase its own saving by even more, further clarifying the intuition for why Z^{i*} expands by more in percentage terms than G^{i*}, as Y^i increases.

Characterizing analytically the effects of the probability of future conflict (q) on spending levels here proves to be challenging, because we cannot sign the effects of a change in q on θ^i and thus cannot identify its effects on the two countries' arming and saving decisions for all $y^i = Y^i/Y^j \in (0, \infty)$. However, numerical analysis shows that an increase in q induces each country to substitute out of saving into arming.[29] In turn, Proposition 1(c) indicates that the effect of an increase in q on the smaller country's arming is proportionately greater

[25] The rankings shown in Proposition 2 can be used to substantiate the presence of the weak form of the paradox of power in our setting. Specifically, those rankings imply $\widehat{G}^{i*} - \widehat{G}^{j*} < \widehat{Y}^i - \widehat{Y}^j$, such that increases in $y^i = Y^i/Y^j$ induce smaller increases in the ratio G^{i*}/G^{j*}. Considering the benchmark where $y^i = 1$ initially, it follows that $1 < G^{i*}/G^{j*} < y^i$ and thus, by (2), $\phi^{i*}/\phi^{j*} < y^i$ for all $y^i > 1$. (By similar reasoning, the result in Proposition 2 that $\widehat{Z}^i - \widehat{Z}^j > \widehat{Y}^i - \widehat{Y}^j$ implies $Z^{i*}/Z^{j*} > Y^i/Y^j$ for all $y^i > 1$.) However, along the lines of Hirshleifer (1991)'s finding in the standard conflict model, the paradox of power can be overturned in our setting when the conflict technology exhibits increasing returns (i.e., $m > 1$). Of course, as noted earlier (footnote 16), allowing for such increasing returns can result in multiple equilibria and thereby complicate our equilibrium analysis in shares. Alternatively, sufficiently strong complementarities between Z^i and Z^j in the production of second-period output (in the event of conflict) along with a sufficiently large degree of relative risk aversion can overturn the paradox of power (a result shown formally in the standard model of conflict by Skaperdas and Syropoulos, 1997).

[26] Proposition B.2(a) presented in Online Appendix B shows country i's current consumption also rises.

[27] As discussed in Online Appendix A, when output is only partially insecure in the event of conflict, the countries' arming choices need not be strategic complements. Nonetheless, even in this case, G^{j*} depends positively on Z^{i*} regardless of which country is larger, and this effect always dominates any strategic substitutability in arming choices so that the results of Proposition 2 continue to hold.

[28] A continuity argument (confirmed by numerical analysis) shows that, even when $\widehat{Y}^j > 0$, country j could reduce its savings, provided that increase in income is sufficiently small. Proposition B.2(b) presented in Online Appendix B indicates the effect of an increase in the opponent's income (Y^i) on country j's current consumption (C^j) is non-monotonic. In particular, as $Y^i \to Y^j$, an increase in Y^i implies C^j rises. However, for extreme differences in initial income where either $Y^i \to 0$ or $Y^i \to \infty$ given $Y^j \in (0, \infty)$ initially, C^j falls with increases in Y^i.

[29] Also see Proposition B.3 presented in Online Appendix B that characterizes these effects on arming, saving, and first-period consumption when incomes across countries become either very similar (i.e., as $Y^i \to Y^j$) or extremely different (i.e., $Y^i \to 0$, while $Y^j \in (0, \infty)$).

to augment that country's relative power.

Although we cannot pin down, in general, the effects on their relative contributive shares to world savings and thus the pool of contestable output, numerical analysis shows further that an increase in q has a disproportionately negative effect on the smaller country's savings, implying an increase in the larger country's relative contribution. Still, these tendencies are consistent with the intuitive idea that increased international tensions can have adverse consequences for growth.

In any case, Proposition 2 clarifies several ambiguities left over from our representation of the problem in terms of shares. Having fully characterized how the two countries' choices (G^{i*} and Z^{i*}) in both levels and shares depend on income levels, we now turn our attention to the more intricate problem of identifying how exogenous income changes affect each country's equilibrium payoff:

Proposition 3 (Income and Equilibrium Payoffs.) *Suppose $q \in (0,1)$, $\delta \in (0,1]$, and $y^i \in (0,\infty)$. Then, for any given Y^j, an exogenous change in country i's income Y^i affects the two countries' equilibrium payoffs, U^{i*} and U^{j*} ($j \neq i$), as follows:*

(a) *$dU^{i*}/dY^i > 0$.*

(b) *There exist threshold income levels $\underline{Y}^i$ and $\overline{Y}^i$ satisfying $\underline{Y}^i \leq \overline{Y}^i (< Y^j)$ such that $dU^{j*}/dY^i < 0$ for all $Y^i < \underline{Y}^i$ whereas $dU^{j*}/dY^i > 0$ for all $Y^i \geq \overline{Y}^i$.*

As suggested by Proposition 2, an increase in country i's first-period income generates both positive and negative welfare effects for both countries. For country i, the increase in Y^i has the direct, positive effect of increasing country i's first-period consumption. At the same time, the other country's ($j \neq i$) responses in terms of increased arming and decreased saving generate indirect, negative effects on country i's payoff. Part (a) establishes the direct, positive effect dominates, such that an increase in country i's first-period income always has a positive net effect on its own payoff.

The more interesting set of effects is for the rival country $j \neq i$. On the one hand, the increase in arming by country i implies a negative security externality for country j. On the other hand, the increase in saving by country i implies a larger pool of future output to be contested, thereby creating a positive externality. By Proposition 2, we know that the growth rate of country i's savings is faster than that of its arming. However, because country i arms much more than it saves when it is very small, its share of the balance of power ϕ^i initially increases by more, in absolute terms, than the share it contributes to future world output θ^i. Eventually, as it becomes close in size to country j, the faster rate of growth in its savings causes θ^i to increase by more than ϕ^i, as shown in Fig 1. Thus, as described in Proposition 3(b), growth in country i can affect its potential rival's payoff adversely, but *only* if country i is small enough in relative terms for the negative externality from its increased arming to dominate the positive externality from its increased saving.[30] Importantly, this finding arises regardless of the possible absence or presence of trade and is unrelated to price (or terms-of-trade) effects. Nevertheless, the result is crucial in our analysis below that demonstrates trade could be unappealing to *ex ante* "larger" countries.

5 Equilibria under Autarky and Trade

With our characterization of how equilibrium outcomes depend on initial income levels (Y^i, Y^j), we can explore the implications of "trade". As discussed in Online Appendix D, our central results regarding trade are quite general, holding for a variety of different trade models, including those that feature trade costs. However, for the sake of transparency, our baseline model adopts a trade framework based on Armington (1969), a relatively simple trade setting in which each country produces a unique intermediate good; furthermore, we abstract from trade costs. In what follows, we first show how this setting implies larger relative income gains for countries initially having smaller resource endowments (or *ex ante* "smaller" countries). We then consider whether, as a result, situations can arise where *ex ante* larger countries will choose not to trade with their smaller rivals.

5.1 Resource Endowments and the "Gains from Trade"

Recall that, at the beginning of period $t = 1$, each country i is endowed with R^i $(i = 1, 2)$ units of a productive resource (e.g., capital). Along the lines of Armington (1969), each country i converts that resource on a one-to-one basis into a unique intermediate good. Specifically, country 1 produces R^1 units of input 1, and country 2 produces R^2 units of input 2. In turn, country i employs this good, alone (in the case of autarky) or in combination with the intermediate good produced by country j (in the case of trade), to produce final output Y^i. Let D^i_j denote the quantity of intermediate good j $(= 1, 2)$ that becomes available domestically to producers of the final good in country i $(= 1, 2)$. The technology for producing the final good takes the following symmetric CES form:

$$Y^i = \left[\sum_{j=1,2} \left(D^i_j\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{13}$$

where $\sigma > 1$ represents the (constant) elasticity of substitution between intermediate inputs. The specification in (13), which is symmetric, increasing, linearly homogeneous, and (provided $\sigma < \infty$) strictly concave in its arguments, reflects the benefit of employing a variety of distinct inputs in production, which is analogous to the "love of variety" exhibited by Dixit-Stiglitz preferences. As the two intermediate goods become more distinct (i.e., as σ falls), this benefit of diversity rises and so do the gains from trade.

[30] Although we cannot pin down the payoff effects of changes in the probability of conflict q, Proposition B.4 shows the effects in the two extreme cases where (i) countries have identical incomes, in which case an increase in q lowers each country's payoff and (ii) one country is infinitesimal, in which case the smaller country gains from an increase in q, while the larger country's payoff is unaffected.

Whether the two countries trade their intermediate goods or not, each country i chooses inputs to maximize its income Y^i in (13) subject to the relevant constraints that depend on the trade regime in place. However, under autarky where each country can employ only its domestically produced intermediate good, $D^i_j = 0$ and $D^i_i = R^i$ hold for $i, j \in \{1, 2\}$, $i \neq j$. Thus, each country i's output is simply $Y^i_A = R^i$. Furthermore, absent trade in period $t = 2$, country i's output in that period, regardless of whether or not conflict breaks out, is given by $\widetilde{Y}^i = \widetilde{Y}^i_A = Z^i$ for $i = 1, 2$.

To study the case of trade in period $t = 1$, let p^i_j denote the price country i pays for input $j = 1, 2$ and μ^i_j denote its expenditure share on that good; then, $\mu^i_i = 1 - \mu^i_j$ denotes country i's expenditure share on the unique input it produces. As one can verify, the country's maximizing choice of inputs implies $\mu^i_j \equiv (p^i_j/P^i)^{1-\sigma}$, where $P^i \equiv [\sum_{k=i,j}(p^i_k)^{1-\sigma}]^{1/(1-\sigma)}$, and furthermore that its demand for input j $(= 1, 2)$ equals $D^i_j = \mu^i_j p^i_i R^i / p^i_j$. That we abstract from trade costs means domestic and world prices coincide, implying that $p^i_j = p^j_j$ and $p^j_i = p^i_i$. Now let $p^i \equiv p^i_j/p^i_i$ denote country i's domestic relative price of intermediate good j $(\neq i)$. By Walras' Law, these relative prices follow from the world market-clearing condition $p^i_j D^i_j = p^j_i D^j_i$ $(i \neq j)$, which implies p^i_T $(= p^i_j/p^j_i) = (R^i/R^j)^{1/\sigma}$. With these equilibrium relative prices, one can then substitute the demand functions D^i_j into (13) to find $Y^i_T = T^i(R^i, R^j)R^i$, where

$$T^i(R^i, R^j) \equiv \left[1 + \left(R^i/R^j\right)^{\frac{1-\sigma}{\sigma}}\right]^{-\frac{1}{1-\sigma}}, \quad \text{for } i, j \in \{1, 2\},\ i \neq j. \tag{14}$$

Given $\sigma > 1$ and $R^i, R^j \in (0, \infty)$, it follows that $T^i(R^i, R^j) > 1$. Hence, both countries realize strictly positive income gains, $Y^i_T/Y^i_A = T^i(R^i, R^j) > 1$, when they trade in period $t = 1$, and abstracting from security considerations (essentially assuming in our model that $q = 0$) enjoy greater overall payoffs. As we will argue, however, if conflict arises with a strictly positive probability $(q > 0)$ in the future in which case second period output is contested, then one country might find trade in the current period unappealing.

But, first, the next proposition offers a more detailed view of how the distribution of endowments translates into the distribution of first-period incomes under autarky and trade and, hence, the distribution of income gains from trade:

Proposition 4 (Relative Incomes and the Gains from Trade.) *Under autarky and trade, the country with the larger resource endowment enjoys a higher first-period income. Yet, country i's income gain from trade, $Y^i_T/Y^i_A = T^i(R^i, R^j) \geq 1$, is decreasing in R^i/R^j with $\lim_{R^i \to 0} T^i(R^i, R^j) = \infty$ and $\lim_{R^i \to \infty} T^i(R^i, R^j) = 1$, such that $R^i/R^j \lesseqgtr 1$ implies $(Y^i_T/Y^i_A)/(Y^j_T/Y^j_A) \gtreqless 1$.*

This proposition establishes that the *ex ante* smaller country (i.e., the country having the relatively smaller resource endowment) always enjoys a larger relative income gain from trade. These results are illustrated in Fig. 2(a), which shows the income level enjoyed under trade and autarky by country i, conditioned on the distribution of initial resources, $R^i \in (0, \overline{R})$ where $\overline{R} \equiv R^i + R^j$ $(i, j \in \{1,2\},\ i \neq j)$. As shown in the proof, the divergence in gains is decreasing in σ.

Following our proof of Proposition 4 in Online Appendix B, we also discuss how a closely related set of results can be obtained if we relax our assumption that technology levels are equivalent across countries (i.e., if $A^i \neq A^j$.) As noted earlier in Section 3, country i's relative gain from trade in this slightly more general case continues to be determined by its relative autarky income level $(A^iR^i)/(A^jR^j)$. As such, the limit results and overall relationship between relative gains and relative endowment sizes remain the same as stated in the proposition, and equivalent results can obtained if we instead consider changes in relative autarky incomes. Online Appendix D describes how these results carry over to other trade-theoretic settings, including settings where trade is costly.

5.2 Trade, Power, and Welfare

We move now to the main objective of our analysis: to see how and when the security considerations brought on by the possibility of future conflict can limit—or even overwhelm completely—the standard gains from trade for either country. To begin, we characterize the security externalities associated with trade, synthesizing our key results thus far for the effects of trade on the balance of power. Let ϕ^i_A $(i = 1, 2)$ denote country i's equilibrium power under autarky and ϕ^i_T $(i = 1, 2)$ denote country i's equilibrium power under trade. To fix ideas, let country i represent the larger *ex ante* country (i.e., with $R^i > R^j$). Proposition 1(a) implies that, under both trade regimes, country i always appropriates a larger share of the contested output in the event of a future conflict: $\phi^i_A > \phi^j_A$ and $\phi^i_T > \phi^j_T$.

Because trade raises the first-period income of both countries relative to their respective autarky incomes, we know further, by Proposition 2, that trade necessarily induces both countries to produce more guns, thereby generating negative security externalities for each country. However, Proposition 4 also establishes that the *ex ante* smaller country j always realizes a relatively larger income gain from trade. Thus, by Proposition 2, the introduction of trade reduces the *ex ante* larger country i's military advantage as compared with autarky, thereby leading to a more equitable division of the contested output in the event of conflict: $\phi^i_A > \phi^i_T > \phi^j_T > \phi^j_A$.

Turning to the savings externalities, the second component of Proposition 1(a) implies that the *ex ante* larger country i contributes a larger share to future income. Following our convention for notation above, we have $\theta^i_A > \theta^j_A$ and $\theta^i_T > \theta^j_T$. Since by Proposition 4 the relatively smaller country j realizes a relatively larger income gain from trade, Proposition 2 implies that trade induces the *ex ante* smaller (larger) country j (i) to become a relatively larger (smaller) contributor to future income as compared with what happens under autarky:

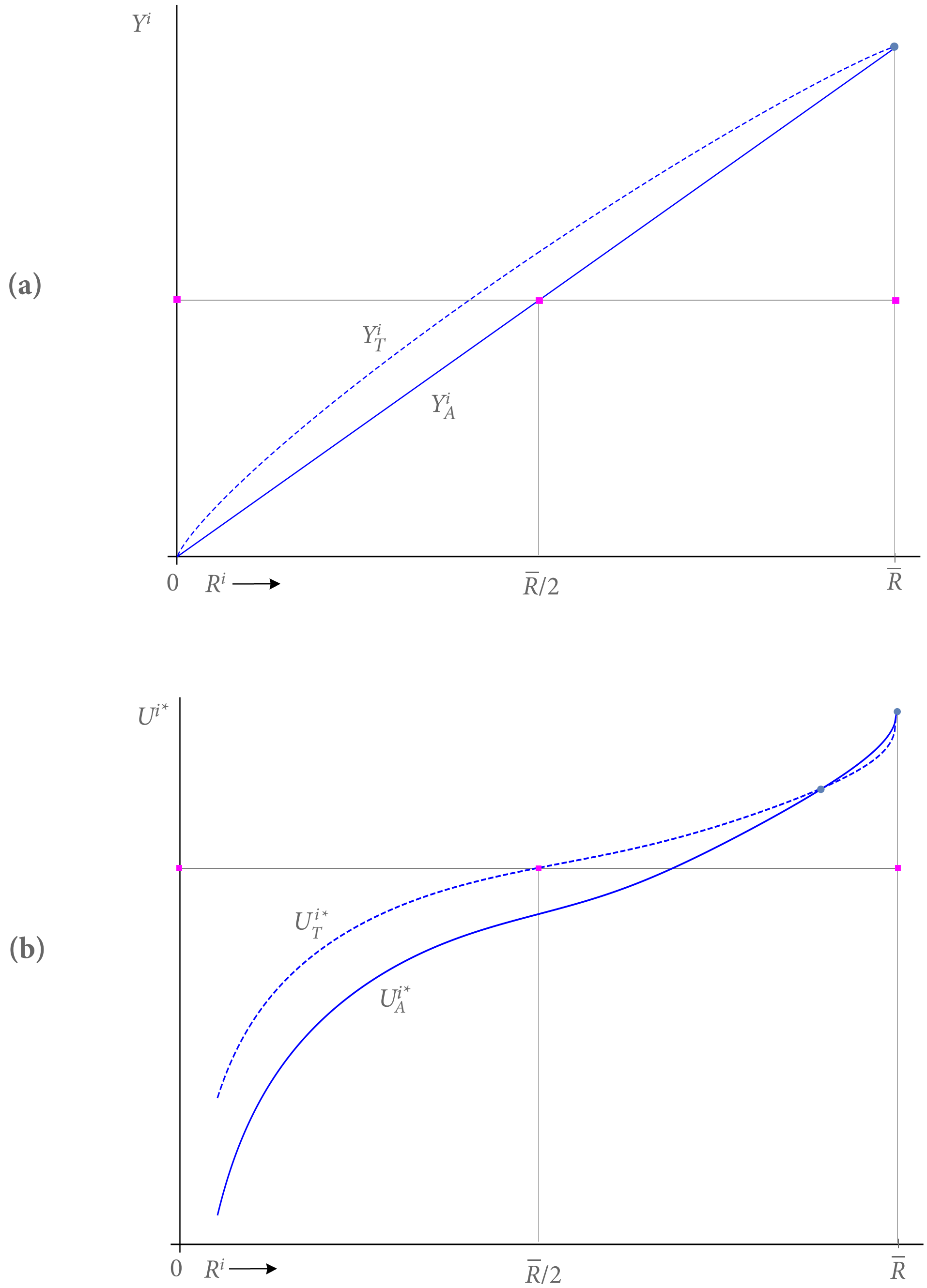

Figure 2: Income and Payoffs under Autarky and Trade, with $\sigma = 4$, $q = .9$ and $m = \delta = 1$

$\theta_A^i > \theta_T^i > \theta_T^j > \theta_A^j$.[31] While the savings externality can be negative for the smaller country if the distribution of initial resource endowments is sufficiently uneven to imply very small gains from trade for the larger country, it is necessarily positive for the larger country.

The next proposition, which accounts for both the security and savings externalities as well as the direct income gains, summarizes the welfare implications of trade when future conflict is possible:

Proposition 5 (Payoffs.) *If the international distribution of initial resource endowments is sufficiently even, introducing trade in period* 1 *improves both countries' equilibrium discounted payoffs. But, if this distribution is sufficiently uneven, then the ex ante larger country will find trade unappealing as compared with autarky.*

As illustrated in Fig. 2(b), which depicts the payoffs to country i under autarky and trade for various distributions of initial resource endowments, the smaller country (i.e., with $R^i < \frac{1}{2}\overline{R}$) necessarily benefits from trade, both through its own income gain and through the net effects of trade-induced changes in the larger country's arming and saving decisions. The larger country also benefits from trade when the initial size difference is not too large. However, as shown in the same figure and consistent with Proposition 5, its payoff under trade eventually falls below its payoff under autarky when the initial size difference becomes sufficiently large.

In discussing this last result, it is important to emphasize that it does not follow immediately from our earlier propositions and instead requires new arguments. Propositions 3 and 4 and our previous discussions thereof are nonethess useful for establishing some of the key intuition. As shown in Proposition 3(b), when the less endowed country is sufficiently small to start, a small increase in its income always reduces the larger country's payoff; as the discussion following Proposition 3 explains, this adverse payoff effect arises from the dominance of the negative security externality induced by the smaller country's increased arming over the positive externality induced by that country's increased saving. Proposition 4 then demonstrates that the larger country's relative income gain from trade is less than the relative income gain enjoyed by its smaller rival and becomes vanishingly small as it becomes increasingly large in relation to its rival. Combining the ideas from these two propositions might appear to be all that is needed to explain why a sufficiently large county will find trade relatively unappealing.

However, this is not the case. In particular, the result in Proposition 3(b) does not directly apply here, since the introduction of trade induces *discrete* changes in *both* countries' income when they are finitely sized (i.e., $R^i \in (0, \infty)$ for $i = 1, 2$). More to the point, this proposition cannot rule out the possibility that the smaller country's income gain from

[31] See Fig. B.1 in Online Appendix B that illustrates the effects of trade on the countries' arming and savings in levels and on the equilibrium appropriative and contributive shares for various distributions of initial resources.

trade could be large enough to generate, for the larger country, a substantial (positive) saving externality that, when combined with its own income gain from trade, dwarfs the (negative) security externality. This possibility arises since, by Proposition 2, the smaller country devotes an increasingly bigger share of its income to saving as it grows larger.

Our solution to this problem, in the proof to Proposition 5, consists of three main components. First, we establish that the larger country's payoff under trade converges to its payoff under autarky as its rival becomes infinitesimal. This convergence follows since the larger country's income gain from trade, the smaller country's saving, and arming by both countries all vanish in this limit. Second, from that starting point, we consider a small increase in the small country's resource endowment, which we know from Proposition 3(b) causes the large country's payoff to decrease under trade as well as under autarky. Here we demonstrate that the decrease in the larger country's payoff is always larger in magnitude under trade than under autarky. Thus, when the smaller country's initial resource base is marginally above 0, the larger country prefers autarky to trade. Finally, we appeal to the continuity of the payoff functions along with Proposition 3(b), to show that the larger country will eventually prefer trade once the smaller country's resource endowment becomes sufficiently large.

Importantly, as we mentioned above (and as we discuss in Online Appendix D), this finding is generally robust across a variety of trade models, including the classical ("Ricardian") and neoclassical ("Heckscher-Ohlin" and "Ricardo-Viner") frameworks as well as the more recent paradigms described in Krugman (1980), Eaton and Kortum (2002), and Melitz (2003), which feature trade costs, incomplete specialization, multiple factors of production, heterogeneous firms, and/or increasing returns. Furthermore, it would remain valid across any of these models if we were to compare payoffs under autarky with payoffs under non-cooperative trade policies or trade agreements (that do not consider the implications for arming and saving) instead of the payoffs under autarky and free trade.

The implications for the extended policy game should be clear: when given the choice to either trade or remain under autarky, a country that is sufficiently larger than its rival will choose not to trade at all, because it relinquishes power without gaining much back in return from trade. However, based on our numerical analysis for given relative incomes described in Section 4, one would expect the negative strategic payoff effects for the larger country to be smaller and the positive indirect payoff effects it enjoys to be larger with decreases in the probability of conflict q. Indeed, additional numerical analysis of payoffs under autarky and trade shows the range of relative endowment sizes for which the larger country prefers trade over autarky tends to expand as q decreases.[32]

[32] These results always hold in the Armington (1969) and Krugman (1980) models and, provided that σ and comparative advantage are sufficiently large, in the classical Ricardian model. However, when comparative advantage and σ are relatively small, the range of relative endowments can expand with an increase in q or the relationship can be non-monotonic.

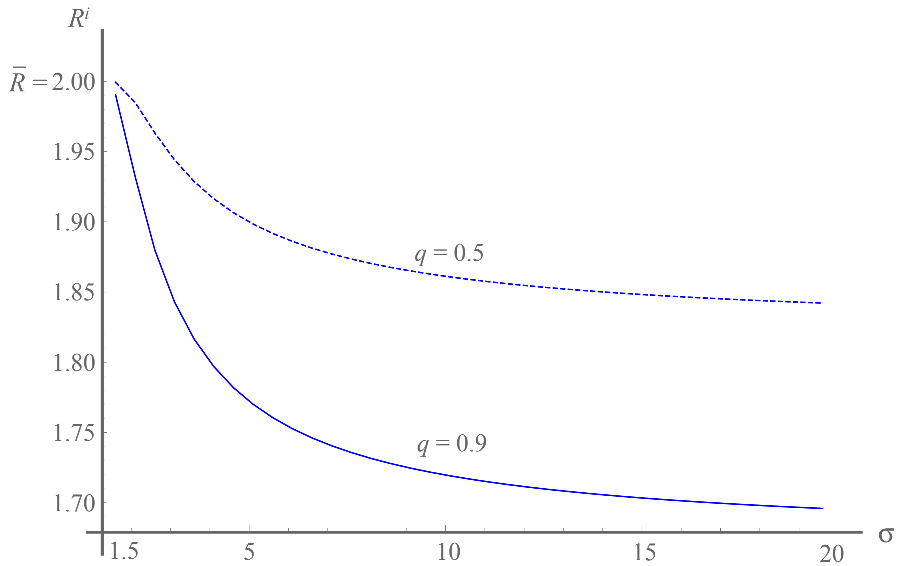

Figure 3: The Range of Resource Distributions that Make Trade Relatively Unappealing for the Large Country i

To give some sense of the magnitudes here, Fig. 3 depicts combinations of country i's resource endowment R^i and the elasticity of substitution between tradeable goods σ for which the larger country i obtains equal payoffs under trade and autarky. The figure highlights the role of the probability of future conflict, by showing these relationships for two possible values of q, $q = .9$ and $q = .5$. Points above each curve (i.e., given q and imposing the constraint $R^i + R^j = 2$) imply $U_A^{i*} > U_T^{i*}$, whereas points below the curve imply $U_A^{i*} < U_T^{i*}$. Thus, as illustrated in the figure, a decrease in q increases the range of resource distributions for which country i prefers trade. The figure also illustrates that, given q, a decrease in the elasticity of substitution σ, which amplifies the gains from trade, increases the range of resource endowments for which the larger country i prefers trade. Additional numerical analysis suggests that decreases in m (implying less effective arming) and/or δ (implying greater discounting of the future) similarly expand the range of relative endowment sizes for which the larger country prefers trade.

The findings of Proposition 5 can be related back to the historical examples discussed above in Section 2. In particular, even when the threat of a future conflict is quite high, the gains from trade can be sufficiently large to render trade preferable over autarky. This was ostensibly the case for Great Britain and Germany in their rivalry leading up to World War I. By contrast, it is plausible that the gains for the U.S. in its trade with Japan leading up to World War II and in its trade with Russia following World War II were not seen as sufficiently large to dominate the negative security externality net of any positive saving externality.

5.3 Possible Extensions and Generalizations

In this section, we consider briefly various extensions that relax some of the simplifying assumptions we have imposed to make the analysis as transparent as possible. In particular, we discuss (i) a different interpretation of our model of conflict resolution, consistent with the idea that countries choose between war and peace; (ii) alternative rules of division under peaceful settlement; (iii) trade in the future; (iv) a longer time horizon; and, (v) a three-country setting. Collectively, these extensions suggest that our analysis above is robust to a variety of alternative assumptions.

Choosing between war and peace. Above, we assumed that the probability of war breaking out in period $t = 2$ is exogenous. Let us now reinterpret q as the probability that a dispute arises between the two countries in period $t = 2$. With probability $1-q$, no dispute arises and each country enjoys all the return from its first-period savings Z^i. If a dispute occurs, it can be resolved either through "war" or through "peaceful settlement" conducted in the shadow of war. In the case of war, each country i deploys the arms G^i it had produced in period $t = 1$ to increase its probability of winning all the contested output, according to ϕ^i shown in (2). So that defeat does not result in zero future consumption, we suppose that only a fraction of future output, denoted by $\kappa \in (0, 1)$, is contestable. When the dispute is instead resolved through settlement, each country agrees on a peaceful division; in this case, ϕ^i gives country i's share of insecure output, along the lines of what we call "conflict" in the baseline model. As shown in Online Appendix E.1, our central results remain intact if the two countries can resolve their dispute only via a winner-take-all contest. The key here is that countries similarly allocate productive resources to arm in the first-period in anticipation of such a contest when a dispute arises, such that the adverse strategic effect associated with a switch to trade in the first period can swamp the benefits for the one country that is sufficiently larger than its potential adversary. Of course, the countries' equilibrium arming choices will depend on whether they expect such a dispute (should one arise) to be resolved through war or settlement. However, our maintained assumption that countries are risk-averse ensures that, given the choice between settlement and war with arming choices having already been made to fix the value of ϕ^i, peaceful settlement strictly dominates. Thus, our analysis above is consistent with the possibility that countries choose between war and peaceful settlement when a dispute arises—they always choose settlement. What's more, allowing for the possibility that the choice of war results in the destruction of a fraction of future output makes the preference for settlement even stronger.

Alternative rules of division. Clearly, our assumption that the peaceful settlement of a dispute involves a rule of division that is based exclusively on ϕ^i simplifies the analysis of the choice between war and peace considerably. In Online Appendix E.2, we show how alternative rules of division of contested output, based on Nash-bargaining and split-the-surplus protocols with the countries' payoffs under a winner-take-all contest representing their respective threat-point payoffs (along the lines of Anbarci et al., 2002), can be incorporated into the analysis without materially affecting our central results. The key point here is, once again, that the possibility of a future dispute, however resolved, is costly, as it induces each country to divert resources away from current consumption and investment for future consumption. Under either division rule, countries arm to gain leverage in future negotiations should a dispute arise. Accordingly, the adverse strategic consequences of trade for the larger country will still overwhelm any positive effects if the difference in initial size is sufficiently large.

Trade in the future. Our argument above that, given a dispute arises in period $t = 2$, the countries would necessarily choose peaceful settlement suggests their decision to settle does not depend on the possibility of future trade. This is not to argue, however, that the possibility of future trade is irrelevant even in our simple setting. First, the fact that war has the costly effect of precluding trade in period t=2 gives an additional reason for the countries to prefer settlement over war, as emphasized by the "liberal peace" hypothesis (see e.g., Polachek, 1980; Martin et al., 2008). Second, and more interestingly, as we show in Online Appendix E.3, future trade matters in shaping the larger country's preference for current trade. Specifically, we suppose that, when peace prevails in period $t = 2$, the two countries go on to freely trade their intermediate goods produced from their previously chosen savings, Z^i and Z^j. When a dispute arises, the two countries enter into a negotiated settlement whereby they trade their intermediate goods freely and then divide the contested pool of output according to (2). The possibility of future trade amplifies the potential benefit of current trade to the larger country, as current trade enables its smaller rival to grow in size and become a larger and more valuable trading partner in the future. Nonetheless, numerical analysis of the model confirms that a shift to trade in period $t = 1$ generates a negative security externality that can swamp current trade's positive effects. That is to say, there exists a set of parameter values, for which a sufficiently uneven distribution of endowments renders trade in the first period unappealing to the larger country. Intuitively, though future settlement ensures the countries enjoy mutual benefits from trade in period $t = 2$, it does not constrain arming choices, which are made ahead of time, and therefore does not resolve the problem that asymmetric income gains from trade in period $t = 1$ have implications for how resources are divided in period $t = 2$. Notably, however, relative resource endowments for which this preference ranking holds vanishes when the elasticity of substitution $\sigma > 1$ becomes sufficiently small to imply large enough compounded gains from trade across the two periods.

Longer time horizon. Our focus on two-period settings naturally raises the question of whether our results survive with longer time horizons. To explore this issue, we turn to a three-period version of the model, presented in Online Appendix F. We assume that a conflict arises with probability $q > 0$ in each of the latter two periods, $t = 2, 3$. In addition, we allow countries to trade in periods $t = 2$ and 3; yet, for simplicity, we now assume trade takes place only in the event of peace that period. Otherwise, conflict ensues with a division of insecure output according to each country's appropriative share, ϕ^i, as in the baseline model. The main complication that arises in this setting is that arming and saving choices made in $t = 1$ now must take into account their effects on the rival's future income and thus on its future arming and saving choices. As a benchmark for comparison, we also produced results for a two-period model with a similar structure, i.e., one where trade occurs in period $t = 2$ only in the event that a dispute does not arise. For both of these models, numerical analysis shows that the range of relative endowment sizes for which the large country prefers not to trade can vanish when σ is sufficiently small, similar to what we found for the above model that also features trade in the future. The main effect of adding another time period is to mitigate the adverse consequences of first-period trade for the larger country when the initial size difference is moderate but magnify them when size differences are sufficiently large. Thus, although adding a third period tends to reduce the range of relative endowment sizes for which the larger country prefers not to trade in the first period, the effect on its preferences toward trade in the limit where the large country becomes infinitely large is generally ambiguous, depending on parameter values.

Three countries. While our central finding that the larger country could prefer not to trade with its potential rival holds in a variety of different trade models, one might wonder if it remains intact in the presence of a third country that does not participate in disputes. In Online Appendix G, we extend our baseline model to allow for three countries, each producing a distinct intermediate good.[33] To fix ideas, we think of countries $i = 1$ and 2 as rivals and country $i = 3$ as the rest of the world (ROW). Furthermore, to keep the analysis as simple as possible and to facilitate comparisons with the baseline model, we return to our two-period setting, assuming that trade can take place only in period $t = 1$ and that the potential conflict between countries $i = 1$ and 2 arises in period $t = 2$. Within this setting, we compare the larger adversarial country's payoffs under 3 alternative trade regimes for period $t = 1$: (i) global free trade; (ii) an embargo on the smaller adversarial country by the larger adversarial country, with free trade between ROW and each of the two adversaries; (iii) a blockade on the smaller adversary, with free trade only between ROW and the larger adversary and without any added costs relative to an embargo. Numerical analysis reveals that, even when trade with a third country (ROW) is possible, the larger adversarial country could prefer to embargo trade with its rival for the same reasons identified in the baseline model. However, the presence of ROW does matter here. In particular, the range of relative resource endowments for which the large country prefers an embargo over free

[33] We thank a referee for suggesting this extension to us.

trade is decreasing in ROW's size. This result largely follows from our existing arguments, since trade with ROW increases the income of the smaller adversary relative to that of the larger one. But, at the same time, the larger country tends to prefer a blockade to an embargo for any initial endowment distribution for which an embargo is preferred to free trade. Furthermore, unlike with an embargo, the range of relative sizes for which a blockade would be preferred to free trade is increasing in the size of ROW. Intuitively, the blockade has the effect of reducing the smaller country's relative size as compared with the case of an embargo—and by more when ROW is larger.

