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Prudence versus predation and the gains from trade

Michelle R. Garfinkel [a,*], Constantinos Syropoulos [b], Thomas Zylkin [c]

[a] University of California-Irvine, 3151 Social Science Plaza, Irvine CA 92697, USA
[b] LeBow College of Business, Drexel University, 3220 Market Street, Philadelphia PA 19104, USA
[c] Robins School of Business, University of Richmond, 102 UR Drive, Richmond, VA 23173, USA

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Abstract

We analyze a dynamic, two-country model that highlights the various trade-offs each country faces between current consumption and competing investments in its future productive and military capacities as it prepares for a possible conflict in the future. Our focus is on the circumstances under which the effects of current trade between the two countries on the future balance of power render trade unappealing to one of them. We find that a positive probability of future conflict induces the country with less resource wealth to "prey" on the relatively more "prudent" behavior of its larger rival, and more so as conflict becomes more likely. While a shift from autarky to trade always raises the current incomes of both countries, the smaller country realizes the relatively larger income gain from trade and also devotes a relatively larger share of its income gain towards arming. Our analysis shows that the larger country rationally chooses not to trade today when the difference in initial resource wealth is sufficiently large and is more likely to prefer autarky when the probability of future conflict is higher.

JEL classification: D30; D74; F10; F20; F51

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* Corresponding author.
E-mail addresses: mrgarfin@uci.edu (M.R. Garfinkel), c.syropoulos@drexel.edu (C. Syropoulos), tzylkin@richmond.edu (T. Zylkin).

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1. Introduction

Despite a resurgence in global tensions in recent years, we live in an era of unprecedented peace between nations. Seventy years have gone by without a repeat of the major conflicts of the early twentieth century; and, since the end of the Cold War, the number of inter-state conflicts has steadily declined to practically zero. The expansion of international trade since World War II, in particular, is credited with ensuring a more secure global order by raising the economic costs of war (Polachek, 1980; Martin et al., 2008; Glick and Taylor, 2010). Nonetheless, global expenditure on defense in 2018 was roughly $1.8 trillion, a 76% increase in real terms from the post-Cold War lows of the late-1990s. Aside from the West's recent interventions in the Middle East, a major driver of this trend is the rapid expansion of defense spending by emerging economies that have become more integrated into the world trading system in the past few decades.[1] Though the growth of these economies due to trade has been welcome on the whole, the accompanying trend in their defense spending underscores the basic point that wealth created by trade is wealth that can be used to invest in one's military capacity.

Economists have recently begun to expand the scope of international trade theory to explore the importance of arming and conflict (see, e.g., Skaperdas and Syropoulos (2001); Garfinkel et al. (2020) discussed below). These theories, however, are silent on why we continue to observe increases in arming in a seemingly peaceful and continually globalizing world. Furthermore, while "realist" security scholars have long expressed the view that the standard "gains from trade" could be outweighed by the negative consequences for a country's future security if these gains are asymmetrically distributed, trade theory has not examined this argument specifically.[2] This oversight diminishes our understanding of why and when nations expand trade with one another. Even in the present day, policymakers openly regard trade policy as an instrument for achieving security-related goals.[3]

In this paper, with an aim to address these and related issues, we present a simple dynamic theory of trade, investment, and arming, focusing on two countries. Central to both our motivation and our analysis is the conceit that, while peace prevails in the present, the two countries make these decisions in the shadow of a future conflict that emerges with some known, positive probability. This setting naturally delivers motivations for costly arming in the midst of an ongoing peace—namely, as necessary (and/or opportunistic) preparations made for an uncertain future. More importantly, the nature of the interaction between the two countries, with each having to balance how its decisions today will affect outcomes under both peace and conflict tomorrow, allows us to examine how differences in the initial distribution of resources translate into dif-

[1] Data from SIPRI (2019), available at www.sipri.org, show China (212%), Vietnam (188%), Russia (77%) and the former communist nations of Eastern Europe (collectively, 81%) have each increased their defense spending tremendously since 2005 alone, continuing long term trends that soon followed the liberalization of their economies during the 1990s. Recent upward trends are also common across Africa, Asia, and Central and South America.

[2] Scholars writing in the "realist" tradition generally treat dependence on trade with other nations as a source of diminished security, especially if the gains from trade are uneven (see Waltz, 1979; Gilpin, 1981; Grieco, 1990). The opposing "liberal" view argues that the efficiency gains from trade should raise the opportunity cost of war (see Polachek, 1980; Martin et al., 2008). A third view presented in Copeland (2015) combines elements of both views, positing that wars can arise because of uncertainty over future trade (see also Bonfatti and O'Rourke, 2018; Morelli and Sonno, 2017). As will become clearer shortly, the main focus of our analysis in this paper is on the realist view.

[3] For example, in a speech in 2015, then-U.S. Secretary of Defense Ash Carter stated, "you may not expect to hear this from a Secretary of Defense, but in terms of a rebalance in the broadest sense, passing TPP [the Trans-Pacific Partnership] is as important to me as another aircraft carrier" (Carter, 2015).

ferences in military power and, ultimately, in preferences toward trade. In doing so, the analysis yields a clear prediction regarding the effectiveness and credibility of security-based arguments for trade restrictions: a sufficiently high threat of conflict can eliminate a larger country's incentive to trade with a smaller rival, but only if the difference in *ex ante* economic size between the two countries is sufficiently large; otherwise, trade in the present remains the best policy for both countries, despite the possibility of future conflict.

Our theory highlights two types of trade-offs that countries face in the midst of an ongoing rivalry. First, both countries must invest some of their current resources to provide for future consumption—a standard "*inter*-temporal" trade-off. Second, they face an "*intra*-temporal" trade-off between two types of investment that support future consumption in distinct ways: "saving," which yields resources for future consumption, and "arming," which determines how these resources will be divided in the event of a future conflict. Of course, countries with larger initial resource endowments are in a better position to satisfy their current consumption needs. As expected, then, a relatively larger country chooses higher levels of arming and saving in equilibrium than its smaller rival. However, the degree to which it enjoys an arming advantage does not depend simply on the difference in endowment sizes (as might be presumed), but rather on how such differences and the likelihood of conflict jointly shape each country's strategic incentives for both arming and saving. In equilibrium, the *ex ante* smaller country allocates a relatively smaller share of its income to saving and a relatively larger share to arming compared with its larger counterpart. Thus, a strictly positive probability of future conflict enables the smaller country to "prey" on the more "prudent" behavior of its counterpart, thereby making it disproportionately powerful as compared with its initial size.

The relevance of trade in this setting derives from its effects on the current income of the two countries. As is true for most static trade models without security concerns—and as we will illustrate using a simple "Armington" example—trade does not reduce the productive efficiency of either country and usually generates real income gains in absolute terms for both. At the same time, we find the intuitive result that the smaller country always gains more from trade than the larger country relative to its initial size. Though unequal distributions of the gains from trade would not ordinarily prevent trade from taking place, our dynamic setting where future security concerns matter highlights a clear set of circumstances under which a larger country will find trade in the present relatively unappealing. More precisely, as the initial size difference between the two countries increases, the larger country's gains from trade become smaller, the smaller country's gains from trade become larger, and—because of how the possibility of conflict affects relative arming choices versus relative saving choices—the smaller country increasingly allocates a larger portion of its income gain towards arming versus saving. To be sure, the presence of dual strategic interactions in both arming and saving makes characterizing the effects of trade in this setting quite complex (and potentially ambiguous). Nonetheless, it is always possible to identify a sufficiently unequal distribution of initial endowments beyond which the larger country prefers autarky to trade.[4] Numerical analysis further clarifies that the larger country's preference for

[4] Remarkably, as shown in the Online Appendix, both this result and the result that the relative gains from trade are inversely related to relative sizes hold not only in our simple Armington setting, but also in many other, more complex trade environments that feature trade costs, incomplete specialization, multiple factors of production, increasing returns and/or heterogeneous firms. To our knowledge, the relationships that we demonstrate between relative sizes and relative gains from trade (independent of the presence of possible conflict) are new results in trade theory.

autarky tends to emerge for a wider range of relative endowment sizes when the probability of conflict is higher.[5]

The history of 20th century military rivalries offers numerous episodes that can be related to the tradeoffs highlighted in our theory. These episodes include the steady continuation of trade between Germany and Great Britain in the lead-up to World War I, the U.S.'s progressive tightening of economic sanctions against Japan before its entry into World War II, and the U.S.'s aggressive containment policies towards the Soviet Union at the beginning of the Cold War. Though we do not intend to position these historical rivalries as proving our model or being fully illustrated by our model, they share several features that our model can explain. In particular, we can document in each instance how decisions whether to restrict trade reflected discussions surrounding the severity of the threat, the economic losses from restricting trade, and the need to build up arms to maintain the balance of power. The relevant details of these episodes will be explained further in Section 2.

Our choice to abstract from the possibility that either country can take actions to try to influence the likelihood of conflict is driven by our perspective in relation to the literature. Theories underlining the "liberal peace" argument that trade can serve as a deterrent to war (e.g., Martin et al., 2008) typically examine how the decision to go to war is affected by exogenous changes in the trade regime.[6] Our approach pursues a kind of converse: even acknowledging that trade can be useful for ensuring peace, it is also worth investigating why and when peace could be necessary for ensuring trade.

In shedding light on these issues, our analysis builds on and synthesizes several disparate strands of the relevant literature. The first of these is the "relative gains" argument for restricting economic cooperation articulated by the "realist" school of international relations. In this tradition, as in our model, "cooperation that creates and distributes wealth affects security as well as welfare" (Liberman, 1996); thus, countries concerned about both security and welfare must be strategic in choosing with whom they cooperate and when. Our own formalization of this idea is related to the contributions of Powell (1991) and Gowa (1995) in that we incorporate the linkages between changes in relative wealth, future security threats, and expected payoffs in a unified game-theoretic framework. In this context, a key distinguishing feature of our analysis is that we explicitly model the endogenous relationships between trade, relative wealth, and relative power as well as the conditions under which these relationships could be stronger or weaker.

Second, we share with Skaperdas and Syropoulos (2001), Dal Bó and Dal Bó (2011), Garfinkel et al. (2008, 2015), and Garfinkel et al. (2020) an interest in how trade affects incentives for arming relative to other, more productive activities. The first four papers focus on a factor-price channel that can render trade by countries involved in conflict unappealing even when the gains from trade are evenly distributed across adversaries. Garfinkel et al. (2020) find,

[5] Our finding that one country might choose not to trade in the present resembles Gonzalez (2005)'s finding that agents might adopt inefficient technologies to discourage future aggression by rivals. Though Gonzalez (2005) also relies on a dynamic model, his work differs from ours in that disincentives for technology adoption arise from how technology adoption shapes future (contested) output, whereas disincentives for trade in our analysis arise from how trade shapes current (secure) output.

[6] The "liberal peace" hypothesis finds some support in the empirical literature—see the recent survey by Morelli and Sonno (2017). However, this literature has also highlighted some interesting exceptions relevant to our analysis. Most notably, Hegre (2004) and Morelli and Sonno (2017) respectively find that the peace-promoting effects of trade could be conditional on asymmetries in trade dependence and on asymmetries in resource wealth. In other related work, Seitz et al. (2015) show that, if trade lowers the probability of conflict, the resulting reduction in the need for defense spending can generate substantial economic benefits on top of the usual gains from trade.

as we do, that trade between adversaries can be relatively unappealing to one country in the presence of sharp resource asymmetries; however, there the mechanism is a terms-of-trade channel.[7] In any case, with an emphasis on how the anticipation of trade influences arming incentives, neither of these two approaches captures the possibility that increases in national income due to trade have direct implications for military spending, as is apparent from the arming statistics cited above. Our analysis, by contrast, centers on an income channel that is more directly relevant for understanding why relative gains from trade might matter.

