Philosophy of Mathematics

LPS 247
Fall 2017 - Winter 2018

Our focus this year is on the fundamental metaphysical and epistemological questions raised by mathematics: what is it about? How do we come to know it? To clarify, we'll largely bypass the important methodological or foundational issues raised by particular branches of the subject -- as last year's seminar focused on set theory -- and the many current discussions in the so-called 'philosophy of mathematical practice' -- explanation, purity of proofs, mathematical depth, etc. Instead we want to better understand what is going on when humans do mathematics: does this practice have a subject matter, like physics or botany? If so, what is the nature of that subject matter and what methods are effective for its investigation? If not, why doesn't just anything go; what constrains the practice? Either way, how and why does mathematics function in application to the natural sciences?

The default requirement for first-year students taking the course for a grade (other than S/U, which involves only reading and attending) is three short papers (750-1250 words) due at the beginning of class in the 4th week, 7th week, and 10th week. These papers should isolate one localized philosophical, conceptual or methodological point within one of the readings (without appeal to outside sources) and offer some analysis and/or critique. The thesis and its defense needn't be earth-shattering in any way; this is really just an exercise in finding a topic of the right size and crafting a thesis and defense to match. I encourage writers to email me their topic and thesis, or even better, a draft introductory paragraph, for discussion well before the due date.

Other options are open to negotiation.

I assume everyone has access to a copy of

Benacerraf and Putnam, ed., *Philosophy of Mathematics*.

Frege, *Foundations of Arithmetic.*

The rest of the assigned readings are available from the syllabus on the course EEE web site. (This is limited to those registered for the course. If you'd like to participate without registering, alternative access can be arranged.) For those in search of quick overviews, aside from the many excellent articles in the online *Stanford Encyclopedia of Philosophy*, Linnebo's *Philosophy of Mathematics* is a good start. There's also Shapiro's *Thinking about Mathematics*, a bit older but still useful.

Please come to the first meeting prepared to discuss the reading in Topic 1.

Topics

Chronology and far-reaching influence dictate that we begin with Kant, but there's no denying that he represents the deep end of the pool in terms of difficulty. (For those largely innocent of Kant, Gardner's *Kant and the Critique of Pure Reason* is an invaluable introduction. The excerpts from *Second Philosophy* in the extra reading to topic 1 include some 'cliff notes'-style exposition.) I'll attempt a brief overview of the background at the beginning of class.

Kant's approach to geometry is well-covered in Jeremy's seminars, so here we focus on his philosophies of arithmetic and algebra.

1. Kant I

Shabel, 'Kant's philosophy of mathematics', pp. 94-113.

Kant, *Critique of Pure Reason*, Introduction V.1: 'Mathematical judgments are all synthetic' (B14-17).

'On the schematism of the pure concepts' (A137/B176-A147/B187)

'Axioms of intuition' (A162/B202-A166/B207).

'The discipline of pure reason in dogmatic use' (A712/B740-A738/B766).

Table of Judgments (A70/B95) and Table of Categories (A80/B106)

Maddy, 'A second philosophy of arithmetic', pp. 238-245.

Shabel provides an overview of Kant's philosophy of mathematics. The selections from Kant are those most relevant to arithmetic and algebra with the Tables added for reference. I give a somewhat heretical reading of Kant on arithmetic.

Extra reading:

Parsons, 'Kant's philosophy of arithmetic'.

Parsons, 'Arithmetic and the categories', pp. 57-68.

Maddy, *Second Philosophy*, §§I.4, III.2.

2. Kant II

Anderson, 'It all adds up after all: Kant's philosophy of arithemtic in light of the traditional logic', pp. 501-511, 515-540.

Shabel, 'Kant on the "symbolic construction" of mathematical concepts'.

These are two classic papers on Kant's philosophy of mathematics: Anderson on arithmetic and Shabel on algebra.

Several mathematical developments of the 19th century -- projective geometry, the use of complex numbers as coordinates, non-Euclidean geometries -- put pressure on Kant's intuition-based account of mathematical epistemology. Various neo-Kantian schools of the late 19th century tried to cope with this. Though Frege was aware of, even participated in, some of this, he regarded intuition as necessary for geometry. But he drew the line at arithmetic, inventing what we now call Logicism.

3. Frege I

Frege, *Foundations of Arithmetic*, Introduction and §§1-27, 45-83, 87-91.

Most of you will have read this wonderful book at some time or another, so this is intended as a review.

Extra reading

Heis, 'The priority principle from Kant to Frege'.

Wilson, 'Frege: the royal road from geometry'.

'Frege's mathematical setting'.

Mancosu, 'Frege's *Grundlagen*, section 64'.