6 Concluding Remarks

Trade and security are inseparable pillars of international policy. Yet the study of international trade largely abstracts from how the vast sums that are spent on national defense are affected by the wealth that is created by international commerce. Similarly, the conflict literature lacks theoretical frameworks that formally model how changes in relative wealth translate into changes in relative power and how this relationship depends on how countries allocate their respective resources across arming versus other, more productive activities. In this paper, we analyze a dynamic, two-country model of trade and arming interactions, where counties arm to prepare for an uncertain future. Notably, we show how arming decisions reflect not only the economic capabilities of each country, but also how the marginal benefit of more productive investments (i.e., saving) varies with the degree of uncertainty.

A key implication of the theory is that larger countries will find trade in the shadow of a possible future conflict with smaller rivals unappealing when the difference in *ex ante* size is sufficiently large. This prediction derives generally from the nonlinearity that occurs, for a given trade regime, in the relationship between "size" and "power" when the probability of future conflict is nonzero. Thus, while the threat of conflict could itself be a source of "power" for an *ex ante* small country, it could also undermine that country's ability to realize the possible gains from trade with larger rivals. This last observation would seem particularly salient for informing conflict management policies in situations where disproportionate compensation to seemingly weaker rivals would be necessary to entice them to improve diplomatic relations.

Our model has, by design, leaned on an income channel as the primary linkage between trade and arming. We conjecture that richer insights could be gained by amending the production structure of our model so that arms are produced from the same resources used to produce the tradable inputs, thereby introducing a terms-of-trade channel and a factor-price channel. With such a modification, these added channels could serve to modulate the costliness of arming to the point where similarly sized economies refuse to trade, whereas in the present paper this result only occurs for sufficiently dissimilar economies. Furthermore, while our comparison of outcomes under autarky and free trade has shed light on the desirability of trade in the shadow of an uncertain future, our analysis has remained silent on the implications of activist trade policies. In particular, countries could influence the security policies of their potential rivals and thus their own power by adjusting trade flows and prices through appropriate unilateral or bilateral commercial policies. We leave these possibilities for future research.

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References

Anbarci, N., Skaperdas, S., and Syropoulos, C. (2002). Comparing bargaining solutions in the shadow of conflict: How norms against threats can have real effects. *Journal of Economic Theory*, 106(1):1–16.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *Staff Papers (International Monetary Fund)*, 16(1):159–178.

Bolt, J. and van Zanden, J. L. (2020). Maddison style estimates of the evolution of the world economy. A new 2020 update. *Maddison Project Working Paper 15.*

Bonfatti, R. and O'Rourke, K. H. (2018). Growth, import dependence, and war. *Economic Journal*, 128:2222–2257.

Brawley, M. R. (2004). The political economy of balance of power theory. In Paul, T. V., Wirtz, J. J., and Fortmann, M., editors, *Balance of Power: Theory and Practice in the 21st Century.* Stanford University.

Cain, F. (2005). Computers and the Cold War: United States restrictions on the export of computers to the Soviet Union and communist China. *Journal of Contemporary History*, 40(1):131–147.

Carter, A. (2015). Remarks on the next phase of the US rebalance to the Asia-Pacific. *McCain Institute, Arizona State University, April*, 6.

Copeland, D. C. (2015). *Economic Interdependence and War*. Princeton University Press.

Dal Bó, E. and Dal Bó, P. (2011). Workers, warriors, and criminals: Social conflict in general equilibrium. *Journal of the European Economic Association*, 9(4):646–677.

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779.

Garfinkel, M. R., Skaperdas, S., and Syropoulos, C. (2008). Globalization and domestic conflict. *Journal of International Economics*, 76(2):296–308.

Garfinkel, M. R., Skaperdas, S., and Syropoulos, C. (2015). Trade and insecure resources. *Journal of International Economics*, 95(1):98–114.

Garfinkel, M. R., Syropoulos, C., and Yotov, Y. V. (2020). Arming in the global economy: The importance of trade with enemies and friends. *Journal of International Economics*, 123.

Gilpin, R. (1981). *War and Change in World Politics*. Cambridge University Press.

Glick, R. and Taylor, A. M. (2010). Collateral damage: Trade disruption and the economic impact of war. *Review of Economics and Statistics*, 92(1):102–127.

Gonzalez, F. M. (2005). Insecure property and technological backwardness. *The Economic Journal*, 115(505):703–721.

Gowa, J. (1995). *Allies, Adversaries, and International Trade*. Princeton University Press.

Grieco, J. M. (1990). *Cooperation Among Nations: Europe, America, and Non-tariff Barriers to Trade*. Cornell University Press.

Hegre, H. (2004). Size asymmetry, trade, and militarized conflict. *Journal of Conflict Resolution*, 48(3):403–429.

Hirshleifer, J. (1991). The paradox of power. *Economics and Politics*, 3(3):177–200.

Hosoya, C. (1968). Miscalculations in deterrent policy: Japanese-US relations, 1938-1941. *Journal of Peace Research*, 5(2):97–115.

Jackson, M. O. and Morelli, M. (2007). Political bias and war. *American Economic Review*, 97(4):1353–1373.

Kennedy, P. (1980). *The Rise of the Anglo-German Antagonism, 1860-1914*. Allen & Unwin.

Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *American Economic Review*, 70(5):950–959.

Liberman, P. (1996). Trading with the enemy: Security and relative economic gains. *International Security*, 21(1):147–175.

Martin, P., Mayer, T., and Thoenig, M. (2008). Make trade not war? *Review of Economic Studies*, 75(3):865–900.

Mastanduno, M. (1985). Strategies of economic containment: US trade relations with the Soviet Union. *World Politics*, 37(4):503–531.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725.

Morelli, M. and Sonno, T. (2017). On economic interdependence and war. *Journal of Economic Literature*, 55(3):1084–97.

Polachek, S. W. (1980). Conflict and trade. *Journal of Conflict Resolution*, 24(1):55–78.

Powell, R. (1991). Absolute and relative gains in international relations theory. *American Political Science Review*, 85(04):1303–1320.

Saltzman, I. Z. (2012). Soft balancing as foreign policy: Assessing American strategy toward japan in the interwar period. *Foreign Policy Analysis*, 8(2):131–150.

Seitz, M., Tarasov, A., and Zakharenko, R. (2015). Trade costs, conflicts, and defense spending. *Journal of International Economics*, 95(2):305–318.

Seligmann, M. S. (2017). Failing to prepare for the Great War? The absence of grand strategy in British war planning before 1914. *War in History*, 24(4):414–437.

SIPRI (2019). Trends in world military expenditure, 2018. Fact Sheet, Stockholm International Peace Research Institute,.

Skaperdas, S. (1996). Contest success functions. *Economic Theory*, 7(2):283–290.

Skaperdas, S. and Syropoulos, C. (1997). The distribution of income in the presence of appropriative activities. *Economica*, 64:101–117.

Skaperdas, S. and Syropoulos, C. (2001). Guns, butter, and openness: On the relationship between security and trade. *American Economic Review*, 91(2):353–357.

Steiner, Z. S. (1977). *Britain and the Origins of the First World War.* St. Martin's Press.

U.S. Munitions Board (1949). National security aspects of export controls. Technical Report CD36-1-1, Records of the Secretary of Defense, RG330, NARA.

Waltz, K. N. (1979). *Theory of International Politics.* Waveland Press.

Williamson, S. R. (1969). *The Politics of Grand Strategy: Britain and France Prepare for War, 1904-1914.* Harvard University Press.

Online Appendix for "Prudence versus Predation and the Gains from Trade"

This online appendix consists of 7 sections. In the first, we present an extended version of the baseline model that allows for partial security of output in the event of conflict. Section B contains proofs of the lemmas and propositions that hold in this more general setting; it also presents supplementary lemmas and propositions (along with their proofs) as well as additional results related to partial output security. In Section C, we provide additional details to help the reader work through some of the more tedious calculations. Section D contains brief notes in support of our claim that the possibility the larger country could prefer autarky over trade extends to other standard models of trade. Section E outlines the key ideas regarding war as a "winner-take-all" contest and peaceful settlement under alternative rules of division; it also explores the implications of the possibility of trade in the second period. In Section F, we analyze numerically a three-period version of the model that shows our central result survives with at least one added period. Finally, Section G checks the robustness of our central result to the addition of a third (friendly) trading partner.

A Allowing for Partial Security of Output under Conflict

The model presented in the main text assumes that all output generated from the countries' first-period saving is contested in the event that conflict emerges in the second period. Here, we present a slighted modified (and more general) version that allows for imperfect security. More precisely, we assume as before that, when peace prevails, which occurs with probability $1-q$, country i enjoys its entire output: $\widetilde{C}^i = Z^i$, $i = 1, 2$. However, if a conflict arises, which occurs with probability q, a fraction $1-\kappa$ of a country i's output continues to be secure; only the remaining fraction goes into a contested pool, $\kappa\left(Z^i + Z^j\right)$, $i, j \in \{1, 2\}$, $i \neq j$, where $\kappa \in (0, 1]$ indexes the degree of output insecurity. Then, country i's expected two-period payoff becomes

$$\begin{aligned} U^i = {} & \ln\left(Y^i - G^i - Z^i\right) \\ & + \delta\left[q \ln\left(\phi^i \kappa\left[Z^i + Z^j\right] + (1-\kappa) Z^i\right) + (1-q)\ln\left(Z^i\right)\right]. \end{aligned}$$

The salience of a possible future conflict is now governed by two parameters: the probability of a future conflict $q > 0$ as before and the insecurity of output in the case of conflict $\kappa > 0$. When either $q = 0$ or $\kappa = 0$, the model simplifies to one without conflict and thus $G^i = 0$ for $i = 1, 2$.

Each country i's first-period arming and saving choices satisfy FOCs analogous to those shown in equation (6) in the main text. The marginal costs of arming and saving, $MC^i_G = MC^i_Z = 1/(Y^i - G^i - Z^i)$, remain unchanged. However, the associated marginal benefits do change:

$$MB^i_G = \delta q \frac{\phi^i_{G^i}\kappa\left(Z^i + Z^j\right)}{\phi^i\kappa\left(Z^i + Z^j\right) + (1-\kappa)Z^i} \tag{A.1a}$$

$$MB^i_Z = \delta\left[\frac{q\left(\phi^i\kappa + 1 - \kappa\right)}{\phi^i\kappa\left(Z^i + Z^j\right) + (1-\kappa)Z^i} + \frac{1-q}{Z^i}\right]. \tag{A.1b}$$

As expected, while MB^i_G is increasing in output insecurity κ, MB^i_Z is decreasing in κ. Importantly, country i's payoff remains strictly concave in G^i (i.e., $U^i_{G^iG^i} < 0$) and in Z^i (i.e., $U^i_{Z^iZ^i} < 0$). Nonetheless, a number of notable differences arise when $\kappa < 1$. First, the qualitative influence of country j's arming on MB^i_G now depends on both countries' arming and saving decisions, as well as the degree of output insecurity κ. In particular, when $\kappa < 1$, $U^i_{G^iG^j} > 0$ continues to hold if either $G^i \geq G^j > 0$ or G^i and G^j are sufficiently similar; otherwise, $U^i_{G^iG^j} < 0$. Second, MB^i_G is increasing in Z^j and decreasing in Z^i, implying that country i tends to be more aggressive in its security policy if its rival j $(\neq i)$ saves more, but arms by less as its own saving increases. Finally, when $\kappa < 1$, MB^i_Z is increasing in G^j and decreasing in G^i.[1]

With the expressions above, the definition of $\theta^i = Z^i/(Z^i + Z^j)$ for $i, j \in \{1, 2\}$, $i \neq j$ and the conflict technology specified in equation (2), we follow the strategy of the main text to obtain our two equilibrium conditions, requiring that $MB^i_G = MC^i_G$ and $MB^i_Z = MC^i_Z$ hold simultaneously:

$$S^i\left(\phi^i, \theta^i; q, \kappa\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}}\left[\frac{1-\kappa+\kappa\left(1-q+q\theta^i\right)\left(\phi^i/\theta^i\right)}{1-\kappa+\kappa\left(1-q+q\theta^j\right)\left(\phi^j/\theta^j\right)}\right] - 1 = 0 \tag{A.2}$$

$$B^i\left(\phi^i, \theta^i; Y^i/Y^j, q, \kappa\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{1/m} - \frac{\gamma^i\left(1+\gamma^j+\zeta^j\right)}{\gamma^j\left(1+\gamma^i+\zeta^i\right)}\left(\frac{Y^i}{Y^j}\right) = 0, \tag{A.3}$$

where the coefficients γ and ζ that govern the allocation per unit of income to arming and saving according to equation (10) are now given by

$$\gamma^i = \gamma^i\left(\phi^i, \theta^i\right) \equiv \delta q m\left(1-\phi^i\right)\frac{\kappa\phi^i}{\kappa\phi^i + (1-\kappa)\theta^i} > 0 \tag{A.4a}$$

$$\zeta^i = \zeta^i\left(\phi^i, \theta^i\right) \equiv \delta\left[q\frac{\kappa\phi^i\theta^i + (1-\kappa)\theta^i}{\kappa\phi^i + (1-\kappa)\theta^i} + 1 - q\right] > 0. \tag{A.4b}$$

As one can easily confirm, these key equations simplify to those in the main text, respectively (9) and (12) with (11), when $\kappa = 1$.

[1] Observe further that allowing for the possibility that conflict destroys some fraction of all period $t = 2$ output would have no effect on the marginal benefits of arming or saving nor on the associated marginal costs. Thus, incorporating conflict's destructive effect would be inconsequential for equilibrium choices.

B Proofs and Additional Lemmas and Propositions

Proof of Lemma 1. For the analysis to follow, we rewrite $S^i = 0$ in (A.2) as:

$$S^i(\cdot) \equiv E^i - H^i = 0, \tag{B.1a}$$

where for $i, j \in \{1, 2\}$, $i \neq j$

$$E^i \equiv \left(\frac{\phi^i}{\phi^j}\right)^{1/m} \tag{B.1b}$$

$$H^i \equiv \left(\frac{\theta^i}{\theta^j}\right)\left(\frac{\Lambda^j}{\Lambda^i}\right) \tag{B.1c}$$

$$\Lambda^r \equiv (1-\kappa)\,\theta^r + \kappa\phi^r\left(1 - q + q\theta^r\right) \in (0,1)\,, \quad r = i, j. \tag{B.1d}$$

Part (a): Partially differentiating the E^i and H^i components of $S^i(\cdot) = 0$ in (B.1), while keeping in mind that $\phi^j = 1 - \phi^i$ and $\theta^j = 1 - \theta^i$, yields

$$\frac{E^i_{\phi^i}}{E^i} = \sum_{r=i,j}\left[\frac{1}{m\phi^r}\right] = \frac{1}{m\phi^i\phi^j} > 0 \tag{B.2a}$$

$$E^i_{\theta^i} = 0 \tag{B.2b}$$

$$\frac{H^i_{\phi^i}}{H^i} = \frac{\Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\Lambda^i_{\phi^i}}{\Lambda^i} = -\sum_{r=i,j}\left[\frac{\kappa\left(1-q+q\theta^r\right)}{\Lambda^r}\right] < 0 \tag{B.2c}$$

$$\frac{H^i_{\theta^i}}{H^i} = \frac{1}{\theta^i\theta^j} + \frac{\Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\Lambda^i_{\theta^i}}{\Lambda^i} = \sum_{r=i,j}\left[\frac{\kappa\left(1-q\right)\phi^r/\theta^r}{\Lambda^r}\right] > 0, \tag{B.2d}$$

for $\phi^i \in (0, 1)$. From (B.1a), we have $S^i_\eta = E^i_\eta - H^i_\eta$ for $\eta = \phi^i, \theta^i$. Then, the signs of the above expressions imply that $S^i_{\phi^i} > 0$, $S^i_{\theta^i} < 0$. By the implicit function theorem and the requirement that $E^i = H^i$ along the S^i-contour, we have

$$\left.\frac{d\theta^i}{d\phi^i}\right|_{S^i=0} = -\frac{S^i_{\phi^i}}{S^i_{\theta^i}} = \frac{\frac{1}{m\phi^i\phi^j} + \sum_{r=i,j}\left[\frac{\kappa(1-q+q\theta^r)}{\Lambda^r}\right]}{\sum_{r=i,j}\left[\frac{\kappa(1-q)\phi^r/\theta^r}{\Lambda^r}\right]} > 0. \tag{B.3}$$

Thus, the S^i-contour is increasing in $\phi^i \in (0, 1)$, as stated in part (a) and depicted in Fig. 1 in the main text.

Part (b): By substituting the expressions for Λ^r shown in (B.1d) into (B.3) and rearranging some, one can verify that the following holds along the S^i-contour:

$$\left.\frac{d\theta^i}{d\phi^i}\right|_{S^i=0} = \left[\frac{\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(1+\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\theta^i}{\theta^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}\right]\frac{\theta^i}{\phi^i}. \tag{B.4}$$

The first point of part (b) can be established by evaluating the expression in the square brackets above as $\phi^i \to 0$, which implies $\phi^j \to 1$ and, from part (d) shown below, $\theta^i/\phi^i \to 0$ and $\theta^j/\phi^j \to 1$ along the S^i-contour, to find it simplifies to a finite term, $1 + \frac{1}{m}$; since $\theta^i/\phi^i \to 0$ as $\phi^i \to 0$, the entire expression in (B.4) goes to 0. One can similarly show that $\lim_{\phi^i \to 1} d\theta^i/d\phi^i = 0$. The last point in part (b) follows by evaluating the expression in (B.4) at $\phi^i = \theta^i = \frac{1}{2}$. After rearranging, the expression becomes

$$\lim_{\phi^i \to \frac{1}{2}} \left(\left. \frac{d\theta^i}{d\phi^i} \right|_{S^i=0} \right) = 1 + \frac{1}{m} + \frac{1 - \kappa + \kappa q \left(1+m\right)/2}{\kappa m \left(1-q\right)} > 1.$$

One should now be able to see that schedule S^i has an inflection point at $\phi^i = 1/2$.

The above relationship reveals that schedule S^i tends to be very steep at $\phi^i \to \frac{1}{2}$ (implying that the flats at both endpoints of the S^i-contour are tend to be long) when either the probability of conflict q is very high or the degree of output insecurity κ is very low. In either case, adjustments along schedule S^i for moderate values of ϕ^i (perhaps due to changes in income levels as analyzed below) will involve primarily changes in saving shares θ^i, with power (captured by ϕ^i) being relatively unresponsive.

Part (c): By the definition of E^i, we have $E^i \gtreqqless 1 \iff \phi^i/\phi^j \gtreqqless 1$. The requirement that $S^i = 0$ implies $E^i = H^i$, and thus $E^i \gtreqqless 1 \iff H^i \gtreqqless 1$ or, equivalently, after combining (B.1c) and (B.1d),

$$E^i = \frac{\phi^i}{\phi^j} \gtreqqless 1 \iff H^i = \frac{1 - \kappa + \kappa \left(1 - q + q\theta^j\right) \left(\phi^j/\theta^j\right)}{1 - \kappa + \kappa \left(1 - q + q\theta^i\right) \left(\phi^i/\theta^i\right)} \gtreqqless 1, \tag{B.5}$$

along the S^i-contour. The second set of inequalities in the line above can be rewritten as follows:

$$\left(1-q\right)\left(\theta^i - \phi^i\right) \gtreqqless q\theta^i\theta^j \left(\phi^i - \phi^j\right). \tag{B.6}$$

Now, suppose that $\phi^i = \phi^j = \frac{1}{2}$, implying that $E^i = 1$ and thus (B.6) must hold as an equality. Accordingly, $\theta^i = \phi^i = \frac{1}{2}$ holds. Next, suppose that $\phi^i > \phi^j$, implying $E^i > 1$. Since $H^i > 1$ must hold, (B.6) requires that $\theta^i > \phi^i$ hold. Conversely, if $\phi^i < \phi^j$, we have $E^i < 1$, requiring that $H^i < 1$, and thus from (B.6) $\theta^i < \phi^i$.

Part (d): Recall that $\phi^j = 1 - \phi^i$, $\theta^j = 1 - \theta^i$ and $m \in (0, 1]$. Now suppose $\phi^i \to 0$ which implies $\phi^j \to 1$. Since $\theta^i < \phi^i$, from part (c), it must the case that θ^i is arbitrarily close to 0 as well; that is, $\theta^i \to 0$ which implies $\theta^j \to 1$. Thus, $\theta^j/\phi^j \to 1$ as $\phi^i \to 0$. Next, observe from (B.1b) that $\phi^i \to 0$ also implies $E^i \to 0$. However, the definition of $S^i = 0$ in (B.1a) implies that H^i must also converge to 0. Since $\phi^j/\theta^j \to 1$, the numerator of H^i in (B.5) is finite. Thus, $H^i \to 0$ only if the denominator of H^i becomes infinitely large as $\phi^i \to 0$. Inspection of (B.5) readily reveals that $\lim_{\phi^i \to 0} H^i = 0$ only if $\phi^i/\theta^i \to \infty$ or, equivalently, if $\theta^i/\phi^i \to 0$. This suggests that the S^i-contour becomes flat as $\phi^i \to 0$. The second portion of part (d) should be obvious. ||

The next lemma builds on Lemma 1 to provide additional implications for ϕ^i and θ^i along the S^i-contour that are useful for proving some of the propositions below. In what follows, we use a hat ($\wedge$) over variables denote percent changes (e.g., $\widehat{x} \equiv dx/x$).

Lemma B.1 *Percent changes in shares, $\widehat{\theta}^i$ and $\widehat{\phi}^i$, along the S^i-contour have the following properties:*

(a) (i) $\lim_{\phi^i \to 0} \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} = 1 + \frac{1}{m}$

(ii) $\lim_{\phi^i \to \frac{1}{2}} \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} = 1 + \frac{1}{m} + \frac{1-\kappa+\kappa q(1+m)/2}{m\kappa(1-q)}$

(iii) $\lim_{\phi^i \to 1} \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} = 0.$

(b) $\lambda \times \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} > 1 + \frac{1}{m}$ *where* $\lambda = 1$ *for* $\phi^i \in \left(0, \frac{1}{2}\right]$ *and* $\lambda = \frac{\theta^i \phi^j}{\theta^j \phi^i}$ *for* $\phi^i \in \left[\frac{1}{2}, 1\right)$.

(c) $\arg\max_{\phi^i} \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} \in \left(0, \frac{1}{2}\right)$.

Proof: Noting $\widehat{\theta}^i/\widehat{\phi}^i = (d\theta^i/d\phi^i)(\phi^i/\theta^i)$, the proof is based on the expression for $d\theta^i/d\phi^i|_{S^i=0}$ shown in (B.4) pre-multiplied by ϕ^i/θ^i:

$$\left.\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\right|_{S^i=0} = \frac{\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(1+\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\theta^i}{\theta^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}. \quad \text{(B.7)}$$

Part (a): The three items of this part are established by taking the appropriate limits of the expression above. (See the proof of Lemma 1(b) for more details.)

Part (b): This part of the lemma divides the parameter space for ϕ^i as follows: (i) $\phi^i \in (0, \frac{1}{2}]$, where we set $\lambda = 1$; and, (ii) $\phi^i \in [\frac{1}{2}, 1)$, where we set $\lambda = \theta^i\phi^j/\theta^j\phi^i$:

(i) For $\phi^i \in (0, \frac{1}{2}]$ (or $\lambda = 1$), Lemma 1(c) implies $\phi^i/\phi^j > \theta^i/\theta^j$. Then, substituting ϕ^i/ϕ^j for θ^i/θ^j in the denominator of (B.7) gives

$$\begin{aligned}\left.\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\right|_{S^i=0} &> \frac{\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(1+\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}} \\ &> 1 + \frac{1}{m}\left[\frac{1+\left(\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}\right] \\ &> 1 + \frac{1}{m}.\end{aligned}$$

The inequality on the first line immediately follows, since $\theta^i, \theta^j > 0$ and $\phi^i/\phi^j > \theta^i/\theta^j$. The inequality in the second line can be confirmed by comparing the various components that appear in the numerator and the denominator in the right hand side (RHS) of the first inequality. The inequality in the third line is obtained by applying the same logic to the expressions inside the square brackets of the second inequality.

(ii) For $\phi^i \in [\frac{1}{2}, 1)$ (or $\lambda = \theta^i\phi^j/\theta^j\phi^i$), Lemma 1(c) implies $\lambda > 1$. Pre-multiply (B.7) by

λ. After some simplifying, the resulting expression shows

$$\begin{aligned}\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\bigg|_{S^i=0} &> 1+\frac{1}{m}\left[\frac{1+\left(\frac{\theta^i}{\theta^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)}+\left(\frac{\theta^i}{\theta^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}\right] \\ &> 1+\frac{1}{m}.\end{aligned}$$

The inequality on the first line can be confirmed by using the facts that $\theta^i/\theta^j > \phi^i/\phi^j$ in this case and $\theta^i, \theta^j > 0$ and comparing the various components of the numerator and denominator of the resulting expression (not shown). The inequality on the second line can similarly be confirmed by comparing the components of the expressions that appear in the numerator and the denominator of the RHS of the first inequality.

Part (c): Parts (a) and (b) suggest that, even though both ϕ^i and θ^i rise along the S^i-contour, the increase in θ^i tapers off after some value of ϕ^i and vanishes as $\phi^i \to \frac{1}{2}$. Part (c) asserts that $\widehat{\theta}^i/\widehat{\phi}^i|_{S^i=0}$ attains a maximum prior to arriving at $\phi^i = \frac{1}{2}$. To see this point, note that $\widehat{\theta}^i/\widehat{\phi}^i|_{S^i=0} = \left(\frac{d\theta^i/d\phi^i}{\theta^i/\phi^i}\right)\Big|_{S^i=0}$ and that its rate of change along the S^i-contour is

$$\left(\widehat{\widehat{\theta}^i/\widehat{\phi}^i}\right) = (\widehat{d\theta^i/d\phi^i}) - (\widehat{\theta^i/\phi^i}).$$

When evaluated at $\phi^i = \frac{1}{2}$, the first term in the RHS equals 0 because, by symmetry, the S^i-contour has an inflection point $\phi^i = \frac{1}{2}$. Furthermore, at $\phi^i = \frac{1}{2}$, $(\widehat{\theta^i/\phi^i}) > 0$ by part (a), suggesting that $\widehat{\theta}^i/\widehat{\phi}^i|_{S^i=0}$ is decreasing as $\phi^i \to \frac{1}{2}$, thus establishing part (c). ||

Proof of Lemma 2. It is convenient to rewrite (A.3) as

$$B^i(\cdot) \equiv E^i - F^i = 0, \tag{B.8a}$$

where E^i was defined in (B.1b) and, for $i, j \in \{1,2\}$, $i \neq j$,

$$F^i \equiv \left(\frac{\Omega^j}{\Omega^i}\right)\left(\frac{Y^i}{Y^j}\right) \tag{B.8b}$$

$$\Omega^r \equiv (1+\delta)(1-\kappa)\theta^r + \kappa\phi^r\left[1+\delta(1-q)+q\delta(\theta^r+m(1-\phi^r)\right] > 0, \; r = i, j. \tag{B.8c}$$

The expression for F^i in (B.8b) is obtained by substituting the values of γ^i and ζ^i described in (A.4) into the second term in (A.3). Partial differentiation of F^i gives

$$\frac{F^i_{\phi^i}}{F^i} = \frac{\Omega^j_{\phi^i}}{\Omega^j} - \frac{\Omega^i_{\phi^i}}{\Omega^i} = -\kappa\sum_{r=i,j}\left[\frac{1+\delta(1-q)+q\delta\left(\theta^r+m-2m\phi^r\right)}{\Omega^r}\right] < 0 \tag{B.9a}$$

$$\frac{F^i_{\theta^i}}{F^i} = \frac{\Omega^j_{\theta^i}}{\Omega^j} - \frac{\Omega^i_{\theta^i}}{\Omega^i} = -\sum_{r=i,j}\left[\frac{(1-\kappa)(1+\delta)+\kappa q\delta\phi^r}{\Omega^r}\right] < 0. \tag{B.9b}$$

By definition (B.8a), $B^i_\eta = E^i_\eta - F^i_\eta$ for $\eta = \phi^i, \theta^i$. Since $E^i_{\phi^i} > 0$ and $F^i_{\phi^i} < 0$, we have $B^i_{\phi^i} > 0$. Similarly, since $E^i_{\theta^i} = 0$ and $F^i_{\theta^i} < 0$, $B^i_{\theta^i} > 0$ holds. Thus, applying the implicit function theorem to (B.8a), while keeping in mind that $E^i = F^i$ where $B^i = 0$, yields

$$\left.\frac{d\theta^i}{d\phi^i}\right|_{B^i=0} = -\frac{B^i_{\phi^i}}{B^i_{\theta^i}} = -\frac{\frac{1}{m\phi^i\phi^j} + \sum_{r=i,j}\left[\frac{\kappa[1+\delta(1-q)+q\delta(\theta^r+m-2m\phi^r)]}{\Omega^r}\right]}{\sum_{r=i,j}\left[\frac{(1-\kappa)(1+\delta)+\kappa q\delta\phi^r}{\Omega^r}\right]} < 0,$$

which implies that θ^i and ϕ^i are negatively related along schedule B^i.

Next, consider the values of ϕ^i along B^i in the extremes where $\theta^i = 0$ and $\theta^i = 1$. One can verify rom (B.8) and (B.1b) that, for $\theta^i = 0$, we have $\lim_{\phi^i \to 0} B^i < 0$ and $\lim_{\phi^i \to 1} B^i > 0$. Since $B^i_{\phi^i} > 0$, there exists a unique value of $\phi^i \in (0,1)$ such that $B^i(\cdot) = 0$. One can also verify that the same argument applies for $\theta^i = 1$. We can thus conclude that, at $\theta^i = 0$ and $\theta^i = 1$, schedule B^i cuts the horizontal axes, as illustrated in Fig. 1 in the main text. ||

Proof of Proposition 1. Totally differentiating the $S^i(\cdot) = 0$ and $B^i(\cdot) = 0$ conditions in (B.1) and (B.8) respectively, while focusing on percent changes, allows us to rewrite the system of equations involving changes in these contours in a more convenient way for the purpose of proving this proposition and others to follow. Letting $y^i \equiv Y^i/Y^j$, we have

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} \widehat{\phi}^{i*} \\ \widehat{\theta}^{i*} \end{pmatrix} + \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix}\widehat{y}^i + \begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix}\widehat{\kappa} + \begin{pmatrix} b_{13} \\ b_{23} \end{pmatrix}\widehat{q} + \begin{pmatrix} b_{14} \\ b_{24} \end{pmatrix}\widehat{\delta} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad \text{(B.10)}$$

From (B.1) and (B.8) with (B.2) and (B.9), the a-coefficients can be written as[2]

$$a_{11} = \frac{\phi^i E^i_{\phi^i}}{E^i} - \frac{\phi^i H^i_{\phi^i}}{H^i} = \frac{\phi^i E^i_{\phi^i}}{E^i} + \frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} - \frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j}$$

$$= \frac{1}{m\phi^j} + \kappa\phi^i\left[\sum_{r=i,j}\frac{1-q+q\theta^r}{\Lambda^r}\right] > 0 \quad \text{(B.11a)}$$

$$a_{12} = \frac{\theta^i E^i_{\theta^i}}{E^i} - \frac{\theta^i H^i_{\phi^i}}{H^i} = -\frac{1}{\theta^j} + \frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j}$$

$$= -\frac{\kappa(1-q)}{\theta^j}\left[\sum_{r=i,j}\frac{\phi^r(1-\theta^r)}{\Lambda^r}\right] < 0 \quad \text{(B.11b)}$$

$$a_{21} = \frac{\phi^i E^i_{\phi^i}}{E^i} - \frac{\phi^i F^i_{\phi^i}}{F^i} = \frac{\phi^i E^i_{\phi^i}}{E^i} + \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}$$

$$= \frac{1}{m\phi^j} + \kappa\phi^i\left[\sum_{r=i,j}\frac{1+(1-q)\delta+q\delta[\theta^r+m(1-2\phi^r)]}{\Omega^r}\right] > 0 \quad \text{(B.11c)}$$

$$a_{22} = \frac{\theta^i E^i_{\theta^i}}{E^i} - \frac{\theta^i F^i_{\phi^i}}{F^i} = \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} - \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}$$

$$= \theta^i\left[\sum_{r=i,j}\frac{(1-\kappa)(1-q)+\kappa q\delta\phi^r}{\Lambda^r}\right] > 0. \quad \text{(B.11d)}$$

[2]For more details on deriving the individual components, see equations (C.2) and (C.3) presented in Appendix C.

We derive the b-coefficients below.

The proof of existence and uniqueness of equilibrium follows readily from Lemmas 1 and 2, which described the properties of the schedules S^i and B^i, respectively. In particular, the shapes of these schedules imply that they will intersect once and only once since

$$\mathcal{D} \equiv \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \overset{(+)}{a_{11}}\overset{(+)}{a_{22}} - \overset{(-)}{a_{12}}\overset{(+)}{a_{21}} > 0. \tag{B.12}$$

That the point $(\phi^{i*}, \theta^{i*}) \in (0,1) \times (0,1)$ is unique means we can recover G^{i*} and Z^{i*} from it using equation (10) in the main text with (A.4), thereby completing this part of the proof. Henceforth, we drop "$*$" from equilibrium values to avoid cluttering.