Third, our model gives rise to a variant of the weak form of Hirshleifer's (1991) concept of the "paradox of power," which states that players with fewer resources devote disproportionately more resources to appropriative activities because they have less to lose and more to gain from distributional conflicts than their larger rivals. While it would be reasonable to conjecture that the presence of this paradox implies a more even distribution of income could be welfare improving for the larger player, Hirshleifer (1991) does not consider this possibility. Our analysis contributes to this line of inquiry by showing that exogenous increases in the size of the smaller player can make both players better off, but only if the initial distribution of resources is sufficiently even. A similar finding also holds for trade. However, because trade has discrete, as opposed to continuous, effects on incomes, our proof of the latter result requires a very different strategy that involves differentiating payoffs under both autarky and trade in the neighborhood of an infinitesimal trading partner. This approach represents a methodological contribution that could also be applied to other settings where trade with a smaller country generates negative externalities, such as via environmental damage or intellectual property theft.

The rest of the paper is organized as follows. The next section provides a discussion of historical military rivalries that evoke the tradeoffs highlighted in our theory. Section 3 describes the basic elements of our model. In Section 4, we then characterize how equilibrium arming, savings, and payoffs respond to (exogenous) changes in first-period incomes. Section 5 closes the model by allowing first-period incomes to be determined by the trade regime and examines each country's preferences towards trade. We also consider a number of extensions that speak to the robustness of our central theoretical findings. Section 6 concludes. All technical details are provided in the Online Appendix.

2. Historical examples: rivalries and trade

In this section, we review three historical military rivalries that illustrate the tradeoffs that our theoretical analysis aims to highlight: Great Britain and Germany before World War I, the U.S. and Japan before the U.S.'s entry into World War II, and the U.S. and the Soviet Union in World War II's immediate aftermath. While we would not go so far as to claim that our analysis offers a comprehensive explanation of any of these episodes, our review of them reveals the salience of several key elements found in our theory. In particular, we will document how the choices that countries in these scenarios made (of whether or not to restrict trade) reflected the degree to

[7] Skaperdas and Syropoulos (2001), Dal Bó and Dal Bó (2011), and Garfinkel et al. (2008, 2015) each study extended Heckscher-Ohlin settings with small countries that trade with the rest of the world where changes in relative world prices induce changes in relative factor prices of capital and labor that in turn influence the relative costs of arming versus producing useful output. Garfinkel et al. (2020) analyze a modified Ricardian model of trade between large adversarial countries, intentionally abstracting from both factor-price and income channels to isolate the importance of a terms-of-trade channel that more often induces both countries to reduce their arms production as they internalize the negative price externality; the resulting payoff effect reinforces the traditional gains from trade.

which their economies benefited from trade with their rivals. The Great Britain/Germany rivalry is useful to focus on first because it shows how countries might continue to allow trade despite their awareness of a growing security threat, whereas the latter two rivalries illustrate cases where the risk of a future conflict with an economically dependent rival induces one country to cut off trade.

Great Britain/Germany In the decades preceding World War I, Britain increasingly perceived Germany's naval buildup as a threat to its power and security. One of the interesting elements about this period of Anglo-German relations for our purposes is the role that bilateral trade played in their arms race. As described by Kennedy (1980), the rapid industrialization in Germany that fueled its naval expansion was fed by imports of raw materials and food from Britain's colonies and was financed by London-based banks. At the same time, British shipbuilders relied on German sheet metal to build their ships, some of which were exported back to Germany. The British military also benefited from German imports of pig iron, optical equipment, precision tools, automobiles, and even khaki dye for uniforms (Kennedy, 1980; Liberman, 1996). Though Britain remained the world's largest exporter, it had also become the largest importer of iron and steel, with much of it coming from fast-growing Germany. Germany, in turn, had become Britain's second largest export market (Steiner, 1977).[8]

In view of this economic interdependence, it is understandable why Britain continued to trade with Germany even as it had explicitly begun to prepare for a possible war in 1912.[9] To be sure, there had been ample domestic pressure for restrictions on trade via the Tariff Reform movement of Joseph Chamberlain. But, ultimately, this pressure was resisted; as contextualized by Liberman (1996), Britain's government believed that it benefited substantially in absolute terms, if not in relative terms, from freer trade with Germany.[10] Consequently, British-German trade grew unabated right up until the start of the war.[11]

U.S./Japan The rivalry between the U.S. and Japan, by contrast, illustrates a case where one adversary was significantly more dependent on trade with the other than vice versa. Between 1937 and 1940, Japan relied on imports for 90% of its petroleum consumption, with 66% of its imported petroleum coming from the U.S.. Because these petroleum imports were crucial for Japan's war effort—alongside its imports of steel, iron, copper, and other raw materials—the U.S. perceived that economic sanctions would be effective in constraining Japan's threat to its own interests in the Asia-Pacific region (Hosoya, 1968; Saltzman, 2012).

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[8] Historical real GDP data from the Maddison Project show that Great Britain was the larger of the two countries in terms of economic size throughout most of this period. Due to its faster growth, Germany briefly caught up to Great Britain in 1912 and 1913 before falling behind again from 1914 onwards (see Bolt and van Zanden, 2020).

[9] As documented in Williamson (1969), by 1912, British foreign policy had committed to supporting France in the event of an unprovoked attack by Germany. In addition, the resumption of German naval expansion prompted the British to shift their own naval forces from the Mediterranean to the North Sea and to increase their own naval forces. The British and French militaries had been coordinating on a strategic response to a German invasion for 8 years by this point, and the two navies were now also discussing how to coordinate naval deployments.

[10] It is worth adding here that representatives from the British Treasury had been of the view that, even if war were to break out, disrupting trade with Germany would be counter-productive, hurting Britain's economy and, along with that, its war effort (Seligmann, 2017).

[11] According to statistics collected by Liberman (1996), trade between the two countries had nearly doubled since 1900, with annual trade growth actually accelerating (to 10%) between 1912 and 1913. Britain was not alone among Germany's rivals in permitting unfettered trade. Remarkably, Germany's trade with France and Russia (enemy countries far less inclined towards free trade than the British) grew even faster between 1900 and 1913.

The U.S.'s embargo on trade with Japan was progressive in nature and at first proceeded in stages. The abrogation of its commercial treaty with Japan in 1939 served as a warning that severe sanctions might follow. After Japan proceeded to invade French Indochina in 1940, and after peace talks that would have ended the sanctions had failed, the U.S. moved increasingly towards a total embargo on trade and began preparing for war in earnest. Between 1939 and 1941, the U.S. moved the bulk of its fleet to the Pacific, doubled its military spending, froze Japanese-owned assets, and cut its trade with Japan by more than half (Liberman, 1996; Saltzman, 2012). In the context of the theory we will soon describe, these actions are consistent with the U.S. believing that war was now highly possible and that restricting trade with its rival was an effective strategy for enhancing its advantage in the event of a conflict or negotiated settlement.

U.S./Soviet Union Interestingly, one of the immediate lessons the U.S. took away from its experience with Japan was that it should have moved more quickly to cut off trade. Cain (2005) describes a political environment, in the early days of the emerging post-World War II rivalry with the Soviet Union, where the U.S.'s continued trade with Japan between 1937 and 1940 was seen as a strategic error not to be repeated. The U.S. moved aggressively to design, by 1949, a set of export controls intended to "prevent or delay further increase in the war potential of Eastern European economies" (U.S. Munitions Board, 1949). As discussed in Brawley (2004), new U.S.-led organizations and initiatives such as NATO, the Marshall Plan, and ANZUS were used as a way of binding other major economies to these trade measures in order to enhance their effectiveness. The GATT agreement likewise had the strategic benefit of excluding the Soviet Union from the increased trade that was created.

To synthesize, these episodes illustrate that countries view restrictions on trade as a strategic instrument for maintaining a relative power advantage over their potential future enemies. However, as the Britain-Germany example suggests, they do so with a clear-eyed view of how reduced trade will affect their own economies and militaries, rather than focusing solely on differences in relative gains from trade. Even in the cases of the sanctions against Japan and the Soviet Union, domestic economic considerations were still seen as salient. Liberman (1996) notes that the U.S. State department listed "economic dislocation" as a valid argument against using sanctions against Japan, but ultimately concluded their economic impact on the U.S. would be limited. In the Soviet Union case, the U.S.'s allies began dropping their own sanctions in 1954, once fears of a "hot" war had passed, to realize the benefits from trade with the untapped Soviet market (Mastanduno, 1985).[12] In general, these examples illustrate that decisions to restrict trade with a military rival reflect not only the absolute economic gains from trade but also the relative gains and their implications for security, with the latter becoming less salient when the risk of conflict subsides. Naturally, we do not claim that these are always the most important considerations surrounding trade in any given rivalry. However, that these elements should matter is intuitive, and our analysis will demonstrate how they can be studied together theoretically.

3. A dynamic model of prudence versus predation

We consider a two-period model of a world economy that is populated by two countries identified by a superscript $i = 1, 2$. The key feature of our setting is that, while peace always prevails in

[12] As discussed in Mastanduno (1985), the U.S. continued to view U.S./Soviet Union trade as primarily a "gift" to the Soviet Union rather than a mutual benefit well into the 1970s and 1980s. This view could explain why the U.S. was much slower than its allies to embrace trade with the Soviet Union.

the first period ($t = 1$), conflict emerges in the second period ($t = 2$) with a strictly positive probability. In the case of peace in period $t = 2$, each country's output, produced using the resources generated from their savings/investments in period $t = 1$, is secure. But, if a conflict emerges, then the combined output of both countries becomes contestable via the force of arms.[13]

Our principal aim in this setting is to explore how the income gains from trade in intermediate inputs in period $t = 1$ influence each country's saving and arming decisions in that same period and how that matters for relative power, growth and, ultimately, national welfare over the two periods. In our initial exposition, we strive to motivate our results for trade in a simple yet general way, deferring formal details regarding trade until Section 5. Accordingly, for now, we generically characterize each country by its possession of an initial resource endowment R^i and a native technology level A^i, such that its final output (or income) under autarky is given by $Y^i = A^i R^i$. The technology parameter A^i is taken to reflect the country's ability to produce intermediate inputs on its own with R^i that it, in turn, employs in the production of final output. To introduce the basis for trade in this general formulation, we suppose there are multiple such inputs, with each country having a comparative advantage in the production of at least one. Then, under trade in inputs, one country—and possibly both—can realize efficiency gains in the production of its final output. That is to say, output under trade can be written instead as $Y^i = T^i R^i$, where $T^i \equiv T^i(R^i, R^j)$ depends on initial resource endowments of both countries as well as technologies and where $T^i \geq A^i$ holds as a strict inequality for at least one country, reflecting the possible gains from trade.[14]

The most salient feature of trade for our current purposes, however, is how these income gains are distributed across countries. Specifically, as we demonstrate below using a simple Armington (1969) trade model and as we can show in other trade models (see Online Appendix D), the smaller of the two countries under autarky can expect to enjoy relatively larger income gains from trade than its larger trading partner:

$$\textit{if } A^j R^j > A^i R^i, \text{ then } \frac{T^i R^i}{A^i R^i} > \frac{T^j R^j}{A^j R^j}.$$

This forthcoming result should be kept in mind as we develop the intuition behind our results for trade by first focusing on exogenous changes in relative output.

Setting aside (for now) the decision to trade, a central component of our analysis is how each country subsequently decides to allocate its first-period income Y^i. Specifically, each country divides its output between current consumption C^i and two distinct types of activities that can augment future consumption $\widetilde{C}^i$: "arming," which we denote by G^i, and "saving," which we

[13] Our assumption that conflict emerges in the future with some positive probability represents an important departure from much of the conflict literature that assumes conflict emerges with certainty. Even in analyses that study the choice between war and peace (e.g., Jackson and Morelli, 2007), war emerges with either probability 1 or probability 0. We view our approach as appealing since it allows us to study both these special cases and intermediate cases where arming is prudent *ex ante*, though not necessarily *ex post*. Furthermore, as discussed below in Section 5.3, our setting can be interpreted as one where (should a dispute arise, which occurs with some probability) countries choose between war and peaceful settlement that amounts to a division of whatever is being contested based on countries' relative arms.