Wilson explains how the use of (what we would regard as) equivalence classes arose in the course of those late 19th century upheavals in geometry, in the very example Frege exploits: directions (as surrogates for points at infinity).

4. Frege II

Yablo, 'Carving content at the joints', pp. 246-262, 266-268.

Yablo explores the puzzle of how a single 'content' can be 'carved' in different ways.

Burge, ‘Frege on knowing the foundation’.

If Frege’s logicism is to reduce the epistemic question for arithmetic to the same question for logic, we naturally wonder how Frege thinks we come to know the truths of logic. Burge gives a strongly rationalistic interpretation of Frege on that point.

Extra reading

Maddy, *Defending the Axioms*, pp. 117-123.

This is a quick sketch of Burge’s take on Frege’s rationalism, examined as a possible defense of Robust Realism in the foundations of set theory.

As we all know, Frege’s brilliant system failed because his famous Basic Law V ($\hat{x}F=\hat{x}G$ iff $\forall x(Fx \equiv Gx)$) generates Russell’s paradox. In desperation, Frege eventually suggested that arithmetic might be founded on geometry (see Frege [1924/5])! The neo-Fregean response begins with Wright’s *Frege’s Conception of Numbers as Objects* (1983), where he shows that the arithmetic consequences Frege draws from Basic Law V can actually be drawn from Hume’s Principle (#F=#G iff F ≈ G). This is now called ‘Frege’s Theorem’ -- and Heck has shown that Frege himself knew this (Heck, ‘The development of arithmetic in Frege’s *Grundgesetze der Arithmetik*’; see also Heck, *Frege’s Theorem*, pp. 9-13 for a quick summary). Neo-Fregeans embrace Frege’s theorem as an avenue for the revival of logicism. The idea is that Hume’s Principle (HP) is analytic, a mere definition, and that arithmetic then follows in (second-order) logic.

5. Neo-Fregeanism

Boolos, ‘Is Hume’s principle analytic?’

For the example of ‘parities’, see Boolos’s paper in the extra reading, pp. 214-215.

Wright, ‘Is Hume’s principle analytic?’, pp. 307-324.

For the example of ‘nuisances’, see Wright’s paper in the extra reading, pp. 290-291.

Heck, ‘The Julius Caesar objection’.

Given that Frege himself knew Frege’s theorem, why didn’t he make use of it in just the way the Neo-logicists do? The answer seems to be that Frege was worried about the Julius Caesar problem, but if it’s so hard to tell that Caesar isn’t a number, why is it so easy to tell that he’s not the extension of some concept? Heck explores the mysteries of the Julius Caesar problem.

Extra reading

Boolos, ‘The standard of equality for numbers’.

Wright, ‘The philosophical significance of Frege’s theorem’.

Heck, ‘On the consistency of second-order contextual definitions’ (with postscript).

Heck shows that the so-called ‘bad company’ objection is even worse than Boolos imagined. His 2011 postscript brings that debate up to 2011.

Heck, *Frege’s Theorem*, pp. 13-27.

This is Heck’s 2011 revisiting of the Caesar problem.

Meanwhile, with the contemporaneous rise of Cantor’s mathematical theory of sets, other forces arose to either curtail or somehow legitimate the theory of the infinite. Here are two attempts.

6. Intuitionism

Brouwer, ‘Intuitionism and formalism’.

Heyting, ‘Disputation’.

Posy, ‘Intuitionism and philosophy’, pp. 318-35.

Extra reading

Brouwer, ‘Consciousness, philosophy, and mathematics’.

Iemhoff, ‘Intuitionism in the philosophy of mathematics’.

7. Formalism

Hilbert, ‘On the infinite’, pp. 369-384, 392.

Detlefsen, *Hilbert’s Program*, pp. 1-22, 77-92.

Extra reading

Zach, ‘Hilbert’s program’.

Meanwhile, the *Tractatus* inspired the Vienna Circle (see Friedman's paper in the extra reading to topic 8), which eventually led to the pivotal disagreement between Carnap and Quine, one of the centerpieces of modern analytic philosophy.

8. Carnap and Quine I

Carnap, *Logical Syntax of Language*, §§1-2, 16-17.

'Empiricism, semantics, and ontology'.

Quine, 'On what there is'.

'Two dogmas', pp. 37-46.

Extra reading

Carnap, *Logical Syntax of Language*, §§50-52.

These are the crucial sections that explain how Carnap intends to draw the analytic/synthetic distinction. Luckily the secondary literature provides a more accessible account (see below).

Quine, 'Posits and reality'.

'Five milestones of empiricism'.

9. Carnap and Quine II

Quine, 'Carnap and logical truth'.

Carnap, 'Reply to Quine'.

Richardson, 'Two dogmas about logical empiricism: Carnap and Quine on logic, epistemology, and empiricism'.