Part (a): As discussed in the text, the S^i-contour is independent of the two countries' first-period relative incomes $y^i \equiv Y^i/Y^j$, always going through the midpoint where $\phi^i = \theta^i = \frac{1}{2}$ for $i, j \in \{1, 2\}$, $i \neq j$. By contrast, the B^i-contour goes through that midpoint only when $y^i = 1$, shifting out as y^i increases above 1 and shifting in as y^i decreases below 1. More formally, to establish this part of the proposition, set $\widehat{\kappa} = \widehat{q} = \widehat{\delta} = 0$ and solve (B.10) to obtain:

$$\begin{pmatrix} \widehat{\phi}^i \\ \widehat{\theta}^i \end{pmatrix} = \frac{1}{\mathcal{D}} \begin{pmatrix} -a_{22}b_{11} + a_{12}b_{21} \\ a_{21}b_{11} - a_{11}b_{21} \end{pmatrix} \widehat{y}^i, \tag{B.13}$$

where, from (B.1) and (B.8), the coefficients on $\widehat{y}^i$ in (B.10) are given by

$$b_{11} = \frac{y^i E^i_{y^i}}{E^i} - \frac{y^i H^i_{y^i}}{H^i} = 0 \tag{B.14a}$$

$$b_{21} = \frac{y^i E^i_{y^i}}{E^i} - \frac{y^i F^i_{y^i}}{F^i} = -1. \tag{B.14b}$$

Then, substituting these expressions along with those for a_{11} and a_{12} shown respectively in (B.11a) and (B.11b) into (B.13) gives

$$\widehat{\phi}^i = -\frac{a_{12}}{\mathcal{D}}\widehat{y}^i = \frac{1}{\mathcal{D}} \left[\frac{\kappa(1-q)}{\theta^j} \left(\sum_{r=i,j} \frac{\phi^r(1-\theta^r)}{\Lambda^r} \right) \right] \widehat{y}^i > 0 \tag{B.15a}$$

$$\widehat{\theta}^i = \frac{a_{11}}{\mathcal{D}}\widehat{y}^i = \frac{1}{\mathcal{D}} \left[\frac{1}{m\phi^j} + \kappa\phi^i \left(\sum_{r=i,j} \frac{1-q+q\theta^r}{\Lambda^r} \right) \right] \widehat{y}^i > 0. \tag{B.15b}$$

Thus, both ϕ^i and θ^i rise with increases in y^i and fall with decreases in y^i.

Part (b): Observe that upward shifts in schedule B^i shown in Fig. 1 in the main text (induced by increases in y^i) trace out the points along the S^i-contour. The limit results in this part of the proposition, then, follow from parts (c) and (d) of Lemma 1. Interestingly, the shape of this contour implies the ratio θ^i/ϕ^i rises initially with increases in y^i, but eventually falls with y^i once y^i crosses a certain threshold.

Part (c): We start by focusing on the first component of this part related to the probability q of a future dispute arising and some additional results related to the degree of output insecurity $\kappa \in (0,1]$ when a dispute does arise. Set $\widehat{y}^i = \widehat{\delta} = 0$ and solve (B.10) for $\widehat{\phi}^i$ to obtain:

$$\widehat{\phi}^i = \frac{1}{\mathcal{D}}\left[\,(-a_{22}b_{12} + a_{12}b_{22})\,\widehat{\kappa} + (-a_{22}b_{13} + a_{12}b_{23})\,\widehat{q}\,\right]. \tag{B.16}$$

Since E^i does not depend on κ, (B.1) and (B.8) imply $b_{12} \equiv -\kappa H^i_\kappa/H^i$ and $b_{22} \equiv -\kappa F^i_\kappa/F^i$ or equivalently[3]

$$\begin{aligned}
b_{12} &= \frac{\kappa\Lambda^i_\kappa}{\Lambda^i} - \frac{\kappa\Lambda^j_\kappa}{\Lambda^j} = \frac{\kappa\phi^i\left(1-q+q\theta^i\right) - \kappa\theta^i}{\kappa\phi^i\left(1-q+q\theta^i\right) + (1-\kappa)\,\theta^i} \\
&\qquad - \frac{\kappa\phi^j\left(1-q+q\theta^j\right) - \kappa\theta^j}{\kappa\phi^j\left(1-q+q\theta^j\right) + (1-\kappa)\,\theta^j} \\
b_{22} &= \frac{\kappa\Omega^i_\kappa}{\Omega^i} - \frac{\kappa\Omega^j_\kappa}{\Omega^j} = \frac{\kappa\phi^i\left[1+\delta\,(1-q) + \delta q\left(\theta^i + m\phi^j\right)\right] - \kappa\,(1+\delta)\,\theta^i}{\kappa\phi^i\left[1+\delta\,(1-q) + \delta q\left(\theta^i + m\phi^j\right)\right] + (1-\kappa)\,(1+\delta)\,\theta^i} \\
&\qquad - \frac{\kappa\phi^j\left[1+\delta\,(1-q) + \delta q\left(\theta^j + m\phi^i\right)\right] - \kappa\,(1+\delta)\,\theta^j}{\kappa\phi^j\left[1+\delta\,(1-q) + \delta q\left(\theta^j + m\phi^i\right)\right] + (1-\kappa)\,(1+\delta)\,\theta^j}.
\end{aligned} \tag{B.17a}$$

Similarly, since E^i does not depend on q, (B.1) and (B.8) imply $b_{13} \equiv -qH^i_q/H^i$ and $b_{23} \equiv -qF^i_q/F^i$ or[4]

$$\begin{aligned}
b_{13} &= \frac{q\Lambda^i_q}{\Lambda^i} - \frac{q\Lambda^j_q}{\Lambda^j} = \frac{\kappa q\theta^i\phi^j}{\kappa\phi^j\left(1-q+q\theta^j\right) + (1-\kappa)\,\theta^j} \\
&\qquad - \frac{\kappa q\theta^j\phi^i}{\kappa\phi^i\left(1-q+q\theta^i\right) + (1-\kappa)\,\theta^i} \\
b_{23} &= \frac{q\Omega^i_q}{\Omega^i} - \frac{q\Omega^j_q}{\Omega^j} = \frac{\kappa q\delta\left(m\phi^j - \theta^j\right)\phi^i}{\kappa\phi^i\left[1+\delta\,(1-q) + \delta q\left(\theta^i + m\phi^j\right)\right] + (1-\kappa)\,(1+\delta)\,\theta^i} \\
&\qquad - \frac{\kappa q\delta\left(m\phi^i - \theta^i\right)\phi^j}{\kappa\phi^j\left[1+\delta\,(1-q) + \delta q\left(\theta^j + m\phi^i\right)\right] + (1-\kappa)\,(1+\delta)\,\theta^j}.
\end{aligned} \tag{B.17b}$$

We have already shown that $a_{11} > 0$, $a_{12} < 0$, $a_{21} > 0$, $a_{22} > 0$ and $\mathcal{D} > 0$. Furthermore, from part (a), we have $\phi^i = \phi^j$ and $\theta^i = \theta^j$ when $Y^i = Y^j$. Thus, inspection of the expressions in (B.17a) and (B.17b) reveals that $b_{12} = b_{22} = b_{13} = b_{23} = 0$ when $Y^i = Y^j$, so that ϕ^{i*} and θ^{i*} are not affected by changes in κ or q in this symmetric case.

To characterize how the equilibrium distribution of power ϕ^i depends on κ and q for asymmetric distributions of income, we focus on the case where $Y^i < Y^j$, and establish that $b_{12} > 0$ and $b_{22} > 0$ in (B.17a), while $b_{13} < 0$ and $b_{23} < 0$ in (B.17b).[5] We start with b_{12},

[3] For more details on deriving the individual components, see equation (C.4) presented in Appendix C.

[4] Again, see equation (C.4) presented in Appendix C.

[5] The proof for $Y^i > Y^j$ can be established along similar lines, showing $b_{12} < 0$ and $b_{22} < 0$, while $b_{13} > 0$ and $b_{23} > 0$ in this case.

which can be simplified, by adding and subtracting $(1-\kappa)\theta^i$ from the numerator of the first term and then adding and subtracting $(1-\kappa)\theta^j$ from the numerator of the second term on the far RHS of the first expression in (B.17a), as follows:

$$b_{12} = 1 - \frac{\theta^i}{\Lambda^i} - 1 + \frac{\theta^j}{\Lambda^j} = \frac{\theta^j}{\Lambda^j} - \frac{\theta^i}{\Lambda^i}. \tag{B.18}$$

To evaluate the sign of this expression, recall from (B.1) that $E^i = H^i$ or equivalently $(\phi^i/\phi^j)^{\frac{1}{m}} \left(\frac{\theta^j \Lambda^i}{\theta^i \Lambda^j}\right) = 1$ along the S^i-contour. Furthermore, we have

$$\frac{\theta^j}{\Lambda^j} > \frac{\theta^i}{\Lambda^i} \iff 1 > \frac{\theta^i \Lambda^j}{\theta^j \Lambda^i}.$$

Now multiply both sides of the second inequality above by $(\phi^i/\phi^j)^{\frac{1}{m}} \left(\frac{\theta^j \Lambda^i}{\theta^i \Lambda^j}\right) = 1$ to find

$$1 > \frac{\theta^i \Lambda^j}{\theta^j \Lambda^i} \times \left[\left(\phi^i/\phi^j\right)^{\frac{1}{m}} \left(\frac{\theta^j \Lambda^i}{\theta^i \Lambda^j}\right)\right] = \left(\phi^i/\phi^j\right)^{\frac{1}{m}}.$$

Our assumption that $Y^i < Y^j$ implies, from part (a), $\phi^i < \phi^j$. Thus, the latter inequality holds, and $b_{12} > 0$.

Turning to b_{22}, we use a similar procedure used above for b_{12}. Specifically, we first add and subtract $(1-\kappa)(1+\delta)\theta^i$ from the numerator of the first term, and then add and subtract $(1-\kappa)(1+\delta)\theta^j$ from the numerator of the second term on the far RHS of the second expression in (B.17a). After simplifying, we obtain:

$$b_{22} = 1 - \frac{(1+\delta)\,\theta^i}{\Omega^i} - 1 + \frac{(1+\delta)\,\theta^j}{\Omega^j} = (1+\delta)\left[\frac{\theta^j}{\Omega^j} - \frac{\theta^i}{\Omega^i}\right]. \tag{B.19}$$

To evaluate the sign of the far RHS expression, first note that

$$\frac{\theta^j}{\Omega^j} > \frac{\theta^i}{\Omega^i} \iff 1 > \frac{\theta^i \Omega^j}{\theta^j \Omega^i}.$$

Following our strategy above to evaluate the sign of b_{12}, we multiply both sides of the second inequality above by $(\phi^i/\phi^j)^{\frac{1}{m}} \times \left(\frac{\theta^j \Lambda^i}{\theta^i \Lambda^j}\right) = 1$ (as required along the S^i-contour) to rewrite it as

$$1 > \frac{\theta^i \Omega^j}{\theta^j \Omega^i} \times \left[\left(\phi^i/\phi^j\right)^{\frac{1}{m}} \left(\frac{\theta^j \Lambda^i}{\theta^i \Lambda^j}\right)\right] = \left(\phi^i/\phi^j\right)^{\frac{1}{m}} \frac{\Omega^j \Lambda^i}{\Omega^i \Lambda^j}.$$

The definitions of the relevant variables imply that the expression on the far RHS of the inequality is a function of (ϕ^i, θ^i). We now argue that the last inequality holds true for values of $\phi^i \in \left(0, \frac{1}{2}\right)$ and $\theta^i \in \left(0, \frac{1}{2}\right)$ and not just the combinations of ϕ^i and θ^i along the S^i-contour. To see this, consider any $\phi^i \in \left(0, \frac{1}{2}\right)$ and suppose that $\theta^i = \phi^i$ initially. It is straightforward for one to verify (after some tedious but straightforward calculations) that

$$\left.\frac{\Omega^j\Lambda^i}{\Omega^i\Lambda^j}\right|_{\theta^i=\phi^i\in\left(0,\frac{1}{2}\right)} = 1+\frac{\kappa q\,(1+\delta m)\,\phi^i\phi^j\,(\phi^i-\phi^j)}{\Omega^i\Lambda^j}<1,$$

thereby suggesting that the inequality of interest is satisfied at any $\phi^i\in\left(0,\frac{1}{2}\right)$ when $\theta^i=\phi^i$. We now show that, for any given $\phi^i\in\left(0,\frac{1}{2}\right)$, $\frac{\Omega^j\Lambda^i}{\Omega^i\Lambda^j}$ is increasing in θ^i:

$$\begin{aligned}\left(\widehat{\frac{\Omega^j\Lambda^i}{\Omega^i\Lambda^j}}\right)/\widehat{\theta^i} &= \frac{\theta^i\Lambda^i_{\theta^i}}{\Lambda^i}-\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}-\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j}+\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\\ &= \frac{\theta^i\left(1-\kappa+\kappa q\phi^i\right)}{\Lambda^i}-\frac{\theta^i\left[(1-\kappa)(1+\delta)+\kappa q\delta\phi^i\right]}{\Omega^i}\\ &\quad+\frac{\theta^i\left(1-\kappa+\kappa q\phi^j\right)}{\Lambda^j}-\frac{\theta^i\left[(1-\kappa)(1+\delta)+\kappa q\delta\phi^j\right]}{\Omega^j}\\ &= \frac{\kappa q\theta^i\phi^i\left[(1-\kappa)\left(1+\delta m\phi^j\right)+\kappa\phi^i\left(1+q\delta m\phi^j\right)\right]}{\Lambda^i\Omega^i}\\ &\quad+\frac{\kappa q\theta^i\phi^j\left[(1-\kappa)\left(1+\delta m\phi^i\right)+\kappa\phi^j\left(1+q\delta m\phi^i\right)\right]}{\Lambda^j\Omega^j}>0.\end{aligned}$$

Since the above expression is positive, a decrease in θ^i implies that the inequality will hold true for all $\theta^i\leq\phi^i\in\left(0,\frac{1}{2}\right)$, including the points along the S^i-contour. In turn, we have that $b_{22}>0$ holds.

Our findings that $b_{12}>0$ and $b_{22}>0$ in the context of (B.16) imply that $d\phi^i/d\kappa<0$ (when $Y^i<Y^j$).[6] An increase in the degree of insecurity κ causes the S^i-contour to rotate clockwise around the point where $\phi^i=\theta^i=\frac{1}{2}$ with the endpoints $(0,0)$ and $(1,1)$ unchanged; the B^i-contour also rotates clockwise with a pivot point located at its intersection with the 45° line when $m=1$ and below that intersection for $m<1$.[7]

Turning to the effect of changes in the probability of conflict q, one can easily confirm from the first expression in (B.17b) that b_{13} can be written as

$$b_{13}=\kappa q\left[\frac{\theta^i}{\Lambda^j/\phi^j}-\frac{\theta^j}{\Lambda^i/\phi^i}\right]. \tag{B.20}$$

To sign this expression, first note that, from part (a) of this proposition, our assumption that $Y^i<Y^j$ implies $\theta^i<\theta^j$. Hence, a sufficient condition for $b_{13}<0$ is that $\Lambda^i/\phi^i<\Lambda^j/\phi^j$.

[6] Applying analogous reasoning, one can show that, when $Y^i>Y^j$, $b_{12}<0$ and $b_{22}<0$, and thus $d\phi^i/d\kappa>0$.

[7] Recall that for points to the right and below the S^i-contour $S^i>0$, while points to the right and above the B^i-contour imply $B^i>0$. To verify the pivot point for the S^i-contour, evaluate the expression for b_{12} in (B.18) at the midpoint, to find that it equals zero. Similarly, to verify the pivot point of the B^i-contour, evaluate the expression in b_{22} in (B.19) at any $\phi^i=\theta^i<\frac{1}{2}$ (i.e., along the lower segment of the 45° line). Some straightforward algebra reveals that the sign of this expression equals sign$\{(\phi^i-\phi^j)(1-m)\}$. Thus, the pivot point for the B^i-contour lies on the 45° line when $m=1$ and (since by assumption $Y^i<Y^j$ and thus $\phi^i<\phi^j$) lies below it when $m<1$.

To proceed, observe that

$$\Lambda^i/\phi^i = (1-\kappa)\left(\theta^i/\phi^i\right) + \kappa\left[1 - q + q\theta^i\right],$$

which enables us to form the difference

$$\Lambda^i/\phi^i - \Lambda^j/\phi^j = (1-\kappa)\left[\frac{\theta^i}{\phi^i} - \frac{\theta^j}{\phi^j}\right] + \kappa q\left[\theta^i - \theta^j\right] < 0.$$

The negative sign of the above expressions follows again from part (a) of the proposition, that $Y^i < Y^j$ implies $\theta^i/\phi^i - \theta^j/\phi^j < 0$ and $\theta^i - \theta^j < 0$. (Note that the inequality above also implies that $\Lambda^i - \Lambda^j < 0$ for $Y^i < Y^j$.) Hence, $b_{13} < 0$.

Turning to the last coefficient of interest, note that b_{23} in the second expression of (B.17b) can be written as

$$b_{23} = \kappa q\delta\left[\frac{\theta^i - m\phi^i}{\Omega^j/\phi^j} - \frac{\theta^j - m\phi^j}{\Omega^i/\phi^i}\right]. \tag{B.21}$$

To confirm that the sign of the expression above is negative, first note that $\theta^i - m\phi^i - (\theta^j - m\phi^j) = \phi^i\left(\frac{\theta^i}{\phi^i} - m\right) - \phi^j\left(\frac{\theta^j}{\phi^j} - m\right)$, which is negative, since $Y^i < Y^j$ implies $\phi^i < \phi^j$ and $\theta^i/\phi^i < \theta^j/\phi^j$ from part (a) of the proposition. Thus, $\theta^i - m\phi^i < \theta^j - m\phi^j$, and we need only to establish that $(\Omega^i/\phi^i) < (\Omega^j/\phi^j)$. Noting that

$$\Omega^i/\phi^i = (1-\kappa)(1+\delta)\left(\theta^i/\phi^i\right) + \kappa\left[1 + (1-q)\delta + q\delta\left(\theta^i + m\phi^j\right)\right]$$

for $i, j \in \{1,2\}$, $i \neq j$, we find

$$\Omega^i/\phi^i - \Omega^j/\phi^j = (1-\kappa)(1+\delta)\left[\frac{\theta^i}{\phi^i} - \frac{\theta^j}{\phi^j}\right] + \kappa q\delta\left[\theta^i + m\phi^j - \theta^j - m\phi^i\right] < 0.$$

The above inequality is obtained from our finding immediately above that $\theta^i - m\phi^i < \theta^j - m\phi^j$ and the assumption that $Y^i < Y^j$ which implies (again, by part (a) of the proposition) that $\theta^i/\phi^i - \theta^j/\phi^j < 0$. (The inequality above also implies that $\Omega^i - \Omega^j < 0$ for $Y^i < Y^j$.) Thus, $b_{23} < 0$.

An application of the observations that $b_{13} < 0$ and $b_{23} < 0$ to (B.16) gives $d\phi^i/dq > 0$ (when $Y^i < Y^j$).[8] The effect of an increase in q can be seen graphically as a counter-clockwise rotation of the S^i-contour around the $\phi^i = \theta^i = \frac{1}{2}$ pivot point; at the same time, the B^i-contour rotates in a counterclockwise direction around a pivot point that lies on its intersection with the 45° line when $m = 1$ and above (below) it when $m < 1$ and $Y^i < Y^j$ $(Y^i > Y^j)$.[9]

[8] Taking a similar approach, one can establish that $b_{13} > 0$ and $b_{23} > 0$ and thus $d\phi^i/dq < 0$ when $Y^i > Y^j$.

[9] Along the same logic spelled out in footnote 7, one can verify these pivot points using the expressions

Let us now consider what happens to the S^i-contour as $\kappa \to 0$ such that property becomes very secure. From the expression for H^i shown in (B.5), we have $\lim_{\kappa\to 0} H^i = 1$ for all $\theta^i \in (0,1)$ or equivalently when $y^i \in (0,\infty)$. Since $E^i = (\phi^i/\phi^j)^{1/m}$, the equilibrium condition $S^i = E^i - H^i = 0$ can be satisfied only for $\phi^i = \phi^j = \frac{1}{2}$. As property becomes perfectly secure, the S^i-contour becomes vertical for all $\theta^i \in (0,1)$ at $\phi^i = \frac{1}{2}$, such that the positioning of the B^i-contour alone determines the equilibrium value of θ^i; specifically, $\theta^i \gtreqless \frac{1}{2}$ when $Y^i \gtreqless Y^j$. Next, to establish the limit result in part (b) of the proposition, consider what happens as $q \to 1$. From the expression for H^i again shown in (B.5), we have $\lim_{q\to 1} H^i = (1-\kappa\phi^i)/(1-\kappa\phi^j)$ for all $\theta^i \in (0,1)$. As before, the equilibrium condition $S^i = E^i - H^i = 0$ can be satisfied only at $\phi^i = \phi^j = \frac{1}{2}$, and the positioning of the B^i-contour in turn pins down the equilibrium value for θ^i, again with $\theta^i \gtreqless \frac{1}{2}$ when $Y^i \gtreqless Y^j$.

To prove the remaining components of part (c) of the proposition related to the discount factor δ, we set $\widehat{y}^i = \widehat{q} = \widehat{\kappa} = 0$ and solve (B.10), whereby we obtain:

$$\begin{pmatrix} \widehat{\phi}^i \\ \widehat{\theta}^i \end{pmatrix} = \frac{1}{\mathcal{D}} \begin{pmatrix} -a_{22}b_{14} + a_{12}b_{24} \\ a_{21}b_{14} - a_{11}b_{24} \end{pmatrix} \widehat{\delta}, \tag{B.22}$$

where the a-coefficients are given in equation (B.11). Since neither E^i nor H^i depends on δ, $b_{14} = 0$. Thus, we need only to sign the coefficient b_{24}.

Let us define $\widehat{\delta} \equiv \frac{d\delta}{1+\delta}$, which implies $b_{24} = -(1+\delta)F^i_\delta/F^i$ or equivalently

$$\begin{aligned} b_{24} &= -\frac{(1+\delta)\,\Omega^j_\delta}{\Omega^j} + \frac{(1+\delta)\,\Omega^i_\delta}{\Omega^i} = \left[\frac{(1+\delta)\,\Omega^j_\delta}{\Omega^j} - 1\right] + \left[\frac{(1+\delta)\,\Omega^i_\delta}{\Omega^i} - 1\right] \\ &= \left[\frac{\kappa q \phi^j\left(\theta^i - m\phi^i\right)}{\Omega^j}\right] - \left[\frac{\kappa q \phi^i\left(\theta^j - m\phi^j\right)}{\Omega^i}\right] \\ &= \kappa q \phi^i \phi^j \left[\frac{\theta^i/\phi^i - m}{\Omega^j} - \frac{\theta^j/\phi^j - m}{\Omega^i}\right]. \end{aligned} \tag{B.23}$$

Recall that $a_{12} < 0$ and $a_{11} > 0$. Therefore, to establish this part of the proof with our maintained focus on the case where $Y^i < Y^j$, it suffices to show that $b_{24} < 0$. From our analysis in the proof of part (b), we know that $Y^i < Y^j$ implies $\Omega^i < \Omega^j$ (which, in turn, implies $1/\Omega^j < 1/\Omega^i$). Furthermore, from part (a), $Y^i < Y^j$ implies $\theta^i/\phi^i < \theta^j/\phi^j$. These two results taken together imply $b_{24} < 0$ and thus $d\phi^i/d\delta > 0$ and $d\theta^i/d\delta > 0$ when $Y^i < Y^j$.[10] The effect in an increase in δ can be illustrated as a counterclockwise rotation of the B^i-contour around a pivot point that lies on the 45° line when $m = 1$ and above (below) it when $Y^i < Y^j$ ($Y^i > Y^j$).[11] $\quad||$

for b_{13} and b_{23} shown respectively in (B.20) and (B.21).

[10] One can similarly establish that when $Y^i > Y^j$, $b_{24} > 0$, so that $d\phi^i/d\delta < 0$ and $d\theta^i/d\delta < 0$ in this case.

[11] Once again, the reader can confirm these pivot points applying the logic spelled out in footnote 7 with (B.23).

Proposition B.1 *The influences of changes in insecurity of property κ and the probability of a future conflict q on equilibrium shares in the cases of perfect symmetry and extreme asymmetry are as follows:*

(a) *When $Y^i = Y^j = Y \in (0, \infty)$, the equilibrium shares are $\phi^{i*} = \theta^{i*} = \frac{1}{2}$ for all $\kappa \in (0, 1]$ and $q \in (0, 1)$.*

(b) *For given $Y^j \in (0, \infty)$, $\lim_{Y^i \to 0} \widehat{\phi}^{i*}/\widehat{\kappa} < 0$, $\lim_{Y^i \to 0} \widehat{\phi}^{i*}/\widehat{q} > 0$, $\lim_{Y^i \to 0} \widehat{\theta}^{i*}/\widehat{\kappa} = 0$, and $\lim_{Y^i \to 0} \widehat{\theta}^{i*}/\widehat{q} < 0$.*

Proof: The system of equations in (B.10) implies

$$\widehat{\phi}^i = \frac{1}{\mathcal{D}}\left[\,(-a_{22}b_{12} + a_{12}b_{22})\,\widehat{\kappa} + (-a_{22}b_{13} + a_{12}b_{23})\,\widehat{q}\,\right] \tag{B.24a}$$

$$\widehat{\theta}^i = \frac{1}{\mathcal{D}}\left[\,(-a_{21}b_{12} + a_{11}b_{22})\,\widehat{\kappa} + (-a_{21}b_{13} + a_{11}b_{23})\,\widehat{q}\,\right], \tag{B.24b}$$

where $\mathcal{D} = a_{11}a_{22} - a_{12}a_{21} > 0$ and the a- and b-coefficients are shown respectively in (B.11) and (B.17).

Part (a): In the proof of Proposition 1(c), we have already established that, when $Y^i = Y^j > 0$, $\phi^i = \theta^i = \frac{1}{2}$, such that from (B.17) $b_{12} = b_{22} = b_{13} = b_{23} = 0$. Thus, evaluating the expressions in (B.24) where $Y^i = Y^j$ shows that changes in in κ and q have no effects on the equilibrium values of ϕ^i and θ^i.

Part (b): For the case of extreme asymmetry, we evaluate the expressions in (B.24) in the limit as $Y^i \to 0$ with $Y^j \in (0, \infty)$. First, using the calculations shown in Appendix C, one can find the appropriate limits of the a-coefficients in (B.11):[12]

$$\lim_{Y^i \to 0} a_{11} = 1 + \frac{1}{m}, \ \lim_{Y^i \to 0} a_{12} = -1, \ \lim_{Y^i \to 0} a_{21} = 1 + \frac{1}{m}, \text{ and } \lim_{Y^i \to 0} a_{22} = 0, \tag{B.25}$$

and

$$\lim_{Y^i \to 0} \mathcal{D} = \lim_{Y^i \to 0} \left[a_{11}a_{22} - a_{12}a_{21}\right] = 1 + \frac{1}{m}. \tag{B.26}$$

Similarly, using (B.17), one can establish the following for the b-coefficients:[13]

$$\lim_{Y^i \to 0} b_{12} = \lim_{Y^i \to 0} b_{22} = 1, \ \lim_{Y^i \to 0} b_{13} = -\frac{q}{1-q}, \text{ and } \lim_{Y^i \to 0} b_{23} = \frac{q\delta(m-1)}{1 + \delta + q\delta(m-1)}.$$

With these results and (B.25) and (B.26), one can take the limit of each expression in (B.24) to find

$$\lim_{Y^i \to 0} \widehat{\theta}^{i*} = -\frac{q}{1-q}\left[\frac{1 + \delta m}{1 + \delta - q\delta\,(1-m)}\right]\widehat{q} \tag{B.27a}$$

[12] The derivation, based on equations (C.6) and (C.7), is shown in (C.8).
[13] For details, see the derivation of (C.10) using (C.9), as presented in Appendix C.

$$\lim_{Y^i \to 0} \widehat{\phi}^{i*} = \frac{m}{1+m}\left[-\widehat{\kappa} + \frac{q\delta\,(1-m)}{1+\delta - q\delta\,(1-m)}\widehat{q}\right], \tag{B.27b}$$

thereby confirming the signs of the limits stated in the proposition. (Note that these findings for $\widehat{\phi}^{i*}$ are consistent with the results stated in Proposition 1(c) in the case that $0 < Y^i < Y^j < \infty$.) ||

Proof of Proposition 2. To begin, we use (A.4) with (B.1d) and (B.8c), to rewrite the expressions for G^j and Z^j shown in equation (10) in the main text:

$$G^j = \kappa m q \left(\frac{\phi^i \phi^j}{\Omega^j}\right) \delta Y^j \tag{B.28a}$$

$$Z^j = \left(\frac{\Lambda^j}{\Omega^j}\right) \delta Y^j, \tag{B.28b}$$

for $j = 1, 2$, where Ω^j was defined in (B.8c) and Λ^j was defined in (B.1d). Henceforth, we assume that $\widehat{Y}^i > \widehat{Y}^j \geq 0$, which implies $\widehat{y}^i > 0$. Noting that $\theta^j = 1 - \theta^i$ and $\phi^j = 1 - \phi^i$, logarithmic differentiation of the above quantities (after some rearranging) gives[14]

$$\widehat{Y}^j - \widehat{G}^j = \left(\frac{\overset{(-)}{\theta^i \Omega^j_{\theta^i}}}{\Omega^j}\right)\widehat{\theta}^i - \left(\frac{\phi^j - \phi^i}{\phi^j} - \frac{\overset{(-)}{\phi^i \Omega^j_{\phi^i}}}{\Omega^j}\right)\widehat{\phi}^i \tag{B.29a}$$

$$\widehat{Z}^j - \widehat{Y}^j = \left(\frac{\overset{(-)}{\theta^i \Lambda^j_{\theta^i}}}{\Lambda^j} - \frac{\overset{(-)}{\theta^i \Omega^j_{\theta^i}}}{\Omega^j}\right)\widehat{\theta}^i + \left(\frac{\overset{(-)}{\phi^i \Lambda^j_{\phi^i}}}{\Lambda^j} - \frac{\overset{(-)}{\phi^i \Omega^j_{\phi^i}}}{\Omega^j}\right)\widehat{\phi}^i, \tag{B.29b}$$

and

$$\widehat{Y}^i - \widehat{G}^i = \left(\frac{\overset{(+)}{\theta^i \Omega^i_{\theta^i}}}{\Omega^i}\right)\widehat{\theta}^i + \left(\frac{\phi^i - \phi^j}{\phi^j} + \frac{\overset{(+)}{\phi^i \Omega^i_{\phi^i}}}{\Omega^i}\right)\widehat{\phi}^i \tag{B.30a}$$

$$\widehat{Y}^i - \widehat{Z}^i = -\left(\frac{\overset{(+)}{\theta^i \Lambda^i_{\theta^i}}}{\Lambda^i} - \frac{\overset{(+)}{\theta^i \Omega^i_{\theta^i}}}{\Omega^i}\right)\widehat{\theta}^i - \left(\frac{\overset{(+)}{\phi^i \Lambda^i_{\phi^i}}}{\Lambda^i} - \frac{\overset{(+)}{\phi^i \Omega^i_{\phi^i}}}{\Omega^i}\right)\widehat{\phi}^i. \tag{B.30b}$$

For clarity, we break the analysis into parts (a) and (b). In part (a) we show $\widehat{Z}^j < \widehat{Y}^j < \widehat{G}^j$ when y^i increases. Then, in part (b), we show $\widehat{G}^i < \widehat{Y}^i < \widehat{Z}^i$ when y^i increases. In both parts, it is necessary to distinguish between two cases: (i) $Y^i \leq Y^j$ and (ii) $Y^i > Y^j$. We complete the proof by showing that $\widehat{G}^j < \widehat{G}^i$ when y^i increases.

[14] More details regarding the individual terms in the coefficients on $\widehat{\phi}^i$ and $\widehat{\theta}^i$ in the expressions to follow in (B.30) and (B.29), including their signs, can be found in (C.2) and (C.3) presented in Appendix C.

Part (a): ($\widehat{Z}^j < \widehat{Y}^j < \widehat{G}^j$) The following calculations fill out the details for the coefficients on $\widehat{\theta}^i$ and $\widehat{\phi}^i$ in the expressions for $\widehat{G}^j$ and $\widehat{Z}^j$ in (B.29):

$$\begin{aligned}
\frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j} &= -\frac{\theta^i \left[(1-\kappa)(1+\delta) + \kappa q \delta \phi^j\right]}{\Omega^j} < 0 \\
\frac{\phi^j - \phi^i}{\phi^j} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j} &= \frac{\theta^j (1-\kappa)(1+\delta)(\phi^j - \phi^i) + \kappa \left[1 + \delta\left(1 - q + q\theta^j\right)\right]\left(\phi^j\right)^2}{\phi^j \Omega^j} \\
\frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j} &= -\frac{\kappa q \theta^i \phi^j}{\Lambda^j \Omega^j}\left[(1-\kappa)\left(1 + m\delta\phi^i\right) + \kappa\phi^j\left(1 + mq\delta\phi^i\right)\right] < 0 \\
\frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j} &= -\frac{\kappa q \phi^i \theta^j}{\Lambda^j \Omega^j}\left[(1-\kappa)\left[m\delta\left(\phi^j - \phi^i\right) - \theta^i\right] + \kappa m \delta\left(1 - q + q\theta^j\right)\phi^j\left(\phi^j/\theta^j\right)\right].
\end{aligned}$$

Using the expressions above in (B.29) reveals that

$$\operatorname{sign}\left\{\widehat{Y}^j - \widehat{G}^j\right\} = -\operatorname{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i + \Xi^j_G\right\} \tag{B.31a}$$

$$\operatorname{sign}\left\{\widehat{Z}^j - \widehat{Y}^j\right\} = -\operatorname{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i + \Xi^j_Z\right\}, \tag{B.31b}$$

where

$$\begin{aligned}
\Xi^j_G &\equiv -\left(\frac{\phi^j - \phi^i}{\phi^j} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}\right) \Big/ \left(\frac{\theta^i \Omega^j_{\theta}}{\Omega^j}\right) \\
&= \frac{(1-\kappa)(1+\delta)\theta^j\left(\phi^j - \phi^i\right) + \kappa\left[1 + \delta\left(1 - q + q\theta^j\right)\right]\left(\phi^j\right)^2}{\theta^i \phi^j \left[(1-\kappa)(1+\delta) + \kappa q \delta \phi^j\right]} \qquad \text{(B.32a)} \\
&> -\frac{\theta^j \phi^i}{\theta^i \phi^j}\left[\frac{(1-\kappa)(1+\delta)}{(1-\kappa)(1+\delta) + \kappa q \delta \phi^j}\right] > -\frac{\theta^j \phi^i}{\theta^i \phi^j}
\end{aligned}$$

and

$$\begin{aligned}
\Xi^j_Z &\equiv \left(\frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}\right) \Big/ \left(\frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}\right) \\
&= \left(\frac{\phi^i \theta^j}{\phi^j}\right)\left\{\frac{(1-\kappa)\left[-\theta^i + m\delta\left(\phi^j - \phi^i\right)\right] + \kappa m \delta\left(1 - q + q\theta^j\right)\phi^j\left(\phi^j/\theta^j\right)}{\theta^i\left[(1-\kappa)\left(1 + m\delta\phi^i\right) + \kappa\phi^j\left(1 + mq\delta\phi^i\right)\right]}\right\}. \qquad \text{(B.32b)}
\end{aligned}$$

(i) $y^i \equiv Y^i/Y^j \leq 1$: Proposition 1(a) implies $\phi^j - \phi^i \geq 0$ which in the context of (B.32a), in turn, implies $\Xi^j_G > 0$. In addition, Lemma B.1(b) implies $\widehat{\theta}^i/\widehat{\phi}^i > 0$ as y^i increases. Therefore, from (B.31a), $\widehat{Y}^j < \widehat{G}^j$ holds when $y^i \leq 1$ initially.