[14] In static versions of many trade models, such as the Armington (1969) model, there is an isomorphism between changes in the resource endowment R^i and changes in productivity A^i in terms of their respective influence on the production of final output under autarky, $Y^i = A^i R^i$. Hence, we can write the $T^i(R^i, R^j)$ function as $T^i = T(A^i R^i, A^j R^j)$. Online Appendix D contains remarks on how the relationship between the relative gains from trade and relative endowments depends on technology differences as well as on trade costs and other similar parameters.

denote by Z^i. (Throughout, we use a tilde ($\sim$) over a variable to indicate its value in the second period, $t = 2$.) This choice must satisfy the following resource constraint:

$$C^i + G^i + Z^i \leq Y^i, \text{ for } i = 1, 2. \tag{1}$$

To simplify the exposition, and without altering any of our key results, we will henceforth assume that the two countries have equivalent technologies under autarky: $A^i = A^j = 1$. First-period output in the absence of trade will thus simply be given by $Y^i = R^i$. Turning to period 2, each country i's first-period saving yields $\widetilde{R}^i = Z^i$ units of the productive resource. Assuming that trade is not possible in period $t = 2$, that resource in turn is transformed into $\widetilde{Y}^i = Z^i$ units of second-period output.[15]

From the perspective of period $t = 1$, the output held by each country in period $t = 2$ and, thus, the return from such saving are subject to uncertainty due to the possibility of future conflict. In the baseline version of the model presented here, the weight of this uncertainty is governed by the probability of conflict, denoted by $q \in (0, 1]$. More precisely, in the event that no conflict arises and thus peace prevails, which occurs with probability $1 - q$, country i enjoys its entire output: $\widetilde{C}^i = Z^i$, $i = 1, 2$. By contrast, if a conflict arises, which occurs with probability q, each country's output goes into a contested pool, $Z^i + Z^j$, $i, j \in \{1, 2\}$, $i \neq j$. In the fuller version of the model presented in Online Appendix A and in some of our extensions discussed in Section 5.3, we allow for the possibility that some of this output is secure even in the event of conflict.

In the case that conflict arises, country i's share ϕ^i of the contested pool in period $t = 2$ depends on arming by both countries (G^i, G^j) chosen in period $t = 1$. This share takes the standard ratio form:

$$\phi^i(G^i, G^j) \equiv \frac{(G^i)^m}{(G^i)^m + (G^j)^m}, \text{ if } G^i + G^j > 0 \text{ for } i, j \in \{1, 2\}, i \neq j, \tag{2}$$

where $m \in (0, 1]$ reflects the effectiveness of arming; if $G^i + G^j = 0$ then $\phi^i = \phi^j = \frac{1}{2}$.[16] This specification implies a country's share is increasing in its own arming (i.e., $\phi^i_{G^i} \equiv \partial\phi^i/\partial G^i = m\phi^i\phi^j/G^i > 0$) and decreasing in the opponent's arming (i.e., $\phi^i_{G^j} \equiv \partial\phi^i/\partial G^j = -m\phi^i\phi^j/G^j < 0$). Furthermore, this conflict technology is symmetric (i.e., $\phi^i(G^i, G^j) = \phi^j(G^i, G^j)$ for any feasible G^i and G^j). The influence of guns on the division of contested output between the two countries can be interpreted as the result of either open conflict (i.e., war without destruction) or a bargaining process with the countries' relative military strength playing a prominent role. Importantly, as discussed below in Section 5.3 (with further details provided in Online Appendix E), our central results to follow remain qualitatively unchanged provided the resolution of conflict—whether it results in a division of contested output as modeled here or is modeled as a "winner-take-all" contest with ϕ^i representing the probability of winning—requires the use of resources to produce arms.

[15] Although a central objective in this paper is to explore the influence that the trade regime in place in $t = 1$ has on current equilibrium allocations to saving and arming when conflict in the next period possibly materializes, our analysis can be extended to consider the possibility of trade also in $t = 2$. This extension, which is discussed in Section 5.3, reveals that future trade favorably influences preferences for current trade.

[16] See Skaperdas (1996), who axiomatizes a more general functional form of this conflict technology. The particular form shown in (2) is commonly used in the contest and conflict literatures. We impose the restriction that $m \leq 1$, which is sufficient to ensure that a unique pure-strategy equilibrium exists. Focusing on pure-strategy equilibria allows us abstract from the possibility of multiple equilibria and the issues that arise as a result.

Each country i chooses its allocation of current income Y^i to arming G^i and saving Z^i to maximize expected lifetime utility: $U^i = u(C^i) + \delta E\{u(\widetilde{C}^i)\}$, where $\delta \in (0, 1]$ represents the common discount factor and $u(\cdot)$ has the usual properties that ensure the quasi-concavity of payoff functions and ensure strictly positive allocations to both saving and arming: $u' > 0$, $u'' < 0$, and $\lim_{C\to 0} u'(C) = \infty$. While the results to follow hold under any function $u(C) = C^{1-\rho}/(1-\rho)$ for $\rho > 0$, where ρ is the coefficient of relative risk aversion and $1/\rho$ is the elasticity of intertemporal substitution, we assume logarithmic preferences ($\rho = 1$) to keep the analysis as simple as possible:

$$U^i = \ln C^i + \delta E\left\{\ln \widetilde{C}^i\right\} \text{ for } i = 1, 2. \tag{3}$$

This maximization problem for each country i, which takes rival country j's choices as given, is subject to the first-period resource constraint (1), the conflict technology (2) and, for $i, j \in \{1, 2\}$, $i \neq j$, the following:

$$\widetilde{C}^i = \begin{cases} Z^i & \text{with probability } 1-q \\ \phi^i(Z^i + Z^j) & \text{with probability } q. \end{cases} \tag{4}$$

As (4) shows, a country's arming matters for future consumption (through ϕ^i) only in the event of conflict. Thus, when $q = 0$, the model simplifies to a standard consumption/investment savings model (i.e., with $G^i = G^j = 0$), a useful benchmark for highlighting the importance of insecurity and uncertainty for such dynamic problems.[17]

The timing of the extended policy game is as follows. First, at the beginning of period $t = 1$, the two countries' policymakers simultaneously and noncooperatively choose their individually preferred trade regimes. If both countries announce "trade" (T), then the two countries exchange their intermediate goods, and each country i's output level is $Y^i = T^i(R^i, R^j)R^i$ for $i, j \in \{1, 2\}$, $i \neq j$; if, however, at least one country announces "autarky" (A), then no trade takes place, and each country i's output level is $Y^i = R^i$. Second, once first-period output levels are determined, each country i chooses G^i and Z^i noncooperatively and simultaneously and consumes the remaining income C^i. In period $t = 2$, each country uses its available resource $\widetilde{R}^i = Z^i$ to produce the intermediate goods and then output $\widetilde{Y}^i = Z^i$. The amount consumed that period by each country depends on whether or not conflict breaks out and of course on both countries' first-period choices as shown in (4).

A key difference between the interaction we have just described and that in standard models of distributive conflict is that each player has more than one instrument it can use to influence payoffs. In standard conflict models, each player is viewed as choosing its quantity of arms only; given the player's initial resources, those choices determine residually the size of the prize. In the present setup, while saving choices alone determine the size of the prize, these choices are jointly determined with arming choices. Our characterization of this more complex problem in the next section—in particular, the "equilibrium in shares" approach described in Section 4.1—therefore represents a methodological contribution of our work to the conflict literature even before considering our central question regarding trade.

[17] We could also modify the model so that conflict, when it arises, destroys a fraction of the contested pool of output. We do not consider this possibility here because it does not substantively alter our conclusions.

4. Equilibrium arming, saving and payoffs given income

Given the dynamic structure of the model, we find the subgame perfect equilibrium by solving the model backwards. Specifically, in this section, we characterize the Nash equilibrium of the simultaneous-move subgame in arming and saving and the associated discounted payoffs given Y^i for $i = 1, 2$, deferring until the next section our discussion of trade. Using equation (1) as an equality (due to non-satiation) together with (4), we can rewrite country i's expected, two-period payoff (3) as follows:

$$U^i = \ln\left(Y^i - G^i - Z^i\right) + \delta\left[q\ln\left(\phi^i[Z^i + Z^j]\right) + (1-q)\ln\left(Z^i\right)\right], \tag{5}$$

for $i, j \in \{1, 2\}$, $i \neq j$, where $\phi^i = \phi^i(G^i, G^j)$ is shown in (2) and where $Y^i = R^i$ under autarky and $Y^i = T^i(R^i, R^j)R^i$ under free trade. Country i's choices of arming G^i and saving Z^i in an interior solution, then, satisfy the following first-order conditions (FOCs):

$$U^i_{G^i} = \delta\left[\frac{q\phi^i_{G^i}}{\phi^i}\right] - \frac{1}{Y^i - G^i - Z^i} = 0 \tag{6a}$$

$$U^i_{Z^i} = \delta\left[\frac{q}{Z^i + Z^j} + \frac{1-q}{Z^i}\right] - \frac{1}{Y^i - G^i - Z^i} = 0, \tag{6b}$$

for $i, j \in \{1, 2\}$, $i \neq j$, which is a system of four equations in four unknowns.

The second terms shown in the expressions for $U^i_{G^i}$ and $U^i_{Z^i}$ in (6a) and (6b) respectively represent the marginal costs to country i of arming (MC^i_G) and saving (MC^i_Z) that arise as such activities reduce current consumption, $C^i = Y^i - G^i - Z^i > 0$. Because G^i and Z^i constitute competing uses of $t = 1$ output and they displace the same quantity of current consumption, their marginal costs are identical (i.e., $MC^i_G = MC^i_Z$) and always reflect the *inter*-temporal trade-off between present and future consumption. In addition, both MC^i_G and MC^i_Z are increasing and convex in G^i and Z^i (respectively) and are decreasing in country i's $t = 1$ output Y^i; that is, $\lim_{G^i + Z^i \to Y^i} MC^i_J = \infty$ and $\partial MC^i_J / \partial Y^i < 0$ for $J = G, Z$.

The first term shown in the expression for $U^i_{G^i}$ in (6a) represents country i's expected, discounted marginal benefit of producing an additional gun (MB^i_G). This benefit derives from the effect of increased arming to expand country i's share of the contested output and thereby augment its future consumption $\widetilde{C}^i$ in the event of conflict. Accordingly, MB^i_G depends positively on the probability of conflict q and the discount factor δ. Next, observe from (2) that $\phi^i_{G^i} = m\phi^i\phi^j / G^i$. Thus, country i's marginal benefit of arming simplifies as $MB^i_G = \delta q m\phi^j / G^i$, which clearly is decreasing in country i's own arming G^i and increasing in the other country's arming G^j. Noting again that MC^i_G is increasing in G^i but is independent of G^j, it follows that country i's payoff is strictly concave in G^i (i.e., $U^i_{G^i G^i} < 0$) and that the two countries' arming choices are strategic complements (i.e., $U^i_{G^i G^j} > 0$).

Country i's expected marginal benefit of saving (MB^i_Z) is captured by the first term shown in the expression for $U^i_{Z^i}$ in (6b). Unlike MB^i_G, this expected benefit derives from two distinct sources, one that matters only in the event of conflict and one that matters only in the event of peace. If conflict arises, increases in savings affect the total pie of insecure future output to be contested. If instead peace prevails, each country's savings then convert entirely to future consumption. Not surprisingly, then, MB^i_Z falls with increases in the likelihood of conflict q and rises with increases in the discount factor δ. Further inspection also reveals MB^i_Z is decreasing in the country's own saving Z^i, and, when $q < 1$, $\lim_{Z^i \to 0} MB_Z^i = \infty$. Thus, provided the probability of future peace is strictly positive (i.e., $q < 1$), both countries choose strictly positive savings: $Z^i > 0$ for $i = 1, 2$. Since MC_Z^i is increasing in Z^i, these properties imply country i's payoff is strictly concave in Z^i (i.e., $U^i_{Z^i Z^i} < 0$). Furthermore, since MB_Z^i is decreasing in Z^j while MC_Z^i is independent of Z^j, the countries' savings choices are strategic substitutes (i.e., $U^i_{Z^i Z^j} < 0$).