Extra reading

Quine, 'Two dogmas in retrospect'.

Friedman, 'Carnap and Wittgenstein's *Tractatus*'.

10. Carnap and Quine III

Hiller, 'Mathematics in science: Carnap vs. Quine'.

Friedman, 'Tolerance and analyticity in Carnap's philosophy of mathematics'.

Ricketts, 'Tolerance and logicism', §III.

Friedman, 'Tolerance, intuition, and empiricism'.

Extra reading

Hillier, 'Analyticity and language engineering in Carnap's *LSL*'.

Maddy, *Second Philosophy*, §§I.5-6.

End of fall quarter/Beginning of winter quarter

Quine's indispensability argument for the existence of mathematical entities went on to a central role in the field. His version relied on conformational holism, but that was soon taken to be a defect. (See *Naturalism*, pp. 135-143, for one account of why.) The debate over different versions of the argument continues from there.

11. Indispensabilty

Putnam, 'Philosophy of logic', §V.

'What is mathematical truth?', pp. 74-75.

Maddy, *Naturalism in Mathematics*, pp. 143-157.

Colyvan, *The Indispensability of Mathematics*, pp. 98-105.

Baker, 'Are there genuine mathematical explanations of physical phenomena?'.

Pincock, *Mathematics and Scientific Representation*, chapter 10, pp. 210-217.

Extra reading

Maddy, *Second Philosophy*, pp. 95, 404-9.

Baker, 'Mathematical explanation in science'.

Baker responds to a range of objections to his cicada example.

Mancosu, 'Explanation in mathematics', pp. 11-17.

This passage from Mancosu gives a useful guide to the development of the explanatory indispensability debate after Baker's paper.

These new indispensability arguments trade on mathematical explanations in science, but mathematical explanations in mathematics are also a current topic of considerable discussion under the general heading of ‘philosophy of mathematical practice’. For example, see Hafner and Mancosu, ‘Varieties of mathematical explanation’. Lange’s recent *Because Without Cause* treats these questions at length. Of particular interest (it seems to me) is his work on the interconnections between explanation and coincidence (chapter 8).

Before moving on to Benacerraf’s famous papers, let step back for a moment to an earlier view that’s fallen out of favor among philosophers but still holds some attractions for mathematicians.

12. If-thenism

Putnam, ‘The thesis that mathematics is logic’, pp. 20-34.

Introduction, p. xiii.

Maddy, ‘Enhanced if-thenism’.

Extra reading

Resnik, *Frege and the Philosophy of Mathematics,* pp. 105-106, 117-119, 131-136.

Quine, ‘Truth by convention’.

Maddy, ‘A second philosophy of logic’

‘A second philosophy of arithmetic’.

These papers fill in some of the background to ‘Enhanced if-thenism’. As noted, this version of if-thenism can be viewed as a generalization of the Arealism of *Defending the Axioms* (chapter 4, see especially p. 99).

13. Benacerraf on truth

Benacerraf, ‘Mathematical truth’.

Field, *Realism, Mathematics, Modality*, pp. 25-30.

Maddy, *Realism in Mathematics*, pp. 28-35, 41-75.

Extra reading

Maddy, *Defending the Axioms*, p. ix.

This page from the preface to *Defending* explains what went wrong with the view in *Realism*.

Faced with the tension between Benacerraf’s epistemological challenge and the Quine/Putnam indispensability arguments, there are two rough options: endorse the indispensability arguments and meet Benacerraf’s challenge head-on or defuse Benacerraf’s challenge by denying the indispensability arguments. In the 1980s, Maddy took the first path (in topic 13); Field the second, by denying the main premise of the indispensability arguments. As we saw in topic 11, the cogency of the indispensability arguments themselves came under fire in the 1990s, opening the way to a less onerous versions of fictionalism. (These are sometimes called ‘hard road’ and ‘easy road’ fictionalism.)

14. Fictionalism

Field, ‘Realism and anti-realism about mathematics’.

Field’s ambitious program in *Science Without Numbers* (1980) produced a vast secondary literature and eventually a book of essays, *Realism, Mathematics, Modality* (1989) largely responding to this outpouring of discussion. I’ll try to give a quick overview of these developments in class.

Leng, *Mathematics and Reality*, chapter 7, ‘Mathematics and make-believe’.

Leng, like Balaguer and Melia in the extra reading, represents the easy road, but she gives a more systematic account.

Extra reading

Maddy, *Realism*, pp. 159-170.

‘Mathematics and Oliver Twist’.

These are some of my efforts to adjudicate between Field’s nominalism and the compromise platonism of *Realism* (see also ‘Physicalistic platonism’).

Balaguer, *Platonism and Anti-Platonism in Mathematics*, pp. 130-142.