Based on (B.29b), one might conjecture that the sign of $\widehat{Z}^j - \widehat{Y}^j$ in (B.31b) is ambiguous. However, because $\phi^i/\phi^j \leq 1$ in this case, the expression inside the curly brackets in (B.32b) implies

$$\Xi^j_Z > -\left(\frac{\phi^i \theta^j}{\phi^j}\right)\left\{\frac{(1-\kappa)}{(1-\kappa)\left(1 + m\delta\phi^i\right) + \kappa\phi^j\left(1 + mq\delta\phi^i\right)}\right\} > -\left(\frac{\phi^i}{\phi^j}\right)\theta^j > -1.$$

The above inequality, together with the finding in Lemma B.1(b) that $\widehat{\theta}^i/\widehat{\phi}^i > 1 + \frac{1}{m}$ for $\widehat{y}^i > 0$ implies $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^j > 0$. Hence, from (B.31b), we have $\widehat{Z}^j < \widehat{Y}^j$. This inequality together with our finding that $\widehat{Y}^j < \widehat{G}^j$ establishes that $\widehat{Z}^j < \widehat{Y}^j < \widehat{G}^j$ holds as y^i increases when $y^i \leq 1$ initially.

(ii) $y^i \equiv Y^i/Y^j > 1$: From Proposition 1(a), we have $\phi^j - \phi^i < 0$ and $\frac{\theta^i\phi^j}{\theta^j\phi^i} > 1$ in this case. Furthermore, from (B.32a) we have $\Xi_G^j > -\left(\frac{\phi^i\theta^j}{\theta^i\phi^j}\right)$. Multiplying $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_G^j$ in the curly brackets of (B.31a) by $\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)$ (> 1) gives

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) + \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_G^j > \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) - 1 > 0.$$

The last inequality in the above expression holds true because $\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) > 1 + \frac{1}{m}$ (part (b) of Lemma B.1 for $\lambda = \frac{\theta^i\phi^j}{\theta^j\phi^i}$). Thus, $\widehat{Y}^j < \widehat{G}^j$ holds in this case, too, when y^i increases.

To identify the sign of $\widehat{Z}^j - \widehat{Y}^j$ when $y^i > 1$ initially, note from (B.32b) that

$$\Xi_Z^j > -\left(\frac{\phi^i\theta^j}{\theta^i\phi^j}\right)\left\{\frac{(1-\kappa)\left[\theta^i + m\delta\phi^i\right]}{(1-\kappa)\left(1+m\delta\phi^i\right) + \kappa\phi^j\left(1+mq\delta\phi^i\right)}\right\} > -\left(\frac{\phi^i\theta^j}{\theta^i\phi^j}\right).$$

Following our strategy above for $\widehat{G}^j$, we multiply $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^j$ in the curly brackets in (B.31b) by $\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)$ to obtain

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) + \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_Z^j > \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) - 1 > 0,$$

where, again, the last inequality follows from part (b) in Lemma B.1. Thus, $\widehat{Z}^j < \widehat{Y}^j$ which, together with our finding that $\widehat{Y}^j < \widehat{G}^j$, implies $\widehat{Z}^j < \widehat{Y}^j < \widehat{G}^j$ as y^i increases for $y^i > 1$ initially.

Part (b): ($\widehat{G}^i < \widehat{Y}^i < \widehat{Z}^i$). The following expressions provide details on the coefficients of $\widehat{\theta}^i$ and $\widehat{\phi}^i$ in the expressions for $\widehat{G}^i$ and $\widehat{Z}^i$ in (B.30):

$$\begin{aligned}
\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i} &= \frac{\theta^i\left[(1-\kappa)(1+\delta) + \kappa q\delta\phi^i\right]}{\Omega^i} > 0 \\
\frac{\phi^i - \phi^j}{\phi^j} + \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i} &= \frac{\theta^i(1-\kappa)(1+\delta)\left(\phi^i - \phi^j\right) + \kappa\left[1 + \delta\left(1 - q + q\theta^i\right)\right]\left(\phi^i\right)^2}{\phi^j\Omega^i} \\
\frac{\theta^i\Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i} &= \frac{\kappa q\phi^i\theta^i}{\Lambda^i\Omega^i}\left[(1-\kappa)\left(1 + m\delta\phi^j\right) + \kappa\phi^i\left(1 + mq\delta\phi^j\right)\right] > 0 \\
\frac{\phi^i\Lambda^i_{\phi^i}}{\Lambda^i} - \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i} &= \frac{\kappa q\phi^i\theta^i}{\Lambda^i\Omega^i}\left[(1-\kappa)\left[m\delta\left(\phi^i - \phi^j\right) - \theta^j\right] + \kappa m\delta\left(1 - q + q\theta^i\right)\phi^i\left(\phi^i/\theta^i\right)\right].
\end{aligned}$$

Using the expressions above in (B.30) gives

$$\operatorname{sign}\left\{\widehat{Y}^i - \widehat{G}^i\right\} = \operatorname{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i + \Xi_G^i\right\} \tag{B.33a}$$

$$\operatorname{sign}\left\{\widehat{Z}^i - \widehat{Y}^i\right\} = \operatorname{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^i\right\}, \tag{B.33b}$$

where

$$\begin{aligned}\Xi_G^i &\equiv \frac{\frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i\Omega_{\phi^i}^i}{\Omega^i}}{\frac{\theta^i\Omega_{\theta^i}^i}{\Omega^i}} = \frac{\theta^i\left(1-\kappa\right)\left(1+\delta\right)\left(\phi^i-\phi^j\right) + \kappa\left[1+\delta\left(1-q+q\theta^i\right)\right]\left(\phi^i\right)^2}{\theta^i\phi^j\left[\left(1-\kappa\right)\left(1+\delta\right)+\kappa q\delta\phi^i\right]} \\ &> -\frac{\left(1-\kappa\right)\left(1+\delta\right)}{\left(1-\kappa\right)\left(1+\delta\right)+\kappa\delta q\phi^i} > -1\end{aligned}$$

and

$$\Xi_Z^i \equiv \frac{\frac{\phi^i\Lambda_{\phi^i}^i}{\Lambda^i} - \frac{\phi^i\Omega_{\phi^i}^i}{\Omega^i}}{\frac{\theta^i\Lambda_{\theta^i}^i}{\Lambda^i} - \frac{\theta^i\Omega_{\theta^i}^i}{\Omega^i}} = \frac{\left[\theta^i\left(1-\kappa\right)\left[-\theta^j + m\delta\left(\phi^i-\phi^j\right)\right] + \kappa m\delta\left(1-q+q\theta^i\right)\left(\phi^i\right)^2\right]}{\theta^i\left[\left(1-\kappa\right)\left(1+m\delta\phi^j\right)+\kappa\phi^i\left(1+mq\delta\phi^j\right)\right]}.$$

(i) $y^i \equiv Y^i/Y^j \leq 1$: Lemma B.1(b) shows $\widehat{\theta}^i/\widehat{\phi}^i > 1 + \frac{1}{m}$ for $\phi^i \in (0, \frac{1}{2}]$, which holds true for $y^i \leq 1$. In addition, as noted in the definition of Ξ_G^i above we have $\Xi_G^i > -1$. Together these inequalities give $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_G^i > \frac{1}{m}$, thereby establishing that $\widehat{G}^i < \widehat{Y}^i$ holds as y^i increases in this case.

Turning to the sign of $\widehat{Z}^i - \widehat{Y}^i$ in (B.33b), from the definition of Ξ_Z^i one can see that

$$\Xi_Z^i > -\frac{\left(1-\kappa\right)\left(\theta^j + m\delta\phi^j\right)}{\left(1-\kappa\right)\left(1+m\delta\phi^j\right)+\kappa\phi^i\left(1+mq\delta\phi^j\right)} > -1.$$

Therefore, $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^i > \widehat{\theta}^i/\widehat{\phi}^i - 1 > \frac{1}{m}$ from part (b) of Lemma B.1, which implies $\widehat{Y}^i < \widehat{Z}^i$ for $y^i \leq 1$ as $y^i \uparrow$. Together the inequalities $\widehat{G}^i < \widehat{Y}^i$ and $\widehat{Y}^i < \widehat{Z}^i$ give us $\widehat{G}^i < \widehat{Y}^i < \widehat{Z}^i$ as y^i increases when $y^i \leq 1$ initially.

(ii) $y^i \equiv Y^i/Y^j > 1$: Since $\phi^i - \phi^j > 0$, the definition of Ξ_G^i reveals that $\Xi_G^i > 0$ in this case. In addition, $\widehat{\theta}^i/\widehat{\phi}^i > 0$ as y^i increases. It follows from that $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^i > 0$, which establishes that $\widehat{G}^i < \widehat{Y}^i$ holds as $y^i \uparrow$ in this case as well.

The sign of $\widehat{Z}^i - \widehat{Y}^i$ in (B.33b) can be obtained by using the definition of Ξ_Z^i to show that

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_Z^i > -\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left[\frac{\left(1-\kappa\right)\theta^j}{\left(1-\kappa\right)\left(1+m\delta\phi^j\right)+\kappa\phi^i\left(1+mq\delta\phi^j\right)}\right] > -\left(\frac{\phi^j}{\phi^i}\right)\theta^i > -1.$$

Multiplying $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^i$ inside the curly brackets in (B.33b) by $\frac{\theta^i\phi^j}{\theta^j\phi^i}$ gives

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) + \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_Z^i > \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) - 1 > 0,$$

where, again, the last inequality follows from part (b) of Lemma B.1. Thus, (B.33b) implies $\widehat{Y}^i < \widehat{Z}^i$ in this case, too. We conclude that $\widehat{G}^i < \widehat{Y}^i < \widehat{Z}^i$ as $y^i \uparrow$ in this case as well.

Finally, observe from the specification of ϕ^i in equation (2) of the main text that $\text{sign}\{\widehat{G}^i - \widehat{G}^j\} = \text{sign}\{\widehat{\phi}^i\}$. Since $\widehat{\phi}^i > 0$ for $\widehat{y}^i > 0$ from (B.15a) in the proof of Proposition 1(a), it follows that $\widehat{G}^j < \widehat{G}^i$, thereby completing the proof of the proposition. ||

Proposition B.2 *If the assumptions of Proposition 1 are satisfied, then an increase in country i's income alone (i.e., $\widehat{Y}^i > 0$ and $\widehat{Y}^j = 0$) affects equilibrium first-period consumption in countries $i, j \in \{1, 2\}$, $i \neq j$ as follows:*

(a) $0 < \widehat{C}^{i*}$.

(b) $\lim_{Y^i \to 0} \widehat{C}^{j*} < 0$, $\lim_{Y^i \to Y^j} \widehat{C}^{j*} > 0$ *and* $\lim_{Y^i \to \infty} \widehat{C}^{j*} < 0$.

Proof: Here we study how current-period consumption for each country responds to a change in Y^i, treating Y^j as fixed. Using (B.28a) and (B.28b) in $C^i = Y^i - G^i - Z^i$ yields

$$C^i = \left(\frac{\kappa \phi^i + (1-\kappa)\,\theta^i}{\Omega^i} \right) Y^i, \tag{B.34}$$

Part (a): Logarithmic differentiation of (B.34) gives

$$\widehat{C}^i = \widehat{Y}^i + \left(\overset{(+)}{\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i + \kappa\phi^i}} - \overset{(+)}{\frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}} \right) \widehat{\theta}^i + \left(\overset{(+)}{\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i + \kappa\phi^i}} - \overset{(+)}{\frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}} \right) \widehat{\phi}^i. \tag{B.35}$$

Substituting the values of $\widehat{\phi}^i$ and $\widehat{\theta}^i$ from (B.15) into (B.35) gives:

$$\begin{aligned}
\widehat{C}^i &= \widehat{Y}^i + \left[\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i + \kappa\phi^i} - \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} \right] \frac{a_{11}}{\mathcal{D}} \widehat{Y}^i + \left[\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i + \kappa\phi^i} - \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} \right] \frac{(-a_{12})}{\mathcal{D}} \widehat{Y}^i \\
&= \frac{\widehat{Y}^i}{\mathcal{D}} \Bigg\{ [a_{11}a_{22} - a_{12}a_{21}] + a_{11} \left[\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i + \kappa\phi^i} - \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} \right] \\
&\qquad - a_{12} \left[\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i + \kappa\phi^i} - \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} \right] \Bigg\} \\
&= \frac{\widehat{Y}^i}{\mathcal{D}} \left\{ a_{11} \left[a_{22} + \frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i + \kappa\phi^i} - \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} \right] - a_{12} \left[a_{21} + \frac{\kappa\phi^i}{(1-\kappa)\,\theta^i + \kappa\phi^i} - \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} \right] \right\}.
\end{aligned}$$

The second line was obtained by substituting in the value of $\mathcal{D} = a_{11}a_{22} - a_{12}a_{21}$ (> 0), and the expression in the third line follows upon factoring out the common terms a_{11} and a_{12}. Then, we substitute in the expressions for a_{22} and a_{21} from (B.11) to rewrite the above equation as follows:

$$\widehat{C}^i = \frac{\widehat{Y}^i}{\mathcal{D}} \left\{ \overset{(+)}{a_{11}} \left[-\overset{(-)}{\frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}} + \overset{(+)}{\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i + \kappa\phi^i}} \right] - \overset{(-)}{a_{12}} \left[\overset{(+)}{\frac{\phi^i E^i_{\phi^i}}{E^i}} - \overset{(-)}{\frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}} + \overset{(+)}{\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i + \kappa\phi^i}} \right] \right\},$$

which is positive. Thus, an increase in Y^i raises country i's period $t=1$ consumption.

Part (b): Logarithmic differentiation of (B.34) for country j keeping Y^j fixed yields

$$\widehat{C}^j = -\left(\overset{(+)}{\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^j + \kappa\phi^j}} + \overset{(-)}{\frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}} \right) \widehat{\theta}^i - \left(\overset{(+)}{\frac{\kappa\phi^i}{(1-\kappa)\,\theta^j + \kappa\phi^j}} + \overset{(-)}{\frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}} \right) \widehat{\phi}^i. \tag{B.36}$$

By rearranging (B.36) one can show that

$$\operatorname{sign}\left\{\widehat{C}^j\right\} = \operatorname{sign}\left\{ \widehat{\theta}^i / \widehat{\phi}^i - \left(\frac{\theta^j/\phi^j}{\theta^i/\phi^i} \right) \Xi^j_C \right\}, \tag{B.37a}$$

where

$$\Xi^j_C \equiv \frac{(1-\kappa)\left(\theta^i + m\phi^j - m\phi^i\right) + \kappa m\phi^j \left(\phi^j/\theta^j\right)}{(1-\kappa)\left(1-m\phi^i\right) + \kappa\phi^j}. \tag{B.37b}$$

Let us first consider the case where countries have identical income levels $Y^i = Y^j$, which implies $\phi^i = \phi^j = \theta^i = \theta^j = \frac{1}{2}$. It is easy to verify that, in this case, we have $\frac{\theta^j/\phi^j}{\theta^i/\phi^i} = 1$ and $\Xi^j_C = \frac{1-\kappa+\kappa m}{1-\kappa+\kappa m+1-m} \le 1$. However, from Lemma B.1(a), we know that $\lim_{Y^i \to Y^j} (\widehat{\theta}^i/\widehat{\phi}^i) > 1 + \frac{1}{m}$. It thus becomes clear from (B.37a) that $\lim_{Y^i \to Y^j} \left(\widehat{C}^j\right) > 0$.

We now consider the extremes cases of (i) $Y^i \to 0$ and (ii) $Y^i \to \infty$, for any given finite $Y^j > 0$.

(i) $Y^i \to 0$: From Proposition 1(b), we have $\phi^i \to 0$, $\theta^i \to 0$ and $\theta^i/\phi^i \to 0$, while $\phi^j \to 1$, $\theta^j \to 0$ and $\theta^j/\phi^j \to 1$ in this case. Thus, $\lim_{Y^i \to 0} \left(\frac{\theta^j/\phi^j}{\theta^i/\phi^i}\right) = \infty$ and $\lim_{Y^i \to 0} \Xi^j_C = m$. Since by Lemma B.1(a) $\lim_{Y^i \to 0} (\widehat{\theta}^i/\widehat{\phi}^i) = 1 + \frac{1}{m}$, (B.37a) implies $\widehat{C}^j < 0$ for Y^i sufficiently close to 0.

(ii) $Y^i \to \infty$: Multiplying the expression inside (B.37a) by $\frac{\theta^i \phi^j}{\theta^j \phi^i}$ does not change the sign of that expression, but allows us to rewrite it as

$$\operatorname{sign}\left\{ \lim_{Y^i \to \infty} \widehat{C}^j \right\} = \operatorname{sign}\left\{ \lim_{Y^i \to \infty} \left[\frac{\theta^i \phi^j}{\theta^j \phi^i} \left(\widehat{\theta}^i / \widehat{\phi}^i \right) \right] - \lim_{Y^i \to \infty} \Xi^j_C \right\}.$$

Let us now study the two components inside the curly brackets, starting with the first.

Multiplying both sides of (B.7) by $(\theta^i\phi^j)/(\theta^j\phi^i)$ yields

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\bigg|_{S^i=0} = \frac{\left(\frac{\theta^i\phi^j}{\phi^i}\right)\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \frac{\theta^i\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(\frac{\theta^i}{\phi^i}\right)}{\frac{\theta^j\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \frac{\theta^i\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}.$$

Next, we take the limit of the expression above as $Y^i \to \infty$. To do so, recall that $\lim_{Y^i\to\infty}\theta^i = \lim_{Y^i\to\infty}\phi^i = 1$, whereas $\lim_{Y^i\to\infty}\theta^j = \lim_{Y^i\to\infty}\phi^j = 0$ and $\lim_{Y^i\to\infty}\theta^j/\phi^j = 0$. One can then verify the following:

$$\begin{aligned}
\lim_{Y^i\to\infty}\left[\left(\frac{\theta^i\phi^j}{\phi^i}\right)\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)\left(\theta^i/\phi^i\right)+\kappa\left(1-q+q\theta^i\right)}\right] &= 0\\
\lim_{Y^i\to\infty}\left[\frac{\theta^i\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)\left(\theta^j/\phi^j\right)+\kappa\left(1-q+q\theta^j\right)}\right] &= 1\\
\lim_{Y^i\to\infty}\left[\frac{1}{m}\left(\frac{\theta^i}{\phi^i}\right)\right] &= \frac{1}{m}\\
\lim_{Y^i\to\infty}\left[\frac{\theta^j\kappa\left(1-q\right)}{(1-\kappa)\left(\theta^i/\phi^i\right)+\kappa\left(1-q+q\theta^i\right)}\right] &= 0\\
\lim_{Y^i\to\infty}\left[\frac{\theta^i\kappa\left(1-q\right)}{(1-\kappa)\left(\theta^j/\phi^j\right)+\kappa\left(1-q+q\theta^j\right)}\right] &= 1.
\end{aligned}$$

Using the expressions above gives

$$\lim_{Y^i\to\infty}\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\bigg|_{S^i=0} = 1+\frac{1}{m}.$$

Thus, to complete the proof, it suffices to show that $\lim_{Y^i\to\infty}\Xi_C^j > 1+\frac{1}{m}$. Take that limit to find

$$\lim_{Y^i\to\infty}\Xi_C^j = 1+\frac{\kappa m}{(1-\kappa)(1-m)}\lim_{Y^i\to\infty}\left[\frac{\phi^j}{\theta^j/\phi^j}\right]. \tag{B.38}$$

Since any change in relative incomes always moves us along the S^i-contour, we use the definition of this contour to establish the following:

$$\frac{\phi^j}{\theta^j/\phi^j} = \left(\phi^j\right)^{1-\frac{1}{m}}\left\{\frac{\left(\phi^i\right)^{\frac{1}{m}}\left[(1-\kappa)\left(\theta^i/\phi^i\right)+\kappa\left(1-q+q\theta^i\right)\right]}{\left(\theta^i/\phi^i\right)\left[(1-\kappa)\left(\theta^j/\phi^j\right)+\kappa\left(1-q+q\theta^j\right)\right]}\right\}.$$

Taking limits gives

$$\lim_{Y^i\to\infty}\left[\frac{\phi^j}{\theta^j/\phi^j}\right] = \lim_{Y^i\to\infty}\left[\left(\phi^j\right)^{1-\frac{1}{m}}\right]\left\{\frac{1}{\kappa\left(1-q\right)}\right\}.$$

Now substitute the above expression into (B.38):

$$\lim_{Y^i\to\infty} \Xi_C^j = 1 + \frac{m}{(1-\kappa)(1-m)(1-q)} \lim_{Y^i\to\infty} \left[\left(\phi^j\right)^{1-\frac{1}{m}}\right].$$

Since $\lim_{Y^i\to\infty} \phi^j = 0$ and $m \in (0,1]$, we have $\lim_{Y^i\to\infty} \Xi_C^j > 1 + \frac{1}{m}$ for all parameter values. $||$

Proposition B.3 *In the two benchmark cases of perfect symmetry and extreme asymmetry in country sizes, changes in the degree of property security $\kappa \in (0,1]$ and in the probability of a future conflict $q \in (0,1)$ influence each country's allocation of income to arming, savings, and first-period consumption as follows :*

(a) *If $Y^i = Y^j \in (0,\infty)$, then*
  (i) $dG^{i*}/d\kappa = dG^{j*}/d\kappa > 0$, $dZ^{i*}/d\kappa = dZ^{j*}/d\kappa < 0$, *and* $dC^{i*}/d\kappa = dC^{j*}/d\kappa > 0$.
  (ii) $dG^{i*}/dq = dG^{j*}/dq > 0$, $dZ^{i*}/dq = dZ^{j*}/dq < 0$, *and* $dC^{i*}/dq = dC^{j*}/dq > 0$.

(b) *For given $Y^j \in (0,\infty)$, we have*
  (i) $\lim_{Y^i\to 0} \widehat{G}^{i*}/\widehat{\kappa} = \lim_{Y^i\to 0} \widehat{Z}^{i*}/\widehat{\kappa} = \lim_{Y^i\to 0} \widehat{C}^{i*}/\widehat{\kappa} = 0$, $\lim_{Y^i\to 0} \widehat{G}^{i*}/\widehat{q} > 0$, $\lim_{Y^i\to 0} \widehat{Z}^{i*}/\widehat{q} < 0$, *and* $\lim_{Y^i\to 0} \widehat{C}^{i*}/\widehat{q} > 0$.
  (ii) $\lim_{Y^i\to 0} \widehat{G}^{j*}/\widehat{\kappa} > 0$ *and* $\lim_{Y^i\to 0} \widehat{G}^{j*}/\widehat{q} > 0$, *while* $\lim_{Y^i\to 0} \widehat{Z}^{j*}/\widehat{\kappa} = \lim_{Y^i\to 0} \widehat{Z}^{j*}/\widehat{q} = \lim_{Y^i\to 0} \widehat{C}^{j*}/\widehat{\kappa} = \widehat{C}^{j*}/\widehat{q} = 0$.

Proof: The effects of changes in κ and q on first-period allocations consist of both direct and indirect effects through the implied changes in θ^{i*} and ϕ^{i*}, and they can be found by appropriately differentiating (B.28) and (B.34) to find

$$\begin{aligned}\widehat{G}^{i*} &= \left(1 - \frac{\kappa\Omega_\kappa^i}{\Omega^i}\right)\widehat{\kappa} + \left(1 - \frac{q\Omega_q^i}{\Omega^i}\right)\widehat{q} \\ &\quad - \left(\frac{\theta^i\Omega_{\theta^i}^i}{\Omega^i}\right)\widehat{\theta}^{i*} - \left(\frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i\Omega_{\phi^i}^i}{\Omega^i}\right)\widehat{\phi}^{i*}\end{aligned} \tag{B.39a}$$

$$\begin{aligned}\widehat{G}^{j*} &= \left(1 - \frac{\kappa\Omega_\kappa^j}{\Omega^j}\right)\widehat{\kappa} + \left(1 - \frac{q\Omega_q^j}{\Omega^j}\right)\widehat{q} \\ &\quad - \left(\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right)\widehat{\theta}^{i*} - \left(\frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right)\widehat{\phi}^{i*}\end{aligned} \tag{B.39b}$$

$$\begin{aligned}\widehat{Z}^{i*} &= \left(\frac{\kappa\Lambda_\kappa^i}{\Lambda^i} - \frac{\kappa\Omega_\kappa^i}{\Omega^i}\right)\widehat{\kappa} + \left(\frac{q\Lambda_q^i}{\Lambda^i} - \frac{q\Omega_q^i}{\Omega^i}\right)\widehat{q} \\ &\quad + \left(\frac{\theta^i\Lambda_{\theta^i}^i}{\Lambda^i} - \frac{\theta^i\Omega_{\theta^i}^i}{\Omega^i}\right)\widehat{\theta}^{i*} + \left(\frac{\phi^i\Lambda_{\phi^i}^i}{\Lambda^i} - \frac{\phi^i\Omega_{\phi^i}^i}{\Omega^i}\right)\widehat{\phi}^{i*}\end{aligned} \tag{B.40a}$$

$$
\begin{aligned}
\widehat{Z}^{j*} &= \left(\frac{\kappa\Lambda^j_\kappa}{\Lambda^j} - \frac{\kappa\Omega^j_\kappa}{\Omega^j}\right)\widehat{\kappa} + \left(\frac{q\Lambda^j_q}{\Lambda^j} - \frac{q\Omega^j_q}{\Omega^j}\right)\widehat{q} \\
&\quad + \left(\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right)\widehat{\theta}^{i*} + \left(\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right)\widehat{\phi}^{i*}
\end{aligned}
\tag{B.40b}
$$

and

$$
\begin{aligned}
\widehat{C}^{i*} &= \left[\frac{\kappa\left(1-\theta^i/\phi^i\right)}{\kappa+(1-\kappa)\left(\theta^i/\phi^i\right)} - \frac{\kappa\Omega^i_\kappa}{\Omega^i}\right]\widehat{\kappa} - \frac{q\Omega^i_q}{\Omega^i}\widehat{q} \\
&\quad + \left[\frac{(1-\kappa)\left(\theta^i/\phi^i\right)}{\kappa+(1-\kappa)\left(\theta^i/\phi^i\right)} - \frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right]\widehat{\theta}^{i*} + \left[\frac{\kappa}{\kappa+(1-\kappa)\left(\theta^i/\phi^i\right)} - \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right]\widehat{\phi}^{i*}
\end{aligned}
\tag{B.41a}
$$

$$
\begin{aligned}
\widehat{C}^{j*} &= \left[\frac{\kappa\left(1-\theta^j/\phi^j\right)}{\kappa+(1-\kappa)\left(\theta^j/\phi^j\right)} - \frac{\kappa\Omega^j_\kappa}{\Omega^j}\right]\widehat{\kappa} - \frac{q\Omega^j_q}{\Omega^j}\widehat{q} \\
&\quad - \left[\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^j+\kappa\phi^j} + \frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right]\widehat{\theta}^{i*} - \left[\frac{\kappa\phi^i}{(1-\kappa)\,\theta^j+\kappa\phi^j} + \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right]\widehat{\phi}^{i*}.
\end{aligned}
\tag{B.41b}
$$

Part (a): As established in Proposition B.1(a), when $Y^i = Y^j > 0$, $\phi^{i*} = \theta^{i*} = \frac{1}{2}$ regardless of the values of $\kappa \in (0,1]$ and $q \in (0,1)$. Therefore, only the direct effects of changes in these two parameters matter for first-period allocations of income. To evaluate those effects, we differentiate Λ^r and Ω^r (for $r = i,j$) with respect to κ and q,[15] and then evaluate those expressions at $\phi^i = \theta^i = \frac{1}{2}$:

$$
\begin{aligned}
\left.\frac{\kappa\Lambda^i_\kappa}{\Lambda^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} &= \left.\frac{q\Lambda^i_q}{\Lambda^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} = -\frac{\frac{1}{2}\kappa q}{1-\frac{1}{2}\kappa q} < 0 \\
\left.\frac{\kappa\Omega^i_\kappa}{\Omega^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} &= \left.\frac{q\Omega^i_q}{\Omega^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} = -\frac{\frac{1}{2}\kappa q\delta\,(1-m)}{1+\delta-\frac{1}{2}\kappa q\delta\,(1-m)} < 0.
\end{aligned}
$$

By substituting these expressions into (B.39)–(B.41) evaluated at $\phi^i = \theta^i = \frac{1}{2}$, one can then confirm the following:

$$
\widehat{G}^{i*} = \widehat{G}^{j*} = \frac{1+\delta}{1+\delta-\frac{1}{2}\kappa q\delta\,(1-m)}\left(\widehat{\kappa}+\widehat{q}\right) > 0 \tag{B.42a}
$$

$$
\widehat{Z}^{i*} = \widehat{Z}^{j*} = -\frac{\frac{1}{2}\kappa q\,(1+m\delta)}{\left(1-\frac{1}{2}\kappa q\right)\left[1+\delta-\frac{1}{2}\kappa q\delta\,(1-m)\right]}\left(\widehat{\kappa}+\widehat{q}\right) < 0 \tag{B.42b}
$$

$$
\widehat{C}^{i*} = \widehat{C}^{j*} = \frac{\frac{1}{2}\kappa q\delta\,(1-m)}{1+\delta-\frac{1}{2}\kappa q\delta\,(1-m)}\left(\widehat{\kappa}+\widehat{q}\right) > 0, \tag{B.42c}
$$

which completes the proof of part (a).

[15] The resulting partial derivatives are shown in (B.17). Also see (C.4) presented in Appendix C.

Part (b): For country i, one can combine equations (B.39)–(B.41) shown above with the limit results in (C.6), (C.7) and (C.9) presented in Appendix C to verify the following:

$$\lim_{Y^i \to 0} \widehat{G}^{i*} = (1-1) \times \widehat{\kappa} + \left(1 + \frac{q\delta(1-m)}{1+\delta-q\delta(1-m)}\right) \times \widehat{q} - 0 \times \widehat{\theta}^i - (0-1+1) \times \widehat{\phi}^i$$
$$= \frac{1+\delta}{1+\delta-q\delta(1-m)}\widehat{q} \tag{B.43a}$$
$$\lim_{Y^i \to 0} \widehat{Z}^{i*} = (1-1) \times \widehat{\kappa} - \left[\frac{q}{1-q} + \frac{q\delta(1-m)}{1+\delta-q\delta(1-m)}\right] \times \widehat{q}$$
$$+ (0-0) \times \widehat{\theta}^{i*} + (1-1) \times \widehat{\phi}^{i*}$$
$$= -\frac{q(1+\delta m)}{(1-q)[1+\delta-q\delta(1-m)]}\widehat{q} \tag{B.43b}$$
$$\lim_{Y^i \to 0} \widehat{C}^{i*} = [1-1] \times \widehat{\kappa} + \left[\frac{q\delta(1-m)}{1+\delta-q\delta(1-m)}\right] \times \widehat{q} + [0-0] \times \widehat{\theta}^{i*} + [1-1] \times \widehat{\phi}^{i*}$$
$$= \frac{q\delta(1-m)}{1+\delta-q\delta(1-m)}\widehat{q}. \tag{B.43c}$$

These results confirm the findings stated in part (b.i) of the proposition. Similarly, for part (b.ii) that focuses on country j, one can verify the following:

$$\lim_{Y^i \to 0} \widehat{G}^{j*} = \widehat{\kappa} + \widehat{q} + \frac{m}{1+m}\left[-\widehat{\kappa} + \frac{q\delta(1-m)}{1+\delta-q\delta(1-m)}\widehat{q}\right]$$
$$= \frac{1}{1+m}\widehat{\kappa} + \left[1 + \frac{mq\delta(1-m)}{(1+m)[1+\delta-q\delta(1-m)]}\right]\widehat{q} \tag{B.44a}$$
$$\lim_{Y^i \to 0} \widehat{Z}^{j*} = (0-0) \times \widehat{\kappa} + (0-0) \times \widehat{q} + (0-0) \times \widehat{\theta}^{i*} + (0-0) \times \widehat{\phi}^{i*} = 0 \tag{B.44b}$$
$$\lim_{Y^i \to 0} \widehat{C}^{j*} = [0-0] \times \widehat{\kappa} - 0 \times \widehat{q} - [0+0] \times \widehat{\theta}^{i*} - [0+0] \times \widehat{\phi}^{i*} = 0, \tag{B.44c}$$

thereby completing the proof. ||

Proof of Proposition 3. *Part (a):* To identify the effect of a change in Y^i on U^i, we differentiate U^i and invoke the envelope theorem to obtain

$$dU^{i*} = \Xi_U^i \left[\widehat{Y}^i - \left(\frac{G^i}{Y^i}\right)\widehat{G}^{j*} + \left(\frac{G^i}{Y^i}\right)\left(\frac{\theta^j}{m\phi^j}\right)\widehat{Z}^{j*}\right],$$

where

$$\Xi_U^i \equiv \frac{\Omega^i}{(1-\kappa)\theta^i + \kappa\phi^i} > 0.$$

Using the expression for G^i from (B.28a), the expressions for $\widehat{G}^{j*}$ and $\widehat{Z}^{j*}$ from (B.30a) and

(B.30b), and the expressions for $\widehat{\theta}^i$ and $\widehat{\phi}^i$ from (B.15) gives

$$\begin{aligned}
\frac{dU^{i*}}{dY^i} &= \frac{\Xi_U^i}{Y^i}\left\{1-\left(\frac{G^i}{Y^i}\right)\left[-\left(\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right)\frac{\widehat{\theta}^i}{\widehat{Y}^i}+\left(1-\frac{\phi^i}{\phi^j}-\frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right)\frac{\widehat{\phi}^i}{\widehat{Y}^i}\right]\right.\\
&\quad\left.+\left(\frac{G^i}{Y^i}\right)\left(\frac{\theta^j}{m\phi^j}\right)\left[\left(\frac{\theta^i\Lambda_{\theta^i}^j}{\Lambda^j}-\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right)\frac{\widehat{\theta}^i}{\widehat{Y}^i}+\left(\frac{\phi^i\Lambda_{\phi^i}^j}{\Lambda^j}-\frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right)\frac{\widehat{\phi}^i}{\widehat{Y}^i}\right]\right\}\\
&= \frac{\Xi_U^i}{Y^i}\left\{1-\frac{\kappa mq\delta\phi^i\phi^j}{\Omega^i}\left[-\left(\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right)\left(\frac{a_{11}}{\mathcal{D}}\right)+\left(1-\frac{\phi^i}{\phi^j}-\frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right)\left(\frac{-a_{12}}{\mathcal{D}}\right)\right]\right.\\
&\quad\left.+\frac{\kappa q\delta\phi^i\theta^j}{\Omega^i}\left[\left(\frac{\theta^i\Lambda_{\theta^i}^j}{\Lambda^j}-\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right)\left(\frac{a_{11}}{\mathcal{D}}\right)+\left(\frac{\phi^i\Lambda_{\phi^i}^j}{\Lambda^j}-\frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right)\left(\frac{-a_{12}}{\mathcal{D}}\right)\right]\right\}.
\end{aligned}$$

Multiply the second equation by $\Omega^i\mathcal{D} > 0$, while recalling that $\mathcal{D} \equiv a_{11}a_{22} - a_{12}a_{21}$. Then, one can rearrange terms by pulling a_{11} (> 0) and $-a_{12}$ (> 0) as common factors to establish

$$\operatorname{sign}\left\{\frac{dU^{i*}}{dY^i}\right\} = \operatorname{sign}\left\{\Gamma_1 a_{11} + \Gamma_2(-a_{12})\right\},$$

where

$$\begin{aligned}
\Gamma_1 &\equiv a_{22}\Omega^i + \kappa mq\delta\phi^i\phi^j\left(\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right) + \kappa q\delta\phi^i\theta^j\left(\frac{\theta^i\Lambda_{\theta^i}^j}{\Lambda^j}-\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right)\\
\Gamma_2 &\equiv a_{21}\Omega^i + \kappa mq\delta\phi^i\phi^j\left(-1+\frac{\phi^i}{\phi^j}+\frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right) + \kappa q\delta\phi^i\theta^j\left(\frac{\phi^i\Lambda_{\phi^i}^j}{\Lambda^j}-\frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right).
\end{aligned}$$

Since $a_{11} > 0$ and $a_{12} < 0$, we need only to demonstrate that $\Gamma_1 > 0$ and $\Gamma_2 > 0$ hold to establish this part of the proposition.