4.1. Equilibrium in shares

Building on the relationships outlined above, we can define and characterize the equilibrium implied by (6). In view of the complexity of this strategic environment, we first reduce the dimensionality of the problem in order to obtain what we call an "equilibrium in shares" representation. This approach enables us to illuminate how country "size" translates into "power" as well as how this relationship is moderated by changes in the probability of conflict q, thereby paving the way for our upcoming analysis of how the trade regime matters for equilibrium outcomes and payoffs.

To proceed, define the share that country i contributes to the (potentially) contested pool of future output as

$$\theta^i(Z^i, Z^j) \equiv \frac{Z^i}{Z^i + Z^j} \text{ if } Z^i + Z^j > 0 \text{ for } i, j \in \{1, 2\}, i \neq j, \tag{7}$$

where $\theta^j = 1 - \theta^i$. One can easily verify that $q < 1$ ensures $\theta^i, \theta^j > 0$, $\theta^i_{Z^i} = \theta^i\theta^j/Z^i > 0$, and $\theta^i_{Z^j} = -\theta^i\theta^j/Z^j < 0$. The definition of θ^i allows us to characterize relative saving choices across countries in terms of a single endogenous parameter: $Z^i/Z^j = \theta^i/\theta^j = \theta^i/(1-\theta^i)$ for $i, j \in \{1, 2\}$, $i \neq j$. Similarly using the conflict technology in (2), we can write relative arming choices as a function of ϕ^i: $G^i/G^j = (\phi^i/\phi^j)^{1/m} = (\phi^i/(1-\phi^i))^{1/m}$ for $i, j \in \{1, 2\}$, $i \neq j$. Using these two relationships, we then transform (6) (a system of four equations in four unknowns) into a system of two equations in just two unknowns—specifically, the appropriative and contributive shares, ϕ^i and θ^i.

To derive the first of these equations, we proceed in two steps. First, we form the *relative* marginal benefits of arming and saving, given respectively by

$$\frac{MB_G^j}{MB_G^i} = \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}+1} \quad \text{and} \quad \frac{MB_Z^j}{MB_Z^i} = \frac{\theta^i}{\theta^j}\left[\frac{1-q+q\theta^j}{1-q+q\theta^i}\right].$$

Keeping in mind that $\phi^j = 1 - \phi^i$ and $\theta^j = 1 - \theta^i$, one can easily see that the expressions above depend only on ϕ^i, θ^i, and q. Second, we exploit the fact that, in any equilibrium, the marginal benefits of arming and saving must equalize for each country (since $MC_G^i = MC_Z^i$ implies $MB_G^i = MB_Z^i$ for $i = 1, 2$). Thus, the following equalities must also hold in equilibrium:

$$\frac{MB_G^j/MB_Z^j}{MB_G^i/MB_Z^i} = \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}+1}\frac{\theta^j}{\theta^i}\left[\frac{1-q+q\theta^i}{1-q+q\theta^j}\right] = \frac{MC_G^j/MC_Z^j}{MC_G^i/MC_Z^i} = 1. \tag{8}$$

Rewriting (8), we obtain the first of two conditions that define an equilibrium:

$$S^i\left(\phi^i, \theta^i; q\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}}\left[\frac{\left(1-q+q\theta^i\right)\left(\phi^i/\theta^i\right)}{\left(1-q+q\theta^j\right)\left(\phi^j/\theta^j\right)}\right] - 1 = 0. \tag{9}$$

The first term in the expression for $S^i(\phi^i, \theta^i; q)$ represents the ratio of the relative marginal benefits of arming and saving across countries, whereas the second term reflects the ratio of the

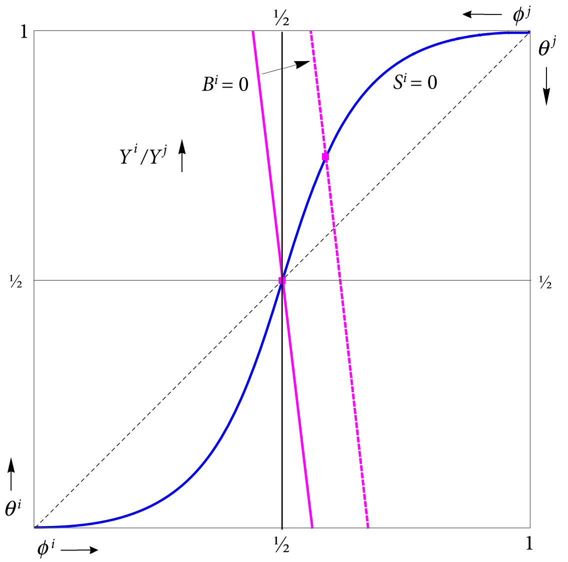

Fig. 1. The determination of countries' equilibrium shares in appropriative and productive investments.

relative marginal costs of arming and saving across countries that must equal 1. Since $\phi^i + \phi^j = 1$ and $\theta^i + \theta^j = 1$, the condition in (9) implicitly defines a relationship between θ^i and ϕ^i that we henceforth refer to as the "S^i-contour" or, alternatively, as "schedule S^i." The lemma below describes the key properties of this schedule, named for its "S" shape as depicted in Fig. 1.

Lemma 1. *The $S^i(\phi^i, \theta^i; \cdot) = 0$ condition in (9) implicitly defines a continuous and increasing relationship between θ^i and ϕ^i that holds true in equilibrium. This relationship is characterized as follows*:

(a) $S^i_{\phi^i} > 0$, $S^i_{\theta^i} < 0$ *and* $d\theta^i/d\phi^i\big|_{S^i=0} = -S^i_{\phi^i}/S^i_{\theta^i} > 0$;
(b) $\lim_{\phi^i \to 0} d\theta^i/d\phi^i\big|_{S^i=0} = \lim_{\phi^i \to 1} d\theta^i/d\phi^i\big|_{S^i=0} = 0$ *and* $\lim_{\phi^i \to \frac{1}{2}} d\theta^i/d\phi^i\big|_{S^i=0} > 1$;
(c) *if* $\phi^i \gtreqless \frac{1}{2}$, *then* $\theta^i \gtreqless \phi^i$;
(d) $\lim_{\phi^i \to 0} \theta^i/\phi^i\big|_{S^i=0} = 0$ *and* $\lim_{\phi^i \to 1} \theta^i/\phi^i\big|_{S^i=0} = 1$.

Part (a) establishes that the S^i-contour is increasing over the entire range of values of ϕ^i, with points to the right (left) and below (above) the contour implying $S^i > 0$ ($S^i < 0$). Yet, from part (b), the contour is "flat" at the endpoints. Part (c) points out that the less powerful country's contributive share to the potentially contested pool of future income is not only less than that of its relatively more powerful rival but also less than its own appropriative share. Finally, combined with parts (b) and (c), part (d) establishes that the contour starts at $(\phi^i, \theta^i) = (0, 0)$, crosses the midpoint $(\phi^i, \theta^i) = (\frac{1}{2}, \frac{1}{2})$ where it is steeper than 1, and ends up at $(\phi^i, \theta^i) = (1, 1)$.

For some intuition regarding the ϕ^i/θ^i relationship along the S^i-contour, recall that it represents a balance between the countries' marginal benefits of arming relative to the marginal benefits of saving and their respective relative marginal costs. As shown in (8), because the ratio of the two countries' marginal costs is fixed at 1, adjustments in ϕ^i/θ^i along the contour are due solely to changes in the ratio of countries' relative marginal benefits. To dig a little deeper, consider the midpoint of the S^i-contour, at $\left(\phi^i, \theta^i\right) = (\frac{1}{2}, \frac{1}{2})$ where $\phi^i/\theta^i = \phi^j/\theta^j = 1$. Now consider how the ratio of relative marginal benefits would change if ϕ^i and θ^i increased proportionately (i.e., if we moved NE along the 45° line in the figure). Since ϕ^i and θ^i increase (and thus ϕ^j and θ^j decrease), while $\phi^i/\theta^i = \phi^j/\theta^j$ remain unchanged at 1, the ratio of marginal benefits in (8) rises above 1, implying $S^i > 0$ and that we have traveled below schedule S^i. Therefore, starting at $\phi^i = \theta^i = \frac{1}{2}$, an increase in a country's appropriative share ϕ^i must be accompanied by a greater increase in its contributive share θ^i (such that $\theta^i > \phi^i$) to keep the value of the ratio of marginal benefits equal to 1 and thus remain on the S^i-contour, as emphasized in Lemma 1(c). However, part (b) establishes that this tendency becomes less pronounced as ϕ^i approaches 1.[18]

As shown in the definition in (9) and as we discuss in detail below, the shape of the S^i-contour also depends on the probability of conflict q. But, it does not depend on income levels Y^i and Y^j or on the discount factor δ; thus, while any equilibrium in (ϕ^i, θ^i) must lie somewhere on the S^i-contour, determining its exact location requires a second condition capturing the influence of these other variables on relative arming and saving decisions.

To derive this second condition, we solve for each country i's arming and saving decisions, G^i and Z^i, from the FOCs in (6), in order to obtain:

$$G^i = \frac{\gamma^i}{1 + \gamma^i + \zeta^i} Y^i \quad \text{and} \quad Z^i = \frac{\zeta^i}{1 + \gamma^i + \zeta^i} Y^i, \text{ for } i = 1, 2, \tag{10}$$

where

$$\gamma^i = \gamma^i\left(\phi^i\right) \equiv \delta q m \left(1 - \phi^i\right) \geq 0 \quad \text{(with equality when } \phi^i = 1) \tag{11a}$$

$$\zeta^i = \zeta^i\left(\theta^i\right) \equiv \delta\left(q\theta^i + 1 - q\right) > 0, \tag{11b}$$

represent weights that jointly determine spending on arming and saving respectively per unit of income spent on current consumption.[19] Clearly, the income share that country i channels into arming and the income share that it channels into saving, shown in (10), depend on both ϕ^i and θ^i through the relationships shown in (11). To proceed, observe from (10) that the ratio G^i/G^j can be written as a function of the two countries' expenditure shares and recall that the specification of ϕ^i in (2) implies $G^i/G^j = (\phi^i/\phi^j)^{1/m}$ for $i, j \in \{1, 2\}$, $i \neq j$. Together, these implications give us our second equilibrium condition:

$$B^i\left(\phi^i, \theta^i; Y^i/Y^j, q\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{1/m} - \frac{\gamma^i\left(1 + \gamma^j + \zeta^j\right)}{\gamma^j\left(1 + \gamma^i + \zeta^i\right)}\left(\frac{Y^i}{Y^j}\right) = 0. \tag{12}$$

Since $\phi^i + \phi^j = 1$ and $\theta^i + \theta^j = 1$, the above equation, with the definitions of γ^i and ζ^i for $i, j \in \{1, 2\}$, $i \neq j$ shown in (11), implicitly defines another relationship between ϕ^i and θ^i, which we call the "B^i-contour" or "schedule B^i." The next lemma characterizes its shape.

[18] Consideration of asymmetries, either in the conflict technology (e.g., $\phi^i = \beta^i G^i/(\beta^i G^i + \beta^j G^j)$ for $\beta^i, \beta^j > 0$) or in production technologies would mainly alter the point where the S-contour meets the 45° line without affecting its behavior at the extremes. As such, allowing for such asymmetries does not affect any of the key limit results that we focus on.

[19] In the fuller version of the model where output is partially secure in the event of conflict, each weight depends on both ϕ^i and θ^i. Either way, the share of income spent on first-period consumption is always given by $1/(1 + \gamma^i + \zeta^i)$, with the weights always being equal to $\gamma^i = G^i MB^i_G$ and $\zeta^i = Z^i MB^i_Z$.

