Philosophy of Mathematics

Fall 2006 - Winter 2007

Our goal over these two quarters is to think through a series of positions on the fundamental metaphysical and epistemological questions about mathematics: what is mathematical truth and how do we come to know it? Answers to both questions must cohere with an account of how pure mathematics is successfully applied in our dealings with the world.

The default requirement for those taking the course for a grade (other than S/U) is three short papers (750-1250 words) due at the beginning of class in the 5th week, 7th week, and 10th week. Each paper should isolate one localized point in the readings and offer some analysis and/or critique. Other options are open to negotiation.

I assume everyone has access to a copy of:

Benacerraf and Putnam, *Philosophy of Mathematics: Selected Readings.*

For quick introductory sketches and suggestions for further reading, see:

Shapiro, *Thinking about Mathematics*.

For more sustained introductions, see:

Shapiro, *The Oxford Handbook of Philosophy of Mathematics and Logic*.

George and Velleman, *Philosophies of Mathematics*.

Another book useful for background (including technical matters) is:

Giaquinto, *The Search for Certainty*.

These and other books of interest will be available in Waldemar Rohloff's office (793 SST). Assigned readings (apart from those in Benacerraf and Putnam) will be available in a box outside my office (759 SST) for photocopying.

Topics

1. Kant

If you haven't read the 'Transcendental Aesthetic' section of the *Critique of Pure Reason*, this would be a good time to do so. Sebastian Gardner's *Guidebook* gives a useful introduction to the *Critique* in general, and to the 'Transcendental Aesthetic in particular in Chapter 4. In class, we'll be discussing:

Shabel, 'Kant's philosophy of mathematics', especially pp. 94-113.

Friedman, *Kant and the Exact Sciences*, chapter 1, especially pp. 55-71.

(Carson, 'Kant on intuition in geometry'.)

Shabel, 'Kant on the foundations of geometry'.

(Shabel, *Mathematics in Kant's Critical Philosophy*, pp. 3-7, 91-114, 157-163.)

(For more on how Kant's views relate to those found in earlier modern thinkers see Shabel, 'Apriority and application: philosophy of mathematics in the modern period'.)

(For more on how Kant argues from geometry to transcendental idealism, see Shabel, 'Kant's argument from geometry' and Rohloff, 'Kant's argument from applicability'.)

2. Kant on arithmetic

Friedman, *Kant and the Exact Sciences*, chapter 2, especially pp. 96-129.

Shabel, 'Kant on the "symbolic construction" of mathematical concepts'.

Anderson, 'It all adds up after all: Kant's philosophy of arithmetic in light of traditional logic'.

3. Frege

Wilson, ‘The royal road from geometry’.

Tappenden, ‘Extending knowledge and “fruitful concepts”: Fregean themes in the foundations of mathematics’, §§I-IV.
‘The Riemannian background to Frege’s philosophy’.

(Those unfamiliar with the structure of Frege’s logicist project should have a look at the brief discussions in Giaquinto, Shapiro, George and Velleman, or perhaps Demopoulos and Clark’s paper on logicism in Shapiro’s *Handbook*.)

(Since the 1980s, there’s been a lively program aimed at reviving the Fregean project in one way or another. For a survey of the mathematical side of this project, see Burgess, *Fixing Frege*, chapter 3; for a survey of its philosophical side, see McBride, ‘Speaking with shadows’, or Hale and Wright, ‘Logicism in the 21st century’.)

4. *Principia Mathematica*

Russell, ‘The regressive method of discovering the premises of mathematics’.

Russell and Whitehead, *Principia Mathematica*, Introduction, especially pp. 37-81 (the Vicious-Circle Principle, ramified type theory, the Axiom of Reducibility, descriptions and the no-class theory).

Ramsey, ‘The foundations of mathematics’, pp. 21-32, 57.

Quine, ‘On the axioms of reducibility’.

Hylton, ‘The vicious circle principle’.

Urquhart, ‘The theory of types’.

(Poincaré also championed the VCP. See chapters IV and V of his *Mathematics and Science: Last Essays*. For discussion of the subsequent development of predicativism, see, Feferman, ‘Predicativism’.)

5. Intuitionism

Brouwer, ‘Intuitionism and formalism’.

Heyting, ‘Disputation’.

Dummett, ‘The philosophical basis of intuitionistic logic’

Posy, ‘Intuitionism and philosophy’.

(For a historical survey of Brouwer’s work, see van Stigt, ‘Brouwer’s intuitionist programme’. For a spirited reply to Dummett, see Burgess, ‘Dummett’s case for intuitionism’.)

6. Formalism

Hilbert, ‘On the infinite’.

Detlefsen, ‘Formalism’.