To proceed, we substitute the values of $a_{22} > 0$ and $a_{21} > 0$ from (B.11) and the definition of Ω^i from (B.8c) into the expressions immediately above. Upon simplifying, we can rewrite Γ_1 and Γ_2 as follows:

$$\begin{aligned}
\Gamma_1 &= (1+\delta)\left[(1-\kappa)\theta^i + \kappa\phi^i\right]\left(-\frac{\theta^i\Omega_{\theta^i}^j}{\Omega^j}\right) + M_1\\
\Gamma_2 &= (1+\delta)\left[(1-\kappa)\theta^i + \kappa\phi^i\right]\left(-\frac{\phi^i\Omega_{\phi^i}^j}{\Omega^j}\right) + M_2,
\end{aligned}$$

where

$$\begin{aligned}
M_1 &\equiv \theta^i\Omega_{\theta^i}^i - \kappa q\delta\phi^i\theta^j\left(-\frac{\theta^i\Lambda_{\theta^i}^j}{\Lambda^j}\right)\\
M_2 &\equiv \kappa\phi^i\left[1+\delta\left(1-q+q\theta^i\right)\right] + \frac{\Omega^i}{m\phi^j} - \kappa q\delta\phi^i\theta^j\left(-\frac{\phi^i\Lambda_{\phi^i}^j}{\Lambda^j}\right).
\end{aligned}$$

Since $\Omega^j_{\theta^i} < 0$ and $\Omega^j_{\phi^i} < 0$, the first terms in Γ_1 and Γ_2 are positive. Hence, to complete the proof, it suffices to show that $M_1 > 0$ and $M_2 > 0$. Expanding the terms in M_1 allows us to rewrite it as

$$\begin{aligned} M_1 &\equiv \theta^i \left\{ (1-\kappa)(1+\delta) + \kappa q \delta \phi^i - \kappa q \delta \phi^i \left[\frac{(1-\kappa)\theta^j + \kappa q \theta^j \phi^j}{(1-\kappa)\theta^j + \kappa(1-q+q\theta^j)\phi^j} \right] \right\} \\ &= \theta^i \left\{ (1-\kappa)(1+\delta) + \kappa q \delta \phi^i \left[\frac{\kappa(1-q)\phi^j}{(1-\kappa)\theta^j + \kappa(1-q+q\theta^j)\phi^j} \right] \right\} > 0. \end{aligned}$$

Hence, $\Gamma_1 > 0$. Turning to M_2, its first term is positive, and the algebraic sum of its last two terms can be written as

$$\begin{aligned} &\frac{(1-\kappa)(1+\delta)\theta^i + \kappa\phi^i[1+(1-q)\delta + q\delta(\theta^i + m\phi^j)]}{m\phi^j} \\ &\qquad -\kappa q \delta \phi^i \theta^j \left[\frac{\kappa(1-q+q\theta^j)\phi^i}{(1-\kappa)\theta^j + \kappa(1-q+q\theta^j)\phi^j} \right] \\ &> \frac{\kappa\phi^i\left[1+(1-q)\delta + q\delta(\theta^i + m\phi^j)\right]}{m\phi^j} - \kappa q \delta \phi^i \theta^j \left[\frac{\phi^i}{\phi^j} \right] \\ &= \frac{\kappa\phi^i}{m\phi^j}\left[1+(1-q)\delta + q\delta\left(\theta^i - m\phi^i\theta^j + m\phi^j\right)\right] > 0. \end{aligned}$$

This last inequality implies $M_2 > 0$ and thus $\Gamma_2 > 0$, thereby completing the proof of part (a).

Part (b): By the envelope theorem, an increase in the income of country j's rival (Y^i) influences its payoff only through the effects on country i's arming and saving. Differentiating U^{j*} appropriately gives

$$dU^{j*} = \frac{\delta q \kappa \phi^i \phi^j}{(1-\kappa)\theta^j + \kappa\phi^j} \left[\left(\theta^i/\phi^i\right) \widehat{Z}^{i*} - m\widehat{G}^{i*} \right],$$

which implies

$$\text{sign}\left\{dU^{j*}/dY^i\right\} = \text{sign}\left\{\left(\theta^i/\phi^i\right)\widehat{Z}^{i*} - m\widehat{G}^{i*}\right\} \tag{B.45}$$

for $\widehat{Y}^i > 0$.

To start, we establish that there exists a threshold level of income for country i, $\overline{Y}^i \leq Y^j$, such that for $Y^i \geq \overline{Y}^i$, $dU^{j*}/dY^i \geq 0$. Recall, from Proposition 2, that $\widehat{Z}^{i*} > \widehat{Y}^i > \widehat{G}^{i*} > 0$. Thus, while both arming and saving rise in the country that grows, $\widehat{Y}^i$ constitutes an upper bound to $\widehat{G}^{i*}$ and a lower bound to $\widehat{Z}^{i*}$. Applying these bounds to the RHS of the expression above gives

$$\frac{\theta^i}{\phi^i}\widehat{Z}^{i*} - m\widehat{G}^{i*} > \frac{\theta^i}{\phi^i}\widehat{Y}^i - m\widehat{Y}^i = \left(\frac{\theta^i}{\phi^i} - m\right)\widehat{Y}^i, \tag{B.46}$$

for $\widehat{Y}^i > 0$ at any $Y^i > 0$. Recall that θ^i/ϕ^i varies continuously with Y^i along the S^i-contour such that $\theta^i/\phi^i \in (0,1]$ for $Y^i \leq Y^j$ while $\theta^i/\phi^i > 1$ for $Y^i > Y^j$. Hence, there must exist a threshold level $\overline{Y}^i \leq Y^j$, depending on m, that causes the far RHS of the above expression to be non-negative. It then follows that the most left hand side (LHS) expression in (B.46) will be non-negative for all $Y^i \geq \overline{Y}^i$, which implies from (B.45) that $dU^{j*}/dY^i > 0$ in this case.

Of course, since (B.46) states that $\frac{\theta^i}{\phi^i}\widehat{Z}^{i*} - m\widehat{G}^{i*} > (\frac{\theta^i}{\phi^i} - m)\widehat{Y}^i$, it is possible for an increase in Y^i to be welfare-enhancing for country j even when $\frac{\theta^i}{\phi^i} < m$. However, we now argue that there exists another threshold level $\underline{Y}^i$ ($\leq \overline{Y}^i$) such that $dU^{j*}/dY^i < 0$ for all $Y^i < \underline{Y}^i$. We do this by studying the behavior of U^{j*} as $Y^i \to 0$. To proceed, recall from the expressions of $\widehat{G}^{i*}$ and $\widehat{Z}^{i*}$ shown respectively in (B.30a) and (B.30b), that arming and saving depend on $\widehat{Y}^i$ directly and indirectly through its effect on shares θ^i and ϕ^i. Using the calculations shown in Appendix C, one can find the direct effects as follows:[16]

$$\lim_{Y^i\to 0}\left(\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right) = \lim_{Y^i\to 0}\left(\frac{\theta^i\Lambda^i_{\theta^i}}{\Lambda^i}\right) = 0,\ \lim_{Y^i\to 0}\left(\frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right) = \lim_{Y^i\to 0}\left(\frac{\phi^i\Lambda^i_{\phi^i}}{\Lambda^i}\right) = 1,\ \text{and}$$
$$\lim_{Y^i\to 0}\left(\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right) = \lim_{Y^i\to 0}\left(\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j}\right) = \lim_{Y^i\to 0}\left(\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right) = \lim_{Y^i\to 0}\left(\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j}\right) = 0. \tag{B.47}$$

As for the indirect effects through $\hat{\theta}^i$ and $\hat{\phi}^i$, we apply our earlier findings for the limits of the a-coefficients and $\mathcal{D}$ as $Y^i \to 0$ shown respectively in (B.25) and (B.26) in (B.15) to establish the following:[17]

$$\lim_{Y^i\to 0}\left(\widehat{\theta}^i/\widehat{Y}^i\right) = \lim_{Y^i\to 0}\left(\frac{a_{11}}{\mathcal{D}}\right) = 1$$
$$\lim_{Y^i\to 0}\left(\widehat{\phi}^i/\widehat{Y}^i\right) = \lim_{Y^i\to 0}\left(-\frac{a_{12}}{\mathcal{D}}\right) = \frac{1}{1+1/m}.$$

Now, going back to the expressions for $\widehat{G}^{i*}$ and $\widehat{Z}^{i*}$ in (B.30a) and (B.30b), the results above with the finding in Proposition 1(b) that $\lim_{Y^i\to 0}(\theta^i/\phi^i) = 0$ give us the following:[18]

$$\lim_{Y^i\to 0}\left(\widehat{Z}^{i*}/\widehat{Y}^i\right) = \lim_{Y^i\to 0}\left(\widehat{G}^{i*}/\widehat{Y}^i\right) = 1.$$

On the basis of the above, we thus have

$$\lim_{Y^i\to 0}\left[\frac{\theta^i}{\phi^i}\left(\widehat{Z}^{i*}/\widehat{Y}^i\right) - m\left(\widehat{G}^{i*}/\widehat{Y}^i\right)\right] = \lim_{Y^i\to 0}\left(\frac{\theta^i}{\phi^i} - m\right) = -m < 0,$$

[16] See specifically the derivations of (C.6) and (C.7).

[17] Notice that the expressions below are consistent with our finding in Lemma B.1(a) that $\lim_{\phi^i\to 0}(\widehat{\theta}^i/\widehat{\phi}^i) = 1 + \frac{1}{m}$.

[18] Noting that $\lim_{Y^i\to 0}((1-\kappa)\theta^i/[(1-\kappa)\theta^i + \kappa\phi^i]) = 0$ and $\lim_{Y^i\to 0}(\kappa\phi^i/[(1-\kappa)\theta^i + \kappa\phi^i]) = 1$, one can also confirm, from (B.35), that $\lim_{Y^i\to 0}(\widehat{C}^{i*}/\widehat{Y}^i) = 1$.

which implies, by (B.45), that $\lim_{Y^i \to 0} dU^{j*}/dY^i < 0$. Since U^{j*} is continuous in Y^i and $dU^{j*}/dY^i > 0$ at a sufficiently high Y^i level, there exists (at least one) threshold level, $\underline{Y}^i \leq \overline{Y}^i$, such that $dU^{j*}/dY^i < 0$ for all $Y^i < \underline{Y}^i$. $\quad ||$

Proposition B.4 *The welfare implications of changes in the degree of insecurity of future income $\kappa \in (0, 1]$ and the probability of a future conflict $q \in (0, 1)$ depend on the relative sizes of the two countries. For the two benchmark cases considered earlier, we have*

(a) $\lim_{Y^i \to Y^j} dU^{i*}/d\kappa = \lim_{Y^i \to Y^j} dU^{j*}/d\kappa < 0$ *and* $\lim_{Y^i \to Y^j} dU^{i*}/dq = \lim_{Y^i \to Y^j} dU^{j*}/dq < 0$*; and*

(b) *given* $Y^j \in (0, \infty)$, $\lim_{Y^i \to 0} dU^{i*}/d\kappa > 0$ *and* $\lim_{Y^i \to 0} dU^{i*}/dq > 0$, *while* $\lim_{Y^i \to 0} dU^{j*}/d\kappa = \lim_{Y^i \to 0} dU^{j*}/dq = 0$.

Proof: The effects of an increase in κ and q on the two countries' payoffs generally consist of both direct effects and indirect effects through their influence on the opponent's choices of arming and savings:

$$dU^{i*} = q\delta \left\{ \left[\frac{\kappa \left(1 - \theta^i/\phi^i\right)}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)} \right] \widehat{\kappa} - \widehat{q} \ln \left[\frac{\left(\theta^i/\phi^i\right)}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)} \right] \right.$$
$$\left. - \left[\frac{\kappa m \phi^j}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)} \right] \widehat{G}^{j*} + \left[\frac{\kappa \theta^j}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)} \right] \widehat{Z}^{j*} \right\} \tag{B.48a}$$

$$dU^{j*} = q\delta \left\{ \left[\frac{\kappa \left(1 - \theta^j/\phi^j\right)}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)} \right] \widehat{\kappa} - \widehat{q} \ln \left[\frac{\left(\theta^j/\phi^j\right)}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)} \right] \right.$$
$$\left. - \left[\frac{\kappa m \phi^i}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)} \right] \widehat{G}^{i*} + \left[\frac{\kappa \theta^i}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)} \right] \widehat{Z}^{i*} \right\}. \tag{B.48b}$$

Part (a): The first lines of the two expressions above show that, when $Y^i = Y^j$ which implies $\phi^i = \theta^i = \frac{1}{2}$, the direct effects of changes in either κ or q equal zero. Turning our attention to the indirect effects, recall from the proof of Proposition B.3(a) that, when $Y^i = Y^j$, the effects of an increase in κ on $G^i = G^j$ and $Z^i = Z^j$ are identical to the respective effects of an increase in q (see equation (B.42)). Denote the corresponding percentage-changes respectively by $\widehat{G}^*$ and $\widehat{Z}^*$. Then, the welfare effects of changes in κ and q in this case can be written as follows:

$$dU^{i*}|_{Y^i = Y^j} = dU^{j*}|_{Y^i = Y^j} = \frac{1}{2} \kappa q \delta \left[-m\widehat{G}^* + \widehat{Z}^* \right] (\widehat{\kappa} + \widehat{q}).$$

From Proposition B.3(a), we know $\widehat{G}^* > 0$ and $\widehat{Z}^* < 0$, implying that an increase in either κ or q reduces payoffs for both countries.

Part (b): When $Y^i \to 0$ given $Y^j \in (0, \infty)$, the welfare effects of an increase in κ and q can include both direct and indirect effects, and generally the combined effects on payoffs are unequal across the two countries. For the calculations to follow, keep in mind that, as

$Y^i \to 0$ for a positive and finite value of Y^j, we have the following: (i) $\phi^i \to 0$, (ii) $\theta^i \to 0$, (iii) $\theta^i/\phi^i \to 0$ and (iv) $\theta^j/\phi^j \to 1$ (see Proposition 1(b)). We start with the smaller country i, taking the limit of dU^{i*} in (B.48a) as $Y^i \to 0$ using (B.44):

$$\begin{aligned}\lim_{Y^i\to 0} dU^{i*} = q\delta\Big\{&1 \times \widehat{\kappa} + \widehat{q} \times \infty \\ &- m \times \left[\frac{1}{1+m}\widehat{\kappa} + \left(1 + \frac{mq\delta\,(1-m)}{(1+m)\,[1+\delta - q\delta\,(1-m)]}\right) \times \widehat{q}\right] + 1 \times 0\Big\} \\ = q\delta\Big\{&\frac{1}{1+m}\widehat{\kappa} + \left[\infty - m\left(1 + \frac{mq\delta\,(1-m)}{(1+m)\,[1+\delta - q\delta\,(1-m)]}\right)\right]\widehat{q}\Big\}.\end{aligned}$$

Hence, an increase in either κ or q is welfare-enhancing for the smaller country. Turning to the larger country (j), we similarly take the limit of dU^{j*} in (B.48b) as $Y^i \to 0$ to find

$$\lim_{Y^i\to 0} dU^{j*} = q\delta\left\{0 \times \widehat{\kappa} - 0 \times \widehat{q} - 0 \times \widehat{G}^{i*} + 0 \times \widehat{Z}^{i*}\right\} = 0.$$

Thus, the larger country's payoff is independent of κ and q. $\quad ||$

Proof of Proposition 4. That the *ex ante* larger country has a higher first-period income than its potential adversary under autarky follows trivally from the fact that $Y_A^i = R^i$. To see that the same country has a higher first-period income under trade, recall that $Y_T^i = T^i(R^i, R^j)R^i$, where $T^i(R^i, R^j)$ is shown in equation (14) in the main text. Then, one can easily verify $Y_T^i/Y_T^j = [R^i/R^j]^{(\sigma-1)/\sigma} \gtreqless 1$ as $R^i \gtreqless R^j$, given $\sigma > 1$.

Moving onto the second main component of the proof, the (income) gains from trade for country i can be written as

$$\frac{Y_T^i}{Y_A^i} = T^i(R^i, R^j) = \left[1 + \left(R^i/R^j\right)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma-1}} \geq 1,\ i, j \in \{1, 2\},\ i \neq j,$$

which are decreasing in R^i/R^j given $\sigma > 1$. The limit results stated in the proposition follow immediately since R^i/R^j approaches 0 as $R^i \to 0$ and approaches ∞ as $R^i \to \infty$. Finally, using this expression for both countries, after some rearranging, gives the gains from trade for country i relative to those for country j:

$$\frac{Y_T^i}{Y_A^i}\bigg/\frac{Y_T^j}{Y_A^j} = \left[\frac{R^j}{R^i}\right]^{1/\sigma} \quad i, j \in \{1, 2\},\ i \neq j,$$

which reveals that relative gains from trade are larger for the *ex ante* smaller country, with the relative difference depending negatively on $\sigma > 1$. $\quad ||$

To add a final remark regarding this proof, note that the details work out in an analogous way if we allow for exogenous differences in technologies by instead assuming $Y_A^i = A^i R^i$ as we did initially in Section 3. In that case, we need only replace R^i with $A^i R^i$ everywhere in the statement of the proposition and in the proof. Doing so reveals a more general result that

$$\frac{Y_T^i}{Y_A^i}\Big/\frac{Y_T^j}{Y_A^j} = \left[\frac{A^j R^j}{A^i R^i}\right]^{1/\sigma} = \left[\frac{Y_A^j}{Y_A^i}\right]^{1/\sigma} \quad i,j \in \{1,2\},\, i \neq j,$$

implying that one can view the autarky size Y_A^i as the relevant measure of "size" in place of R^i and the proposition otherwise stays the same. That said, it is important to point out that our statement from Proposition 4 that each country's relative gain from trade is decreasing in its relative endowment size continues to hold. Likewise, it continues to be true that country i's gain from trade becomes infinite as $R^i \to 0$. Appendix D contains more details on how these results for the relative gains from trade generalize to other trade settings, including if we introduce trade costs.

Our next task is to prove Proposition 5. Although Proposition 5 is valid for a variety of canonical models of trade in the presence of trade costs, our proof to follow focuses on the Armington model with no trade costs as presented in the text. See Proposition D.1 presented in Appendix D for a more general statement.

Proof of Proposition 5. We know from Proposition 3 that the smaller country always benefits from an increase in its own income as well as from an increase in the larger country's income; thus, a smaller country will always prefer trade to autarky. However, we also know from Proposition 3(b) that, while the larger country likewise always benefits from an increase in its own income, it does not necessarily benefit from an increase in the smaller country's income when the probability of a future war is positive. Building on this proposition and the result from Proposition 4 that the larger country's relative income gain is decreasing in its relative initial size, we focus here on the larger country's possible preference for autarky over trade.[19]

As outlined in the main text, we study the effects of the introduction of trade locally in the neighborhood of where the small country i is infinitesimal (i.e., $R^i \to 0$). First, observe, from Proposition 4, that $\lim_{R^i \to 0}(Y_T^j/Y_A^j) = 1$. Thus, the gains from trade for an infinitely large country equal zero. Second, as $R^i \to 0$, $G^{i*} \to 0$ and $Z^{i*} \to 0$ under both autarky and trade, implying that a infinitesimal rival poses no threat to the large country nor does it contribute to world output in the second period under either trade regime. Combining two observations implies $\lim_{R^i \to 0} U_T^{j*} = \lim_{R^i \to 0} U_A^{j*}$.

By the reasoning provided in the proof to Proposition 3(b) while admitting the possibility that a country's own income can change too, we write the change in country j's payoff

[19] As mentioned in the main text, this possibility does not directly follow from Proposition 3(b), because trade induces *discrete* changes in *both* countries' incomes. Our proof effectively shows that, for sufficiently uneven distributions, larger country's payoff gain from trade and from the output expansion in the future (due the smaller country's increased saving) are dominated by the reduction in its payoff due to the adverse strategic effect of the smaller country's increased arming.

generally as

$$dU^{j*} = \frac{Y^j}{Y^j - G^j - Z^j}\widehat{Y}^j + \frac{\delta q \kappa \phi^i}{(1-\kappa)\left(\theta^j/\phi^j\right)+\kappa}\left[\left(\theta^i/\phi^i\right)\widehat{Z}^i - m\widehat{G}^i\right]. \tag{B.49}$$

Now consider a marginal increase in country i's resource base R^i, keeping R^j fixed. Since $Y_A^j = R^j$, $\widehat{Y}_A^j = 0$ holds, implying that the first term in the RHS of (B.49) vanishes and any changes in payoffs under autarky U_A^{j*} are due only to the changes in Z_A^{i*} and G_A^{i*} induced by changes in R^i. Under trade, since $Y_T^j = T^j(R^i, R^j)R^j$, we have

$$\widehat{Y}_T^j = \frac{\frac{1}{\sigma}\left(R^i\right)^{\frac{\sigma-1}{\sigma}}\widehat{R}^i}{\left(R^i\right)^{\frac{\sigma-1}{\sigma}} + \left(R^j\right)^{\frac{\sigma-1}{\sigma}}} \Longrightarrow \lim_{R^i\to 0}\left(\widehat{Y}_T^j/\widehat{R}^i\right) = 0.$$

Thus, although the large country j normally benefits from an expansion of i's resource base under trade, this effect vanishes as country i becomes infinitesimal; analogous to what we just saw under autarky in this limit, changes in payoffs under trade U_T^{j*} are due strictly to changes in Z_T^{i*} and G_T^{i*} induced by changes in R^i.

We now turn to characterize $\lim_{R^i\to 0}(dU_T^{j*}/dU_A^{j*})$, which tells us how U_T^{j*} changes relative to U_A^{j*} for a small increase in R^i above 0. Since as we have just shown changes in U^{j*} are driven entirely by changes in rival i's arming and savings choices under both autarky and trade, we can write

$$\begin{aligned}\lim_{R^i\to 0}\left(\frac{dU_T^{j*}}{dU_A^{j*}}\right) &= \lim_{R^i\to 0}\left(\frac{\phi_T^i}{\phi_A^i}\right) \times \frac{\lim_{R^i\to 0}\left[(1-\kappa)\left(\theta_A^j/\phi_A^j\right)+\kappa\right]}{\lim_{R^i\to 0}\left[(1-\kappa)\left(\theta_T^j/\phi_T^j\right)+\kappa\right]} \times \\ &\quad \frac{\lim_{R^i\to 0}\left[\left(\theta_T^i/\phi_T^i\right)\left(\widehat{Z}_T^{i*}/\widehat{Y}_T^i\right) - m\left(\widehat{G}_T^{i*}/\widehat{Y}_T^i\right)\right]}{\lim_{R^i\to 0}\left[\left(\theta_A^i/\phi_A^i\right)\left(\widehat{Z}_A^{i*}/\widehat{Y}_A^i\right) - m\left(\widehat{G}_A^{i*}/\widehat{Y}_A^i\right)\right]} \times \lim_{R^i\to 0}\left(\frac{\widehat{Y}_T^i/\widehat{R}^i}{\widehat{Y}_A^i/\widehat{R}^i}\right).\end{aligned}$$

Note that $\lim_{R^i\to 0} Y^i = 0$ under both autarky and free trade, which in turn implies, by Proposition 1(b), that $\lim_{Y^i\to 0}\left(\theta^j/\phi^j\right) = 1$. Thus, the second multiplicative term in the RHS of the first line equals 1. The third term of this equation also equals 1, because $\lim_{R^i\to 0}\left(\theta^i/\phi^i\right) = 0$ and because, as shown in the proof to Proposition 3(b), $\lim_{R^i\to 0}(\widehat{Z}^{i*}/\widehat{Y}^i) = \lim_{R^i\to 0}(\widehat{G}^{i*}/\widehat{Y}^i) = 1$ under both autarky and trade.[20] Thus, we have

$$\lim_{R^i\to 0}\left(\frac{dU_T^{j*}}{dU_A^{j*}}\right) = \lim_{R^i\to 0}\left(\frac{\phi_T^i}{\phi_A^i}\right) \times \lim_{R^i\to 0}\left(\frac{\widehat{Y}_T^i}{\widehat{Y}_A^i}\right), \quad \text{for } \; i,j\in\{1,2\},\, i\neq j. \tag{B.50}$$

But, since $\lim_{R^i\to 0}\phi_T^i = \lim_{R^i\to 0}\phi_A^i = 0$, we need to verify that $\lim_{R^i\to 0}\left(\phi_T^i/\phi_A^i\right)$ exists.[21]

Using the definition of the B^i-contour in (B.8), we can multiply E^i and F^i by ϕ^i/ϕ^j in

[20] The limit results mentioned here and thus (B.50) hold more generally when trade costs are present in the trade models considered in Appendix D as well as in the Armington model.

[21] As will become obvious below, $\lim_{R^i\to 0}(\widehat{Y}_T^i/\widehat{Y}_A^i)$ in the same equation is finitely positive in the Armington model. The same is true in the alternative trade models considered in Appendix D.

order to obtain the following relationship:

$$\left(\frac{\phi^i}{\phi^j}\right)^{\frac{1+m}{m}} = \left(\frac{Y^i}{Y^j}\right)\frac{\left[(1-\kappa)\left(1+\delta\right)\left(\theta^j/\phi^j\right)+\kappa\left(1+\left(1-q\right)\delta+q\delta\left(\theta^j+m\left(1-\phi^j\right)\right)\right)\right]}{\left[(1-\kappa)\left(1+\delta\right)\left(\theta^i/\phi^i\right)+\kappa\left(1+\left(1-q\right)\delta+q\delta\left(\theta^i+m\left(1-\phi^i\right)\right)\right)\right]},$$

which holds true both under trade and under autarky. Evaluating the above expression under trade and autarky, and taking ratios appropriately, we next obtain

$$\begin{aligned}
\frac{\phi_T^i}{\phi_A^i} &= \left(\frac{\phi_T^j}{\phi_A^j}\right)\left(\frac{Y_T^iY_A^j}{Y_A^iY_T^j}\right)^{\frac{m}{1+m}}\left(K^j/K^i\right)^{\frac{m}{1+m}}, \text{ where} \\
K^j &\equiv \frac{\left[(1-\kappa)\left(1+\delta\right)\left(\theta_T^j/\phi_T^j\right)+\kappa\left(1+\left(1-q\right)\delta+q\delta\left(\theta_T^j+m(1-\phi_T^j)\right)\right)\right]}{\left[(1-\kappa)\left(1+\delta\right)\left(\theta_A^j/\phi_A^j\right)+\kappa\left(1+\left(1-q\right)\delta+q\delta\left(\theta_T^j+m(1-\phi_T^j)\right)\right)\right]}.
\end{aligned}$$

From Proposition 1(b), $\lim_{R^i\to 0} Y^i = 0$ implies $\lim_{R^i\to 0}\left(\theta^j/\phi^j\right) = 1$, $\lim_{R^i\to 0}\left(\theta^i/\phi^i\right) = 0$, $\lim_{R^i\to 0}\theta^i = \lim_{R^i\to 0}\phi^i = 0$, and $\lim_{R^i\to 0}\theta^j = \lim_{R^i\to 0}\phi^j = 1$ under both trade regimes. Then, from the expression above, we have $\lim_{R^i\to 0} K^j = \lim_{R^i\to 0} K^i = 1$ and $\lim_{R^i\to 0}(\phi_T^j/\phi_A^j) = 1$, which in turn give

$$\lim_{R^i\to 0}\left(\frac{\phi_T^i}{\phi_A^i}\right) = \lim_{R^i\to 0}\left(\frac{Y_T^iY_A^j}{Y_A^iY_T^j}\right)^{\frac{m}{1+m}}. \tag{B.51}$$

Since Proposition 4 implies $\lim_{R^i\to 0}(Y_T^j/Y_A^j) = 1$, the expression above in (B.51) implies that $\lim_{R^i\to 0}\left(\phi_T^i/\phi_A^i\right) = \lim_{R^i\to 0}(Y_T^i/Y_A^i)^{m/(m+1)}$, which allows us to rewrite (B.50) as

$$\lim_{R^i\to 0}\left(\frac{dU_T^{j*}}{dU_A^{j*}}\right) = \lim_{R^i\to 0}\left(\frac{Y_T^i}{Y_A^i}\right)^{\frac{m}{1+m}} \times \lim_{R^i\to 0}\left(\frac{\widehat{Y}_T^i}{\widehat{Y}_A^i}\right), \text{ for } i,j\in\{1,2\},\ i\neq j. \tag{B.52}$$

However, Proposition 4 also implies $\lim_{R^i\to 0}\left(Y_T^i/Y_A^i\right) = \infty$. Turning to the the second limit on the RHS of (B.52), first we logarithmically differentiate $Y_T^i = T^i(R^i,R^j)R^i$ with respect to R^i using equation (14). This yields:

$$\widehat{Y}_T^i = \frac{\left[\left(R^i\right)^{\frac{\sigma-1}{\sigma}} + \frac{\sigma-1}{\sigma}\left(R^j\right)^{\frac{\sigma-1}{\sigma}}\right]\widehat{R}^i}{\left(R^i\right)^{\frac{\sigma-1}{\sigma}} + \left(R^j\right)^{\frac{\sigma-1}{\sigma}}}, \quad \text{for } i,j\in\{1,2\},\ i\neq j.$$

After dividing the expression above by $\widehat{Y}_A^i = \widehat{R}^i$, one can confirm $\lim_{R^i\to 0}(\widehat{Y}_T^i/\widehat{Y}_A^i) = \frac{\sigma-1}{\sigma} \in (0,\infty)$, for $\sigma\in(1,\infty)$. Hence, $\lim_{R^i\to 0}(dU_T^{j*}/dU_A^{j*}) = \infty$.