Lemma 2. *The $B^i(\phi^i, \theta^i; \cdot) = 0$ condition in (12) defines implicitly a continuous and decreasing relationship between ϕ^i and θ^i that holds true in equilibrium. Specifically, $B^i_{\phi^i} > 0$, $B^i_{\theta^i} > 0$ and thus $d\theta^i/d\phi^i\big|_{B^i=0} = -B^i_{\phi^i}/B^i_{\theta^i} < 0$.*

Schedule B^i is the negatively sloped curve in Fig. 1 that, drawn for $Y^i = Y^j$, goes through the midpoint where $\phi^i = \theta^i = \frac{1}{2}$.[20] Points to the right (left) and above (below) the curve imply $B^i > 0$ ($B^i < 0$).

Observe that, like the definition of schedule S^i, the definition of schedule B^i uses both countries' FOCs. However, its derivation relies more directly on the two countries' arming decisions, which in turn explains why the ratio of incomes $y^i \equiv Y^i/Y^j$ and (through the parameters γ^i and ζ^i) the discount factor δ appear in the second term of the expression for B^i shown in (12). Observe further, as revealed by inspection of (12) using (11), the shape and location of the B^i-contour also depend on the probability of conflict q.

Using Lemmas 1 and 2, we now turn to the determination of shares, ϕ^i and θ^i, and their properties in a pure-strategy equilibrium.

Proposition 1 *(*Equilibrium in shares*). Suppose $q \in (0, 1)$, $\delta \in (0, 1]$ and $y^i \in (0, \infty)$. Then, a unique equilibrium $\left(\phi^{i*}, \theta^{i*}\right) \in (0, 1) \times (0, 1)$ in appropriative and contributive shares ($i = 1, 2$) exists, with the following properties*:

(a) *$d\phi^{i*}/dy^i > 0$ and $d\theta^{i*}/dy^i > 0$; furthermore, $\theta^{i*} \gtreqless \phi^{i*} \gtreqless \frac{1}{2}$ as $y^i \gtreqless 1$;*
(b) *$\lim_{y^i \to 0} \phi^{i*} = \lim_{y^i \to 0} \theta^{i*} = 0$, $\lim_{y^i \to 0} \theta^{i*}/\phi^{i*} = 0$, and $\lim_{y^i \to \infty} \theta^{i*}/\phi^{i*} = 1$;*
(c) *$\partial\phi^{i*}/\partial q \lesseqgtr 0$, $\partial\phi^{i*}/\partial\delta \lesseqgtr 0$ and $\partial\theta^{i*}/\partial\delta \lesseqgtr 0$ as $y^i \gtreqless 1$.*

Fig. 1 illustrates the equilibrium shares in appropriative and productive investments, depicted by the intersection the B^i- and S^i-contours that have been derived from the FOCs for arming and saving. As illustrated in the figure, when $Y^i = Y^j$ (or $y^i = 1$), the intersection of the two schedules occurs at the midpoint where the two countries are equally powerful as well as equal contributors to future income ($\phi^{i*} = \theta^{i*} = \frac{1}{2}$). An increase in Y^i given Y^j (equivalently, an increase in y^i) relaxes country i's *inter*-temporal trade-off between current and future consumption, thereby reducing its marginal costs of arming and saving and causing the B^i-contour to shift rightward. The equilibria induced by such changes in Y^i then trace out the S^i-contour, reflecting changes in the relative marginal benefits of arming and saving as country i changes in size. Thus, as pointed out in part (a), when country i is initially larger (i.e., $Y^i > Y^j$), the B^i-contour intersects the S^i-contour to the right and above the midpoint, implying it is more powerful than country j and an even bigger relative contributor to future income (i.e., $\theta^{i*} > \phi^{i*} > \frac{1}{2}$). While the smaller country is less powerful in equilibrium (i.e., $\phi^{j*} < \phi^{i*}$) and contributes less to future output (i.e., $\theta^{j^*} < \theta^{i*}$), it obtains a larger share of that future output in the event of conflict relative to its contribution (i.e., $\theta^{j*} < \phi^{j*} < \frac{1}{2}$). This latter result, which reflects the smaller country's ability to "prey" on the more prudent behavior of its larger rival when future conflict is possible, is reminiscent of (though distinct from) the weak form of Hirshleifer's (1991) "paradox

[20] That $\phi^i = \theta^i = \frac{1}{2}$ is a point on schedule B^i when $Y^i = Y^j$ can be confirmed using equation (11) with the condition $B^i = 0$ in (12).

of power."[21] Part (b) of the proposition, which characterizes the relative limiting behaviors of ϕ^{i*} and θ^{i*}, establishes that as size differences become infinitely large, the smaller country's contributive share vanishes faster than its power share.

The first component of part (c) reveals how the influence of the probability of future conflict q on each country's *intra*-temporal trade-off between arming and saving weighs on the balance of power. Specifically, it establishes that a deterioration of international relations ($q \uparrow$) tends to diminish differences in power. Since, as mentioned earlier, the marginal benefit of arming MB^i_G shown as the first term in the expression for $U^i_{G^i}$ in (6a) is increasing in q while the marginal benefit of saving MB^i_Z shown as the first term in the expression for $U^i_{Z^i}$ in (6b) is decreasing in q, an increase in q raises MB^i_G/MB^i_Z or equivalently reduces the opportunity cost of arming for each country i. The result that an increase in q reduces the disparity in power across countries given income levels suggests that the opportunity cost of arming falls by more for the smaller country.

To tease out some intuition here, observe that, due to the symmetry of the conflict technology (2), the marginal benefit of arming MB^i_G depends symmetrically on country i's own arms G^i and those of its rival G^j. The marginal benefit of saving MB^i_Z, meanwhile, is nearly symmetric across countries i. The sole difference appears in the second term, $(1-q)/Z^i$. Underscoring the importance of saving for the possibility of peace, this term governs the relationship between differences in country size and the *intra*-temporal trade-off. To fix ideas, suppose $Y^i > Y^j$, which implies by part (a) of the proposition that $\theta^i > \frac{1}{2}$ and thus $Z^i > Z^j$. Accordingly, all else the same, this second term is smaller for the larger country (i), which means its opportunity cost of shifting resources from saving to arming is smaller, thereby giving it a military advantage. As the probability of conflict rises ($q \uparrow$), both $(1-q)/Z^i$ and $(1-q)/Z^j$ fall, but the latter falls by more, thereby weakening the larger country's military advantage.[22] Although the larger country remains more powerful, this result suggests that a greater likelihood of future conflict amplifies the smaller country's predatory stance through its more aggressive arming relative to its saving that contributes to future income.[23]

Finally, the last two components of part (c) of the proposition show that an increase in the discount factor δ tends to reduce differences in power ϕ^{i*} and in contributive shares θ^{i*} across

[21] In fact, as shown below, the weak form of the paradox of power that states $Y^j/Y^i < \phi^{j*}/\phi^{i*} < 1$ holds in our setting (see footnote 25).

[22] In the limit as $q \to 1$, both MB^i_G and MB^i_Z become symmetric across i, meaning that both FOCs in (6) can be satisfied as strict equalities (required for an interior solution) for both countries only when $G^i = G^j$, which implies $\phi^i = \phi^j = \frac{1}{2}$. In this special case, the larger country necessarily saves more, such that $C^i = C^j$ despite differences in first-period incomes; and, since $\phi^i = \frac{1}{2}$ for $i = 1, 2$ and $q = 1$ by assumption, $\widetilde{C}^i = \widetilde{C}^j$ also holds, such that two countries enjoy identical payoffs in any interior solution, again despite differences in first-period incomes. However, this possibility arises only when those differences in income are not too pronounced. Otherwise, a corner solution arises in which the smaller country does not save at all and its appropriative share is less than $\frac{1}{2}$.

[23] The effects of an increase in q on the *intra*-temporal trade-off can be visualized in the setting of Fig. 1 as a counterclockwise rotation of the S^i-contour around the midpoint $(\frac{1}{2}, \frac{1}{2})$, with the endpoints unchanged, thereby making the relationship between saving (and thus country size) and power less linear. At the same time, the B^i-contour also rotates in a counterclockwise direction around the point where it intersects the 45° line when $m = 1$ or above (below) that intersection when $Y^i < Y^j$ ($Y^i > Y^j$) and $m < 1$. When $Y^i = Y^j$, the curves rotate as just described, but around their intersection at the midpoint, such that ϕ^{i*} is not affected. Part (a) of Proposition B.1, presented in Online Appendix B, states further that θ^{i*} is similarly independent of q when $Y^i = Y^j$. Although we cannot pin down the influence of this parameter on θ^{i*} for all Y^i and Y^j, part (b) of Proposition B.1 shows that, in the case of an extreme asymmetry as $Y^i \to 0$ for given $Y^j \in (0, \infty)$, θ^i is decreasing in q.

countries. As discussed earlier, an increase in δ magnifies the marginal benefits of both arming and saving for each country. This magnification effect is larger for the smaller country, however, causing it to become more aggressive and, at the same time, more prudent relative to its larger rival.[24]

In sum, Proposition 1 tells us how the countries' appropriative and contributive shares (or relative arming and saving) are related to relative incomes, as well as how the distribution of power adjusts to changes in the probability of future conflict and in time preferences. But, it leaves unanswered the question of how such changes influence the arming and saving decisions of each country in levels. For example, while we know that an increase in country i's relative income makes that country more powerful, it is unclear whether each country devotes more or less resources to arming. Similarly, while we know that country i's saving rises relative to that of its rival, we do not know yet whether the two countries save more or less.

Nonetheless, an appealing feature of our "equilibrium in shares" approach is that the share variables ϕ^{i*} and θ^{i*} pin down the fractions of current income allocated to arming and saving via (11) with (10), allowing us to recover equilibrium spending choices by each country i on G^{i*} and Z^{i*} as functions of ϕ^{i*} and θ^{i*}. Then, having identified the effects of changes in relative income Y^i/Y^j on ϕ^{i*} and θ^{i*}, we can characterize their effects on G^{i*} and Z^{i*}. This characterization not only allows us to flesh out further the implications of Proposition 1, but also prepares the groundwork for our study of the effects of trade on equilibrium arming, saving and payoffs.

4.2. Income changes and equilibrium arming, saving, and payoffs

We now turn to examine the implications of changes in one or both countries' incomes for their equilibrium choices and payoffs. Letting a caret ($\wedge$) over variables denote percent changes (e.g., $\widehat{x} \equiv dx/x$), the following proposition characterizes these effects.

Proposition 2 *(*Equilibrium arming and saving*). Suppose $q \in (0, 1)$, $\delta \in (0, 1]$, and $y^i \in (0, \infty)$. Then, an exogenous change in the countries' incomes with $0 \leq \widehat{Y}^j < \widehat{Y}^i$ for $j \neq i$ imply the following responses in saving and arming:*

$$\widehat{Z}^{j*} < \widehat{Y}^j < \widehat{G}^{j*} < \widehat{G}^{i*} < \widehat{Y}^i < \widehat{Z}^{i*}.$$

Generally speaking, an increase in country i's income (given Y^j) generates positive, direct effects on country i's own arming and saving primarily by reducing the marginal cost of both activities and thereby directly relaxing the country's *inter*-temporal trade-off—i.e., between present and future consumption. At the same time, there are further, indirect effects reflecting how changes in country i's arming and saving levels induce the rival country (j) to adjust its own arming and saving as well as how these adjustments feed back into country i's choices. That arming and saving do not increase proportionately with the change in income for either country reflects the combined influence of these indirect effects.[25]

[24] The effect of an increase in δ can be visualized as a counterclockwise rotation of the B^i-contour around the point where it intersects the 45° line when $m = 1$ or above (below) that intersection when $m < 1$ and $Y^i < Y^j$ $(Y^i > Y^j)$.