(Those unfamiliar with the particulars of Hilbert’s program should have a look at chapters IV and V of Giaquinto.)

(For an historical survey of Hilbert’s formalism during the 1920s, see Mancosu, ‘Hilbert and Bernays on metamathematics’. For evaluations of Hilbert’s legacy, see Seig, ‘Hilbert’s program 60 years later’, Simpson, ‘Partial realizations of Hilbert’s program’, and Feferman, ‘Hilbert’s program relativized’.)

7. Carnap/Quine/Putnam: the indispensability argument

Carnap, ‘Empiricism, semantics and ontology’.

Quine, ‘On what there is’, especially pp. 16-19.
‘Two dogmas of empiricism’, especially pp. 42-46.
(‘Carnap and logical truth’)

Putnam, ‘Philosophy of logic’, especially §VIII.

Maddy, *Naturalism in Mathematics*, pp. 95-107, 133-160.

Colyvan, *The Indispensability of Mathematics*, chapter 5.

Resnik, *Mathematics as a Science of Patterns*, pp. 43-48.
‘Quine and the web of belief’, pp. 429-432.

(For another critique of the indispensability argument, see Sober, ‘Mathematics and indispensability’. Colyvan gives a reply in chapter 6 of his book cited above. See also Resnik’s book, chapter 7, especially pp. 119-120.)

8. Field's nominalism

Field, *Science Without Numbers*, Preface and pp. 1-19, 92-106, 107-112, 125-127.
'Realism and anti-realism about mathematics'.

Urquhart, 'The logic of physical theory'.

Burgess, 'Notes on Field'
'Synthetic physics and nominalistic realism'.

(For a guide to the (large) secondary literature on Field's book, see Burgess and Rosen, *A Subject with no Object*, pp. 190-196.)

(Charles Chihara endorses a different nominalization strategy. For discussion, see his 'Nominalism'.)

9. Gödel's realism

Parsons, 'Introductory note'.
Gödel, 'Russell's mathematical logic'.

Moore, 'Introductory note'.
Gödel, 'What is Cantor's continuum problem?'

Boolos, 'Introductory note'.
Gödel, 'Some basic theorems on the foundations of mathematics and their implications', especially pp. 311-323.

10. Benacerraf's epistemological challenge

Benacerraf, 'Mathematical truth'.

Steiner, *Mathematical Knowledge*, pp. 109-116.

Hart, Review of Steiner, pp. 123-126.

Field, *Realism, Mathematics and Modality*, pp. 25-30.

Burgess, 'Epistemology and nominalism'.

Maddy, *Realism in Mathematics*, pp. 36-48.

Casullo, *A Priori Justification*, pp. 125-146.

(For antiquarians, Maddy offers a platonistic 'solution' to Benacerraf's epistemological challenge in *Realism in Mathematics*, chapter 2. See *Naturalism in Mathematics*, pp. 108-109, for a sketch of set theoretic realism and a list of its critics. For

the record, Maddy thinks the undoing of the view is its dependence on the indispensability argument.)

11. Structuralism I

Benacerraf, ‘What numbers could not be’.

Shapiro, *Philosophy of Mathematics: Structure and Ontology*, chapters 3 and 4.

(For antiquarians again, Maddy offers a platonistic ‘solution’ to Benacerraf’s metaphysical challenge in *Realism in Mathematics*, chapter 3. The idea there -- that both the von Neumann and the Zermelo ordinals provide measuring sticks for the sizes of finite sets -- more or less carries forward to the context of *Second Philosophy*, except that the things measured would be KF-structures not sets.)

12. Structuralism II

Resnik, *Mathematics as a Science of Patterns*, chapters 10 and 11.

Hellman, ‘Modal-structural mathematics’.

Hale, ‘Structuralism’s unpaid epistemological debts’.

13. Structuralism III

Parsons, ‘The structuralist view of mathematical objects’.
‘Structuralism and the concept of set’.
‘Structuralism and metaphysics’.

14. Fictionalism

Yablo, ‘The myth of seven’.

Balaguer, *Platonism and Anti-Platonism in Mathematics*, pp. 98-100, 128-148.

Rosen, ‘Nominalism, naturalism, epistemic relativism’.

Burgess, ‘Mathematics and *Bleak House*’.

15. Azzouni’s nominalism I

Azzouni, *Deflating Existential Consequence*
Introduction and Part I.

16. Azzouni's nominalism II

Azzouni, *Deflating Existential Consequence*, Part II.

17. Second philosophy

Maddy, *Second Philosophy*, §§IV.4 and IV.5.
'What does science tell us about how to do mathematics?'
'Defending the axioms'.

(Maddy, *Second Philosophy*, §§IV.2 and IV.3.)

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