Melia, ‘Weaseling away the indispensability argument’, pp. 466-471.

Yablo, ‘The myth of seven’.

Balaguer and Melia give easier versions of the easy road than Leng. Yablo is a precursor to Leng.

Criticism of Field’s nominalism/fictionalism falls under three main headings. Representative samples of each:

Resnik, 'How nominalist is Hartry Field's nominalism?'.

The first part of this paper examines the status of spacetime points.

Shapiro, 'Conservativeness and incompleteness'.

Here Shapiro establishes the technical shortcomings of both the first- and second-order versions of the view. (Field has a response in his 1989 collection.)

Malament, 'Review of *Science Without Numbers'*.

Malament argues that not all physically important assertions can be formulated in Field's nominalistic version of Newtonian gravitation theory and that many other physical theories are even less amenable to this sort of nominalization. (He also makes some points about technical matters and about spacetime points and regions that are related to those of Shapiro and Resnik.) This review is best known for the second point, and current discussions of Field's program are mostly addressed to this worry, especially for the case of quantum mechanics.

It's widely agreed at this point that the holistic indispensability argument is based on a flawed picture of how mathematics works in application. Here are some subsequent approaches.

15. Applications

Maddy, 'How applied mathematics became pure'.

Maddy, *Defending the Axioms*, pp. 89-96.

Notice that the enhanced if-thenism of topic 12 could be regarded as a version of easy-road fictionalism: in the face of the indispensability arguments it posits no abstracta, and yet it makes no effort like hard-road fictionalism to reconstrue mathematized science. These excerpts fill in the alternative picture of applications evoked in passing there.

Pincock, *Mathematics and Scientific Representation*, pp. 3-12, 16, 21-2, 25-33.

Extra reading

Liston, 'Understanding scientific representations' (critical review of Pincock's *Mathematics and Scientific Representation*).

One of the most widely discussed questions about the way mathematics applies in natural science is the one posed by Eugene Wigner.

16. Wigner’s miracle

Wigner, ‘The unreasonable effectiveness of mathematics in the natural sciences’.

Maddy, *Second Philosophy*, pp. 329-343.

Steiner, *The Applicability of Mathematics as a Philosophical Problem*, pp. 1-11.

Bangu, *The Applicability of Mathematics in Science*, pp. 111-132.

Extra reading

Wilson, ‘The unreasonable uncooperativeness of mathematics in the natural sciences’.

Maddy, *Naturalism in Mathematics*, pp. 116-128, 206-208.

Here is my treatment of the rise and fall of Definabilism.

Bangu, *The Applicability of Mathematics in Science*, chapter 7.

This is Bangu’s response to Wigner.

Now to Benacerraf’s other challenge and its aftermath.

17. Structuralism I

Benacerraf, ‘What numbers could not be’.

Shapiro, *Philosophy of Mathematics*, pp. 71-106, 109-120.

Extra reading

Benacerraf, ‘Recantation’.

Maddy, *Realism*, pp. 170-177.

18. Structuralism II

Shapiro, *Philosophy of Mathematics*, pp. 129-136.

Hale, ‘Structuralism’s unpaid epistemological debts’.

Burgess, *Rigor and Structure*, pp. 126-158.

Extra reading

Putnam, 'Mathematics without foundations'.

Hellman, 'Structuralism'.

Putnam proposes that there are many 'equivalent descriptions' of the mathematical realm, one of which is 'mathematics as modal logic'. In his *Mathematics Without Numbers*, Hellman develops this idea into a position he calls 'modal structuralism' (see pp. 551-560 of the above survey article for a brief sketch).

Shapiro, 'Identity, indiscernibility, and *ante rem* structuralism'.

Maddy, *Second Philosophy*, pp. 160-161.

Finally, a look at what developmental psychology and cognitive science might tell us about how humans come to their mathematical beliefs, especially about arithmetic.

19. Cognitive science of mathematics I

Maddy, 'Second philosophy of arithmetic', pp. 223-7.

Second Philosophy, §III.5.

The first selection sets the background (in the philosophy of logic) to the second, which summarizes some of the developmental work on object perception and closely related topics.

Second Philosophy, pp. 319-328.

'A second philosophy of arithmetic', pp. 231-238, 245-247.

'Psychology and the a priori disciplines'.

Extra reading

Heck, 'Cardinality, counting, and equinumerosity'.

20. Cognitive science of mathematics II

vanMarle, 'What happens when a child learns to count?'.

Schlimm, 'Numbers through numerals', pp. 9-20.

Relaford-Doyle and Núñez, 'Looking into the unnaturalness of natural numbers'.

These three papers, as well as 'Psychology and the a priori disciplines', are a sampling from a forthcoming/new book edited by Bangu: *Naturalizing Logico-Mathematical Knowledge*.

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