[25] The rankings shown in Proposition 2 can be used to substantiate the presence of the weak form of the paradox of power in our setting. Specifically, those rankings imply $\widehat{G}^{i*} - \widehat{G}^{j*} < \widehat{Y}^i - \widehat{Y}^j$, such that increases in $y^i = Y^i/Y^j$ induce smaller increases in the ratio G^{i*}/G^{j*}. Considering the benchmark where $y^i = 1$ initially, it follows that $1 < G^{i*}/G^{j*} < y^i$ and thus, by (2), $\phi^{i*}/\phi^{j*} < y^i$ for all $y^i > 1$. (By similar reasoning, the result in Proposition 2 that

For greater clarity, let us suppose that $\widehat{Y}^j = 0$. This case is especially relevant for our upcoming analysis, since a shift from autarky to trade implies $\widehat{Y}^j = 0$ when country i is infinitesimal. As the proposition shows, a given increase in country i's first-period income induces an increase in both its saving and arming, with a larger (percentage) change in saving.[26] The reasoning here for the larger effect on country i's saving builds on the set of strategic interactions we discussed earlier in connection with the FOCs (6). Specifically, the countries' saving choices are always strategic substitutes, and their arming choices are always strategic complements.[27] Thus, when $\widehat{Y}^j = 0$, country j responds to the increases in G^{i*} and Z^{i*} induced by an increase in Y^i by shifting resources from saving to arming.[28] The reduction in country j's saving induces country i to increase its own saving by even more, further clarifying the intuition for why Z^{i*} expands by more in percentage terms than G^{i*}, as Y^i increases.

Characterizing analytically the effects of the probability of future conflict (q) on spending levels here proves to be challenging, because we cannot sign the effects of a change in q on θ^i and thus cannot identify its effects on the two countries' arming and saving decisions for all $y^i = Y^i/Y^j \in (0, \infty)$. However, numerical analysis shows that an increase in q induces each country to substitute out of saving into arming.[29] In turn, Proposition 1(c) indicates that the effect of an increase in q on the smaller country's arming is proportionately greater, thereby augmenting that country's relative power.

Although we cannot pin down, in general, the effects on their relative contributive shares to world savings and thus the pool of contestable output, numerical analysis shows further that an increase in q has a disproportionately negative effect on the smaller country's savings, implying an increase in the larger country's relative contribution. Still, these tendencies are consistent with the intuitive idea that increased international tensions can have adverse consequences for growth.

In any case, Proposition 2 clarifies several ambiguities left over from our representation of the problem in terms of shares. Having fully characterized how the two countries' choices (G^{i*} and Z^{i*}) in both levels and shares depend on income levels, we now turn our attention to the more intricate problem of identifying how exogenous income changes affect each country's equilibrium payoff:

$\widehat{Z}^i - \widehat{Z}^j > \widehat{Y}^i - \widehat{Y}^j$ implies $Z^{i*}/Z^{j*} > Y^i/Y^j$ for all $y^i > 1$.) However, along the lines of Hirshleifer's (1991) finding in the standard conflict model, the paradox of power can be overturned in our setting when the conflict technology exhibits increasing returns (i.e., $m > 1$). Of course, as noted earlier (footnote 16), allowing for such increasing returns can result in multiple equilibria and thereby complicate our equilibrium analysis in shares. Alternatively, sufficiently strong complementarities between Z^i and Z^j in the production of second-period output (in the event of conflict) along with a sufficiently large degree of relative risk aversion can overturn the paradox of power (a result shown formally in the standard model of conflict by Skaperdas and Syropoulos, 1997).

26 Proposition B.2(a) presented in Online Appendix B shows country i's current consumption also rises.

27 As discussed in Online Appendix A, when output is only partially insecure in the event of conflict, the countries' arming choices need not be strategic complements. Nonetheless, even in this case, G^{j*} depends positively on Z^{i*} regardless of which country is larger, and this effect always dominates any strategic substitutability in arming choices so that the results of Proposition 2 continue to hold.

28 A continuity argument (confirmed by numerical analysis) shows that, even when $\widehat{Y}^j > 0$, country j could reduce its savings, provided that increase in income is sufficiently small. Proposition B.2(b) presented in Online Appendix B indicates the effect of an increase in the opponent's income (Y^i) on country j's current consumption (C^j) is non-monotonic. In particular, as $Y^i \to Y^j$, an increase in Y^i implies C^j rises. However, for extreme differences in initial income where either $Y^i \to 0$ or $Y^i \to \infty$ given $Y^j \in (0, \infty)$ initially, C^j falls with increases in Y^i.

29 Also see Proposition B.3 presented in Online Appendix B that characterizes these effects on arming, saving, and first-period consumption when incomes across countries become either very similar (i.e., as $Y^i \to Y^j$) or extremely different (i.e., $Y^i \to 0$, while $Y^j \in (0, \infty)$).

Proposition 3 *(*Income and equilibrium payoffs*). Suppose $q \in (0, 1)$, $\delta \in (0, 1]$, and $y^i \in (0, \infty)$. Then, for any given Y^j, an exogenous change in country i's income Y^i affects the two countries' equilibrium payoffs, U^{i*} and U^{j*} ($j \neq i$), as follows:*

(a) $dU^{i*}/dY^i > 0$.

(b) *There exist threshold income levels $\underline{Y}^i$ and $\overline{Y}^i$ satisfying $\underline{Y}^i \leq \overline{Y}^i (< Y^j)$ such that $dU^{j*}/dY^i < 0$ for all $Y^i < \underline{Y}^i$ whereas $dU^{j*}/dY^i > 0$ for all $Y^i \geq \overline{Y}^i$.*

As suggested by Proposition 2, an increase in country i's first-period income generates both positive and negative welfare effects for both countries. For country i, the increase in Y^i has the direct, positive effect of increasing country i's first-period consumption. At the same time, the other country's ($j \neq i$) responses in terms of increased arming and decreased saving generate indirect, negative effects on country i's payoff. Part (a) establishes the direct, positive effect dominates, such that an increase in country i's first-period income always has a positive net effect on its own payoff.

The more interesting set of effects is for the rival country $j \neq i$. On the one hand, the increase in arming by country i implies a negative security externality for country j. On the other hand, the increase in saving by country i implies a larger pool of future output to be contested, thereby creating a positive externality. By Proposition 2, we know that the growth rate of country i's savings is faster than that of its arming. However, because country i arms much more than it saves when it is very small, its share of the balance of power ϕ^i initially increases by more, in absolute terms, than the share it contributes to future world output θ^i. Eventually, as it becomes close in size to country j, the faster rate of growth in its savings causes θ^i to increase by more than ϕ^i, as shown in Fig. 1. Thus, as described in Proposition 3(b), growth in country i can affect its potential rival's payoff adversely, but *only* if country i is small enough in relative terms for the negative externality from its increased arming to dominate the positive externality from its increased saving.[30] Importantly, this finding arises regardless of the possible absence or presence of trade and is unrelated to price (or terms-of-trade) effects. Nevertheless, the result is crucial in our analysis below that demonstrates trade could be unappealing to *ex ante* "larger" countries.

5. Equilibria under autarky and trade

With our characterization of how equilibrium outcomes depend on initial income levels (Y^i, Y^j), we can explore the implications of "trade." As discussed in Online Appendix D, our central results regarding trade are quite general, holding for a variety of different trade models, including those that feature trade costs. However, for the sake of transparency, our baseline model adopts a trade framework based on Armington (1969), a relatively simple trade setting in which each country produces a unique intermediate good; furthermore, we abstract from trade costs. In what follows, we first show how this setting implies larger relative income gains for countries initially having smaller resource endowments (or *ex ante* "smaller" countries). We then consider whether, as a result, situations can arise where *ex ante* larger countries will choose not to trade with their smaller rivals.

[30] Although we cannot pin down the payoff effects of changes in the probability of conflict q, Proposition B.4 shows the effects in the two extreme cases where (i) countries have identical incomes, in which case an increase in q lowers each country's payoff and (ii) one country is infinitesimal, in which case the smaller country gains from an increase in q, while the larger country's payoff is unaffected.

5.1. Resource endowments and the "gains from trade"

Recall that, at the beginning of period $t = 1$, each country i is endowed with R^i $(i = 1, 2)$ units of a productive resource (e.g., capital). Along the lines of Armington (1969), each country i converts that resource on a one-to-one basis into a unique intermediate good. Specifically, country 1 produces R^1 units of input 1, and country 2 produces R^2 units of input 2. In turn, country i employs this good, alone (in the case of autarky) or in combination with the intermediate good produced by country j (in the case of trade), to produce final output Y^i. Let D^i_j denote the quantity of intermediate good j $(= 1, 2)$ that becomes available domestically to producers of the final good in country i $(= 1, 2)$. The technology for producing the final good takes the following symmetric CES form:

$$Y^i = \left[\sum_{j=1,2} \left(D^i_j \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{13}$$

where $\sigma > 1$ represents the (constant) elasticity of substitution between intermediate inputs. The specification in (13), which is symmetric, increasing, linearly homogeneous, and (provided $\sigma < \infty$) strictly concave in its arguments, reflects the benefit of employing a variety of distinct inputs in production, which is analogous to the "love of variety" exhibited by Dixit-Stiglitz preferences. As the two intermediate goods become more distinct (i.e., as σ falls), this benefit of diversity rises and so do the gains from trade.

Whether the two countries trade their intermediate goods or not, each country i chooses inputs to maximize its income Y^i in (13) subject to the relevant constraints that depend on the trade regime in place. However, under autarky where each country can employ only its domestically produced intermediate good, $D^i_j = 0$ and $D^i_i = R^i$ hold for $i, j \in \{1, 2\}$, $i \neq j$. Thus, each country i's output is simply $Y^i_A = R^i$. Furthermore, absent trade in period $t = 2$, country i's output in that period, regardless of whether or not conflict breaks out, is given by $\widetilde{Y}^i = \widetilde{Y}^i_A = Z^i$ for $i = 1, 2$.

To study the case of trade in period $t = 1$, let p^i_j denote the price country i pays for input $j = 1, 2$ and μ^i_j denote its expenditure share on that good; then, $\mu^i_i = 1 - \mu^i_j$ denotes country i's expenditure share on the unique input it produces. As one can verify, the country's output-maximizing choice of inputs implies $\mu^i_j \equiv (p^i_j / P^i)^{1-\sigma}$, where $P^i \equiv [\sum_{k=i,j} (p^i_k)^{1-\sigma}]^{1/(1-\sigma)}$, and furthermore that its demand for input j $(= 1, 2)$ equals $D^i_j = \mu^i_j p^i_i R^i / p^i_j$. That we abstract from trade costs means domestic and world prices coincide, implying that $p^i_j = p^j_j$ and $p^j_i = p^i_i$. Now let $p^i \equiv p^i_j / p^i_i$ denote country i's domestic relative price of intermediate good j $(\neq i)$. By Walras' Law, these relative prices follow from the world market-clearing condition $p^i_j D^i_j = p^j_i D^j_i$ $(i \neq j)$, which implies p^i_T $(= p^i_j / p^j_i) = (R^i / R^j)^{1/\sigma}$. With these equilibrium relative prices, one can then substitute the demand functions D^i_j into (13) to find $Y^i_T = T^i(R^i, R^j) R^i$, where

$$T^i(R^i, R^j) \equiv \left[1 + \left(R^i / R^j \right)^{\frac{1-\sigma}{\sigma}} \right]^{-\frac{1}{1-\sigma}}, \quad \text{for } i, j \in \{1, 2\}, i \neq j. \tag{14}$$

Given $\sigma > 1$ and $R^i, R^j \in (0, \infty)$, it follows that $T^i(R^i, R^j) > 1$. Hence, both countries realize strictly positive income gains, $Y^i_T / Y^i_A = T^i(R^i, R^j) > 1$, when they trade in period $t = 1$, and abstracting from security considerations (essentially assuming in our model that $q = 0$) enjoy greater overall payoffs. As we will argue, however, if conflict arises with a strictly positive probability ($q > 0$) in the future in which case second period output is contested, then one country might find trade in the current period unappealing.