Since $\lim_{R^i\to 0} U_T^{j*} = \lim_{R^i\to 0} U_A^{j*}$ and since the proof to Proposition 3(b) implies $dU^{j*} < 0$ for sufficiently small Y^i under either trade regime, it follows that a marginal increase in R^i will reduce j's payoff under trade by more than it reduces j's payoff under autarky. For sufficiently small R^i (>0), we therefore must have $U_T^{j*} < U_A^{j*}$. Finally, since $R^i \geq R^j$ implies $Y_T^i \geq Y_T^j$, Proposition 3(b) implies further that, as R^i rises sufficiently and approaches R^j,

U_T^{j*} and U_A^{j*} eventually cross such that $U_T^{j*} > U_A^{j*}$.[22] ||

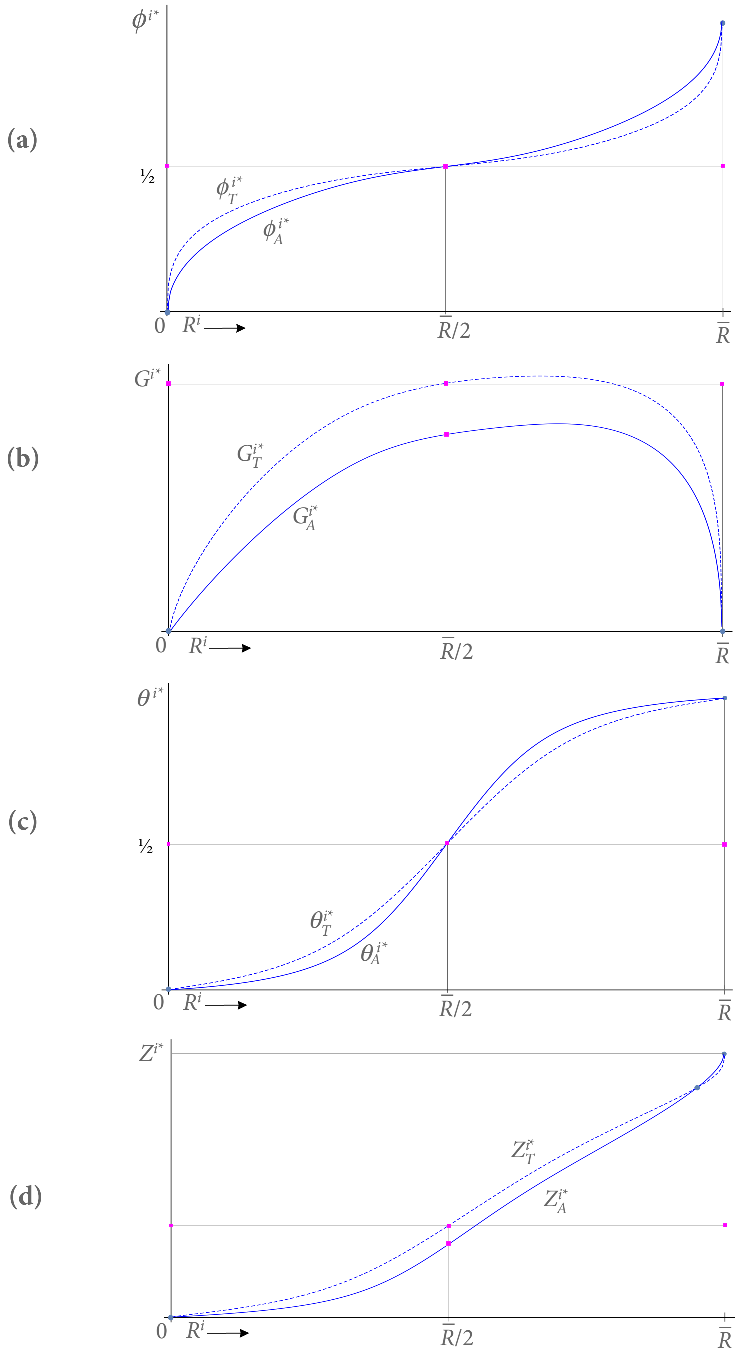

Figure B.1: The Effects of Trade on Equilibrium Arming, Saving and Respective Shares, with $\sigma = 4$, $q = .9$ and $m = \delta = 1$

[22] Although we cannot demonstrate analytically that $\underline{Y}^i = \overline{Y}^i$ in Proposition 3(b) such that the crossing is unique, extensive numerical analysis confirms that it is.

C More Computational Details

In this part of the Appendix, we provide more details regarding some of the calculations used in the Appendix B. Recall that

$$E^i \equiv \left(\phi^i/\phi^i\right)^{1/m} > 0 \tag{C.1a}$$

$$\Lambda^i \equiv (1-\kappa)\,\theta^i + \kappa\left(1 - q + q\theta^i\right)\phi^i > 0 \tag{C.1b}$$

$$\Omega^i \equiv (1-\kappa)\,(1+\delta)\,\theta^i + \kappa\phi^i\left[1 + (1-q)\,\delta + q\delta\left(\theta^i + m(1-\phi^{i)}\right)\right] > 0. \tag{C.1c}$$

Using these definitions and keeping in mind that $\phi^j = 1 - \phi^i$ and $\theta^j = 1 - \theta^i$ (where $i \neq j$), we take and sign the following derivatives:

$$\frac{\phi^i E^i_{\phi^i}}{E^i} = \frac{1}{m\phi^j} > 0 \tag{C.2a}$$

$$\frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} = \frac{\kappa\phi^i\left[1 - q + q\theta^i\right]}{\Lambda^i} > 0 \tag{C.2b}$$

$$\frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j} = -\frac{\kappa\phi^i\left[1 - q + q\theta^j\right]}{\Lambda^j} < 0 \tag{C.2c}$$

$$\frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} = \frac{\theta^i\left[1 - \kappa + \kappa q\phi^i\right]}{\Lambda^i} > 0 \tag{C.2d}$$

$$\frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} = -\frac{\theta^i\left[1 - \kappa + \kappa q\phi^j\right]}{\Lambda^j} < 0, \tag{C.2e}$$

and

$$\frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} = \frac{\kappa\phi^i\left[1 + (1-q)\,\delta + q\delta\left(\theta^i - m\phi^i + m\phi^j\right)\right]}{\Omega^i} > 0 \tag{C.3a}$$

$$\frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j} = -\frac{\kappa\phi^i\left[1 + (1-q)\,\delta + q\delta\left(\theta^j - m\phi^j + m\phi^i\right)\right]}{\Omega^j} < 0 \tag{C.3b}$$

$$\frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} = \frac{\theta^i\left[(1-\kappa)\,(1+\delta) + \kappa q\delta\phi^i\right]}{\Omega^i} > 0 \tag{C.3c}$$

$$\frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j} = -\frac{\theta^i\left[(1-\kappa)\,(1+\delta) + \kappa q\delta\phi^j\right]}{\Omega^j} < 0. \tag{C.3d}$$

For our calculations regarding the effects of changes in κ and q, observe from the definitions of Λ^r and Ω^r respectively shown in (B.1d) and (B.8c) that

$$\frac{\kappa\Lambda^i_\kappa}{\Lambda^i} = \frac{\kappa\phi^i\left(1 - q + q\theta^i\right) - \kappa\theta^i}{\Lambda^i} \tag{C.4a}$$

$$\frac{\kappa\Omega^i_\kappa}{\Omega^i} = \frac{\kappa\phi^i\left[1 + \delta + q\delta\phi^j\left(m - \theta^j/\phi^j\right)\right] - \kappa\theta^i\,(1+\delta)}{\Omega^i} \tag{C.4b}$$

$$\frac{q\Lambda^i_q}{\Lambda^i} = -\frac{\kappa q\phi^i\theta^j}{\Lambda^i} \tag{C.4c}$$

$$\frac{q\Omega^i_q}{\Omega^i} = \frac{\kappa q\delta\phi^i\phi^j\left(m - \theta^j/\phi^j\right)}{\Omega^i}. \tag{C.4d}$$

The following are useful for calculating the limits of the a- and b-coefficients in (B.10), as $Y^i \to 0$ for finite $Y^j > 0$, which, by Proposition 1(b), implies $\phi^i \to 0$ while $\phi^j \to 1$, $\theta^i \to 0$ while $\theta^j \to 1$, and $\theta^i/\phi^i \to 0$ while $\theta^j/\phi^j \to 1$:

$$\lim_{Y^i\to 0}\left(\Lambda^i/\phi^i\right) = \lim_{Y^i\to 0}\left\{(1-\kappa)\left(\theta^i/\phi^i\right) + \kappa\left(1 - q + q\theta^i\right)\right\} = \kappa(1-q) \tag{C.5a}$$

$$\lim_{Y^i\to 0}\left(\Lambda^j/\phi^j\right) = \lim_{Y^i\to 0}\left\{(1-\kappa)\left(\theta^j/\phi^j\right) + \kappa\left(1 - q + q\theta^j\right)\right\} = 1 \tag{C.5b}$$

$$\begin{aligned}\lim_{Y^i\to 0}\left(\Omega^i/\phi^i\right) &= \lim_{Y^i\to 0}\left\{(1-\kappa)(1+\delta)\left(\theta^i/\phi^i\right) + \kappa\left[1 + (1-q)\delta + q\delta\left(\theta^i + m\phi^j\right)\right]\right\}\\ &= \kappa\left[1 + \delta + q\delta(m-1)\right]\end{aligned} \tag{C.5c}$$

$$\begin{aligned}\lim_{Y^i\to 0}\left(\Omega^j/\phi^j\right) &= \lim_{Y^i\to 0}\left\{(1-\kappa)(1+\delta)\left(\theta^j/\phi^j\right) + \kappa\left[1 + (1-q)\delta + q\delta\left(\theta^j + m\phi^i\right)\right]\right\}\\ &= 1 + \delta\end{aligned} \tag{C.5d}$$

$$\lim_{Y^i\to 0}\left(\Lambda^i/\theta^i\right) = \lim_{Y^i\to 0}\left\{(1-\kappa) + \kappa\left(1 - q + q\theta^i\right)\left(\phi^i/\theta^i\right)\right\} = \infty \tag{C.5e}$$

$$\lim_{Y^i\to 0}\left(\Lambda^j/\theta^j\right) = \lim_{Y^i\to 0}\left\{(1-\kappa) + \kappa\left(1 - q + q\theta^j\right)\left(\phi^j/\theta^j\right)\right\} = 1 \tag{C.5f}$$

$$\begin{aligned}\lim_{Y^i\to 0}\left(\Omega^i/\theta^i\right) &= \lim_{Y^i\to 0}\left\{(1-\kappa)(1+\delta) + \kappa\left[1 + (1-q)\delta + q\delta\left(\theta^i + m\phi^j\right)\right]\left(\phi^i/\theta^i\right)\right\}\\ &= \infty\end{aligned} \tag{C.5g}$$

$$\begin{aligned}\lim_{Y^i\to 0}\left(\Omega^j/\theta^j\right) &= \lim_{Y^i\to 0}\left\{(1-\kappa)(1+\delta) + \kappa\left[1 + (1-q)\delta + q\delta\left(\theta^j + m\phi^i\right)\right]\left(\phi^j/\theta^j\right)\right\}\\ &= 1 + \delta.\end{aligned} \tag{C.5h}$$

Next, we evaluate the limits as $Y^i \to 0$ of the expressions in (C.2) and (C.3), relying on the above:

$$\lim_{Y^i\to 0}\frac{\phi^i E^i_{\phi^i}}{E^i} = \lim_{Y^i\to 0}\frac{1}{m\phi^j} = \frac{1}{m} \tag{C.6a}$$

$$\lim_{Y^i\to 0}\frac{\phi^i\Lambda^i_{\phi^i}}{\Lambda^i} = \lim_{Y^i\to 0}\frac{\kappa\left[1 - q + q\theta^i\right]}{\Lambda^i/\phi^i} = 1 \tag{C.6b}$$

$$\lim_{Y^i\to 0}\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j} = -\lim_{Y^i\to 0}\frac{\kappa\frac{\phi^i}{\phi^j}\left[1 - q + q\theta^j\right]}{\Lambda^j/\phi^j} = 0 \tag{C.6c}$$

$$\lim_{Y^i\to 0}\frac{\theta^i\Lambda^i_{\theta^i}}{\Lambda^i} = \lim_{Y^i\to 0}\frac{\left[1 - \kappa + \kappa q\phi^i\right]}{\Lambda^i/\theta^i} = 0 \tag{C.6d}$$

$$\lim_{Y^i\to 0}\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j} = -\lim_{Y^i\to 0}\frac{\frac{\theta^i}{\theta^j}\left[1 - \kappa + \kappa q\phi^j\right]}{\Lambda^j/\theta^j} = 0, \tag{C.6e}$$

and

$$\lim_{Y^i\to 0}\frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}=\lim_{Y^i\to 0}\frac{\kappa\left[1+(1-q)\,\delta+q\delta\left(\theta^i-m\phi^i+m\phi^j\right)\right]}{\Omega^i/\phi^i}=1 \tag{C.7a}$$

$$\lim_{Y^i\to 0}\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}=-\lim_{Y^i\to 0}\frac{\kappa\frac{\phi^i}{\phi^j}\left[1+(1-q)\,\delta+q\delta\left(\theta^j-m\phi^j+m\phi^i\right)\right]}{\Omega^j/\phi^j}=0 \tag{C.7b}$$

$$\lim_{Y^i\to 0}\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}=\lim_{Y^i\to 0}\frac{\left[(1-\kappa)\,(1+\delta)+\kappa q\delta\phi^i\right]}{\Omega^i/\theta^i}=0 \tag{C.7c}$$

$$\lim_{Y^i\to 0}\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}=-\lim_{Y^i\to 0}\frac{\frac{\theta^i}{\theta^j}\left[(1-\kappa)\,(1+\delta)+\kappa q\delta\phi^j\right]}{\Omega^j/\theta^j}=0. \tag{C.7d}$$

With these expressions, we can evaluate the limits of the a-coefficients shown in (B.11) as $Y^i\to 0$:

$$\lim_{Y^i\to 0}a_{11} = \lim_{Y^i\to 0}\left\{\frac{\phi^i E^i_{\phi^i}}{E^i}+\frac{\phi^i\Lambda^i_{\phi^i}}{\Lambda^i}-\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j}\right\}=\frac{1}{m}+1 \tag{C.8a}$$

$$\lim_{Y^i\to 0}a_{12} = \lim_{Y^i\to 0}\left\{-\frac{1}{\theta^j}+\frac{\theta^i\Lambda^i_{\theta^i}}{\Lambda^i}-\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j}\right\}=-1 \tag{C.8b}$$

$$\lim_{Y^i\to 0}a_{21} = \lim_{Y^i\to 0}\left\{\frac{\phi^i E^i_{\phi^i}}{E^i}+\frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}-\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right\}=1+\frac{1}{m} \tag{C.8c}$$

$$\lim_{Y^i\to 0}a_{22} = \lim_{Y^i\to 0}\left\{\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}-\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right\}=0. \tag{C.8d}$$

Next, we turn to the b-coefficients shown in (B.17). First, we evaluate the limits of the expressions in (C.4) as $Y^i\to 0$, again using (C.5):

$$\lim_{Y^i\to 0}\frac{\kappa\Lambda^i_\kappa}{\Lambda^i}=\lim_{Y^i\to 0}\frac{\kappa\left(1-q+q\theta^i\right)-\kappa(\theta^i/\phi^i)}{\Lambda^i/\phi^i}=1 \tag{C.9a}$$

$$\lim_{Y^i\to 0}\frac{\kappa\Lambda^j_\kappa}{\Lambda^j}=\lim_{Y^i\to 0}\frac{\kappa\left(1-q+q\theta^j\right)-\kappa(\theta^j/\phi^j)}{\Lambda^j/\phi^j}=0 \tag{C.9b}$$

$$\lim_{Y^i\to 0}\frac{\kappa\Omega^i_\kappa}{\Omega^i}=\lim_{Y^i\to 0}\frac{\kappa\left[1+\delta+q\delta\phi^j\left(m-\theta^j/\phi^j\right)\right]-\kappa\,(1+\delta)\,(\theta^i/\phi^i)}{\Omega^i/\phi^i}=1 \tag{C.9c}$$

$$\lim_{Y^i\to 0}\frac{\kappa\Omega^j_\kappa}{\Omega^j}=\lim_{Y^i\to 0}\frac{\kappa\left[1+\delta+q\delta\phi^i\left(m-\theta^j/\phi^i\right)\right]-\kappa\,(1+\delta)\,(\theta^j/\phi^j)}{\Omega^j/\phi^j}=0 \tag{C.9d}$$

$$\lim_{Y^i\to 0}\frac{q\Lambda^i_q}{\Lambda^i}=-\lim_{Y^i\to 0}\frac{\kappa q\theta^j}{\Lambda^i/\phi^i}=-\frac{q}{1-q} \tag{C.9e}$$

$$\lim_{Y^i\to 0}\frac{q\Lambda^j_q}{\Lambda^j}=-\lim_{Y^i\to 0}\frac{\kappa q\theta^i}{\Lambda^j/\phi^j}=0 \tag{C.9f}$$

$$\lim_{Y^i\to 0}\frac{q\Omega^i_q}{\Omega^i}=\lim_{Y^i\to 0}\frac{\kappa q\delta\phi^j\left(m-\theta^j/\phi^j\right)}{\Omega^i/\phi^i}=\frac{q\delta(m-1)}{1+\delta+q\delta(m-1)} \tag{C.9g}$$

$$\lim_{Y^i \to 0} \frac{q\Omega_q^j}{\Omega^j} = \lim_{Y^i \to 0} \frac{\kappa q \delta \phi^i \left(m - \theta^i/\phi^i\right)}{\Omega^j/\phi^j} = 0. \tag{C.9h}$$

Then, using (B.17), one can confirm the following:

$$\lim_{Y^i \to 0} b_{12} = \lim_{Y^i \to 0} \left\{ \frac{\kappa \Lambda_\kappa^i}{\Lambda^i} - \frac{\kappa \Lambda_\kappa^j}{\Lambda^j} \right\} = 1 \tag{C.10a}$$

$$\lim_{Y^i \to 0} b_{22} = \lim_{Y^i \to 0} \left\{ \frac{\kappa \Omega_\kappa^i}{\Omega^i} - \frac{\kappa \Omega_\kappa^j}{\Omega^j} \right\} = 1 \tag{C.10b}$$

$$\lim_{Y^i \to 0} b_{13} = \lim_{Y^i \to 0} \left\{ \frac{q \Lambda_q^i}{\Lambda^i} - \frac{q \Lambda_q^j}{\Lambda^j} \right\} = -\frac{q}{1-q} \tag{C.10c}$$

$$\lim_{Y^i \to 0} b_{23} = \lim_{Y^i \to 0} \left\{ \frac{q \Omega_q^i}{\Omega^i} - \frac{q \Omega_q^j}{\Omega^j} \right\} = \frac{q\delta(m-1)}{1+\delta+q\delta(m-1)}. \tag{C.10d}$$

D Alternative Models of Trade

One might reasonably ask whether the possibility that the larger country prefers autarky over trade in the first period extends to other trade models aside from the Armington model. In these brief notes, we explain that the answer is yes.

We focus specifically on five trade models: (1) the classical Ricardian model; (2) neoclassical trade models; (3) Armington (1969); (4) Krugman (1980); and (5) Melitz (2003) – Chaney (2008).[23] We also allow for the possibility that trade could be subject to (possibly asymmetric) "iceberg"-type trade costs, thereby distinguishing the results we show here for the Armington model from those we have already presented. We focus on the one-sector versions of models (3)-(5). In addition, we allow for general technological differences (i.e., $A^i \neq A^j$) in models (1) and (3)-(5), as we have done in our proof of Proposition 4. In model (2), we consider a general neoclassical model with multiple resources and internationally diversified production.

As we show below, the larger country j's payoff under trade converges to that under autarky as its rival becomes infinitesimal (i.e., $\lim_{R^i \to 0} U_T^{j*} = \lim_{R^i \to 0} U_A^{j*}$). Thus, a crucial step in demonstrating that Proposition 5 holds for all 5 models of trade is to show

Proposition D.1 *When country $i \in \{1, 2\}$ is infinitesimal, a marginal increase in its scale (i.e., $R^i \uparrow$) above 0 reduces the payoff of its larger rival ($j \neq i$) under trade by more than under autarky:* $\lim_{R^i \to 0}(dU_T^{j*}/dU_A^{j*}) > 1$.

Proof: That $\lim_{R^i \to 0}(dU^{j*}/dR^i) < 0$ under both trade regimes follows from Proposition 3(b). Let us assume (for now) that an increase in the infinitesimal country's resource base

[23] The second category includes the Heckscher-Ohlin and Specific Factors models. Note further, because the equilibrium conditions of the Armington model and the many-good Ricardian model of Eaton and Kortum (2002) are largely isomorphic to one another, our results extend to this latter model as well. The main difference is that the trade elasticity in Eaton and Kortum (2002) is not given by $\sigma - 1$, but rather by a shape parameter from the Fréchet distribution that governs the dispersion of productivities across goods.

has no implications for the large country's first-period income under both autarky and trade (i.e., $\lim_{R^i \to 0} \widehat{Y}^j / \widehat{R}^i = 0$). Furthermore, assume (again, for now) that $\lim_{R^i \to 0} Y_T^j = Y_A^j$. Then, we can proceed as in our proof to Proposition 5. Specifically, the expression shown in (B.52) and reproduced here for convenience,

$$\lim_{R^i \to 0} \left(\frac{dU_T^{j*}}{dU_A^{j*}} \right) = \lim_{R^i \to 0} \left(\frac{Y_T^i}{Y_A^i} \right)^{\frac{m}{1+m}} \times \lim_{R^i \to 0} \left(\frac{\widehat{Y}_T^i}{\widehat{Y}_A^i} \right), \text{ for } i, j \in \{1, 2\},\ i \neq j, \tag{D.1}$$

holds across all of the trade models under consideration. Thus, proving that Proposition D.1 holds amounts to showing that the expression above is greater than 1.

To fill in the remaining blanks, we start with a description of general results for the consequences of trade that hold across the 5 trade models. Details on how these results map onto each specific model are shown in Table D.1.[24] For the general framework, let $p^i \equiv p_j^i / p_i^i$ be the internal relative price of i's importable, measured in units of its exportable. This price can differ from the corresponding world price ($\pi^i \equiv p_j^j / p_i^i$) due to trade costs. Also let $I^i(p_i^i, p_j^i, V^i)$ be the maximized value of i's production of intermediate goods (i.e., the "revenue function"), given its technology and the vector of its factor endowments V^i. To aim for generality, we assume that, even if the endowment vector V^i has multiple elements (as in neoclassical trade models), increases in a country's "scale" R^i raise all elements of V^i with the same "scale elasticity" $\varepsilon_s \geq 1$.[25] Furthermore, to also admit models with monopolistic competition, such as the Krugman and Melitz models, we treat the price terms p_i^i and p_j^i as generally representing the *price indices* of domestically-produced and imported intermediate bundles, potentially reflecting both the number of available varieties within each bundle and the prices of each underlying variety.

Assume the production function for final goods is (CRS) and identical across countries. Then, under both autarky and trade, national income can be written as

$$Y^i = \eta(p_i^i, p_j^i) I^i, \tag{D.2}$$

where the marginal utility of income $\eta(p_i^i, p_j^i)$ acts as an inverse price index for final output and is decreasing in each of its arguments. Also, note that $I^i = I^i(p_i^i, p_j^i, V^i)$ has the standard properties: linearly homogeneous, increasing and convex in prices (p_i^i, p_j^i). By the envelope theorem, country i's production of any good can be derived as $Q_j^i = I_{p_j^i}^i$. Let $D_j^i(p_i^i, p_j^i, I^i)$ denote its Marshallian demand function for good j and $X_j^i = D_j^i - Q_j^i$ its excess demand.

[24] Additional details are available on request.

[25] To fix ideas, $\varepsilon_s = 1$ in any model with constant returns. In the CES-variants of the Krugman and Melitz models we consider, $\varepsilon_s = \sigma/(\sigma - 1) > 1$, where σ is the elasticity of substitution between varieties. See Table D.1 for further details clarifying how revenue functions differ across models.

Differentiating (D.2), we obtain

$$
\begin{aligned}
dY^i &= Y^i_{p^i_i} dp^i_i + Y^i_{p^i_j} dp^i_j + Y^i_{I^i} \left(I^i_{p^i_i} dp^i_i + I^i_{p^i_j} dp^i_j + \sum\nolimits_k I^i_{V^i_k} dV^i_k \right) \\
&= Y^i_{I^i} \left[Y^i_{p^i_i} / Y^i_{I^i} dp^i_i + Y^i_{p^i_j} / Y^i_{I^i} dp^i_j + I^i_{p^i_i} dp^i_i + I^i_{p^i_j} dp^i_j + \sum\nolimits_k V^i_k I^i_{V^i_k} \widehat{V}^i_k \right] \\
&= Y^i_{I^i} \left[\left(-D^i_i + Q^i_i \right) dp^i_i + \left(-D^i_j + Q^i_j \right) dp^i_j + \varepsilon_s I^i \widehat{R}^i \right],
\end{aligned}
$$

where it is useful to note that $Y^i_{p^i_j} / Y^i_{I^i} = -D^i_j$ (by Roy's identity) and that $\sum_k V^i_k I^i_{V^i_k} = \varepsilon_s I^i$, with $\varepsilon_s \geq 1$ as defined above. To simplify further, note that balanced trade requires that $p^i_i (D^i_i - Q^i_i) = p^i_j (D^i_j - Q^i_j) = p^i_j X^i_j$ and that $Y^i_{I^i} \cdot I^i = Y^i$. Thus, percentage changes in Y^i are always given by

$$
\widehat{Y}^i = \varepsilon_s \widehat{R}^i - \mu^i_j \widehat{p}^i_T, \tag{D.3}
$$

where $\mu^i_j \equiv p^i_j X^i_j / I^i$ denotes country i's expenditure share on imported goods; likewise, $\mu^i_i \equiv 1 - \mu^i_j$ denotes the expenditure share on domestically produced goods.[26] Under autarky, clearly $\mu^i_j = 0$, so that $\widehat{Y}^i_A = \varepsilon_s \widehat{R}^i$. For the larger country j, the expression above also implies that $\lim_{R^i \to 0} (\widehat{Y}^j_T / \widehat{R}^i) = 0$, since $\lim_{R^i \to 0} \mu^j_i = 0$ and $\lim_{R^i \to 0} (\widehat{p}^j_T / \widehat{R^i})$ is finite.[27] In addition, based on the results above and drawing from Arkolakis et al. (2012), one can verify that country j's income under trade converges to that under autarky when its rival's scale becomes infinitesimal: $\lim_{R^i \to 0} Y^j_T = Y^j_A$.[28]

Let us consider the first multiplicative term in (D.1) that depends on Y^i_T / Y^i_A. As country i becomes infinitesimal, its income under both trade regimes approaches zero. However, as indicated in Table D.1, the infinitesimal country always benefits from trade with its larger rival, so that $\lim_{R^i \to 0} (Y^i_T / Y^i_A) > 1$, implying that the first multiplicative term is also greater than 1 across all models of trade under consideration.

Let us now turn to the second multiplicative term in (D.1). Since the log-change in a country's "gains from trade" can be written as $\widehat{Y}^i_T - \widehat{Y}^i_A$, it follows that a country's relative

[26] Note that the definition of μ^i_j here differs slightly from that in the main text, particularly in models where countries do not completely specialize in production.

[27] Since $\widehat{p}^j_T / \widehat{R}^i = -\widehat{p}^i_T / \widehat{R}^i$, this second limit can be seen in the last column of Table D.1 for the Ricardian and neoclassical models. Similarly, for each of the other models, the result can be confirmed by taking the limit of $\widehat{p}^i_T / \widehat{R}^i$, shown in the 5th column of the table with the value of Δ shown in the last column, as $R^i \to 0$.

[28] For models (3)–(5) that exhibit complete specialization in production, Arkolakis et al. (2012) show that we can write $Y^j_T = (\mu^j_j)^{-1/\epsilon} Y^j_A$, where $\epsilon > 0$ represents the (absolute value of the) elasticity of import demand with respect to variable trade costs. In the Armington and Krugman models, $\epsilon = \sigma - 1$; in the Melitz-Chaney model, ϵ denotes the shape parameter of the Pareto distribution of firm-level productivity. Then, the result follows from the fact that $\lim_{R^i \to 0} \mu^j_j = 1$. In the Ricardian model (1), the large country j diversifies its production when its rival's resource base is sufficiently small. Thus, in this case, we once again have that, as R^i falls below a certain threshold, $Y^j_T = Y^j_A$ holds. Turning to the neoclassical models, the result follows from the fact that $\lim_{R^i \to 0} \pi^i_T X^i_j = 0$ (since $\lim_{R^i \to 0} X^i_j = 0$ and since π^i_T is finite and bounded by the two countries' autarky prices) and from the world market-clearing condition (which implies $X^j_i = 0$ and which materializes only when $p^i_T = p^i_A$).

gains from trade are inversely related to its relative size if $\widehat{p}^i_T/\widehat{R}^i > 0$, which Table D.1 shows to be true across all the models we consider. It also follows that

$$\lim_{R^i \to 0} \frac{\widehat{Y}^i_T}{\widehat{Y}^i_A} = 1 - \lim_{R^i \to 0} \frac{\mu^i_j \widehat{p}^i_T}{\varepsilon_s \widehat{R}^i}, \tag{D.4}$$

which will be strictly positive so long as $\lim_{R^i \to 0} \mu^i_j \widehat{p}^i_T/\varepsilon_s \widehat{R}^i < 1$. To say more about the limit shown above, we must characterize more fully how $\widehat{p}^i_T$ is determined and its limit as $R^i \to 0$. For each model, let τ^{ji} be the amount of country i's importable that must be shipped from country j for 1 unit to arrive in country i—i.e., inclusive of trade costs. For the Ricardian, neoclassical, Armington, and Krugman models, this means we always have that $p^i_j = p^j_j \tau^{ji}$ and, furthermore, the world market-clearing condition is always given by

$$p^i_i \tau^{ij} X^j_i = p^i_j X^i_j \Rightarrow p^i_T = \frac{\tau^{ij} X^j_i}{X^i_j}. \tag{D.5}$$

The Melitz model also features other types of trade costs, however, which are described further in Table D.1. But, regardless of the assumed model, we can obtain $\lim_{R^i \to 0} \widehat{p}^i_T/\widehat{R}^i \geq 0$, by differentiating the relevant world market-clearing condition.[29] Then, one can confirm that $\lim_{R^i \to 0}(\widehat{Y}^i_T/\widehat{Y}^i_A) > 0$ for each of the 5 models of trade as shown in Table D.1. More importantly, combining the specific result for $\lim_{R^i \to 0}(Y^i_T/Y^i_A)$ with the corresponding result for $\lim_{R^i \to 0}(\widehat{Y}^i_T/\widehat{Y}^i_A)$ in each model (shown respectively in the 6th and 7th columns) confirms that the expression in (D.1) is greater than 1 for all models considered. ||

To establish that Proposition 5 holds across all 5 models of trade, we need only to verify that the large country j's payoff under trade converges to that under autarky as its rival becomes infinitesimal. We have already shown above that $\lim_{R^i \to 0} Y^j_T = Y^j_A$, so that the large country realizes no gains from trade in this limit. At the same time, since $\lim_{R^i \to 0} Y^i_A = \lim_{R^i \to 0} Y^i_T = 0$, equation (10) in the text implies that $\lim_{R^i \to 0} Z^{i*} = \lim_{R^i \to 0} G^{i*} = 0$, so that country i contributes nothing to future output and poses no threat to its larger rival in this limit under either trade regime. It, thus, follows that $\lim_{R^i \to 0} U^j_T = \lim_{R^i \to 0} U^j_A$.

Characterizing the relative gains from trade. We can easily show that several of the results stated in Proposition 4 generalize across these different trade settings as well. To be precise, we focus here on documenting that $Y^i_T/Y^i_A = T^i(R^i, R^j)$ is generally decreasing in R^i/R^j (though in the Ricardian model it ceases to be true once either country is large enough that it no longer completely specializes) and that $\lim_{R^i \to 0} T^i(R^i, R^j) > 1$ and $\lim_{R^i \to \infty} T^i(R^i, R^j) = 1$ (equivalently, $\lim_{R^i \to 0} T^j(R^j, R^i) = 1$), such that an infinitesimal country always realizes a larger relative gain from trade than its larger trading partner

[29] In the Krugman and Melitz models, this result occurs through the effect of an increase in R^i on the number of varieties produced in country i, which lowers the overall price index for these varieties p^i_i, even though the price charged for each variety actually increases through the home market effects inherent to these models.

($\lim_{R^i \to 0}(Y_T^i/Y_A^i)/(Y_T^j/Y_A^j) > 1$). The latter set of results is weaker than the corresponding results stated in Proposition 4 because we now admit models where country i's gain from trade does not become infinite in the limit as $R^i \to 0$. In addition, because we allow for asymmetric trade costs, the smaller country need not always gain more from trade.[30]

To see that country i's gain from trade Y_T^i/Y_A^i is generally decreasing in its relative endowment size, note first from (D.3) and from Table D.1 that the log-change in Y_T^i/Y_A^i for small changes in R^i is given by

$$d \ln\left(\frac{Y_T^i}{Y_A^i}\right) = \widehat{Y}_T^i - \widehat{Y}_A^i = -\mu_j^i \widehat{p}_T^i = -\frac{\mu_j^i \widehat{R}^i}{\Delta}, \tag{D.6}$$

where Δ is a positive term that differs across models depending on elasticity parameters as shown in the last column of Table D.1. To accommodate the switch to incomplete specialization that occurs in the Ricardian model, it should be noted Δ ceases to be finite whenever one country is sufficiently larger than the other, such that $\widehat{p}_T^i$ becomes zero.

Next, to more readily consider changes in *relative* endowment sizes, as is the focus in Proposition 4, the above formula can be generalized to

$$\widehat{Y}_T^i - \widehat{Y}_A^i = -\mu_j^i \widehat{p}_T^i = -\frac{\mu_j^i \left(\widehat{R}^i - \widehat{R}^j\right)}{\Delta}, \tag{D.7}$$

which follows from the symmetry of how p_T^i and p_T^j are defined.[31] Thus, it is straightforward to observe that, with the exception of the incomplete specialization cases that arise in the Ricardian model, an increase in the relative endowment ratio R^i/R^j (such that $\widehat{R}^i - \widehat{R}^j > 0$) will decrease country i's gains from trade while increasing country j's.

Furthermore, as we have already shown for all of these models, $\lim_{R^i \to 0} Y_T^j/Y_A^j = 1$, whereas $\lim_{R^i \to 0} Y_T^i/Y_A^i > 1$, thereby ensuring that a country with a sufficiently small relative initial endowment gains relatively more from trade, i.e., $\lim_{R^i \to 0}(Y_T^i/Y_A^i)/(Y_T^j/Y_A^j) > 1$. Notably, it is straightforward to show that none of these results depend on technology differences, trade costs, or other parameters inherent to these models.

Finally, a related question to ask is whether we can extend the above result to include the effects of changes in autarky size through changes in technology levels, i.e., as we have done in our proof of Proposition 4.[32] In extended notes that are available on request, we show that, for the Armington, Krugman, and Melitz models, the change in the gains from

[30]For moderate size differences where asymmetries in trade costs favor the exports of the larger country, the larger country could have the larger relative gain. Nonetheless, Proposition D.1 implies that there continues to exist a range of more uneven differences in initial size that render autarky more appealing to the large country; in this range, the small country necessarily enjoys the larger relative gain from trade.

[31]It can also be obtained by differentiating the balanced trade conditions for each model shown in the 5th column of Table D.1.