But, first, the next proposition offers a more detailed view of how the distribution of endowments translates into the distribution of first-period incomes under autarky and trade and, hence, the distribution of income gains from trade:

Proposition 4 *(*Relative incomes and the gains from trade*). Under autarky and trade, the country with the larger resource endowment enjoys a higher first-period income. Yet, country i's income gain from trade,* $Y_T^i/Y_A^i = T^i(R^i, R^j) \geq 1$*, is decreasing in* R^i/R^j *with* $\lim_{R^i \to 0} T^i(R^i, R^j) = \infty$ *and* $\lim_{R^i \to \infty} T^i(R^i, R^j) = 1$*, such that* $R^i/R^j \lesseqqgtr 1$ *implies* $(Y_T^i/Y_A^i)/(Y_T^j/Y_A^j) \gtreqqless 1$.

This proposition establishes that the *ex ante* smaller country (i.e., the country having the relatively smaller resource endowment) always enjoys a larger relative income gain from trade. These results are illustrated in Fig. 2(a), which shows the income level enjoyed under trade and autarky by country i, conditioned on the distribution of initial resources, $R^i \in (0, \overline{R})$ where $\overline{R} \equiv R^i + R^j$ ($i, j \in \{1, 2\}$, $i \neq j$). As shown in the proof, the divergence in gains is decreasing in σ.

Following our proof of Proposition 4 in Online Appendix B, we also discuss how a closely related set of results can be obtained if we relax our assumption that technology levels are equivalent across countries (i.e., if $A^i \neq A^j$.) As noted earlier in Section 3, country i's relative gain from trade in this slightly more general case continues to be determined by its relative autarky income level $(A^iR^i)/(A^jR^j)$. As such, the limit results and overall relationship between relative gains and relative endowment sizes remain the same as stated in the proposition, and equivalent results can be obtained if we instead consider changes in relative autarky incomes. Online Appendix D describes how these results carry over to other trade-theoretic settings, including settings where trade is costly.

5.2. *Trade, power, and welfare*

We move now to the main objective of our analysis: to see how and when the security considerations brought on by the possibility of future conflict can limit—or even overwhelm completely—the standard gains from trade for either country. To begin, we characterize the security externalities associated with trade, synthesizing our key results thus far for the effects of trade on the balance of power. Let ϕ_A^i ($i = 1, 2$) denote country i's equilibrium power under autarky and ϕ_T^i ($i = 1, 2$) denote country i's equilibrium power under trade. To fix ideas, let country i represent the larger *ex ante* country (i.e., with $R^i > R^j$). Proposition 1(a) implies that, under both trade regimes, country i always appropriates a larger share of the contested output in the event of a future conflict: $\phi_A^i > \phi_A^j$ and $\phi_T^i > \phi_T^j$.

Because trade raises the first-period income of both countries relative to their respective autarky incomes, we know further, by Proposition 2, that trade necessarily induces both countries to produce more guns, thereby generating negative security externalities for each country. However, Proposition 4 also establishes that the *ex ante* smaller country j always realizes a relatively larger income gain from trade. Thus, by Proposition 2, the introduction of trade reduces the *ex ante* larger country i's military advantage as compared with autarky, thereby leading to a more equitable division of the contested output in the event of conflict: $\phi_A^i > \phi_T^i > \phi_T^j > \phi_A^j$.

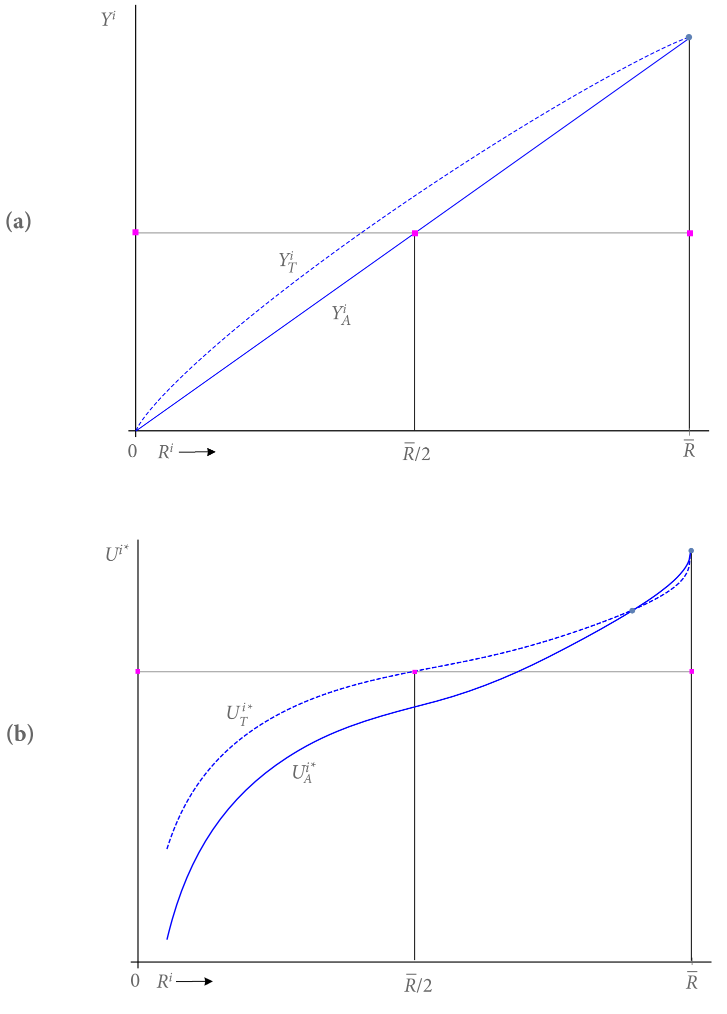

Fig. 2. Income and payoffs under autarky and trade, with $\sigma = 4$, $q = .9$ and $m = \delta = 1$.

Turning to the savings externalities, the second component of Proposition 1(a) implies that the *ex ante* larger country i contributes a larger share to future income. Following our convention for notation above, we have $\theta^i_A > \theta^j_A$ and $\theta^i_T > \theta^j_T$. Since by Proposition 4 the relatively smaller country j realizes a relatively larger income gain from trade, Proposition 2 implies that trade induces the *ex ante* smaller (larger) country j (i) to become a relatively larger (smaller) contributor to future income as compared with what happens under autarky: $\theta^i_A > \theta^i_T > \theta^j_T > \theta^j_A$.[31] While the savings externality can be negative for the smaller country if the distribution of initial resource endowments is sufficiently uneven to imply very small gains from trade for the larger country, it is necessarily positive for the larger country.

[31] See Fig. B.1 in Online Appendix B that illustrates the effects of trade on the countries' arming and savings in levels and on the equilibrium appropriative and contributive shares for various distributions of initial resources.

The next proposition, which accounts for both the security and savings externalities as well as the direct income gains, summarizes the welfare implications of trade when future conflict is possible:

Proposition 5 (Payoffs). *If the international distribution of initial resource endowments is sufficiently even, introducing trade in period* 1 *improves both countries' equilibrium discounted payoffs. But, if this distribution is sufficiently uneven, then the ex ante larger country will find trade unappealing as compared with autarky.*

As illustrated in Fig. 2(b), which depicts the payoffs to country i under autarky and trade for various distributions of initial resource endowments, the smaller country (i.e., with $R^i < \frac{1}{2}\overline{R}$) necessarily benefits from trade, both through its own income gain and through the net effects of trade-induced changes in the larger country's arming and saving decisions. The larger country also benefits from trade when the initial size difference is not too large. However, as shown in the same figure and consistent with Proposition 5, its payoff under trade eventually falls below its payoff under autarky when the initial size difference becomes sufficiently large.

In discussing this last result, it is important to emphasize that it does not follow immediately from our earlier propositions and instead requires new arguments. Propositions 3 and 4 and our previous discussions thereof are nonetheless useful for establishing some of the key intuition. As shown in Proposition 3(b), when the less endowed country is sufficiently small to start, a small increase in its income always reduces the larger country's payoff; as the discussion following Proposition 3 explains, this adverse payoff effect arises from the dominance of the negative security externality induced by the smaller country's increased arming over the positive externality induced by that country's increased saving. Proposition 4 then demonstrates that the larger country's relative income gain from trade is less than the relative income gain enjoyed by its smaller rival and becomes vanishingly small as it becomes increasingly large in relation to its rival. Combining the ideas from these two propositions might appear to be all that is needed to explain why a sufficiently large county will find trade relatively unappealing.

However, this is not the case. In particular, the result in Proposition 3(b) does not directly apply here, since the introduction of trade induces *discrete* changes in *both* countries' income when they are finitely sized (i.e., $R^i \in (0, \infty)$ for $i = 1, 2$). More to the point, this proposition cannot rule out the possibility that the smaller country's income gain from trade could be large enough to generate, for the larger country, a substantial (positive) saving externality that, when combined with its own income gain from trade, dwarfs the (negative) security externality. This possibility arises since, by Proposition 2, the smaller country devotes an increasingly bigger share of its income to saving as it grows larger.

Our solution to this problem, in the proof to Proposition 5, consists of three main components. First, we establish that the larger country's payoff under trade converges to its payoff under autarky as its rival becomes infinitesimal. This convergence follows since the larger country's income gain from trade, the smaller country's saving, and arming by both countries all vanish in this limit. Second, from that starting point, we consider a small increase in the small country's resource endowment, which we know from Proposition 3(b) causes the large country's payoff to decrease under trade as well as under autarky. Here we demonstrate that the decrease in the larger country's payoff is always larger in magnitude under trade than under autarky. Thus, when the smaller country's initial resource base is marginally above 0, the larger country prefers autarky to trade. Finally, we appeal to the continuity of the payoff functions along with Proposition 3(b),

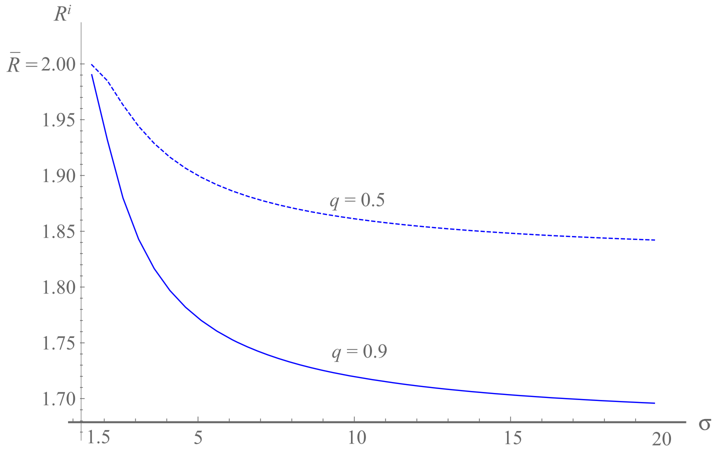

Fig. 3. The range of resource distributions that make trade relatively unappealing for the large country i.

to show that the larger country will eventually prefer trade once the smaller country's resource endowment becomes sufficiently large.

Importantly, as we mentioned above (and as we discuss in Online Appendix D), this finding is generally robust across a variety of trade models, including the classical ("Ricardian") and neoclassical ("Heckscher-Ohlin" and "Ricardo-Viner") frameworks as well as the more recent paradigms described in Krugman (1980), Eaton and Kortum (2002), and Melitz (2003), which feature trade costs, incomplete specialization, multiple factors of production, heterogeneous firms, and/or increasing returns. Furthermore, it would remain valid across any of these models if we were to compare payoffs under autarky with payoffs under non-cooperative trade policies or trade agreements (that do not consider the implications for arming and saving) instead of the payoffs under autarky and free trade.

The implications for the extended policy game should be clear: when given the choice to either trade or remain under autarky, a country that is sufficiently larger than its rival will choose not to trade at all, because it relinquishes power without gaining much back in return from trade. However, based on our numerical analysis for given relative incomes described in Section 4, one would expect the negative strategic payoff effects for the larger country to be smaller and the positive indirect payoff effects it enjoys to be larger with decreases in the probability of conflict q. Indeed, additional numerical analysis of payoffs under autarky and trade shows the range of relative endowment sizes for which the larger country prefers trade over autarky tends to expand as q decreases.[32]

To give some sense of the magnitudes here, Fig. 3 depicts combinations of country i's resource endowment R^i and the elasticity of substitution between tradeable goods σ for which the larger country i obtains equal payoffs under trade and autarky. The figure highlights the role of the probability of future conflict, by showing these relationships for two possible values of q, $q = .9$ and $q = .5$. Points above each curve (i.e., given q and imposing the constraint $R^i + R^j = 2$)

[32] These results always hold in the Armington (1969) and Krugman (1980) models and, provided that σ and comparative advantage are sufficiently large, in the classical Ricardian model. However, when comparative advantage and σ are relatively small, the range of relative endowments can expand with an increase in q or the relationship can be non-monotonic.

imply $U_A^{i*} > U_T^{i*}$, whereas points below the curve imply $U_A^{i*} < U_T^{i*}$. Thus, as illustrated in the figure, a decrease in q increases the range of resource distributions for which country i prefers trade. The figure also illustrates that, given q, a decrease in the elasticity of substitution σ, which amplifies the gains from trade, increases the range of resource endowments for which the larger country i prefers trade. Additional numerical analysis suggests that decreases in m (implying less effective arming) and/or δ (implying greater discounting of the future) similarly expand the range of relative endowment sizes for which the larger country prefers trade.

The findings of Proposition 5 can be related back to the historical examples discussed above in Section 2. In particular, even when the threat of a future conflict is quite high, the gains from trade can be sufficiently large to render trade preferable over autarky. This was ostensibly the case for Great Britain and Germany in their rivalry leading up to World War I. By contrast, it is plausible that the gains for the U.S. in its trade with Japan leading up to World War II and in its trade with Russia following World War II were not seen as sufficiently large to dominate the negative security externality net of any positive saving externality.

5.3. *Possible extensions and generalizations*

In this section, we consider briefly various extensions that relax some of the simplifying assumptions we have imposed to make the analysis as transparent as possible. In particular, we discuss (i) a different interpretation of our model of conflict resolution, consistent with the idea that countries choose between war and peace; (ii) alternative rules of division under peaceful settlement; (iii) trade in the future; (iv) a longer time horizon; and, (v) a three-country setting. Collectively, these extensions suggest that our analysis above is robust to a variety of alternative assumptions.

Choosing between war and peace Above, we assumed that the probability of war breaking out in period $t = 2$ is exogenous. Let us now reinterpret q as the probability that a dispute arises between the two countries in period $t = 2$. With probability $1 - q$, no dispute arises and each country enjoys all the return from its first-period savings Z^i. If a dispute occurs, it can be resolved either through "war" or through "peaceful settlement" conducted in the shadow of war. In the case of war, each country i deploys the arms G^i it had produced in period $t = 1$ to increase its probability of winning all the contested output, according to ϕ^i shown in (2). So that defeat does not result in zero future consumption, we suppose that only a fraction of future output, denoted by $\kappa \in (0, 1)$, is contestable. When the dispute is instead resolved through settlement, each country agrees on a peaceful division; in this case, ϕ^i gives country i's share of insecure output, along the lines of what we call "conflict" in the baseline model. As shown in Online Appendix E.1, our central results remain intact if the two countries can resolve their dispute only via a winner-take-all contest. The key here is that the countries similarly allocate productive resources to arm in the first-period in anticipation of such a contest when a dispute arises, such that the adverse strategic effect associated with a switch to trade in the first period can swamp the benefits for one of them when it is sufficiently larger than its potential adversary. Of course, the countries' equilibrium arming choices will depend on whether they expect such a dispute (should one arise) to be resolved through war or settlement. However, our maintained assumption that countries are risk-averse ensures that, given the choice between settlement and war with arming choices having already been made to fix the value of ϕ^i, peaceful settlement strictly dominates. Thus, our analysis above is consistent with the possibility that countries choose between war and peaceful settlement when a dispute arises—they always choose settlement. What's more, we could also allow for the possibility that the choice of war results in the destruction of a fraction of future output; this would only make the preference for settlement stronger.

Alternative rules of division Clearly, our assumption that the peaceful settlement of a dispute involves a rule of division that is based exclusively on ϕ^i simplifies the analysis of the choice between war and peace considerably. In Online Appendix E.2, we show how alternative rules of division of contested output, based on Nash-bargaining and split-the-surplus protocols with the countries' payoffs under a winner-take-all contest representing their respective threat-point payoffs (along the lines of Anbarci et al., 2002), can be incorporated into the analysis without materially affecting our central results. The key point here is, once again, that the possibility of a future dispute, however resolved, is costly, as it induces each country to divert resources away from current consumption and investment for future consumption. Under either division rule, countries arm to gain leverage in future negotiations should a dispute arise. Accordingly, the adverse strategic consequences of trade for the larger country will still overwhelm any positive effects if the difference in initial size is sufficiently large.

Trade in the future Our argument above that, given a dispute arises in period $t = 2$, the countries would necessarily choose peaceful settlement suggests their decision to settle does not depend on the possibility of future trade. This is not to argue, however, that the possibility of future trade is irrelevant even in our simple setting. First, the fact that war has the costly effect of precluding trade in period $t = 2$ gives an additional reason for the countries to prefer settlement over war, as emphasized by the "liberal peace" hypothesis (see e.g., Polachek, 1980; Martin et al., 2008). Second, and more interestingly, as we show in Online Appendix E.3, future trade matters in shaping the larger country's preference for current trade. Specifically, we suppose that, when peace prevails in period $t = 2$, the two countries go on to freely trade their intermediate goods produced from their previously chosen savings, Z^i and Z^j. When a dispute arises, the two countries enter into a negotiated settlement whereby they trade their intermediate goods freely and then divide the contested pool of output according to (2). The possibility of future trade amplifies the potential benefit of current trade to the larger country, as current trade enables its smaller rival to grow in size and become a larger and more valuable trading partner in the future. Nonetheless, numerical analysis of the model confirms that a shift to trade in period $t = 1$ generates a negative security externality that can swamp current trade's positive effects. That is to say, there exists a set of parameter values, for which a sufficiently uneven distribution of endowments renders trade in the first period unappealing to the larger country. Intuitively, though future settlement ensures the countries enjoy mutual benefits from trade in period $t = 2$, it does not constrain arming choices, which are made ahead of time, and therefore does not resolve the problem that asymmetric income gains from trade in period $t = 1$ have implications for how resources are divided in period $t = 2$. Notably, however, the range of relative resource endowments for which this preference ranking holds vanishes when the elasticity of substitution $\sigma > 1$ becomes sufficiently small to imply large enough compounded gains from trade across the two periods.

Longer time horizon Our focus on two-period settings naturally raises the question of whether our results survive with longer time horizons. To explore this issue, we turn to a three-period version of the model, presented in Online Appendix F. We assume that a conflict arises with probability $q > 0$ in each of the latter two periods, $t = 2, 3$. In addition, we allow countries to trade in periods $t = 2$ and 3; yet, for simplicity, we now assume trade takes place only in the event of peace in that period. Otherwise, conflict ensues with a division of insecure output according to each country's appropriative share, ϕ^i, as in the baseline model. The main complication that arises in this setting is that arming and saving choices made in $t = 1$ now must take into account their effects on the rival's future income and thus on its future arming and saving choices. As a benchmark for comparison, we also produced results for a two-period model with a similar structure, i.e., one where trade occurs in period $t = 2$ only in the event that a dispute does not arise. For both of these models, numerical analysis shows that the range of relative endowment sizes for which the large country prefers not to trade can vanish when σ is sufficiently small, similar to what we found for the above model that also features trade in the future. The main effect of adding another time period is to mitigate the adverse consequences of first-period trade for the larger country when the initial size difference is moderate but magnify them when size differences are sufficiently large. Thus, although adding a third period tends to reduce the range of relative endowment sizes for which the larger country prefers not to trade in the first period, the effect on its preferences toward trade in the limit where the large country becomes infinitely large is generally ambiguous, depending on parameter values.

Three countries While our central finding that the larger country could prefer not to trade with its potential rival holds in a variety of different trade models, one might wonder if it remains intact in the presence of a third country that does not participate in disputes. In Online Appendix G, we extend our baseline model to allow for three countries, each producing a distinct intermediate good.[33] To fix ideas, we think of countries $i = 1$ and 2 as rivals and country $i = 3$ as the rest of the world (ROW). Furthermore, to keep the analysis as simple as possible and to facilitate comparisons with the baseline model, we return to our two-period setting, assuming that trade can take place only in period $t = 1$ and that the potential conflict between countries $i = 1$ and 2 arises in period $t = 2$. Within this setting, we compare the larger adversarial country's payoffs under 3 alternative trade regimes for period $t = 1$: (i) global free trade; (ii) an embargo on the smaller adversarial country by the larger adversarial country, with free trade between ROW and each of the two adversaries; (iii) a blockade on the smaller adversary, with free trade only between ROW and the larger adversary and without any added costs relative to an embargo. Numerical analysis reveals that, even when trade with a third country (ROW) is possible, the larger adversarial country could prefer to embargo trade with its rival for the same reasons identified in the baseline model. However, the presence of ROW does matter here. In particular, the range of relative resource endowments for which the large country prefers an embargo over free trade is decreasing in ROW's size. This result largely follows from our existing arguments, since trade with ROW increases the income of the smaller adversary relative to that of the larger one. But, at the same time, the larger country tends to prefer a blockade to an embargo for any initial endowment distribution for which an embargo is preferred to free trade. Furthermore, unlike with an embargo, the range of relative sizes for which a blockade would be preferred to free trade is increasing in the size of ROW. Intuitively, the blockade has the effect of reducing the smaller country's relative size as compared with the case of an embargo—and by more when ROW is larger.

[33] We thank a referee for suggesting this extension to us.

6. Concluding remarks

Trade and security are inseparable pillars of international policy. Yet the study of international trade largely abstracts from how the vast sums that are spent on national defense are affected by the wealth that is created by international commerce. Similarly, the conflict literature lacks theoretical frameworks that formally model how changes in relative wealth translate into changes in relative power and how this relationship depends on how countries allocate their respective resources across arming versus other, more productive activities. In this paper, we analyze a dynamic, two-country model of trade and arming interactions, where counties arm to prepare for an uncertain future. Notably, we show how arming decisions reflect not only the economic capabilities of each country, but also how the marginal benefit of more productive investments (i.e., saving) varies with the degree of uncertainty.

A key implication of the theory is that larger countries will find trade in the shadow of a possible future conflict with smaller rivals unappealing when the difference in *ex ante* size is sufficiently large. This prediction derives generally from the nonlinearity that occurs, for a given trade regime, in the relationship between "size" and "power" when the probability of future conflict is nonzero. Thus, while the threat of conflict could itself be a source of "power" for an *ex ante* small country, it could also undermine that country's ability to realize the possible gains from trade with larger rivals. This last observation would seem particularly salient for informing conflict management policies in situations where disproportionate compensation to seemingly weaker rivals would be necessary to entice them to improve diplomatic relations.

Our model has, by design, leaned on an income channel as the primary linkage between trade and arming. We conjecture that richer insights could be gained by amending the production structure of our model so that arms are produced from the same resources used to produce the tradable inputs, thereby introducing a terms-of-trade channel and a factor-price channel. With such a modification, these added channels could serve to modulate the costliness of arming to the point where similarly sized economies refuse to trade, whereas in the present paper this result only occurs for sufficiently dissimilar economies. Furthermore, while our comparison of outcomes under autarky and free trade has shed light on the desirability of trade in the shadow of an uncertain future, our analysis has remained silent on the implications of activist trade policies. In particular, countries could influence the security policies of their potential rivals and thus their own power by adjusting trade flows and prices through appropriate unilateral or bilateral commercial policies. We leave these possibilities for future research.

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Appendix A. Supplementary material

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