[32]The relevant aggregate technology parameter in the 4 models that feature technological differences—i.e., except for the neoclassical setting—can be derived using $A^i = Y_A^i/R^i$. In each case it is possible to write an expression for Y_A^i that depends on only R^i and exogenous parameters. Notes are available on request.

trade from (D.7) can be re-expressed in the form

$$\widehat{Y}_T^i - \widehat{Y}_A^i = -\mu_j^i \widehat{p}_T^i = -\frac{\mu_j^i \left(\widehat{Y}_A^i - \widehat{Y}_A^j \right)}{\Delta}, \tag{D.8}$$

such that the limit results for a change in relative autarky output levels—including through changes in marginal productivities, entry costs, and/or fixed production costs—are indeed the same as for a change in relative endowment sizes. By following the same steps used in Table D.1, we also have that, for each of these models, $\lim_{Y_A^i \to 0} Y_T^j / Y_A^j = 1$ and $\lim_{Y_A^i \to 0} Y_T^i / Y_A^i = \infty$, implying $\lim_{Y_A^i \to 0} (Y_T^i / Y_A^i)/(Y_T^j / Y_A^j) = \infty$. For the Ricardian model, it is straightforward to show that a uniform improvement in country i's technology level (i.e., a proportional decrease in the unit cost parameters α_i^i and α_j^i) has the same effects as an increase in R^i, just as in the Armington model. However, when α_i^i and α_j^i change independently of one another, it is possible for the degree of comparative advantage to change in such a way that one country becomes relatively larger in terms of autarky output while also realizing a larger relative gain from trade. Because the neoclassical model does not feature technological differences, we do not consider them here.

References

Arkolakis, C., Costinot, A., and Rodríguez-Clare, A. (2012). New Trade Models, Same Old Gains? *American Economic Review*, 102(1):94–130.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *Staff Papers (International Monetary Fund)*, 16(1):159–178.

Chaney, T. (2008). Distorted gravity: The intensive and extensive margins of international trade. *American Economic Review*, 98(4):1707–1721.

Demidova, S. and Rodríguez-Clare, A. (2013). The Simple Analytics of the Melitz Model in a Small Economy. *Journal of International Economics*, 90(2):266–272.

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779.

Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *American Economic Review*, 70(5):950–959.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725.

Syropoulos, C. (2002). Optimum Tariffs and Retaliation Revisited: How Country Size Matters. *Review of Economic Studies*, 69(3):707–727.

Table D.1: Key results for different trade models

Model	Price indices (p_i^i,p_j^i)	Revenue function (I^i)	Scale elasticity (ε_s)	World market-clearing condition	Relative price change ($\widehat{p}_T^i$), with $\widehat{R}^j = 0$	$\lim_{R^i \to 0} Y_T^i / Y_A^i$	$\lim_{R^i \to 0} \widehat{Y}_T^i / \widehat{Y}_A^i$	Notes
Ricardian	$p_i^i = \alpha_i^i r^i$; $p_j^i = p_j^j \tau^{ji}$	$I^i = r^i R^i = (\alpha_i^i)^{-1} p_i^i R^i$ (if completely specialized) $= (\alpha_i^i)^{-1} p_i^i R_i^i + (\alpha_j^i)^{-1} p_j^i R_j^i$ (if not)	1	$p_T^i = \tau^{ij} X_i^j / X_j^i$	$\widehat{p}_T^i = \widehat{R}^i / \Delta_r$	> 1 if $p_T^i \neq p_A^i$	1	R_j^i denotes the amount of country i's endowment employed in the production of good j. α_j^i is the amount of resources needed to produce a unit of good j in country i. We assume $\alpha_j^i/(\alpha_i^i \tau^{ij}) > (\alpha_j^j \tau^{ji})/\alpha_i^j$ such that country i exports good i when there is trade. This also ensures $\lim_{R^i \to 0} Y_T^i / Y_A^i > 1$. $\Delta_r \equiv \varepsilon^i + \varepsilon^j - 1$, where $\varepsilon^i \equiv -\frac{p_j^i \partial X_j^i}{X_j^i \partial p_j^i}$ is the price elasticity of country i's import demand. Note $\lim_{R^i \to 0} p_T^j = p_A^j$ implies that $\lim_{R^i \to 0} \varepsilon^j = \infty$ and, in turn, that $\lim_{R^i \to 0} \widehat{p}^i / \widehat{R}^i = 0$.
Neoclassical	$p_i^i = \sum_k \alpha_{ki}^i r_k^i$; $p_j^i = \sum_k \alpha_{kj}^i r_k^i = p_j^j \tau^{ji}$	$I^i = \sum_k r_k^i V_k^i$	1	$p_T^i = \tau^{ij} X_i^j / X_j^i$	$\widehat{p}_T^i = \widehat{R}^i / \Delta_n$	> 1 if $p_T^i \neq p_A^i$	1	We consider an increase in R^i as increasing all endowments proportionately: $\widehat{V}_k^i / \widehat{R}^i = 1$ for all V_k^i (as in Syropoulos, 2002). This is what ensures $\varepsilon_s = \widehat{I}^i / \widehat{R}^i = 1$. r_k^i is the factor payment received by owners of factor V_k^i in country i and α_{kj}^i is the amount of factor V_k^i required to produce a unit of good j in country i. $\Delta_n \equiv \varepsilon^i + \varepsilon^j - 1$, where $\varepsilon^i \equiv -\frac{p_j^i \partial X_j^i}{X_j^i \partial p_j^i}$ is the price elasticity of country i's import demand. Note $\lim_{R^i \to 0} p_T^j = p_A^j$ implies that $\lim_{R^i \to 0} \varepsilon^j = \infty$ and, in turn, that $\lim_{R^i \to 0} \widehat{p}^i / \widehat{R}^i = 0$.
Armington	$p_i^i = (A^i)^{-1} r^i$; $p_j^i = p_j^j \tau^{ji}$.	$I^i = r^i R^i$; $p_i^i = (A^i)^{-1} r^i \implies Q_i^i = A^i R^i$	1	$\mu_i^j r^j R^j = \mu_j^i r^i R^i \implies$ $p_T^i = \left[\frac{1+\left(p_T^i \tau^{ji}/\tau^{ij}\right)^{1-\sigma}}{1+\left(1/p_T^i\right)^{1-\sigma}}\right] \left[\frac{\tau^{ji} A^i R^i}{A^j R^j}\right]$	$\widehat{p}_T^i = \widehat{R}^i / \Delta_a$	$\lim_{R^i \to 0} (\mu_i^i)^{\frac{1}{1-\sigma}} = \infty$	$\lim_{R^i \to 0} 1 - \mu_j^i \frac{\widehat{p}_T^i}{\widehat{R}^i}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\Delta_a}$ $= \frac{\sigma - 1}{\sigma} > 0$	$\sigma > 1$ is the elasticity of substitution. A^i gives the technology level in country i. The overall price index is given by $P^i \equiv [(p_i^i)^{1-\sigma} + (p_j^i)^{1-\sigma}]^{1/(1-\sigma)}$. Expenditure shares are given by: $\mu_i^i = (p_i^i/P^i)^{1-\sigma}$; $\mu_j^i = (p_j^i/P^i)^{1-\sigma}$. $\Delta_a \equiv 1 + (\sigma - 1)(\mu_i^i + \mu_j^j) > 0$. The Armington model may be thought of as a limiting case of a Ricardian model where the unit cost of producing a country's importable becomes prohibitive, in which case $A^i = \lim_{\alpha_j^i \to \infty} [1 + (\alpha_j^i/\alpha_i^i)^{1-\sigma}]^{\frac{-1}{1-\sigma}} / \alpha_i^i = 1/\alpha_i^i$.
Krugman	$p_i^i \equiv \left[\int_{N^i} p(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} =$ $(N^i)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} a^i r^i =$ $\left(\frac{R^i}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} a^i r^i$ $p_j^i \equiv p_j^j \tau^{ji}$ (Note free entry implies that $N^i = \frac{R^i}{\sigma f^i}$.)	$I^i = r^i R^i =$ $(a^i)^{-1} \left(\frac{\sigma-1}{\sigma}\right) (N^i)^{\frac{1}{\sigma-1}} p_i^i R^i$ $\implies I^i =$ $(a^i)^{-1} (\sigma f^i)^{\frac{1}{1-\sigma}} \left(\frac{\sigma-1}{\sigma}\right) p_i^i (R^i)^{\frac{\sigma}{\sigma-1}}$	$\frac{\sigma}{\sigma-1} > 1$	$\mu_i^j r^j R^j = \mu_j^i r^i R^i \implies$ $p_T^i =$ $\left[\frac{1+\left(p_T^i \tau^{ji}/\tau^{ij}\right)^{1-\sigma}}{1+\left(1/p_T^i\right)^{1-\sigma}}\right] \left[\frac{\tau^{ji} a^j (f^i)^{\frac{1}{1-\sigma}}}{a^i (f^j)^{\frac{1}{1-\sigma}}}\right] \left[\frac{R^i}{R^j}\right]^{\frac{\sigma}{\sigma-1}}$	$\widehat{p}_T^i = \widehat{R}^i / \Delta_k$	$\lim_{R^i \to 0} (\mu_i^i)^{\frac{1}{1-\sigma}} = \infty$	$\lim_{R^i \to 0} 1 - \mu_j^i \frac{\widehat{p}_T^i}{\varepsilon_s \widehat{R}^i}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\varepsilon_s \Delta_k}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\Delta_a}$ $= \frac{\sigma-1}{\sigma} > 0$	$\sigma > 1$ is the elasticity of substitution. a_i is the marginal cost of production in terms of units of R^i. The overall price index is given by $P^i \equiv [(p_i^i)^{1-\sigma} + (p_j^i)^{1-\sigma}]^{1/(1-\sigma)}$. Expenditure shares are given by: $\mu_i^i = (p_i^i/P^i)^{1-\sigma}$; $\mu_j^i = (p_j^i/P^i)^{1-\sigma}$. N^i is the mass of firms in i. f^i is the fixed cost of production (paid in units of R^i). $\Delta_k \equiv [(\sigma-1)/\sigma]\Delta_a \in (0, \Delta_a)$.
Melitz	$p_i^i \equiv \left[\Lambda_1 \left(\frac{I^i}{f_d^i}\right)^{\xi} N^i \underline{\varphi}_i^{\omega} (r^i)^{-\omega-\xi}\right]^{-\frac{1}{\omega}}$ where $\xi \equiv \frac{\omega-(\sigma-1)}{\sigma-1}$ and $\Lambda_1 \equiv (\frac{\sigma-1}{\sigma})^{-\omega} \sigma^{-\xi} \frac{\omega}{\omega-(\sigma-1)}$. $p_j^i \equiv p_j^j \left(\frac{I^i}{I^j}\right)^{-\frac{\xi}{\omega}} \left(\frac{f_x^{ji}}{f_d^j}\right)^{\frac{\xi}{\omega}} (\tau^{ji})$. (Note the overall price index is given by $P^i = [(p_i^i)^{-\omega} + (p_j^i)^{-\omega}]^{-\frac{1}{\omega}}$.)	$I^i = r^i R^i =$ $\left[\Lambda_1 \left(\frac{R^i}{f_d^i}\right)^{\xi} N^i \underline{\varphi}_i^{\omega}\right]^{\frac{1}{\omega}} p_i^i R^i =$ $\Lambda_2^{\frac{1}{\omega}} \underline{\varphi}_i (f_e^i)^{-\frac{1}{\omega}} (f_d^i)^{-\frac{\xi}{\omega}} p_i^i (R^i)^{\frac{\sigma}{\sigma-1}}$ where $\Lambda_2 = \frac{\sigma-1}{\omega\sigma}\Lambda_1 > 0$. (Note free entry delivers $N^i = \frac{R^i(\sigma-1)}{f_e \omega \sigma}$.)	$\frac{\sigma}{\sigma-1} > 1$	$\mu_i^j r^j R^j = \mu_j^i r^i R^i \implies$ $\frac{r^j}{r^i} = \frac{\mu_j^i R^i}{\mu_i^j R^j} =$ $\left[\frac{\overline{\tau}^{ji} + \left(R^j/R^i\right)\left(r^j/r^i\right)^{-\omega-\xi}}{\overline{\tau}^{ij} + (R^i/R^j)(r^i/r^j)^{-\omega-\xi}}\right] \left[\frac{\overline{\tau}^{ij} R^i}{\overline{\tau}^{ji} R^j}\right]$ where $\overline{\tau}^{ij} \equiv$ $(\tau^{ij})^{-\omega} (f_x^{ij}/f_d^i)^{-\xi} (\underline{\varphi}_i/\underline{\varphi}_j)^{\omega} (f_e^j/f_e^i)$	$\widehat{p}_T^i = \widehat{R}^i / \Delta_m$	$\lim_{R^i \to 0} (\mu_i^i)^{-\frac{1}{\omega}} = \infty$	$\lim_{R^i \to 0} 1 - \mu_j^i \frac{\widehat{p}_T^i}{\varepsilon_s \widehat{R}^i}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\varepsilon_s \Delta_m}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\varepsilon_s \Delta_k + \varepsilon_s [\omega - (\sigma-1)]}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\Delta_a + \frac{\omega\sigma}{\sigma-1} - \sigma} =$ $1 - \frac{\sigma-1}{\omega\sigma} \in (0,1)$	$\sigma > 1$ is the elasticity of substitution. $\omega > \sigma - 1$ is the shape parameter of the Pareto firm productivity distribution, with pdf $g^i(\varphi) = \omega \underline{\varphi}_i^{\omega} \varphi^{-(\omega+1)}$. Expenditure shares are given by: $\mu_i^i = (p_i^i/P^i)^{-\omega}$; $\mu_j^i = (p_j^i/P^i)^{-\omega}$. f_e^i, f_d^i, and f_x^{ij} respectively are the fixed costs of entry, production, and exporting. $\Delta_m \equiv \Delta_k + [\omega - (\sigma-1)](\mu_i^i + \mu_j^j) > \Delta_k > 0$.

For completeness, we also include results for the Armington model. The key result is whether $\lim_{R^i \to 0} (Y_T^i / Y_A^i)^{m/(1+m)} \times \lim_{R^i \to 0} \widehat{Y}_T / \widehat{Y}_A > 1$, which is satisfied for all models shown. Detailed derivations of this result for each of these models are available by request. The "scale elasticity" (ε_s) can be obtained in each model as $\varepsilon_s \equiv \partial \ln I^i / \partial \ln R^i$, after first writing the revenue function in the form $I^i \equiv I(p_i^i, p_j^i, R^i)$ and applying the relevant definitions of p_i^i and p_j^i for each model. Some additional useful references include Arkolakis et al. (2012) and Demidova and Rodríguez-Clare (2013).

E War, Peaceful Settlement, and Future Trade

In what follows, we sketch out the determination of the equilibrium in shares (ϕ^i, θ^i) for war as a winner-take-all (*WTA*) contest and for peaceful settlement under alternative rules of division based on the Nash-bargaining (*NB*) and splitting the surplus (*SS*) protocols that take the outcome under war as the threat point. Finally, we allow for trade in the second period assuming a rule of division of contested output, in the case that a dispute arises, according to ϕ^i. Throughout, we interpret q as the probability of a dispute arising in period $t = 2$, and continue to admit the possibility that that future output can be partially secure in this event (i.e., $\kappa \in (0, 1]$). Furthermore, we conduct our analysis in the context of the Armington (1969) model, continuing to assume that $A^i = A^j = 1$.

E.1 War as a Winner-Take-All Contest

When the two countries anticipate that any future dispute will be resolved through a *WTA* contest, their payoffs are defined as follows:

$$\begin{aligned} U^i &= \ln(Y^i - G^i - Z^i) \\ &\quad + \delta \left\{ q \left[\phi^i \ln \left(Z^i + \kappa Z^j \right) + \left(1 - \phi^i \right) \ln \left((1-\kappa) Z^i \right) \right] + (1-q) \ln \left(Z^i \right) \right\}, \end{aligned}$$

for $i, j \in \{1, 2\}$, $i \neq j$. Their first-period choices of G^i and Z^i at an interior optimum satisfy respectively the following conditions (which are the associated FOCs multiplied respectively by G^i and Z^i):

$$G^i U^i_{G^i} = \delta q \phi^i_{G^i} G^i \left[\ln \frac{Z^i + \kappa Z^j}{(1-\kappa) Z^i} \right] - \frac{G^i}{Y^i - G^i - Z^i} = 0 \tag{E.1a}$$

$$Z^i U^i_{Z^i} = \delta \left[q \left(\frac{\phi^i Z^i}{Z^i + \kappa Z^j} + 1 - \phi^i \right) + (1-q) \right] - \frac{Z^i}{Y^i - G^i - Z^i} = 0, \tag{E.1b}$$

for $i, j \in \{1, 2\}$, $i \neq j$.[33] Similar to the analysis in the main text, these two equations give us a system of four equations in four unknowns. Recalling from (2) in the text that $\phi^i_{G^i} = m \phi^i \phi^j / G^i$ and using our definition of $\theta^i = Z^i/(Z^i + Z^j)$, the first terms in (E.1a) and (E.1b) can be written respectively as

$$\gamma^i \equiv \gamma^i(\phi^i, \theta^i) = G^i MB^i_G = \delta q \phi^i \phi^j m \ln \left[\frac{\theta^i + \kappa \theta^j}{\theta^i (1-\kappa)} \right] > 0 \tag{E.2a}$$

$$\zeta^i \equiv \zeta(\phi^i, \gamma^i) = Z^i MB^i_Z = \delta \left[1 - \frac{\kappa \theta^j}{\theta^i + \kappa \theta^j} q \phi^i \right] > 0. \tag{E.2b}$$

[33] Observe that, as is the case where a dispute is resolved with a division of contested output, allowing for the possibility that a dispute resolved through a *WTA* contest destroys some fraction of all future output would have no effect on the FOCs above and, thus, would be inconsequential for equilibrium arming and saving choices that satisfy those conditions.

In turn, these definitions allow us to write the FOCs in (E.1a) and (E.1b) respectively as

$$G^i U^i_{G^i} \;\; = \;\; \gamma^i - \frac{G^i}{Y^i - G^i - Z^i} = 0 \tag{E.3a}$$

$$Z^i U^i_{Z^i} \;\; = \;\; \zeta^i - \frac{Z^i}{Y^i - G^i - Z^i} = 0, \tag{E.3b}$$

from which we can solve for G^i and Z^i:

$$G^i \;\; = \;\; \frac{\gamma^i}{1 + \gamma^i + \zeta^i} Y^i \tag{E.4a}$$

$$Z^i \;\; = \;\; \frac{\zeta^i}{1 + \gamma^i + \zeta^i} Y^i. \tag{E.4b}$$

Recalling that $G^i/G^j = \left(\phi^i/\phi^j\right)^{1/m}$ and $Z^i/Z^j = \theta^i/\theta^j$ for $i, j \in \{1, 2\}$, $i \neq j$, we use the FOCs in (E.3) to write the S^i- and B^i-contours as

$$S^i\left(\phi^i, \theta^i; q, \kappa\right) \;\; \equiv \;\; \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}} - \frac{\theta^i \zeta^j \gamma^i}{\theta^j \zeta^i \gamma^j} = 0 \tag{E.5a}$$

$$B^i\left(\phi^i, \theta^i; Y^i/Y^j, q, \kappa\right) \;\; \equiv \;\; \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}} - \frac{\gamma^i\left(1 + \gamma^j + \zeta^j\right)}{\gamma^j\left(1 + \gamma^i + \zeta^i\right)} \left(\frac{Y^i}{Y^j}\right) = 0. \tag{E.5b}$$

The expression for B^i above is precisely what we have in the baseline model (A.3). Furthermore, as one can confirm by using the values of γ^i and ζ^i shown in (A.4a) and (A.4b), the expression for S^i above is identical to that shown above in (A.2). Hence, the equations in (E.5) define the equilibrium whether the countries resolve their dispute (given one arises) through a *WTA* contest or through a division of the contested output according to ϕ^i. What differs, of course, are the specific values of γ^i and ζ^i. Nonetheless, the qualitative part of our analysis in the paper remains intact. As before, we can show that the S^i and B^i schedules are well-behaved and generate a unique equilibrium in the interior of the strategy space. Moreover, the dependence of U^i on Y^j is U-shaped, such that a sufficiently large country prefers not to trade in the first period when a second-period dispute is expected to escalate to a *WTA* conflict.

E.2 Peaceful Settlement under Alternative Rules of Division

We now turn to two alternative rules of division under peaceful settlement in the event a dispute arises in period $t = 2$: *SS* that follows from a split of the surplus and *NB* that follows from the Nash bargaining. Here, we show that the S^i- and B^i-contours are again as shown in (E.5), but with different values of γ^i and ζ^i. As will become clear shortly, the solutions for the rules of division under both bargaining protocols depend on the countries' threat-point payoffs given by those under war (a *WTA* contest), as well as the resource allocations G^i and Z^i $(i = 1, 2)$ made in period $t = 1$. For now, let $s^i = s^i(G^i, G^j, Z^i, Z^j)$ denote country i's $(\neq j = 1, 2)$ share of the contested output $\kappa\left(Z^i + Z^j\right)$ produced in period

$t = 2$.

Then, under either *SS* or *NB*, country i's expected, two-period payoff can be written as

$$U^i = \ln(Y^i - G^i - Z^i) + \delta\left[q\ln\left(s^i\kappa(Z^i + Z^j) + (1-\kappa)\,Z^i\right) + (1-q)\ln\left(Z^i\right)\right],$$

for $i, j \in \{1, 2\}$, $i \neq j$, where $Y^i = R^i$ under autarky and $Y^i = T^i(R^i, R^j)R^i$ under free trade. Country i's optimizing choices of G^i and Z^i in period $t = 1$ satisfy the following conditions at an interior solution:

$$G^i U^i_{G^i} = \delta q \frac{\kappa\left(Z^i + Z^j\right)\left(G^i s^i_{G^i}\right)}{s^i\kappa\left(Z^i + Z^j\right) + (1-\kappa)Z^i} - \frac{G^i}{Y^i - G^i - Z^i} = 0 \tag{E.6a}$$

$$Z^i U^i_{Z^i} = \delta\left[\frac{q\left((1-\kappa+\kappa s^i)Z^i + \kappa(Z^i + Z^j)Z^i s^i_{Z^i}\right)}{s^i\kappa(Z^i + Z^j) + (1-\kappa)Z^i} + 1 - q\right] - \frac{Z^i}{Y^i - G^i - Z^i} = 0, \tag{E.6b}$$

which again give us a system of four equations in four unknowns.

Let us now define the elasticities of $s^i(\cdot)$ with respect to country i's arming and saving as follows:

$$\varepsilon^i_G \equiv \frac{\partial s^i/\partial G^i}{s^i/G^i} \quad \text{and} \quad \varepsilon^i_Z \equiv \frac{\partial s^i/\partial Z^i}{s^i/Z^i} \quad \text{for } i = 1, 2.$$

As expected, these elasticities depend on arming and saving. Using the expressions above, one can verify that the FOCs in (E.3) and thus the conditions $S^i = 0$ and $B^i = 0$ in (E.5) continue hold under peaceful settlement with either rule of division, but where now

$$\gamma^i = \gamma(\phi^i, \theta^i) = \delta q \frac{\kappa s^i}{\kappa s^i + (1-\kappa)\,\theta^i}\varepsilon^i_G \tag{E.7a}$$

$$\zeta^i = \zeta(\phi^i, \theta^i) = \delta\left\{q\left[\frac{\left(1-\kappa+\kappa s^i\right)\theta^i + \kappa s^i\varepsilon^i_Z}{\kappa s^i + (1-\kappa)\theta^i}\right] + 1 - q\right\}. \tag{E.7b}$$

Observe that, under a rule of division according to $s^i = \phi^i$ specified in (2), $\varepsilon^i_G = m\phi^j$ and $\varepsilon^i_Z = 0$. Thus, the expressions for γ^i and ζ^i shown in (E.7) simplify to those shown in the text in (A.4a) and (A.4b) when $s^i = \phi^i$.

Next, we find the specific expressions for γ^i and ζ^i as they depend on the rule of division according to *SS* or *NB*. Both rules are derived on the basis of the following second-period payoffs contingent on a dispute arising:

$$v^i = v^i\left(s^i; \cdot\right) \equiv \ln\left(s^i\kappa\left(Z^i + Z^j\right) + (1-\kappa)\,Z^i\right) \tag{E.8a}$$

$$u^i = u^i\left(\phi^i; \cdot\right) \equiv \phi^i\ln\left(Z^i + \kappa Z^j\right) + \phi^j\ln\left((1-\kappa)\,Z^i\right). \tag{E.8b}$$

The first expression v^i shows the second-period payoff under the rule of division s^i; the second expression u^i shows the second-period payoff under a *WTA* contest and serves as the threat-point payoff for both bargaining protocols.

Splitting the surplus. Taking into account the fact that $s^i + s^j = 1$, s^i under the *SS* protocol is defined implicitly as the solution to $v^i - u^i = v^j - u^j$. Using the definitions of the payoffs in (E.8a) and (E.8b), we can write $v^i - u^i$ as

$$v^i - u^i = \ln\left(\frac{s^i\kappa\left(Z^i + Z^j\right) + (1-\kappa)\,Z^i}{\left[Z^i + \kappa Z^j\right]^{\phi^i}\left[(1-\kappa)\,Z^i\right]^{\phi^j}}\right), \quad \text{for} \quad i,j \in \{1,2\},\ i \neq j.$$

With the definition of $\theta^i = Z^i/(Z^i + Z^j)$, the condition $v^i - u^i = v^j - u^j$ can be written as

$$\frac{v^i - u^i}{v^j - u^j} = 1 \implies \frac{\ln\left(\frac{s^i\kappa + (1-\kappa)\theta^i}{\left[\theta^i + \kappa\theta^j\right]^{\phi^i}\left[(1-\kappa)\theta^i\right]^{\phi^j}}\right)}{\ln\left(\frac{s^j\kappa + (1-\kappa)\theta^j}{\left[\theta^j + \kappa\theta^i\right]^{\phi^j}\left[(1-\kappa)\theta^j\right]^{\phi^i}}\right)} = 1, \tag{E.9}$$

or equivalently as

$$\left[\frac{\kappa s^i + (1-\kappa)\,\theta^i}{\kappa s^j + (1-\kappa)\,\theta^j}\right]\left[\frac{\theta^j + \kappa\theta^i}{(1-\kappa)\,\theta^i}\right]^{\phi^j}\left[\frac{(1-\kappa)\,\theta^j}{\theta^i + \kappa\theta^j}\right]^{\phi^i} = 1. \tag{E.10}$$

These equations implicitly define s^i under *SS* as a function of ϕ^i and θ^i. With the definitions of ϕ^i and θ^i, we totally differentiate (E.10) and use the implicit function theorem to characterize the dependence of s^i on G^i and Z^i under this protocol as follows:

$$\varepsilon^i_G \equiv \frac{\partial s^i/\partial G^i}{s^i/G^i} = \frac{m\phi^i\phi^j}{\kappa s^i\chi^i}\ln\left(\left[\frac{\theta^i + \kappa\theta^j}{(1-\kappa)\,\theta^i}\right]\left[\frac{\theta^j + \kappa\theta^i}{(1-\kappa)\,\theta^j}\right]\right) > 0 \tag{E.11a}$$

$$\varepsilon^i_Z \equiv \frac{\partial s^i/\partial Z^i}{s^i/Z^i} = \frac{1}{s^i\chi^i}\left[\frac{s^i\theta^j}{\kappa s^i + (1-\kappa)\,\theta^i} + \frac{s^j\theta^i}{\kappa s^j + (1-\kappa)\,\theta^j} - \frac{\phi^i\theta^j}{\theta^i + \kappa\theta^j} - \frac{\phi^j\theta^i}{\theta^j + \kappa\theta^i}\right], \tag{E.11b}$$

where

$$\chi^i \equiv \frac{1}{\kappa s^i + (1-\kappa)\,\theta^i} + \frac{1}{\kappa^j s^j + (1-\kappa)\,\theta^j} > 0.$$

As shown above, under *SS*, an increase in country i's arming raises its share of the contested output in period $t = 2$. While the sign of the effect of country i's saving on s^i appears to be ambiguous, one can verify that $\varepsilon^i_Z > 0$ when κ is large enough.[34]

To find the equilibrium values of ϕ^i and θ^i under *SS*, we can substitute the expressions for ε^i and η^i into the values of γ^i and ζ^i in (E.7) which we can then use in (E.5). This is a system of two equations in three unknowns: ϕ^i, θ^i and s^i. However, with the condition that implicitly defines s^i, we have a system of three equations in three unknowns.

[34]Specifically, suppose that $\kappa = 1$ so that all future output is contestable. In this case, the sign of the expression inside the brackets in (E.11b) equals sign$\{1 - \phi^i\theta^j - \phi^j\theta^i\}$, which is strictly positive.

Nash bargaining. Focusing on the case of equal bargaining weights, s^i under the *NB* protocol is chosen to maximize

$$\left[v^i\left(s^i\right)-u^i\right]^{\frac{1}{2}}\left[v^j\left(s^j\right)-u^j\right]^{\frac{1}{2}},$$

where again $s^i+s^j=1$. The FOC for this program can be written as

$$\frac{v^i-u^i}{v^j-u^j}=\frac{\partial v^i/\partial s^i}{\partial v^j/\partial s^j} \implies \frac{\ln\left(\frac{s^i\kappa+(1-\kappa)\theta^i}{\left[\theta^i+\kappa\theta^j\right]^{\phi^i}\left[(1-\kappa)\theta^i\right]^{\phi^j}}\right)}{\ln\left(\frac{s^j\kappa+(1-\kappa)\theta^j}{\left[\theta^j+\kappa\theta^i\right]^{\phi^j}\left[(1-\kappa)\theta^j\right]^{\phi^i}}\right)}=\frac{s^j\kappa+(1-\kappa)\,\theta^j}{s^i\kappa+(1-\kappa)\,\theta^i}, \tag{E.12}$$

which implicitly defines s^i in this case. Note the similarity between (E.9) and (E.12). The key difference is that, under *NB*, the slope of the payoff frontier (shown in the LHS of the second expression, which is endogenous) also matters.

To compute ε_G^i and ε_Z^i under *NB*, we apply the implicit function theorem to (E.12):

$$\begin{aligned}\varepsilon_G^i \;&=\; \frac{m\phi^i\phi^j}{\kappa s^i\left(2+\chi^i\right)}\left\{\left[\kappa s^i+(1-\kappa)\,\theta^i\right]\ln\left(\frac{\theta^i+\kappa\theta^j}{(1-\kappa)\,\theta^i}\right)\right.\\ &\qquad\left.+\left[\kappa s^j+(1-\kappa)\,\theta^j\right]\ln\left(\frac{\theta^j+\kappa\theta^i}{(1-\kappa)\,\theta^j}\right)\right\} \qquad\qquad (\text{E.13a})\\ \varepsilon_Z^i \;&=\; \frac{1}{\kappa s^i\left(2+\chi^i\right)}\left\{\kappa\theta^i\left[s^j-\frac{\phi^j\left(\kappa s^j+(1-\kappa)\,\theta^j\right)}{\theta^j+\kappa\theta^i}\right]\right.\\ &\qquad\left.+\,\kappa\theta^j\left[s^i-\frac{\phi^i\left(\kappa s^i+(1-\kappa)\,\theta^i\right)}{\theta^i+\kappa\theta^j}\right]-(1-\kappa)\,\theta^i\theta^j\chi^i\right\}, \qquad (\text{E.13b})\end{aligned}$$

where now

$$\chi^i\equiv\frac{1}{2\left[\kappa s^i+(1-\kappa)\,\theta^i\right]}\ln\left(\frac{\kappa s^j+(1-\kappa)\,\theta^j}{\left[(1-\kappa)\,\theta^j\right]^{\phi^i}\left(\kappa\theta^i+\theta^j\right)^{\phi^j}}\right).$$

As in the case of *SS*, we can flesh out the implications of the above for the equilibrium values of ϕ^i and θ^i, by substituting the expressions for ε^i and η^i above into the values of γ^i and ζ^i in (E.7), which we can then use in (E.5). With (E.12) that implicitly defines s^i, we have a system of three equations in three unknowns: ϕ^i, θ^i and s^i.

Discussion. The similarity of the expressions that lead to the determination of s^i under *SS* and *NB* makes it possible to present both forms in a unified way and compare them. Still, their complexity makes it difficult to derive precise analytical results. Nevertheless, numerical analysis suggests that, while payoffs under both rules behave similarly to the case where $s^i=\phi^i$, the range of relative endowments under which the larger country prefers autarky to trade under *SS* and *NB* tends to be smaller and shrink faster as the elasticity of substitution σ falls.

E.3 Trade in the Future

As described in Section 5.3 of the text, we allow for the possibility of free trade in period $t = 2$ when no dispute arises (with probability $1-q$) and suppose the two countries are able to enter into an agreement that ensures both trade and peace when a dispute does occur (with probability q). In either event, one can apply the analysis of trade for period $t = 1$ in Section 5.1 to show that world output in period $t = 2$ is given by $\widetilde{Y} = [\sum_k (Z^k)^b]^{1/b}$, where Z^i and Z^j equal the countries' respective savings in period $t = 1$ and where $b \equiv \frac{\sigma-1}{\sigma}$.[35] If no dispute occurs, each country i enjoys $\widetilde{Y}^i = \psi^i \widetilde{Y}$ units of the final good, where $\psi^i = (Z^i)^b / [\sum_k (Z^k)^b]$ represents country i's "competitive share" based on its own input production and on its terms of trade.[36] In the case of a dispute, we assume the two countries enter into a negotiated settlement in which they again trade their intermediate goods freely and then divide the contested pool of output $\kappa \widetilde{Y}$ according to ϕ^i; thus, each country i consumes $\phi^i \kappa \widetilde{Y}$ plus its secure output $(1-\kappa)\,\widetilde{Y}^i$ $(= \psi^i (1-\kappa)\,\widetilde{Y})$.[37]

Accordingly, country i's expected, two-period payoff can be written as

$$U^i = \ln(Y^i - G^i - Z^i) + \delta \left[q \ln \left(\phi^i \kappa \widetilde{Y} + \psi^i (1-\kappa)\, \widetilde{Y} \right) + (1-q) \ln \left(\psi^i \widetilde{Y} \right) \right].$$

Based on the payoffs shown above, we can proceed as before. The FOCs associated with each country i's choice of G^i and Z^i can be written precisely as shown earlier in (E.3), but where γ^i and ζ^j take on new values:

$$\gamma^i = \delta q \frac{\kappa \phi^i}{\kappa \phi^i + (1-\kappa)\, \psi^i} m \phi^j > 0 \tag{E.14a}$$

$$\zeta^i = \delta \left\{ qb \left[\frac{(1-\kappa+\kappa\phi^i)\, \psi^i}{\kappa\phi^i + (1-\kappa)\psi^i} \right] + b\,(1-q) + (1-b)\, \psi^i \right\} > 0. \tag{E.14b}$$

Upon substituting these values for γ^i and ζ^i into the expressions for the S^i- and B^i-contours in (E.5), one can solve for the equilibrium values of ϕ^i and θ^i. An equilibrium analysis of the very simple case where $\kappa = 1$ (and thus $\gamma^i = \delta q m \phi^j$ and $\zeta^i = \delta[\psi^i + (1-q) b \psi^j]$) and related ones shows that, given any pair of values for $\kappa > 0$ and $q > 0$, there exists a threshold value of the elasticity of substitution $\sigma > 1$ above which the set of relative endowments that make the large country prefer autarky over trade in period $t = 1$ is non-empty. Conversely, though, the presence of future trade implies that for sufficiently small σ, this set is empty. Hence, the possibility for trade in the future can matter for the larger country's preferences

[35] Recall that a tilde ("$\sim$") above a variable indicates its value in the second period.

[36] One can verify $\widetilde{Y}^i$ is structurally similar to first-period output, $Y^i = T^i(R^i, R^j)R^i$, implied by (14) in the text.

[37] By virtue of the linear homogeneity of $\widetilde{Y}$, settlement could be thought of as a trade agreement implemented through free trade of κZ^i and κZ^j (thus generating insecure world income of $\kappa \widetilde{Y}$) with appropriate *ex post* transfers that give country i $\phi^i \kappa \widetilde{Y}$ units of output; at the same time, each country would obtain the competitive rewards associated with free trade of $(1-\kappa)\, Z^i$ and $(1-\kappa)\, Z^j$, which for country i equals $\psi^i (1-\kappa)\, \widetilde{Y}$.

over current trade.

F Extended Time-Horizon

To extend the time horizon beyond two periods, we make the following adjustments. Let $t \in \{1, 2, ..., \mathcal{T}\}$ be a superscript for the time period. We assume, along the lines of our baseline model, that conflict never arises in period $t = 1$; in all subsequent periods ($t > 1$), conflict arises with probability $q > 0$, in which case some fraction of output in that period $\kappa \in (0, 1]$ is contestable. Furthermore, the two countries can trade their intermediate goods freely beyond the initial period, though only when a conflict does not arise; otherwise, trade cannot occur and the pool of contested output is divided according to ϕ^i as in the baseline model. The full sequence of events for a given time period in this version of the model can be described as follows:

- For period $t = 1$, countries are endowed with resource levels $R^{i,1}$ and $R^{j,1}$, for $i, j \in \{1, 2\}$, $i \neq j$. Since by assumption peace prevails that period, their income $Y^{i,1}$ depends only on whether they both choose to trade then or not:

$$Y^{i,1} = \begin{cases} T^{i,1} R^{i,1} & \text{if they trade} \\ R^{i,1} & \text{otherwise,} \end{cases} \tag{F.15}$$

  where $T^{i,t} \equiv T(R^{i,t}, R^{j,t}) > 1$ represents the relative income gains from trade based on the Armington trade model.

- For periods $t > 1$, resource wealth is determined by savings from the previous period: $R^{i,t} = Z^{i,t-1}$. Income levels depend on that wealth and on whether a conflict arises in that period. As in baseline model, if a dispute arises, it ends with each country receiving a share $\phi^{i,t}$ of contestable world output $\kappa \left(Z^{i,t-1} + Z^{j,t-1}\right)$, where

$$\phi^{i,t} = \frac{\left(G^{i,t-1}\right)^m}{\left(G^{i,t-1}\right)^m + \left(G^{j,t-1}\right)^m} \qquad \text{for } t > 1, \tag{F.16}$$

  and where $Z^{i,t-1}$ and $G^{i,t-1}$ respectively denote country i's saving and arming choices from the previous period. If a dispute does not arise, countries trade and realize the income gains from trade. Hence, income levels for periods $t > 1$ are given by

$$Y^{i,t} = \begin{cases} \phi^{i,t} \kappa \left(Z^{i,t-1} + Z^{j,t-1}\right) + (1 - \kappa) Z^{i,t-1} & \text{w.p. } q \\ T^{i,t} Z^{i,t-1} & \text{w.p. } 1 - q \end{cases} \qquad \text{for } t > 1, \tag{F.17}$$

  where $T^{i,t} \equiv T^i(Z^{i,t-1}, Z^{j,t-1}) > 1$ gives the gains from trade from an Armington model, as above.

- At the end of each period, countries allocate their income between consumption ($C^{i,t}$), savings ($Z^{i,t}$), and arming ($G^{i,t}$, subject to the period t resource constraint:

$$Y^{i,t} = C^{i,t} + G^{i,t} + Z^{i,t}. \tag{F.18}$$

The roles of the parameters σ, δ, and m in this model are analogous to the roles they play in the main text. By the same token, the role of κ is analogous to the role played in the more general model outlined in Appendix A and studied in Appendix B. To keep the analysis compact, we will henceforth assume that $\mathcal{T} = 3$, implying a 3-period game. Each country $i = 1, 2$ aims to maximize

$$U^i = \ln C^{i,1} + \delta E\left[\ln C^{i,2}\right] + \delta^2 E\left[\ln C^{i,3}\right],$$

where the expectations operator $E[\cdot]$ is taken over information available at the end of period $t = 1$, subject to (F.15), (F.16), (F.17), (F.18), and given values for $R^{i,1}$ and $R^{j,1}$. Importantly, we assume countries always choose to trade in later periods ($t > 1$) when given the opportunity. This assumption allows us to pursue a simple comparison of how the central results regarding the countries' preferences over trade in the first period vary when we allow for additional periods. Focusing on the simple $\mathcal{T} = 3$ case also helps to illustrate some basic mechanics that can be then used to extend the game to allow for $\mathcal{T} > 3$ time periods.

With these modifications, it is convenient to first focus on the subgame that begins at the end of period $t = 2$, when countries choose their arming and saving levels that will become operative in period $t = 3$. Let $V^{i,2}$ denote the payoff function of this subgame. Given period $t = 2$ income levels, $Y^{i,2}$ and $Y^{j,2}$, as determined by (F.17) and prior choices, each player's objective at this stage in the game is to maximize

$$\begin{aligned} V^{i,2} &= V(G^{i,2}, G^{j,2}, Z^{i,2}, Z^{j,2}, Y^{i,2}) \\ &= \ln(Y^{i,2} - G^{i,2} - Z^{i,2}) + \delta\Big[q \ln\left[\phi^{i,3}\kappa\left(Z^{i,2} + Z^{j,2}\right) + (1-\kappa) Z^{i,2}\right] \\ &\quad + (1-q)\ln\left(T^{i,3}Z^{i,2}\right)\Big], \end{aligned}$$

for $i, j \in \{1, 2\}$, $i \neq j$, This expected payoff function is nearly isomorphic to the expected payoff in our baseline model. The only difference is the presence of the gains from trade in period $t = 3$ in the event of peace, $T^{i,3}$, that the countries take into account when choosing how much of their incomes to allocate towards saving.[38]

Next, define $V^{i,2*} = V(G^{i,2*}, G^{j,2*}, Z^{i,2*}, Z^{j,2*}, Y^{i,2})$ as the expected payoff for player i in this subgame when both players optimally choose their period $t = 2$ arming and saving

[38] A similar effect arises in the model of the previous section with future trade, but in the case of peace (and free trade) and in the case of conflict (and a negotiated trade settlement). Given our focus in that setting, we did not emphasize those effects; but, we do so here.

levels. The FOCs that give rise to these optimal arming and saving choices are given by

$$V^{i,2}_{G^{i,2}} = \delta q \frac{\phi^{i,3}_{G^{i,2}}\kappa\left(Z^{i,2}+Z^{j,2}\right)}{\phi^{i,3}\kappa\left(Z^{i,2}+Z^{j,2}\right)+(1-\kappa)Z^{i,2}} - \frac{1}{Y^{i,2}-G^{i,2}-Z^{i,2}} = 0, \tag{F.19}$$

$$V^{i,2}_{Z^{i,2}} = \delta\left[q\left(\frac{\phi^{i,3}\kappa+(1-\kappa)}{\phi^{i,3}\kappa\left(Z^{i,2}+Z^{j,2}\right)+(1-\kappa)Z^{i,2}}\right)+(1-q)\left(\frac{1-\frac{1}{\sigma}\mu^{i,3}_{j}}{Z^{i,2}}\right)\right] - \frac{1}{Y^{i,2}-G^{i,2}-Z^{i,2}} = 0, \tag{F.20}$$

for $i,j\in\{1,2\}$, $i\neq j$, where

$$\mu^{i,3}_{j} \equiv (Z^{i,2}/Z^{j,2})^{\frac{1-\sigma}{\sigma}}/[1+(Z^{i,2}/Z^{j,2})^{\frac{1-\sigma}{\sigma}}]$$

represents country i's expenditure share on good imported from country j. The term $-\frac{1}{\sigma}\mu^{i,3}_{j}<0$ in (F.20) reflects how an increase in saving in period $t=2$ worsens country i's terms of trade (TOT) in period $t=3$. The magnitude of the adverse TOT effect is increasing in the expenditure share $\mu^{i,3}_{j}$ and decreasing in the elasticity of substitution σ.[39] Otherwise, these first-order conditions are identical to those from the original model, for all intents and purposes. To highlight the underlying role of income levels, we let $G^{i,2*}\equiv G(Y^{i,2},Y^{j,2})$ and $Z^{i,2*}\equiv Z(Y^{i,2},Y^{j,2})$ denote arming and saving choices that solve respectively (F.19) and (F.20) for both countries $i,j\in\{1,2\}$, $i\neq j$. We can then make use of the following derivatives:

$$\frac{dV^{i,2*}}{dY^{i,2}} = \frac{\partial V^{i,2*}}{\partial G^{j,2*}}\frac{dG^{j,2*}}{dY^{i,2}} + \frac{\partial V^{i,2*}}{\partial Z^{j,2*}}\frac{dZ^{j,2*}}{dY^{i,2}} + \frac{\partial V^{i,2*}}{\partial Y^{i,2}},$$
$$\frac{dV^{i,2*}}{dY^{j,2}} = \frac{\partial V^{i,2*}}{\partial G^{j,2*}}\frac{dG^{j,2*}}{dY^{j,2}} + \frac{\partial V^{i,2*}}{\partial Z^{j,2*}}\frac{dZ^{j,2*}}{dY^{j,2}}.$$

We, thus, treat $V^{i,2*}=V^{i,2*}(Y^{i,2},Y^{j,2})$ as a value function that only depends on country i's income $Y^{i,2}$ and the rival's income $Y^{j,2}$, taking into account how they shape the rival's arming and saving choices via strategic effects as well as the direct effect of increasing $Y^{i,2}$ on country $i's$ future consumption, captured by $\partial V^{i,2*}/\partial Y^{i,2}$.[40]

With $V^{i,2*}$ defined, the expected payoff function for the full game can be re-written as

$$U^{i} = \ln C^{i,1} + \delta q V^{i,2*}_{(q)} + \delta(1-q)V^{i,2*}_{(1-q)}, \tag{F.21}$$

where $V^{i,2*}_{(q)} = V^{i,2*}(Y^{i,2}_{(q)},Y^{j,2}_{(q)})$ and $V^{i,2*}_{(1-q)} = V^{i,2*}(Y^{i,2}_{(1-q)},Y^{j,2}_{(1-q)})$ respectively denote the expected continuation payoffs that are operative in the event of a dispute (with probability

[39] More precisely, the TOT effect is given by $T^{i,3}_{Z^{i,2}} = -\frac{1}{\sigma Z^{i,2}}T^{i,3}\mu^{i,3}_{j}$. Since it lowers the marginal benefit of saving, it induces both countries to arm more than they would otherwise. Thus, in comparison with our baseline model, this future TOT channel could lower the two-period expected payoff to one or both countries; however, were we to consider a choice over second-period trade, the conditions under which the larger country would refuse to trade would still be qualitatively the same as in the baseline model.

[40] Any term that depends on $\partial V^{i,2*}/\partial G^{i*}$or $\partial V^{i,2*}/\partial Z^{i*}$ equals zero here by the envelope theorem.

q) in the second period and in the event of no dispute (with probability $1-q$). To elaborate, the two possible income levels these payoff functions depend on are given by

$$\begin{aligned} Y^{i,2}_{(q)} &= \phi^{i,2}\kappa\left(Z^{i,1}+Z^{j,1}\right)+(1-\kappa)Z^{i,1}, \\ Y^{i,2}_{(1-q)} &= T^{i,2}Z^{i,1}. \end{aligned}$$

The FOCs for country i's period $t=1$ arming and saving choices can then be written as

$$U^i_{G^{i,1}} = \delta q\left(\frac{dV^{i,2*}_{(q)}}{dY^{i,2}}\frac{\partial Y^{i,2}_{(q)}}{\partial G^{i,1}}+\frac{dV^{i,2*}_{(q)}}{dY^{j,2}}\frac{\partial Y^{j,2}_{(q)}}{\partial G^{i,1}}\right)-\frac{1}{Y^{i,1}-G^{i,1}-Z^{i,1}}=0 \tag{F.22}$$

$$\begin{aligned} U^i_{Z^{i,1}} = \delta q&\left(\frac{dV^{i,2*}_{(q)}}{dY^{i,2}}\frac{\partial Y^{i,2}_{(q)}}{\partial Z^{i,1}}+\frac{dV^{i,2*}_{(q)}}{dY^{j,2}}\frac{\partial Y^{j,2}_{(q)}}{\partial Z^{i,1}}\right) \\ &+\delta(1-q)\left(\frac{dV^{i,2*}_{(1-q)}}{dY^{i,2}}\frac{\partial Y^{i,2}_{(1-q)}}{\partial Z^{i,1}}+\frac{dV^{i,2*}_{(1-q)}}{dY^{j,2}}\frac{\partial Y^{j,2}_{(1-q)}}{\partial Z^{i,1}}\right)-\frac{1}{Y^{i,1}-G^{i,1}-Z^{i,1}}=0. \end{aligned} \tag{F.23}$$

The main additional complication that arises here, relative to the two-period subgame, is that arming and saving choices in period $t=1$ affect the rival's income in period $t=2$ and, thus, expected continuation payoffs from the end of period $t=2$ onward. In particular, arming in period $t=1$ has the added effect of lowering the rival's income in $t=2$ and thereby influencing the rival's period $t=2$ arming and saving choices. Of course, these effects materialize only in the case that conflict arises in period $t=2$. A country's period $t=1$ saving choice also generates some strategic effects. The relevant channels of influence are operative both in the event of conflict (by adding to the pool of contested output) and in the event of peace (through trade). More generally, guided by the results of Proposition 3, one would naturally expect actions that increase a country's own future income to raise its expected continuation payoff (i.e, $dV^{i,2*}/dY^{i,2}>0$), whereas increases in the rival's future income have an ambiguous effect ($dV^{i,2*}/dY^{j,2} \lesseqqgtr 0$) that depends on the magnitude of the difference in relative incomes.[41] Thus, it is plausible that the adverse consequences of trade could be magnified for extreme differences in initial size but could be mitigated when size differences are more moderate. However, this reasoning ignores the fact that the effects of first-period trade on $Y^{i,2}$ and $Y^{j,2}$ are discrete as opposed to continuous; once again, we must carefully verify whether there exists a range of initial resource distributions for which the larger country refuses to trade in the first period.

The various complexities of this setup make it difficult to characterize the equilibrium outcome and payoffs analytically. As in the other sections of this appendix, we proceed numerically. First, we compute $V^{i,2*}$ for many different combinations of functions of $Y^{i,2}$

[41] Strictly speaking, Proposition 3 does not apply here perfectly, largely due to the future TOT effects that are absent from our baseline model. Indeed, these TOT effects make it impossible to derive analytically an analogue of Proposition 3 in this setting.

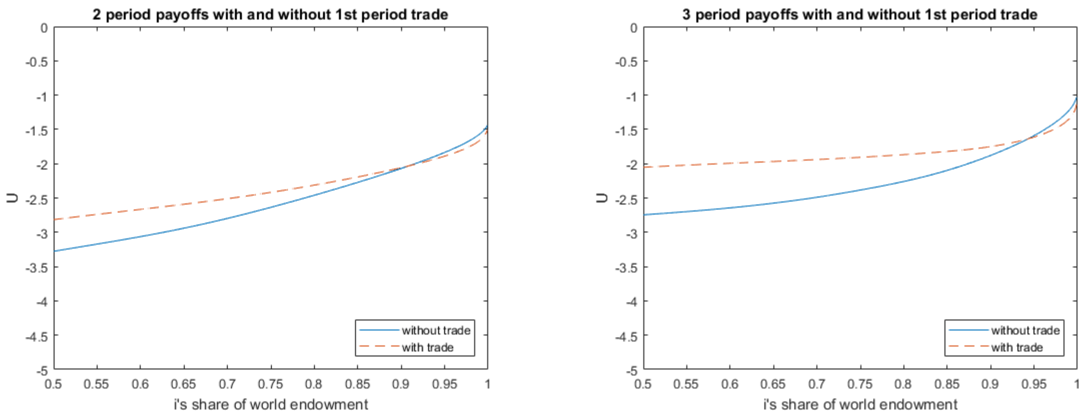

Figure F.1: A comparison of payoff rankings for the 3-period model versus a 2-period model

and $Y^{j,2}$.[42] This approach allows us to approximate the derivatives of $V^{i,2*}$ with respect to $Y^{i,2}$ and $Y^{j,2}$ to obtain the $dV^{i,2*}/dY^{i,2}$ and $dV^{i,2*}/dY^{j,2}$ terms that appear in (F.22) and (F.23). This method of solving model can be extended further to allow for $\mathcal{T} > 3$ (even $\mathcal{T} = \infty$), using recursive methods. However, for simplicity's sake, we focus here only on the implications of adding an additional period to see what, if anything, changes.

Figure F.1, which assumes $q = \kappa = .9$, $\delta = m = 1$, and $\sigma = 4$, shows expected payoffs under autarky and trade for the three-period model as well as for a similar two-period model that also allows for trade in the future in the event of peace. As this comparison illustrates, extending the model to include a third period tends to amplify the benefits of trade for the larger country when the two countries are similar in size. It also flattens the relationship between the larger country's expected payoff under trade and its initial relative size when the two countries are somewhat similar in size but causes it to become steeper in relation to its autarky payoff as size differences become more extreme. The net effect shown in the figure is to reduce the range of relative endowment sizes for which the larger country rationally chooses not to trade in period $t = 1$.[43] We have also found, for both models, that the range of relative endowment sizes for which the large country prefers not to trade can vanish when σ is sufficiently small, similar to what we found for the model with future trade described in Appendix E. Additional results shown in Table F.1 illustrate that the relative appeal of trade in $t = 1$ in the 3-period model is more sensitive to changes in σ in this regard. The table also shows, however, that extending the time horizon does not necessarily augment the larger country's preference for trade, especially when q and κ are small, while σ is not. Comparing the last two lines of the table, in particular, reveals

[42] More specifically, if we increase both $Y^{i,2}$ and $Y^{j,2}$ by the same proportion, all G, Z, and C terms also scale proportionally. Thus, we can write $V^{i*} = (1+\delta)\ln(Y^{i,2} + Y^{j,2}) + V^{i*}(\vartheta^{i,2}, \vartheta^{j,2})$ where $\vartheta^{i,2} \equiv Y^{i,2}/(Y^{i,2} + Y^{j,2})$. We focus on approximating the function $V^{i*}(\vartheta^{i,2}, \vartheta^{j,2}) = V^{i*}(\vartheta^{i,2}, 1 - \vartheta^{i,2})$, which only has one argument. We do so by solving the two-period subgame repeatedly for many different values of $\vartheta^{i,2}$.

[43] While this net effect holds for much of the parameter space, it does not hold always, as discussed below.

that reducing the probability of conflict and/or lowering degree of insecurity does more to mitigate the adverse consequences of first-period trade with a very small country when there is only one future period in which conflict may occur as compared with two future periods.

Table F.1: Multi-period model results ($R^{i,1}/R^{j,1} = 1e6$.)

	Larger country prefers trade in 1st period?	
Parameters	Two-period model	Three-period model
$q = \kappa = .4$, $\sigma = 4$	no	no
$q = \kappa = .4$, $\sigma = 2$	no	yes
$q = \kappa = .9$, $\sigma = 2$	no	no
$q = \kappa = .9$, $\sigma = 1.25$	yes	yes
$q = \kappa = .1$, $\sigma = 4$	no	no
$q = \kappa = .05$, $\sigma = 4$	yes	no

The other parameters are $\delta = m = 1$. Results are reported for the case where the larger country is 1 million times the size of the smaller country. To enable consistent comparisons, the two-period model allows for trade in the second period in the event that a dispute does not occur.

G Three Countries

In what follows, we consider the inclusion of a third country that is not directly involved in conflict. We continue to identify countries 1 and 2 as rivals; one can think of country 3 as ROW. For clarity and to allow for an easier comparison with the baseline model consisting of just two countries, we assume that trade can take place only in period $t = 1$. We consider, in particular, three alternative trade regimes for period $t = 1$: (i) global free trade; (ii) an embargo on one adversary $i = 1$ or 2, by the other adversary j ($\neq i$), with free trade between ROW and each of the two adversaries; (iii) a blockade on one adversary $i = 1$ or 2, with free trade between the other adversary j ($\neq i$) and ROW.

Extending our baseline model of trade based on Armington (1969) to three countries, we assume each one $i = 1, 2, 3$ is endowed with an initial resource R^i that yields, on a one-to-one basis, a distinct and potentially tradable intermediate good, respectively $j = 1, 2, 3$. The production function for the final good in country i takes the CES form

$$Y^i = \left[\sum_{j \in \mathcal{N}^i} \left(D^i_j\right)^b\right]^{\frac{1}{b}}, \tag{G.1}$$

where as previously defined $b = \frac{\sigma-1}{\sigma}$; $\mathcal{N}^i$ denotes the set of countries j with whom country i trades as well as itself; and, D^i_j denotes the intermediate good originating in country j and employed by firms in country i. Let p^i_j and μ^i_j respectively denote the price country i pays for good $j = 1, 2, 3$ and (as in the baseline model) its expenditure share on that good. It is straightforward to show the country i's demand functions for goods $j \in \mathcal{N}^i$ are given by $D^i_j = \mu^i_j p^i_i R^i / p^i_j$. While the precise value of μ^i_j depends on the trade regime under consideration, we can write it generally for $j \in \mathcal{N}^i$ as $\mu_j^i = (p_j^i)^{1-\sigma}/\left[\sum_{k\in N^i}(p_j^i)^{1-\sigma}\right]$. For future reference, it is useful to observe that, in all trade regimes, we can write country i's maximized value of income contingent on prices and resources as $Y^i = (\mu_i^i)^{1/(1-\sigma)}R^i$.[44] With these details in hand, we now turn to the specific trade regimes:

Global free trade. In this case, $\mathcal{N}^i = \mathcal{N} = \{1,2,3\}$ for each i, and all 3 countries face identical prices. Accordingly, we have $p_j^i = p_j$ for all $i \in \mathcal{N}$ and $\mu_j^i = \mu_j = p_j^{1-\sigma}/\left[\sum_{k\in\mathcal{N}} p_k^{1-\sigma}\right]$. Furthermore, the demand functions can be written as $D_j^i = \mu_j p_i R^i/p_j$.

Let us choose good $j = 3$ as the numeraire (such that $p_3 = 1$). Then, the world market-clearing conditions for goods $j = 1,2$ are $\sum_{i\in\mathcal{N}} D_j^i = R_j$. After substituting in the expressions for D_j^i and rearranging, this condition can be rewritten as $\mu_j\left[\sum_{i\in\mathcal{N}} p_i R^i\right] = p_j R^j$ for $j = 1,2$. With our normalization of p_3, these expressions, in turn, deliver the following pricing relationships:

$$p_j = \mu_j R^3/\mu_3 R^j, \quad \text{for } j = 1,2. \tag{G.2}$$

Substitution of the expressions for μ_j and μ_3 above, after rearranging terms, yields the following equilibrium prices for goods $j = 1,2$ in units of good 3:

$$p_j = \left(R^3/R^j\right)^{\frac{1}{\sigma}}, \quad \text{for } j = 1,2. \tag{G.3}$$

Turning to equilibrium output, we substitute (G.3) into the expression for μ_j and then use $Y^i = (\mu_i)^{1/(1-\sigma)}R^i$ to derive a more compact expression for country i's income: $Y^i = \psi^i Y$ where $Y = \left[\sum_{j\in\mathcal{N}}(R^j)^b\right]^{1/b}$ equals world output, and $\psi^i = (R^i)^b/\left[\sum_{j\in N}(R^j)^b\right]$ equals country i's share of it.

Next, consider the S^i- and B^i-contours that inform the equilibrium interactions between the two rivals, $i = 1,2$. Recall the S^i-contour does not depend on incomes. But, the B^i-contour does. Specifically, it depends on $Y^i/Y^j = \left(R^i/R^j\right)^b$ ($i,j \in \{1,2\}$, $i \neq j$) as in the two-country setting where the two rivals engage in free trade.[45] Since Y^i/Y^j is independent of R^3, so are the equilibrium shares (ϕ^i, θ^i). Indeed, we can show that each rival's arming and saving rises in proportion to its national income that is increasing in R^3 due to an improvement in its terms of trade with ROW. However, the ratios G^i/Y^i and Z^i/Y^i for $i = 1,2$ are constant. We explore numerically the welfare implications below.

Embargo. Under this regime, where the two rivals $i = 1,2$ do not trade with each other but do trade with ROW, we have $\mathcal{N}^i = \{i,3\}$ for $i = 1,2$. In this case, while (G.1) continues to describe the production function for ROW, the production functions for countries $i = 1,2$

[44]In particular, it is well known that country i's revenue function can be written as $Y^i = p_i^i R^i/P^i$, where $P^i = [\sum_{j\in N^i}(p_j^i)^{1-\sigma}]^{1/(1-\sigma)}$. The expression in the text follows from the fact that μ_i^i can be written as $\mu_i^i = (p_i^i/P^i)^{1-\sigma}$.

[45]In the special case of no trade between the rivals, Y^i/Y^j is given by the same expression, but with $b = 1$.

are now given by

$$Y^i = \left[\left(D_i^i\right)^b + \left(D_3^i\right)^b\right]^{\frac{1}{b}}.$$

Once again, choosing good $j = 3$ as the numeraire and following the same procedure as above, one can verify that the market-clearing conditions are:

$$p_i = \mu_i^3 R^3 / \mu_3^i R^i, \text{ for } i = 1, 2, \tag{G.4}$$

where $p_i = p_i^i$, $p_3 = p_3^i$ for $i = 1, 2$ as before. However, p_2^1 and p_1^2 do not enter the calculus here, because countries $i = 1$ and 2 do not trade with each other. Expenditure shares are now defined as follows:

$$\begin{aligned}
\mu_3^i &= \frac{p_3^{1-\sigma}}{p_i^{1-\sigma} + p_3^{1-\sigma}} \\
\mu_i^3 &= \frac{p_i^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma} + p_3^{1-\sigma}},
\end{aligned}$$

for $i = 1, 2$. While the expressions in (G.4) with the expenditure shares do not yield closed form solutions for equilibrium prices, we can show a unique equilibrium exists and characterize its dependence on the countries' initial resource endowments, including R^3 and the distribution of resources across countries $i = 1$ and 2.

We are particularly interested in the influence of R^3 on the S^i- and B^i-contours in this case. Since

$$Y^i / Y^j = \left(\mu_i^i / \mu_j^j\right)^{\frac{1}{1-\sigma}} \left(R^i / R^j\right),\ i, j \in \{1, 2\},\ i \neq j$$

enters the B^i-contour, R^3 affects relative incomes for the rivals as follows:

$$\frac{d\left(Y^i / Y^j\right)}{dR^3} \gtreqqless 0 \ \text{ as } \ R^i \lesseqqgtr R^j, \ \ i, j \in \{1, 2\},\ i \neq j.$$

In words, if the size of ROW's resource endowment expands, the relatively smaller rival experiences a relatively larger increase in its income in $t = 1$.

Blockade. Under this regime, where the one rival $i = 1$ or 2 trades with ROW while the other rival $j \neq i$ does not, we have $\mathcal{N}^i = \{i, 3\}$ for $i = 1$ or 2. Equilibrium output levels for the two rivals are as follows:

$$\begin{aligned}
Y^i &= \psi^i \left[\left(R^i\right)^b + \left(R^3\right)^b\right]^{1/b}, \ \text{ where } \psi^i = \frac{\left(R^i\right)^b}{\left(R^i\right)^b + \left(R^3\right)^b} \\
Y^j &= R^j,
\end{aligned}$$

for $i = 1$ or 2 and $j \neq i, 3$. ROW's output is given by

$$Y^3 = (1 - \psi^i) \left[\left(R^i \right)^b + \left(R^3 \right)^b \right]^{1/b}.$$

This case, though very simple, is interesting in that it captures both the case where one country i (perhaps a relatively large country) manages to impose a blockage on country j and the case where one country j chooses to isolate itself from the world. Either way, we can use the expressions above directly in the B^i-contour as in the other trade regimes.

Discussion. We compare three trade options for a large country i ($R^i > \bar{R}/2$, where $\bar{R} = R^i + R^j$) whose rival is country j. Denote the payoff that country i obtains when it participates in "global free trade" by V_T^i, its payoff when it imposes a "trade embargo" on its rival j by V_E^i, and its payoff when it effectively imposes a "blockade" on its rival j by V_B^i. Fig. G.1 depicts the ratios V_T^i/V_E^i and V_B^i/V_E^i as a function of $R^i \in [\bar{R}/2, \bar{R}]$ for two values of ROW's resource endowment: $R^3 = \bar{R}/2$ (which implies that ROW is half the size of the rival's combined resource base and no larger than country i) and when $R^3 = 5\bar{R}$ which captures the case where ROW is substantially larger than country i as well as i and j combined. (The schedules in Fig. G.1 are based on the following parameter values: $\sigma = 4$, $q = \kappa = 0.9$ and $\delta = m = 1$.)

Inspection of Fig. G.1 reveals the following. First, the presence of a third country (ROW) does not necessarily eliminate the appeal to country i of foreclosing on its trade with a rival $j \neq i$. Interestingly, the figure suggests that increasing the size of ROW expands the range of country i's relative sizes that renders trade more appealing than an embargo (compare the two green dots). Second, abstracting from possible non-trade costs that could be needed to implement a blockade, country i always finds a blockade on country j more appealing than an embargo when ROW is sufficiently small (as can be seen by the solid blue schedule); however, when ROW is larger, country i prefers a blockade on country j only if i is not too much larger than its rival j (as shown by the dotted blue schedule). Finally, country i views global free trade as dominant over the other options (i.e., an embargo and a blockade) if it is not sufficiently large relative to its rival j. The relevant range of resource endowments under which this preference holds for country i is larger when ROW is smaller (compare the pink dots).

Finally, extensive numerical analysis of the model with three countries shows that the larger rival i would never prefer isolation (and thus foregoing the gains from trade with both a friend and its rival) over an embargo on country j (and just foregoing those gains in trading with its rival). While the larger country might prefer isolation over free trade, this preference ranking arises only when ROW is extremely small relative to the combined size of countries i and j, $\bar{R}$.

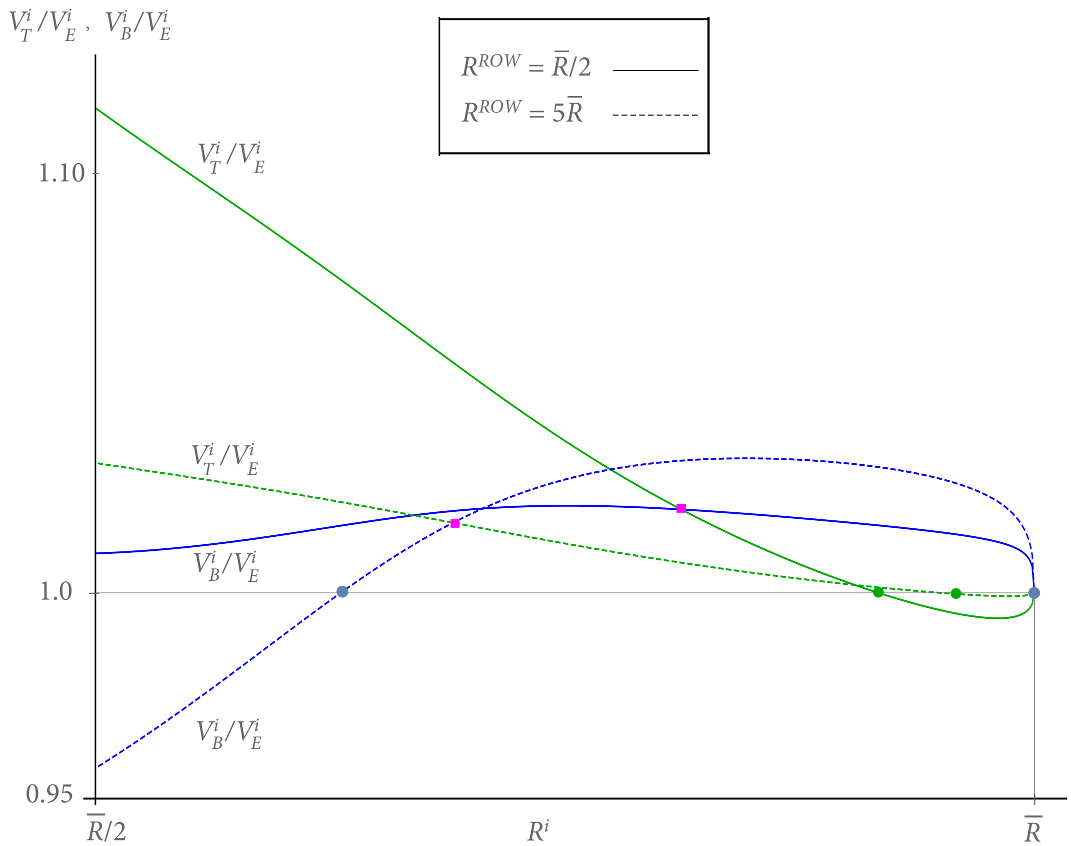

Figure G.1: Relative Payoffs for the Larger Adversary in the Case of Three Countries